

Experiment - 3

* Aim - Exercise to find the eigen value and eigen vectors in Scilab.

* Materials - A Computer with Scilab installed.

* Procedure -

(i) Basic matrix operations -

- familiarize yourself with basic matrix creation and manipulation in Scilab, create a simple matrix by enclosing elements in square brackets separated with commas for ex.

type $A = [[1, 2], [3, 4]]$ to make a 2×2 matrix A .

- Access individual elements using row and col. indices for ex type $A(1, 2)$ to see the element at row 1 and col 2

- practice basic operations like matrix addition, subtraction, multiplication and transposition.

$\rightarrow A = [[1, 2], [3, 4]]$

$A = \begin{matrix} 1. & 2. \\ 3. & 4. \end{matrix}$

$\rightarrow \text{disp}(A(1, 2))$

$\rightarrow \cancel{B = [[5, 6], [7, 8]]}$

$B = \begin{matrix} 5. & 6. \\ 7. & 8. \end{matrix}$

$\rightarrow C.\text{add} = A + B$

$C.\text{add} = \begin{matrix} 6. & 8. \end{matrix}$

$\begin{matrix} 10. & 12. \end{matrix}$

$\rightarrow \text{disp}(C.\text{add})$

$\begin{matrix} 6. & 8. \end{matrix}$

$\begin{matrix} 10. & 12. \end{matrix}$

$$\rightarrow C_{-sub} = A - B$$

$$C_{-sub} = \begin{bmatrix} -4. & -4. \\ -4. & -4. \end{bmatrix}$$

$$\rightarrow \text{diag}(C_{-sub})$$

$$\begin{bmatrix} -4. & -4. \\ -4. & -4. \end{bmatrix}$$

$$\rightarrow C_{-transpose} = A'$$

$$C_{-transpose} = \begin{bmatrix} 1. & 3. \\ 2. & 4. \end{bmatrix}$$

$$\rightarrow \text{diag}(C_{-transpose})$$

$$\begin{bmatrix} 1. & 3. \\ 2. & 4. \end{bmatrix}$$

(ii) Finding Eigen Values -

- Matlab provides the builtin function (spec) to find eigen value of the matrix. For example type eig to get a vector containing the eigen of matrix.
- Explain the obtained eigen values and understand their relationship to the matrix.
- Experiment with different types of matrices including symmetric, triangular and diagonal and observe how their eigen values are affected.

$$\rightarrow A = \begin{bmatrix} 4. & -2. \\ 1. & 1. \end{bmatrix}$$

$$A = \begin{bmatrix} 4. & -2. \\ 1. & 1. \end{bmatrix}$$

$$\rightarrow \text{eigenvalue} = \text{spec}(A)$$

$$\text{eigenvalue} =$$

$$\begin{bmatrix} 3. & +0.1 \\ 2. & +0.1 \end{bmatrix}$$

$$\begin{bmatrix} 3. & +0.1 \\ 2. & +0.1 \end{bmatrix}$$

$$\rightarrow \text{Symmetric matrix} = \begin{bmatrix} 2. & 1. \\ 1. & 3. \end{bmatrix}$$

$$\text{Symmetric matrix} =$$

$$\begin{bmatrix} 2. & 1. \\ 1. & 3. \end{bmatrix}$$

$$\begin{bmatrix} 2. & 1. \\ 1. & 3. \end{bmatrix}$$

$$\rightarrow \text{eigen-symmetric} = \text{spec}(\text{symmetric-matrix})$$

$$\text{eigen-symmetric} = \begin{matrix} 1.3819660 \\ 3.6180340 \end{matrix}$$

$$\rightarrow \text{triangular-matrix} = [1, 2; 0, 3]$$

$$\text{triangular-matrix} = \begin{matrix} 1. & 2. \\ 0. & 3. \end{matrix}$$

$$\rightarrow \text{eigen-triangular} = \text{spec}(\text{triangular-matrix})$$

$$\text{eigen-triangular} = \begin{matrix} 1. + 0.i \\ 3. + 0.i \end{matrix}$$

$$\rightarrow \text{exp}(\text{eigen-triangular})$$

$$\begin{matrix} 1. + 0.i \\ 3. + 0.i \end{matrix}$$

$$\rightarrow \text{diagonal-matrix} = [4, 0; 0, -1]$$

$$\text{diagonal-matrix} = \begin{matrix} 4. & 0. \\ 0. & -1. \end{matrix}$$

$$\rightarrow \text{eigen-diagonal} = \text{spec}(\text{diagonal-matrix})$$

$$\text{eigen-diagonal} = \begin{matrix} -1. \\ 4. \end{matrix}$$

$$\rightarrow \text{exp}(\text{eigen-diagonal})$$

$$\begin{matrix} -1. \\ 4. \end{matrix}$$

(iii) Finding Eigenvectors -

- we use the `eigs` function to find the eigenvectors corresponding to the eigen values. For example type `eigs(A)` to get the matrix containing the eigen vectors as columns.
- Each column in the obtained matrix corresponds to the eigenvector associated with a corresponding eigen value.

- Verify the relation between eigenvectors and eigenvalues using the equation $A * v = \lambda v$ where v is an eigenvector and λ is corresponding eigenvalue

$$\rightarrow A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\rightarrow [v, \lambda] = \text{spec}(A)$$

$$v = \begin{bmatrix} 0.8944272 + 0.1i & 0.7071068 + 0.1i \\ 0.4472136 + 0.1i & 0.7071068 + 0.1i \end{bmatrix}$$

$$\lambda = \begin{bmatrix} 3 + 0.1i & 0 + 0.1i \\ 0 + 0.1i & 2 + 0.1i \end{bmatrix}$$

$$\rightarrow \text{for } i = 1 : \text{size}(\lambda, 1)$$

$$> \lambda = \lambda(i, i)$$

$$> t = v(i, i)$$

$$> \text{result} = A * t - \lambda * t$$

$$> \text{disp}(\text{result})$$

$$> \text{end}$$

$$0.$$

$$0.$$

$$0.$$

$$0.$$

(iv) Application of Eigenvalues and eigenvectors -
 Discuss and research some real world engineering applications of eigenvalues and eigenvectors such as model analysis of vibration, principal component analysis (PCA) for the data compression and data image processing.
 Consider how these concepts can be applied in your specific engineering discipline.

→ import numpy as np

Example 1. Model Analysis of vibrations.

Eigenvalues and eigenvectors are commonly used in model analysis to study vibration in structure.

Define a symmetric matrix corresponding to the structure stiffness or mass matrix.

→ structure - matrix = np.array([[4, -1], [-1, 3]])

Calculate eigen values and eigenvectors

→ eigenvalues, eigenvectors = np.linalg.eig(structure-matrix)

→ print("Example 1 - Model Analysis of vibrations")

→ print("Eigenvectors:")

→ print(eigenvectors)

⇒ Example 1 → Model Analysis and Vibrations

Eigenvalues - [4.61803399 2.38196608]

Eigenvectors - [[0.85065081 0.52573111]

[-0.58573111 0.85065081]]

Example - 2 Principal components Analysis (PCA) for data compression.

Create a sample dataset

→ data = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])

Calculate the covariance matrix

→ covariance - matrix : np.cov(data, rowvar = false)

Calculate eigenvalues and eigenvectors

→ eigenvalues - pca, eigenvectors - pca = np.linalg.eig(covariance-matrix)

→ print("\n Example 2 - PCA for data compression")

→ print("Eigenvalues (PCA):", eigenvalues - pca)

→ print("Eigenvalues (PCA): ")

→ print(eigenvectors - pca)

→ Example 2 → PCA for data compression

Eigenvalue (PCA): $[0.27, 0.]$

Eigenvector (PCA): $\begin{bmatrix} -0.81649658 & 0.57735 \\ 0.40824829 & 0.57735 \\ 0.408248 & 0.57735 \end{bmatrix}$

Example 3: Image Processing -

Consider a grayscale image as 3×3 matrix

→ image-matrix = np.array([[10, 20, 30],
[40, 50, 60], [70, 80, 90]])

Calculate eigenvalue and eigenvector

→ eigenvalue_image, eigenvector_image =
np.linalg.eig(image-matrix)

→ print("In Example 3- Image Processing: ")

→ print("Eigenvalue (Image): ", eigenvalue_image)

→ print("Eigenvector (Image): ")

→ print(eigenvector_image)

→ Example 3 → Image Processing

Eigenvalue (Image): $[1.61168, -1.116843, 2.0680]$

Eigenvector (Image): $\begin{bmatrix} 0.23197 & -0.7858 & 0.4082 \\ -0.52532209 & -0.0867513 & -0.316227766 \\ -0.8186735 & 0.612327 & 0.40824869 \end{bmatrix}$

* Conclusion- The experiment provides a fundamental understanding of finding eigenvalue and eigenvector in Scilab. By continuing to explore these concepts and their applications, you will unlock powerful tools for solving complex engineering problems involving matrix analysis and data interpretation.