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CS 441 - HW3: PDFs and Outliers

Complete the sections below. You do not need to fill out the checklist.

Total Points Available	**[ ] / 160**

1. Estimating PDFs
   a. Segmentation with per-channnel PDFs [ ] / 15
   b. Segmentation with clustered value PDFs [ ] / 15
   c. Segmentation with GMMs [ ] / 20
2. Robust Estimation
   a. Assume no noise [ ] / 10
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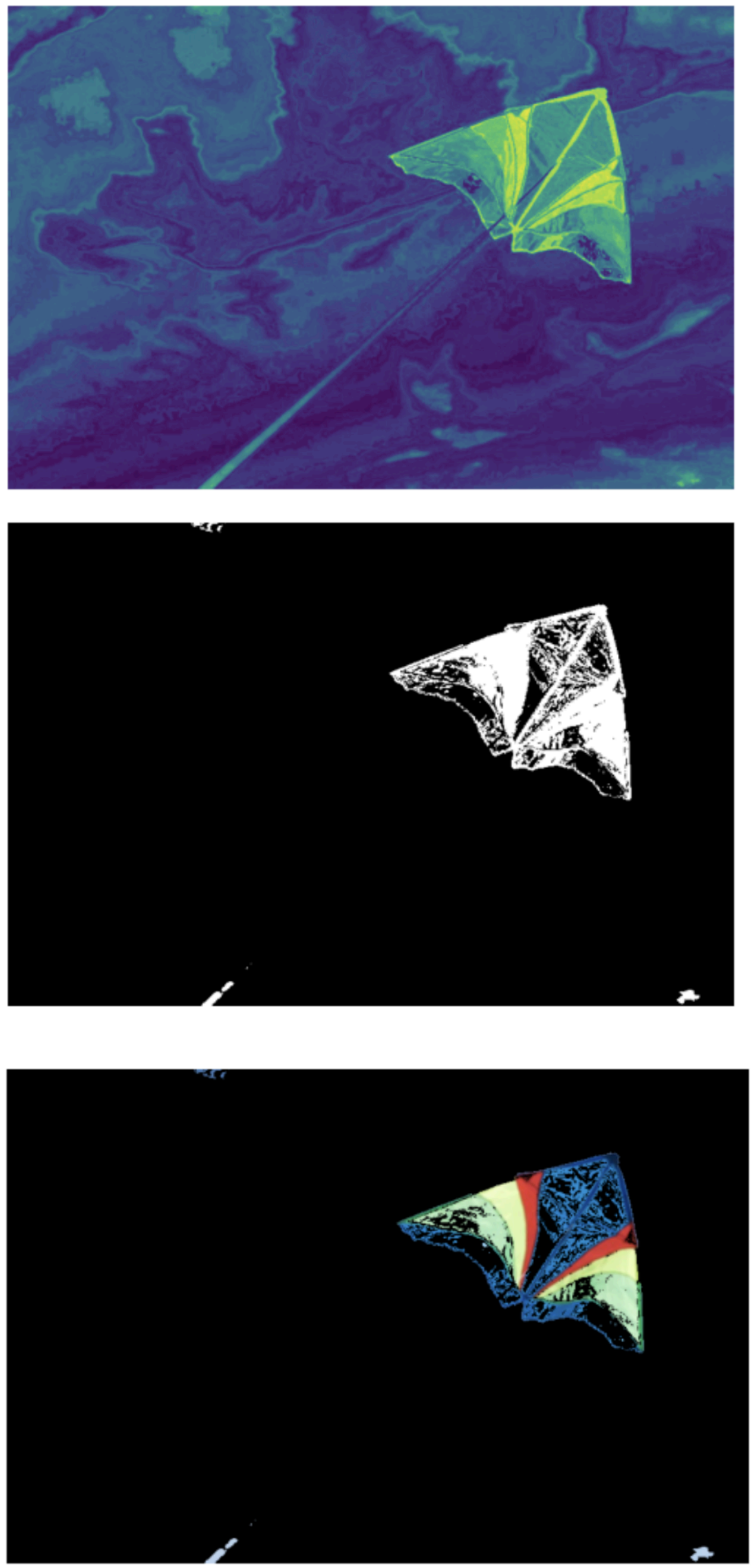
3. Stretch Goals
   a. Impact of school on salary [ ] / 20
   b. Impact of experience on salary [ ] / 20
   c. Mutual information: discrete pdf [ ] / 10
   d. Mutual information: GMM [ ] / 10

1. Estimating PDFs

Include the generated images (score map and thresholded RGB) from the display code. List any parameters.

Note: Not sure exactly which images I should include, so I included all the displayed images by the helper function.

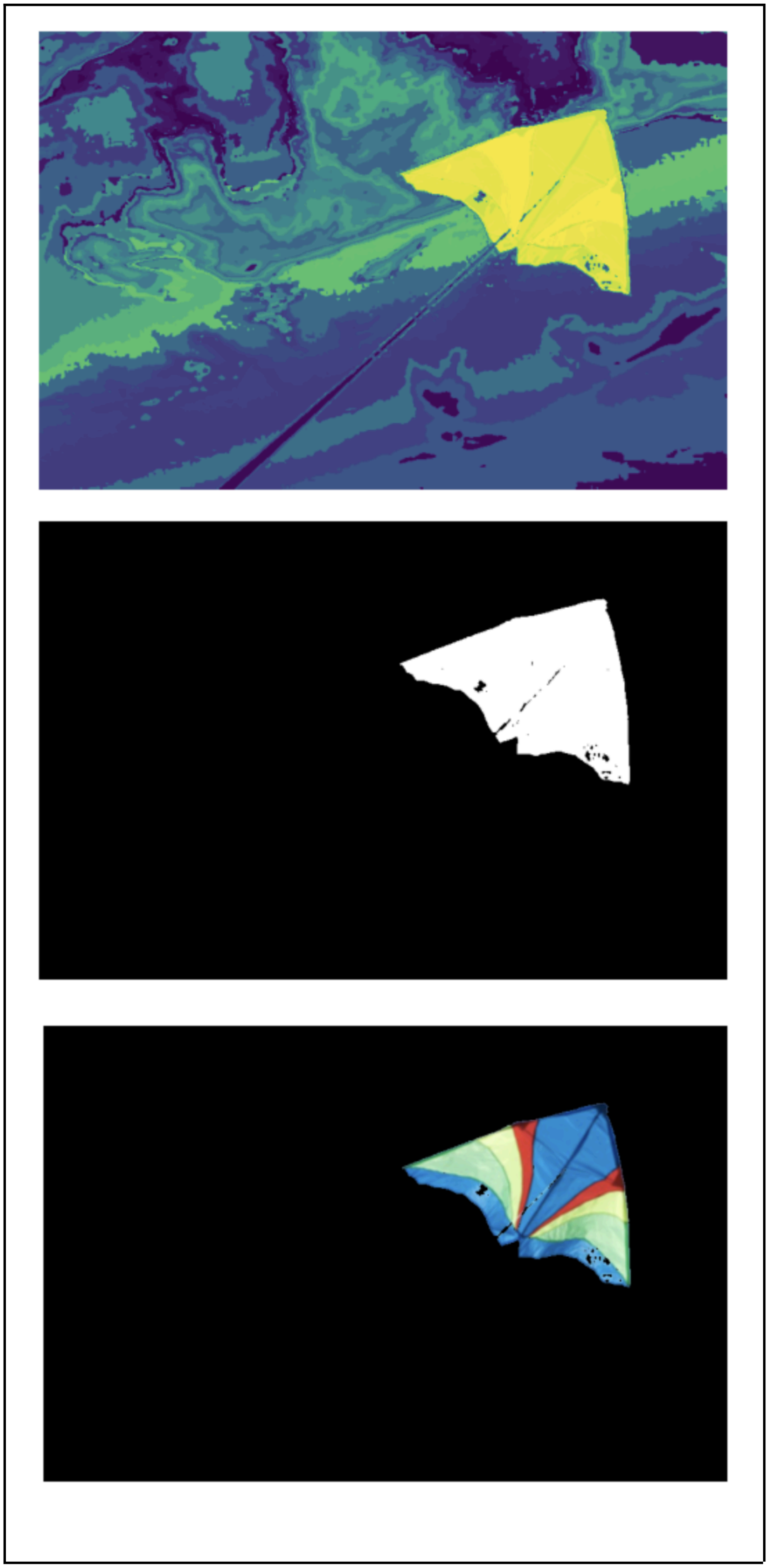
a. Method 1 (Per-channel discrete):



Number of bins / discrete values per channel, threshold

Nbins = 256
Threshold = 2

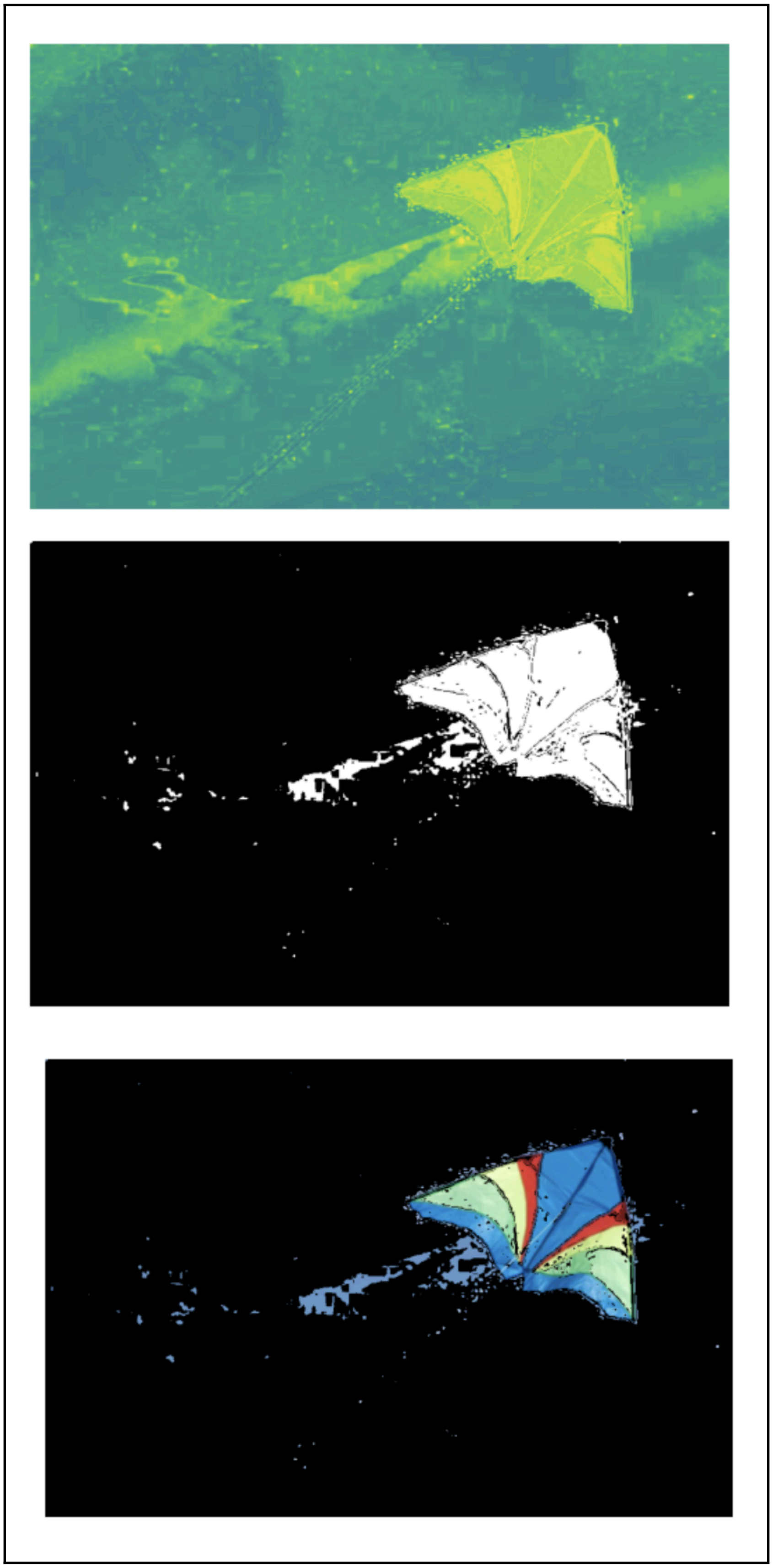
b. Method 2 (Clustering, discrete):



Number of clusters, threshold

Clusters= 64
Threshold = 1

c. Method 3 (Gaussian Mixture Model):



Number of components, variance model, threshold

Components: 10 Covariance: full Threshold: 1.25

2. Robust Estimation

Round to nearest whole number.

	a. No noise	b. Percentiles	c. EM
Min	64694	75494	64694
Mean	123750	113879	111984
Std	61954	15876	17966
Max	611,494	159,901	169,008

First five indices of invalid data (based on EM solution, you add last 3)

18	28	49	127	128

3. Stretch Goals

a. Impact of school on salary

Report mean salary overall and for each school

	Average Salary
Overall	
School 0 (UIUC)	
School 1 (MIT)	
School 2 (Cornell)	

Describe your approach to estimate this.

b. Impact of years of experience on salary

How much are salaries expected to increase with one year of experience?

$ 1107.17

Describe your approach to estimate this.

I used the EM algorithm from the previous part to filter out samples which were likely to be invalid. To account for school, I added the school as a numerical feature (-1,0,1,2). Then I trained a linear regressor and took the coefficient for the experience feature, since this represents the change in salary per change in years of experience.

c. Mutual information of sex and age, discrete approach

Mutual information (base natural log)

0.092593

d. Mutual information of sex and age, GMM approach

Mutual information (base natural log)

0.02427

Acknowledgments / Attribution

Code for EM is taken from lecture slides and adapted to the problem.

CS441_SP24_HW3_Starter

March 6, 2024

0.1 CS441: Applied ML - HW 3

0.1.1 Part 1: Estimating PDFs

[34]:
```
# initalization code

import numpy as np
from matplotlib import pyplot as plt
%matplotlib inline
# from google.colab import drive
# from google.colab.patches import cv2_imshow
import cv2

# read images
# drive.mount('/content/drive')
# datadir = "/content/drive/My Drive/CS441/24SP/hw3/"
datadir = "./"
im = cv2.imread(datadir + 'kite.jpg')  # this is the full image
im = cv2.cvtColor(im, cv2.COLOR_BGR2RGB)/255
im = cv2.blur(im, (3, 3)).clip(0,1)


crop = cv2.imread(datadir + 'kite_crop.jpg')  # this is the cropped image
crop = cv2.cvtColor(crop, cv2.COLOR_BGR2RGB)/255
crop = cv2.blur(crop, (3, 3)).clip(0,1)

# displays a single image
def display_image(im):
  plt.imshow(im)
  plt.axis('off')
  plt.show()

# displays the image, score map, thresholded score map, and masked image
def display_score(im, score_map, thresh):
  display_image(im)
  display_image(np.reshape(score_map, (im.shape[:2])))
  plt.imshow(np.reshape(score_map>thresh, (im.shape[0], im.shape[1])),
 ↪cmap='gray')
```

```
  plt.axis('off')
  plt.show()
  display_image(np.tile(np.reshape(score_map>thresh, (im.shape[0], im.shape[1],␣
 ↪1)), (1,1,3))*im)

print('Whole image')
display_image(im)

print('Foreground')
display_image(crop)
```

```
Whole image
```

```
Foreground
```

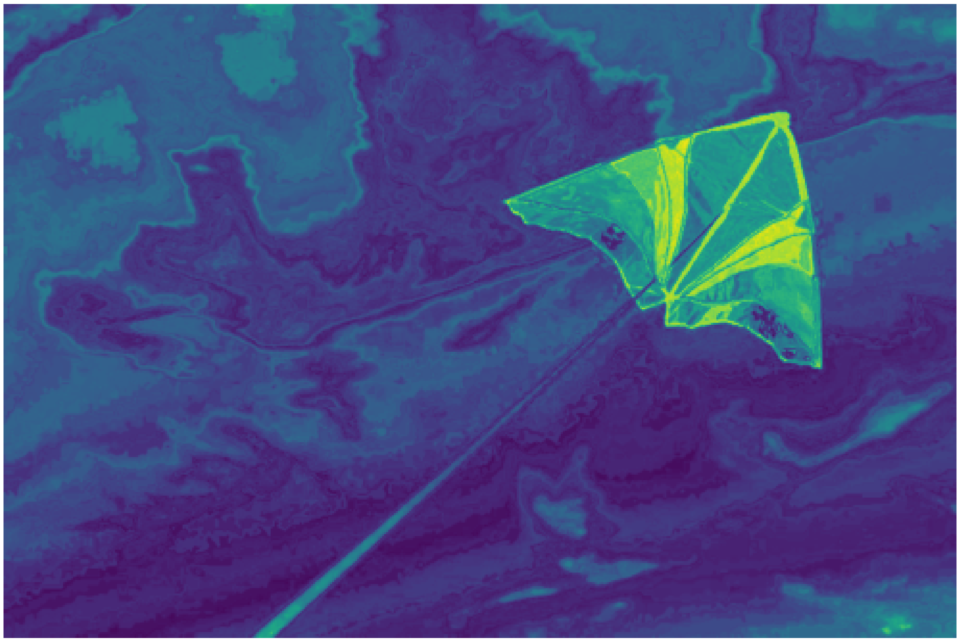
Method 1 (per channel hist)

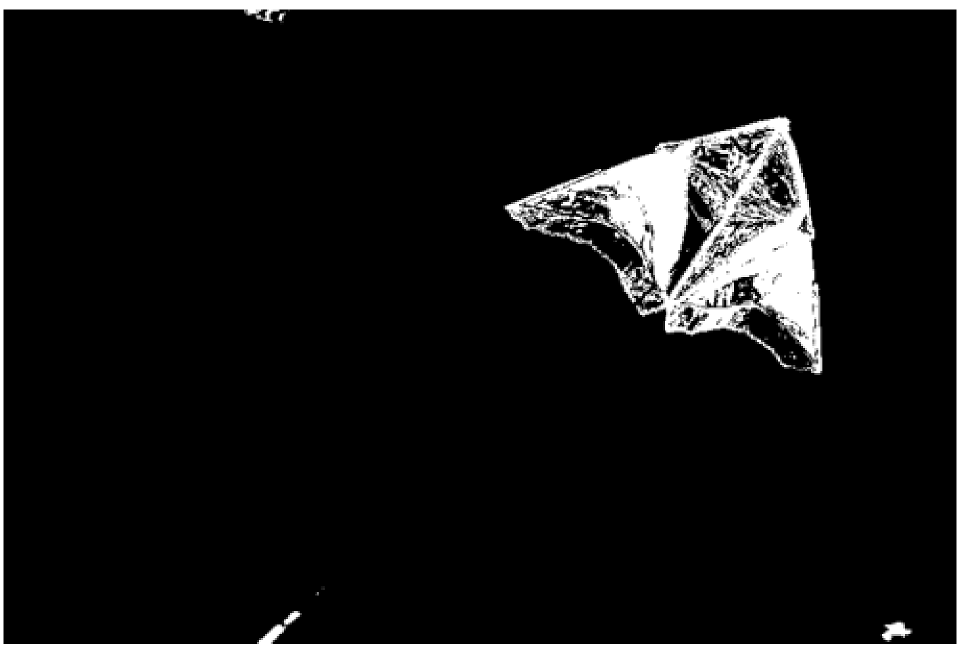
[32]:
```
# estimate discrete pdf
def estimate_discrete_pdf(values, nvalues, prior=1):
  '''
    Estimate P(values=v) for each possible v in (0, nvalues)
    Input:
       values: the values of the data
       nvalues: range of values, such that 0 <= values < nvalues
       prior: initial count used to prevent any value from having zero␣
 ↪probability
    Output:
       p[nvalues,]: P(values=v) for each v
  '''
  hist, edges = np.histogram(values,bins=nvalues)
  hist = np.clip(hist, prior,None)
  return hist / len(values)
```

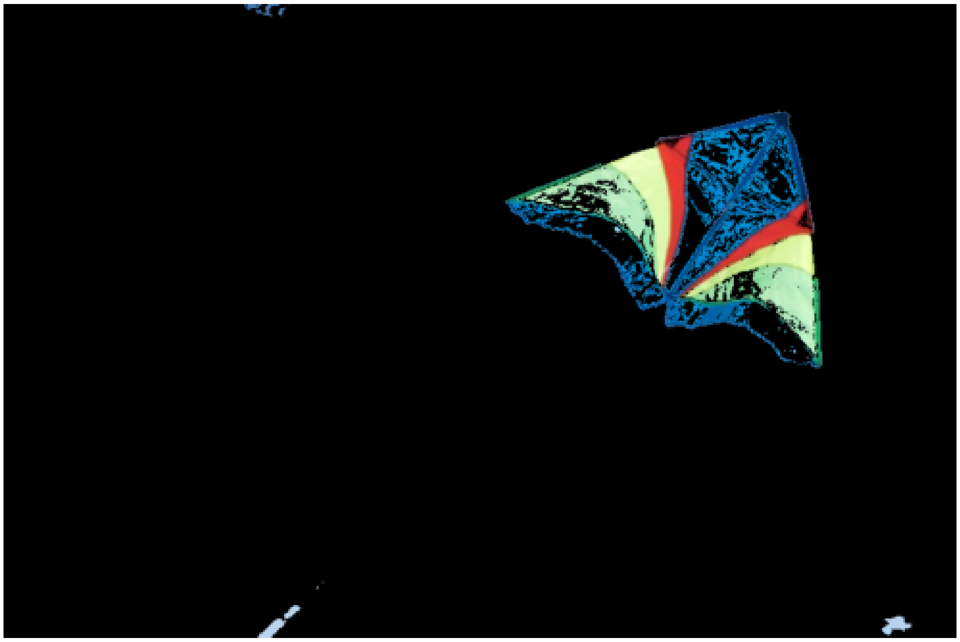
[33]:
```
# reshape so number of rows is number of pixels and number of columns is 3 (for␣
 ↪RGB)
im_3 = np.reshape(im, (im.shape[0]*im.shape[1], 3))
crop_3 = np.reshape(crop, (crop.shape[0]*crop.shape[1], 3))

print(im_3.shape,crop_3.shape)
# estimate PDFs and compute score per pixel
bins = 256
```

```
idcs = (im_3 * bins).astype(int).clip(0,bins-1)

pdf_im = estimate_discrete_pdf(im_3[:,0],bins)[idcs[:,0]]
pdf_im *= estimate_discrete_pdf(im_3[:,1],bins)[idcs[:,1]]
pdf_im *= estimate_discrete_pdf(im_3[:,2],bins)[idcs[:,2]]


pdf_crop = estimate_discrete_pdf(crop_3[:,0],bins)[idcs[:,0]]
pdf_crop *= estimate_discrete_pdf(crop_3[:,1],bins)[idcs[:,1]]
pdf_crop *= estimate_discrete_pdf(crop_3[:,2],bins)[idcs[:,2]]


score = np.log(pdf_crop/pdf_im)
t = 2
display_score(im, score_map=score, thresh=t)
```

```
(425068, 3) (26999, 3)
```





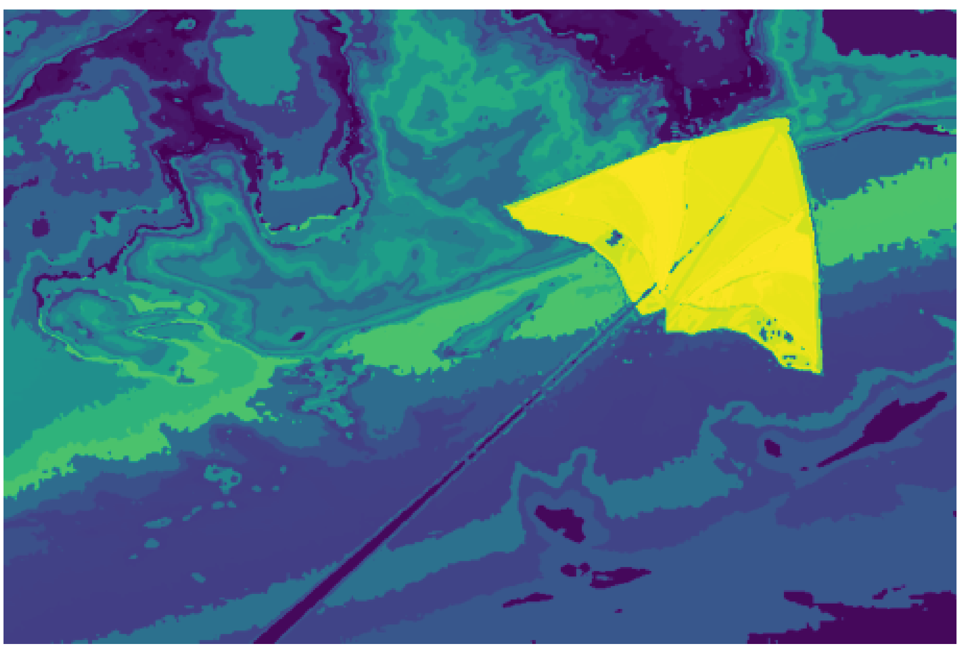


Method 2 (Kmeans)

[35]:
```
# init
# !apt install libomp-dev > /dev/null 2>&1
# !pip install faiss-cpu > /dev/null 2>&1
import faiss
```

[36]:
```
def kmeans_fast(X, K, niter=10):
    X = X.astype(np.float32)
    d = X.shape[1]
    cluster_centers = X[:K].copy()
    index = faiss.IndexFlatL2(d)

    rmse_list = []

    for i in range(niter):
        index.reset()
        index.add(cluster_centers)
        D, I = index.search(X, 1)  # D is the squared distances

        for k in range(K):
            points_in_cluster = X[I.squeeze() == k]
            if len(points_in_cluster) > 0:
                cluster_centers[k] = np.mean(points_in_cluster, axis=0)

    return cluster_centers, rmse_list
```

[37]:
```
# reshape so number of rows is number of pixels and number of columns is 3 (for␣
↪RGB)
im_3 = np.reshape(im, (im.shape[0]*im.shape[1], 3))
crop_3 = np.reshape(crop, (crop.shape[0]*crop.shape[1], 3))



# estimate PDFs and compute score per pixel

K = 64

# Apparently this shit is wrong
# def get_pdf(img):
#   centers, _ = kmeans_fast(img,K,niter=100)
#   index = faiss.IndexFlatL2(img.shape[1])
#   index.add(centers)
#   D, I = index.search(im_3, 1)
#   I = I.squeeze()
#   freqs = (np.bincount(I)) / len(I)
#   return freqs[I]
# pdf_crop = get_pdf(crop_3)
# pdf_im = get_pdf(im_3)

centers, _ = kmeans_fast(im_3,K,niter=100)
index = faiss.IndexFlatL2(im_3.shape[1])
index.add(centers)
D, I_im = index.search(im_3, 1)
I_im = I_im.squeeze()
freqs = (1+np.bincount(I_im)) / len(I_im)

pdf_im = freqs[I_im]

D, I = index.search(crop_3, 1)
I = I.squeeze()
freqs = (1+np.bincount(I)) / len(I)

pdf_crop = freqs[I_im]




score = np.log(pdf_crop/pdf_im)
t= 1
display_score(im=im, score_map=score, thresh=t)
```



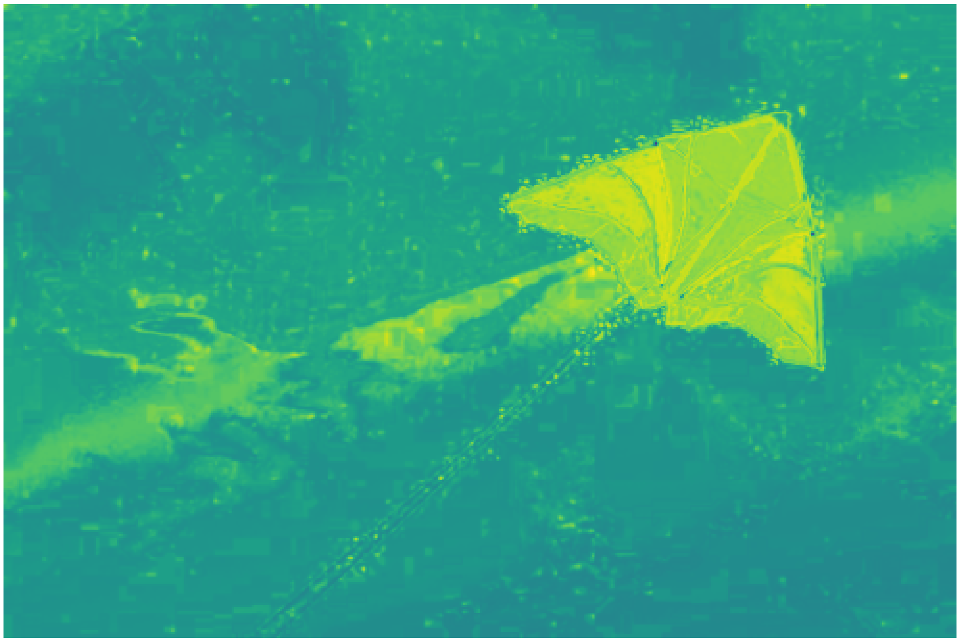
Method 3 (GMM)

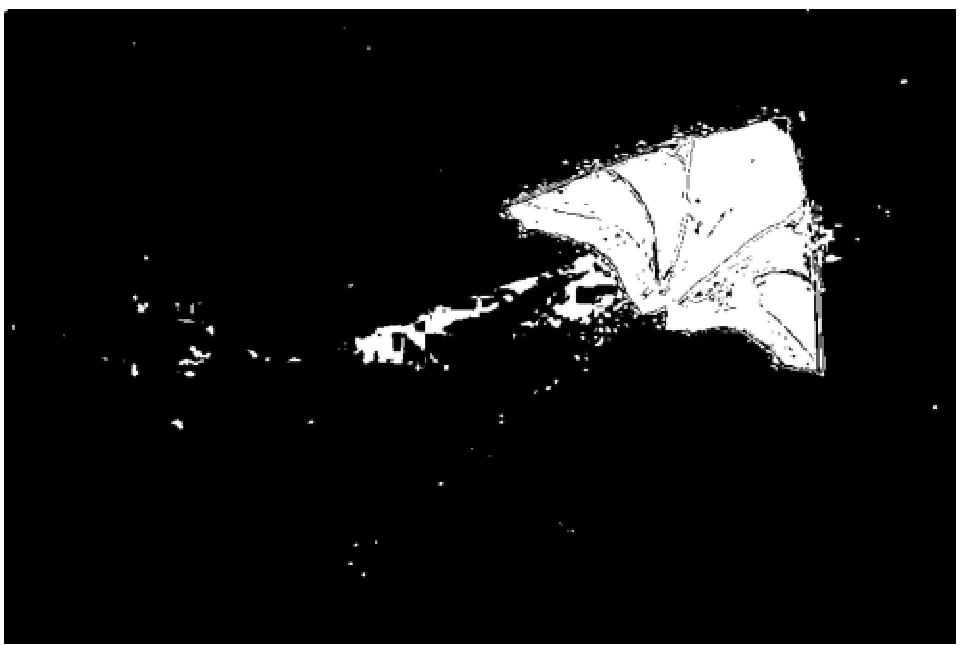
[43]:
```
from sklearn.mixture import GaussianMixture
```

```
# reshape so number of rows is number of pixels and number of columns is 3 (for␣
↪RGB)
im_3 = np.reshape(im, (im.shape[0]*im.shape[1], 3))
crop_3 = np.reshape(crop, (crop.shape[0]*crop.shape[1], 3))


# estimate PDFs and compute score per pixel (can use gmm.score_samples where␣
↪gmm is the mixture model)
K = 10
covariance_type='full'
gm = GaussianMixture(n_components=K,␣
↪random_state=0,covariance_type=covariance_type).fit(im_3)
```

[86]:
```
gm2 = GaussianMixture(n_components=K, random_state =␣
↪0,covariance_type=covariance_type).fit(crop_3)
```

[92]:
```
log_pdf_crop = gm2.score_samples(im_3)
log_pdf_im = gm.score_samples(im_3)
score = log_pdf_crop - log_pdf_im
t= 1.25
display_score(im=im, score_map=score, thresh=t)
```





0.2 Part 2: Robust Estimation

[2]:
```
import numpy as np
from matplotlib import pyplot as plt
# from google.colab import drive

# drive.mount('/content/drive')
# datadir = "/content/drive/My Drive/CS441/24SP/hw3/"

datadir = "./"

# load data
T = np.load(datadir + 'salary.npz')
(salary, years, school) = (T['salary'], T['years'], T['school'])
```

1. No noise Compute the statistics for the data as a whole

[3]:
```
# TO DO
salary_mu = salary.mean()
salary_std = np.std(salary)
salary_min = salary.min()
salary_max = salary.max()
print('Mean: {}  Std: {}  Min: {}   Max: {}'.format(salary_mu, salary_std,␣
↪salary_min, salary_max))
```

```
Mean: 123749.835  Std: 61953.77348723623  Min: 64694.0   Max: 611494.0
```

2. Percentiles Assume valid data will fall between the 5th and 95th percentile.

[4]:
```python
pct = 0.05

percentile_5 = np.percentile(salary, 5)
percentile_95 = np.percentile(salary, 95)

filtered_data = salary[(salary >= percentile_5) & (salary <= percentile_95)]

salary_mu = filtered_data.mean()
salary_std = np.std(filtered_data)
salary_min = np.percentile(salary, 5) - (np.percentile(salary, 95)-np.
 ↪percentile(salary, 5))*(0.05/0.9)
salary_max = np.percentile(salary, 95) + (np.percentile(salary, 95)-np.
 ↪percentile(salary, 5))*(0.05/0.9)
# TO DO

print('Mean: {}  Std: {}  Min: {}   Max: {}'.format(salary_mu, salary_std,␣
 ↪salary_min, salary_max))
```

```
Mean: 113878.65  Std: 15876.450453939286  Min: 75493.8   Max: 159900.79999999973
```

3. EM Assume valid data follows a Gaussian distribution, while the fake data has a uniform distribution between the minimum and maximum value of salary.

[13]:
```python
niter = 20

# initialize by assuming that all scores are good
N = 1
M = len(salary)
print(M)
scores = salary.reshape((N,M))
score_mean = scores.mean(axis=1).reshape((len(scores), 1))  # mu_i
score_std = np.sqrt(np.sum((scores-score_mean)**2, axis=None)/N/M)  # sigma

pz = 0.5 # P(z=1) = 0.5 initially

# print initial estimate
# plot_est(true_score, score_mean)
```

```
200
```

[20]:
```python
for t in range(niter):
  last_mean = score_mean.copy()

  # E-step
```

```
  # update probability that each annotator is good
  p_good = np.zeros((scores.shape[1],1))  # w_a = P(z_a=1 | scores, theta_t)
  for a in range(M):
    p_s_good = pz  # P(s_ia | z=1, mu_i, std)P(z_a=1)
    p_s_bad = 1-pz # P(s_ia | z=0)P(z_a=0)
    for i in range(N):
      p_s_good *= 1/np.sqrt(2*np.pi)/score_std * np.exp(-1/2 *␣
↪(scores[i,a]-score_mean[i])**2/score_std**2)
      p_s_bad *= 1/(salary.max()-salary.min()) # uniform in range [0, 10]
    p_good[a] = p_s_good / (p_s_good + p_s_bad)

  print('\niter {}'.format(t))
  print(np.round(p_good.transpose()*1000)/1000)

  # M-step
  # assign parameters that maximize likelihood under latent variable likelihoods
  for i in range(N):
    # estimate mean for each image
    w_score_sum_i = 0
    for a in range(M):
      w_score_sum_i += scores[i,a]*p_good[a]
    score_mean[i] = w_score_sum_i / np.sum(p_good)

  # estimate std
  w_sqdiff_sum = 0
  for i in range(N):
    for a in range(M):
      w_sqdiff_sum += p_good[a]*(scores[i,a] - score_mean[i])**2
  score_std = np.sqrt(w_sqdiff_sum / np.sum(p_good) / N)

  # estimate pz
  pz = np.mean(p_good)
  # plot_est(true_score, score_mean)
  print('Std: {:0.3f}'.format(score_std[0]))

  if np.all(np.abs(last_mean-score_mean)<0.00001): # check for convergence
    break
```

```
iter 0
[[0.993 0.995 0.995 0.994 0.993 0.995 0.995 0.992 0.994 0.995 0.994 0.986
  0.975 0.995 0.986 0.99  0.995 0.993 0.    0.969 0.988 0.96  0.994 0.995
  0.986 0.995 0.994 0.991 0.    0.967 0.993 0.994 0.995 0.994 0.994 0.995
  0.994 0.98  0.995 0.993 0.995 0.993 0.995 0.962 0.989 0.949 0.994 0.994
  0.994 0.    0.995 0.995 0.995 0.994 0.995 0.995 0.995 0.995 0.995 0.993
  0.995 0.995 0.918 0.86  0.992 0.983 0.995 0.994 0.955 0.995 0.975 0.99
  0.994 0.995 0.995 0.994 0.995 0.981 0.991 0.995 0.994 0.995 0.994 0.995
```

```
 0.994 0.995 0.993 0.993 0.995 0.985 0.987 0.993 0.995 0.918 0.995 0.995
 0.995 0.994 0.989 0.99  0.994 0.993 0.993 0.983 0.991 0.982 0.981 0.995
 0.993 0.989 0.985 0.995 0.994 0.99  0.995 0.995 0.995 0.995 0.994 0.969
 0.99  0.993 0.994 0.995 0.994 0.99  0.991 0.    0.    0.981 0.994 0.994
 0.    0.994 0.993 0.994 0.995 0.995 0.995 0.994 0.995 0.995 0.944 0.993
 0.99  0.994 0.985 0.561 0.984 0.987 0.995 0.995 0.995 0.945 0.993 0.994
 0.994 0.    0.977 0.995 0.995 0.995 0.993 0.993 0.973 0.995 0.992 0.986
 0.994 0.993 0.938 0.995 0.969 0.993 0.    0.982 0.994 0.001 0.995 0.994
 0.994 0.995 0.994 0.992 0.991 0.994 0.994 0.994 0.991 0.988 0.994 0.959
 0.992 0.995 0.983 0.991 0.994 0.994 0.995 0.993]]
Std: 17966.363
```

[36]:
```
goodsals = salary[(p_good >= 0.5).squeeze()]
salary_min = goodsals.min()
salary_max = goodsals.max()


salary_mu = score_mean.item()
salary_std=score_std[0]
```

[37]:
```
# TO DO


print('Mean: {}  Std: {}  Min: {}   Max: {}'.format(salary_mu, salary_std,␣
 ↪salary_min, salary_max))

# print the first five indices of salaries that are not likely to be valid
p_valid_given_s = p_good.copy()
print(np.where(p_valid_given_s<0.5)[0][:5])
```

```
Mean: 111984.38462960183  Std: 17966.36279750227  Min: 64694.0   Max: 169008.0
[ 18  28  49 127 128]
```

0.3 Part 4: Stretch Goals

Include all your code used for any stretch goals in this section. Add headings where appropriate.

1 A

[ ]:
```

```

2 B

[72]:
```
from sklearn.linear_model import LinearRegression
good_entries = (p_good >= 0.5).squeeze()
```

```
X = np.vstack((years,school)).T[good_entries]
Y = salary[good_entries]

model = LinearRegression().fit(X,Y)
print(model.coef_[0])
```

1107.1731484452903

C

[38]:
```
from sklearn.datasets import load_diabetes
X = load_diabetes(as_frame=True,scaled=False)['data']
Y = load_diabetes(as_frame=True,scaled=False)['target']
```

[39]:
```
age = X['age'].values.astype(int)
sex = X['sex'].values.astype(int)
```

[40]:
```
x_size = age.max()+1
y_size = sex.max()+1
probs = np.zeros((x_size,y_size))
for x,y in zip(age,sex):
  probs[x,y]+=1
probs = probs / probs.sum()
py = probs.sum(axis=0)
px = probs.sum(axis=1)
```

[41]:
```
ans =  0
for x in range(x_size):
  for y in range(y_size):
    if probs[x,y]==0:
      continue
    ans += probs[x,y] * np.log(probs[x,y] / px[x] / py[y])
ans
```

[41]: 0.09259264627746933

3 D

[88]:
```
import pandas as pd
df = pd.DataFrame({"age":age,'sex':sex})
ages1 = df[df['sex']==1]['age'].values.reshape(-1,1)
ages2 = df[df['sex']==2]['age'].values.reshape(-1,1)
gm1 = GaussianMixture(n_components=3,␣
 ↪random_state=0,covariance_type=covariance_type).fit(ages1)
gm2 = GaussianMixture(n_components=3,␣
 ↪random_state=0,covariance_type=covariance_type).fit(ages2)
```

```
ps1 = (sex==1).mean()
ps2 = (sex==2).mean()
ages1.size / (ages1.size+ages2.size), ps1
```

[88]: (0.5316742081447964, 0.5316742081447964)

[91]:
```
da = 0.01

ans = 0


for a in np.arange(0,100,da):
  pas1 = np.exp(gm1.score_samples(a.reshape(1,1)))
  pas2 = np.exp(gm2.score_samples(a.reshape(1,1)))

  pa1 = pas1 * ps1
  pa2 = pas2 * ps2

  pa = pa1 + pa2

  ans += pa1 * np.log(pa1/pa/ps1) * da
  ans += pa2 * np.log(pa2/pa/ps2) * da


ans
```

[91]: array([0.0242696])

[78]:
```
# from https://gist.github.com/jonathanagustin/b67b97ef12c53a8dec27b343dca4abba
# install can take a minute

import os
# @title Convert Notebook to PDF. Save Notebook to given directory
NOTEBOOKS_DIR = "/content/drive/My Drive/CS441/24SP/hw2" # @param {type:
↪"string"}
NOTEBOOK_NAME = "CS441_SP24_HW2_Solution.ipynb" # @param {type:"string"}
#---------------------------------------------------------------------------#
from google.colab import drive
drive.mount("/content/drive/", force_remount=True)
NOTEBOOK_PATH = f"{NOTEBOOKS_DIR}/{NOTEBOOK_NAME}"
assert os.path.exists(NOTEBOOK_PATH), f"NOTEBOOK NOT FOUND: {NOTEBOOK_PATH}"
!apt install -y texlive-xetex texlive-fonts-recommended texlive-plain-generic >␣
↪/dev/null 2>&1
!jupyter nbconvert "$NOTEBOOK_PATH" --to pdf > /dev/null 2>&1
NOTEBOOK_PDF = NOTEBOOK_PATH.rsplit('.', 1)[0] + '.pdf'
assert os.path.exists(NOTEBOOK_PDF), f"ERROR MAKING PDF: {NOTEBOOK_PDF}"
```

```
print(f"PDF CREATED: {NOTEBOOK_PDF}")
```

```
---------------------------------------------------------------------------
ModuleNotFoundError                       Traceback (most recent call last)
Cell In[78], line 9
      7 NOTEBOOK_NAME = "CS441_SP24_HW2_Solution.ipynb" # @param {type:"string"}
      8␣
 ↪#---------------------------------------------------------------------------#
----> 9 from google.colab import drive
     10 drive.mount("/content/drive/", force_remount=True)
     11 NOTEBOOK_PATH = f"{NOTEBOOKS_DIR}/{NOTEBOOK_NAME}"

ModuleNotFoundError: No module named 'google.colab'
```

```
[ ]:
```