Dynamics and Structure of Stratified Flows Around Wedge: from Diffusion Induced up to Transient Vortex Regime

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DYNAMICS AND STRUCTURE OF STRATIFIED FLOWS AROUND WEDGE: FROM DIFFUSION INDUCED UPTO TRANSIENT VORTEX REGIME

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Abstract

The problem of evolution of continuously stratified flow on wedge is investigated. The flow is characterized by a wide range of values of internal scales that are absent in a homogeneous liquid. Solution of the problem is constructed numerically in the complete non-linear formulation using finite volume method in the open source package OpenFOAM. The calculation results of flows in a stratified fluid around the stationary and moving wedge are shown. High-gradient interfaces and pattern of internal waves have been identified like observed in laboratory experiments.

Keywords: stratification, wedge, numerical simulation

1 Introduction

The observations showed that all natural systems (atmosphere of stars and planets, Earth's hydrosphere) as well as liquids and gases in the laboratory have a "fine structure". Areas with relatively slow changes in the parameters are separated by thin high gradient interfaces. Especially interesting are the spatially ordered structure having a high degree of internal symmetry and specific shapes.

The atmosphere and hydrosphere often stably stratified due to gravity separation of a non-uniform fluid. Density profile is determined by profiles of temperature, pressure and admixture concentration. Non-equilibrium medium with molecular flows of stratified admixture is at rest only when the density gradient parallel to the gravity. Breaking of the molecular flow on impermeable boundaries of arbitrary shape generates flow induced by diffusion. The currents include interfaces, large slow vortices and dissipative-gravity waves. More complex structures essentially depending on parameters of the problem (stratification value, obstacle geometry and flow velocity) are formed at the beginning of a body motion.

Stratification effects playing an important role in hydrodynamics of natural systems are actively investigated during the last 50 years both theoretically and experimentally. Scientific interest in the problem is due to the need to study a number of phenomena in the environment, including diffusion induced flows on topography [1], mild and intensive valley or mountain winds in the atmosphere [2, 3], slope flows in the ocean [4] and the self-motion of biological objects [5]. Under normal conditions the disturbances, which are concentrated in thin layers at impermeable surfaces, can reach storm values when large temperature gradients are formed in the atmosphere in the mountain systems, near the glaciers or on steep slopes in the oceans and laboratory [1, 4].

The original theory was developed for stationary flows induced by diffusion on a sloping plane in gases and liquids [1, 4]. Later unsteady problems of flows on the plane and the plate of finite length began to be investigated. Numerical models for the process of stabilization of diffusion induced flows on various obstacles such as a cylinder [6], sphere [7], horizontal and sloping plate [8] were developed together with the analytical studies [1, 4]. The observations in the laboratory by using sensitive colour schlieren technique supported theoretical studies [9]. The system of non-stationary dissipative gravity waves was observed in experiments at the obstacle boundaries together with large vortices.

A great practical interest lies in computations of diffusion induced flows on asymmetric obstacles, which enable producing self-motion under action of the buoyancy forces. Diffusion induced flows provide self-motion of free neutral buoyancy bodies that was demonstrated by uniform movement of horizontal wedge in a quiescent stably stratified medium [10]. Such flows, being intensified by fluxes of additional substances due to the natural metabolism, play an important role in the dynamics of self-motion of phyto- and zooplankton (“diffusion fish”).

2 Problem statement and solving method

Unsteady 2-D problem of the evolution of a flow of continuously stratified fluid flow around the horizontally placed wedge is solved in this study by numerical methods.

Mathematical modelling of the problem is based on the equation of incompressible stratified fluid mechanics set with undisturbed density distribution $\rho_0(z)$ given by a stable salinity profile $S_0(z)$ where the axis $0z$ is directed upright. The governing system includes equations of state for density, continuity, Navier-Stokes accounting for the gravity in the Boussinesq approximation, diffusion of stratifying agent as well as no-slip and no-flux boundary conditions on solid boundaries

$$\rho = \rho_{00}\left(-z/\Lambda + s\right), \tag{1}$$

$$\nabla \mathbf{v} = 0, \tag{2}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\nabla\right)\mathbf{v} = -\frac{1}{\rho_{00}}\nabla P + \nu\Delta\mathbf{v} - s\mathbf{g}, \tag{3}$$

$$\frac{\partial s}{\partial t} + \mathbf{v}\cdot\nabla s = \kappa_S \Delta s + \frac{v_z}{\Lambda}, \tag{4}$$

$$v_{x,z}\big|_{\Sigma} = 0,\ s\big|_{x,z\to\infty} = 0,\ v_x\big|_{x,y\to\infty} = U_0,\ v_z\big|_{x,z\to\infty} = 0, \tag{5}$$

$$\left\|\frac{\partial S}{\partial n}\right\|_{\Sigma} = -\frac{1}{\Lambda}\frac{\partial z}{\partial n} + \left\|\frac{\partial s}{\partial n}\right\|_{\Sigma} = 0. \tag{6}$$

Here, $S = S_0(z) + s$ is total salinity including the salt contraction coefficient, s is the salinity perturbation, ρ_{00} is density at zero (neutral buoyancy horizon), $\mathbf{v}$ is velocity vector, P is the pressure except for the hydrostatic one, ν and κ_S are the constant kinematic viscosity and salt diffusion coefficients, t is time, ∇ and Δ are Hamilton and Laplace operators, respectively, $\mathbf{g}$ is the gravity acceleration, $\Lambda = \left|d\ \ln\rho_0(z)/dz\right|^{-1}$, $N = \sqrt{g/\Lambda}$ and $T_b = 2\pi/N$ are the buoyancy scale, frequency and period, respectively, n is external normal to the wedge's surface Σ, U_0 is external flow velocity.

The set of equation (1-4) with the initial and boundary conditions (5, 6) is characterized by a number of characteristic scales including time scale, $t = T_b$, velocity scales ($U_N^{\nu} = \sqrt{\nu N}$, $U_N^{\kappa_S} = \sqrt{\kappa_S N}$, U_0) and length scales (buoyancy scale Λ, horizontal and vertical dimensions of the obstacle L and h, the length of the gravity surface waves $\lambda_s = 2\pi U^2/g$, and internal ones $\lambda_i = UT_b$, viscous $\delta_N^{\nu} = \sqrt{\nu/N}$ and diffusion $\delta_N^{\kappa_S} = \sqrt{\kappa_S/N}$ microscales). A variety of length scales and differences in their values (4-6 orders of magnitude) indicate the complexity of internal structure which arises due to the non-uniformity of molecular flux of the stratifying agent. Macroscales characterize the size of the computational domain, which should contain all the components of the course studied. Microscales determine the spatial resolution of the computational grid.

Solution of the equation system (1-6) is constructed numerically in the complete non-linear formulation using finite volume method in the open computational package OpenFOAM (www.openfoam.com). The original solver with the standard package icoFoam was supplemented with the original program modules accounting for the stratification and diffusion effects [8, 11]. New variables (ρ, s), the additional equations (1, 3), and new physical parameters (N, Λ, κ_S, $\mathbf{g}$) were supplemented to the standard solver icoFoam. The boundary condition of the salinity perturbation (6) has been realized by using the extended utility funkySetBoundaryField. It allows defining analytical expressions for the physical variables. Vorticity, forceCoeffs, funkySetFields and other utilities were used to calculate additional physical variables.

Parallel computations of the problem were carried out in the web-laboratory UniHUB (www.unihub.ru) and Research Computing Centre "Lomonosov" (www.parallel.ru). The following values of the problem parameters were used in the calculations: $\rho_{00} = 1020\ \mathrm{kg/m^2}$, $g_z = 9,8\ \mathrm{m/s^2}$, $\nu = 10^{-6}\ \mathrm{m/s^2}$, $\kappa_S = 1,41\cdot 10^{-9}\ \mathrm{m/s^2}$, $T_b = 6.28$ s, $L = 10$ cm, $h = 2$ cm.

3 Computational results and discussions

Construction of numerical solutions of the problems for the stratified flows formations was started from calculations of the diffusion induced flow on motionless obstacle, which was instantaneously introduced without perturbation of the initial salinity profile [11]. After phase of fast adaptation almost stationary pattern of disturbances is formed.

In fields of salinity and horizontal component of its gradient perturbations, produced by stationary diffusion induced flow, which are shown in Fig.1 and Fig. 2, the areas of surplus (green) and deficit (blue) of values are presented. The most fine structures are placed near the sharp corners of the wedge. The stationary fields of diffusion induced flows around the motionless wedge were selected as the initial conditions for the problem of flow formation with starting of the wedge motion.

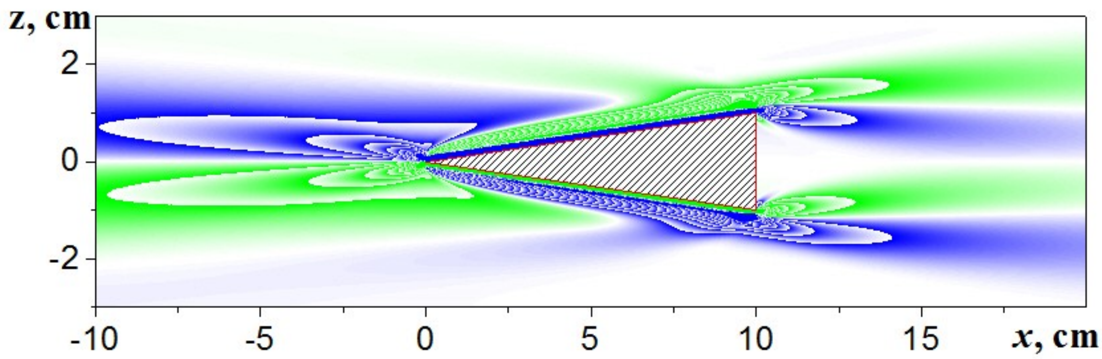

Figure 1: Field of salinity perturbations ($L = 10$ cm , $h = 2$ cm, $T_b = 6.28$ s, $U_0 = 0$).

Field of salinity perturbation gradients reflects the complex periodic structure of the diffusion induced flow (Fig. 2). The horizontal extent of the structure does not contradict the experimental data of the refractive index in a lab tank ("color schlieren method" with a horizontal slit and grating) for bodies of other geometric shapes [9]. The overall structure of the image is typical for stratified flows, in which the forces of buoyancy inhibit vertical motion. Inhomogeneity of the vertical flux of the molecular substances caused by impermeable bodies in the fluid interior or slopes of its boundary creates horizontal components of the density gradient that forms the flow even in the absence of additional force factors. In formation of flow structure the important role played by edge effects.

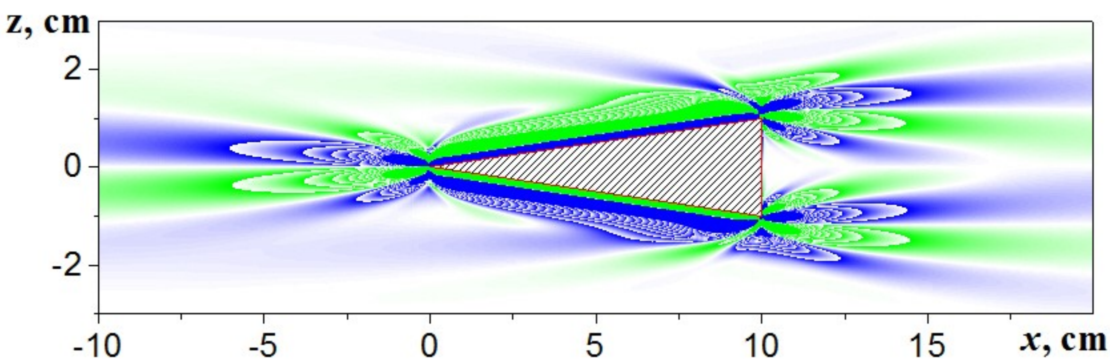

Figure 2: Field of horizontal component of salinity gradient perturbations $\partial s/\partial x$ ($L = 10$ cm , $h = 2$ cm, $T_b = 6.28$ s, $U_0 = 0$).

Around the corner points of the wedge the additional fine-scale components are formed. Flowing out from sharp edges thin streams forming along each side of the wedge, generate zero frequency internal waves. High longitudinal gradient perturbations of salinity of the order $\left|\partial s/\partial x\right|_{\max} = 4\cdot 10^{-2}$ recorded near corner points of the wedge where the values of the disturbance of salinity are 10^{-5} . With the going away from the obstacles perturbation of salinity gradient decreases sharply and reaches values 10^{-6} at horizontal distances of about 5 cm and vertical of 0.5 cm.

The evolution of the fields of the horizontal component of gradient for salinity disturbances around wedge which abruptly started to move uniformly from rest at a velocity U = 1 cm /s in the fluid to the period of buoyancy $T_b = 6.28$ s is shown in Fig. 3.

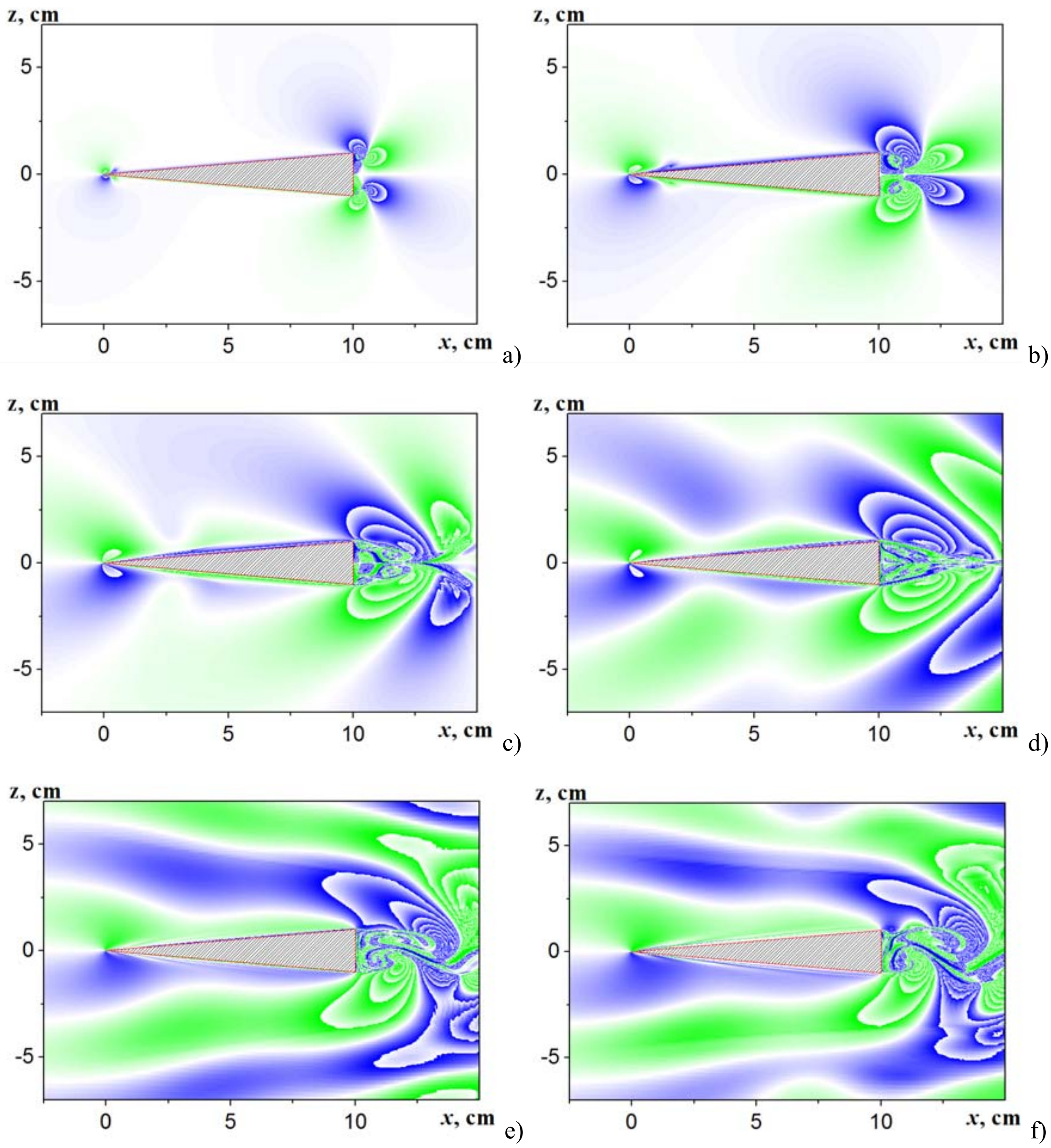

Figure 3: Temporal evolution of the fields of perturbations for horizontal component of salinity gradient $\partial s/\partial x$ ($L = 10$ cm , $h = 2$ cm, $T_b = 6.28$ s, $U_0 = 1$ cm/s): $(a-f)$ $\tau = t / T_b =$ 0.1, 0.3, 0.8, 2.2, 10.0, 30.0.

Since the start of the wedge motion inside the continuously stratified fluid begin to form upstream and downstream disturbances, fine interfaces near upper and lower sides of the wedge, rosettes of transient and fields of attached internal waves and vortex wake as past the moving strip [12]. Well defined vortex systems are formed past the extreme points of the wedge corners (Fig. 3, a). In the bottom part of the flow inside the field of four initial vortices forming pair of attached rear vortices cam be identified. They are separated from outer vortex flow by bounding high gradient interface (Fig. 3, b). Internal waves around rear part of the wedge, where they are created by the body and expanding vortex system are more expressed than near the leading edge (Fig. 3, c). Leading and rear internal wave systems interfere and are vanished in the central part of the wedge (Fig. 3, d). Shape of the rear vortex boundary is gradually deformed into triangle (Fig. 3, d). System of split interfaces forming in vicinity of the leading edge gradually fill the whole flow near the side surface of the wedge supplementing flow components of the large scales (Fig. 3, e). Different vortex components inside the wake are bounded by high gradient interfaces having typical mushroom shapes.

With time the flow pattern became well organized (Fig. 3, b, c) and consistent with the results of experimental and numerical studies of stratified fluid flow around bodies with other geometric shapes [12]. The sources of the internal waves are the wedge corners, generating intense vertical displacement of fluid. A deviation of fluid layers from the original position of neutral buoyancy creates consequently their periodic oscillations. Irregularities of crests and troughs of internal wave shapes reflect complex pattern of interference of growing transient and attached internal waves.

All flow components both of the large and small scales simultaneously interact with each other generating new growing and decaying vortices, waves and fine interfaces. As results the rear vortex pinch off the body forming waving transient wake (Fig. 3, e, f).

4 Conclusions

Universal code to calculate flow past 2D uniformly moving wedge in continuously stratified in a wide range of the fluid parameters developed. Fine structure of diffusion induced flow and transient vortex regime calculated and used as initial conditions in further calculations. All observed in experiment flow components including upstream disturbances; fine interfaces, internal waves and downstream wake with submerged transient vortices well reproduced. Rich fine flow structure visualized in vicinity and far from the obstacle.

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