GAME THEORY

کاری از:
محمد حسن شماخی
مسعود مردانشاهی

بسم الله الرحمن الرحیم

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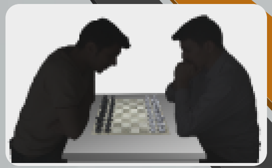

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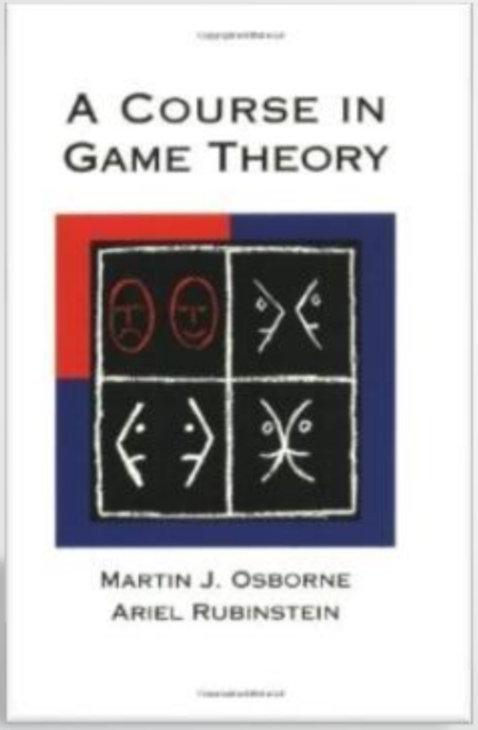

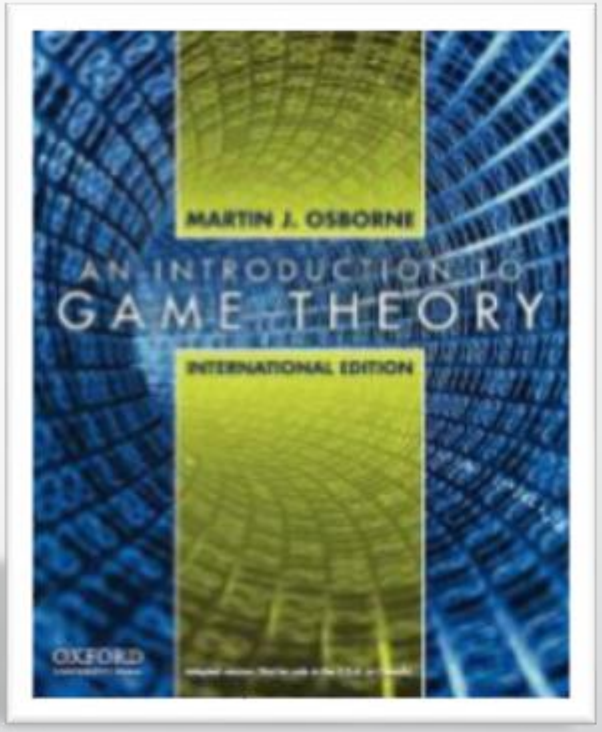

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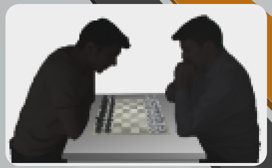
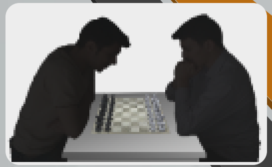


WHAT IS GAME THEORY?

WHAT IS
GAME THEORY?

RATIONAL INDIVIDUAL
MAX PAY OFF



GAME COMPONENTS:

PLAYERS

STRATEGY (ACTION)

STRATEGY PROFILE

ORDER OF PLAY

INFORMATION SET

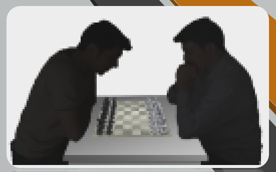
OUTCOME

PAYOFF

Player 2

Player 1

Matching Pennies	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

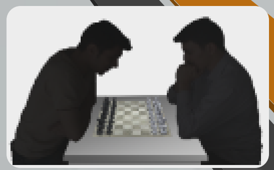


Number of players

2 players

Matching Pennies:

3 players

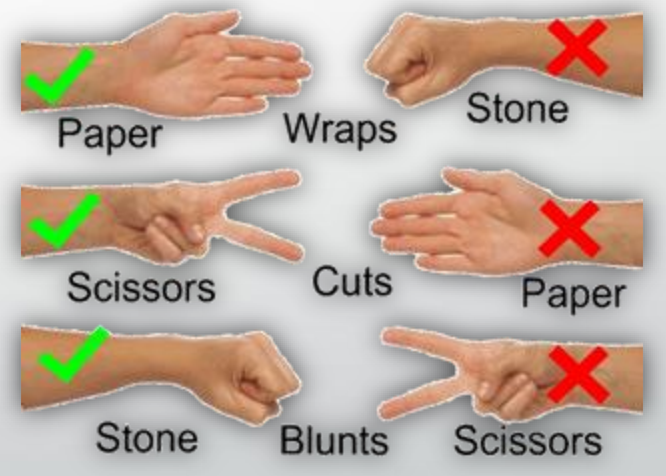


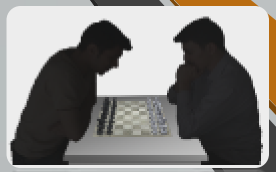
GAME COMPONENTS:

Strategy

Matching Pennies:

Rock, Scissors, Paper





تعاریف

Simultaneous or Ordinal

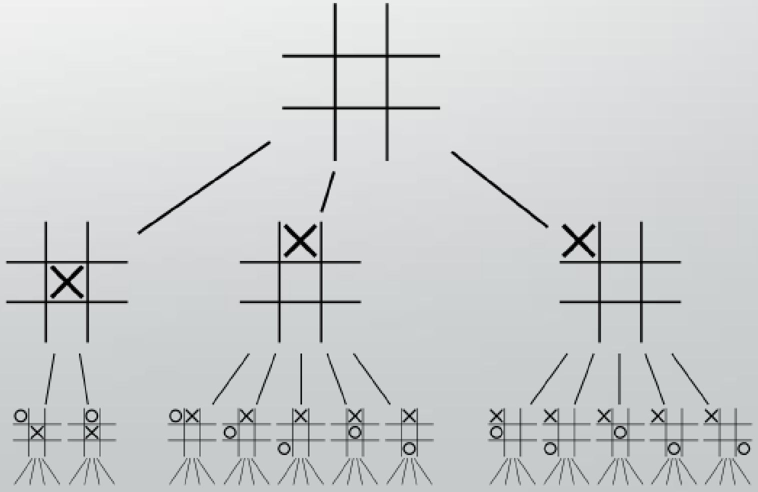
Matching Pennies:

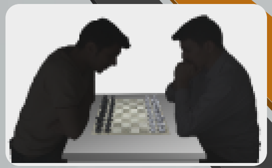
Player 2

Matching Pennies	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

Player 1

Dots & Crosses (Tick-Tack-Toe)

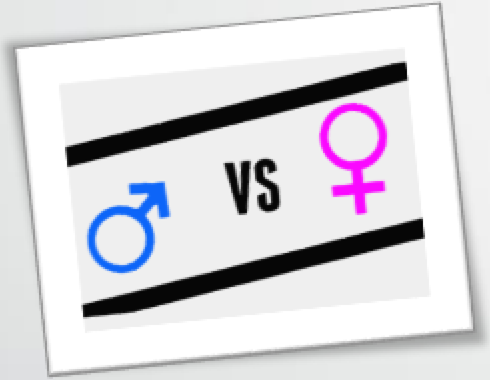




تعاريف

Cooperative (Coalitional)

Battle of Sexes:



Wife

	Cinema	Theatre
Cinema	2 , 1	0 , 0
Theatre	0 , 0	1 , 2

Husband

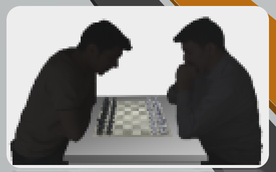
Non-Cooperative

Chicken Game
(Hawk-Dove game)

Driver 2

	Swerve	Straight
Swerve	0 , 0	-1 , 2
Straight	2 , -1	-5 , -5

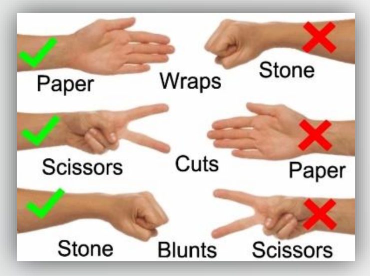
Driver 1



تعاريف

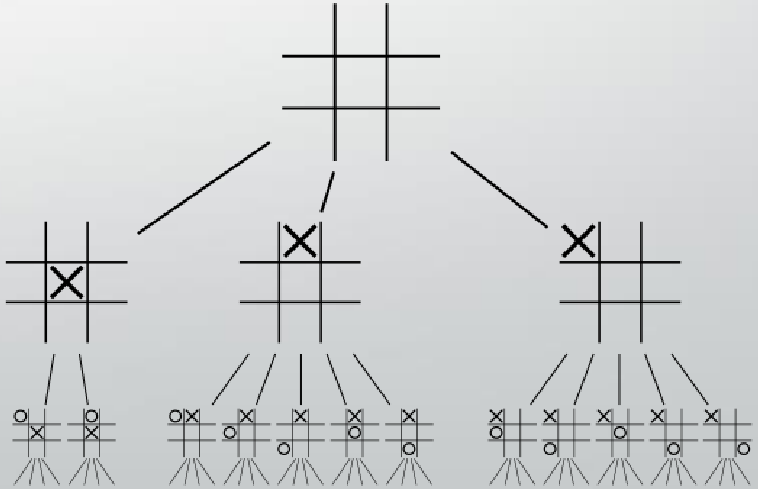
Strategic or Extensive

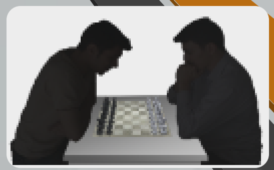
Rock, Scissors, Paper



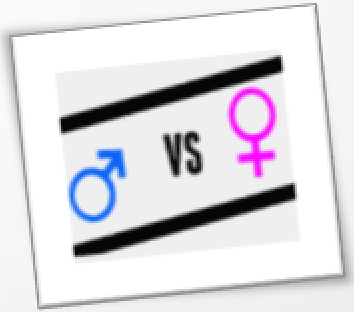
		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0 0	1 -1	-1 1
	Paper	-1 1	0 0	1 -1
	Scissors	1 -1	-1 1	0 0

Dots & Crosses (Tick-Tack-Toe)





Zero-sum or Non Zero-sum



Player 2

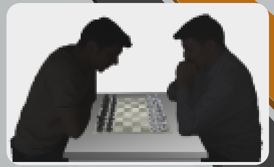
Player 1

Matching Pennies	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

Wife

Husband

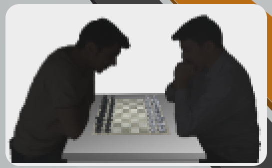
	Cinema	Theatre
Cinema	2 , 1	0 , 0
Theatre	0 , 0	1 , 2



NASH EQUILIBRIUM

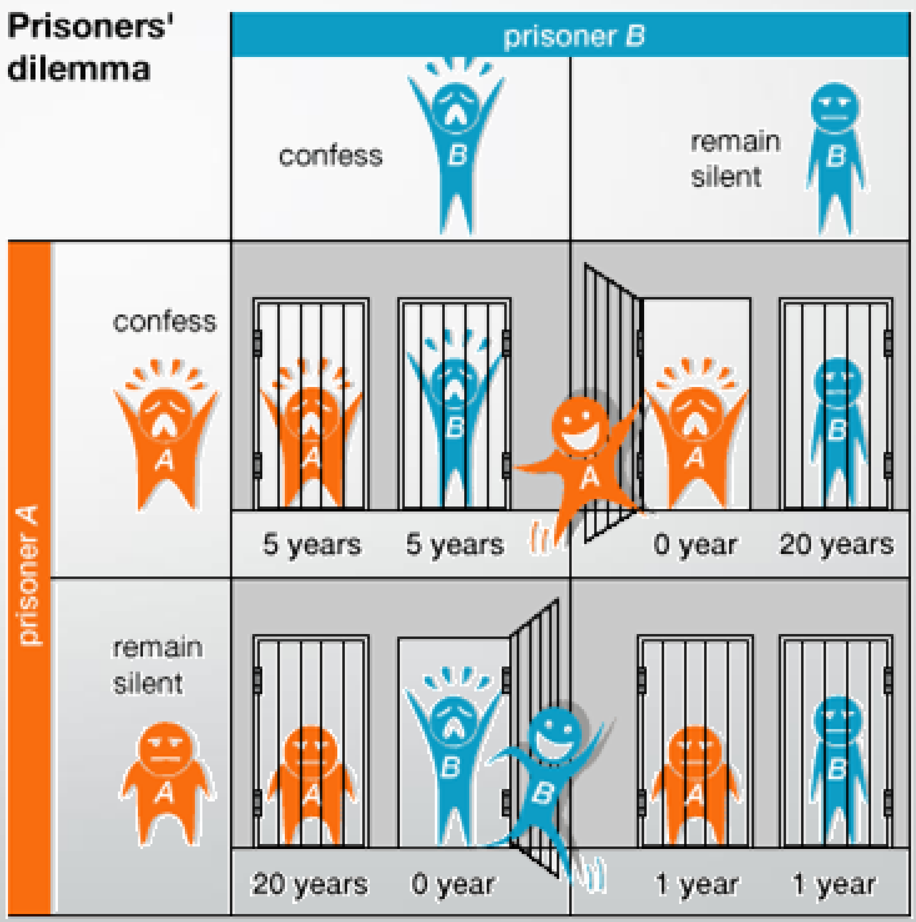
$u_i\, a_* \geq u_i\, a_i, a_{-i*}$ for every action a_i of player i.

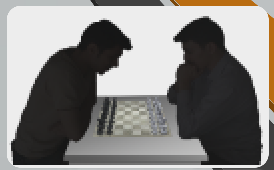
Where u_i. is a payoff function representing player i's preferences



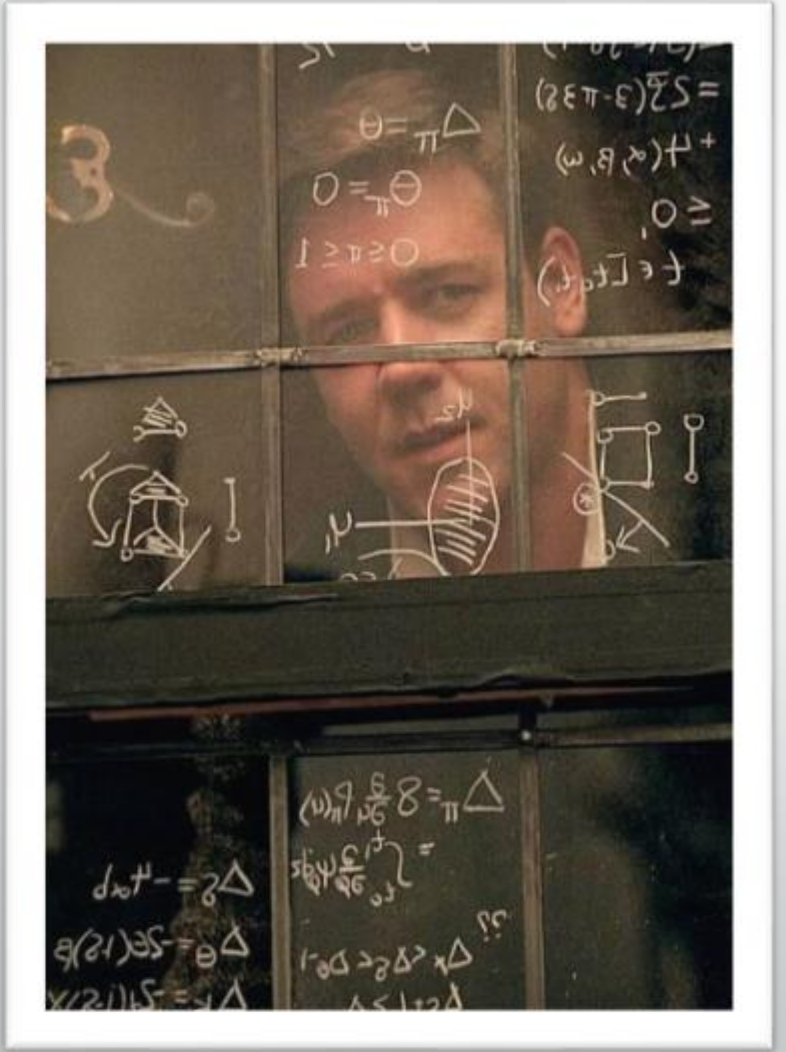
معمای زندانی ها:

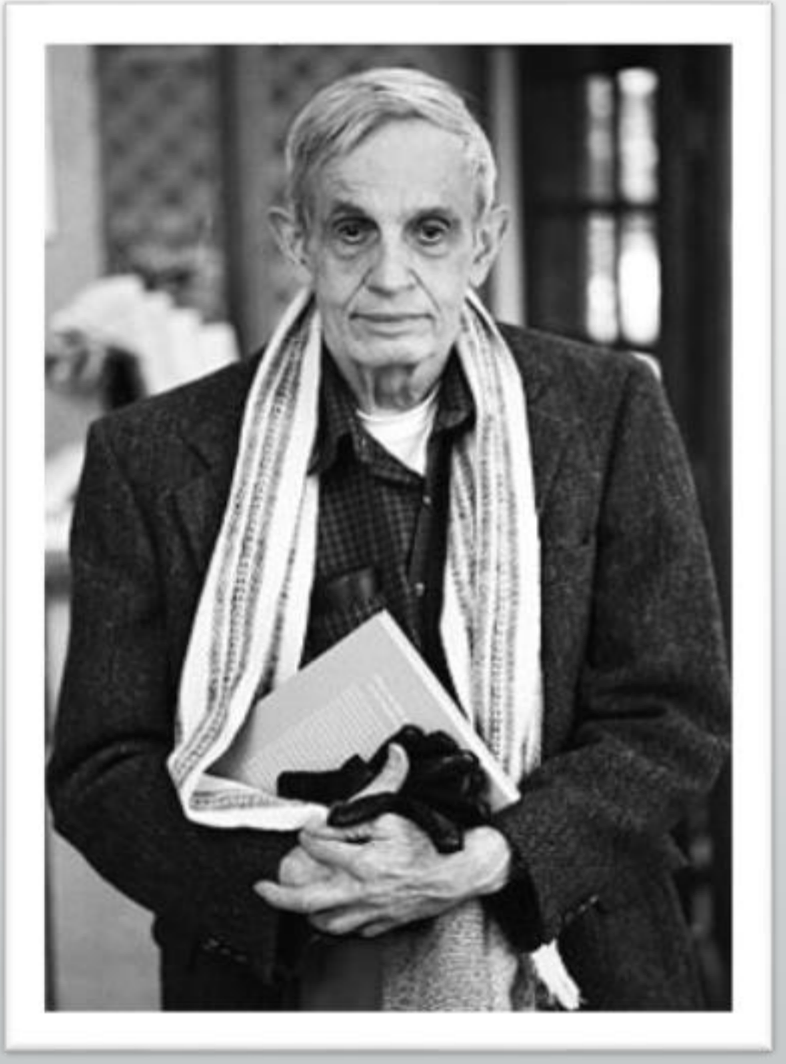
Prisoner 1	Prisoner 2: Confess	Prisoner 2: Don't Confess
Confess	-4 , -4	-1 , -10
Don't Confess	-10 , -1	-2 , -2





John Nash





EXAMPLES:

Matching Pennies:

Matching Pennies	Player 2: Head	Player 2: Tail
Player 1: Head	1,-1	-1,1
Player 1: Tail	-1,1	1,-1

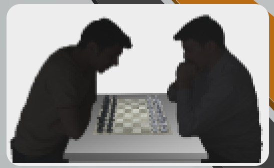
EXAMPLES:

The money sharing game:

B

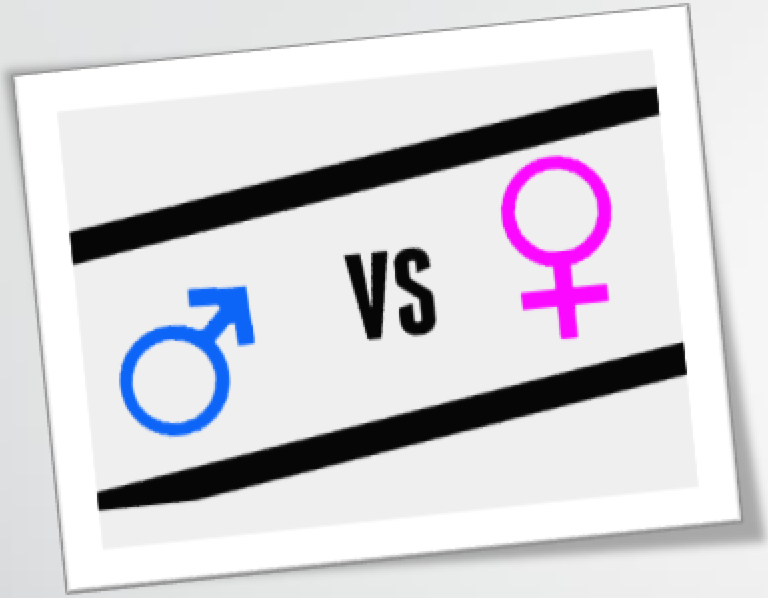
A

	Share	Grab
Share	M/2 , M/2	0 , M
Grab	M , 0	0 , 0

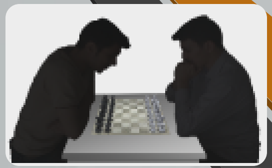


EXAMPLES:

Battle of Sexes:



		Wife	
		Cinema	Theatre
Husband	Cinema	2 , 1	0 , 0
	Theatre	0 , 0	1 , 2



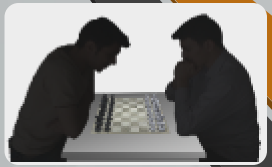
EXAMPLES:

Chicken Game
(Hawk-Dove game)

Driver 2

Driver 1

	Swerve	Straight
Swerve	0 , 0	-1 , 2
Straight	2 , -1	-5 , -5

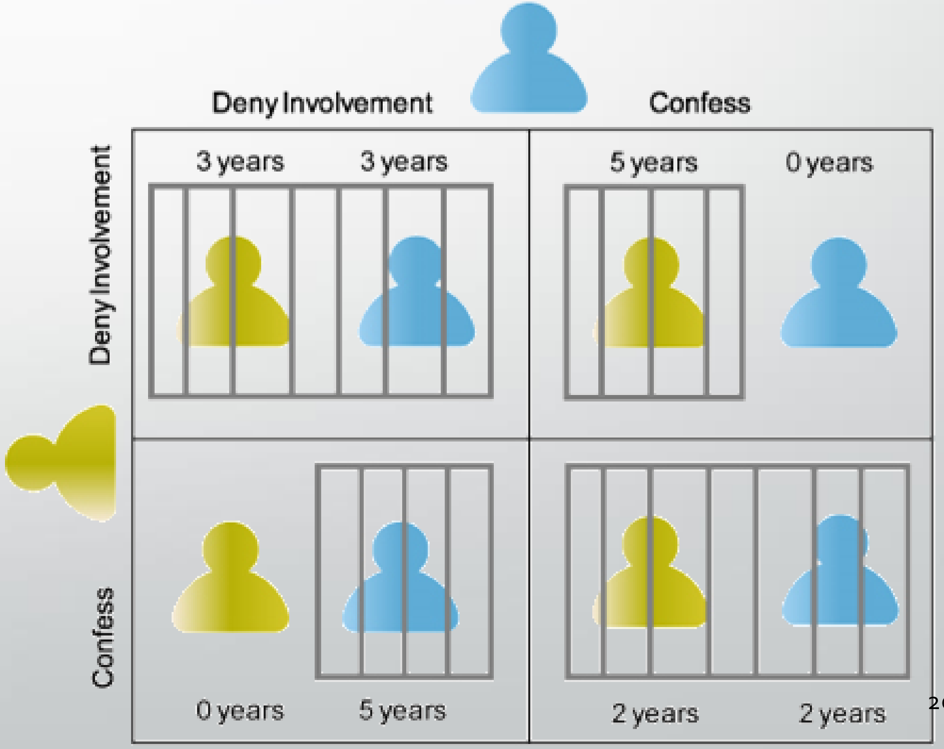


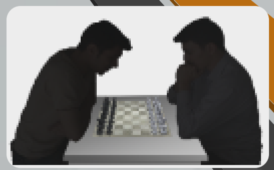
STRICT NASH EQUILIBRIUM

Definition: The action profile $a^* \in A$ in a strategic game is a ***Strict Nash Equilibrium*** if for every player i:

$$u_i(a^*) > u_i(a_i, a^*_{-i}) \quad \text{for every action } a_i \text{ of player } i.$$

The money sharing game:





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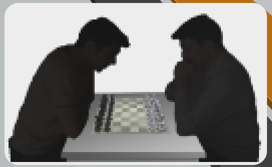
B

	Share	Grab
Share	M/2 , M/2	0 , M
Grab	M , 0	0 , 0

Prisoner 2

Prisoner 1	Confess	Don't Confess
Confess	-4 , -4	-1 , -10
Don't Confess	-10 , -1	-2 , -2



PROVISION OF A PUBLIC GOOD

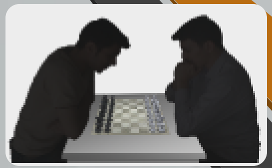
تامین کالای عمومی

Players: $N = \{1, \cdots, n\}$

Strategies: $A_i = \{C, NC\}$

C : Contribute for the public good

NC : Don't contribute for the public good



تامین کالای عمومی

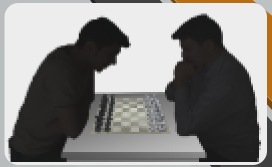
Payoff:

$$u_i(a_i = C; Out = 0) < u_i(a_i = NC; Out = 0) < u_i(a_i = C; Out = 1) < u_i(a_i = NC; Out = 1)$$

- Is there a NE where more than k players contribute? ✗

- Is there a NE where exactly k players contribute? ✓

- Is there a NE where less than k players contribute? ?



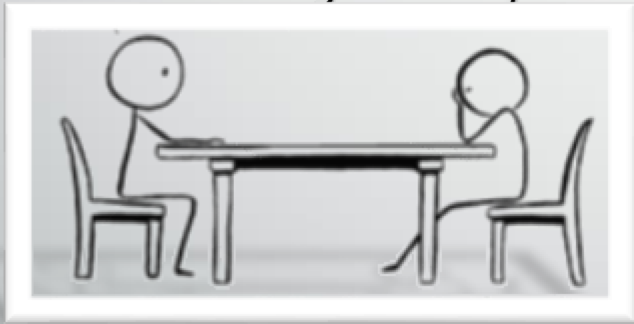
Best Response Functions

روش سریع تر یافتن تعادل نش

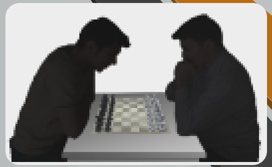
Definition: Player i's ***best response function (correspondence)*** in a strategic game is the function that assigns to each $a_{-i} \in A_{-i}$ the set:

$$BR_i(a_{-i}) = \{a_i \in A_i \,:\, u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}); \forall a_i' \in A_i\}.$$

Working on a Project



		Player 2	
		Work hard	Don't Bother
Player 1	Work hard	2 , 2	0 , 3
	Don't Bother	3 , 0	1 , 1



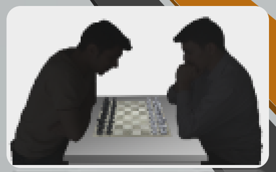
USING BR FUNCTION TO FIND NE

There are n hunters. Only m hunters are enough to catch a stag where $2 \leq m < n$. Assume there is only a single stag.

What is the NE of the game if :

a) Each hunter prefers the fraction $1/_n$ of the stag to a hare.

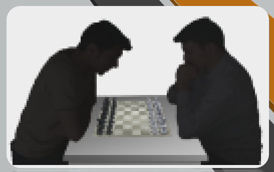
b) Each hunter prefers the fraction $1/_k$ of the stag to a hare ($m \leq k \leq n$), but prefers a hare to any smaller fraction of the stag.



Stag Hunt with n Hunters

DUOPOLY

Firm 1 \ Firm 2	High	Low
High	1000 , 1000	-200 , 1200
Low	1200 , -200	600 , 600



COURNAT DUOPOLY

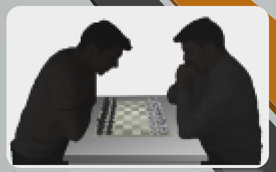
Two firms, 1 and 2, producing a homogeneous good

The inverse demand function for the good is $P = 10 - \frac{1}{10}Q$.

They choose quantity $q_i \geq 0$ simultaneously.

For simplicity suppose marginal costs are zero.

Total quantity $Q = q_1 + q_2$ is placed on the market and determines the



COURNAT DUOPOLY

Firm 1's Profit is:

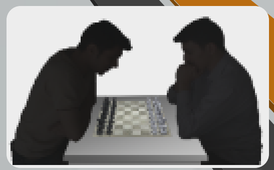
$$\pi_1 = q_1 \cdot P = q_1(10 - 0.1q_1 - 0.1q_2) = 10q_1 - 0.1q_1^2 - 0.1q_1q_2$$

Suppose firm 2 fixes his production level at $\hat{q}_2$; then the best response by firm 1 should satisfy the first order condition:

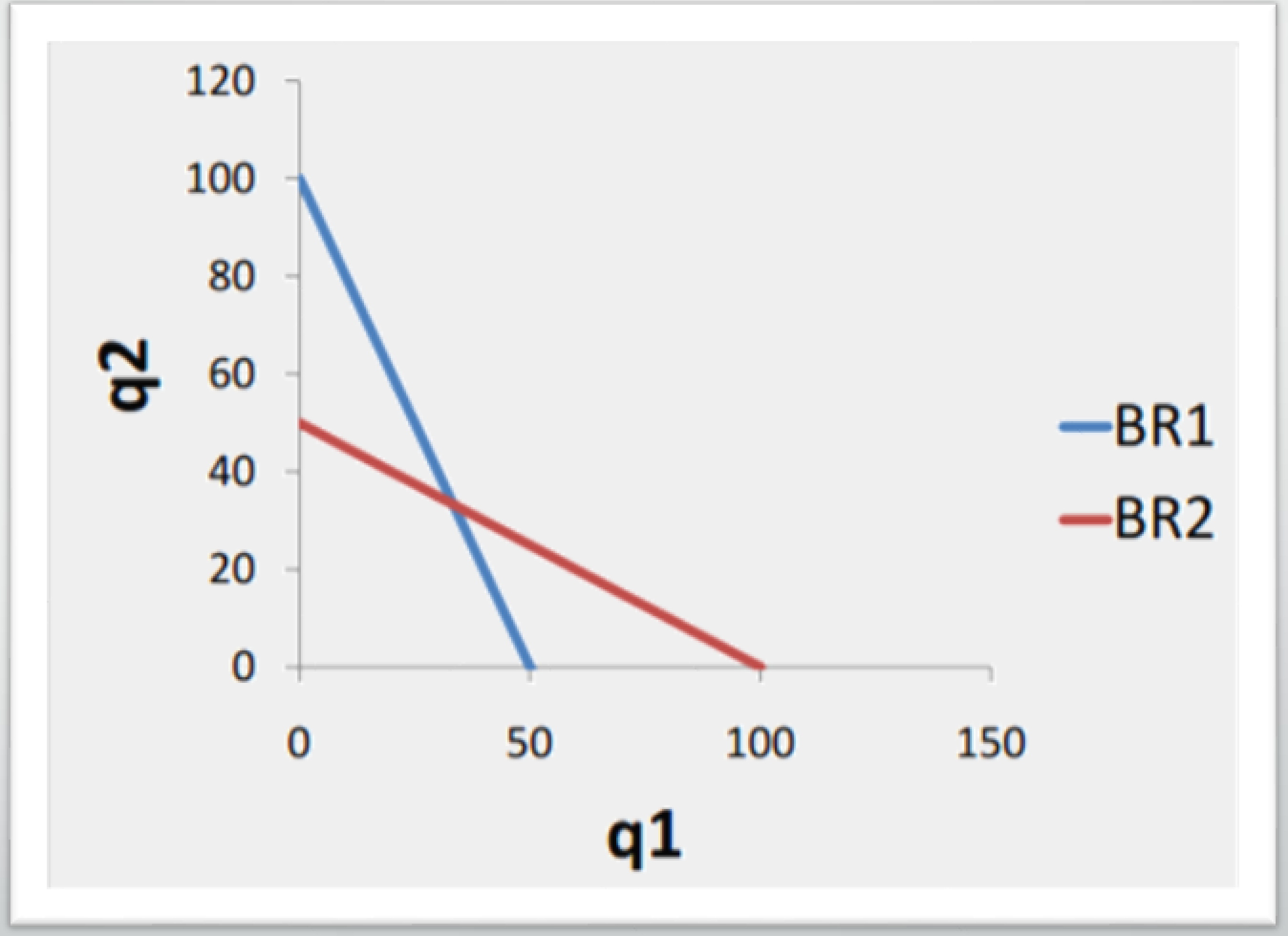
$$\partial\pi_1/\partial q_1 = 0 \quad \Longrightarrow \quad 10 - 0.2q_1 - 0.1\hat{q}_2 = 0 \quad \text{or} \quad q_1 = 50 - 0.5\hat{q}_2$$

Then the BR function for firm 1 is:

$$q_1 = BR_1(\hat{q}_2) = 50 - 0.5\hat{q}_2$$



COURNAT DUOPOLY



COURNAT DUOPOLY

$$\pi_1 = q_1 \cdot P = q_1(10 - 0.1q_1 - 0.1q_2) = 10q_1 - 0.1q_1^2 - 0.1q_1q_2$$

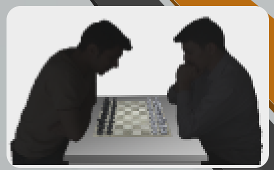
Remember the monopoly quantity is: $q^M = 50$.

Easy to calculate that:

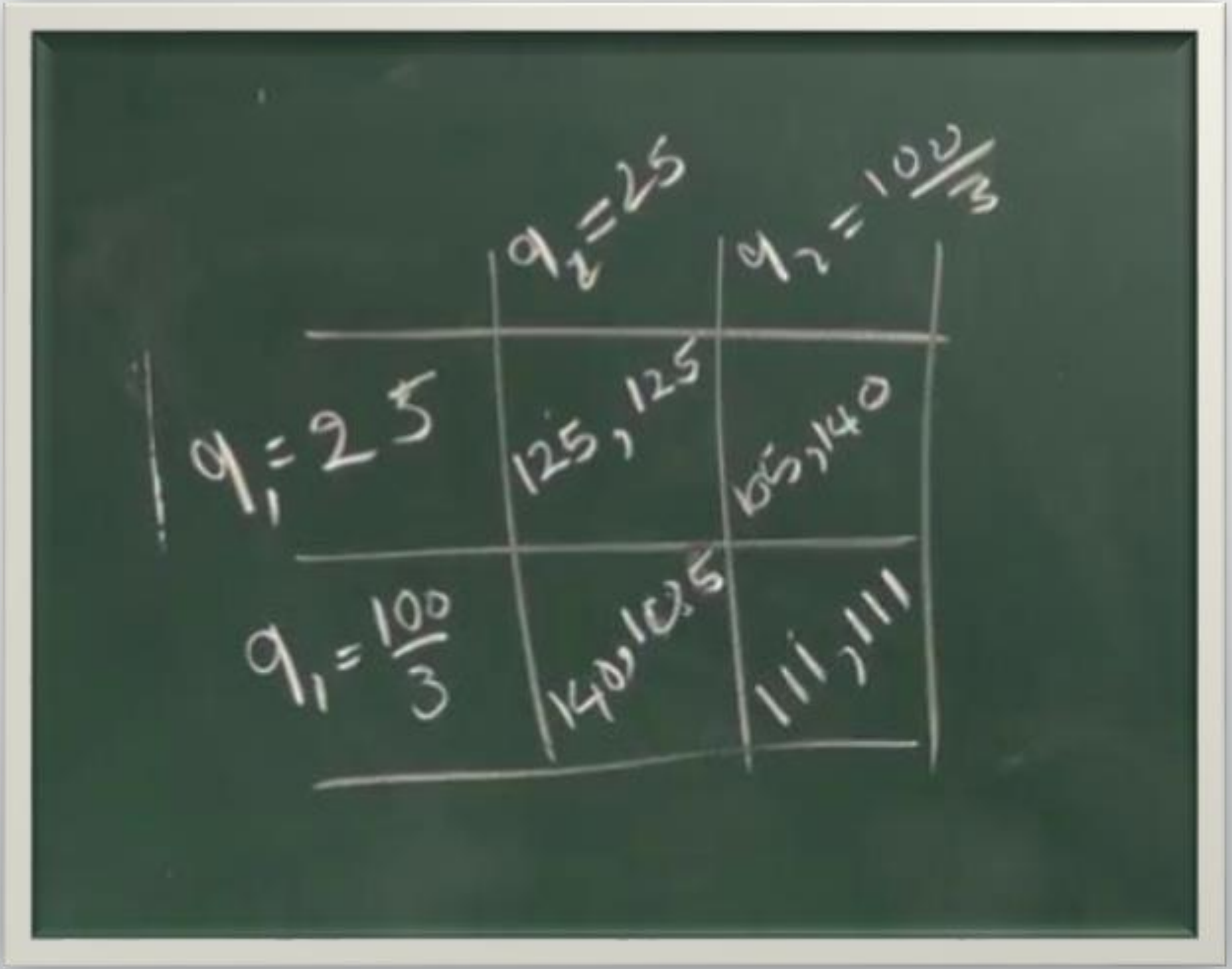
$$P^M = 5, \pi^M = 250$$

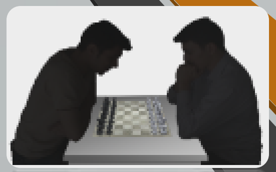
And

$$P^C = \frac{10}{3} = 3.33, \pi_1^C = \pi_2^C = 111.1$$



COURNAT DUOPOLY





BERTRAND DUOPOLY

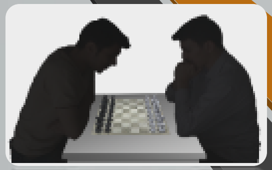
Same context, but firms choose prices. Prices can be continuously varied, i.e. p_i is any real number. Firms have the same marginal cost of mc.

If prices are unequal, all consumers go to lower price firm.

If equal, market is shared.

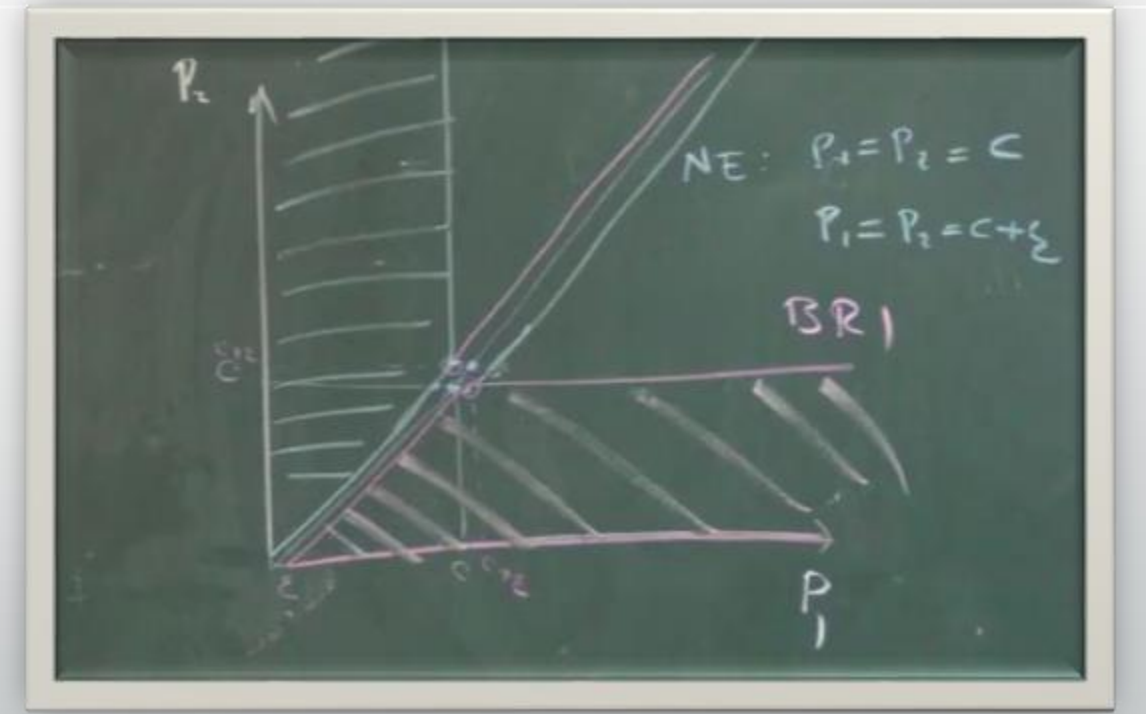
1) There is a Nash equilibrium where $p_1 = p_2 = mc$: None of the firms has incentive to deviate from this strategy.

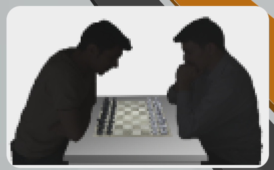
2) There is no other Nash equilibrium in pure strategies.



BERTRAND DUOPOLY

$$\pi_i = \begin{cases} (\alpha - p_i)(p_i - c) & \text{if} \quad p_i < p_j \\ \frac{1}{2}(\alpha - p_i)(p_i - c) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$





COURNAT OLIGOPOLY

Players: $N = \{1, \cdots, n\}$; n firms all produce a homogenous good.

Strategy: $q_i \geq 0 \quad i = 1, \cdots, n$

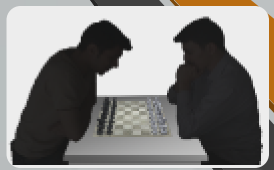
Output: The total output is $\sum_i q_i = Q$ and the price is determined by this inverse demand function:

$$P = \begin{cases} \alpha - Q & \text{if} \quad Q \leq \alpha \\ 0 & \text{if} \quad Q > \alpha \end{cases}$$

Payoffs: $\pi_i = Pq_i - C_i(q_i)$

For simplicity assume $C_i(q_i) = cq_i \quad \forall i$

Show that the symmetric NE of the game is $q_i = \frac{1}{n+1}(\alpha - c)$.



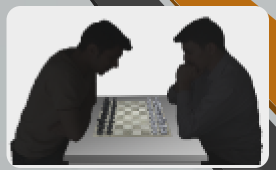
BERTRAND OLIGOPOLY

Players: $N = \{1, \cdots, n\}$; n firms all produce a homogenous good.

Strategy: $p_i \geq 0 \quad i = 1, \cdots, n$

Payoffs: ($D(p)$ is the demand function at price p)

$$\pi_i = \begin{cases} p_i \dfrac{D(p_i)}{m} - C_i\left(\dfrac{D(p_i)}{m}\right) & \text{if } i \text{ is one of } m \text{ firms with the lowest price} \\ 0 & \text{otherwise} \end{cases}$$



BERTRAND OLIGOPOLY

In the simplest version:

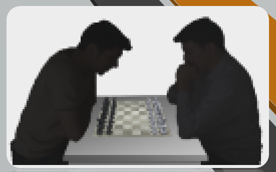
$$n = 2\,,$$

$$D(p) = \begin{cases} \alpha - p & \text{if} \quad p \leq \alpha \\ 0 & \text{if} \quad p > \alpha \end{cases},$$

$$C_i(q_i) = cq_i \qquad\qquad i = 1\,,2 \;\; (c < \alpha)\,.$$

Then

$$\pi_i = \begin{cases} (\alpha - p_i)(p_i - c) & \text{if} \quad p_i < p_j \\ \dfrac{1}{2}(\alpha - p_i)(p_i - c) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$



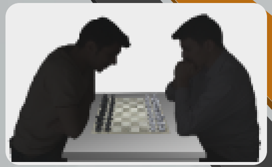
BERTRAND OLIGOPOLY

The best response function is (p_m is the monopolistic price):

$$BR_i(p_j) = \begin{cases} \{p_i | p_i > p_j\} & \text{if} \quad p_j < c \\ \{p_i | p_i \geq p_j\} & \text{if} \quad p_j = c \\ \emptyset & \text{if} \quad c < p_j < p_m \\ p_m & \text{if} \quad p_j > p_m \end{cases}$$

And the only NE of the game is:

$$(p_1, p_2) = (c, c)$$



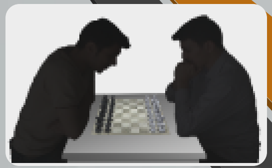
MATCH

حدس دو سوم میانگین کلاس

Rule: The player who has the closest a_i to $\frac{2}{3}$ of the average is the winner.

$$\text{Payoffs: } \pi_i = \begin{cases} 1 & i \text{ is the only winner} \\ \frac{1}{m} & i \text{ is the joint winner in a } m-\text{way tie} \\ 0 & i \text{ is not a winner} \end{cases}$$

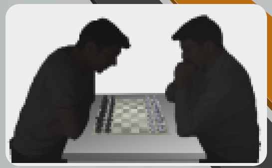
What is the NE of this game?



AUCTIONS

حراج ها

- Simultaneous or sequential bidding?
- The rule determining the winner?
- How much the winner should pay?
- Private value or common value (with asymmetric information sets)?
- How many units of the object are auctioned off?
- How to break ties?
- Is there a reserve price (Is the reserve price common knowledge)?



تعادل نش حراجی در کجا قرار دارد؟

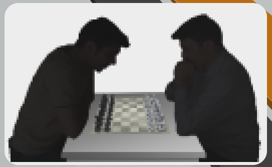
$$b_i = v_i \quad i = 1, \cdots, n$$

Strategy: bi≥0 i=1 ,⋯,n ; Each bidder submits a non-negative bid

Pay-offs:

$$\pi_i(b_i, \bar{b}_{-i}, v_i) = \begin{cases} v_i - \bar{b}_{-i} & b_i > \bar{b}_{-i} \\ \frac{1}{m}(v_i - b_i) & b_i = \bar{b}_{-i} \quad (m \text{ way tie}) \\ 0 & b_i < \bar{b}_{-i} \end{cases}$$

$$\text{where} \quad \bar{b}_{-i} = \max_{j \neq i} b_j$$



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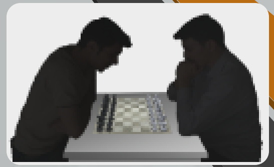

The End

تهیه و تنظیم:

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