ASEN 6519-007 Decision Making under Uncertainty
Homework 1: Probabilistic Models

January 15, 2020

1 Conceptual Questions

Question 1. There is a 1% chance there is both life and liquid water on Mars, a 1% chance there is life but no water, and a 4% chance there is no life and no water. What is the probability that there is life on Mars given that there is water?[1]

Question 2. Suppose that a stochastic process $\{x_t\}$ is defined by the following equation: $x_{t+1} = x_t + x_{t-1} + v_t$ where v_t are i.i.d. noise random variables.

(a) Is this process Markov if the state is defined as x_t?

(b) What would need to be included in the state at time t to make this a Markov process?

Question 3. Suppose your company has a dataset with the following variables:

- S: a binary variable indicating whether it has snowed in the last two days.
- T: a binary variable indicating whether the temperature is above or below freezing.
- D: a binary variable indicating whether the delay for a commute is more than 10 minutes.

Consider the following two Bayes Nets:
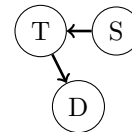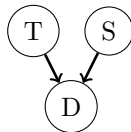
(a) How many independent parameter values are required to fully define each distribution?

(b) Suppose you are an app developer who wants to warn users how likely it is that they will be delayed based on the temperature and snow history. Your data scientist fits a model in the form on the right because "it has fewer parameters so it is better" and gives it to you. How do you respond?

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[1]These probabilities are totally made up. If you have strong feelings about the real probability of life on Mars, feel free to speculate in your answer :)

Question 4. Consider the Bayes Net below:
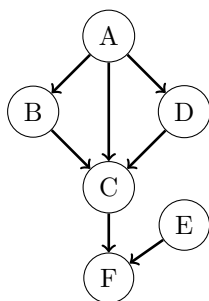
(a) Suppose that you want to know the value of D with the highest precision possible. Allie knows the value of A, Brian knows the value of B. You can only talk to one of these people. Who should you ask?

(b) Suppose that you already know the values of C and A and want to guess the value of D. Eric knows the value of E and Brian knows the value of B. You can only talk to one of these people. Who should you ask?[2]

2 Exercises

Question 5. Consider two stochastic processes, $\{x_t\}$ and $\{y_t\}$, defined by $x_t = f_x(x_0, x_1..., x_{t-1}, v_t)$ and $y_t = f_y(y_0, y_1..., y_{t-1}, v_t)$ where v_t are independent, identically distributed random variables that introduce noise. The `HW1` module in `DMUStudent` contains two Julia functions, `fx` and `fy`, which can sample from the stochastic processes, i.e. `fx([x1, x2])` will return a sample of x_3 given that $x_1 = $ `x1` and $x_2 = $ `x2`.

Suppose you know that one of these processes is Markov and the other is not. By drawing samples with the Julia functions, determine which one is Markov.

3 Challenge Problem

Question 6. Write a function in Julia that takes in two arguments and uses the Pythagorean Theorem to compute the hypotenuse of a right triangle with side lengths specified by the arguments. Submit this function with `DMUStudent.submit`. A score of 1 will receive full credit.[3]

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[2]Hint: Consider potential conditional independence relationships using the d-separation rules.

[3]This particular "Challenge Problem" is not meant to be challenging; it is meant to test that everyone can download the code and submit. It should require only 2 lines. Future problems in this section will be quite challenging.