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Homework 1

Decision Making under Uncertainty
ASEN 6519-007
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Conceptual Questions

Problem 1: Life on Mars

Given

- 1% probability of life and water: $P(\text{Life}, \text{H}_2\text{O}) = 0.01$
- 1% probability of life but no water: $P(\text{Life}, \text{H}_2\text{O}') = 0.01$
- 4% probability of no life and no water: $P(\text{Life}', \text{H}_2\text{O}') = 0.04$

Find
Probability of life, given that there is water: $P(\text{Life}|\text{H}_2\text{O}) = ?$

Solution
Given the probabilities above, the probability that there is water, but no life is 94%:

$$P(\text{Life}', \text{H}_2\text{O}) = 1 - \left[P(\text{Life}, \text{H}_2\text{O}) + P(\text{Life}, \text{H}_2\text{O}') + P(\text{Life}', \text{H}_2\text{O}')\right] = 0.94$$

By the law of total probability, the marginal probability there is water is 95%:

$$P(\text{H}_2\text{O}) = P(\text{Life}, \text{H}_2\text{O}) + P(\text{Life}', \text{H}_2\text{O}) = 0.01 + 0.94 = 0.95$$

The definition of conditional probability states $P(A|B) = \frac{P(A,B)}{P(B)}$, thus:

$$P(\text{Life}|\text{H}_2\text{O}) = \frac{P(\text{Life}, \text{H}_2\text{O})}{P(\text{H}_2\text{O})} = \frac{0.01}{0.95} = 0.0105$$

There is a 1.5% chance that there is life on Mars, given that there is liquid water.

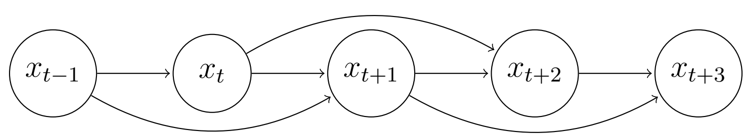
Problem 2: A Markov Process?

Given
A stochastic process, $\{x_t\}$ defined by: $x_{t+1} = x_t + x_{t-1} + v_t$, where v_t are iid.

Find
(a) Is the process Markov if the state is defined as x_t?
(b) How can we make the process Markov?

Solution

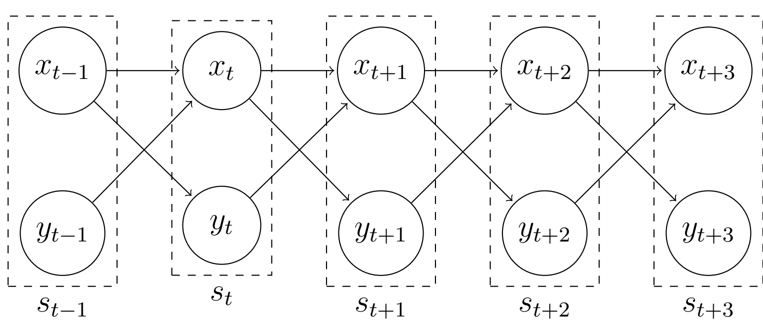
(a) The above process can be drawn as shown below. If we take the state to be x_t, each state depends on the previous **two** timesteps, thus the process is not Markov.

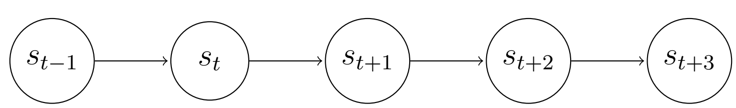


(b) By introducing a new variable, y_t, we can re-write the problem as so:

$$x_{t+1} = x_t + y_t$$
$$y_t = x_{t-1} + v_t$$

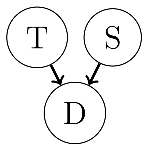
which yields a depiction as below (by ignoring the noise term). If we take our state to be $s_t = (x_t, y_t)$ then each state depends only on the previous, and the process is now Markov.

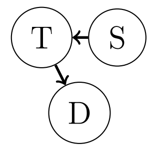




Problem 3: Two Bayes Nets

Given
The two Bayes nets below.





Find
(a) How many independent parameters are required to fully describe each distribution?
(b) Respond to the comment "The model on the right has fewer parameters, so it is better."

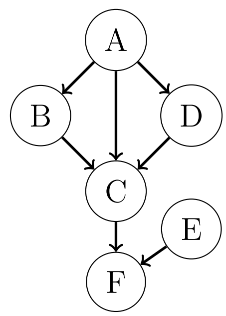
Solution

(a)
- For the model on the left, six independent parameters are needed: Since each probability is binary, $P(T)$ and $P(S)$ both each need one parameter. For D, which is dependent on both T and S, $P(D|T,S)$, four independent parameters are needed.
- For the model on the right, only five independent parameters are needed: $P(S)$ needs only one, while $P(T|S)$ and $P(D|T)$ both each need two parameters each.

(b) Fewer parameters doesn't necessarily give a "better" model. In the second model, D and S are independent, given T. If we know that the temperature is currently below freezing, then knowing that it snowed in the last two days doesn't give us any more information about a possible delay or not.

Problem 4: A More Complicated Bayes Net

Given
The Bayes net below.



Find
(a) If you want to know the value of D with the highest precision, should you learn the value of A or B?
(b) If you know the value of A and C, and want to know the value of D, should you learn the value of B or E?

Solution

(a) **To know the value of D, I would want to ask Allie, who knows A.**

B and D are conditionally independent, given A, and so asking Brian, who knows B, will not get us any closer to knowing D. We can see this independence because B and D are "d-separated" by A, since there is a fork in diagram between them: B $\leftarrow$ A $\rightarrow$ D.

(b) **To know the value of D, I would want to ask Brian, who knows B.**

Since we now know the value of C, learning B will tell us something about D. This is because B $\rightarrow$ C $\leftarrow$ D form a V-structure, indicating that B and D are conditionally dependent given C. Thus, knowing B will provide additional information about D if we already know C.

Knowing E will not get us any more information about D, if we already know C. This is because there exists a chain in the path between D and E: D $\rightarrow$ C $\rightarrow$ F $\leftarrow$ E, thus, D and E and conditionally independent, given C.

Exercises

Problem 5: Sampling from Stochastic Processes

Given
Two stochastic processes, $\{x_t\}$ and $\{y_t\}$, one of which is Markov.

Find
By sampling from each, determine which of the two is Markov.

Solution
The Markov process is $\{y_t\}$.

See the code below that I wrote to determine this, along with sample output from my program. I calculated the expected fourth term of the series, given both the previous three terms, and given just the last term, then compared the two predictions using the squared error. I did this a number of times (N), with different starting values, reporting the total sum of the squared errors. In all cases I got that the sum of squared errors on the $\{y_t\}$ process was about half that of the $\{x_t\}$, leading me to think that the $\{x_t\}$ process is not Markovian, since a longer history leads to a better prediction. Increasing the length of the history did not change the magnitude of the difference, which makes me think that $\{x_t\}$ is determined only by the previous three values, and not more.

```
using DMUStudent.HW1
using Plots

function GenerateGuesses(f, x1, x2, x3)
    x4 = f([x1, x2, x3])
    x4r = f([x3])
    return x4, x4r
end

function CalculateErrors(f, n, xs = [], xrand = [], sse = [])
    for i in 1:n
        x4, x4r = GenerateGuesses(f, rand(1:20), rand(1:20), rand(1:20))
        se = sqrt((x4-x4r)^2)
        push!(sse, se)
    end
    return sum(sse)
end

N = 100000
x_errors = CalculateErrors(fx, N)
y_errors = CalculateErrors(fy, N)
println("Number of samples: ", N)
println("Total x errors: ", x_errors)
println("Total y errors: ", y_errors)

if x_errors > y_errors
    print("y is the Markov Process")
```

```
elseif x_errors < y_errors
    println("x is the Markov Process")
else
    println("Not enough information to tell")
end
```

CODE OUTPUT

```
    Number of samples: 100000
    Total x errors: 219735.0
    Total y errors: 117026.0
    y is the Markov Process
```

Challenge Problem

Problem 6: Pythagorean Theorem

Submitted through the leader-board.