CHAPTER 10

INTERNAL MIXING PROCESSES

The mechanisms responsible for mixing in the interior of a stratified fluid are even less well understood than those described in chapter 9, since the sources of energy are not so obvious, and several different processes must be taken into account simultaneously. The important ideas have already been introduced in earlier sections, but it seems appropriate in this final chapter to take a broader and less detailed view, and to consider together the whole array of mixing phenomena which can be relevant in large natural bodies of stratified fluid. Geophysical examples have figured prominently in this book, and indeed the range of subject matter has been chosen with this final synthesis in mind. The basic facts requiring explanation are somewhat scattered, however, and in order to collect them together and to define the problems to be treated here, a brief summary will now be given of the observed structure of the ocean and atmosphere.

This field is developing rapidly, and the interpretation given here must necessarily be a somewhat tentative and personal one. Nevertheless, it seems useful to sketch how our present knowledge of the separate components can be fitted into a self-consistent picture, at the same time extending some of the earlier arguments so that they can be applied in this wider context.

10.1. The observational data

Routine density profiles made using reversing bottles and thermometers show that the ocean is everywhere stably stratified, except for limited regions where bottom water forms intermittently as the water column becomes convectively unstable. Near the surface, there is characteristically a layer of more nearly uniform density, bounded below by a thermocline, or region of rapid variation of temperature and density (see §9.2). This structure varies on a daily

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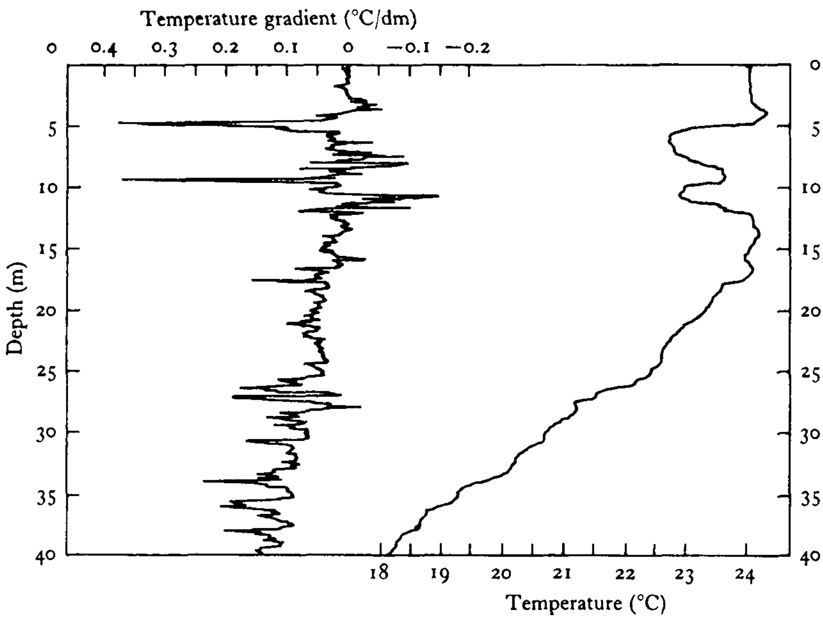

Fig. 10.1. Temperature and temperature gradient profiles recorded in the upper part of the summer thermocline near Malta by J. D. Woods (private communication). Over the depths where there are large temperature inversions (between 3 and 12 m) there must be compensating gradients of salinity since the density must increase with increasing depth, except in localized regions of active overturning.

and seasonal timescale, in a manner which depends on the local wind and heat flux, and on the vertical and horizontal gradients of temperature and salinity. (Heat is both added and removed at the surface of the ocean, which is a very different distribution of sources and sinks from that in the atmosphere.) At a depth of some hundreds of metres, there is a weaker permanent thermocline. Below this again is a region extending to the bottom in which the density gradient becomes progressively weaker with increasing depth. Nevertheless, the 'overall Richardson number' for the deep ocean (based on the geostrophic velocities produced by the measured density differences on the rotating earth) are typically very large, at least of order 10^2.

Recent detailed measurements of temperature and salinity profiles, made with continuously recording instruments in various parts

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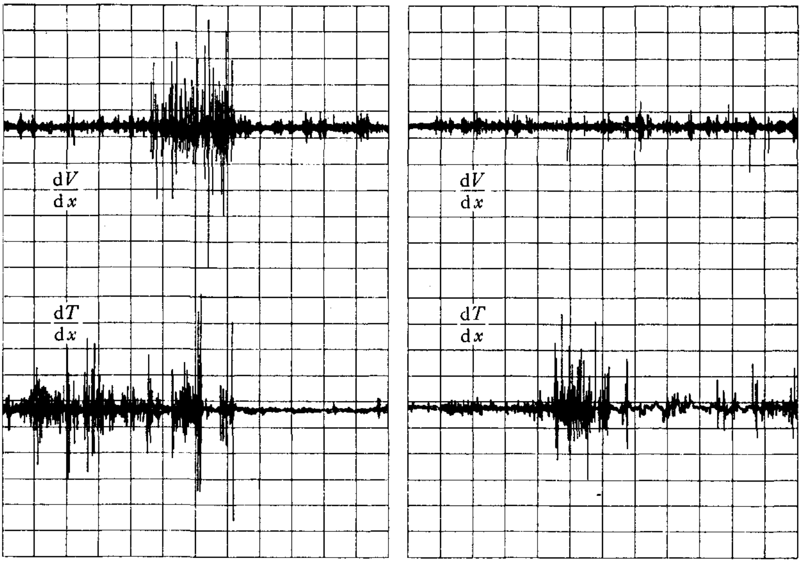

Fig. 10.2. Velocity and temperature derivatives in the deep ocean, obtained by towing sensors at about 200 m in the permanent halo-thermocline. The record length represents one minute or a distance of about 100 m. (*a*) Correlated velocity and temperature microstructure, indicative of active small scale mixing. (*b*) 'Fossil turbulence', or temperature microstructure unaccompanied by any visible velocity structure. The velocity signals shown are all caused by vibrations in the towing system. (Unpublished data from P. W. Nasmyth.)

of the ocean, have shown that the density distribution is not smooth, but contains significant variations on very small lengthscales even at great depths. Particularly when vertical gradients (or differences over a few centimetres) are recorded, rather than properties at a point, it becomes clear that the vertical density profile is often step-like, with layers of more uniform density separated by 'sheets' or interfaces where the gradients are large. (See fig. 10.1.) This is true not only when double-diffusive processes are important (§8.3.4), but small scale layering can occur even in situations where variations of one property alone determines the density. (Similar records of temperature variations have been obtained in the ocean and in fresh water lakes; see Simpson and Woods 1970.) Measurements of the vertical shear near the surface and in the deeper ocean also have a

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very non-uniform character, and give further evidence of the step-like structure. It has been shown, for example by Grant, Moilliet and Vogel (1968) in a tidal estuary, and by Woods (1969) in the summer thermocline near Malta, that turbulence in the interior is confined to thin, elongated patches which occur intermittently in space and time. Records of velocity and temperature fluctuations, obtained by towing a sensor through the upper ocean, reveal regions of vigorous small scale turbulence separated by others which are essentially laminar (fig. 10.2*a*). Patches of 'fossil turbulence', i.e. temperature microstructure remaining after the turbulence has decayed, have also been observed. (See Stewart 1969, and fig. 10.2*b*.)

Temperature profiles measured using radiosondes in the atmosphere also reveal a mainly stable stratification, but because heat is on average added to the atmosphere at the ground, and removed at higher levels by long wave radiation, the lowest layers are often convectively unstable. Dry convection occurs immediately above the ground, while higher up clouds are the more important mechanism for transporting heat and water vapour into the upper atmosphere. At the top of the surface layer there can be a sharp temperature inversion, implying a very rapid potential density change, or there may be a smoother transition to a stable gradient. The mean gradient increases with increasing height above this level. Again there is much fine structure, and a tendency towards layering, with turbulence occurring intermittently and in patches rather than uniformly through the atmosphere.

It is worth remarking that some of the observed non-uniformities of density gradient may be transient, and associated with internal waves rather than mixing. Even if a region is smoothly stratified, the propagation of a group of waves through it will cause a local squeezing and separation of the surfaces of constant density, which would be interpreted as variations in the vertical gradient. Repeated profiles made through regular layered structures also give evidence of spatial and temporal variations which can be attributed to waves.

It has been suggested (see, for example, Iselin 1939 and Munk 1966) that a major part of oceanic mixing might be due to boundary effects, that is, to turbulence produced at the margin of continents or at the surface, followed by a density-driven flow into the interior.

Large scale advection of this kind could indeed lead to vertical transports and the production of some of the layering which is observed, and both in the atmosphere and in the ocean the motion of fronts will have similar results. The effectiveness of these processes is limited by the horizontal constraint of rotation, however, and the sharpness of the observed gradients and the speed with which the microstructure can change show that local vertical mixing is often important. Only these smaller scale, more rapid processes will in fact be considered in this chapter, and again we use the assumption (already introduced in chapter 9) that they can for the most part be treated as one-dimensional. It would take us too far afield to give a proper discussion of horizontal diffusion and mixing, and the 'shear effect' referred to in § 5.3, whereby the horizontal spread is strongly influenced by the interaction between vertical mixing and shear, but there is no doubt that these processes play a significant role. (See Pingree (1971), already mentioned in §8.2.3; Woods and Wiley (1971); and the papers by Okubo (1968, 1970).)

10.2. Critical Richardson number criteria

The central difficulty of the internal mixing problem is associated with the apparently very great stability of the geophysical flows. As was pointed out in § 10.1, the overall Richardson numbers (based on the mean density and velocity differences $\Delta\rho$ and ΔU over the depth of the ocean, for instance) are typically very large. How, then, in view of the criterion for the breakdown of a shear flow discussed in §4.1.3, and that for the equilibrium state proposed in § 5.1.4, can turbulence ever be produced and sustained in the interior of such a region?

A qualitative answer to these questions can be found by developing the ideas expressed in §§ 5.1.4 and 6.2.4 and paying special attention to the sources of turbulent energy. Long (1970) has gone further and proposed a detailed theory which combines several of the features discussed below, but it seems preferable here just to describe the essential ingredients of such a solution, rather than to give a particular recipe for their combination.

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10.2.1. *Examples of equilibrium conditions*

It is less difficult to understand what is happening when localized processes can be identified which impose the differences $\Delta\rho$ and ΔU over a much smaller lengthscale than the whole depth of the ocean or atmosphere. When a sufficiently large shear is applied across a density interface and is such that the gradient Richardson number falls below a critical value of about $\frac{1}{4}$, Kelvin–Helmholtz waves will grow and overturn to produce patches of turbulent mixing. (The results given in §4.1.3 will be followed up in the geophysical context in §10.3.1.) If the shear is not sustained, this turbulence will decay as the mixed layer spreads out (§5.3.2). If further kinetic energy is provided, however, the arguments of §5.1.4 have shown that there should be an 'equilibrium', self-adjusting value of the gradient Richardson number Ri_e at which turbulence can be maintained. In this state no external lengthscale is relevant, and the mixing region adjusts its thickness to accommodate the imposed $\Delta\rho$ and ΔU.

The earlier applications to wakes (§5.3) and especially to the outer edge of inclined plumes (§6.2.4) were in accord with these ideas, and suggested that Ri_e is of order 0.1 (somewhat smaller than the critical value for the original breakdown). Various observers (see, for example, Browning 1971) have been accumulating evidence in the atmosphere which shows that the structure of frontal zones can be described in a similar way (cf. case (*e*) of fig. 4.19). They have combined Doppler radar measurements of winds with frequent temperature profiles made using radiosondes to deduce values of minimum Ri lying consistently in the range 0.15–0.3 in these regions (fig. 10.3), though Ri is poorly defined elsewhere.

An equilibrium value of Ri can be sustained in this case, sometimes for several hours, because the motion of a front tends to sharpen the gradients and decrease Ri, while mixing acts to increase it again. The numerical values quoted for the atmosphere can only be upper limits, since they were obtained by averaging vertically over 200 m and horizontally over several km diameter (see the discussion relating to (10.2.1)), but the principle of a limiting structure controlled by a Richardson number criterion seems to be confirmed. The regions above and below jet streams in the high atmosphere, and intruding layers in the ocean, should also be

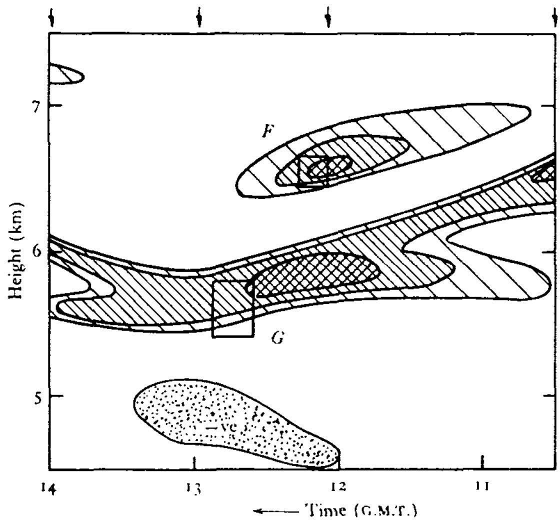

Fig. 10.3. Time–height pattern of Richardson number (defined over layers 200 m deep), which was obtained by Browning (1971) in the vicinity of two 'billow' events. These events were observed by radar and are indicated in duration and vertical extent by the rectangular frames marked F and G; the times of radiosonde ascents are marked by arrows. The increasingly dense cross hatching represents values of Ri of 0.3–0.5, 0.2–0.3 and 0.1–0.2 respectively, and in the stippled region $Ri < 0$.

governed by this criterion. In other situations, such as the atmosphere near the ground where the wind typically 'dies away' at night (as described in § 5.3.2), energy cannot be supplied rapidly enough to the shear layer and so the turbulence must decay.

10.2.2. *Non-equilibrium conditions: step formation*

When $\Delta\rho$ and ΔU are imposed over a given depth Δz, it is most unlikely that the external boundary conditions will exactly match the equilibrium 'internal' criterion, i.e. that

$$Ri_0 \equiv g\Delta\rho\,\Delta z/\rho(\Delta U)^2 = Ri_e.$$

Only in this case, however, could the equilibrium state be maintained, with linear gradients of density and velocity extending through the whole depth, and turbulent transports of buoyancy and momentum independent of position. If $Ri_0 < Ri_e$, the shear will

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dominate over the density differences, and the mixing will be an 'external' process, controlled directly by the boundary turbulence (cf. case (*a*) of fig. 4.19). The distance from the boundary z then is a relevant parameter, and one must use the z/L scaling described in §5.1.2. Of most interest here is the opposite situation where $Ri_0 > Ri_e$, which is strongly satisfied in deep layers of the ocean or atmosphere.

In this latter case, the stratification (and therefore internal processes) must certainly dominate, but if 'equilibrium Richardson number' ideas are still to apply, another smaller lengthscale must become relevant. Stewart (1969) has pointed to a way out of this dilemma. If the gradients $\partial\rho/\partial z$ and $\partial u/\partial z$ are very non-uniform, then a constant value of the gradient Richardson number (say Ri_e everywhere) is compatible with *any* value of Ri_0 – essentially because it is always true that

$$(\Delta U/\Delta z)^2 \leqslant \overline{(\partial u/\partial z)^2}. \tag{10.2.1}$$

If $Ri = Ri_e$ is the preferred state of a turbulent stratified fluid, then this can be achieved by a rearrangement of the density and velocity structure, for any given overall differences.

There are, of course, infinitely many ways of splitting up a given $\Delta\rho$ and ΔU to produce a constant $Ri = Ri_e$ (even if one ignores for the present the patchy nature of the observed distributions, and assumes horizontal uniformity), but a particular example will help to fix our ideas. Let us suppose that initially linear distributions of density and velocity, corresponding to $Ri_0 = \frac{1}{3}$, are to be transformed into 'well-mixed layers' and 'interfaces', subject to the conservation of mass and momentum. In both of these there is to be the same constant $Ri = Ri_e$, and the gradients are assumed to remain linear (but necessarily of very different magnitudes). Adopting for illustration the value $Ri_e = \frac{1}{16}$ suggested by the laboratory experiments described in §6.2.4, fig. 10.4 shows the various forms which can be taken by the velocity and density profiles for different ratios r of interface to total (layer plus interface) thickness H; the absolute value of H is arbitrary. Note that the contrast between the density gradients through the interface and in the layers is much greater than it is for the velocity gradients.

This idea resolves a basic difficulty, and shows that turbulence

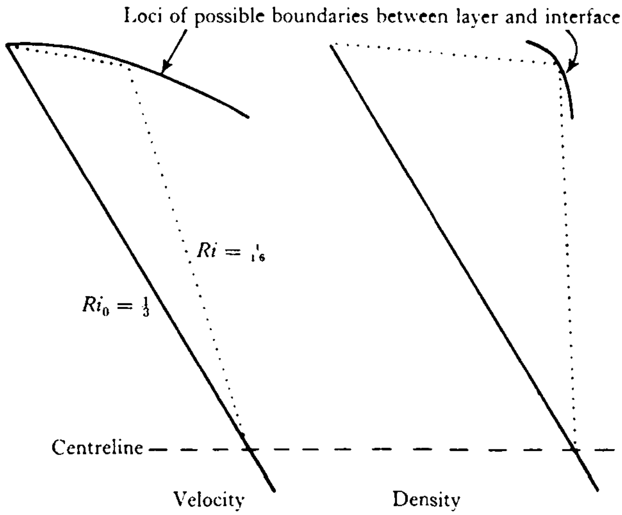

Fig. 10.4. Showing (half) the velocity and density profiles which are consistent with an initial layer Richardson number of $Ri_0 = \frac{1}{3}$, and have a linear 'layer' and 'interface' structure with $Ri = \frac{1}{16}$ everywhere.

need not be inhibited in a stratified fluid simply because the Richardson number is too high; a redistribution of properties can always reduce its value. It does, however, raise other questions which can be answered in principle but not yet in detail. The arguments of § 5.1.4 imply that in the 'equilibrium' state, the ratio of the buoyancy flux to the momentum flux is proportional to the velocity gradient (5.1.20). If this gradient is non-uniform, as it must be according to the arguments given above, then the transport of momentum is relatively less efficient at the interfaces where the velocity gradient is larger, and it will be impossible to have steady turbulent fluxes of both properties through the whole depth. One must either abandon the one-dimensional and steady state assumptions (and it is conceivable that the transport processes might be essentially unsteady, with moving interfaces and horizontal spreading playing the dominant roles) or take into account other processes, besides the *turbulent* stresses implied in (5.1.20), which can transport momentum.

For this, and other reasons to be outlined below, one is led to consider the transports due to internal waves (as foreshadowed by

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the earlier discussion of case (g), fig. 4.19). The generation of interfacial waves provides just the right kind of complementary mechanism to transport momentum across the steep interfacial gradients without correspondingly increasing the buoyancy flux, and a steady state can plausibly be achieved by a combination of these two mechanisms. Thus a stratified fluid with large overall Richardson number can be turbulent at most intermittently and in patches, while a large fraction of its volume will be in a state of laminar, wave-like motion.

10.2.3. *Energetics of a layered system*

There are other physical constraints on the possible non-uniform profiles which it is again convenient to illustrate using the horizontally uniform 'layer and interface' model pictured in fig. 10.4. The condition for the layers to have constant velocity and density, while $Ri = Ri_e$ in the interfaces, is just

$$r = r_0 = Ri_e/Ri_0, \tag{10.2.2}$$

using the earlier notation. Thus if Ri_e is regarded as given, and reversals of velocity gradients are excluded as implausible in a flow which has developed from a unidirectional shear, the interfaces when they first form will be relatively thinner the larger Ri_0 becomes. (Any horizontal advection which follows the local formation of a layer will of course tend to thicken the interfaces.)

Now consider the energy changes implied by the transition from linear gradients $u = \alpha z$, $\rho = -\beta z + \rho_0$ to a well mixed layer of depth H, bounded by sharp steps above and below. The change in kinetic energy is $T = \frac{1}{24}\alpha^2 H^3$ and in potential energy $V = -\frac{1}{12} g\beta H^3$; the decrease in T just balances the increase in V when

$$Ri_0 = g\beta/\alpha^2 = \tfrac{1}{2}. \tag{10.2.3}$$

This result may readily be extended to the case where the transition between the well mixed layers takes place over an 'interface' of finite thickness (in which $r = r_0$ and $Ri = Ri_e$, as in (10.2.2) above) to give

$$Ri_0 = \tfrac{1}{2}(Ri_e + 1) \tag{10.2.4}$$

as the condition for equality of T and V. At smaller values of Ri_0 than this, $T > V$ at $r = r_0$, and $T = V$ for $r < r_0$. Thus for each Ri_e there is an upper limit to Ri_0 such that all the energy for mixing could in principle have come from the *local* kinetic energy. The important point to emphasize is that the *overall* Richardson number Ri_0 is the parameter which determines the stability of the flow in the absence of an external energy supply, and in this sense flows with $Ri_0 \gg 1$ are stable. Note that the 'critical' Ri_0 values implied by (10.2.4) are somewhat larger than the limit $Ri_0 = \frac{1}{4}$ for infinitesimal disturbances found in §4.1.3, since a particular finite change in the distributions has been imposed. (See also Businger (1969) and Hines (1971) for discussions of this point. The latter has also shown that the energy condition for generation of turbulence is less stringent when the velocity vector is changing direction with height.)

A similar consideration of the overall energy balance also sets an upper limit on the thickening of an interface due to local turbulent mixing following the breakdown of an interfacial wave at $Ri < \frac{1}{4}$ (cf. §4.3.3). The profiles can at first become very non-uniform, with turbulence sustained at $Ri = Ri_e$, but when the thickness h of the mixing interface increases to a value such that $Ri_0 = g\Delta\rho h/\rho(\Delta U)^2$ is comparable with unity, the kinetic energy made available by the fluid entrained into the interfacial region will no longer be sufficient to change the potential energy (even if one neglects the reduction in turbulent energy due to viscous decay).

It seems appropriate to comment again here on the common tendency to confuse the several possible meanings of the 'Richardson number' (see §1.4). Not only is the term used without proper definition, but the numerical differences between various measured values are often disregarded once these have fallen well below some presumed critical value. The experiments described in §4.1.4, and the arguments given above show, however, that the growth rate of an instability and the amount of kinetic energy available for subsequent mixing are both strong functions of the magnitude of the relevant Richardson number, so it certainly is desirable to measure and quote this accurately. This question would probably not arise if an inverse parameter were in common use (10 and 100 are clearly different while 0.1 and 0.01 are lumped together as small!), and this is perhaps another argument in favour of using the 'gradient Froude number' F_g.

It is also of interest to calculate the Reynolds numbers in the layer and interface under various circumstances. The interface Reynolds number Re_i (based on its thickness and the velocity difference across it) is a maximum at $r = r_0$, when the corresponding layer Reynolds number Re_l is zero. For $r < r_0$, Ri_i decreases slowly and Re_l increases more rapidly, and at a particular $r = r_1$ it can be shown that

$$\begin{aligned} Re_i = Re_l &= r_1(1-r_1)H\Delta U/\nu \\ &= \frac{Ri_e}{(3Ri_e+Ri_0)}\frac{H\Delta U}{\nu}. \end{aligned} \qquad (10.2.5)$$

This can be much less than the overall Reynolds number

$$Re = H\Delta U/\nu$$

when $Ri_e \ll Ri_0$. The above result makes more explicit another suggestion due to Stewart (1969), that turbulence in a stably stratified fluid may be inhibited by viscosity, essentially because Re_i is too low. The density gradient enters indirectly, because large Ri_0 implies a small value of r and a restricted scale of motion. (At very low Reynolds numbers, turbulence cannot even arise, since the shear instability is inhibited – see §4.1.4.)

The results of this section, though based on a particular example, are more generally valid. Most important is the conclusion that when $Ri_0 \gg 1$, sufficient kinetic energy is just *not* available in the mean motion to change a smooth shear flow, having roughly similar profiles of velocity and density, to a step-like structure. To produce the non-uniform profiles, which are commonly observed in nature and which seem to be necessary if turbulence is to exist in the interior of large bodies of stratified fluid, some other supply of energy is needed. In chapter 8 it was shown how layers can be formed by double-diffusive processes which draw on the potential energy of an unstably distributed component. In the absence of such a local source of convective energy (and thus in a fluid stratified with temperature or salinity alone), one is forced to take account of energy supplied at the boundaries, or at a discontinuity of properties such as a front, which propagates into the interior in the form of internal gravity waves. No lengthscale for the layering due to mechanical mixing has emerged from the arguments so far given, and this too must be a consequence of the particular mechanism which supplies the energy to the interior.

10.3. Wave-induced mixing

Now that the role of wave energy in the interior mixing process has been more firmly established, we must retrace our steps and develop in this geophysical context some of the ideas introduced earlier, especially in chapter 4. The most important application will be to the phenomenon of 'clear air turbulence', an all-embracing name applied to various processes which also occur in the ocean. We will review in turn the mechanisms leading to the intermittent formation of turbulent patches, to the production of a non-uniform structure from smooth gradients, and to the possibility of 'saturation' of the wave amplitude. The questions of the likely sources of the wave energy and the distinction between waves and turbulence will also be raised briefly.

10.3.1. *Mixing at existing interfaces*

Atmospheric examples of the production of turbulence due to the growth of interfacial waves have already been described in chapter 4. A sufficiently large steady shear leads to an instability of the Kelvin–Helmholtz form, which can be observed using radar in the clear air (fig. 4.15 pl. IX) or is made visible because cloud is present (fig. 4.14 pl. X). Of more general importance is the fact that similar instabilities can be produced in regions of relatively large density gradient by long internal waves propagating through the fluid (§4.3.3). The shear is a maximum and the gradient Richardson number *Ri* a minimum at such interfaces, and modest wave amplitudes, acting in combination with mean shears due to larger scale motions, can reduce *Ri* to the critical value for instability. The wave-induced growth of K–H billows, and their subsequent breakdown to give patches of 'billow turbulence', has been beautifully documented by Woods (1968*a*, *b*) in the upper ocean (fig. 4.21 pl. X).

Woods and Wiley (1971) have gone further, and proposed that the whole of the vertical mixing in the ocean might be described in terms of this process, following the formation of a few sharp inter-

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faces in some other way, for example by wind mixing or by larger scale intrusion of one water mass into another. The local breakdown of a single 'sheet' will produce a patch of relatively well-mixed fluid which is elongated horizontally by the vertical shear and by spreading along surfaces of constant density. At first this patch will be turbulent, but for energetic reasons (§ 10.2.3) the turbulence must decay as fluid is entrained from above and below. The net result, they argue, is to replace a single sharp interface by two interfaces, separated by a mixed layer (cf. fig. 4.11 p. 105). (The process is probably not as clear-cut as this, since a single breaking event may produce multiple interfaces, as suggested by fig. 4.12 pl. VIII; but this complication does not affect the substance of the argument.) The passage of subsequent long waves will produce further 'splitting' of the sheets, until in the mature state the whole water column is filled with microstructure which has developed from the original sharp interfaces, by a combination of splitting and superposition of horizontally non-uniform structures. Molecular diffusion must eventually act to limit and smooth out the microstructure. At any one time, only a small fraction of the volume contains turbulent patches, but the net effect of many such events is to give mean transports of heat, salt and momentum which are much greater than the molecular rates. The fluxes will of course depend on the large scale processes producing the shear, and will be largest in the neighbourhood of fronts.

This picture is consistent both with observations such as those of Woods and Wiley and with the theoretical constraints outlined in § 10.2.2. A major part of the vertical flux of horizontal momentum in the ocean (and also the atmosphere) must be carried by internal waves. The fraction is largest near density interfaces, where interfacial waves can grow, and as these intermittently break, they also produce a smaller turbulent transport of the scalar quantities. Plausible estimates of the various contributions to the total transports can be obtained from the existing data, but in the absence of a fully quantitative theory these will not be pursued here.

10.3.2. *Formation of layers from a smooth gradient*

When interfaces exist, they are the first regions to become unstable to internal waves, but waves can also lead to the formation of turbulent patches in a smoothly stratified fluid, with non-uniformities of the vertical gradients developing only as a result of the subsequent horizontal spreading of the mixed regions. The overturning of the streamlines in lee waves and the production of rotors is the most obvious example of such a process.

When the mean horizontal velocity u varies in the vertical, the 'critical layer' mechanism (§2.3.2) can lead to the growth of internal gravity waves and the absorption of energy in limited regions. At levels where u equals the horizontal phase velocity of the waves, the group velocity relative to the fluid also tends to zero and wave energy accumulates. The amplitude grows, dissipation is increased either because of viscosity or by wave breaking, and eventually wave energy and momentum are transferred to the mean flow. The distortion of the velocity profile produced in this way was remarked on earlier in connection with the laboratory experiment illustrated in fig. 2.15 pl. IV. Small scale jets, which have been interpreted by Woods (private communication) in terms of the local addition of momentum to the mean flow when a wave breaks, are also a common feature of the oceanic thermocline.

Bretherton (1969*a*, *b*) has shown that a substantial fraction of the drag exerted by an obstacle can be effective well away from the boundary: for example the drag exerted by the Welsh mountains on the airflow over them often acts at great heights. Most of the momentum is added at the critical layer, but it can also be absorbed elsewhere in the flow, in any region where friction dissipates the waves. Thus patches of turbulence (i.e. locally well-mixed layers) could be self-sustaining so long as wave energy is propagating into them, even if they do not remain near a critical level. There is also the possibility that a more gradual absorption of wave momentum will so distort the velocity profile that a critical layer develops where there was none before. An internal gravity wave could in this way produce the optimum conditions to hasten its own decay.

Another mechanism for the concentration of energy in a smoothly stratified flow involves weak resonant interactions, which are most

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important in just the conditions of interest here (when the overall Richardson number is large). If is often difficult to separate this from the result of superposition of waves from several sources, so both effects will be considered together. A striking illustration of wave breakdown of this kind has been provided by the experiments of McEwan (1971), the earlier stages of which were described in §2.4.3, and it will be useful to enlarge on them here. He generated a standing internal wave in a laboratory tank, and observed that for sufficiently large amplitudes, the waveform became modulated by other waves having a frequency different from the forcing frequency (fig. 10.5*a* pl. XXIV). In most cases the higher mode waves could be identified with an unstable pair which formed a resonant triplet with the forced wave. They grew by extracting energy from the forced mode, until the superposition of the several motions produced visible local disturbances of the smooth gradient. These were of two kinds, either an overturning or a violent horizontal convergence followed by vertical divergence. In both cases there was a subsequent collapse to form sharp density discontinuity layers, which seemed to be 'contagious', i.e. they tended to spread in consecutive cycles of oscillation in groups round the original discontinuity, as the first disturbances contributed to the kinematic distortion elsewhere. Such disturbances were often self-stabilizing, because the small scale structure so produced gave enough viscous dissipation to limit the growth and suppress further breaking. With stronger forcing, however, patches of turbulent fluid could be formed and sustained, as shown by the fine structure in shadowgraph pictures (fig. 10.5*b*).

McEwan also demonstrated that, although the conditions leading to permanent distortion are realized more easily when growing resonant modes are produced, similar effects can be observed without such interactions provided the forcing amplitude is sufficiently large. The photographs of fig. 10.5*c* and *d* pl. XXIV were taken in an experiment where the lowest 1/1 mode was generated (a case for which there was evidently no unstable interaction), and they show the growth of disturbances in time, and the final breakdown. Even in this simpler case, for which the amplitudes of the fundamental and its harmonics could be calculated, it was not possible to relate the kinematic conditions for breakdown to any of

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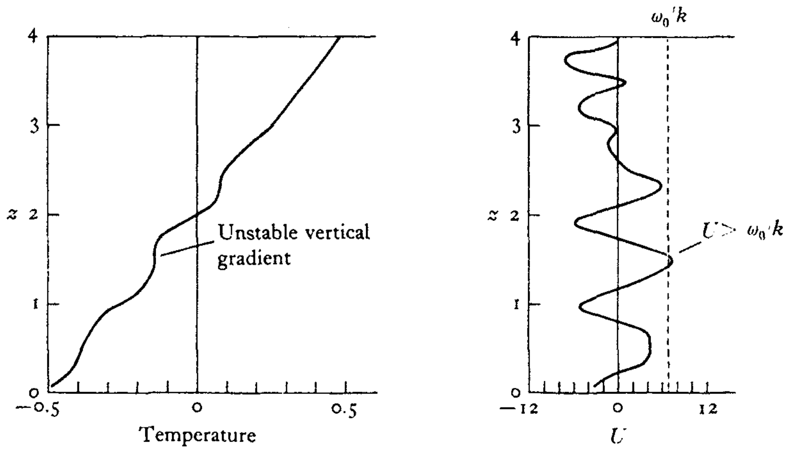

Fig. 10.6. Vertical profiles of temperature and horizontal velocity in the vicinity of an overturning event, according to the numerical calculation of Orlanski and Bryan (1969). Internal waves were generated by resonant forcing in the top quarter of the region, but at the instant shown the amplitude was largest in the lower part. (From Orlanski and Bryan (1969), *J. Geophys. Res.* **74**, 6975–83.)

those which apply in other better understood situations. The maximum slope of the constant density surfaces was always small, and the calculated minimum gradient Richardson number remained large (of order 5), so that neither an overturning nor a shear instability would have been predicted on the basis of earlier work.

A numerical model leading to an overturning instability has been described by Orlanski and Bryan (1969). They too considered a fluid with constant N^2 contained in a two-dimensional rectangular region, and assumed a disturbance consisting of an oscillating body force applied at $t = 0$ to the upper quarter of the fluid. Using the inviscid equations of motion in the Boussinesq approximation, they followed the growth from rest of a wave resonating with the forcing function, with a dominant vertical scale smaller than the total depth. Ultimately, non-linear effects dominated, and higher wavenumbers appeared, which by superposition led to overturning, at points half way between crests and troughs and between horizontal nodal lines. As shown in fig. 10.6, at particular times the points of instability can be remote from the generating region. The condition for overturning corresponds to a state where the disturbance velocity

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just exceeds the phase velocity locally (which is consistent with the 'critical layer' arguments of §2.3.3).

The energy input near the boundary, which is required to produce overturning in this numerical model, decreases as the vertical scale of motion is reduced. When viscosity is taken into account, however, the dissipation also increases rapidly because of the larger shears associated with the smaller scales. (See Johns and Cross 1970, McEwan 1971.) These two effects combine to give a sharp minimum, thus defining a scale for which the energy needed is a minimum, and Orlanski and Bryan have suggested that this will be of the order of tens of meters in the main thermocline. Though the numerical values are subject to revision, this does seem to be a promising approach to the question of a preferred layer depth.

10.3.3. *Statistical aspects of wave generation and breaking*

So far, only relatively simple generation and interaction processes have been considered, involving particular forced modes and resonant triads whose behaviour could in principle be calculated explicitly. Now we must recognize the possibility of interaction between many waves generated in different places, and abandon any attempt to follow each of them in detail. Broadly one can say that non-linear interactions will tend to transfer energy from large scale internal waves into shorter and shorter wavelengths, and eventually there will be a whole spectrum of resonant modes, which can only be treated by statistical methods. Associated with the smaller vertical lengthscale of the higher modes will be an increased shear $\partial u/\partial z$, and so the gradient Richardson number *Ri* will be decreased in localized regions, spaced a small distance apart compared with the total depth. Eventually there will be a breakdown to turbulence in randomly distributed patches, where the superposition of many contributions to the local shear and density gradient has led to *Ri* falling to some specified value, say below a critical value of about $\frac{1}{4}$.

Bretherton (1969*c*) has proposed a model of this process which is based on an analogy between weak resonant interactions and the theory of colliding molecules in a dilute gas. Using a linear superposition and assuming that the magnitudes of velocity and density gradients are normally distributed, he showed that the probability that $Ri < \frac{1}{4}$ depends only on the overall N^2 and on $\overline{(\partial u/\partial z)^2}$ through the parameter

$$\sigma^2 = \overline{(\partial u/\partial z)^2}/N^2, \qquad (10.3.1)$$

a kind of inverse Richardson number for the wavefield as a whole. For small σ the probability of breakdown is

$$P \sim \frac{\sigma}{\sqrt{\pi}} \mathrm{e}^{-(1/2\sigma^2)}. \qquad (10.3.2)$$

Though this must certainly be an underestimate (since non-linear effects can only increase the distortion and the number of turbulent spots, and McEwan (1971) has demonstrated that instabilities can occur for much smaller shears) the conclusion that a substantial change in P can be associated with a small variation in σ will probably remain valid. Thus it seems likely that a statistically steady state could be achieved, in which the energy input to large waves is balanced by the dissipation in turbulent patches, with a minor change in the motions on intermediate scales. That is, (10.3.2) suggests that large differences of energy input can be accommodated by increasing the volume of the dissipating regions, while (for a given N^2) $\overline{(\partial u/\partial z)^2}$ changes little.

A similar argument due to Garrett and Munk (private communication) has gone a step further, and relates the input of wave energy to the rate of vertical mixing. They have derived a universal energy spectrum describing internal waves in the deep ocean, which is based on observations extending over the inertial range as well as gravity wave scales and frequencies. Using this to estimate the probability of breakdown, together with mechanistic models of the mixing produced by each breaking wave event, they arrive at estimates of the vertical eddy diffusivity K which are very sensitive functions of the shear. This again supports the view that the suggested universal spectrum could be a consequence of a saturation effect. The rather narrow range of K (of order 1 cm^2/s) deduced from observations in the deep ocean (Munk 1966) implies that the mean rate of input of wave energy also varies little, at least over long time scales.

Some kind of saturation of the quasi-horizontal motions due to waves of large horizontal and small vertical scale is also indicated by certain more detailed observations in the ocean, which it therefore seems relevant to mention here. The form in which these are presented is most easily understood by quoting first a result for two-dimensional wave motions which follows from (2.2.5). The vertical velocity through a region in which N^2 is a slowly varying function of z, rather than constant, is

$$\hat{w}(z) \sim AN^{-\frac{1}{2}} \exp\left\{\mathrm{i}\int (Nk/\omega)\,\mathrm{d}z\right\} \qquad (10.3.3)$$

(see Phillips 1966*a*, p. 174). Here A is a dimensional constant determined by the boundary conditions, but the whole of the vertical dependence of the amplitudes of the velocity fluctuations for any high order mode is contained in the function $N^{-\frac{1}{2}}(z)$. Using the continuity equation with (10.3.3) shows that the corresponding horizontal velocity is

$$u(z) \propto N^{\frac{1}{2}}(z); \qquad (10.3.4)$$

that is, in regions of large density gradient, horizontal motions are increased while vertical motions are decreased.

Fofonoff and Webster (1971) have analysed current records at various depths in the deep ocean and have found that below the surface layer the horizontal kinetic energy in the band of frequencies associated with gravity waves is proportional to N, that is

$$\tfrac{1}{2}\rho(\overline{u^2} + \overline{v^2}) \propto N(z), \qquad (10.3.5)$$

in agreement with (10.3.4). Even the spectra can be brought to an identical form by scaling with the local value of N, as is shown in fig. 10.7. Moreover – and this is a point of special interest here – Webster (1971) has shown that the constant of proportionality varies surprisingly little with geographical position or with the strength of the local winds. This suggests again that the intensity is internally limited, and also that surface disturbances have little direct effect on the deeper motions.

Very little is known about the sources of the wave energy in the ocean, but some ideas about the relative effectiveness of generating waves at various levels can be obtained using an extension of the above arguments. For given external boundary conditions, it can be shown that σ^2 defined by (10.3.1) is an increasing function of N, so that the probability of breakdown is increased where the density

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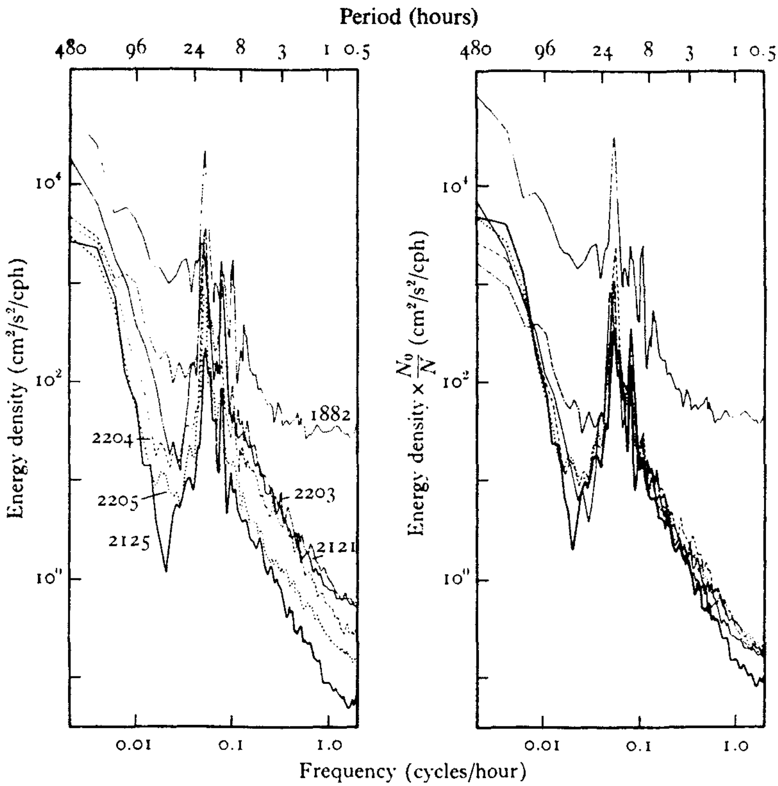

Fig. 10.7. Energy density spectra computed from measurements of horizontal velocities at various depths in the ocean, as reported by Fofonoff and Webster (1971). On the left are the directly measured values and on the right are the same data normalized using the local value of the buoyancy frequency. Note that the curves for all depths below the surface layer are superimposed in the range of frequencies above the inertial peak. The record numbers correspond to the following depths: 1882, 7 m; 2121, 50 m; 2203, 106 m; 2204, 511 m; 2205, 1013 m; and 2125, 1950 m.

gradient is large (cf. §4.3.2, where a similar result was quoted for interfaces). This means that, if energy is added at the top of a thermocline where the density gradients are a maximum, no instability can be produced at greater depths unless 'breaking' is already present nearer the surface. The observations suggest that a wind-mixed surface layer (§9.2.1) can have immediately below it a gradient region in which intermittent mixing is driven by wave energy propagating downwards from the interface (see, for example,

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Kitaigordskii and Miropol'kii 1970). The thickness of this layer, which is indirectly but simultaneously mixed by the surface processes, must be strictly limited, however, since so much energy will be lost near the source that the amplitude of the propagating waves will fall below the level necessary to produce instability lower down. Wave energy can nevertheless be fed in this way from the surface into the deeper ocean, to interact later with that originating in other regions and produce intermittent mixing which cannot be identified in space or time with particular sources.

On the other hand, waves generated on the ocean bottom or at the edge of a shelf, which are regions of minimum N^2, will tend to become more unstable as they propagate upwards. Some fraction of the energy can be absorbed at each level due to breaking, while still allowing enough through to make the higher layers unstable in their turn. No such difficulty arises in the discussion of wave generation in the atmosphere, since the major wave generating processes (at the surface) are naturally associated with the weakest density gradients. Note again too that the increasing instability with increasing height (and increasing N^2) will be enhanced in the atmosphere by the decrease in air density (see §4.3.5).

10.3.4. *Waves and turbulence in large scale flows*

The interpretations of geophysical phenomena suggested above have of course been deliberately simplified, so that the physical processes could be identified and discussed individually, and always with buoyancy forces taken as the major controlling factor. We will now consider the relation between the various phenomena described, and comment briefly on some of the effects which have been omitted.

It is apparent that 'clear air turbulence' (and its equivalent in the ocean) can encompass a great variety of physically distinct phenomena, each of which may become important if the circumstances are right. Sometimes it can be associated with a particular part of the boundary (where separation occurs behind an obstacle, for example, or an internal hydraulic jump is generated downstream). Strong enough shears, especially across density interfaces, will lead to a local instability and the production of turbulence. With the generation of waves, the effects of boundaries can be propagated far into the

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interior of a stratified fluid. In different conditions, stable wave motions of large amplitude, those which are in the process of overturning, and those which have actually broken down to give smaller scale turbulence may have similar effects on aircraft, but the various alternatives must be distinguished if one is to obtain a proper understanding of what is observed.

This raises the more general question of the criteria which can be used, in principle and in practice, to decide whether motions in a stably stratified fluid should be called 'waves' or 'turbulence'. It is clearly inefficient to lump everything together. We must learn to recognize the characteristic features of the two kinds of motion so that we can design observations more intelligently and analyse them in the most suitable way. As pointed out by Stewart (1969),† the distinction is not always clearcut. Internal waves of large amplitude produce turbulent patches (§§4.3, 10.3.2) and turbulence can generate waves (§§7.3.4, 9.2.4), and when such non-linear effects can continuously exchange energy between the two kinds of motion it is difficult to draw a firm line between them.

Many phenomena can, however, be put unambiguously in one category or the other, and the distinctive features can be listed. The most important property of turbulence is its ability to produce mixing, thereby transporting scalar quantities such as heat or salt both along and across surfaces of constant density. Associated with this is the fact that turbulence is a strong interaction phenomenon, and highly dissipative. There is little connection between the motion at well separated points in the fluid, and energy propagates slowly, with the speed of the fluid motion. Waves, on the other hand, may distort the density distribution to produce an apparent, transient layering (§10.1), but they cannot permanently change the stratification unless they 'break' to produce turbulence. Waves are characterized by a dispersion relation, a definite relation between their speed and scale. They transfer energy more rapidly through the fluid, varying only gradually as they do so, and preserve phase relations over several wavelengths at least.

Both observation and theory support the view that in the interior of strongly stratified fluids, such as the atmosphere and ocean,

† Presented at a Colloquium on 'Spectra of Meteorological Variables', the whole of which is recommended to those interested in this field.

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turbulence as defined in this way can occur only sporadically and in isolated patches. Through most of the volume, wave theories should therefore be the more appropriate, whereas inside these patches one should change to a description in terms of turbulence (cf. §5.2). Sometimes, by using tracer techniques for instance (§4.3.3), it may be possible to pick out these regions directly, but more often the state of motion will have to be diagnosed from temperature and velocity records like those of fig. 10.2. Analysis of the cross spectra of these variables seems the most promising method so far proposed. Wave motions will give a high coherence between temperature and vertical velocity, but since the fluctuations are 90° out of phase (§2.2), no vertical transport is implied. (See, for example, Axford (1971), who has analysed observations of internal gravity waves in this way.) Phase angles near zero or 180°, implying a much stronger co-spectrum than quadrature spectrum and a non-zero vertical transport of heat, will be indicative of motions which are mostly turbulence.

Patches of turbulence in the ocean or atmosphere can arise as a result of the superposition of motions from many sources and on many scales. A completely deterministic theory is therefore unlikely, and detailed forecasting of clear air turbulence will always be very difficult, but there are obvious ways in which the discussion given here can be extended and made more realistic. The mean shears certainly contribute to the internal breakdown mechanism, and one must also include the larger scale, quasi-horizontal motions associated with inertial waves. A full description of even the smaller scale motions will strictly involve both stratification and rotation, with the theory of inertio-gravitational waves replacing that of pure gravity waves. It seems likely, for instance, that inertial waves could lead to the critical conditions for turbulence being achieved simultaneously everywhere in a horizontally elongated patch, rather than in a more localized region which then spreads sideways.

More detailed measurements will be needed before we can even properly describe the transport mechanisms in the atmosphere and ocean. Already in recent years there has been a notable change in attitude to small scale observations and the associated theoretical and laboratory work, and each new technique has led to further

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advances. The characteristic layered and patchy structure, and the importance of density interfaces, have become widely appreciated, and more information will come with increased resolution. There is still a great need, however, for more systematic surveys, designed not only to learn more about individual mixing events but also to determine how these are distributed in space and time, and how they are related to the local gradients and the energy inputs. Remote sensing techniques based on radar and acoustic sounding offer the best hope for rapid progress, and an example of the kind of measurement which can already be made is given in fig. 10.8 pl. XXII. (See also Beran, Little and Willworth 1971.) More attention should also be given to the problem of identification of significant events on a record, before embarking on routine analyses (such as the calculation of spectra) which may sometimes be quite inappropriate.

The detailed description of the ocean and atmosphere will undoubtedly change as more information becomes available and additional effects, especially those introduced by rotation, are added to those considered here. We can be confident, however, that microphysical processes will always be important, and that they will be described using concepts closely related to those which have been discussed. It is only by building systematically on our theoretical and laboratory knowledge of individual physical processes that we can hope first to recognize, and then to understand, the fascinating array of more complex natural phenomena.