Name:	
	Matthew Tang

Netid:

mhtang2

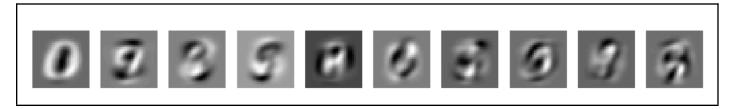
CS 441 - HW2: PCA and Linear Models

Complete the sections below. You do not need to fill out the checklist.

Total	Points .	Available	[]/160
1.	PCA c	on MNIST	
	a.	Display 10 principal component vectors	[]/5
	b.	Display scatterplot	[]/5
	C.	Plot cumulative explained variance	[]/5
	d.	Compression and 1-NN experiment	[]/15
2.	MNIS	Γ Classification with Linear Models	
	a.	LLR / SVM error vs training size	[]/20
	b.	Error visualization	[]/10
	C.	Parameter selection experiments	[]/15
3.	Tempe	erature Regression	
	a.	Linear regression test	[]/10
	b.	Feature selection results	[]/15
4.	Stretc	h Goals	
	a.	PR and ROC curves	[]/10
	b.	Visualize weights	[]/10
	C.	Other embeddings	[]/15
	d.	One city is all you need	[]/15
	e.	SVM with RBF kernel	[]/10

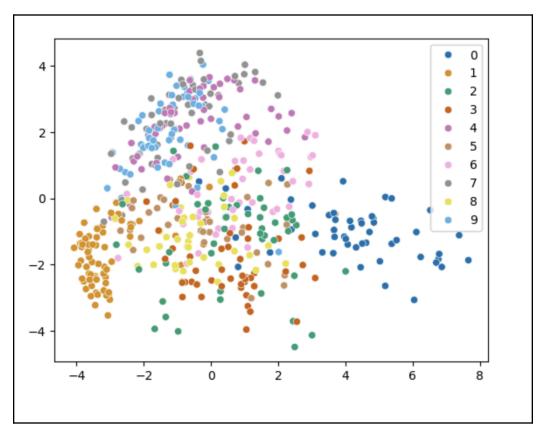
1. PCA on MNIST

a. Display 10 principal component vectors

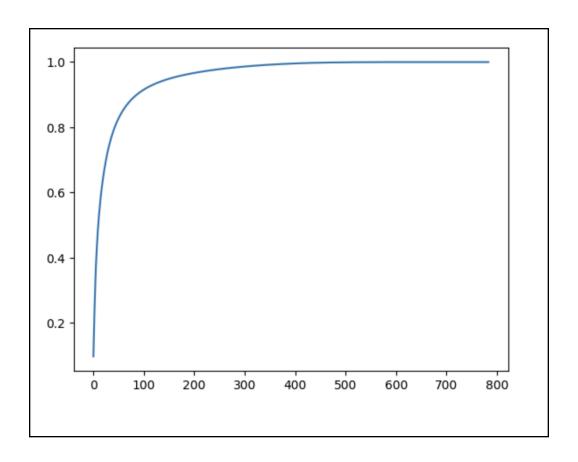


b. Display scatterplot

Scatterplot $x_{train[:500]}$ for the first two PCA dimensions. Show a different color for each label.



c. Plot cumulative explained variance



d. Compression and 1-NN experiment

Number of components selected

	Total Time (s)	Test Error (%)	Dimensions
Brute Force (PCA)	4.545	2.73%	87
Brute Force	36.031	3.09%	784

2. MNIST Classification with Linear Models

a. LLR / SVM error vs training size

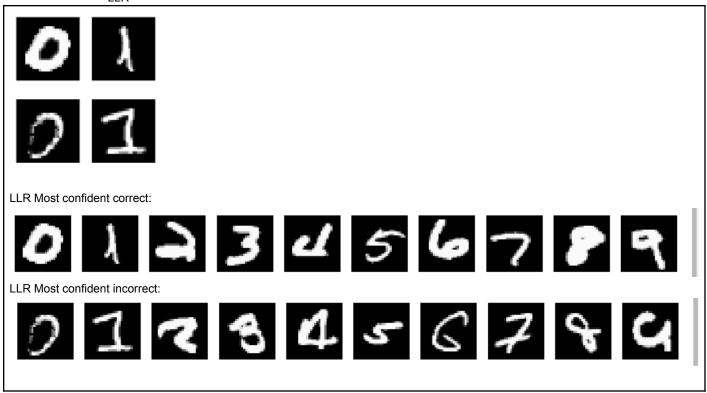
Test error (%)

# training samples	LLR	SVM
100	32.5%	32.4%
1,000	13.64 %	16.11 %

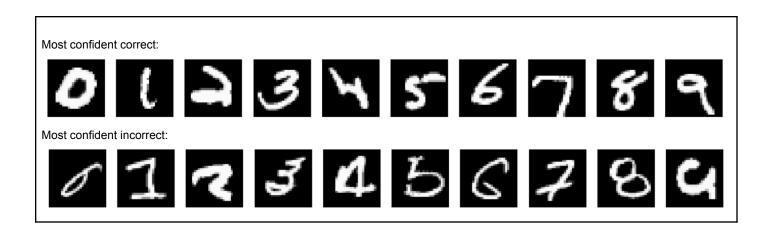
10,000	9.5 %	11.12 %
60,000	7.38 %	8.17 %

b. Error visualization

LLR



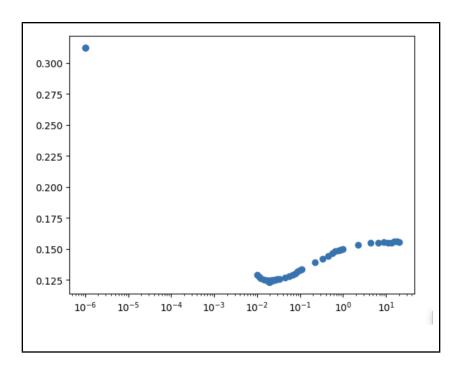
SVM



c. Parameter selection experiments

	SVM
Best C value	0.0196
Validation error (%)	12.35 %
Test error (%)	13.58 %

Plot C value vs validation error for values tested



3. Temperature Regression

a. Linear regression test

Test RMSE

	Linear regression
Original features	2.1608
Normalized features	2.1631

Why might normalizing features in this way not be as helpful as it is for KNN?

Because KNN uses distances, it is sensitive to the scale of features. Linear regression can learn weights to scale features so it is less sensitive. This normalization essentially centers/rescales features.

b. Feature selection results

Feature Rank	Feature number	City	Day
1	334	Chicago	-1
2	347	Minneapolis	-1
3	405	Grand Rapids	-1
4	366	Kansas City	-1
5	361	Cleveland	-1
6	307	Omaha	-2
7	367	Indianapolis	-1
8	264	Minneapolis	-2
9	9	Boston	-5
10	236	Springfield	-3

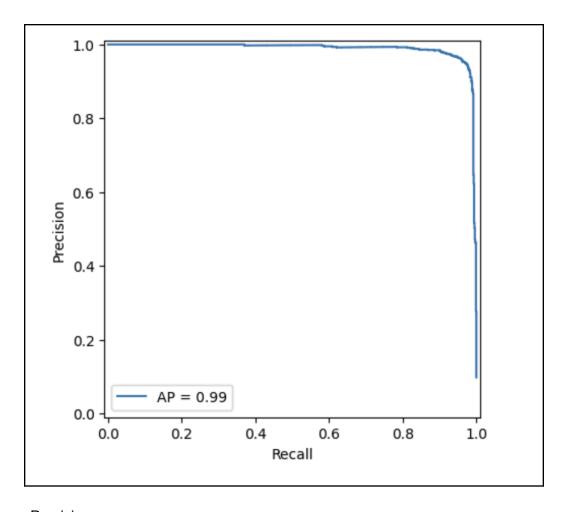
Test error using only the 10 most important features for regression

	Linear Regression
RMS Error	2.0621

4. Stretch Goals

a. PR and ROC curves

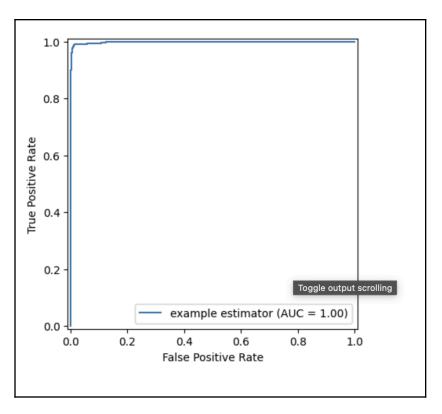
PR plot



Average Precision

0.98883

ROC plot



Area under the curve (AUC)

0.9983

b. Visualize weights

LLR - L2













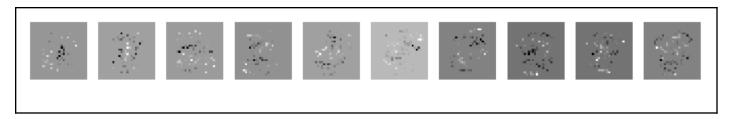




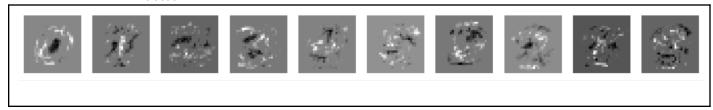




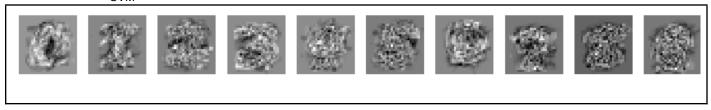
LLR - L1



LLR - elastic



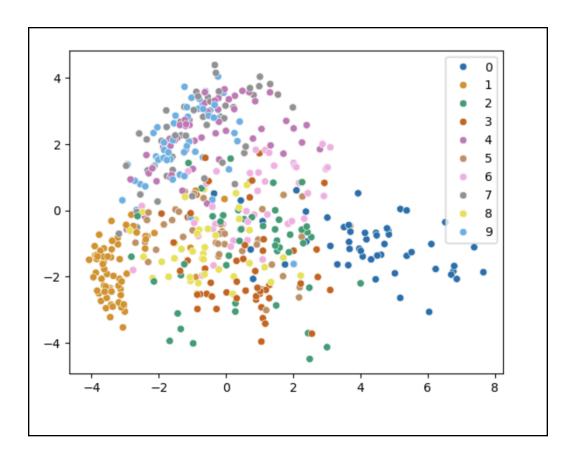
SVM



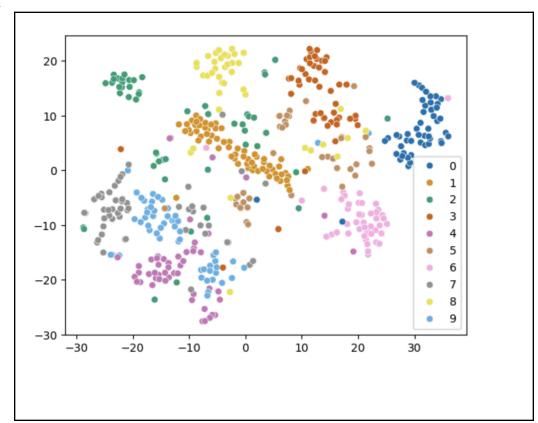
c. Other embeddings

Display 2+ plots for TSNE, MDA, and/or LDA, and copy PCA plot from 1b here.

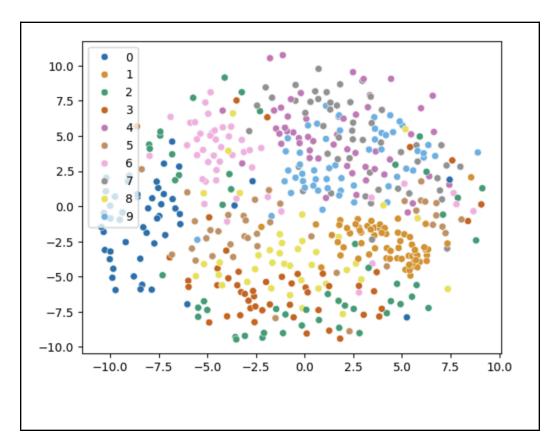
PCA



TNSE



MDS



d. One city is all you need

City

St. Louis

Test error using features only from that city

3.1263 RMSE

Explain your process (in words):

Iterated through every city, and trained a Ridge model (no feature normalization, default parameters) using only days from that city as input features. Then I took the validation error for each of these models, and took the city with the minimum validation error.

e. Compare linear SVM and SVM with RBF kernel

Test accuracy (%)

# training samples	SVM-Linear	SVM-RBF
100	32.4%	34.4%
1,000	16.11 %	9.17%
10,000	11.12 %	4.06%
60,000	8.17 %	2.08%

Acknowledgments / Attribution

List any outside sources for code or ideas or "None". Stackoverflow + sklearn docs

CS441 SP24 HW2 Starter

February 20, 2024

0.1 CS441: Applied ML - HW 2

0.1.1 Parts 1-2: MNIST

Include all the code for generating MNIST results below

```
[253]: # initialization code
       import numpy as np
       from keras.datasets import mnist
       %matplotlib inline
       from matplotlib import pyplot as plt
       from scipy import stats
       from sklearn.linear_model import LogisticRegression
       !apt install libomp-dev > /dev/null 2>&1
       !pip install faiss-cpu > /dev/null 2>&1
       import faiss
       import time
       def load mnist():
         Loads, reshapes, and normalizes the data
         (x_train, y_train), (x_test, y_test) = mnist.load_data() # loads MNIST data
        x_train = np.reshape(x_train, (len(x_train), 28*28)) # reformat to 768-d_
        \rightarrowvectors
        x_test = np.reshape(x_test, (len(x_test), 28*28))
        maxval = x train.max()
        x_train = x_train/maxval # normalize values to range from 0 to 1
        x_test = x_test/maxval
        return (x_train, y_train), (x_test, y_test)
       def display_mnist(x, subplot_rows=1, subplot_cols=1):
         111
         Displays one or more examples in a row or a grid
         if subplot_rows>1 or subplot_cols>1:
           fig, ax = plt.subplots(subplot_rows, subplot_cols, figsize=(15,15))
           for i in np.arange(len(x)):
```

```
ax[i].imshow(np.reshape(x[i], (28,28)), cmap='gray')
ax[i].axis('off')
else:
   plt.imshow(np.reshape(x, (28,28)), cmap='gray')
   plt.axis('off')
plt.show()
```

0.1.2 Part 1: PCA and Data Compression

```
[254]: from sklearn.decomposition import PCA
  import matplotlib.pyplot as plt

  (x_train, y_train), (x_test, y_test) = load_mnist()

# Compute the first 10 principal components using x_train
  pca_model = PCA()
  pca_model.fit(x_train)
  principal_components = pca_model.components_[:10]

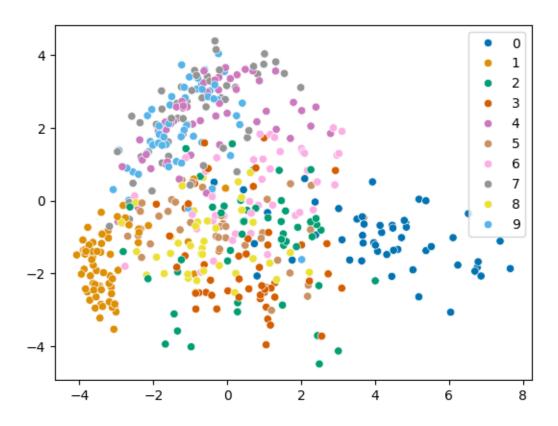
# Display First 10 Components
  display_mnist(principal_components, 1, 10)
```



```
[57]: # Scatter plot of first two PCA dimensions
import seaborn as sns

# use pca.transform
x = pca_model.transform(x_train[:500])
# TO DO
ind = np.arange(500)
sns.scatterplot(x=x[ind,0],y=x[ind,1], hue=y_train[ind], palette="colorblind")

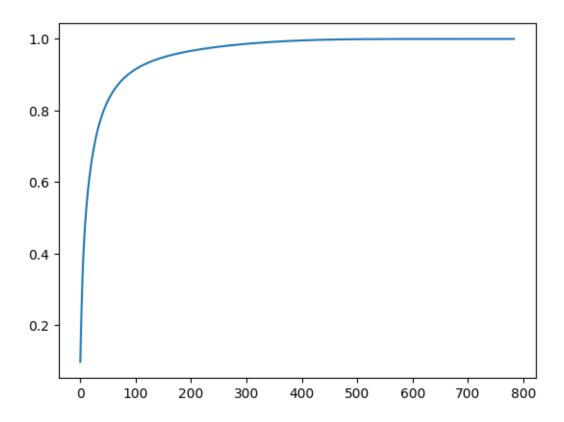
[57]: <Axes: >
```



```
[58]: # Plot cumulative explained variance ratio
# cumsum and pca.explained_variance_ratio_ will be useful

# TO DO
cum_var = np.cumsum(pca_model.explained_variance_ratio_)
plt.plot(cum_var)
```

[58]: [<matplotlib.lines.Line2D at 0x7f3a416a2190>]



```
[59]: # Select number of dimensions that explains 90% of variance, according to your
       ⇔plot above
      m = (cum_var >= 0.9).argmax() + 1
      print(m)
      pca_model = PCA(n_components=m).fit(x_train)
      # Get time and error when using original features with brute force 1-NN
      # TO DO
      start = time.time()
      index = faiss.IndexFlatL2(x_train.shape[1])
      index.add(x_train)
      dist, idx = index.search(x_test,1)
      pred = y_train[idx.squeeze()]
      err = (pred != y_test).mean()
      print(err)
      print(time.time() - start)
      # Get time and error when using compressed features with brute force 1-NN
      # TO DO
      x_train_comp = pca_model.transform(x_train)
      x_test_comp = pca_model.transform(x_test)
```

```
start = time.time()
index = faiss.IndexFlatL2(x_train_comp.shape[1])
index.add(x_train_comp)
dist, idx = index.search(x_test_comp,1)
pred = y_train[idx.squeeze()]
err = (pred != y_test).mean()
print(err)
print(time.time() - start)
```

87 0.0309 36.031790018081665 0.0273 4.545729160308838

0.1.3 Part 2: MNIST Classification with Linear Models

```
[6]: from sklearn.linear_model import LogisticRegression from sklearn.svm import LinearSVC
```

LLR/SVM vs training size

```
[14]: # LLR
for n in [100, 1000, 10000, 60000]:
    LLR = LogisticRegression(max_iter = 10000).fit(x_train[:n],y_train[:n])
    pred = LLR.predict(x_test)
    err = 100*(pred != y_test).mean()
    print(err)
```

```
[7]: # SVM
for n in [100, 1000, 10000, 60000]:
    SVM = LinearSVC(max_iter = 10000).fit(x_train[:n],y_train[:n])
    pred = SVM.predict(x_test)
    err = 100*(pred != y_test).mean()
    print(err)
```

/home/matty/.local/lib/python3.11/site-packages/sklearn/svm/_classes.py:31:
FutureWarning: The default value of `dual` will change from `True` to `'auto'`
in 1.5. Set the value of `dual` explicitly to suppress the warning.
 warnings.warn(
/home/matty/.local/lib/python3.11/site-packages/sklearn/svm/_classes.py:31:
FutureWarning: The default value of `dual` will change from `True` to `'auto'`

```
in 1.5. Set the value of `dual` explicitly to suppress the warning.
       warnings.warn(
     32.35
     16.11
     /home/matty/.local/lib/python3.11/site-packages/sklearn/svm/_classes.py:31:
     FutureWarning: The default value of `dual` will change from `True` to `'auto'`
     in 1.5. Set the value of `dual` explicitly to suppress the warning.
       warnings.warn(
     11.12
     /home/matty/.local/lib/python3.11/site-packages/sklearn/svm/_classes.py:31:
     FutureWarning: The default value of `dual` will change from `True` to `'auto'`
     in 1.5. Set the value of `dual` explicitly to suppress the warning.
       warnings.warn(
     8.17
     Error visualization
[54]: # to get scores for logistic regression use: scores = model_lr.
       \hookrightarrow predict_proba(x_test)
      def makeplots(scores):
        mostcorrect = np.empty((10, x_test.shape[1]))
        mostwrong = mostcorrect.copy()
        for i in range(10):
          print(f"idx {i}")
          probs = scores[:,i]
          islabel = (y_test == i).astype(int)
          maxidx = (probs * islabel).argmax()
          minidx = ((1-probs) * islabel).argmax()
          print(probs[maxidx], probs[minidx])
          # display_mnist(x_test[minidx])
          # display mnist(x test[maxidx])
          mostcorrect[i] = x_test[maxidx]
          mostwrong[i] = x_test[minidx]
        display mnist(mostcorrect, subplot cols=10)
        display_mnist(mostwrong, subplot_cols=10)
      makeplots(LLR.predict_proba(x_test))
      # to get scores for SVM use: scores = model sum.decision_function(x test)
      makeplots(SVM.decision_function(x_test))
     idx 0
     0.999999963899942 0.00025819211990930365
     idx 1
     0.9996567977883941 3.0877626333995e-07
```

- idx 2
- $\tt 0.999999744316168\ 8.143784266293726e-08$
- idx 3
- 0.9999989329684581 2.559199561611065e-05
- idx 4
- 0.999997898596669 1.332051810719343e-07
- idx 5
- 0.9999795469879909 0.00011521805519352722
- idx 6
- 0.9999992170012151 9.196212267621756e-05
- idx 7
- 0.9999978520581978 2.8968218320809687e-06
- idx 8
- 0.9999939555425639 2.7513009347682197e-05
- idx 9
- 0.9999710313820391 1.0870533123166027e-07



- idx 0
- 4.61895263661966 -2.6070372181055292
- idx 1
- 2.4273329555515355 -5.02282639157594
- idx 2
- 4.3457220804306536 -4.503737615673133
- idx 3
- 4.294225965182193 -4.2211613082993695
- idx 4
- 3.769328884272958 -4.4923061471241255
- idx 5
- 4.0326785525818565 -4.385248350034097
- idx 6
- 3.9589625796426087 -3.189884761323814
- idx 7
- 5.565132589995729 -4.641314083206345
- idx 8
- 2.6371411849356683 -2.8530038389350647

3.0858680559435063 -4.077201020006369





Parameter selection

```
[255]: cs = []
       errs = []
[264]: | # Try multiple C parameters, select one that minimizes validation error
       # Often, you need to try a few values and see those results to determine what
       ⇔other values to try
       def geterr(c):
         model = LinearSVC(C=c, dual='auto').fit(x_train[:1000], y_train[:1000])
         pred = model.predict(x_train[50000:])
         err = (pred != y_train[50000:]).mean()
         print(c, err)
         cs.append(c)
         errs.append(err)
         return err
       minval = 1000
       minc = -1
       for c in np.linspace(0.0185,0.0199,10):
         err = geterr(c)
         if err < minval:</pre>
           minval = err
          minc = c
       print(minc, minval)
```

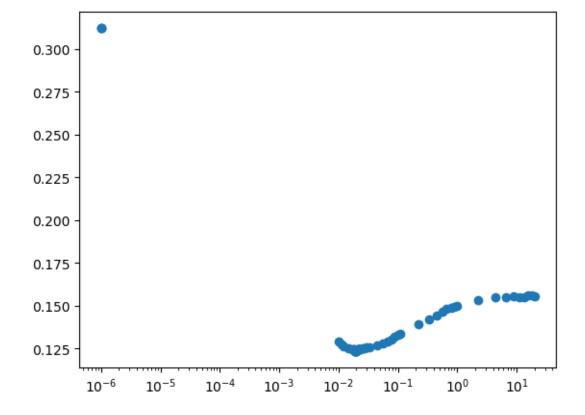
- 0.0185 0.1239
- 0.01865555555555554 0.1238

```
0.01896666666666666 0.1236
0.019122222222222 0.1237
0.0192777777777778 0.1238
0.019433333333333334 0.1237
0.0195888888888889 0.1235
0.019744444444444446 0.1237
0.0199 0.1238
0.01958888888888889 0.1235
```

```
[265]: # Get test result for selected parameter
plt.semilogx(cs,errs, marker='o', linestyle='')
# TO DO

c = cs[np.argmin(errs)]
print(min(errs))
model = LinearSVC(C=c, dual='auto').fit(x_train[:1000], y_train[:1000])
pred = model.predict(x_test)
err = (pred != y_test).mean()
print(err)
```

0.1235 0.1358



0.2 Part 3: Temperature Regression

```
[125]: import numpy as np
       # from google.colab import drive
       %matplotlib inline
       from matplotlib import pyplot as plt
       from sklearn.linear_model import Ridge
       from sklearn.linear_model import Lasso
       # load data (modify to match your data directory or comment)
       def load temp data():
         # drive.mount('/content/drive')
         # datadir = "/content/drive/My Drive/CS441/24SP/hw1/"
        datadir = './'
         T = np.load(datadir + 'temperature data.npz')
         x_train, y_train, x_val, y_val, x_test, y_test, dates_train, dates_val,_

dates_test, feature_to_city, feature_to_day = \

        T['x_train'], T['y_train'], T['x_val'], T['y_val'], T['x_test'], T['y_test'],
        GT['dates_train'], T['dates_val'], T['dates_test'], T['feature_to_city'], μ

¬T['feature_to_day']
        return (x_train, y_train, x_val, y_val, x_test, y_test, dates_train,_
        dates_val, dates_test, feature_to_city, feature_to_day)
       # plot one data point for listed cities and target date
       def plot_temps(x, y, cities, feature_to_city, feature_to_day, target_date):
        nc = len(cities)
        ndays = 5
         xplot = np.array([-5, -4, -3, -2, -1])
         yplot = np.zeros((nc,ndays))
         for f in np.arange(len(x)):
           for c in np.arange(nc):
             if cities[c] == feature to city[f]:
               yplot[feature_to_day[f]+ndays,c] = x[f]
        plt.plot(xplot,yplot)
        plt.legend(cities)
        plt.plot(0, y, 'b*', markersize=10)
        plt.title('Predict Temp for Cleveland on ' + target_date)
        plt.xlabel('Day')
        plt.ylabel('Avg Temp (C)')
        plt.show()
```

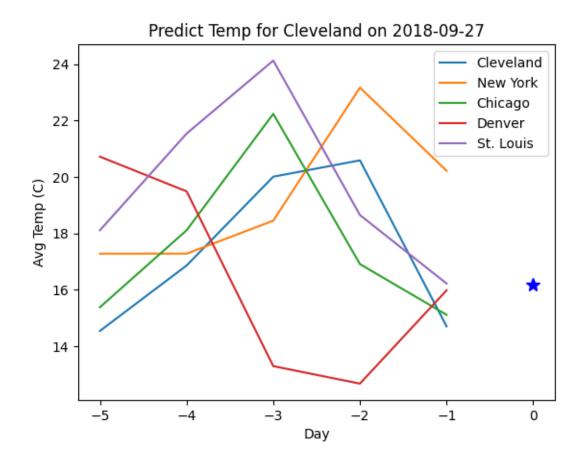
```
x val, y val: features and target value for each validation sample (used \Box
 ⇒to select hyperparameters, such as regularization and K)
      x_{test}, y_{test}: features and target value for each test sample (used to \Box
 ⇔evaluate final performance)
      dates_xxx: date of the target value for the corresponding sample
      feature_to_city: maps from a feature number to the city
      feature_to_day: maps from a feature number to a day relative to the __
 ⇒target value, e.g. -2 means two days before
     Note: 361 is the temperature of Cleveland on the previous day
f = 361
print('Feature {}: city = {}, day= {}'.format(f,feature_to_city[f],__

¬feature_to_day[f]))
baseline_rmse = np.sqrt(np.mean((y_val[1:]-y_val[:-1])**2)) # root mean squared_
 \rightarrow error
print('Baseline - prediction using previous day: RMSE={}'.format(baseline_rmse))
# plot first two x/y for val
plot_temps(x_val[0], y_val[0], ['Cleveland', 'New York', 'Chicago', 'Denver', _

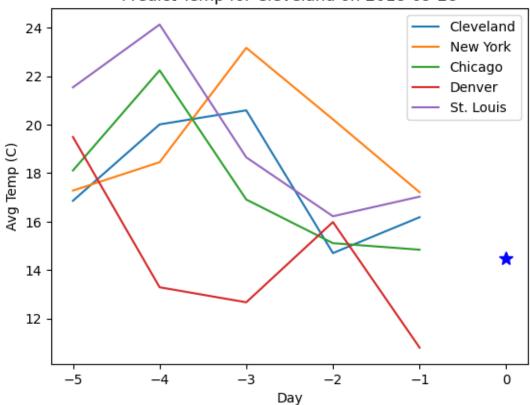
¬'St. Louis'], feature_to_city, feature_to_day, dates_val[0])

plot_temps(x_val[1], y_val[1], ['Cleveland', 'New York', 'Chicago', 'Denver', _
```

Feature 361: city = Cleveland, day= -1
Baseline - prediction using previous day: RMSE=3.460601246750482







Linear regression test

```
[135]: # linear regression (use Ridge)

# original features
model = Ridge().fit(x_train,y_train)
pred = model.predict(x_test)
```

```
print(np.sqrt(np.square(pred-y_test).mean()))

# normalized features
fnum=361
xnorm_train,ynorm_train = normalize_features(x_train,y_train,fnum)
xnorm_test, ynorm_test = normalize_features(x_test,y_test,fnum)

y_pred = Ridge().fit(xnorm_train,ynorm_train).predict(xnorm_test)

print(np.square(y_pred - ynorm_test).mean()**0.5)
# TO DO
```

- 2.1608605260795755
- 2.1630698027575606

Feature selection

```
[169]: # feature analysis (select important features using Lasso)
# TO DO
model = Lasso().fit(x_train,y_train)
fidx = (np.abs(model.coef_)>0.001)
fidx = np.arange(x_train.shape[1])[fidx]

import pandas as pd
df = pd.DataFrame({
  'coef': np.abs(model.coef_[fidx]),
        'fnum':fidx,
  'city':feature_to_city[fidx],
        'day':feature_to_day[fidx],
}
df = df.sort_values('coef',ascending=False)
df
```

```
[169]:
              coef fnum
                                 city day
          0.279599
                    334
                              Chicago
                                       -1
      6
      7
          0.225942
                    347
                          Minneapolis
                                       -1
      12 0.180130
                    405 Grand Rapids
                                       -1
      10 0.162950
                    366
                          Kansas City
                                       -1
      9
          0.138357
                    361
                            Cleveland
                                       -1
          0.111248
                                Omaha
                    307
                                       -2
      11 0.046103
                    367 Indianapolis
                                       -1
          0.043280
      3
                    264
                          Minneapolis
                                       -2
      0
          0.025518
                     9
                               Boston
                                       -5
      2 0.025198
                    236
                          Springfield
                                       -3
                           Providence
                                       -2
      4
          0.015140
                    290
          0.005399
                               Boston
                    175
                                       -3
```

```
8 0.001785 354 St. Louis -1
```

```
[267]: # predict using best features
idxs = df['fnum'][:10].values
model = Ridge().fit(x_train[:,idxs],y_train)
pred = model.predict(x_test[:,idxs])
print(np.sqrt(np.square(pred-y_test).mean()))
```

2.7350935573443653

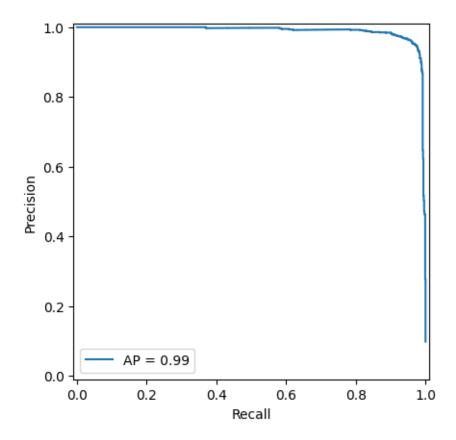
0.3 Part 4: Stretch Goals

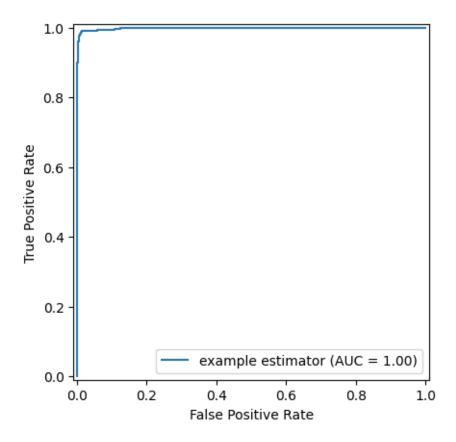
Include all your code used for any stretch goals in this section. Add headings where appropriate.

1 4a

```
[192]: probs = model.predict_proba(x_test)[:,1]
       average_precision = average_precision_score(y_test, probs)
       print('Average Precision (AP):', average_precision)
       auroc = roc_auc_score(y_test,probs)
       print('Area under the ROC curve (AuROC):', auroc)
       precision, recall, _ = precision_recall_curve(y_test, probs)
       disp = PrecisionRecallDisplay(precision=precision, recall=recall,_
        →average_precision=average_precision)
       disp.plot()
       precision, recall, _ = precision_recall_curve(y_test, probs)
       fpr, tpr, thresholds = metrics.roc_curve(y_test, probs)
       auroc = roc_auc_score(y_test,probs)
       display = metrics.RocCurveDisplay(fpr=fpr, tpr=tpr,
        →roc_auc=auroc,estimator_name='example estimator')
       display.plot()
       fpr, tpr, _ = roc_curve(y_test, probs)
```

Average Precision (AP): 0.9888354276370754 Area under the ROC curve (AuROC): 0.9983122539481425





2 4b

```
[212]: (x_train, y_train), (x_test, y_test) = load_mnist()

def go(model):
    model.fit(x_train[:1000],y_train[:1000])
    display_mnist(model.coef_, subplot_cols=10)

go(LogisticRegression(penalty='l1',solver='liblinear'))
go(LogisticRegression(penalty='l2',solver='liblinear'))
go(LogisticRegression(penalty='elasticnet',solver='saga',l1_ratio=0.5))
go(LinearSVC())
```









































/home/matty/.local/lib/python3.11/sitepackages/sklearn/linear_model/_sag.py:350: ConvergenceWarning: The max_iter was
reached which means the coef_ did not converge
 warnings.warn(





















/home/matty/.local/lib/python3.11/site-packages/sklearn/svm/_classes.py:31:
FutureWarning: The default value of `dual` will change from `True` to `'auto'`
in 1.5. Set the value of `dual` explicitly to suppress the warning.
warnings.warn(















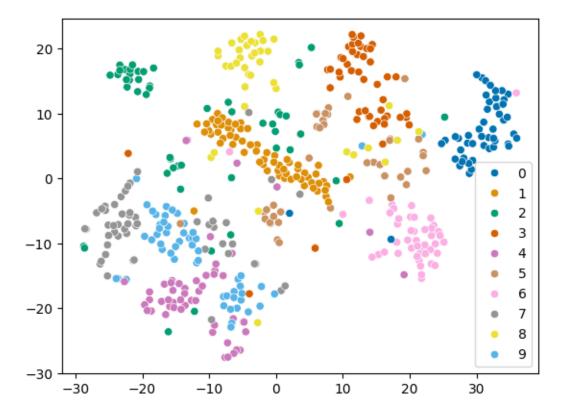






3 4c

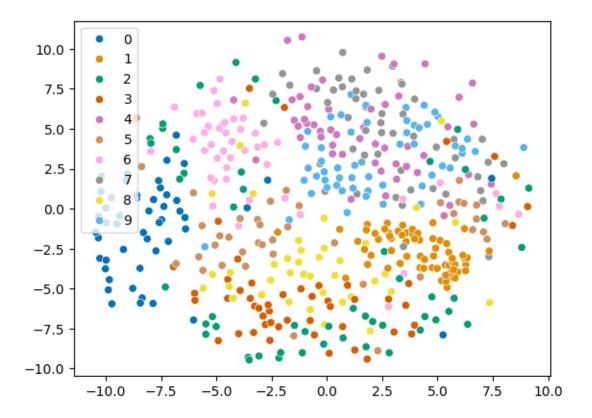
[218]: <Axes: >



```
[217]: sns.scatterplot(x=X_mds[:, 0], y=X_mds[:, 1], hue=y_train[:500], u

→palette="colorblind")
```

[217]: <Axes: >



4 4D

```
[246]: # load data
       (x_train, y_train, x_val, y_val, x_test, y_test, dates_train, dates_val, __
        dates_test, feature_to_city, feature_to_day) = load_temp_data()
       def RMSE(a,b):
         return np.sqrt(np.square(a-b).mean())
       errs = []
       for i in range(83):
         x = x_train[:, i:415:83]
        xv = x_val[:,i:415:83]
        model = Ridge().fit(x, y_train)
        pred = model.predict(xv)
        err = RMSE(pred, y_val)
         errs.append(err)
       imin = np.argmin(errs)
       print(imin, feature_to_city[imin])
       x = x_{train}[:, imin:415:83]
```

```
model = Ridge().fit(x, y_train)
pred = model.predict(x_test[:,imin:415:83])
err = RMSE(pred, y_test)
print(err)

22 St. Louis
3.1263272970544764
```

5 4e

```
[220]: from sklearn.svm import SVC
       # SVM
       for n in [100, 1000, 10000, 60000]:
        SVM = SVC(max_iter = 10000).fit(x_train[:n],y_train[:n])
        pred = SVM.predict(x_test)
         err = 100*(pred != y_test).mean()
         print(err)
      34.410000000000004
      9.17
      4.06
      2.08
 []: # from https://gist.github.com/jonathanagustin/b67b97ef12c53a8dec27b343dca4abba
       # install can take a minute
       import os
       # @title Convert Notebook to PDF. Save Notebook to given directory
       NOTEBOOKS DIR = "/content/drive/My Drive/CS441/24SP/hw2" # @param {type:
       ⇔"string"}
       NOTEBOOK NAME = "CS441_SP24_HW2_Solution.ipynb" # @param {type:"string"}
       from google.colab import drive
       drive.mount("/content/drive/", force_remount=True)
       NOTEBOOK_PATH = f"{NOTEBOOKS_DIR}/{NOTEBOOK_NAME}"
       assert os.path.exists(NOTEBOOK_PATH), f"NOTEBOOK_NOT FOUND: {NOTEBOOK_PATH}"
       !apt install -y texlive-xetex texlive-fonts-recommended texlive-plain-generic >∪
       →/dev/null 2>&1
       !jupyter nbconvert "$NOTEBOOK_PATH" --to pdf > /dev/null 2>&1
       NOTEBOOK_PDF = NOTEBOOK_PATH.rsplit('.', 1)[0] + '.pdf'
       assert os.path.exists(NOTEBOOK_PDF), f"ERROR MAKING PDF: {NOTEBOOK_PDF}"
       print(f"PDF CREATED: {NOTEBOOK_PDF}")
```

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