Deep-Sea Research, 1970, Vol. 17, pp. 293 to 301. Pergamon Press. Printed in Great Britain.

On oceanic boundary mixing

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(*Received* 13 *August* 1969)

Abstract—The basic stratification and the sloping boundaries of the ocean induce a mean upwelling velocity at the walls in order to satisfy the no flux condition. In the oceanographically most interesting case, the effect is confined to a boundary layer of thickness $0\ ((\nu\kappa)^{\frac{1}{4}}/N^{\frac{1}{2}})$ where ν, κ are the austausch coefficients for momentum and heat respectively, and N is the local Brunt–Väisälä frequency. The effect of strong rotation is to induce an accompanying thermal wind in the interior of the ocean.

If boundary mixing processes are sufficiently strong, then this upwelling process might account for most of the vertical velocity needed in the abyssal circulation. This is in accord with earlier suggestions that the boundaries play the essential role in the vertical exchange of oceanic properties.

1. INTRODUCTION

THE PURPOSE of this note is two-fold. First, we discuss the boundary conditions at a sloping wall in a stratified ocean. These conditions are of some interest when it is recognized that the condition of no flux of salt or heat at a boundary requires the respective isolines to encounter a wall at a right angle. The adjustment from the quasi-horizontal isolines of the oceanic interior to normal incidence at the boundary can take place either as a slow slope throughout the interior of the ocean, or as a boundary layer adjustment near the wall.

We show that either case is possible, but that the most common situation should be a boundary layer adjustment region of order of the (Rayleigh Number)$^{-\frac{1}{4}}$. Consequently, there is no state of rest possible in a stratified fluid with sloping walls.

Second, using these results, we examine the role of boundary mixing in accounting for the observed vertical diffusion in the ocean, as suggested by MUNK (1966). The importance of the results to some extent depends upon the still as yet undetermined stirring rates at oceanic boundaries, but they are at least potentially important. Some evidence in favor of the hypothesis is given by WUNSCH (1970).

BARCILON and PEDLOSKY (1967a, b, c) and VERONIS (1967a, b), have considered the boundary layers on vertical walls of rotating and stratified fluids in considerable detail. This note may be considered a slight extension and application of their results. The mathematical details omitted here may be found in their work. An earlier partial discussion may also be found in PRANDTL (1952) and GILL (1966).

2. NO ROTATION

We consider the situation shown in Fig. 1, a stably stratified fluid (for convenience we will use ' temperature ' as a generic term for the stratification parameter, be it heat or salt). The fluid has a temperature gradient $T_{0z}\ (z)$ imposed at $x = \infty$ and

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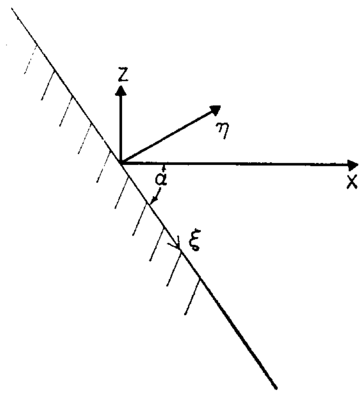

Fig. 1. The two coordinate systems.

maintained in some fashion into which we will not inquire. For convenience in using his results, we will follow the notation and scaling of VERONIS (1967a). In some ways the scaling is not completely appropriate, but this is more than offset by the insight to be gained from Veronis' results.

The pressure, temperature and velocity are scaled, with the star denoting a dimensional quantity:

$$p^* = g\alpha L\rho\Delta T_c \sqrt{\sigma}\, p, \quad \frac{\partial T_0^*}{\partial z} = \frac{4\Delta T}{L}\frac{\partial T_0}{\partial z}$$

$$T^* = z\Delta T_c \sqrt{\sigma}\, T, \quad (x^*, z^*) = L\,(x, z)$$

$$\bar{v}^* = \left(\frac{\Delta T_c}{\Delta T}\right)^{\frac{1}{2}} (g\alpha\Delta T_c\, L)^{\frac{1}{2}}\, \bar{v}$$

where L is a characteristic length (presumably the scale height of the interior temperature stratification), α the coefficient of 'thermal' expansion; $\sigma = \nu/\kappa$, the Prandtl number; $4\Delta T/L$ is the basic gradient; ΔT_c is the (unknown) temperature perturbation due to the wall. T_{0z} is the imposed temperature gradient, and T a perturbation. We also define the Brunt–Vaisala frequency,

$$N = \left(\frac{g\alpha\Delta T}{L}\right)^{\frac{1}{2}}.$$

For steady motion, the scaled equations become:

$$\sigma^{-\frac{1}{2}}\,\epsilon\,(\bar{v}\cdot\nabla)\,\bar{v} = -\nabla p + 2T\hat{k} + R\nabla^2\bar{v} \tag{1}$$

$$\sigma^{\frac{1}{2}}\,\epsilon\,(\bar{v}\cdot\nabla)\,T + 2w = R\nabla^2 T \tag{2}$$

$$\nabla\cdot\bar{v} = 0 \tag{3}$$

$$\epsilon = \frac{\Delta T_c}{\Delta T}, \; R = \left(\frac{\nu\kappa}{g\alpha\Delta T L^3}\right)^{\frac{1}{2}} = (\text{Rayleigh No.})^{-\frac{1}{2}}.$$

$\hat{k}$ is a unit vector in the z-direction. R is generally small. We will assume ϵ is negligible (to be checked later), and that $\sigma = 0\,(1)$, linearizing the equations. The 2-dimensional component equations are then:

$$-p_x + R\,\nabla^2 u = 0, \tag{4}$$

$$-2T = -p_z + R\,\nabla^2 w, \tag{5}$$

$$2w\,\frac{\partial T_0}{\partial z} = R\,\nabla^2 T, \tag{6}$$

$$u_x + w_z = 0. \tag{7}$$

Let the boundary surface be $z = -\gamma x$ at an angle α to the horizontal (Fig. 1). If $T_{0z} = 1$, then the temperature perturbation T satisfies:

$$R^2\,\nabla^6 T + 4T_{xx} = 0. \tag{8}$$

An interior equation is then:

$$T^I{}_{xx} = 0$$

and we choose the trivial solution $T^I = 0$.

To treat the boundary, we introduce coordinates ξ, η parallel and normal to it, such that:

$$\xi = x\cos\alpha - z\sin\alpha \tag{9}$$

$$\eta = z\cos\alpha + x\sin\alpha. \tag{10}$$

Equation (8) is now:

$$R^2\left\{\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right\}^3 T + 4\left\{\cos^2\alpha\,\frac{\partial^2}{\partial\xi^2} + \sin^2\alpha\,\frac{\partial^2}{\partial\eta^2} + 2\sin\alpha\cos\alpha\,\frac{\partial^2}{\partial\eta\partial\xi}\right\} T = 0. \tag{11}$$

The boundary conditions are $\partial T_s/\partial\eta = 0$, on $\eta = 0$ where T_s is the sum of the basic temperature plus the perturbation, and $u = w = 0$. Introduce a stretched coordinate $\zeta = R^{-\frac{1}{2}}\eta$. Then as long as $\sin\alpha > R^{\frac{1}{2}}$ (i.e. the wall may be almost, but not quite, horizontal) the boundary layer is described by:

$$\frac{\partial^6\bar{T}}{\partial\zeta^6} + 4\sin^2\alpha\,\frac{\partial^2\bar{T}}{\partial\zeta^2} = 0 \tag{12}$$

where we use the bar to denote a boundary layer variable. Applying the boundary conditions, we easily find that to $0\,(R^{\frac{1}{2}})$;

$$\bar{T} = R^{\frac{1}{2}}\frac{\cos\alpha}{(\surd\sin\alpha)}\,\sigma^{-\frac{1}{2}}\left(\frac{\Delta T}{\Delta T_c}\right)\exp\left\{-(\surd\sin\alpha)\,R^{-\frac{1}{2}}\eta\right\}\cos\left\{\surd\sin\alpha)\,R^{-\frac{1}{2}}\eta\right\}, \tag{13}$$

$$\bar{w} = R^{\frac{1}{2}}\cos\alpha\,(\surd\sin\alpha)\,2\sigma^{-\frac{1}{2}}\left(\frac{\Delta T}{\Delta T_c}\right)\exp\left\{-(\surd\sin\alpha)\,R^{-\frac{1}{2}}\eta\right\}\sin\left\{(\surd\sin\alpha)\,R^{-\frac{1}{2}}\eta\right\}, \tag{14}$$

$$\bar{u} = R^{\frac{1}{2}}\,\frac{\cos^2\alpha}{(\surd\sin\alpha)}\,2\,\sigma^{-\frac{1}{2}}\left(\frac{\Delta T}{\Delta T_c}\right)\exp\left\{-(\surd\sin\alpha)\,R^{-\frac{1}{2}}\eta\right\}\sin\left\{(\surd\sin\alpha)\,R^{-\frac{1}{2}}\eta\right\}. \tag{15}$$

This solution represents the now well-known ' buoyancy layer ', the stratified analogy to the Ekman layer. We note that the interior is at rest to this order, all the motion being induced by the pressure gradient in the boundary layer.

The solution is perhaps a little more useful in dimensional form:

$$\bar{T}^* = 2\,\Delta T\,R^{\frac{1}{2}}\frac{\cos\alpha}{(\surd\sin\alpha)}\exp\left\{-(\surd\sin\alpha)\,\frac{N^{\frac{1}{2}}}{(\nu\kappa)^{\frac{1}{4}}}\,\eta^*\right\}\times\sin\left\{(\surd\sin\alpha)\,\frac{N^{\frac{1}{2}}}{(\nu\kappa)^{\frac{1}{4}}}\,\eta^*\right\} \tag{16}$$

$$\bar{w}^* = (\nu\kappa)^{\frac{1}{4}} N^{\frac{1}{2}} 2\sigma^{-\frac{1}{2}} (\sqrt{\sin\alpha}) \cos\alpha \exp\left\{-(\sqrt{\sin\alpha}) \frac{N^{\frac{1}{2}}}{(\nu\kappa)^{\frac{1}{4}}} \eta^*\right\} \sin\left\{(\sqrt{\sin\alpha}) \frac{N^{\frac{1}{2}}}{(\nu\kappa)^{\frac{1}{4}}} \eta^*\right\} \quad (17)$$

$$\bar{u}^* = (\nu\kappa)^{\frac{1}{4}} N^{\frac{1}{2}} 2\sigma^{-\frac{1}{2}} \frac{\cos^2\alpha}{(\sqrt{\sin\alpha})} \exp\left\{-(\sqrt{\sin\alpha}) \frac{N^{\frac{1}{2}}}{(\nu\kappa)^{\frac{1}{4}}} \eta^*\right\} \sin\left\{(\sqrt{\sin\alpha}) \frac{N^{\frac{1}{2}}}{(\nu\kappa)^{\frac{1}{4}}} \eta^*\right\}. \quad (18)$$

Note that $\epsilon \cong 2\cos\alpha/(\sqrt{\sin\alpha}) R^{\frac{1}{2}}$, and the linearization is justified until $\sin\alpha = R < R^{\frac{1}{2}}$. The stability of a similar buoyancy layer has recently been discussed by GILL and DAVEY (1969), though in the case being considered here, there is a stabilizing force normal to the wall. We may also note that in their terms, the local Reynolds number is order one, and the boundary layer presumably stable.

The buoyancy layer balance breaks down when $\sin\alpha = O(R^{\frac{1}{2}})$. From the analogy between stratified fluids and rotating fluids demonstrated by Veronis, we anticipate the presence of Stewartson layers of thickness $R^{\frac{1}{3}}$ and $R^{\frac{1}{4}}$ for these small angles. There is one subtlety. If the boundary is precisely horizontal, it is clear that a purely diffusive interior will, to lowest order, adjust the temperature field so as to satisfy the no flux boundary condition. This represents a large local modification of the initially imposed temperature gradient in a thin non-linear boundary layer. To retain a linear problem it is necessary to perturb about a basic gradient that would satisfy the boundary conditions were the boundary, in fact, horizontal, though we then only *model* the basic state.

For example, we can use $T_0(z) = z^2$. The details of the boundary layers are uninteresting. The $R^{\frac{1}{4}}$ layer is required to bring the average value of $\partial T_0/\partial\eta$ to zero at the wall, and the $R^{\frac{1}{3}}$ layer to satisfy the boundary conditions on the velocities induced in the $R^{\frac{1}{4}}$ layer, plus the boundary conditions on the non-average properties of $\partial T_0/\partial\eta$. A closed problem is obtained by matching the $R^{\frac{1}{4}}$ layer onto buoyancy layers at any 'vertical' walls. The details of the solutions may be obtained from GREENSPAN (1968) or the very clear account by HOWARD (1968).

3. ROTATION

This is discussed at some length by VERONIS (1967b) (cf. BARCILON and PEDLOSKY, 1967c). Generally speaking, for the scales appropriate here, the relative importance of rotation to stratification is given by the internal rotational Froude number $F = f/2N$, where f is the Coriolis parameter. In most of the ocean this is a very small number, and rotational effects are negligible. However, in the abyss F may approach 1 and it is interesting to briefly consider the effect rotation has on the above solutions. The cited papers provide a more detailed analysis.

There is an entire hierarchy of solutions depending upon the relative magnitudes of F and R. We will consider, as an example, the case $F = O(1)$ and R small. The 2 dimensional equations of motion are:

$$\begin{aligned} -2Fv &= -p_x + R\,\nabla^2 u \\ 2Fu &= R\,\nabla^2 v \\ -p_z + 2T + R\,\nabla^2 w &= 0 \\ 2\,w &= R\,\nabla^2 T \\ u_x + w_z &= 0 \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \end{aligned} \quad (19)$$

An equation in the temperature is:

$$\nabla^2 \left\{ R^2 \nabla^6 T + 4T_{xx} + 4F^2 T_{zz} \right\} = 0. \tag{20}$$

Hence, an interior equation is:

$$\nabla^2 \left\{ T^I_{xx} + F^2 T^I_{zz} \right\} = 0, \tag{21}$$

for which we again accept the trivial solution $T^I = 0$.
In the rotated coordinate system (9–10), we have

$$R^2 \left\{ \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right\}^3 T + 4 \left\{ \cos^2 \alpha \frac{\partial^2}{\partial \xi^2} + \sin^2 \alpha \frac{\partial^2}{\partial \eta^2} + 2 \sin \alpha \cos \alpha \frac{\partial^2}{\partial \eta\, \partial \xi} \right\} T$$
$$+ 4 F^2 \left\{ \sin^2 \alpha \frac{\partial^2}{\partial \xi^2} + \cos^2 \alpha \frac{\partial^2}{\partial \eta^2} + 2 \sin \alpha \cos \alpha \frac{\partial^2}{\partial \eta\, \partial \xi} \right\} T = 0. \tag{22}$$

In the stretched system $\zeta = R^{-\frac{1}{2}} \eta$, we have approximately:

$$\frac{\partial^6 \bar{T}}{\partial \zeta^6} + (4 \sin^2 \alpha + 4F^2 \cos^2 \alpha) \frac{\partial^2 \bar{T}}{\partial \zeta^2} = 0 \tag{23}$$

valid now for all α. The solutions for $\bar{T}$, $\bar{u}$, $\bar{w}$, are the same as (13–15) with $\sin \alpha$ replaced with $(\sin^2 \alpha + F^2 \cos^2 \alpha)^{\frac{1}{2}}$. As $\alpha \to \pi/2$ we have a buoyancy layer and as $\alpha \to 0$ a classical Ekman layer. There will be no Stewartson layers here since the Ekman layer renders them unnecessary. There is now an azimuthal velocity

$$\bar{v} = R^{\frac{1}{2}} \sigma^{-\frac{1}{2}} \frac{\Delta T}{\Delta T_c} \frac{\cos^2 \alpha}{(\sin^2 \alpha + F^2 \cos^2 \alpha)^{\frac{3}{4}}} \exp \left\{ - (\sin^2 \alpha + F^2 \cos^2 \alpha)^{\frac{1}{4}} R^{-\frac{1}{2}} \eta \right\} \tag{24}$$
$$\times \cos \left\{ (\sin^2 \alpha + F^2 \cos^2 \alpha)^{\frac{1}{4}} R^{-\frac{1}{2}} \eta \right\}$$

which does not vanish at the boundary. This induces an interior velocity $v^I = 0\,(R^{\frac{1}{2}})$ to satisfy the boundary condition. This in turn distorts the interior isotherms at $O\,(R^{\frac{1}{2}})$ by the thermal wind relation, subject to (21) or:

$$\nabla^2 \left\{ v^I_{xx} + F^2 v^I_{zz} \right\} = 0. \tag{25}$$

An alternative and more attractive point of view is the following. In (21), we have, except for a scale factor, Laplace's equation (actually the biharmonic equation but the lower order equation suffices) for the interior. This equation is of sufficiently high order that its solutions satisfy the no flux condition at the boundary, reducing to $T = 0$ at $x = \infty$. The isotherms bend slowly throughout the interior to approach the boundary at a right angle. The rotation induces a thermal wind of $O\,(R^{\frac{1}{2}})$ which is then brought to zero in a boundary layer on the wall.

The case $F = O\,(R^{\frac{1}{2}})$ may be deduced from Veronis (1967b) and the case $F \gg 1$ from Barcilon and Pedlosky (1967c). The boundary layer intricacies are of no particular interest here.

4. OCEANIC MIXING

Before attempting to apply these results to the ocean, it is instructive to evaluate the size of the quantities involved. One is immediately faced with the fact that ν, κ must be interpreted as eddy coefficients, and as is usual, there are no obvious values to be assigned to them. Various estimates have been made of mixing coefficients in the ocean, but all are unreliable to some extent. Furthermore, oceanic mixing is generally supposed highly anisotropic, so that horizontal eddy coefficients are many orders of magnitude greater than vertical ones. The model considered here can, of course, be reformulated using different vertical and horizontal eddy coefficients; this is a refinement that does not at the moment appear to be worthwhile. Some order of magnitude estimates are useful though.

The thickness of the buoyancy layer is

$$\delta \sim (\nu\kappa)^{\frac{1}{4}}/N^{\frac{1}{2}}.$$

A common estimate for the vertical rates of mixing is 1 cm²/sec.
Using $N = 2 \times 10^{-3}$, we have

$$\delta \sim 20\ \text{cm}.$$

If we take ν, $\kappa \sim 10^4$, then

$$\delta \sim 20\ \text{m}.$$

The induced vertical velocity w is given by (17) of order

$$\bar{w}^* \sim (\nu\kappa)^{\frac{1}{4}} N^{\frac{1}{2}}$$

[we will take all trigonometrical values as O (1)] which is about

$$4{\cdot}5 \times 10^{-2}\ \text{cm/sec} \quad \nu, \kappa = 1$$

$$4{\cdot}5\ \text{cm/sec} \quad \nu, \kappa = 10^4.$$

In a stimulating paper, MUNK (1966) pointed out that boundary mixing processes might account for a substantial fraction of the vertical stirring required to explain observed tracer properties in the ocean, though he did not pursue the subject. There is indeed some evidence that boundary mixing may, in fact, be important. Figure 2 is taken from WUNSCH (1970) and shows the systematic change in the temperature gradient as measured with an S–T–D as one approaches the Bermuda islands. The appearance of the large erratic steps indicates that some process is tending to strongly stir the water near the islands. Several mechanisms suggest themselves; the possibility that it is due to breaking internal waves is discussed in WUNSCH and DAHLEN (1970). In principle, with sufficient temperature resolution, such curves can be used to estimate the local eddy mixing. The S–T–D does not have sufficient vertical resolution (about 1 cm is required) but this limitation should soon be overcome. In the meantime, the indication that boundary mixing is important makes the matter worth pursuing.

SCHIFF (1966) made an attempt to model boundary stirring by posing the following problem. An ' ocean ' is heated from above and cooled from below (temperature T_2 and T_1 respectively are imposed). The effect of stirring at the walls is modelled by imposing a constant temperature $T_0 = (T_1 + T_2)/2$ there. One asks for the distribution of temperature that results in the interior of the conducting viscous fluid, and a physical understanding of how heat is carried from the top to the bottom. Schiff linearized the problem about a mean interior gradient, and deduced the presence of $R^{\frac{1}{4}}$ layers at top and bottom. Unfortunately, his interior solution is isothermal, rendering the original linearization invalid. His solution is probably a correct limiting case for large depth to width ratios, but the problem needs more work.

It is not necessary, however, to deal with such an extreme nonlinear case, for the

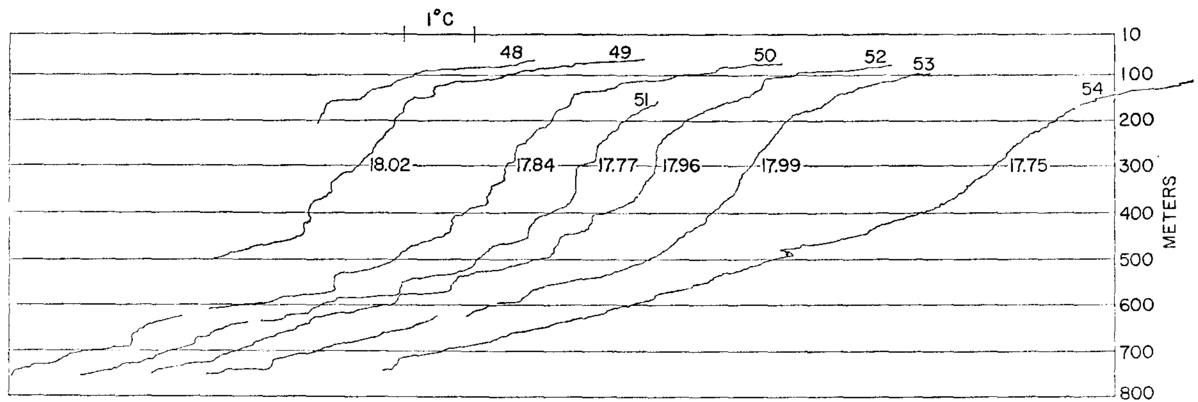

Fig. 2. A Salinity–Temperature–Depth (S–T–D) section made on a northeasterly line approximately normal to the bottom contours at Bermuda. The island is at the left and the different traces were taken at depth intervals of approximately 200 fathoms beginning with the 100 fathom line. Since the horizontal displacements on the plot are arbitrary, the measured temperature at the 300 m level is shown. The numbers at the top are station numbers. This is an extreme case.

boundary layers deduced above provide a systematic means of studying the boundary mixing process.

In an ocean modelling the Pacific, of area $1 \cdot 37 \times 10^{18}$ cm^2, MUNK (1966) deduced* a total upward transport of $1 \cdot 89 \times 10^{13}$ cm^3/sec. Let us compute, for comparison, the total upwelling in the buoyancy layers. Consider a square ocean $1 \cdot 17 \times 10^9$ cm on a side. Then an estimate of the total upwelling is:

$$4 \times 1 \cdot 17 \times 10^9 \int_0^\infty \bar{w}^* \, d\eta^* = 4 \cdot 68 \times 10^9 (\nu\kappa)^{\frac{1}{2}} \sigma^{-\frac{1}{2}} \cos \alpha.$$

(independent of N). If $\sigma^{-\frac{1}{2}} \cos \alpha$ is $O(1)$, and we take ν, $\kappa \sim 1$, we account for a fraction of 4×10^{-4} of Munk's estimate. If we wish to account for all of it, we need ν, $\kappa \sim 3 \times 10^4$ which is approaching values used for horizontal diffusivity. Presumably the truth is somewhere in between. Note that we have probably underestimated the actual length of oceanic shoreline.

The 'standard' oceanic thermocline theory (ROBINSON and WELANDER, 1961) requires an upwelling oceanic interior, and it is difficult to fit the picture of upwelling boundaries into such a theory. However, WELANDER (1969) has recently proposed a perfect fluid thermocline model with a downwelling interior. The fluid must be returned upward in some manner (unspecified) either in boundary layers, or other essentially singular regions. Whether a model such as Welander's can be fitted in detail to the upwelling boundaries proposed here remains to be investigated.

VERONIS (private communication) has pointed out that the diffusive thermocline models can be superposed on Welander's, so that both such models may be correct.

The actual mixing of the oceanic interior comes through the divergence of the buoyancy layer in strict analogy to the Ekman layer divergence. In this case it would be due to either slight variations in the vertical temperature gradient or to variations in the angle α of the walls. In either case, at $O(R)$, there would be an influx and efflux at various levels of fluid from the interior to the boundary layers where mixing would actually take place. VERONIS (1967a) has shown how this would work for small temperature inhomogeneities on the boundaries, and the physics is precisely the same here.

In conclusion it should be pointed out that the mechanism postulated here is probably most applicable to the Pacific where both the temperature and salinity gradients are generally stable. An unstable salinity gradient, as is found in the Atlantic, formally would tend to drive a flow down the wall rather than up. On the other hand, J. S. TURNER (private communication) has shown that the combination of a stable temperature gradient and an unstable salinity gradient on a sloping boundary leads to many complex and interesting results in which the boundary layers found here probably play only a minor role.

Acknowledgements—This work was completed while I was a visitor at the Department of Applied Mathematics and Theoretical Physics, University of Cambridge. The Office of Naval Research provided support under Contract Nonr-1841 (74) with the Massachusetts Institute of Technology.

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*Munk's estimate of vertical velocities is based on a questionable interpretation of carbon-14 data. His value of a mean vertical velocity of $1 \cdot 4 \times 10^{-5}$/sec is, however, consistent with other such estimates.

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Note added in proof: I have recently seen a manuscript by O. M. PHILLIPS where the results of Section 2 are independently derived. Phillips makes the important point that to this order of approximation, the non-linear terms in the equations vanish identically (this would be true of the solutions of Section 3 as well), which has interesting implications for fluids of very small Prandtl number.