Sinusoidal Steady-State Analysis Steps

① Transform the circuit to the phasor or frequency domain.

Transform all independent sources to their phasor equivalent.

$u \; i \longrightarrow \dot{U} \; \dot{I}$

Calculate the impedance (Z) of all passive circuit elements.

$R, L, C \longrightarrow Z, Y$

② Apply analysis method in the frequency domain.

③ Transform the resulting phasor to the time domain.

Nodal and Mesh Analysis, Superposition, Source Transformations, and Thevenin's theorem...

9.8 Phasor Diagrams

Provide a graphical (geometrical) method for solving circuit.

Key 1: Choose appropriate reference phasor

Current for series circuits; voltage for parallel circuits

Key 2: Current and voltage relation for elements and branches

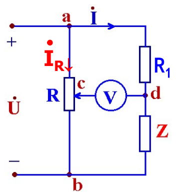
Resistor	i and u in phase;
Inductor	i lags u by 90°;
Capacitor	i leads u by 90°.

RL branches, i lags u by φ.

RC branches, i leads u by φ;

$0 < \varphi < 90°$

Example 9. In the following circuit, U=100V, R=20Ω, R_1=6.5 Ω. C is moveable. If R_{ac}=4Ω, the minimum U_{cd} can be reached and the value is 30V. Try to find Z.



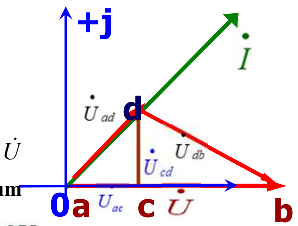
Solution 1. Assume

$\dot{U} = 100\angle 0°$

$\dot{I}_R = \dfrac{\dot{U}}{R} = 5\angle 0° A$

If Z=R_o-jX_o, $\dot{I}$ leads $\dot{U}$

When Ucd is minimum



$U_{cd} \perp U_{ac}$ $\quad U_{cd} = 30V$

voltage divider $U_{ac} = 20V$

$U_{ad} = 36.1V \quad I = \dfrac{36.1}{6.5} = 5.55A$

$\because \; \Delta adc \cong \Delta abe \quad \dfrac{U_{ad}}{U_{cd}} = \dfrac{U}{U_{eb}}$ $\quad U_{ad} = 36.1V \quad U_{cd} = 30V \quad U_{ac} = 20V \Rightarrow U_{eb} = 83.1V$

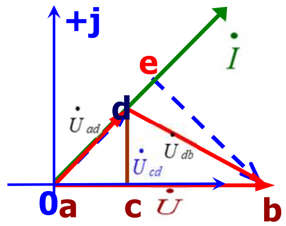
$Z = R_0 + jX_0$

$x_o = \dfrac{U_{eb}}{I} = 15\Omega$

$\because \; \Delta adc \cong \Delta abe \quad \dfrac{U_{ae}}{U_{ab}} = \dfrac{U_{ac}}{U_{ad}}$

$U_{ae} = 55.4V$

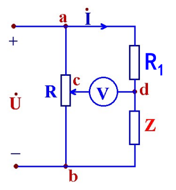
$R_o = \dfrac{U_{ae}}{I} - R_1 = 3.51\Omega$



$\therefore \; Z = 3.5 - j15(\Omega)$

In the similar way, $Z = 3.5 + j15(\Omega)$

Solution 2.Assume $\dot{U} = 100\angle 0°$



$$\dot{U}_{ac} = \frac{R_{ac}}{R}\dot{U} \qquad \dot{U}_{ad} = \frac{R_1}{R_1 + Z}\dot{U}$$

$$\dot{U}_{cd} = \dot{U}_{ad} - \dot{U}_{ac}$$

$$= \left(\frac{R_1}{R_1 + Z} - \frac{R_{ac}}{R}\right)\dot{U}$$

R_{ac} changes accompanying with the movement of C.

If U_{cd} is minimum

$$\left(\frac{6.5}{6.5 + Z} - \frac{4}{20}\right)100\angle 0° = \pm j30$$

$$\therefore \quad Z = 3.5 \pm j15(\Omega)$$

84

9.9 AC Circuit Power Analysis

(交流电路功率分析)

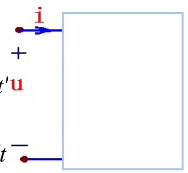
9.9.1 Instantaneous Power (瞬时功率)

p(t) = u(t)i(t) **(associated direction is assumed)**

$$R: p(t) = u(t)i(t) = i^2(t)R = \frac{u^2(t)}{R}$$

$$C: \ p(t) = u(t)i(t) = cu(t)\frac{du(t)}{dt} = \frac{1}{c}i(t)\int_{-\infty}^{t} i(t')dt'$$

$$L: \ p(t) = u(t)i(t) = Li(t)\frac{di(t)}{dt} = \frac{1}{L}u(t)\int_{-\infty}^{t} u(t')dt'$$



Network: $p(t) = u(t)i(t)$

85

Power due to sinusoidal excitation

$$u(t) = \sqrt{2}U\cos\omega t \quad i(t) = \sqrt{2}I\cos(\omega t - \varphi) \quad \varphi = \varphi_u - \varphi_i$$

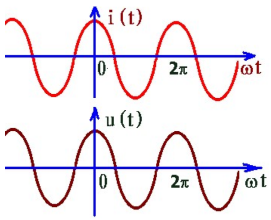
$$p(t) = u(t)i(t) = 2UI\cos\omega t\cos(\omega t - \varphi)$$

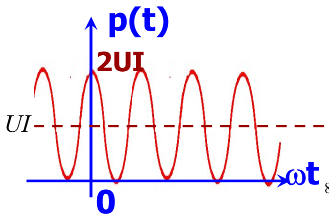
Thus $= UI\cos\varphi + UI\cos(2\omega t - \varphi)$ **Unit: W**

constant

sinusoidal
T=T/2

R: $p(t) = UI + UI\cos 2\omega t$



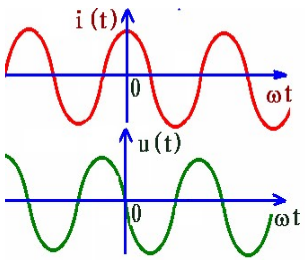


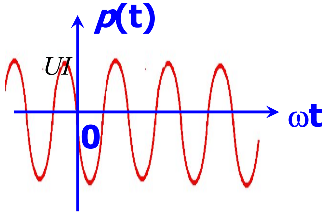
86

L: $\varphi = \varphi_u - \varphi_i = 90°$

$$p = UI\cos\varphi + UI\cos(2\omega t - \varphi)$$

$$= UI\cos(2\omega t - 90°)$$

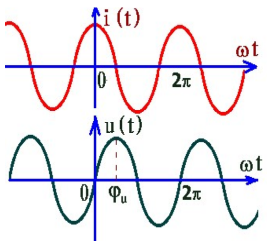


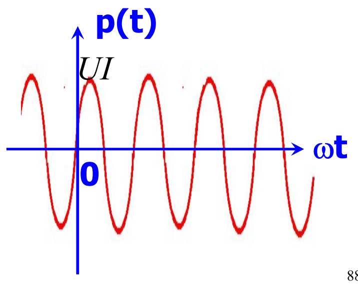


87

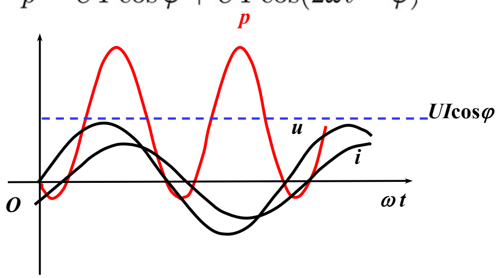
C: $\varphi = \varphi_u - \varphi_i = -90^\circ$

$$p = UI\cos\varphi + UI\cos(2\omega t - \varphi)$$
$$= UI\cos(2\omega t + 90^\circ)$$





$$p = UI\cos\varphi + UI\cos(2\omega t - \varphi)$$



$p(t)$ could be positive or negative for different time

$p(t) > 0$, absorbed power;（吸收功率）

$p(t) < 0$, developed power ;（发出功率）

9.9.2 Average Power

(平均功率，有功功率-Active Power)

$$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt$$

1. Average energy for periodic waveforms

$$P = \frac{1}{T}\int_0^T p(t)dt$$

2. Average energy in the sinusoidal steady state

$$u(t) = \sqrt{2}U\cos\omega t \quad i(t) = \sqrt{2}I\cos(\omega t - \varphi) \quad \varphi = \varphi_u - \varphi_i$$

$$p = UI\cos\varphi + UI\cos(2\omega t - \varphi)$$

$$P = \frac{1}{T}\int_0^T p(t)dt = UI\cos\varphi \qquad \text{Unit: W}$$

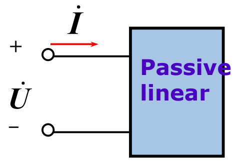
$\cos\varphi$: power factor(功率因数);

$\varphi = \psi_u - \psi_i$: power-factor angle

R: $P_R = UI\cos\varphi = UI\cos 0^\circ = UI = I^2R = U^2/R$

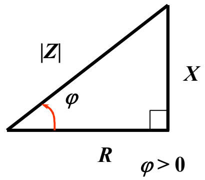
L: $P_L = UI\cos\varphi = UI\cos 90^\circ = 0$

C: $P_C = UI\cos\varphi = UI\cos(-90^\circ) = 0$



$Z = R + jX$

$U = I|Z|$



$$P = UI\cos\varphi = I|Z|I\cos\varphi = I^2|Z|\cos\varphi = I^2R$$

Average Power is the power absorbed by equivalent resistance. $\boxed{\text{Active Power}}$

$Z = R + jX$

$$\begin{cases} \cos\varphi = 1,\ X = 0 \\ \cos\varphi = 0,\ R = 0 \end{cases}$$

Generally, $0 \le \cos\varphi \le 1$

$X > 0$, $\varphi > 0$, inductive, (current) lagging power-factor;

$X < 0$, $\varphi < 0$, capacitive, (current) leading power-factor;

$\cos\varphi = 0.5$（滞后）， $\varphi = 60^o$;
$\cos\varphi = 0.5$（超前）， $\varphi = -60^o$;

92

Example 1 **$u(t) = 707\cos 10\omega t$(V)，$i(t) = 1.41\cos(\omega t - 53.1°)$**

Find P

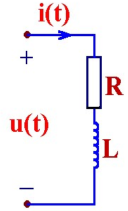
Solution 1: $P = UI\cos\varphi = \frac{707}{\sqrt{2}} \times \frac{1.41}{\sqrt{2}} \cos 53.1^o = 300\ W$

Solution 2:

$\dot{I} = 1\angle -53.1^0 \quad \dot{U} = 500\angle 0^0 V$

$Z = \frac{\dot{U}}{\dot{I}} = 500\angle 53.1^0 = 300 + j400\,\Omega$

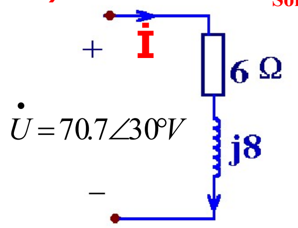
$P = I^2 R = 1 \times 300 = 300W$



93

Example 2 **$f = 50$Hz，Find P**

1)



Solution :

$\dot{I} = \frac{\dot{U}}{Z} = \frac{70.7\angle 30^0}{6 + j8} = \frac{70.7\angle 30}{10\angle 53.1^0}$

$= 7.07\angle -23.1°$

$P = UI\cos\varphi = 300W$

or

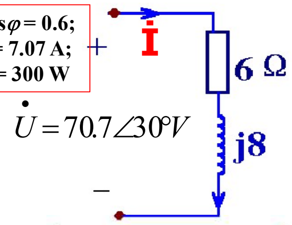
$P = P_R = I^2 R = 300W$

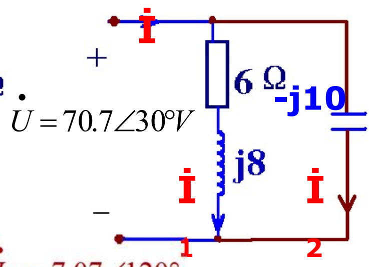
$\boxed{\cos\varphi = \cos(53.1^o) = 0.6}$

94

2) Find *P*.

$\boxed{\begin{array}{l}\cos\varphi = 0.6; \\ I = 7.07\text{ A}; \\ \text{P} = 300\text{ W}\end{array}}$





$\dot{I}_1 = 7.07\angle -23.1° \quad \dot{I}_2 = 7.07\angle 120°$

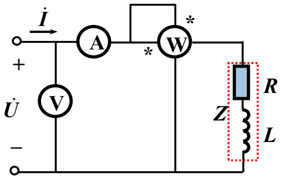
$\dot{I} = \dot{I}_1 + \dot{I}_2 = 4.47\angle 48.43°$ $\boxed{I = 4.47\text{ A}}$

$P = UI\cos\varphi = 300W$ $\boxed{\cos\varphi = \cos(-18.43^o) = 0.9482}$

95

Example 3 $f = 50\text{Hz}, U=50\text{V}, I=1\text{A}, P=3\text{W}$

Find R, L.



Solution:

$$P = I^2 R \qquad \therefore R = \frac{P}{I^2} = \frac{30}{1^2} = 30\Omega$$

$$|Z| = \frac{U}{I} = \frac{50}{1} = 50\Omega$$

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

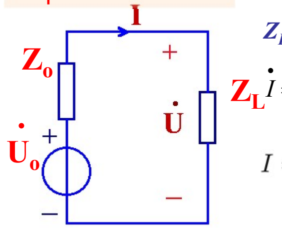
$$L = \frac{1}{\omega}\sqrt{|Z|^2 - R^2} = \frac{1}{314}\sqrt{50^2 - 30^2} = \frac{40}{314} = 0.127\text{H}$$

96

3. Maximum power transfer

Impedance load

$$Z_L = R_L + jX_L \qquad Z_o = R_o + jX_o$$



Z_L= ? P reaches P_{max}

$$\dot{I} = \frac{\dot{U}_o}{Z_o + Z_L} \qquad \dot{I} = \frac{U_o\angle\varphi_u}{(R_o + R_L) + j(X_o + X_L)}$$

$$I = \frac{U}{\sqrt{(R_0 + R_L)^2 + (X_o + X_L)^2}}$$

$$P = I^2 R_L = \frac{R_L U^2}{(R_0 + R_L)^2 + (X_0 + X_L)^2} \qquad \boxed{X_L = -X_o}$$

$$\frac{d}{dR_L}\left[\frac{R_L}{(R_o + R_L)^2}\right] = 0 \qquad \frac{R_o - R_L}{(R_o + R_L)^3} = 0 \qquad \boxed{R_L = R_o}$$

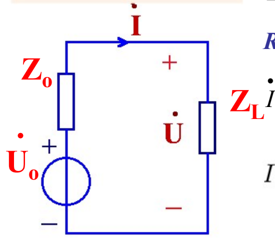
$$\therefore Z_L = R_o - jX_o = Z_o^* \qquad P_{max} = \frac{U_o^{\,2}}{4R_o}$$

97

3. Maximum power transfer

Resistive load

$$Z_L = R_L \qquad Z_o = R_o + jX_o$$



R_L= ? P reaches P_{max}

$$\dot{I} = \frac{\dot{U}_o}{Z_o + Z_L} = \frac{U_o\angle\varphi_u}{(R_o + R_L) + jX_o}$$

$$I = \frac{U}{\sqrt{(R_0 + R_L)^2 + X_o^2}}$$

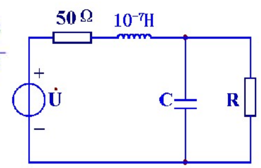
$$P = I^2 R_L = \frac{R_L U^2}{(R_0 + R_L)^2 + X_0^2}$$

$$R_L = \sqrt{R_o^{\,2} + X_o^{\,2}} = |Z_o| \qquad P_m = \frac{U_o^{\,2}|Z_o|}{(R_o + |Z_o|)^2 + X_o^{\,2}}$$

98

Example . $\dot{U} = 0.1\angle 0°V,\ f = 100MHz.$

Find 1) What are the values of R and C if it is known that R is absorbing maximum power P_{max}?



$$Z_o = 50 + j62.8 \qquad Z_L = \frac{R - j\omega CR^2}{1 + (\omega CR)^2}$$

Apply the maximum power transfer theorem

$$Z_L = \overset{*}{Z_o}$$

So $$\frac{R}{1 + (\omega CR)^2} = 50 \qquad \frac{\omega CR^2}{1 + (\omega CR)^2} = 62.8$$

$$\omega CR = 1.256 \quad \therefore \quad R = 128.8768\Omega$$

$$C = 15.5pF \qquad P_m = \frac{U^2}{4R_o} = 50\mu W$$

99

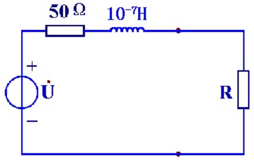
Example . $\dot{U}=0.1\angle 0°V, f=100MHz.$

Find 2) Remove C, What is the value of R to absorb the maximum power?

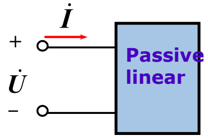
$Z_o=50+j62.8$

$R=|Z_o|=80.2735\Omega$

$P_m=\dfrac{U_o^2|Z_o|}{(R_o+|Z_o|)^2+X_o^2}=38.38\mu W$

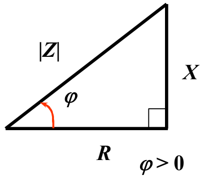


9.9.3 Reactive Power (无功功率)



$Z=R+jX$

$U=I|Z|$



$P=UI\cos\varphi=I^2R$

Define

$Q=UI\sin\varphi=I|Z|I\sin\varphi=I^2|Z|\sin\varphi=I^2X$

R: $Q_R=UI\sin 0°=0$ **Unit: Var**

L: $Q_L=UI\sin 90^0=UI=U^2/X_L=X_LI^2>0$, **absorbed reactive power**

C: $Q_C=UI\sin 90^0=UI=U^2/X_C=X_CI^2<0$, **developed reactive power**

Time rate of energy flow back and forth between the source and the reactive loads. Compensation between capacitor and inductor.

9.9.4 Complex Power (复功率)

Define 共轭复数

$\tilde{S}=\dot{U}\,\overset{*}{\dot{I}}$

$\tilde{S}=U\angle\varphi_u I\angle-\varphi_i=UI\angle\varphi_u-\varphi_i$

$=UI\cos(\varphi_u-\varphi_i)+jUI\sin(\varphi_u-\varphi_i)$ $\quad\varphi=\psi_u-\psi_i$

$=UI\cos\varphi+jUI\sin\varphi$

$=P+jQ$ **Unit: VA**

The complex sum of active power and reactive power

9.9.5 Power Factor(功率因数) and Apparent Power (视在功率)

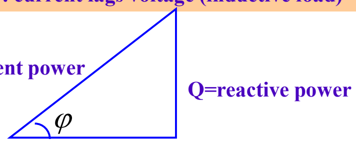
Power Factor Angle $\varphi=\varphi_u-\varphi_i$

Power Factor $\cos\varphi$

Leading Power Factor: current leads voltage (capacitive load)

Lagging Power Factor: current lags voltage (inductive load)

$tg\varphi=\dfrac{Q}{P}$



A Power triangle

Apparent Power: $S=\sqrt{P^2+Q^2}=UI$ **Unit: VA**

Apparent power represent the volt-amps capacity required to supply the average power.