2.4 Equivalent Subcircuits

- A generally useful strategy in analyzing electric circuits is to *simplify wherever possible*.
  - Replacing a part of a circuit with a simple ***subcircuit*** contains fewer elements, without altering any current or voltage outside that part (or region).
  - The simpler circuit can then be analyzed, and the results will apply equally to the original, more complex, circuit.
- Such a beneficial trade is possible only when the original and replacement subcircuits are equivalent to one another in specific sense to be defined.
- Subcircuit: A subcircuit is any part of a circuit.
- Two-terminal subcircuit: A subcircuit containing any number of interconnected elements, but with two accessible terminals, is called two-terminal subcircuit

- Terminal voltage and terminal current: The voltage across and current into these terminals are called the terminal voltage and terminal current of the two-terminal subcircuit , as in Fig.2.13

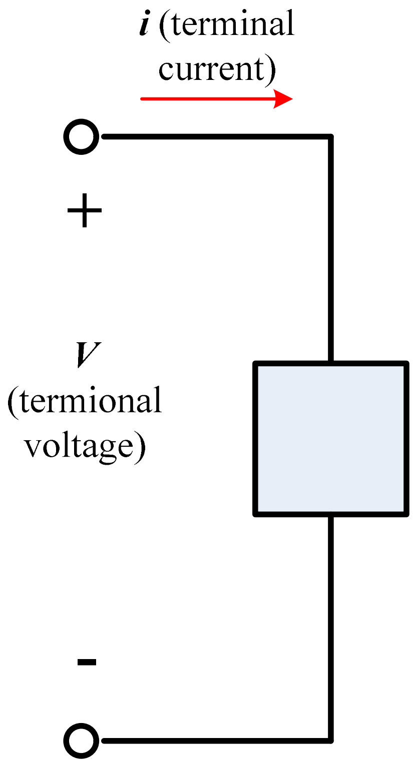


Fig.2.13

- The behavior of a two-terminal subcircuit(what it "does" to any circuit containing it, is completely described by its ***terminal law***(the terminal law is a function of the form $v = f(i)$ or $i = g(v)$).
- *Two two-terminal subcircuit are said to be equivalent if they have same terminal law.*

- While the currents and voltage *external* to the equivalent subcircuit will not be changed when one is exchanged for the other in any circuit, the *internal* behavior of the equivalents may be quite different.
- The use of equivalent circuits will prove very helpful in simplifying circuit problems.

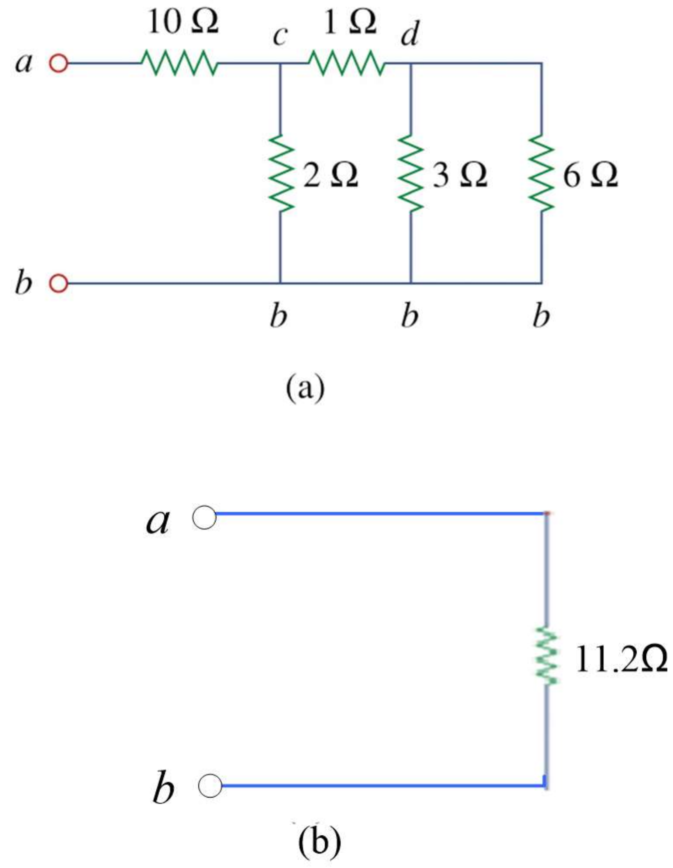


Fig. 2.14

2.5 Series Resistors and Voltage Division

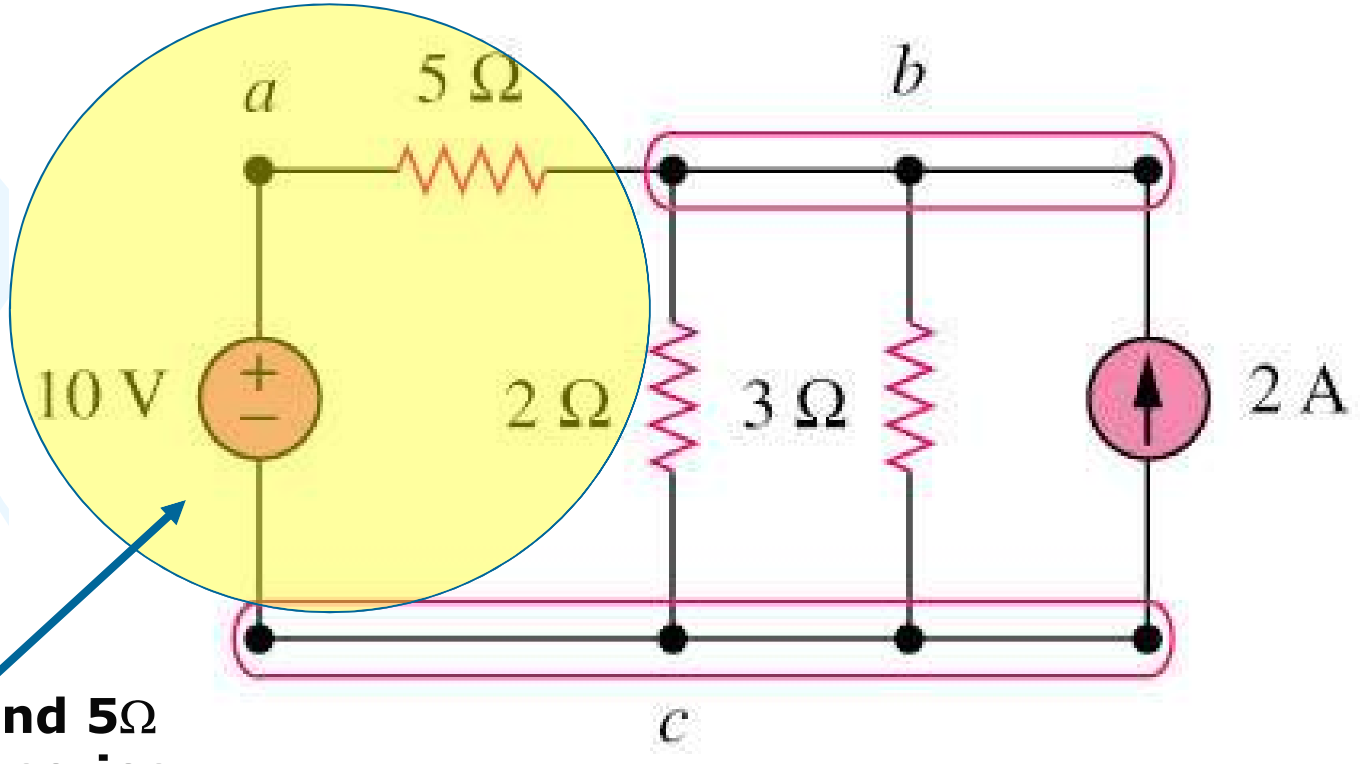
- ***Series:*** Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.
- The equivalent resistance of any number of resistors connected in a series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

- The voltage divider can be expressed as

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Example 12



10V and 5Ω are in series

Fig. 2.15

2.6 Parallel Resistors and Current Division (1)

- Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.
- The equivalent resistance of a circuit with N resistors in parallel is:

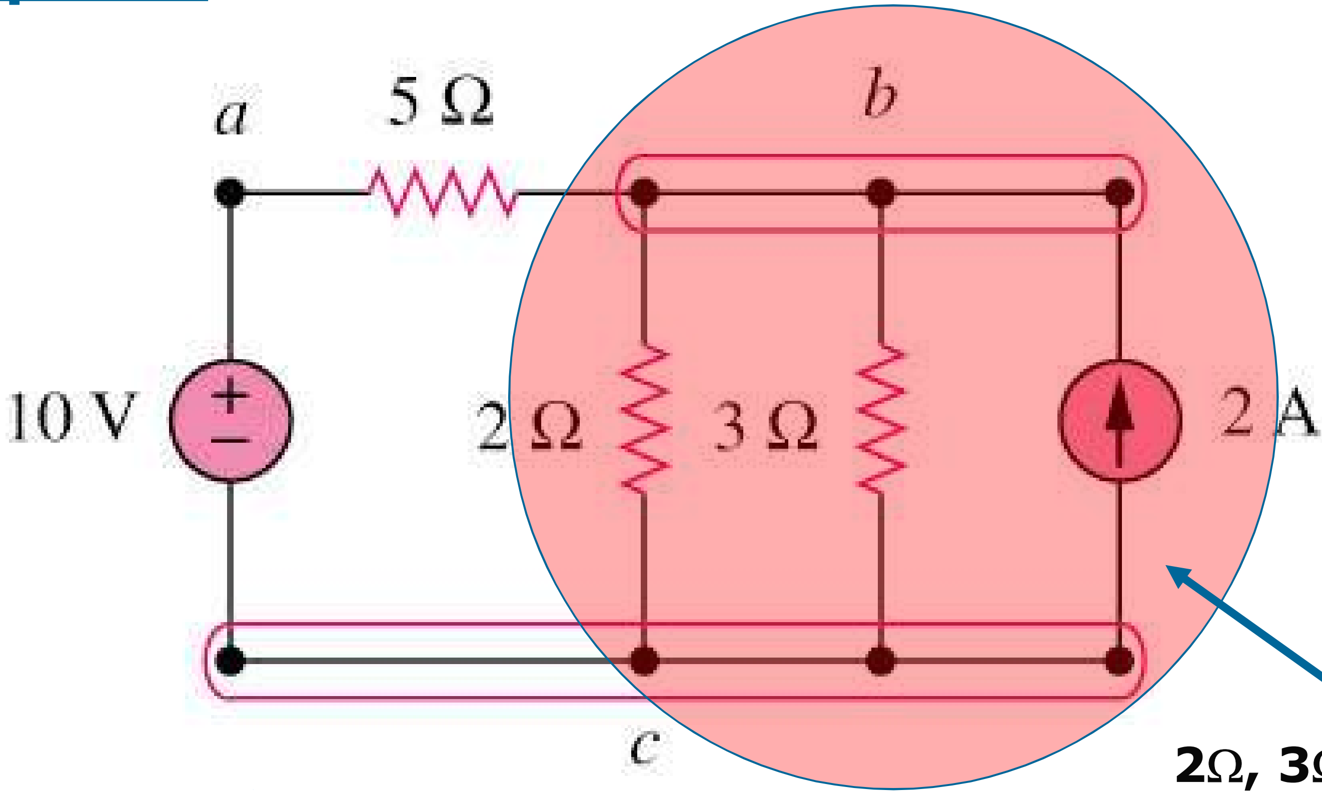
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- The total current i is shared by the resistors in inverse proportion to their resistances. The current divider can be expressed as:

$$i_n = \frac{v}{R_n} = \frac{iR_{eq}}{R_n}$$

2.6 Parallel Resistors and Current Division (2)

Example 13



2Ω, 3Ω and 2A are in parallel

Fig. 2.16

Example 13

Find R_{eq} for the circuit shown in Fig. 2.17

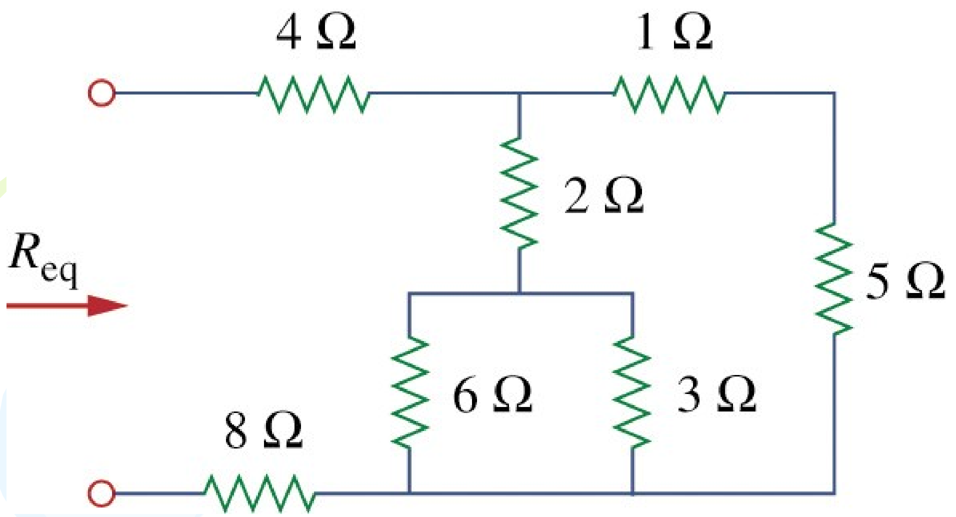


Fig. 2.17

$\Rightarrow$

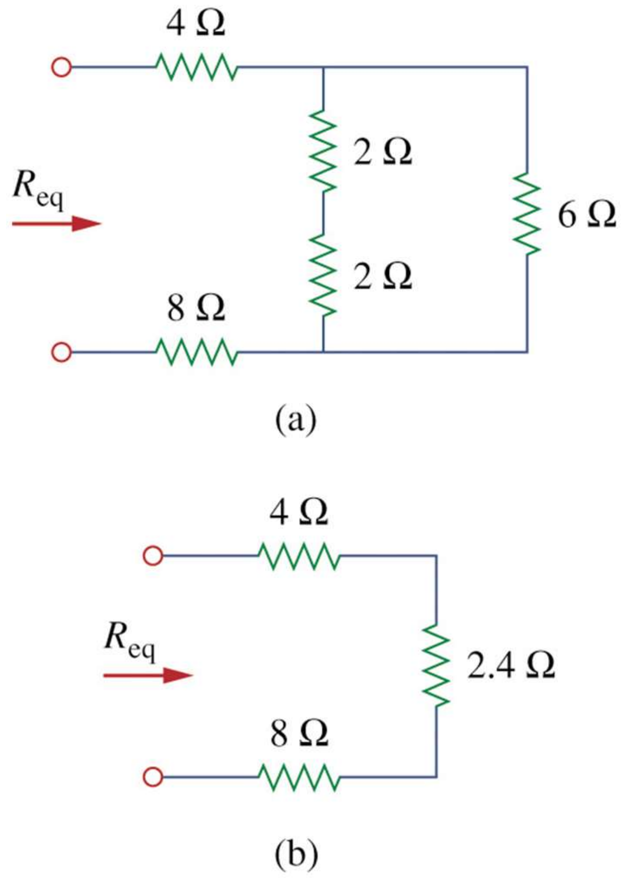


Fig. 2.18

R_{eq}=14.4Ω

Practice

By combining the resistors in Fig.2.19(a), find R_{eq}

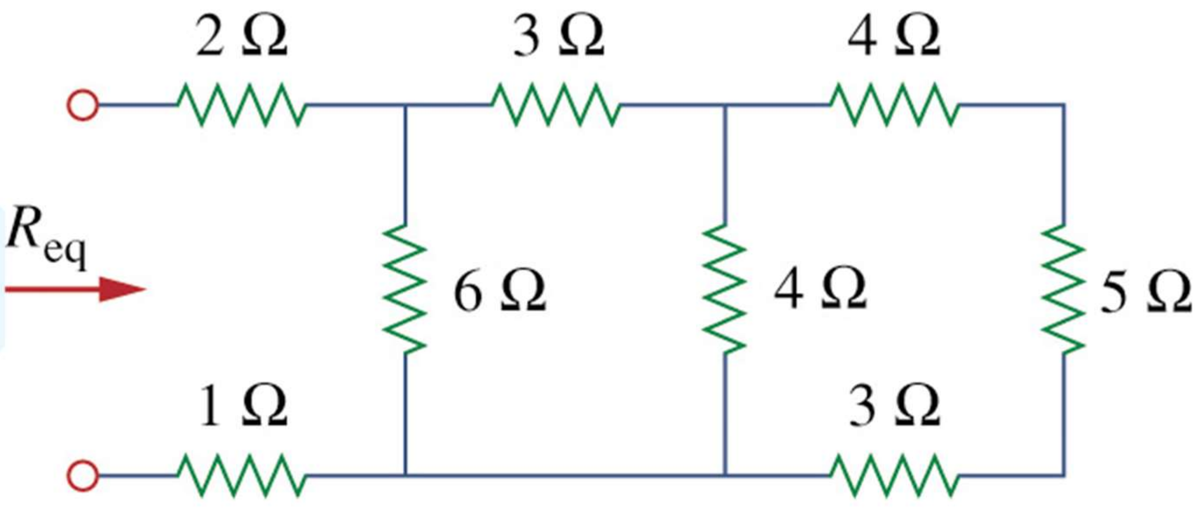


Fig. 2.19 (a)

Ans: 6Ω

Example 14

Calculate the equivalent resistance R_{ab} in the circuit in Fig.2.19(b)

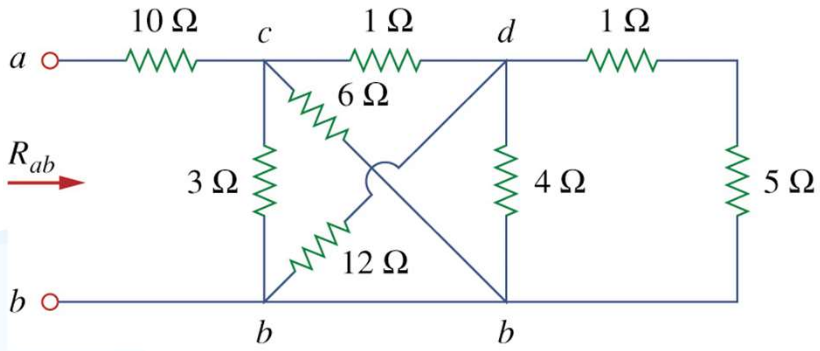
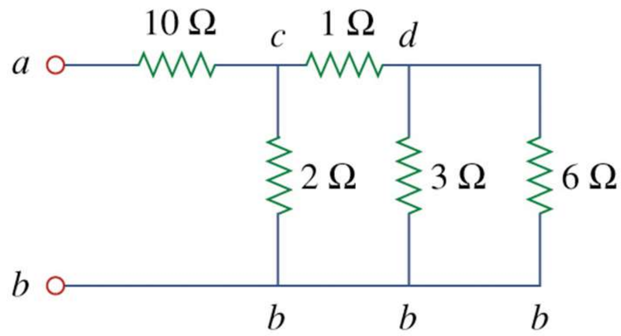
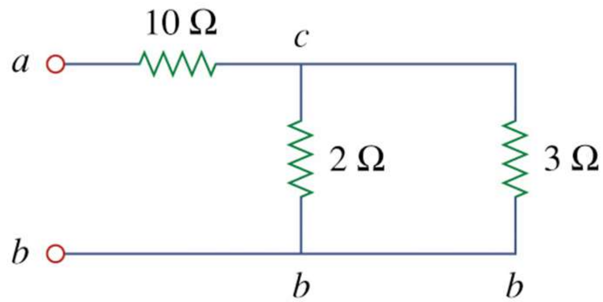


Fig.2.19 (b)



(a)



(b)

Fig 2.20

R_{ab}=11.2 Ω

Practice

Find R_{ab} for the circuit in Fig.2.21.

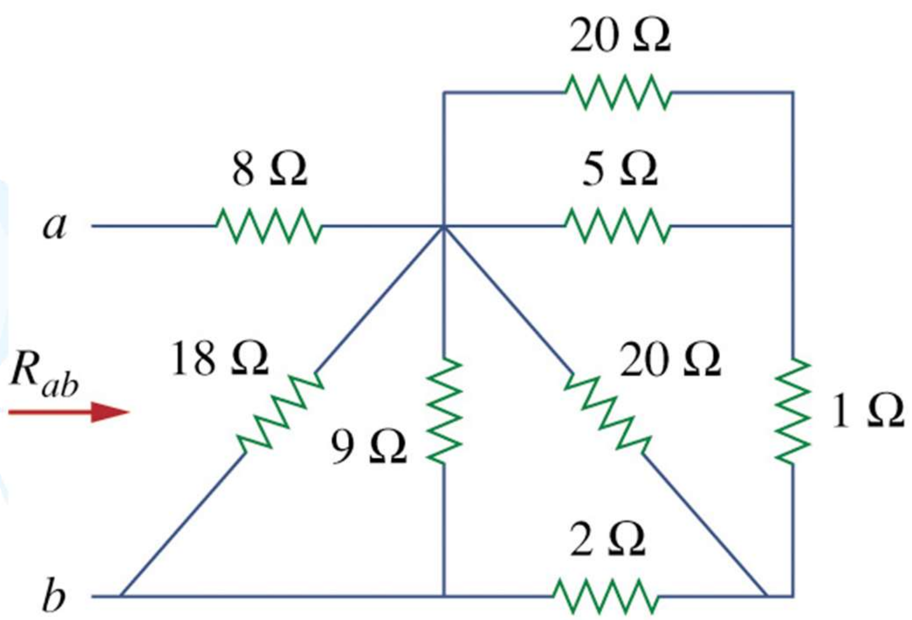


Fig. 2.21

Example 15

Find i_0 and v_0 in the circuit shown in Fig.2.22. calculate the power dissipated in the 3-Ω resistor.

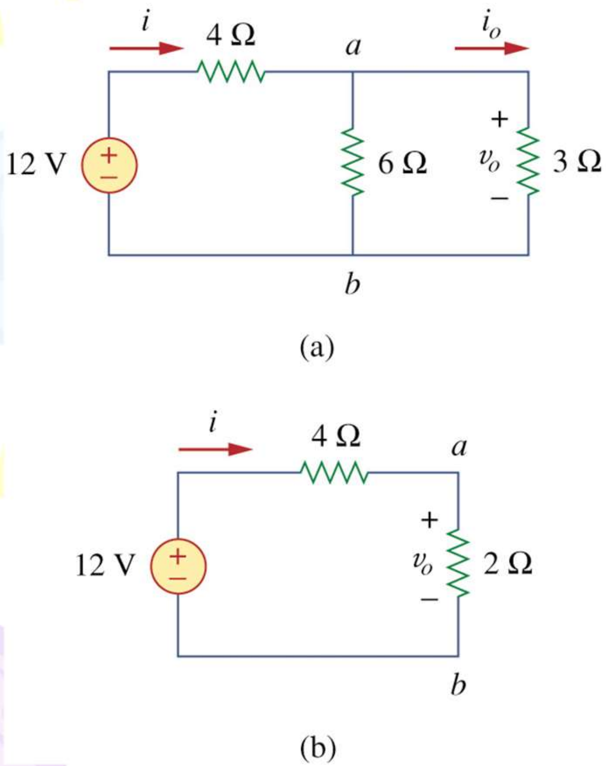


Fig. 2.22

i = 2A

v_0 = 4V

i_0 = 4/3A

p_0 = 5.333W

Practice

For the circuit shown in Fig.2.23.,find (a) v_1 and v_2, (b) the power dissipated in 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.

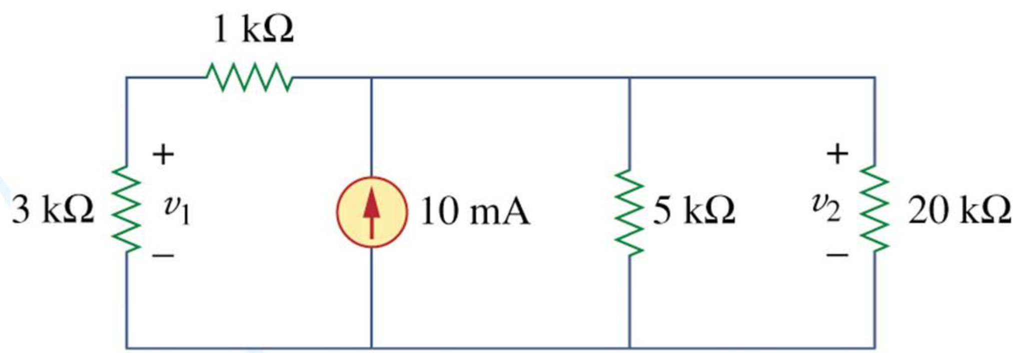
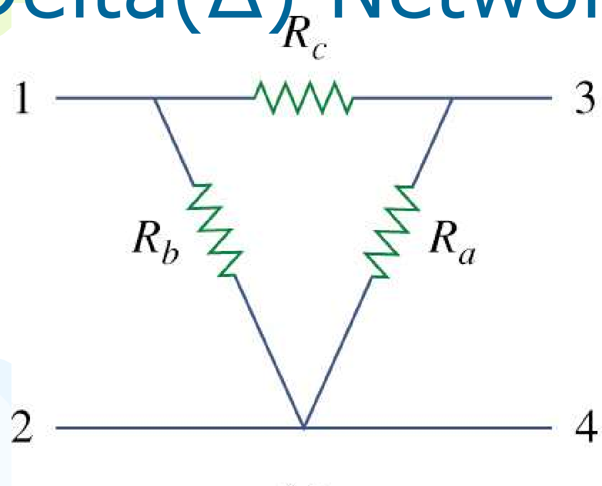


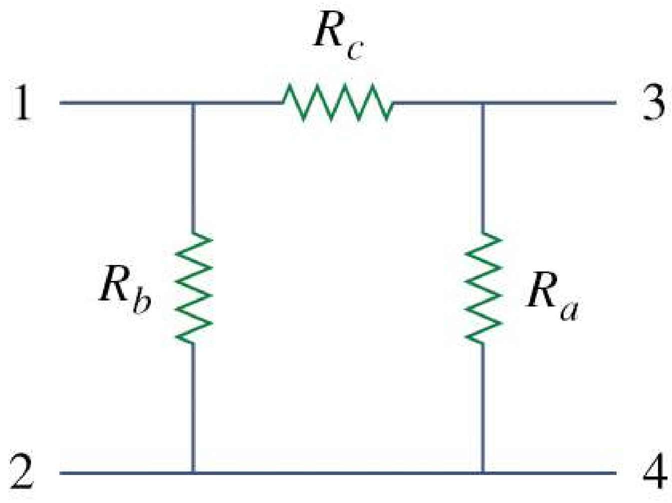
Fig. 2.23

2.7 Wye-Delta Transformations(1)

- Delta(Δ) Network

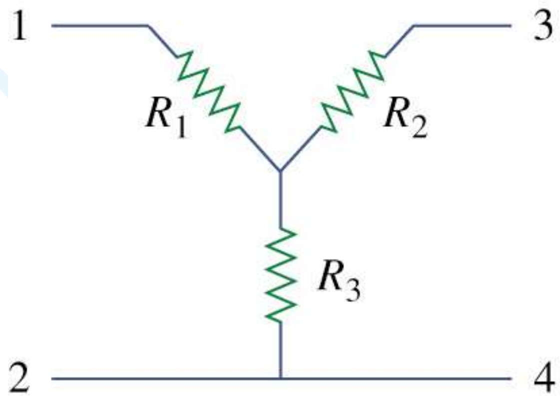


(a)

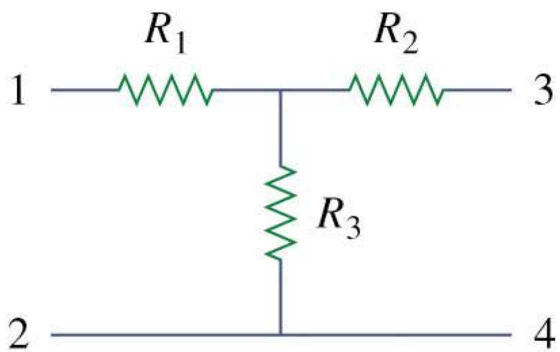


(b)

- Wye(Y or T) Network

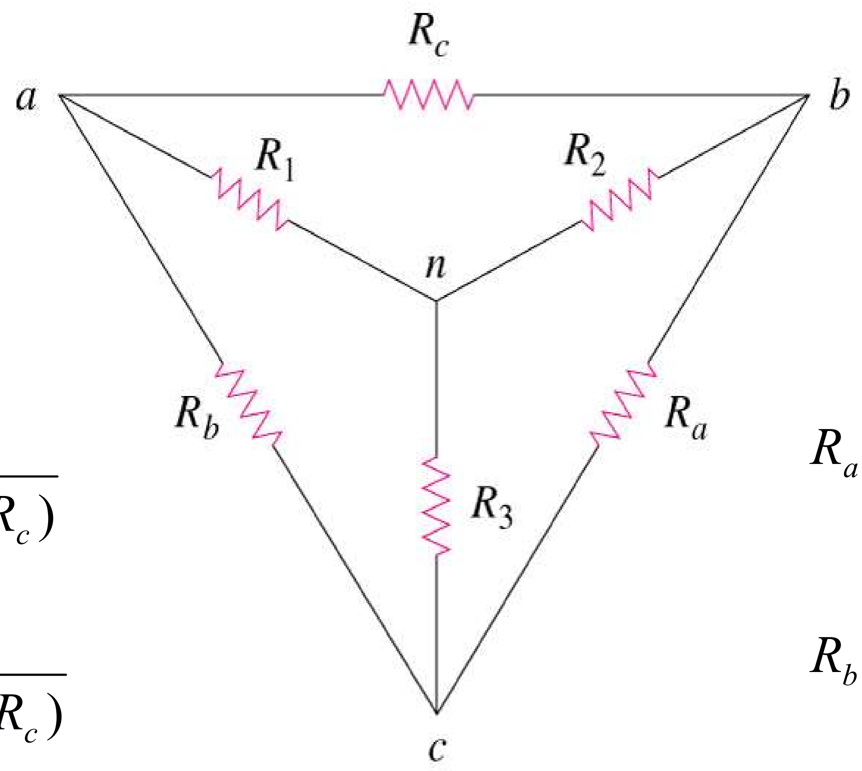


(a)



(b)

2.7 Wye-Delta Transformations(2)



Delta -> Star

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Star -> Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Proof

For the two networks to be equivalent at each corresponding pair of terminals, it is necessary that the resistance at the corresponding terminals be equal (e.g., the resistances at terminals a and b with c open-circuited must be the same for both networks). Therefore, if we equate the resistances for each corresponding set of terminals, we obtain the following equations:

Delta to Wye Conversion

For Delta Network

$$R_{ab}(Y) = R_1 + R_2 \tag{2.7.1}$$

$$R_{bc}(Y) = R_2 + R_3 \tag{2.7.2}$$

$$R_{ca}(Y) = R_1 + R_3 \tag{2.7.3}$$

For Wye Network

$$R_{ab}(\Delta) = R_c \| (R_a + R_b) = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \tag{2.7.4}$$

$$R_{bc}(\Delta) = R_a \| (R_b + R_c) = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{2.7.5}$$

$$R_{ab}(\Delta) = R_c \| (R_a + R_b) = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \tag{2.7.6}$$

Setting $R_{ab}(\Delta) = R_{ab}(Y)$ gives

$$R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \qquad (2.7.7)$$

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \qquad (2.7.8)$$

$$R_1 + R_3 = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} \qquad (2.7.9)$$

Subtracting Eq(2.7.9) from Eq(2.7.8), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \qquad (2.7.10)$$

Adding Eqs(2.7.10) and (2.7.7) gives

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad (2.7.11)$$

and subtracting Eq(2.7.7) from Eq(2.7.10), we obtains

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad (2.7.12)$$

Subtracting Eq(2.7.11) from Eq(2.7.9) yields

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad (2.7.13)$$

Each resistor in the Y network is the product of the resistors in the two adjacent Δbranches, divided by the sum of the threeΔresistors

Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs(2. 7.11) to (2.7.13) that

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \qquad (2.7.14)$$

$$= \frac{R_aR_bR_c}{(R_a + R_b + R_c)}$$

Dividing Eq(2.7.14) by each of Eqs(2. 7.11) to (2.7.13) leading to the following equations.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \quad (2.7.15)$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad (2.7.16)$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad (2.7.17)$$

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at time, divided by the opposite Y resistor.

If $R_1=R_2=R_3=R_Y$ and $R_a=R_b=R_c=R_\Delta$, the Y and Δ networks are said to *be balanced*

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \qquad or \qquad R_\Delta = 3R_Y$$

Example 16

Obtain the equivalent resistance R_{ab} for the circuit shown in Fig.2.24. and use it to calculate current i.

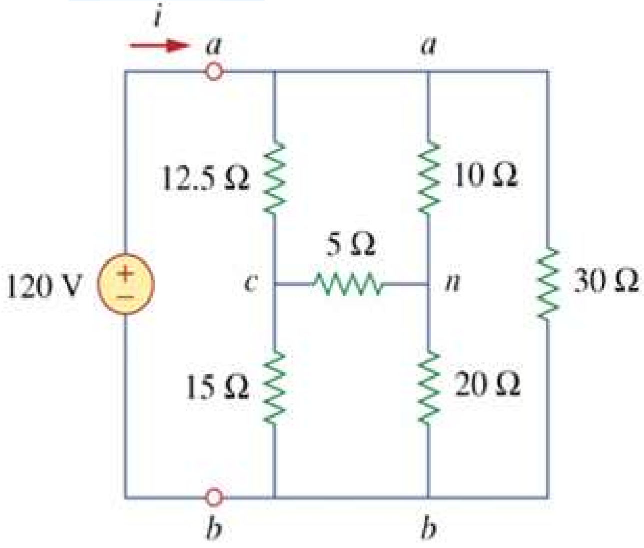


Fig.2.24(a)

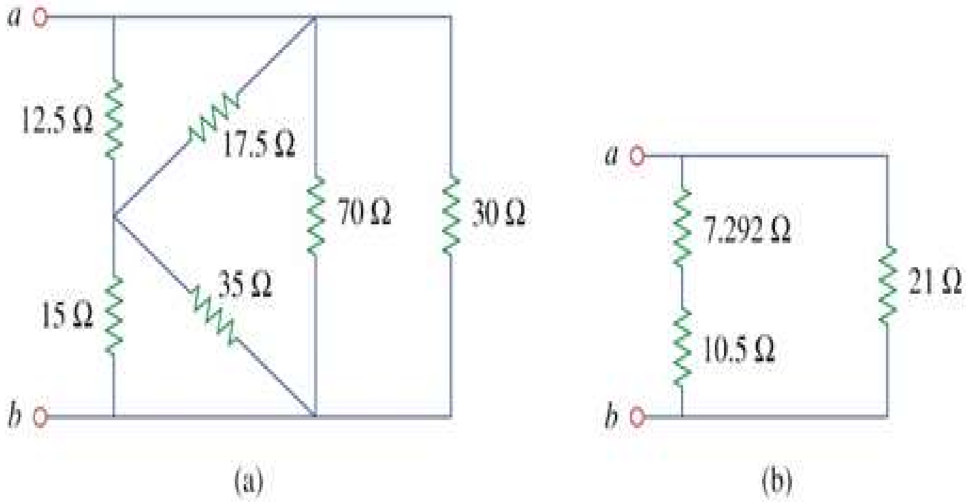


Fig.2.24(b)

Practice

For the bridge network in Fig.2.25.,find R_{ab} and i.

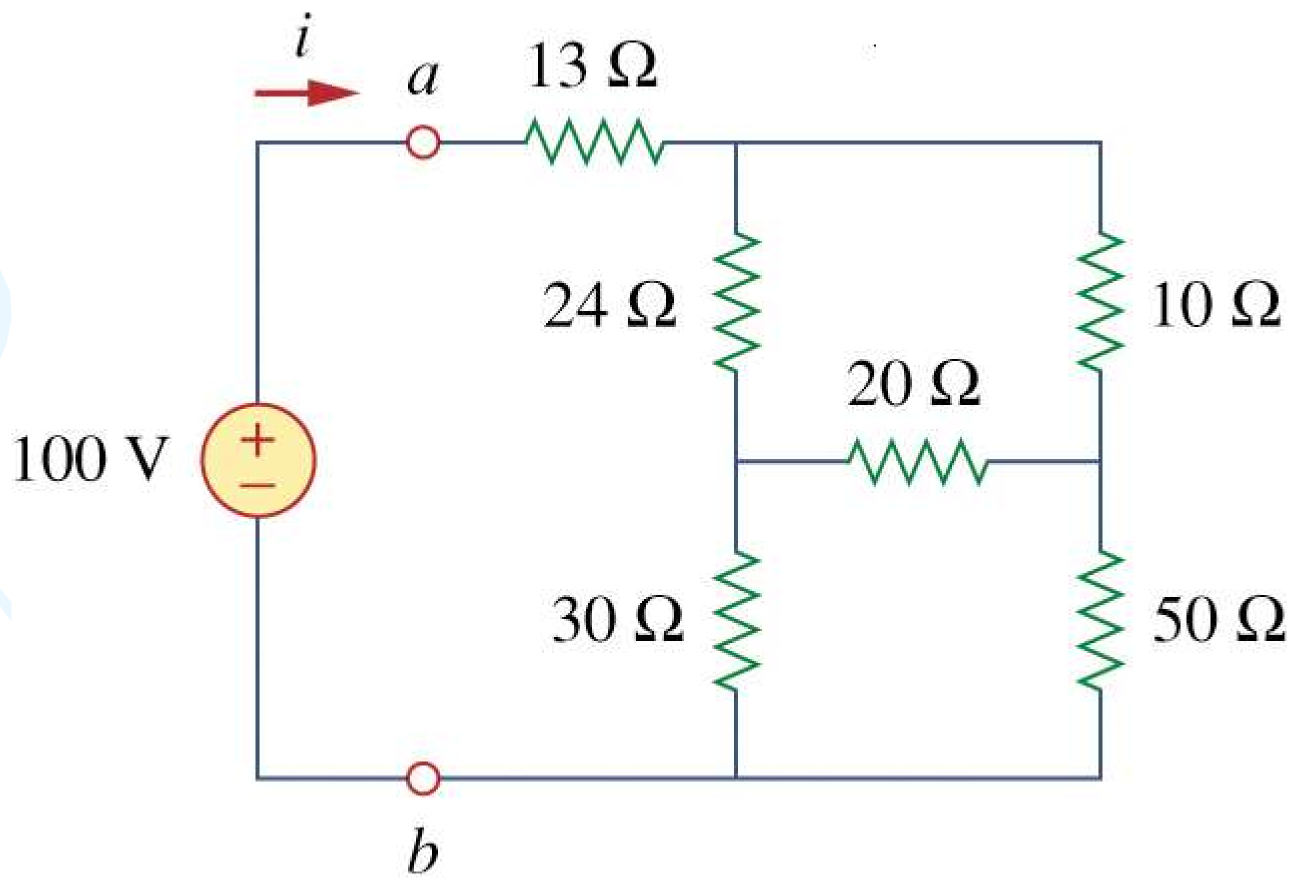


Fig. 2.25

Example 17

Three lightbulbs are connected to 9-V battery as shown in Fig.2.26(a). Calculate: (a) the total current supplied by the battery, (b0 the current through each bulb, (c) the resistance of each bulb.

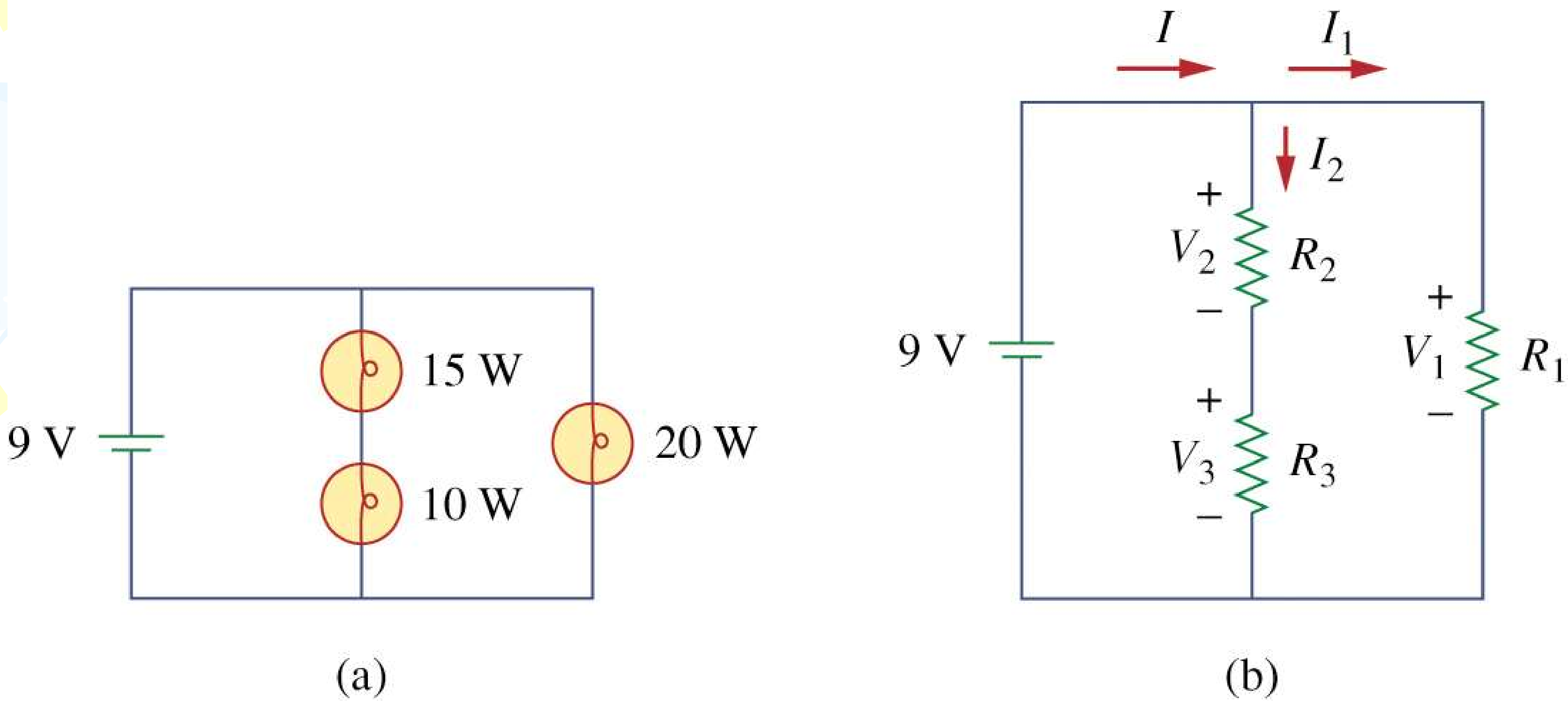


Fig. 2.26

Summary and Review

- Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering any node is zero.
- Kirchhoff's voltage law (KVL) states that the algebraic sum of the voltage around closed path in a circuit is zero.
- All elements in a circuit that carry the same current are said to be connected in series.
- Elements in a circuit having a common voltage across them are said to be connected in parallel.

- A series combination of N resistors can be replaced by a single resistor having the value
- $R_{eq} = R_1 + R_2 + \ldots + R_N.$
- A parallel combination of N resistors can be replaced by a single resistor having the value

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

- Voltage sources in series can be replaced by a single source, provided care is taken to note the individual polarity of each source.

- Current sources in parallel can be replaced by a single source, provided care is taken to note the direction of each current arrow.
- Voltage division allows us to calculate what fraction of the total voltage across a series string of resistors is dropped across any one resistor (or group of resistors).
- Current division allows us to calculate what fraction of the total current into a parallel string of resistors flows through any one of the resistors.

Objectives For This Chapter

In this chapter, we seek to develop our

- Understanding of the distinction between nodes, paths, loops, and branches
- Ability to employ Kirchhoff's current law (KCL)
- Ability to employ Kirchhoff's voltage law (KVL)
- Skills in analyzing simple series and parallel circuits
- Ability to simplify series and parallel connected sources
- Competence at reducing series and parallel resistor combinations
- Intuitive understanding of voltage and current division