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Exercise sheet 4: Constraint Satisfaction Problems

Due on *24/11/2017*, 2pm.

Question 1: Constraint Satisfactory Problem I (3P)

Suppose that Timo, Biagio, Uta, and Sabine are eating in a sushi bar and sit next to each other (in line).

a) Uta drives a Kia.

b) Sabine has a monkey.

c) Biagio drinks Cola.

d) Timo sits on the most right sit.

e) The Plymouth driver sits on the most left sit.

f) The BMW driver sits to the left of the beer drinker.

g) The tuna eater and the salmon eater sit next to each other.

h) The shrimp eater has a neighbor with a pet llama.

i) The dog owner and the tuna eater are not neighbors.

j) The fish owner has a neighbor who drinks Fanta.

k) The dog owner eats gyoza.

l) The BMW driver drinks Sprite.

m) The Peugeot driver sits to the right of the shrimps eater.

n) The gyoza eater drinks beer.

o) The Fanta drinker keeps a llama as a pet.

p) Everyone keeps a different pet.

q) Everyone enjoys a different drink.

r) Everyone drives a different make of car.

s) Everyone eats a different type of meal.

t) Everyone sits in a different spot.

Question 1: Represent this problem as a CSP. That means define the variables and their domains as well as the constraints using a fixed set of relations between the variables.
For example : person X sits next ot Y could be represented as X *nextto* Y .

Question 2: Who eats shrimps?

Question 2: Constraint Satisfactory Problem II **(3P)**

Suppose that we have a grid of $n \times n$ cells. In each cell (i, j), there is a material of constant density $d_{i,j} \geq 0$.
We assume that we can measure the density in a *tomographic* manner, i.e. we are able to measure the sum of densities along each horizontal or vertical line.
Example for $n = 3$:

$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	h_1
$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	h_2
$d_{3,1}$	$d_{3,2}$	$d_{3,3}$	h_3
v_1	v_2	v_3	

e.g. $h_1 = \sum_{i=1}^{3} d_{1,i}$.

It is convenient to enumerate the densities $d_{i,j}$ with a single index:

d_1	d_2	d_3
d_4	d_5	d_6
d_7	d_8	d_9

a) Formulate the problem of determining d from h and v as a constraint satisfaction problem (for $n = 3$). Try to find a compact description. Is this CSP solvable? How many solutions exist?
b) In the case of multiple solutions we would like to choose one with smallest norm

$$F(d) := \|d\|_1 = \sum_{i=1}^{9} |d_i|.$$

Reformulate $F(d)$ as $F(d) = w^\top d$ for some vector $w \in \mathbb{R}^9$, keeping in mind that only non-negative densities are allowed. Formulate the corresponding problem of minimizing $F(d)$ over all vectors d satisfying the CSP.

please turn over

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Question 3: Constraint Satisfactory Problem III **(4P)**

In a cryptarithmetic problem **each letter stands for a distinct digit** $(0 - 9)$. The aim is to find a digit for each letter such that the resulting sum is arithmetically correct with the added restriction that **no leading zeros** are allowed.

Example: Suppose you are performing an addition starting in the right column

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

with adding up O and O. Then the resulting constraint for right column can be written as

$$O + O = R + 10 \cdot C_1$$

$$O \neq R$$

with $domain(O) = domain(R) = \{0...9\}$ and $domain(C_1) = \{0...1\}$ where C_1 is an auxiliary variable representing the digit carried over into the tens column. The constraints for the other columns can be formulated in a similar way.

Given the following cryptarithmetic problem

$$\begin{array}{cc} & \mathrm{A} \\ + & \mathrm{B} \\ \hline \mathrm{B} & \mathrm{C} \end{array}$$

a) Rewrite the problem as a constraint satisfaction problem so that the addition is correct. Give the domains of the variables A, B, C, U (auxiliary variable for carry-over) and specify the constraints.

b) Apply the Backtracking-Search Algorithm to the problem. Suppose the method Select-Unassigned-Variable gives back the variables in the following order: C, B, A, U. Furthermore the method Order-Domain-Values provides the possible values in ascending order.

c) Now apply additional Forward-Checking and check at which points general Constraint-Propagation would have reduced the search space.

Note: Submit exactly one ZIP file and one PDF file via Moodle before the deadline. The ZIP file should contain your executable code. Make sure that it runs on different operating systems and use relative paths. Non-trivial sections of your code should be explained with short comments, and variables should have self-explanatory names. The PDF file should contain your written code, all figures, explanations and answers to questions. Make sure that plots have informative axis labels, legends and captions.