Fundamentals of Electric Circuits

2020.04

Chapter 4
Circuit Theorems

Chapter 4 Circuit Theorems

4.1 Motivation

4.2 Linearity Property

4.3 **Superposition**

4.4 **Source Transformation**

4.5 Substitution theorem

4.6 Simplification of a one-port network contains no independent sources

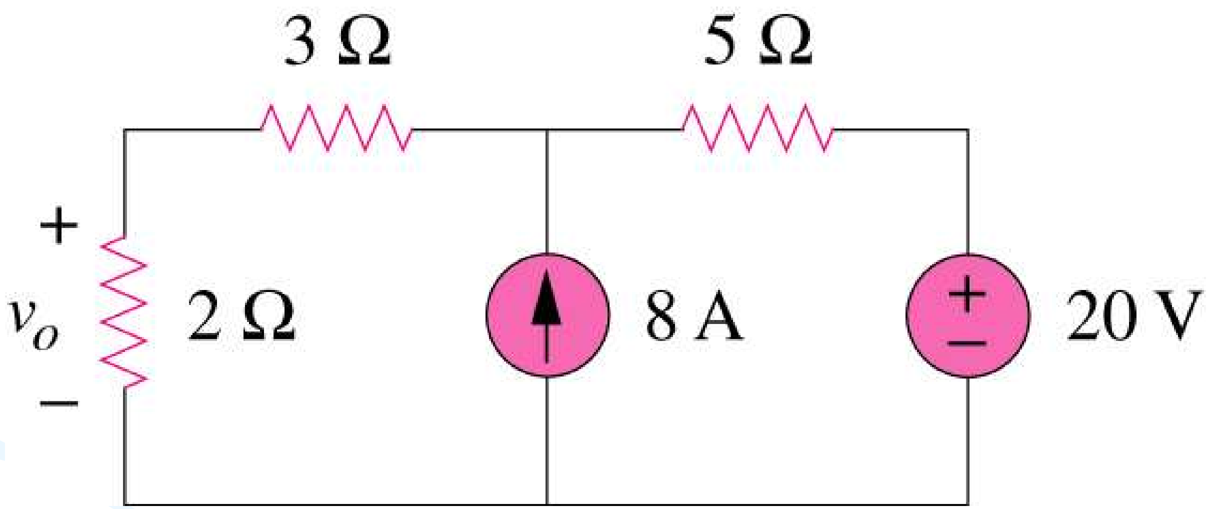
4.7 **Thevenin's Theorem**

4.8 **Norton's Theorem**

4.9 Maximum Power Transfer

4.1 Motivation

If you are given the following circuit, are there any other alternative to determine the voltage across 2Ω resistor?



What are they? And how?

Can you work it out by inspection?

4.2 Linearity Property (1)

It is the property of an element describing ***a linear relationship between cause and effect*.**

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Homogeneity (scaling) property

$$\boldsymbol{v = i\,R \quad \longrightarrow \quad k\,v = k\,i\,R}$$

Additive property

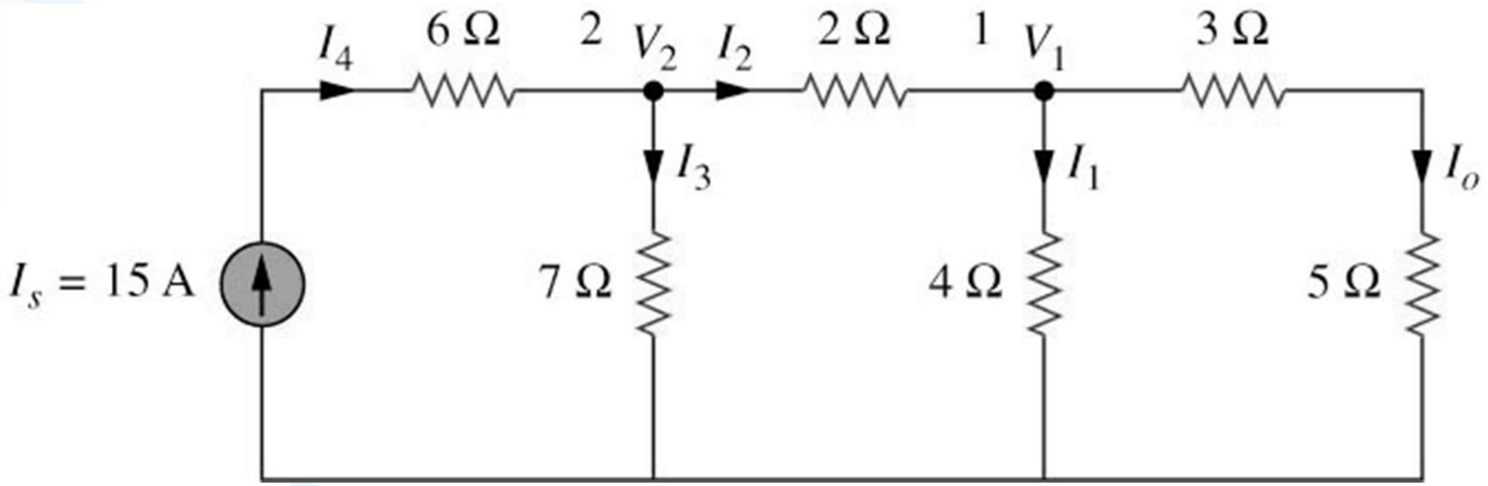
$$\boldsymbol{v_1 = i_1\,R \text{ and } v_2 = i_2\,R}$$

$$\boldsymbol{\longrightarrow \quad v = (i_1 + i_2)\,R = v_1 + v_2}$$

4.2 Linearity Property (2)

Example 1

By assume $I_o = 1$ A, use linearity to find the actual value of I_o in the circuit shown below.



Answer $I_o = 3$A

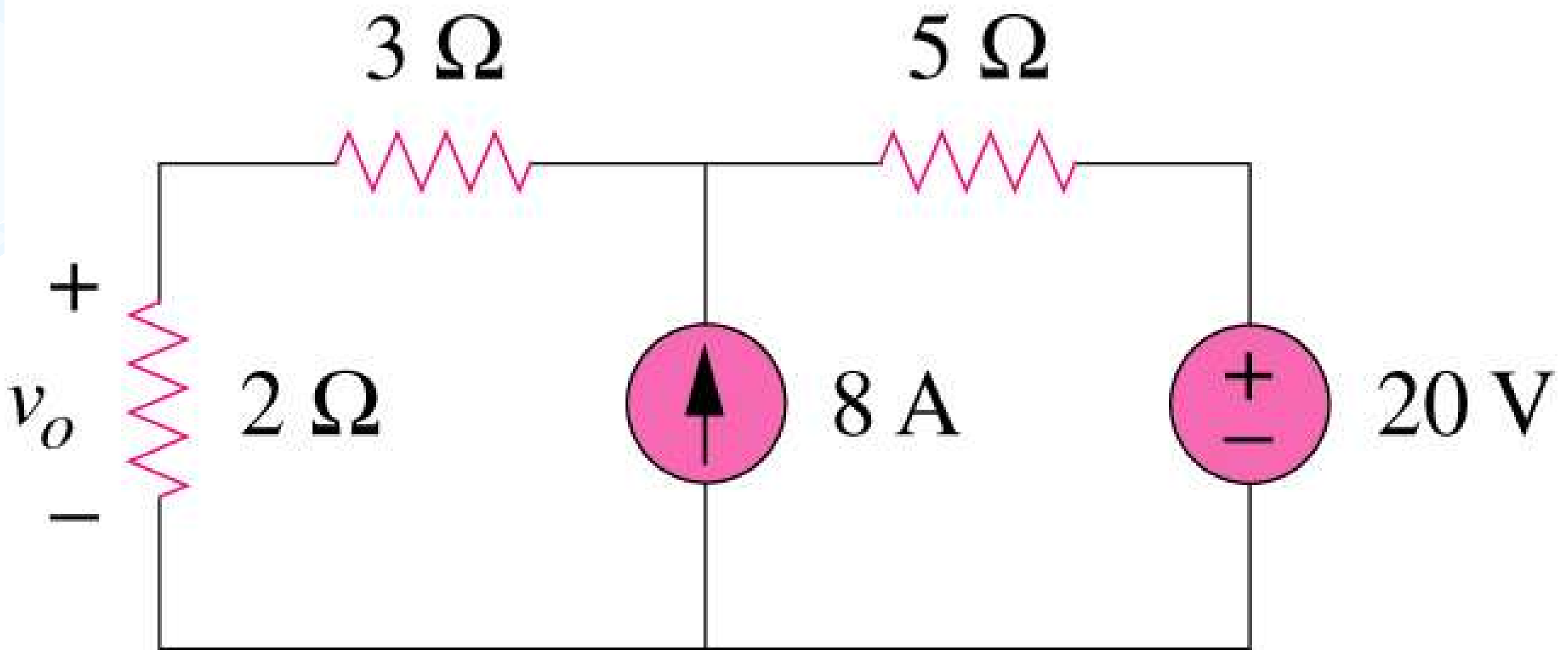
4.3 Superposition Theorem (1)

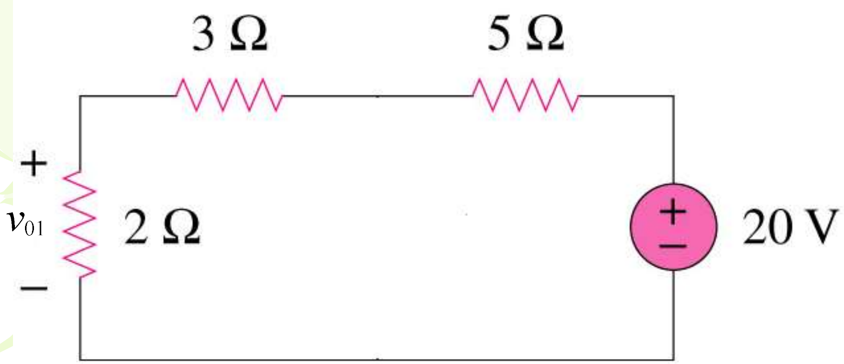
It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

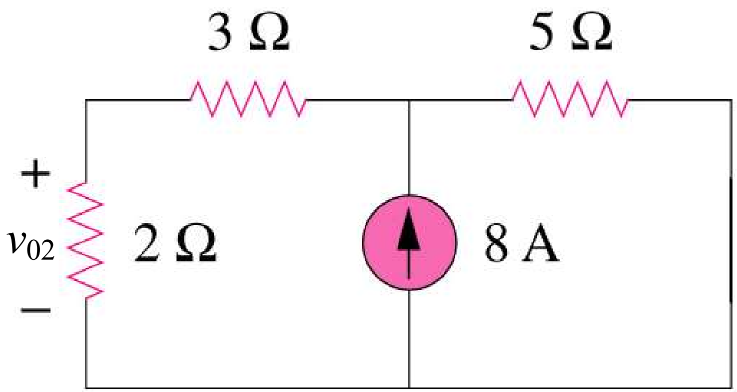
4.3 Superposition Theorem (2)

We consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.





(a)



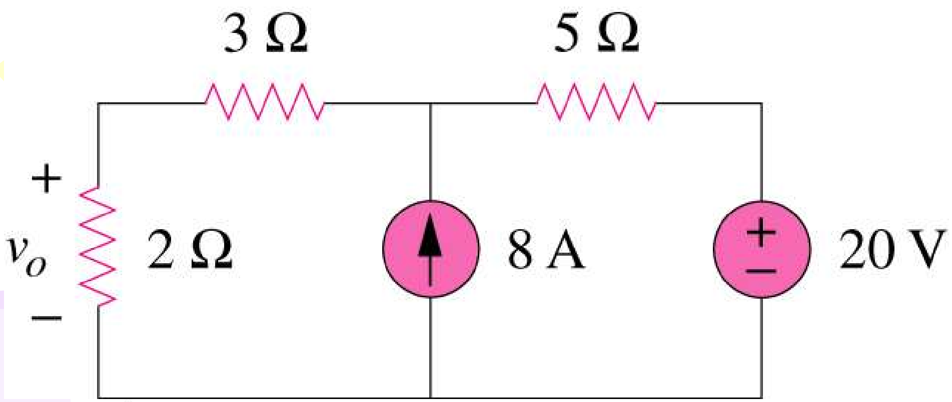
(b)

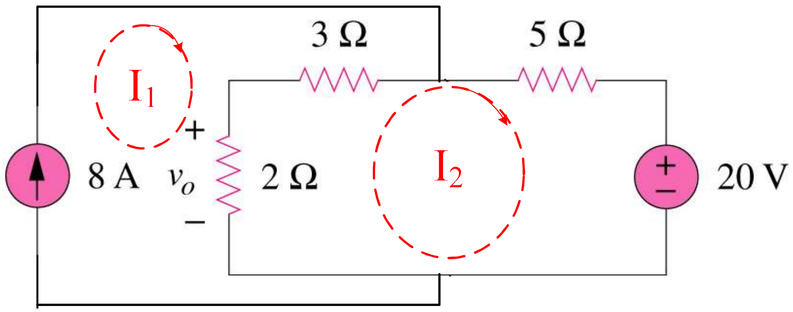
By simple analyzing we obtain

v_{01}=4V v_{02}=8V

$$I_1 = 8$$

$$-5I_1 + 10I_2 = -20$$

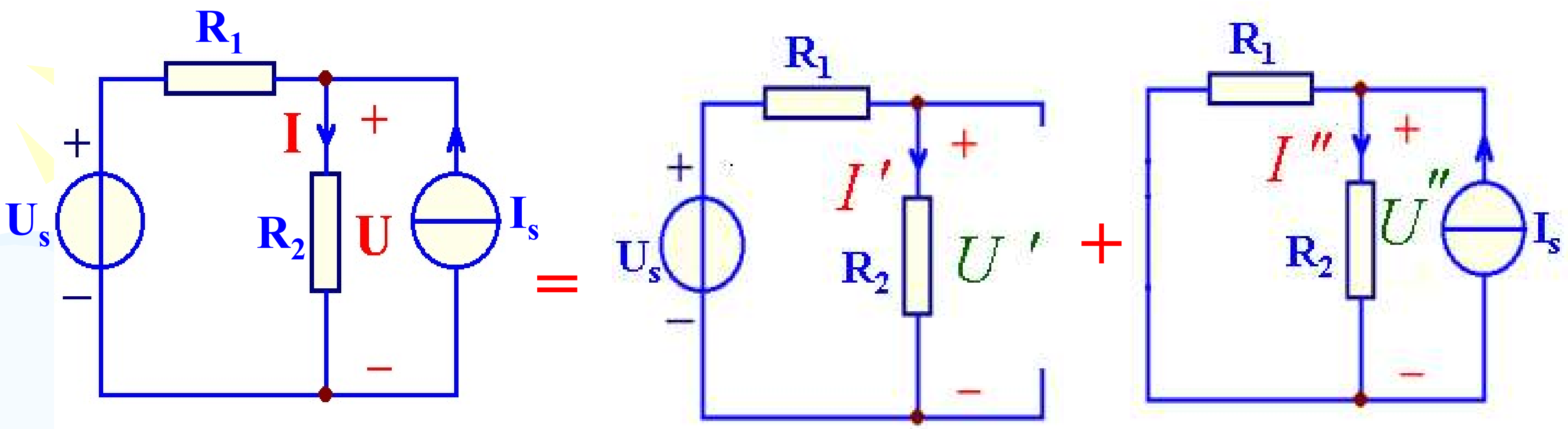




$$v_0 = 12V$$

The Superposition Principle(叠加原理)

Find *U* and *I*.



$$U=\frac{U_s/R_1+I_s}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$$

$$=\frac{U_sR_2+R_1R_2I_s}{R_1+R_2}$$

$$U'=\frac{R_2}{R_1+R_2}U_s \qquad U''=\frac{R_2R_1}{R_1+R_2}I_s$$

$$I'=\frac{U_s}{R_1+R_2} \qquad I''=\frac{R_1}{R_1+R_2}I_s$$

$$U=U'+U'' \qquad I=I'+I''$$

$$=\alpha_1U_s+\alpha_2I_s \qquad =\beta_1U_s+\beta_2I_s$$

4.3 Superposition Theorem (3)

Steps to apply superposition principle

1. ***Turn off*** all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. ***Repeat step 1*** for each of the other independent sources.
3. ***Find*** the total contribution by adding ***algebraically*** all the contributions due to the independent sources.

4.3 Superposition Theorem (4)

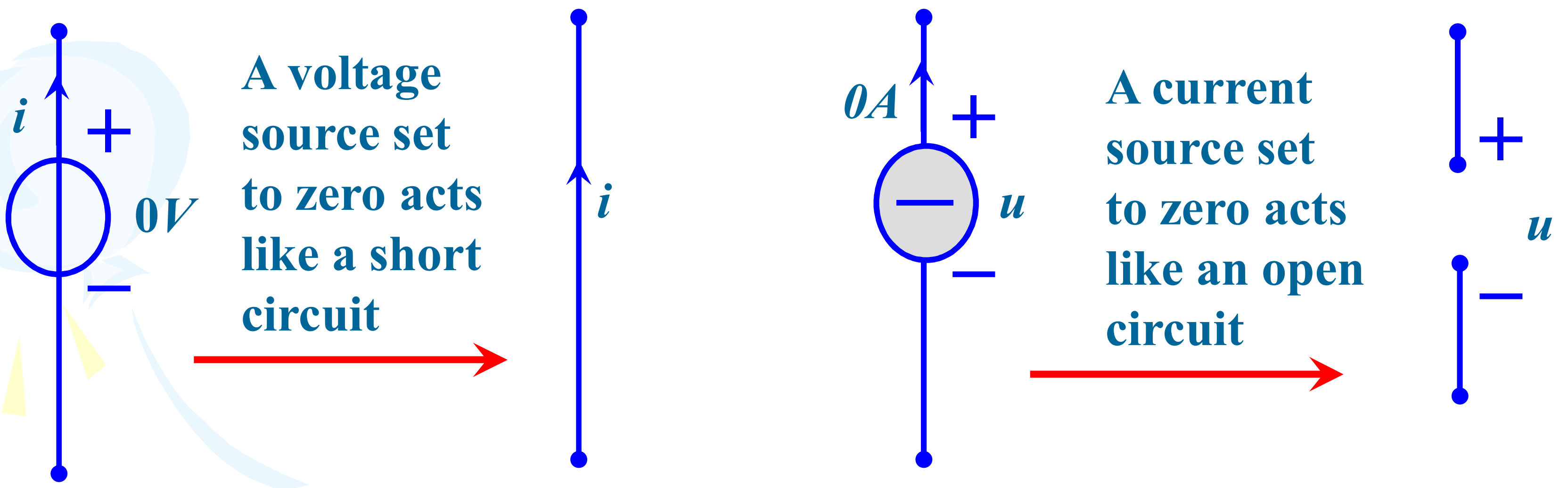
Two things have to be keep in mind:

When we say turn off all other independent sources:

- Independent voltage sources are replaced by 0 V (short circuit) and
- Independent current sources are replaced by 0 A (open circuit).
- Dependent sources are left intact because they are controlled by circuit variables.

Note: 1) Superposition is based on linearity.

Note: 2) acting alone means other independent sources “inactive” ,“turned off” or “zeroed out”, dependent sources are in general active.



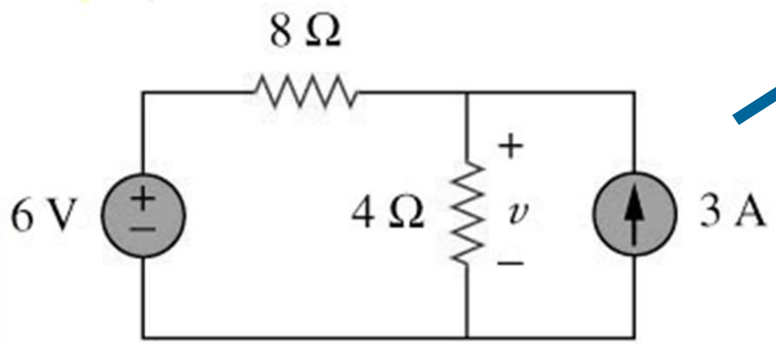
Note: 3) The reference direction

Note: 4) Not applicable to the effect on power .

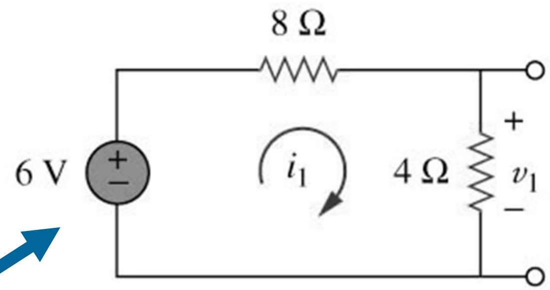
4.3 Superposition Theorem (5)

Example 2

Use the superposition theorem to find v in the circuit shown below.

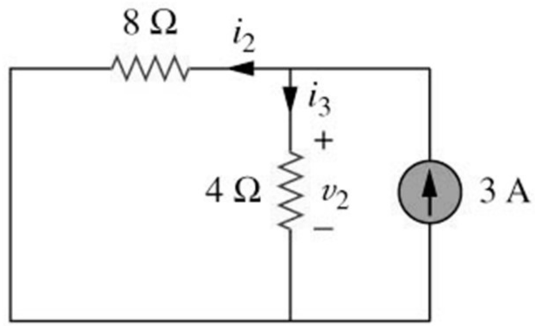


3A is discarded by open-circuit



(a)

6V is discarded by short-circuit

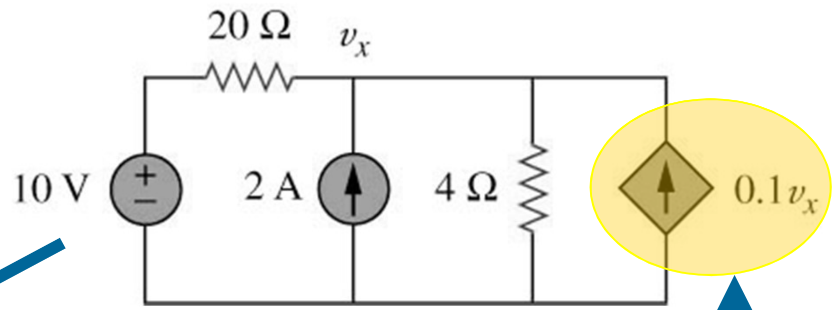


(b)

Answer v = 10V

Example 3

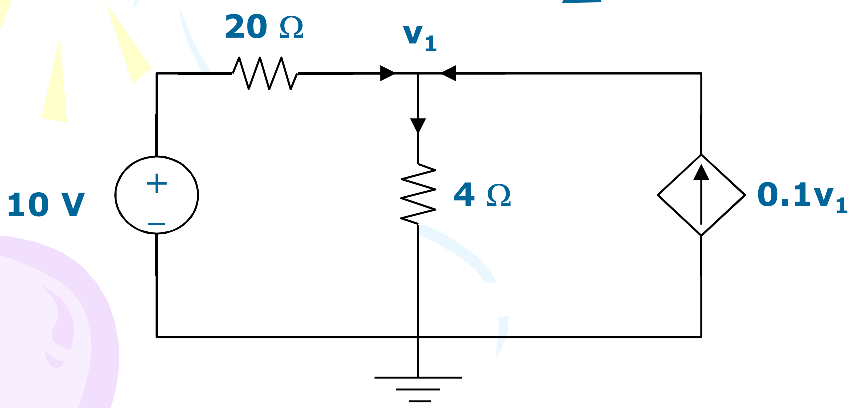
Use superposition to find v_x in the circuit below.



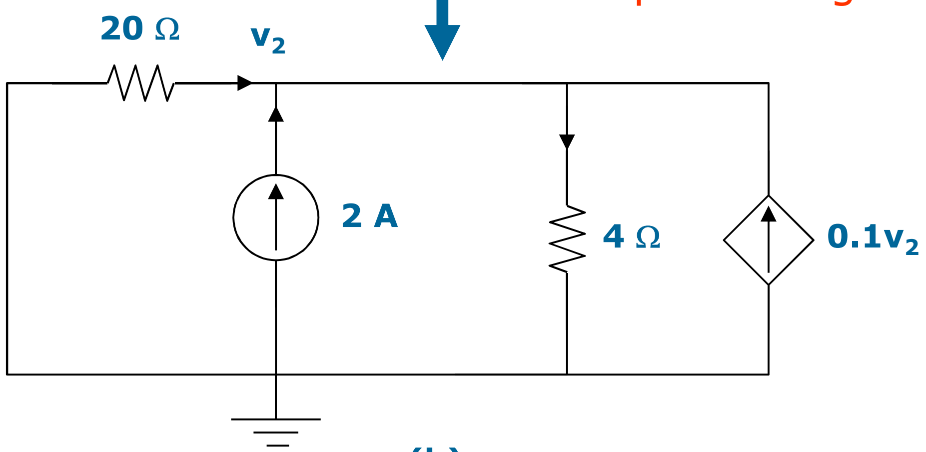
2A is discarded by open-circuit

10V is discarded by open-circuit

Dependent source keep unchanged



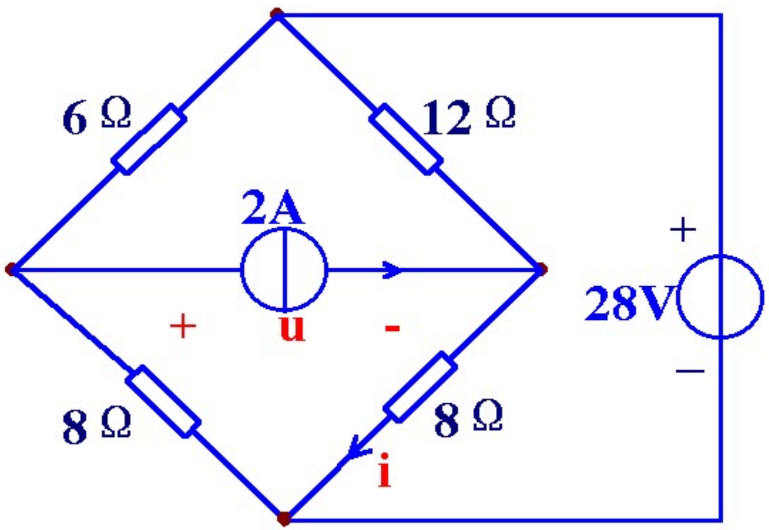
(a)

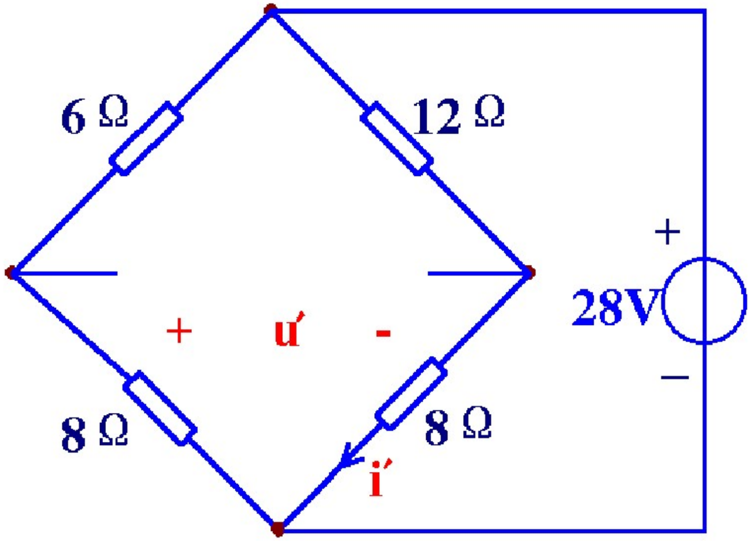


(b)

Answer Vx = 12.5V

Example 1. Use superposition to compute *u* and *i* in the circuit shown in following Fig. .





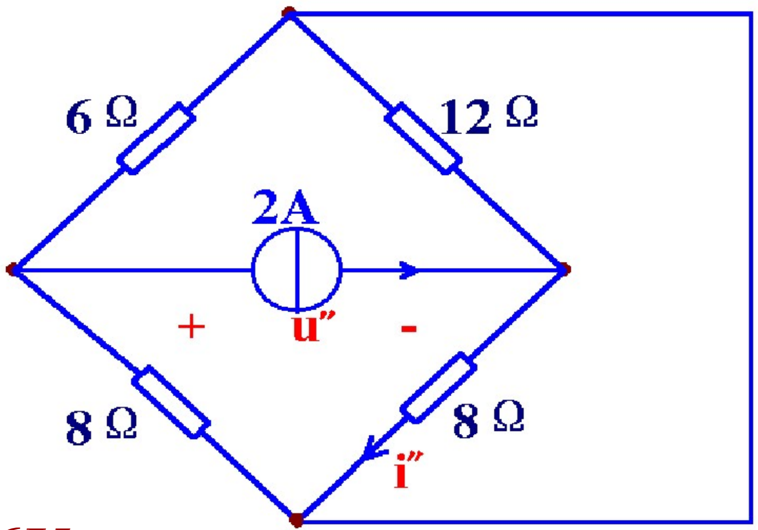
Solution:

Let $28V$ source act alone,

$$i' = \frac{28}{12+8} = 1.4A \quad u' = 4.8V$$

Let $2A$ source act alone,

$$i'' = \frac{12}{12+8} \times 2 = 1.2A \quad u' = -16.46V$$



Thus

$$i = 2.6A \qquad u = -11.66V$$

Example 3. In the following circuit, if U_s=1V, I_s=1A: U_2= 0 ; if U_s=10V, I_s=0: U_2= 1V . If U_s=0, I_s=10A, find U_2.

Solution:

Suppose the following equation using the superposition principle

$$U_2 = K_1 I_s + K_2 U_s$$

Therefore

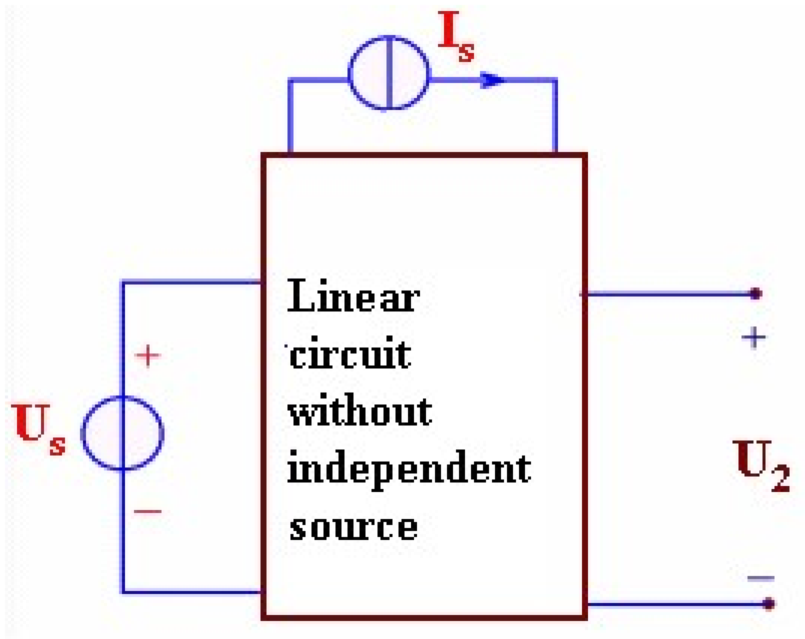
$$0 = K_1 \bullet 1 + K_2 \bullet 1$$

$$1 = K_1 \bullet 0 + K_2 \bullet 10$$

Solving the equations, we have

$$K_1 = -0.1 \qquad K_2 = 0.1$$

$$\therefore \quad U_2 = -0.1 I_s + 0.1 U_s$$



Thus if **Us=0, Is=10A,**

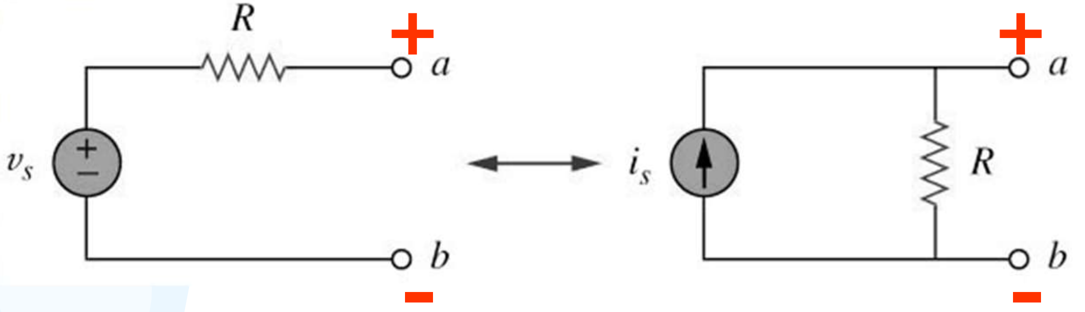
$$U_2 = -1V$$

4.4 Source Transformation (1)

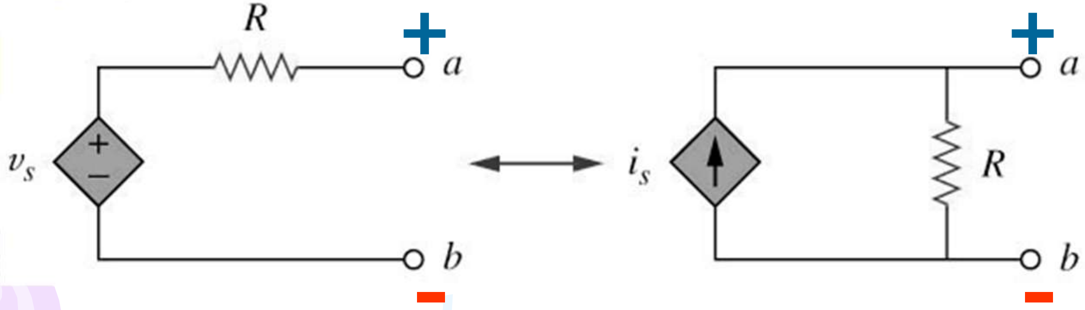
- An equivalent circuit is one whose *v-i* characteristics are identical with the original circuit.

- It is the process of replacing ***a voltage source v_s in series with a resistor R*** **by a** ***current source i_s in parallel with a resistor*** *R*, or vice versa.

4.4 Source Transformation (2)

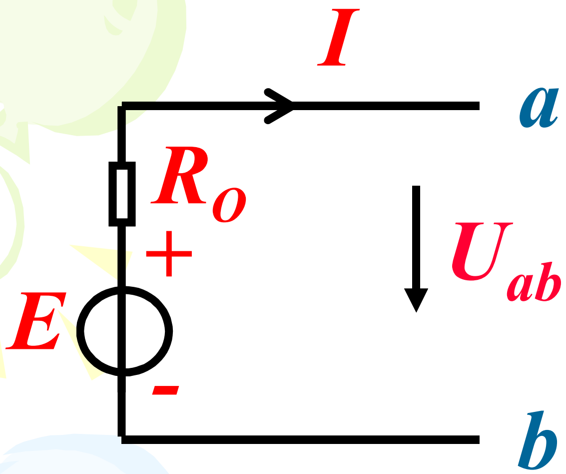


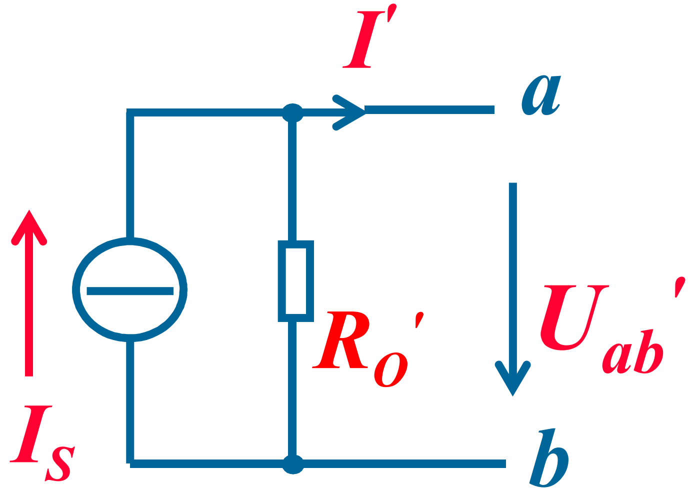
(a) Independent source transform



(b) Dependent source transform

- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.





$$U_{ab} = E - I \cdot R_o$$

$$U_{ab}' = \left(I_s - I'\right) \cdot R_o' = I_s \cdot R_o' - I' \cdot R_o'$$

if

$$I = I'$$

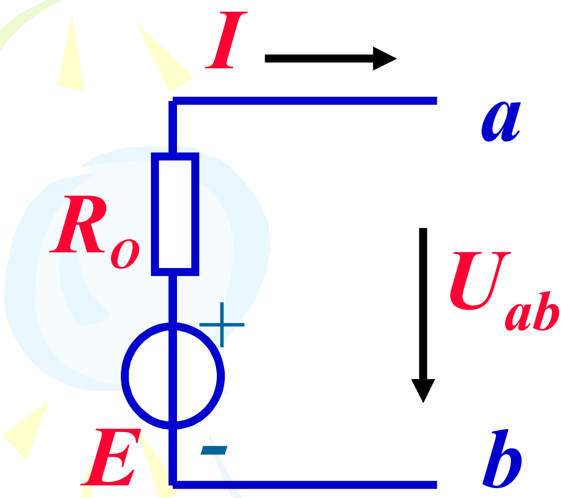
$$U_{ab} = U_{ab}'$$

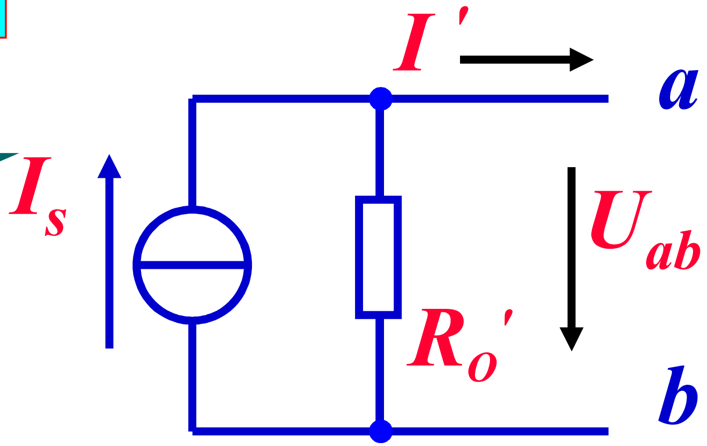
then $E - I \cdot R_o = I_s \cdot R_o' - I' \cdot R_o'$

$$E = I_s \cdot R_o' \qquad R_o = R_o'$$

$$I_s = E \big/ R_o$$

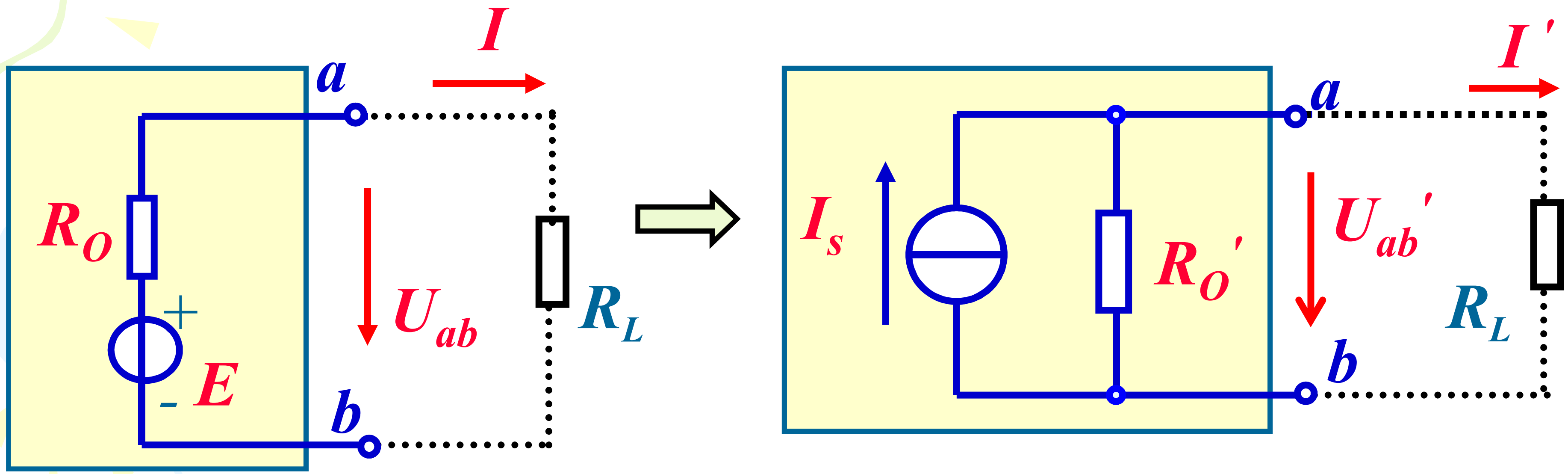
$$R_o' = R_o$$



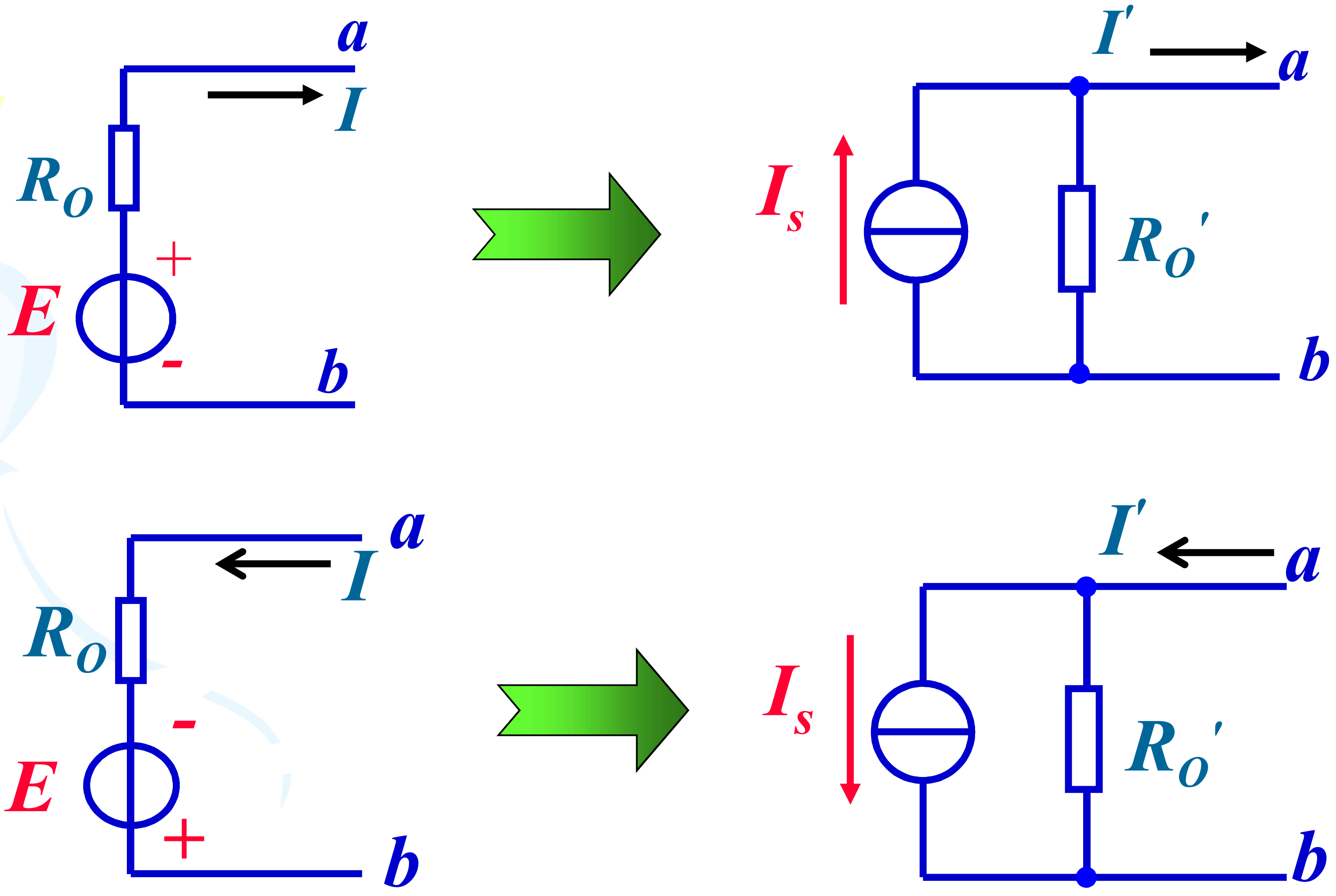


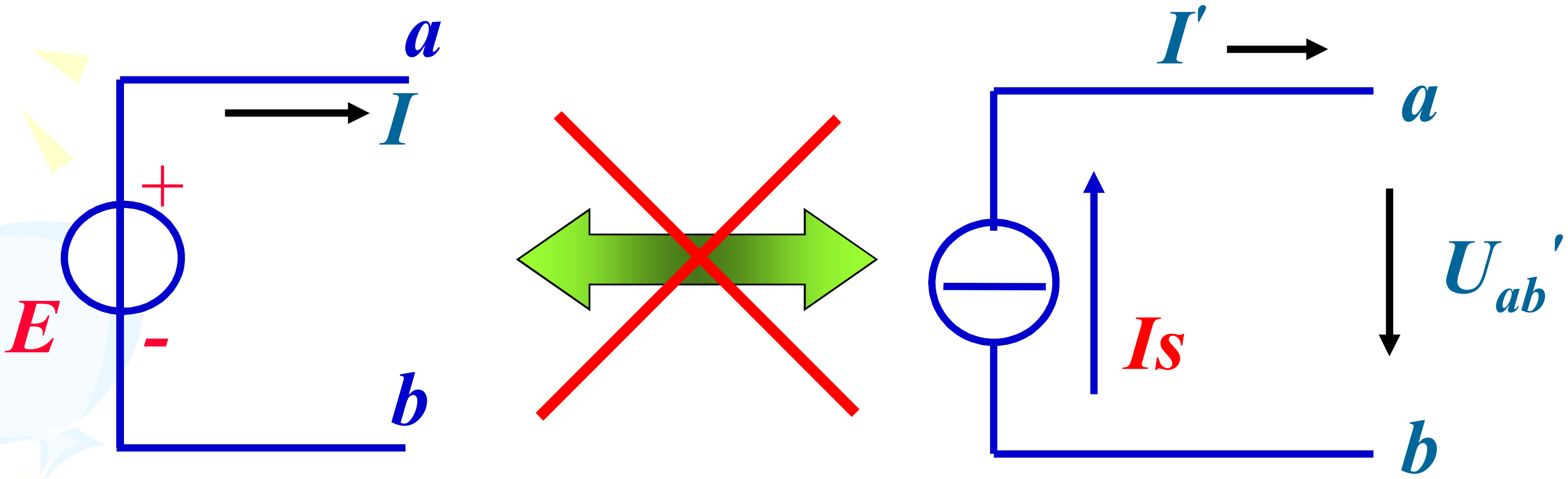
$$E = I_s \cdot R_o'$$

$$R_o = R_o'$$

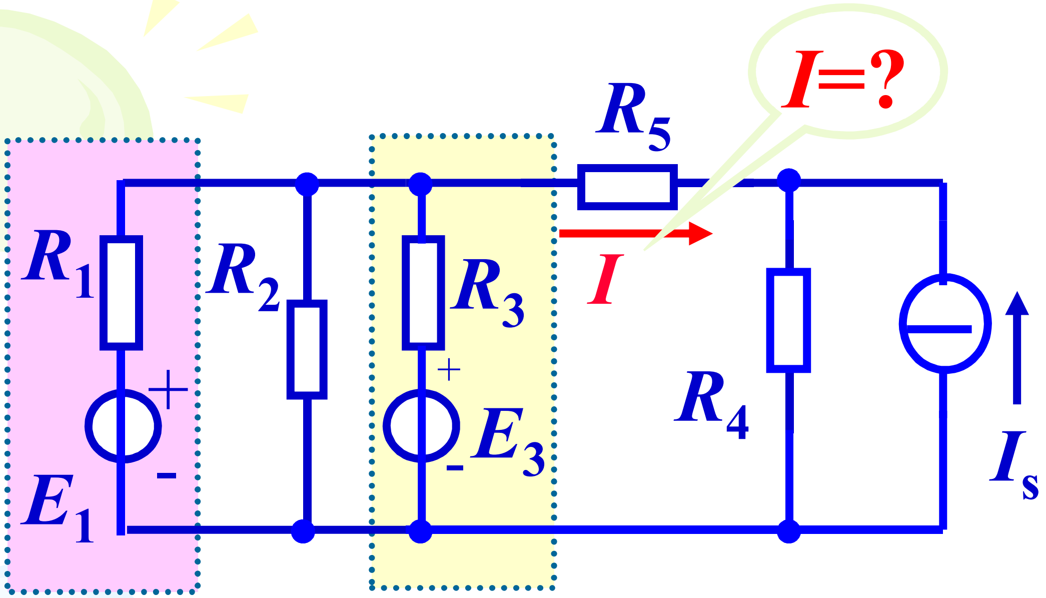


I
a
R_O
+
E
-
b
U_{ab}
R_L
I'
a
I_s
R_O'
U_{ab}'
R_L
b


a
I
R_O
+
E
-
b
I′
a
I_s
R_O'
b
a
I
R_O
-
E
+
b
I′
a
I_s
R_O'
b

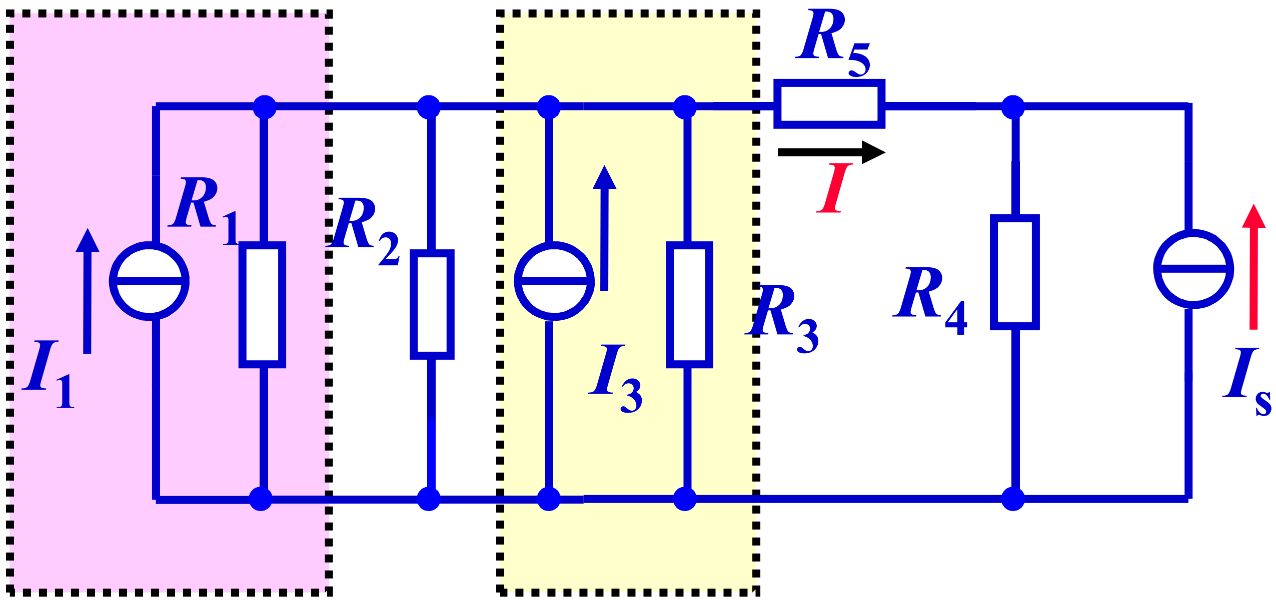


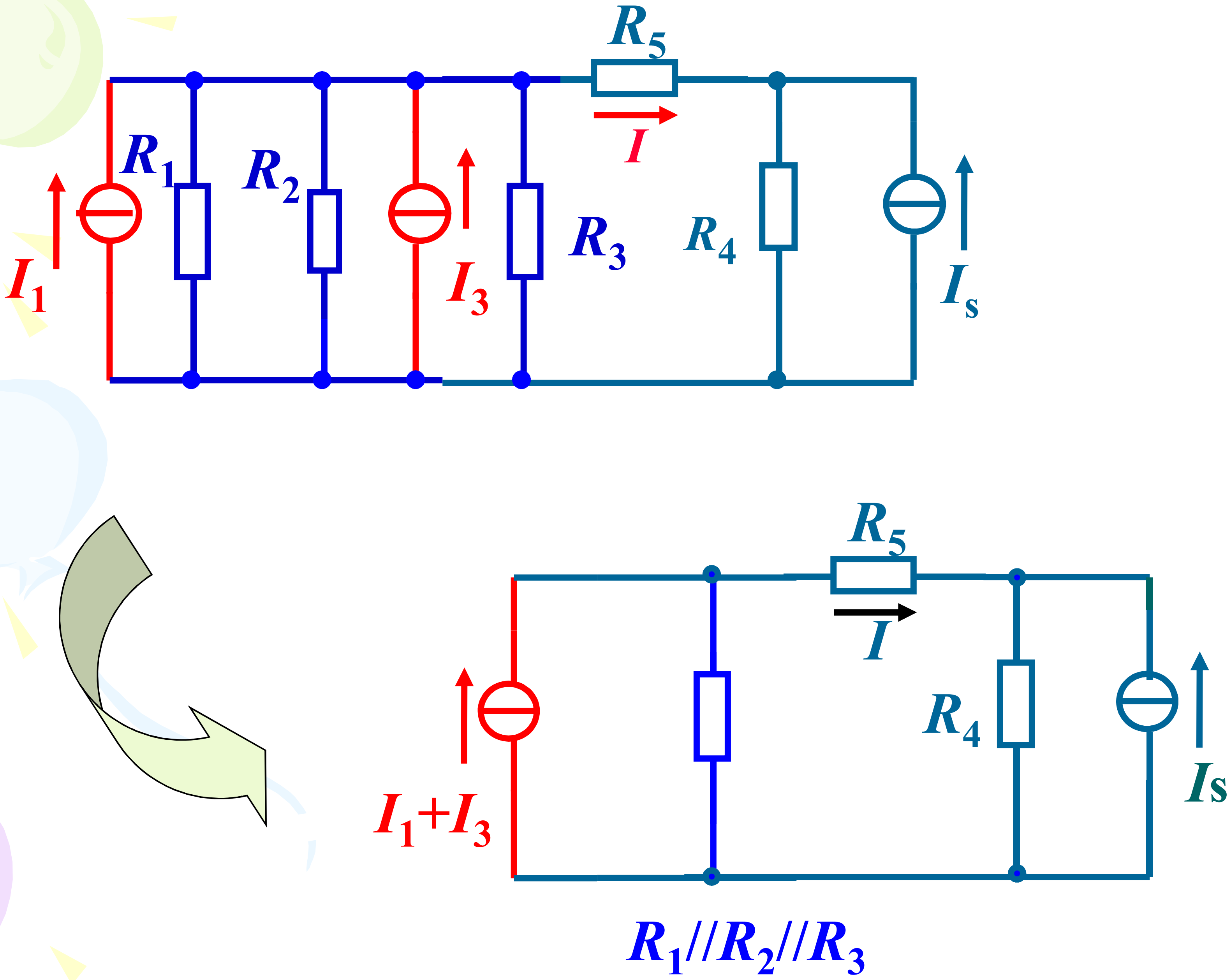
$$I_S = \frac{E}{R_o} = \frac{E}{0} = \infty$$

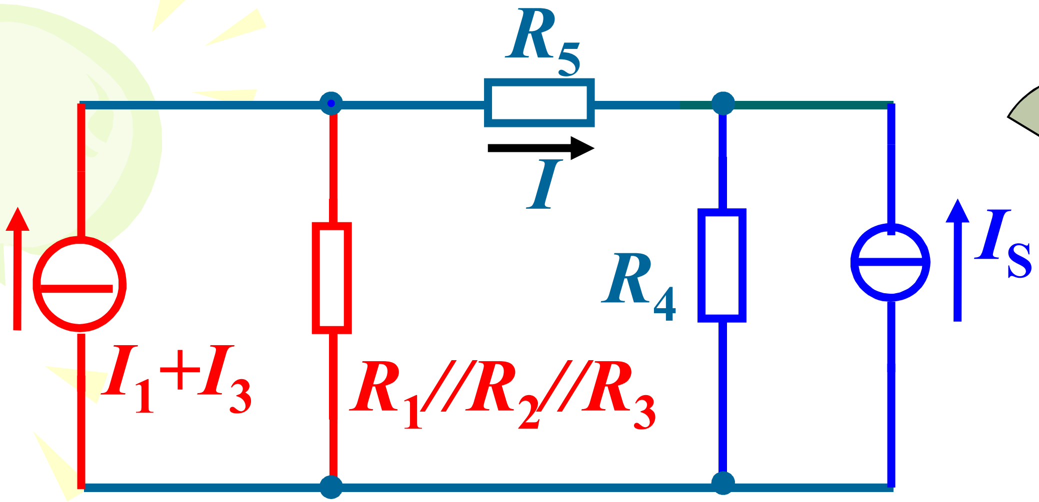


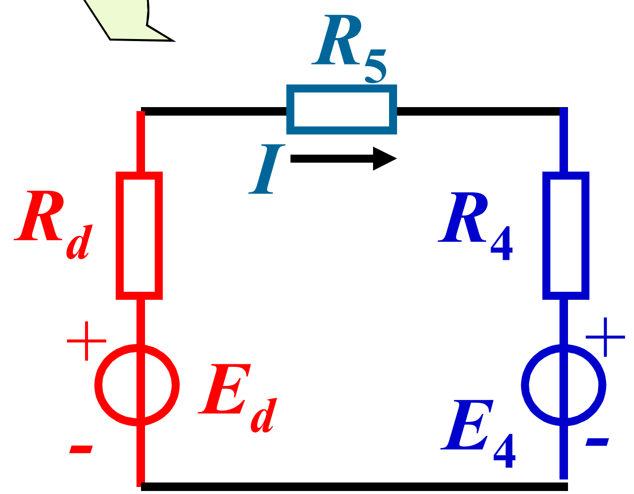
$$I_1 = \frac{E_1}{R_1}$$

$$I_3 = \frac{E_3}{R_3}$$




R5
I
R1
R2
R3
R4
I1
I3
Is
R5
I
R4
Is
I1+I3
R1//R2//R3



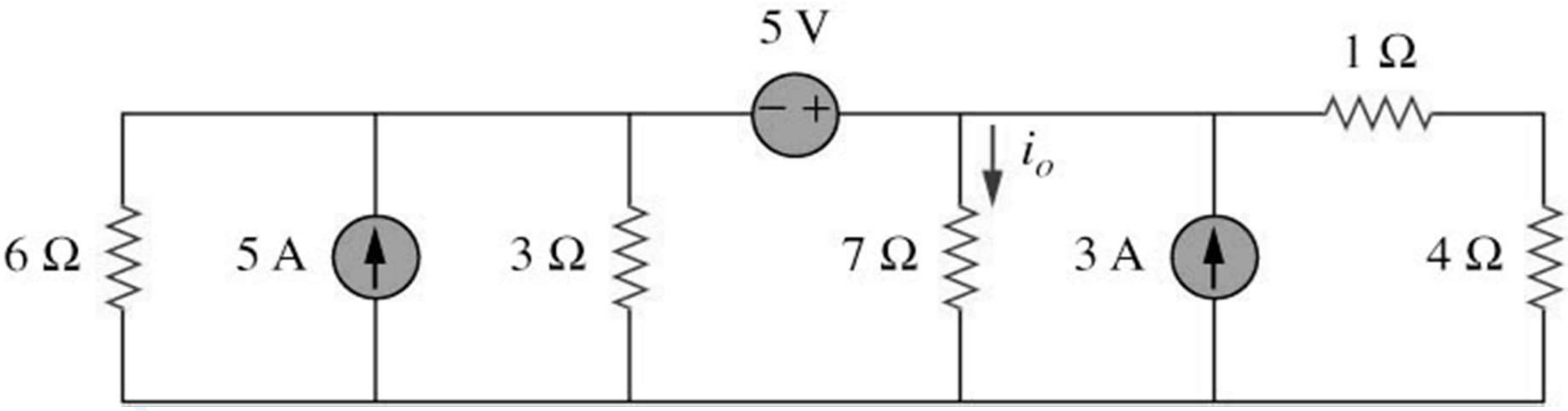


$$
\begin{cases}
E_d = (I_1 + I_3) \cdot (R_1 // R_2 // R_3) \\
R_d = R_1 // R_2 // R_3 \\
E_4 = I_S \cdot R_4
\end{cases}
$$

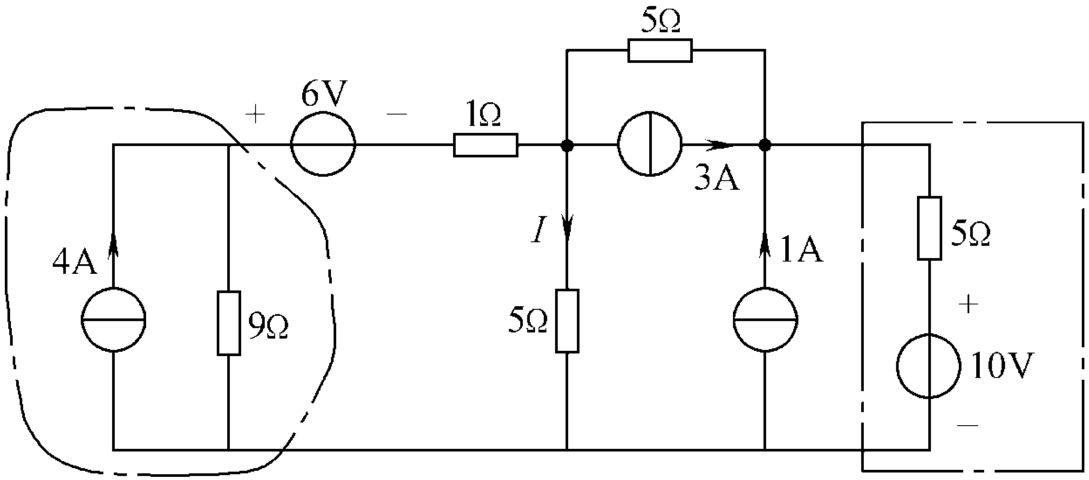
$$
\boxed{I = \frac{E_d - E_4}{R_d + R_5 + R_4}}
$$

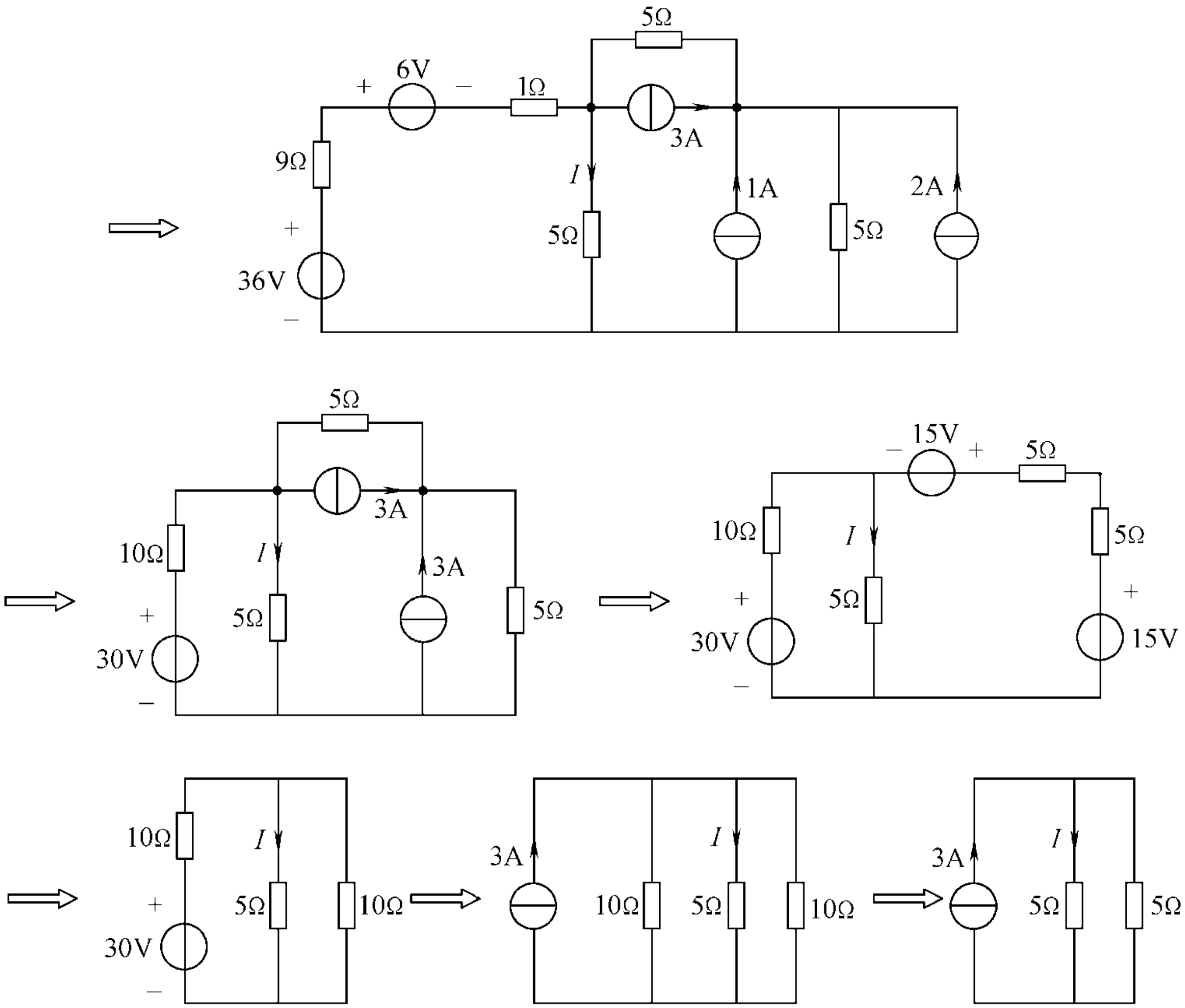
Example 4

Find i_o in the circuit shown below using source transformation.



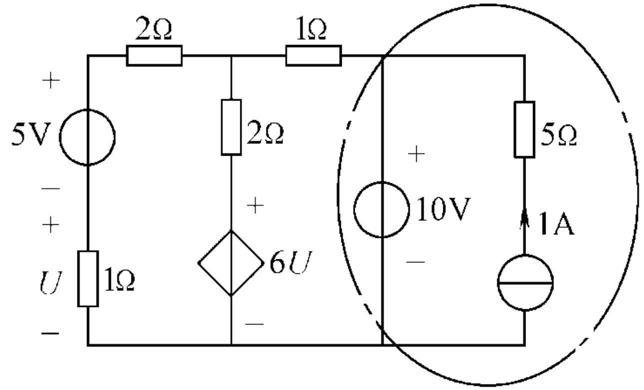
i_0 = 1.78A


5Ω
6V
+
−
1Ω
3A
4A
9Ω
I
5Ω
1A
5Ω
+
10V
−

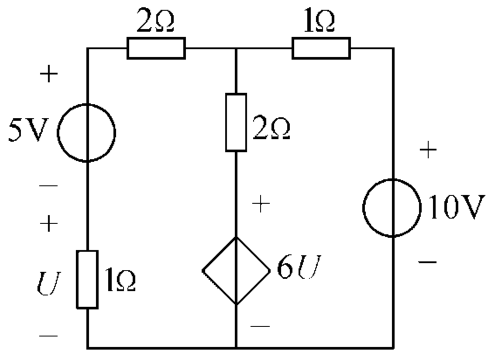

5Ω
6V
1Ω
9Ω
36V
I
3A
1A
5Ω
2A
5Ω
5Ω
10Ω
30V
I
3A
3A
5Ω
5Ω
15V
5Ω
10Ω
I
5Ω
30V
5Ω
15V
10Ω
I
5Ω
10Ω
30V
3A
10Ω
I
5Ω
10Ω
3A
I
5Ω
5Ω

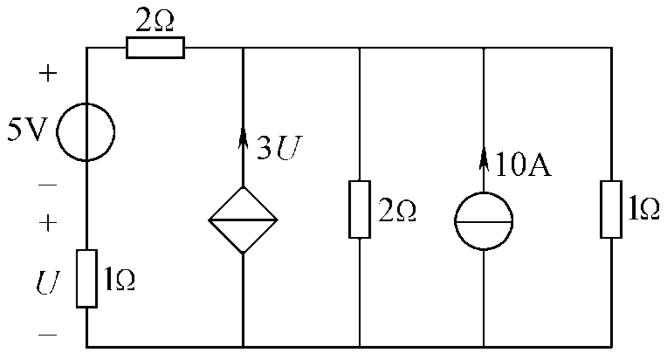
$$I = 1.5\text{A}$$

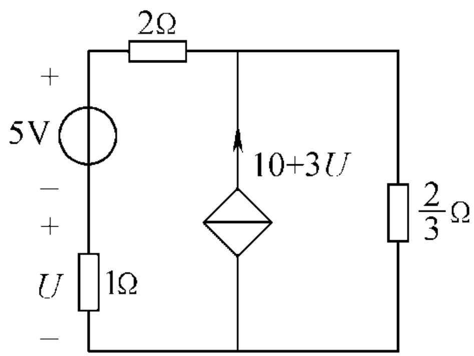
$$U = \frac{(10+3U)\cdot\frac{2}{3}-5}{3+\frac{2}{3}}\times 1$$

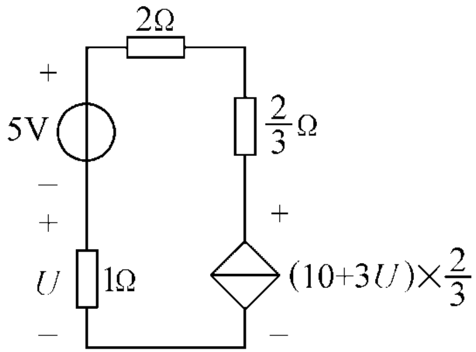


$$U = 1\text{V}$$



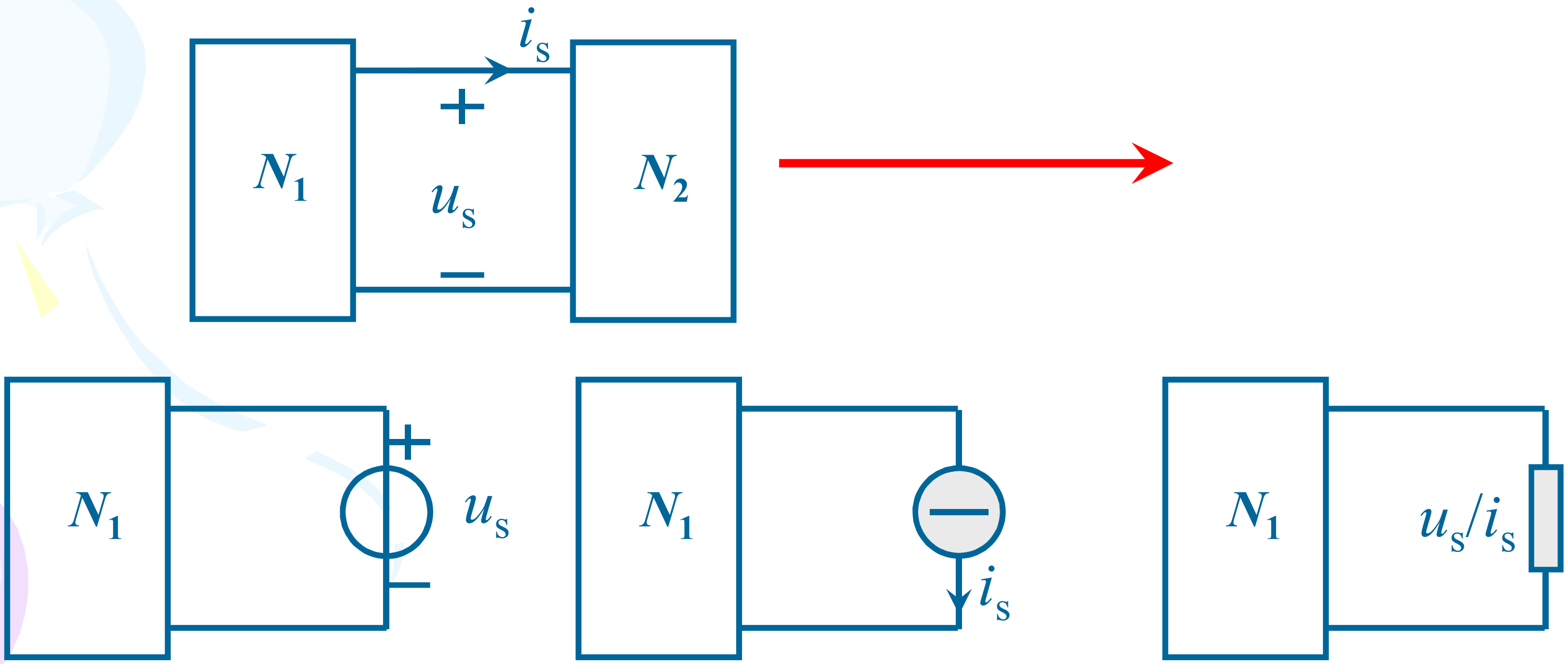






4.5 Substitution theorem(替代定理)

- If the voltage across and current through any branch of a dc network with two terminals are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.



替代定理：

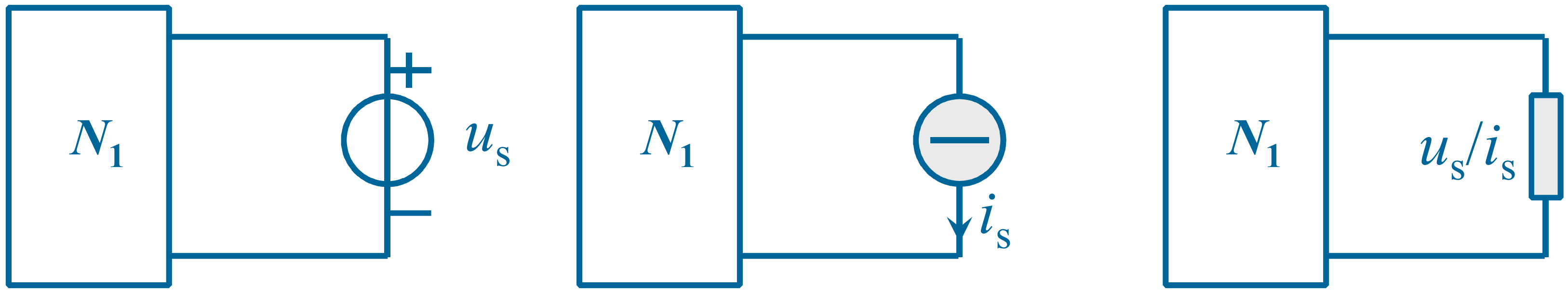
在任意线性和非线性、定常和时变的网络中，如果某k支路的电压为U_k，电流为I_k，只要该支路和网络的其它支路之间无耦合，即k支路不是非独立电源支路，总可以用下列任一元件去替代该支路：

电压为$U_{sk}=U_k$，极性与U_k相同的独立电压源；

电流为$I_{sk}=I_k$，方向与I_k一样的独立电流源；

电阻为$R_k=U_k/I_k$的线性电阻器（假设U_k和I_k有关联参考方向）。

替代后整个网络中的电流和电压都保持不变。



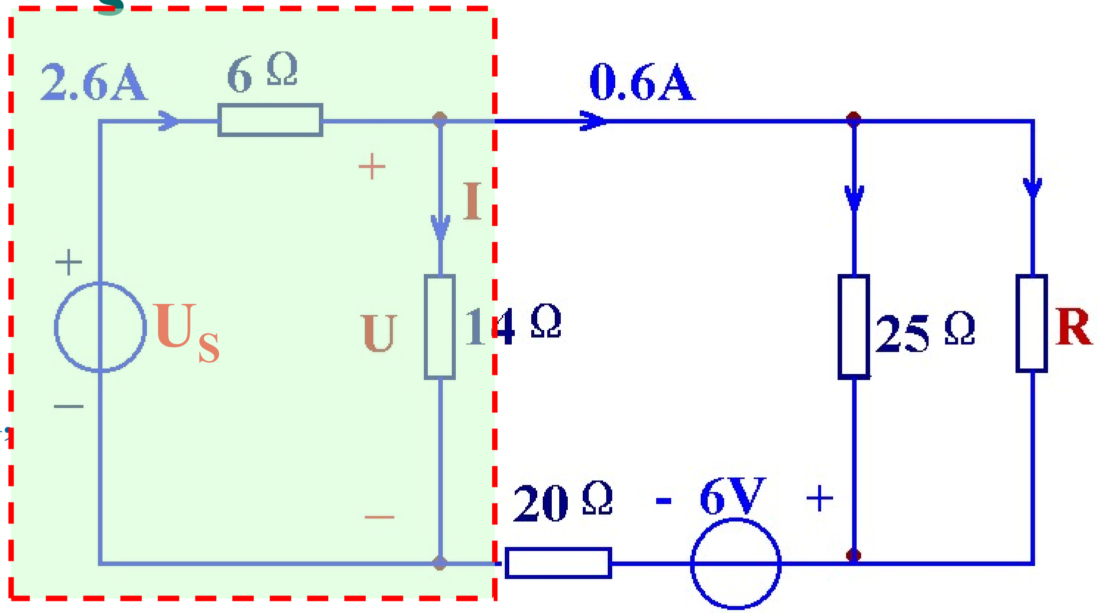
Example: Find U_s and R.

Solution:

I=2A U=28v

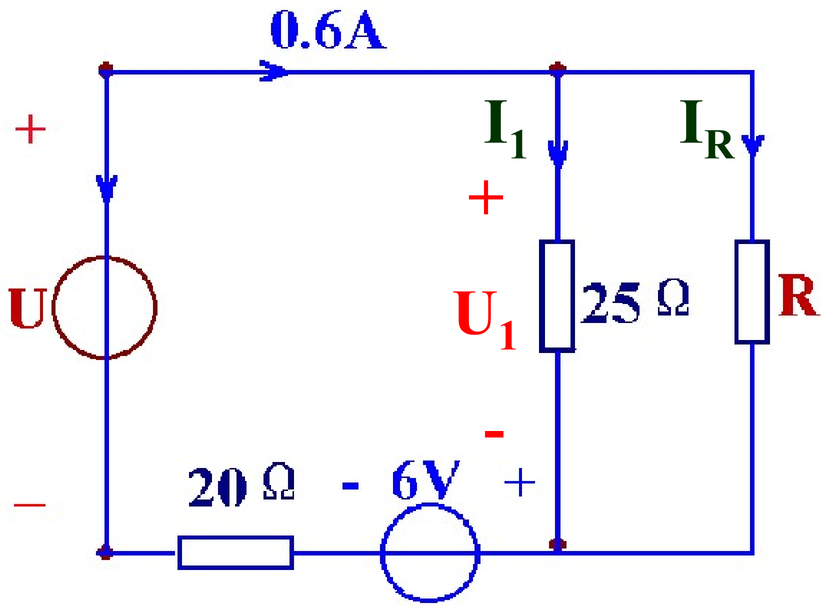
U_S=43. 6v

Apply Substitution theorem, we have



$$U_1 = 28 - 20 \times 0.6 - 6$$

=10v I_1=0. 4A

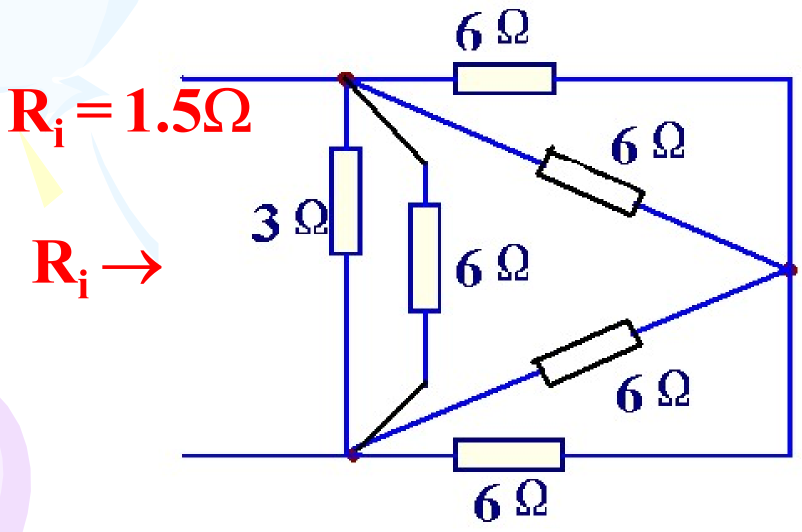


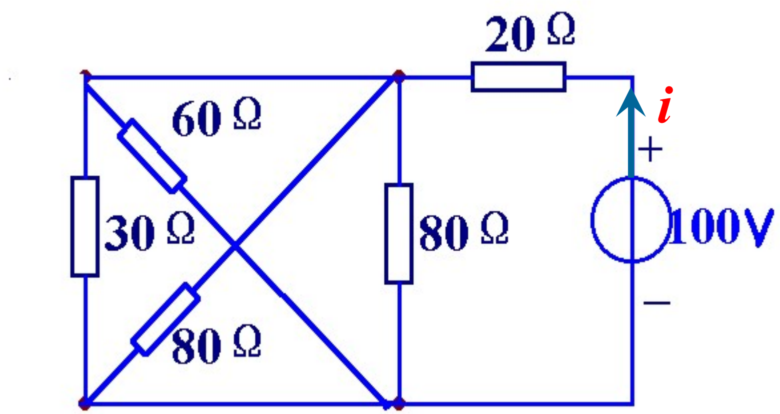
I_R=0. 6−0. 4=0. 2A

∴ **R=50Ω.**

4.6 Simplification of a one-port network contains no independent sources

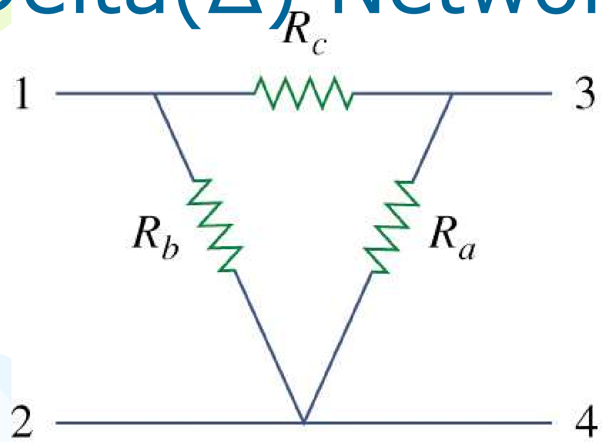
Case 1: contains no dependent sources (its equivalence is a resistor)



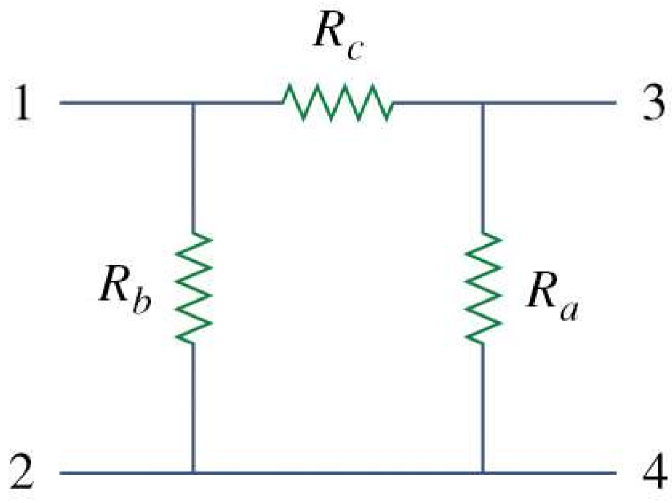


Wye-Delta Transformations(1)

- Delta(Δ) Network

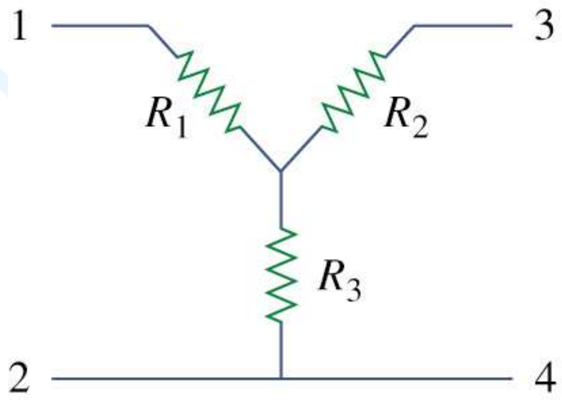


(a)

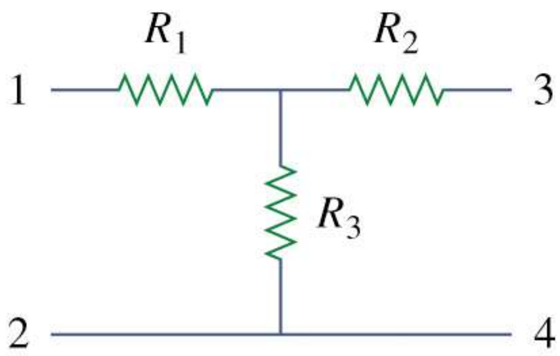


(b)

- Wye(Y or T) Network

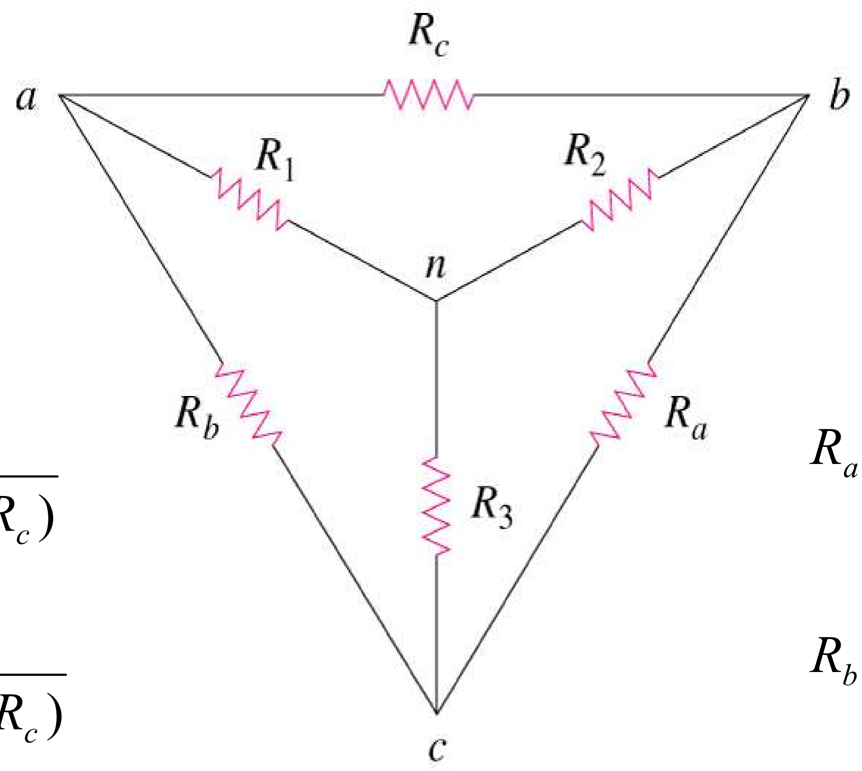


(a)



(b)

Wye-Delta Transformations(2)



Delta -> Star

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

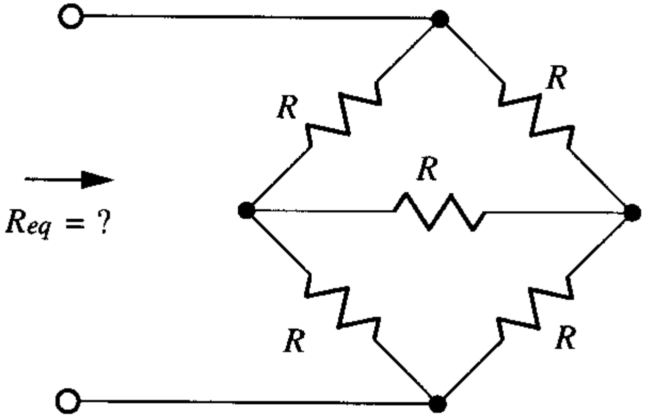
$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Star -> Delta

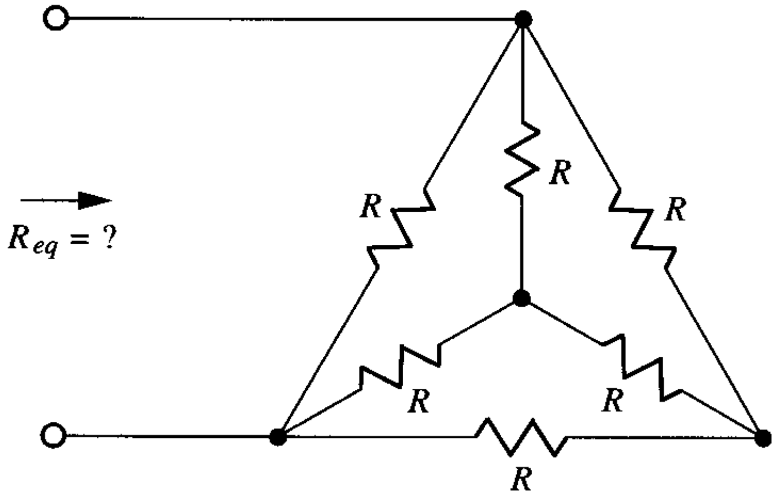
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

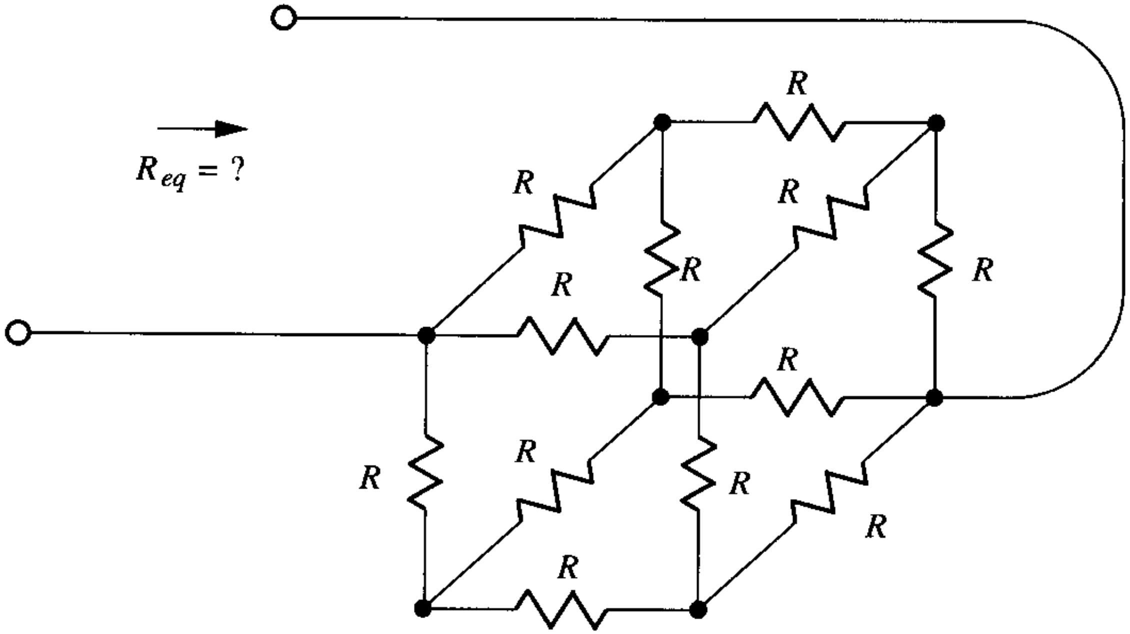
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$



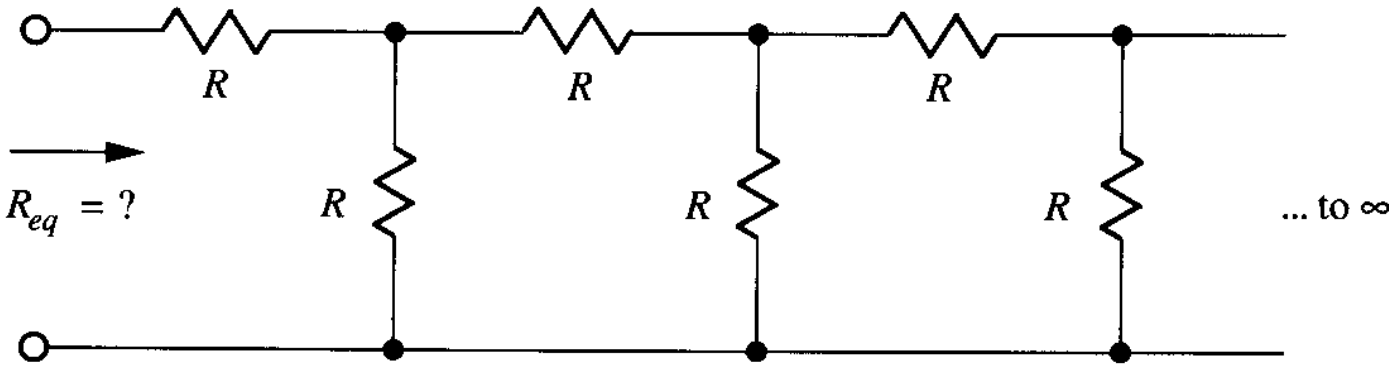
(a)



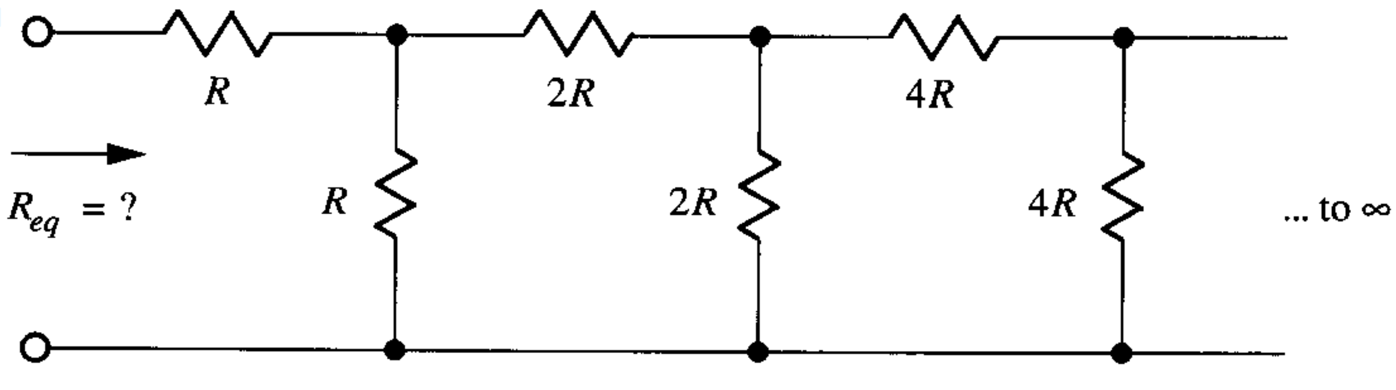
(b)



(c)



(a)

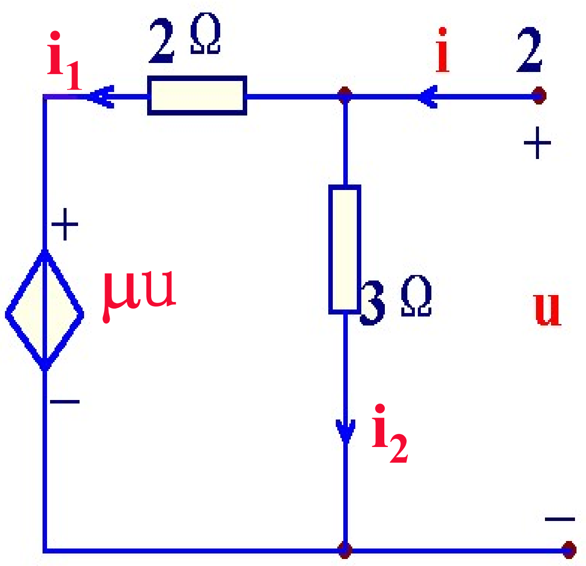


(b)

Case 2: contains dependent sources (its equivalence is a resistor)

Method: Apply either a test voltage source or a test current source to the terminals of the one-port network, the ratio of the voltage across the test source to the current delivered by the test source equals to the equivalent resistance.

Example1: Simplify the following circuit.

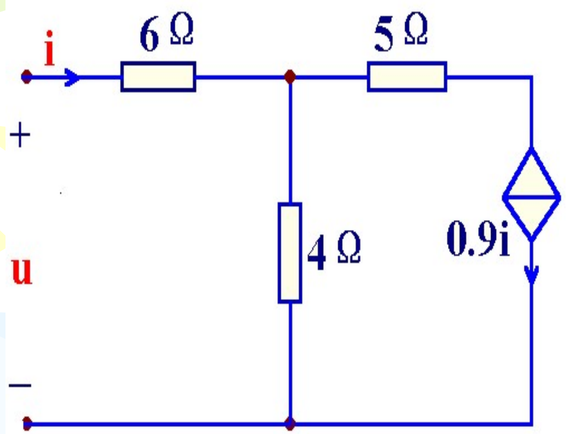


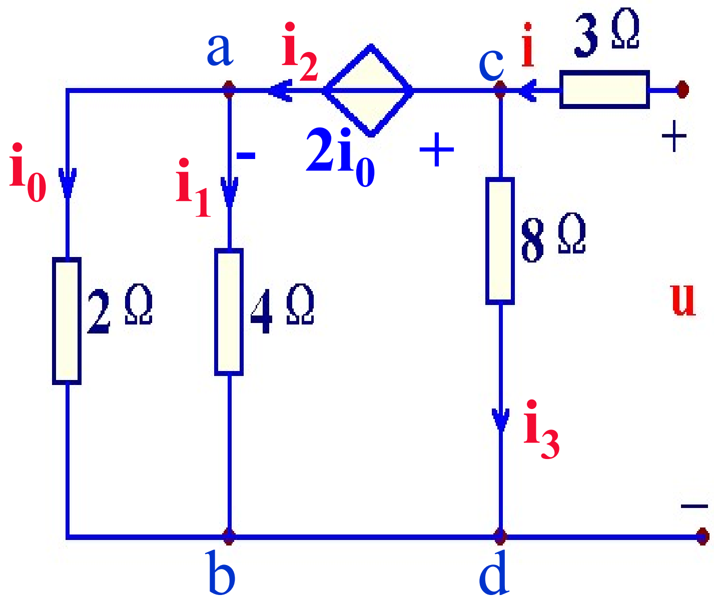
Solution: Apply a test voltage source,

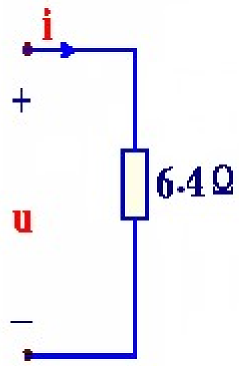
$$i_2 = \frac{u}{3} \qquad i_1 = \frac{u - \mu u}{2}$$

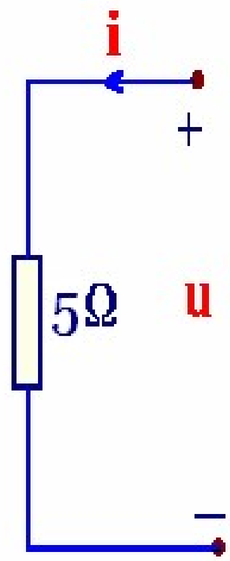
$$i = i_1 + i_2 = \frac{u}{3} + \frac{u - \mu u}{2} = \left(\frac{1}{3} + \frac{1-\mu}{2}\right)u$$

$$R = \frac{u}{i} = \frac{1}{\frac{1}{3} + \frac{1-\mu}{2}} = \frac{6}{5 - 3\mu}$$


i
6 Ω
5 Ω
+
u
4 Ω
0.9i
–


a
i_2
c
i
3 Ω
i_0
-
i_1
$2i_0$
+
+
8 Ω
2 Ω
4 Ω
u
i_3
–
b
d


i
+
u
6.4 Ω
–


i
+
5Ω
u
–