What's next?

（1）Frequency characteristics; Resonance

（2）Magnetically coupled circuits; Transformers

（3）Three-Phase circuits;

（4）Periodic, nonsinusoidal excitations

1

Fundamentals of Electric Circuits 2020.05

Chapter13 Magnetically Coupled Circuits

2

Chapter13　Magnetically Coupled Circuits

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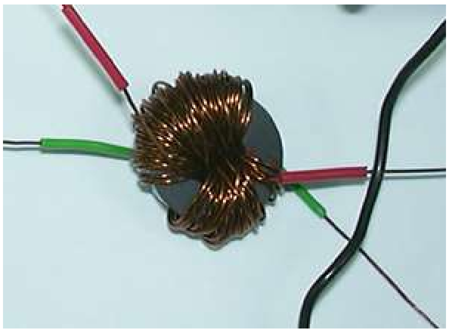
在环形磁芯上用漆包线绕一个耦合电感，初级60匝，次级30匝，如图所示。



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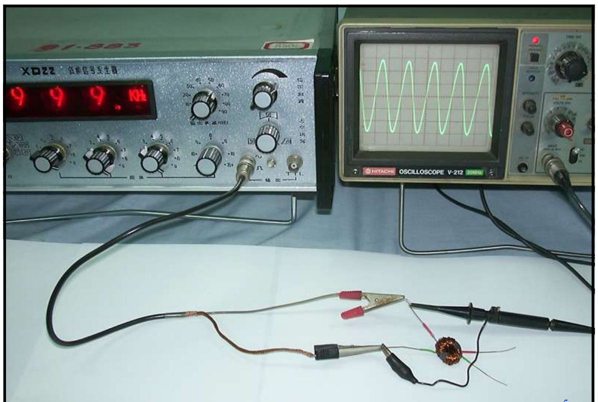
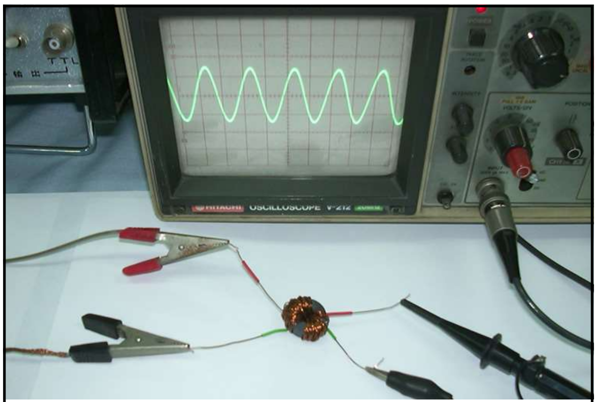
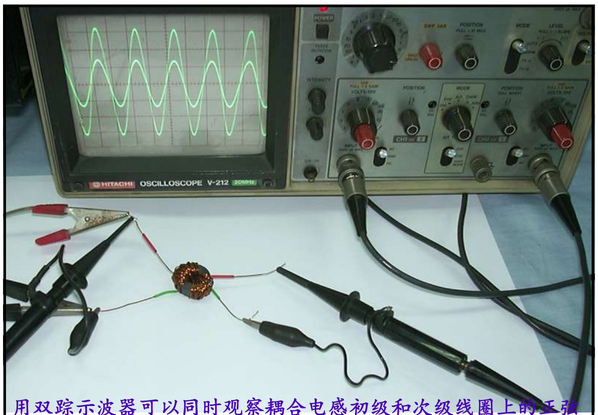


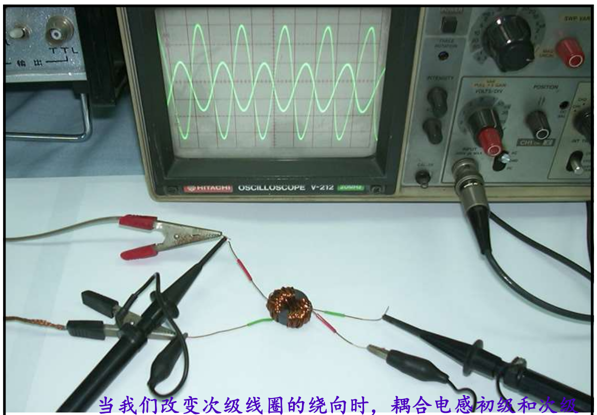
在初级加上999kHz的正弦信号，用示波器观察到正弦波形 5



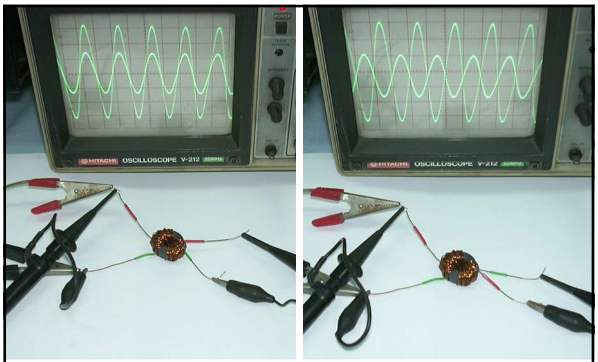
在耦合电感的次级上，可以观察到正弦波形，其幅度约为初级电压的一半。 6



用双踪示波器可以同时观察耦合电感初级和次级线圈上的正弦电压波形，它们的相位是相同的。 7



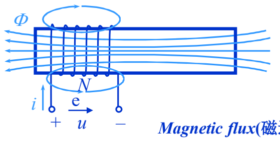
当我们改变次级线圈的绕向时，耦合电感初级和次级线圈上电压波形的相位是相反的。 8

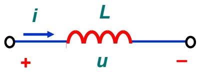


为了区别这两种情况，需确定耦合电感的同名端，图示耦合电感线圈的两个红色(或绿色)端钮是一对同名端。当初次级电压参考方向的正极都在同名端时，它们的相位相同。

13.1 Mutual Inductance and Mutual Inductance Voltage

Inductor 互感和互感电压





$i \to B \to \Phi \to \Psi$

Magnetic flux(磁通量)

Magnetic linkage(磁链)

$$u = L\frac{di}{dt}$$

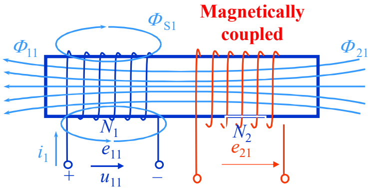
Linear inductance(线性电感) $L = \dfrac{\Psi}{i}$

Right hand rule: i、e，Φ

Faraday law of electromagnetic induction

$$e = -L\frac{di}{dt}$$

Mutual inductance



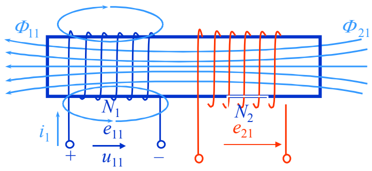
$i_1 \to B \to \Phi_{11} \to \Psi_{11}$ $\quad L_1 = \dfrac{\Psi_{11}}{i_1}$ **Self-induction(自感)**

$\Phi_{11}=\Phi_{21}+\Phi_{S1}$

$\Phi_{21} \to \Psi_{21}=N_2\Phi_{21}$ $\quad M_{21} = \dfrac{\Psi_{21}}{i_1}$ **Mutual inductance(互感)**

M_{21} is the mutual inductance of coil 2 with respect to coil 1

线圈1对线圈2的互感



$$L_1 = \frac{N_1\Phi_{11}}{i_1} \quad L_2 = \frac{N_2\Phi_{22}}{i_2}$$

$$M_{21} = \frac{N_2\Phi_{21}}{i_1} \quad M_{12} = \frac{N_1\Phi_{12}}{i_2}$$

Unit: H

Notice:

(1) For linear inductor，$M_{12}= M_{21}= M$;

(2) M is determined by the dimensions, number of turns, mutual position and the permeability of the surrounding medium, but independent on the current.

Coupling Coefficient 耦合系数

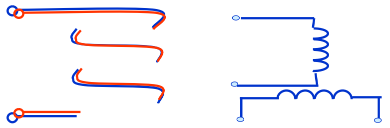
$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}} \qquad k \le 1$$

$$L_1 = \frac{N_1\Phi_{11}}{i_1} \qquad L_2 = \frac{N_2\Phi_{22}}{i_2} \qquad \Phi_{11}=\Phi_{21}+\Phi_{S1}$$

$$M_{21} = \frac{N_2\Phi_{21}}{i_1} \qquad M_{12} = \frac{N_1\Phi_{12}}{i_2} \qquad \Phi_{22}=\Phi_{12}+\Phi_{S2}$$

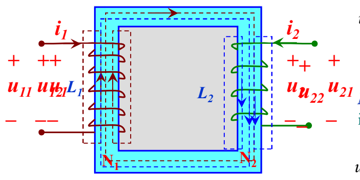
k=1
perfectly coupled
全耦合

$k \approx 1$ $\qquad$ $k \approx 0$



Coupling Coefficient can beajusted by mutual position of the two coils.

Mutual Inductance Voltage(互感电压)



$$u_{11} = L_1 \frac{di_1(t)}{dt}$$ Self-Induction voltage (自感电压)

$$u_{21} = M_{21} \frac{di_1(t)}{dt}$$ mutual voltage (互感电压)

M_{21},the coefficient of mutual inductance, (互感系数)

$$u_{22} = L_2 \frac{di_2(t)}{dt}$$

$$u_{12} = M_{12} \frac{di_2(t)}{dt}$$

i_1 and i_2 act together

$$u_1(t) = L_1 \frac{di_1(t)}{dt} + M_{12}\frac{di_2(t)}{dt}$$

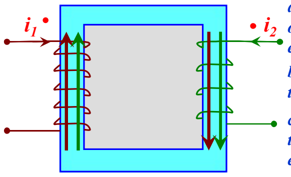
$$u_2(t) = L_2 \frac{di_2(t)}{dt} + M_{21}\frac{di_1(t)}{dt}$$

$M_{12} = M_{21} = M$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$ The Coupling Coefficient 耦合系数

Dot Convention(同名端)

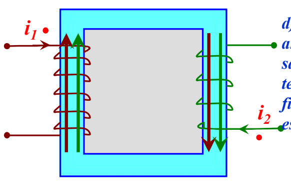
The procedure for determining dot markings



a) Arbitrary select one terminal of one coil and mark it with a dot .(for example, 1) .

b) Assign a current into the dotted terminal .(for example, i_1)

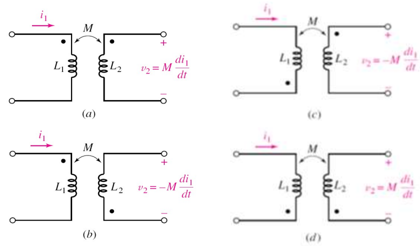
c) Using the right-hand rule to determine the direction of the magnetic field established by i_1 inside the coupled coils .



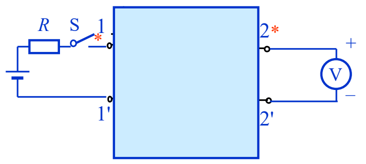
d) Pick one terminal of the second coil and place a dot on it. The terminal satisfies: if a current i_2 enters the terminal, the direction of the magnetic field established by i_2 is the same as that established by i_1.

The rule for using the dot convention to determine the polarity of mutually induced voltage

When the reference direction for a current enters the dotted terminal of one coil, the reference polarity of the voltage it induces in the other coil is positive at its dotted terminal.



Measure the dot convention by a simple test



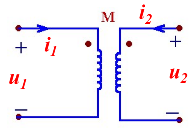
Close the switch, i increases,

$$\frac{di}{dt} > 0, \qquad u_{22'} = M\frac{di}{dt} > 0 \quad V > 0$$

13.2 The Voltage-current relationship of the Mutual inductance

互感的伏安关系

1. Time domain



$$u_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$u_2(t) = M\frac{di_1(t)}{dt} + L_2\frac{di_2(t)}{dt}$$

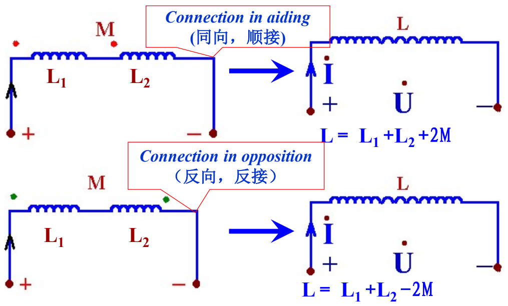
2. Frequency domain

$$\dot{U}_1 = j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2$$

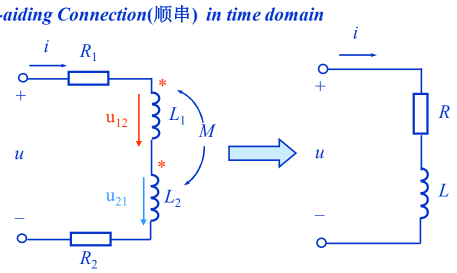
$$\dot{U}_2 = j\omega M\dot{I}_1 + j\omega L_2\dot{I}_2$$

13.3 The Decoupling Equivalent Circuit of the Mutual inductance 互感的去耦等效电路

1. series connection（串联）



Series-aiding Connection(顺串) in time domain

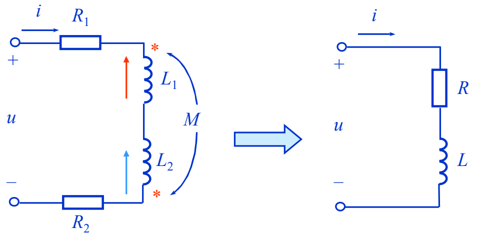


$$u = R_1 i + L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + R_2 i$$

$$= (R_1 + R_2)i + (L_1 + L_2 + 2M)\frac{di}{dt} = Ri + L\frac{di}{dt}$$

$$R = R_1 + R_2 \qquad L = L_1 + L_2 + 2M$$

Series-opposing Connection(反串) in time domain

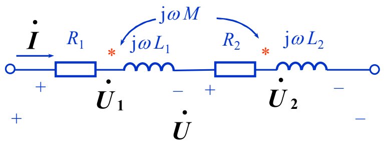


$$u = R_1 i + L_1\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + R_2 i$$

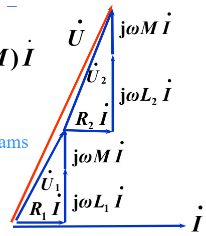
$$= (R_1 + R_2)i + (L_1 + L_2 - 2M)\frac{\mathrm{d}i}{\mathrm{d}t} = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$\therefore\ R = R_1 + R_2 \qquad L = L_1 + L_2 - 2M$$

Series-aiding Connection in frequency domain

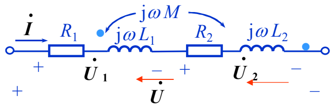


$$\dot{U} = (R_1 + R_2)\dot{I} + \mathrm{j}\omega(L_1 + L_2 + 2M)\dot{I}$$

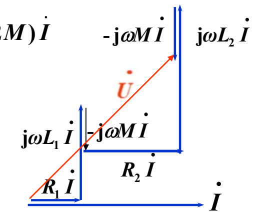


Phasor Diagrams

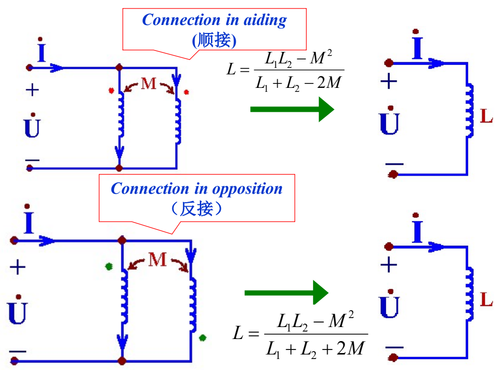
Series-opposing Connection in frequency domain



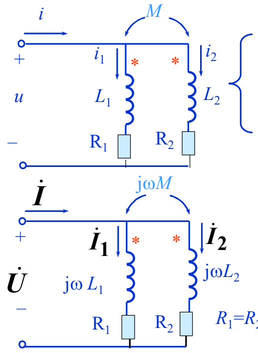
$$\dot{U} = (R_1 + R_2)\dot{I} + \mathrm{j}\omega(L_1 + L_2 - 2M)\dot{I}$$



2. parallel connection(并联)



***Parallel-aiding Connection*(同侧并联)**



$$\begin{cases} \dot{U} = (R_1 + j\omega L_1)\dot{I}_1 + j\omega M\,\dot{I}_2 \\ \dot{U} = j\omega M\,\dot{I}_1 + (R_2 + j\omega L_2)\dot{I}_2 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

$$\dot{U} = Z_1\dot{I}_1 + Z_M\dot{I}_2$$

$$\dot{U} = Z_M\dot{I}_1 + Z_2\dot{I}_2$$

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_M^2}{Z_1 + Z_2 - 2Z_M}$$

$R_1 = R_2 = 0$

$$L_{eq} = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 - 2M}$$

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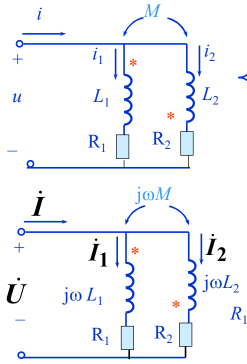
***Parallel-aiding Connection*(异侧并联)**



$$\begin{cases} \dot{U} = (R_1 + j\omega L_1)\dot{I}_1 - j\omega M\,\dot{I}_2 \\ \dot{U} = -j\omega M\,\dot{I}_1 + (R_2 + j\omega L_2)\dot{I}_2 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

$$\dot{U} = Z_1\dot{I}_1 - Z_M\dot{I}_2$$

$$\dot{U} = -Z_M\dot{I}_1 + Z_2\dot{I}_2$$

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_M^2}{Z_1 + Z_2 + 2Z_M}$$

$R_1 = R_2 = 0$

$$L_{eq} = \frac{(L_1 L_2 - M^2)}{L_1 + L_2 + 2M}$$

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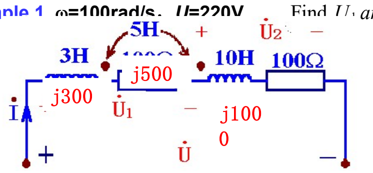
Example 1 ω=100rad/s, U=220V Find $\dot{U}_1$ and $\dot{U}_2$。



Solution:

$$Let\ \dot{U} = 220\angle 0°$$

$$\dot{I} = \frac{220\angle 0°}{200 + j300} = j0.61\angle -56.31° A$$

$$\dot{U}_1 = j300\,\dot{I} - j500\,\dot{I} + 100\,\dot{I} = 136.4\angle -119.74° V$$

$$\dot{U}_2 = j1000\,\dot{I} - j500\,\dot{I} + 100\,\dot{I} = 311.04\angle 22.38° V$$

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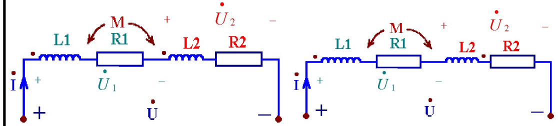
Example 2. Two coupled inductors are connected to a sinusoidal source with an effective value of 220V and the frequency of 50Hz。If the two inductors are connected ***in aiding***, I=2.7A,P=218.7W； if they are connected ***in opposition***, I=7A。Find the mutual inductance M=?



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