Review

(1) Instantaneous power (瞬时功率)

$p(t) = u(t)\ i(t)$ W

(2) Average power, active power (有功功率)

$P = UI\cos\varphi = I^2R$ W

$\cos\varphi$: power factor(功率因数)

(3) Reactive power (无功功率)

$Q = UI\sin\varphi = I^2X$ Var

(4) Complex Power (复功率)

$\tilde{S} = P + jQ$ VA

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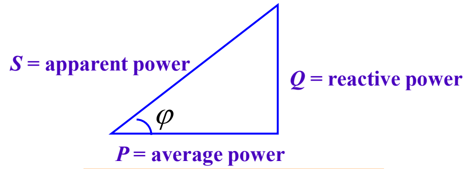
9.9.5 Apparent Power (视在功率)

Apparent Power: $S = \sqrt{P^2 + Q^2} = UI$ Unit: VA

$S = |\tilde{S}|$



$\cos\varphi = \dfrac{P}{S}$

Power triangle（功率三角形）

Apparent power represent the volt-amps capacity required to supply the average power. 变压器容量(100kVA, 5000kVA)

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Transformers

10 kV, 100kVA

500 kV, 450MVA

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Transformers

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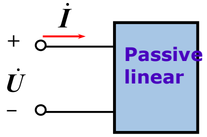
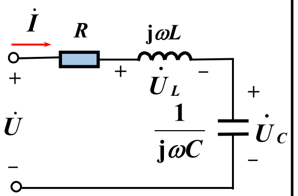
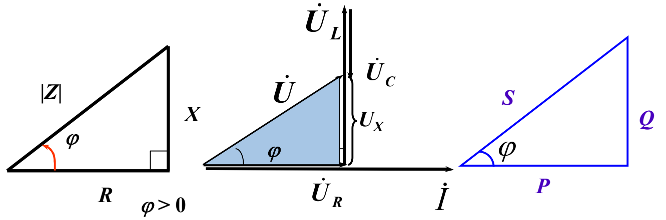
Similar triangles



$Z = R + jX$

$U = I|Z|$





Impedance triangle, voltage triangle and power triangle are similar triangles

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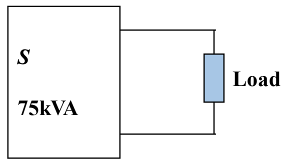
Apparatus Capacity and average power



$P = S\cos\varphi$

$\cos\varphi = 1$, $P = S = 75\text{kW}$

$\cos\varphi = 0.7$, $P = 0.7S = 52.5\text{kW}$

$\cos\varphi = 0$, $P = 0\text{kW}$

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Example

An electrical load operates at 240V rms (effective value). The load absorbs an average power of 8kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.
b) Calculate the impedance of the load.

Solution: a)The load is inductive, so $\cos\varphi = 0.8$, $\sin\varphi = 0.6$, therefore

$$S = \frac{P}{\cos\varphi} = \frac{8kW}{0.8} = 10kVA \qquad Q = S\sin\varphi = 6kVAR$$

and $\tilde{S} = 8 + j6kVA$

b) $P = UI\cos\varphi = 240I(0.8) = 8000W$

Solving for I $\quad I = 41.67A$

$$|Z| = \frac{U}{I} = \frac{240}{41.67} = 5.76$$

$$Z = |Z|\cos\varphi + j|Z|\sin\varphi = 4.608 + j3.456\Omega$$

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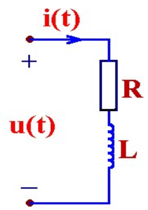
Example 1) u=707cos10ωt(V)，i=1.41cos(ωt-53.1°)(A)。

Find P、Q、S。



Solution

$$\dot{U} = \frac{707}{\sqrt{2}}\angle 0^0 = 500V \qquad \dot{I} = \frac{1.414}{\sqrt{2}}\angle -53.1^0 A$$

So $\tilde{S} = \dot{U}\overset{*}{\dot{I}} = 500\angle 53.1^0 = 300 + j400VA$

and $S = |\tilde{S}| = 500VA$

$P = 300W \qquad Q = 400VAR$

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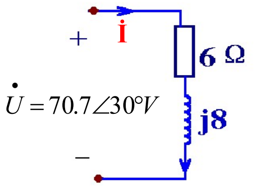
2) Find P、Q、S、$\cos\varphi$.

Solution



So

$$\dot I = \frac{\dot U}{Z} = 7.07\angle -23.1^\circ$$

$$\tilde S = \dot U \overset{*}{I} = 70.7\angle 30^0 \times 7.07\angle 23.1^0$$
$$= 500\angle 53.1^0$$
$$= 300 + j400VA$$

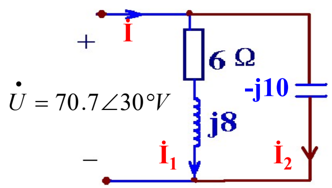
and $S = |\tilde S| = 500VA$

$P = 300W$ $\quad Q = 400VAR$

$$\cos\varphi = \frac{P}{S} = 0.6$$

3) Find P、Q、S、$\cos\varphi$.

Solution



$$\dot I_1 = 7.07\angle -23.1^\circ$$
$$\dot I_2 = 7.07\angle 120^\circ$$
$$\dot I = \dot I_1 + \dot I_2 = 4.47\angle 48.43^\circ$$

So

$$\tilde S = \dot U \overset{*}{I} = 70.7\angle 30^0 \times 4.47\angle -48.43^0$$
$$= 316\angle -18.43^0 VA$$
$$= 300 - j100VA$$

and S=316VA, P=300W, Q=-100VAR,

$\cos\varphi = P/S$=300/316=0.949.

Power Factor Correction (功率因数校正-PFC)

We need higher of lower Power Factor? Why? How?

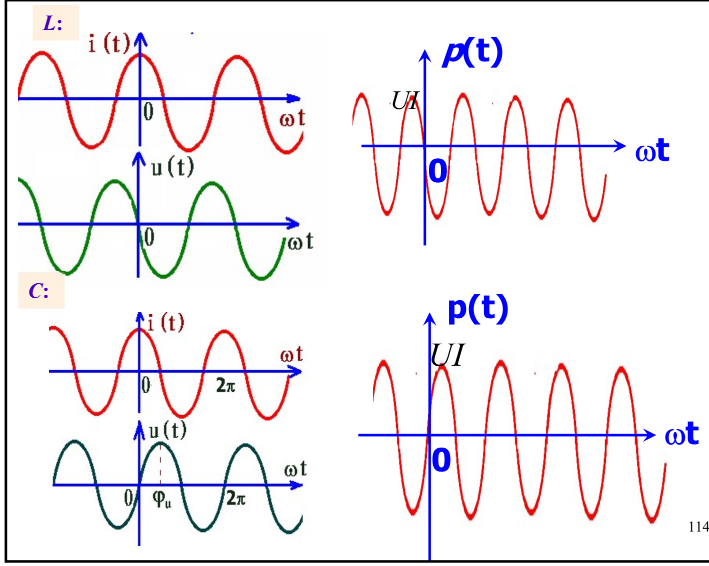
We need higher higher Power Factor, but it is always low for many apparatus.

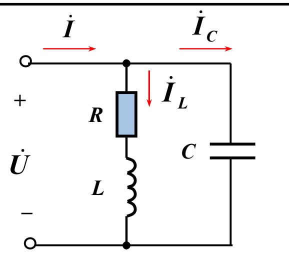
Motors: no-load $\cos\varphi$ =0.2~0.3
full-load $\cos\varphi$ =0.7~0.85

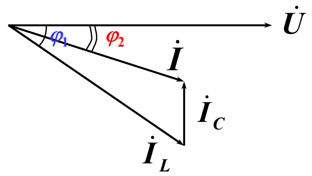
Fluorescent lamps : $\cos\varphi$=0.45~0.6

Benefits:(1)Make full use of equipment capacity; (2) Reduce the current and line loss.

Most industrial loads are inductive, operate at a lagging power factor. So we need to corrected the power factor by adding a parallel capacitor.



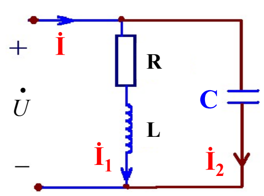


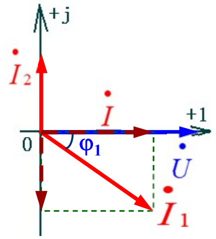


$$UI_L\cos\varphi_1 = UI\cos\varphi_2 \qquad P_1 = P_2$$
$$UI_L\sin\varphi_1 > UI\sin\varphi_2 \qquad Q_1 > Q_2 \quad S_1 > S_2$$

Increasing C reactive compensation（无功补偿）
- under-compensation（欠补偿） √
- full-compensation（全补偿）
- over-compensation（过补偿）

Appropriate value for power factor ~ 0.9 .





***P* and *U* are known. To improve the power factor from cos φ_1 to cos φ_2, find C.**

Before C is connected to the circuit:

$$Q_L = Ptg\varphi_1$$

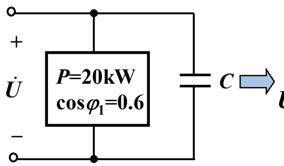
After C is connected to the circuit:

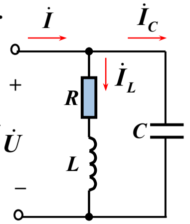
$$Q = Ptg\varphi_2 \qquad Q = Q_L - Q_C \qquad Q_C = U^2\omega C$$

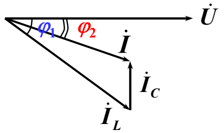
Thus

$$Ptg\varphi_2 = Ptg\varphi_1 - U^2\omega C \qquad C = \frac{P}{\omega U^2}(tg\varphi_1 - tg\varphi_2)$$

Example: *f*=50Hz, *U*=380V, *P*=20kW, cosφ_1=0.6(lagging). Find *C*, if the power factor is to be improved to 0.9.







Solution 1

$$由\cos\varphi_1 = 0.6 \quad 得 \quad \varphi_1 = 53.13^\circ$$
$$由\cos\varphi_2 = 0.9 \quad 得 \quad \varphi_2 = 25.84^\circ$$

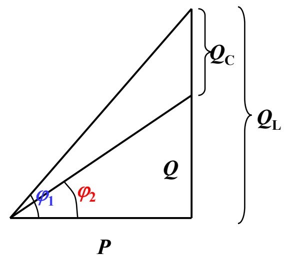
$$I_C = I_L\sin\varphi_1 - I\sin\varphi_2 \qquad |I\sin\varphi_2$$

$$I_L = \frac{P}{U\cos\varphi_1} = \frac{20\times10^3}{380\times0.6} = 87.72$$

$$I = \frac{P}{U\cos\varphi} = \frac{20\times10^3}{380\times0.9} = 58.48$$

$$I_C = I_L\sin\varphi_1 - I\sin\varphi_2 = 44.69 \quad C = \frac{I_C}{\omega U} = 375\mu F$$

Solution 2: By using the **power triangle**



$$|Q_C| = |Q_L - Q| = P(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi_2)$$
$$|Q_C| = \omega CU^2$$
$$\therefore \quad C = \frac{P}{\omega U^2}(\mathrm{tg}\varphi_1 - \mathrm{tg}\varphi_2)$$
$$C = \frac{20\times10^3}{314\times380^2}(\mathrm{tg}53.13^\circ - \mathrm{tg}25.84^\circ)$$
$$= 375\,\mu F$$

Conservation of AC power (功率的守恒)

Notice: $$\tilde{S} = \tilde{S}_1 + \tilde{S}_2 + \tilde{S}_3 + \dots$$
$$= \dot{U}_1 \dot{I}_1^* + \dot{U}_2 \dot{I}_2^* + \dot{U}_3 \dot{I}_3^* + \dots$$

$Q = Q1 + Q2 + Q3$…….:

$P = P1 + P2 + P3$…….;

but $S \neq S1 + S2 + S3$…….

Brief summary

（1）Instantaneous power (瞬时功率)

（2）Average power, active power (有功功率）

（3）Reactive power (无功功率)

（4）Complex Power (复功率)

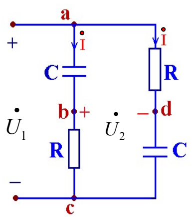
（5）Apparent Power (视在功率)

（6）Power Factor Correction (功率因数的提高)

（7）Maximum power transfer (最大功率传输）

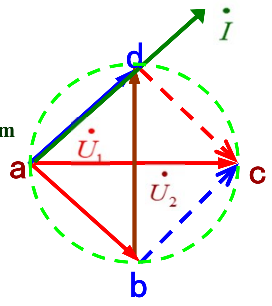
Example 10. Illustrate that when ω changes from 0 to ∞, $U_2=U_1$, φ_2 changes from $180°+\varphi_1$ to φ_1.

Solution 1. Assume

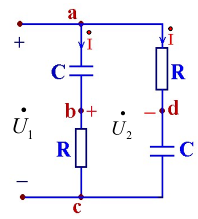


$$\dot{U}_1 = U_1 \angle 0°$$

When ω changes from 0 to ∞, the track of d and b is a circle and bd is the diameter, $U_{bd} = U_2 = U_1$



Solution 2. Assume



$$\dot{U}_2 = \frac{R}{R + \frac{1}{j\omega C}} \dot{U}_1 - \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \dot{U}_1$$

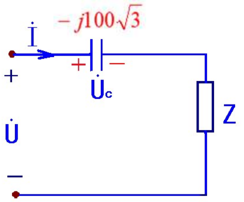
$$\dot{U}_1 = U_1 \angle \varphi_1, \dot{U}_2 = U_2 \angle \varphi_2 = \frac{j\omega CR - 1}{j\omega CR + 1} \dot{U}_1$$

$$= \frac{R - \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \dot{U}_1 \quad \therefore \quad \dot{U}_2 = \dot{U}_1 \angle (180° - 2\arctan CR\omega)$$
$$= U_1 \angle (\varphi_1 + 180° - 2\arctan CR\omega)$$

Exercise 1. U=100V，

$U_c = 100\sqrt{3}V$, $X_c = 100\sqrt{3}\Omega$,

$\varphi_Z = 60°$ **Find Z.**



Solution

$I = \frac{U_c}{X_C} = 1A$ $\dot{I} = 1\angle 0°$

$Z = R + jX$

$\dot{U} = (-j100\sqrt{3} + R + jX) \bullet 1\angle 0° = R + j(X - 100\sqrt{3})$

$\therefore\ 100^2 = R^2 + (X - 100\sqrt{3})^2$ $\therefore\ R = \begin{cases} 100\Omega \\ 50\Omega \end{cases}$

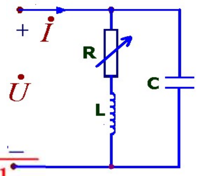
$\frac{X}{R} = tg60° = 1.732$ $X = \begin{cases} 173.2\Omega \\ 86.6\Omega \end{cases}$

$\boxed{Z = \begin{cases} 100 + j173.2\Omega \\ 50 + j86.6\Omega \end{cases}}$

Exercise 2. Find the relationship of L、C、ω, to make I constant when R changes.?????

If R=0: $\dot{I} = j(\omega C - \frac{1}{\omega L})\dot{U}$

If R=∞: $\dot{I} = j\omega C\dot{U}$

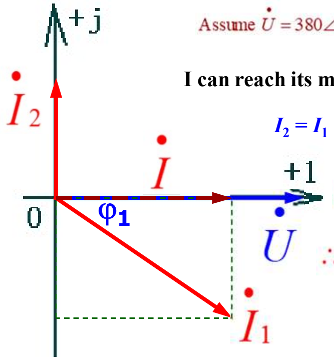


$\left|\omega C - \frac{1}{\omega L}\right| = \omega C$

$\frac{1}{\omega L} - \omega C = \omega C\ \ \therefore\ \ \omega = \sqrt{\frac{1}{2LC}}$

$\omega C - \frac{1}{\omega L} = \omega C$ （No solution）

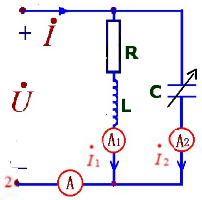
Exercise 3. U=380V，f=50Hz。The minimum reading of the current meter A is 2.59A when C=80.85 μF. Find the reading of current meter A1 and A2.



Assume $\dot{U} = 380\angle 0°$

I can reach its minimum when

$I_2 = I_1 \sin\varphi_1$



$I_1^2 = I^2 + I_2^2$

$\therefore\ \ I_1 = \sqrt{I^2 + I_2^2} = 9.952A$

Summary

（1）amplitude, angular frequency, initial phase angle；Phase angle difference；Root mean square, RMS；

（2）Definition of Phasor；

（3）Passive circuit elements in the frequency domain; impedance;

（4）Sinusoidal steady-state analysis; Phasor diagrams

（5）AC circuit power analysis

What's next?

（1）Frequency characteristics; Resonance

（2）Magnetically coupled circuits; Transformers

（3）Three-Phase circuits;

（4）Periodic, nonsinusoidal excitations

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正误判断

在电阻电路中：

瞬时值

有效值

$i = \frac{U}{R}$ ✗

$I = \frac{U}{R}$ √

$i = \frac{u}{R}$ √

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正误判断

在电感电路中：

$i = \frac{u}{X_L}$ ✗

$i = \frac{u}{\omega L}$ ✗

$I = \frac{U}{\omega L}$ √

$\frac{U}{I} = j\omega L$ ✗

$\frac{\dot U}{\dot I} = X_L$ ✗

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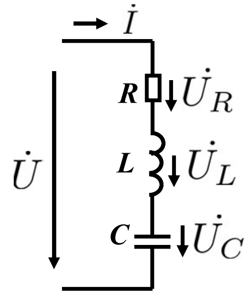
正误判断

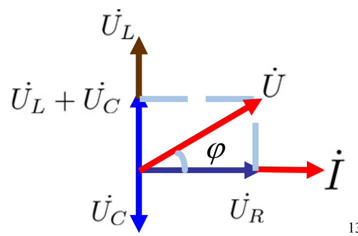
在*R-L-C*串联电路中

$U = U_R + U_L + U_C = IR + I(X_L - X_C)$ ✗

因为交流物理量除有效值外还有相位。



$$\dot U = \dot U_R + \dot U_L + \dot U_C$$



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正误判断

$$\dot{U} = \dot{I}\dot{Z}$$ ✗

$\dot{U}$, $\dot{I}$ 反映的是正弦电压或电流，

而复数阻抗只是一个运算符号。**Z 不能加 “•”**

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正误判断 在 $R-L-C$ 正弦交流电路中

$I = \dfrac{U}{|Z|}$ ✓

$i = \dfrac{u}{|Z|}$ ✗

$I = \dfrac{U}{Z}$ ✗

$\dot{I} = \dfrac{\dot{U}}{Z}$ ✓

$\dot{I} = \dfrac{\dot{U}}{|Z|}$ ✗

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正误判断

在 ***R-L-C*** 串联电路中，假设 $\dot{I} = I\angle 0^\circ$

$$U = \sqrt{U_R^2 + U_L^2 + U_C^2}$$ ✗

$$U = I\sqrt{R^2 + (X_L - X_C)^2}$$ ✓

$$\dot{U} = \dot{I}[R + j(X_L - X_C)]$$ ✓

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正误判断在*R-L-C*串联电路中，假设 $\dot{I} = I\angle 0^\circ$

$\varphi = tg^{-1}\dfrac{X_L - X_C}{R}$ ✓

$\varphi = tg^{-1}\dfrac{U_L - U_C}{U}$ ✗

$\varphi = tg^{-1}\dfrac{U_L - U_C}{U_R}$ ✓

$\varphi = tg^{-1}\dfrac{\omega L - \omega C}{R}$ ✗

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