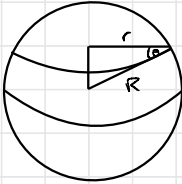


17.5/20

1)  $h$  = height,  $\theta$  = latitude,  $\vec{\omega}$  Earth's rot.



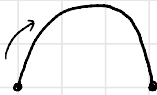
$$r = R \cos \theta$$

$$\omega = \frac{2\pi}{24h} = \frac{2\pi}{24 \cdot 3600s} \Rightarrow \omega = 7.27 \times 10^{-5} \frac{1}{s}$$

Then the velocity  $v = \omega r = \omega \cdot R \cos \theta$   
 $= 7.27 \times 10^{-5} \frac{1}{s} \cdot 6.371 \times 10^6 m \cdot \cos \theta$

•  $s = v \cdot t$

Further  $h = v_0 t - \frac{1}{2} g t^2$  but it is a parabola



$$\Rightarrow h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

Thus the total time is  $t_{tot} = 2 \sqrt{\frac{2h}{g}}$

then  $s = v \cdot t = 7.27 \times 10^{-5} \frac{1}{s} \cdot 6.371 \times 10^6 m \cdot \cos \theta \cdot 2 \sqrt{\frac{2h}{g}} = \dots$

to the west.

3.5/4

## 2) Floating Iceberg

Find fraction of volume of iceberg below sea level.

let  $m_i$  and  $V_i$  be its mass and volume.

Mass density of ice is  $M/V = \rho_i = 0.917 \text{ g/cm}^3$  ✓

let  $V_w$  be the volume below water = volume of water displaced,  
its mass:  $m_w = \rho_w V_w$  with  $\rho_w = 1 \text{ g/cm}^3$  (water)

Then the weight of the iceberg is given by

$$W_i = m_i \cdot g, \quad g = 9.81 \frac{\text{m}}{\text{s}^2} \quad \checkmark$$

And buoyant force equals weight of displaced water, thus

$$W_w = M_w \cdot g \quad \checkmark$$

and since it floats

$$W_w = W_i$$

$$\Rightarrow M_w g = M_i g$$

$$V_w \rho_w = V_i \rho_i$$

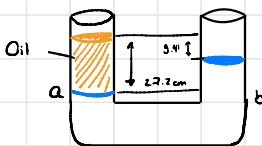
$$\Rightarrow V_w = \rho_i / \rho_w \cdot V_i$$

$\Rightarrow \frac{V_w}{V_i}$  (fraction of the iceberg underwater) is given by  
 $\rho_i / \rho_w = 0.917 \Rightarrow \approx 91\%$  of the iceberg is underwater. ✓

1/1

## 3. Water and oil

• Density of oil



$$\rho_w = \rho_{oil}, \text{ Then } \rho_w \cdot h_w \cdot g = \rho_{oil} \cdot h_{oil} \cdot g$$

$$\Rightarrow \rho_{oil} = \frac{\rho_w \cdot h_w}{h_{oil}} = \frac{1000 \frac{\text{kg}}{\text{m}^3} \cdot \overbrace{0.1779\text{m}}^{27.7\text{cm} - 9.41\text{cm}}}{0.272\text{m}} = \underline{\underline{654 \frac{\text{kg}}{\text{m}^3}}}$$

#### 4. Pipe in Building

pressure = 3.8 atm, speed  $0.6 \frac{\text{m}}{\text{s}}$ , diameter<sub>street</sub> 5cm  
 diameter<sub>top</sub> 2.6cm, 18m to top

Flow velocity and gauge pressure on the top floor.

•  $P + \rho g h + \frac{1}{2} \rho v^2 = \text{constant}$

•  $I_v = v A = \text{constant}$

$$\Rightarrow \frac{v_1 A_1}{v_2} = \frac{A_2 v_1}{A_2} = \frac{v_1 \cancel{\pi r_1^2}}{\cancel{\pi r_1^2}} = \frac{0.6 \frac{\text{m}}{\text{s}} \cdot (0.025\text{m})^2}{(0.013\text{m})^2} = \underline{\underline{2.22 \frac{\text{m}}{\text{s}}}}$$

$$P_{\text{before}} + \rho \cdot g \cdot h + \frac{1}{2} \rho v_1^2 = P_{\text{after}} + \rho \cdot g \cdot h + \frac{1}{2} \rho v_2^2$$

$$P_{\text{after}} = P_{\text{before}} + \frac{1}{2} \rho v_1^2 - \rho \cdot g \cdot h - \frac{1}{2} \rho v_2^2$$

$$= P_{\text{before}} + \frac{1}{2} \rho (v_1^2 - v_2^2) - \rho \cdot g \cdot h$$

$$= 3.838 \cdot 10^5 \text{ Pa} + \frac{1}{2} \cdot 1000 \frac{\text{kg}}{\text{m}^3} \left( \left( 0.6 \frac{\text{m}}{\text{s}} \right)^2 - \left( 2.22 \frac{\text{m}}{\text{s}} \right)^2 \right) - 1000 \frac{\text{kg}}{\text{m}^3} \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot 18\text{m}$$

$$= 2.05 \cdot 10^5 \text{ Pa}$$

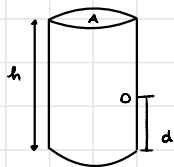
$$(1\text{atm} \sim 101325 \text{ Pa})$$

$$\Rightarrow \underline{\underline{2.02 \text{ atm}}}$$

## 5. Water shooting out of tank

density  $\rho$ , Area  $A$ , height  $h$ , piston mass  $m$

$A_0$  area of spout.



level a

level b

$$(p_{atm} + p_{piston}) + \rho g \cdot h + \frac{1}{2} \rho v_0^2 = p_{atm} + \rho g \cdot h + \frac{1}{2} \rho v_0^2 \quad \checkmark$$

$$\Rightarrow p_{piston} + \rho g \cdot h = \frac{1}{2} \rho v_0^2 \quad \checkmark$$

$$\Rightarrow v_0 = \sqrt{\frac{2}{\rho} (p_{piston} + \rho g \cdot h)} \quad \checkmark$$

vertical

$$s = \frac{1}{2} g t^2, \text{ then } d = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2d}{g}} \quad \checkmark$$

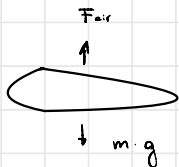
horizontal

$$s = v_0 \cdot t = \sqrt{\frac{2}{\rho} (p_{piston} + \rho g \cdot h)} \cdot \sqrt{\frac{2d}{g}}$$

$$= \sqrt{\frac{4d}{\rho g} (p_{piston} + \rho g \cdot h)} \quad \checkmark$$

□

## 6. Airplane



$$\sum \vec{F}_y = 0 \Rightarrow F_{air} - mg = 0$$

$$\bullet P + \rho g \cdot h + \frac{1}{2} \rho v^2 = \text{constant} \quad \checkmark$$

$$\text{Then } P_{low} + \rho g \cdot h + \frac{1}{2} \rho v_{low}^2 = P_{up} + \rho g \cdot h + \frac{1}{2} \rho v_{up}^2 \quad \checkmark$$

$$\Rightarrow P_{low} - P_{up} = \frac{1}{2} \rho v_{up}^2 - \frac{1}{2} \rho v_{low}^2 \quad \checkmark$$

$$\Rightarrow P_{low} - P_{up} = \frac{1}{2} \rho (v_{up}^2 - v_{low}^2) \quad \checkmark$$

$$\rho_{air} = 1.293 \frac{\text{kg}}{\text{m}^3}$$

3/3

$$\frac{2(P_{\text{low}} - P_{\text{up}})}{\rho_{\text{air}}} + v_{\text{low}}^2 = v_{\text{up}}^2$$

Further  $P = \frac{F}{A} \Rightarrow F = \Delta P \cdot A$

$$\Delta P \cdot A - mg = 0$$

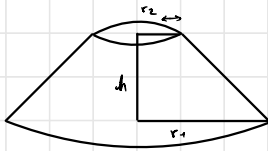
$$(P_{\text{low}} - P_{\text{up}}) = \Delta P = \frac{mg}{A} = 16'350 \text{ Pa}$$

$$v_{\text{up}} = \sqrt{\frac{2 \cdot \Delta P}{\rho_{\text{air}}} + v_{\text{low}}^2} = \sqrt{\frac{2 \cdot 16'350}{1.293 \frac{\text{kg}}{\text{m}^3}} + \left(95 \frac{\text{m}}{\text{s}}\right)^2}$$

$$= \underline{\underline{185.2 \frac{\text{m}}{\text{s}}}} \quad \checkmark$$

3/3

7. Truncated cone submerged



density  $\rho$ ,  $L > h$

$$\rho_L > \rho$$

$$V = \frac{h\pi}{3} (R_1^2 + R_1 R_2 + R_2^2)$$

$$F_L + F_{mg} = F_B \Rightarrow m \cdot g = \rho_L V g + \dots$$

$$\Rightarrow V \rho \cdot g = \rho_L V g$$

$$\Rightarrow \rho = \rho_L$$

Very good!

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