Fundamentals of Electric Circuit

Chapter -12

Three-Phase Circuits

CHAPTER 12 THREE-PHASE CIRCUITS (三相电路)

1. Introduction

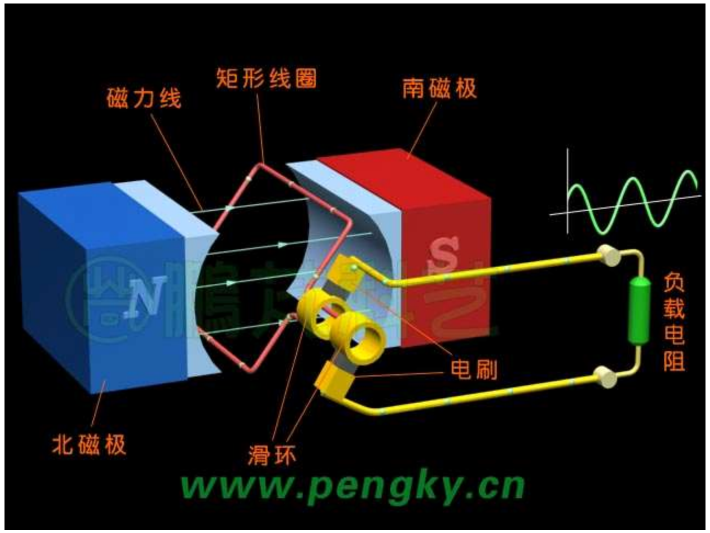
Thomas Alva Edison

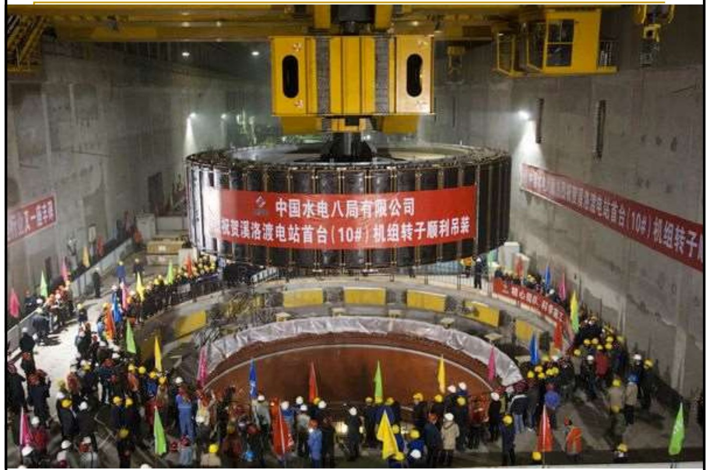
(1847-1931)

A balanced three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

Three-phase systems are important for at least three reasons:

- Nearly all electric power is generated and distributed in three-phase.
- The instantaneous power in a three-phase system can be constant.
- For the same amount of power, the three-phase system is more economical than the single-phase.

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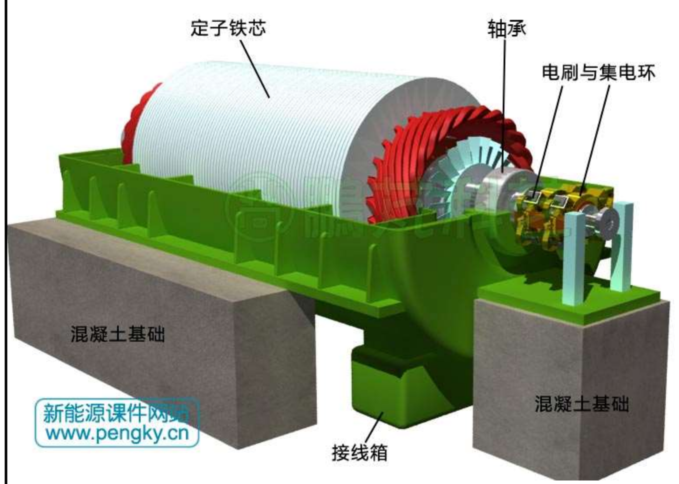
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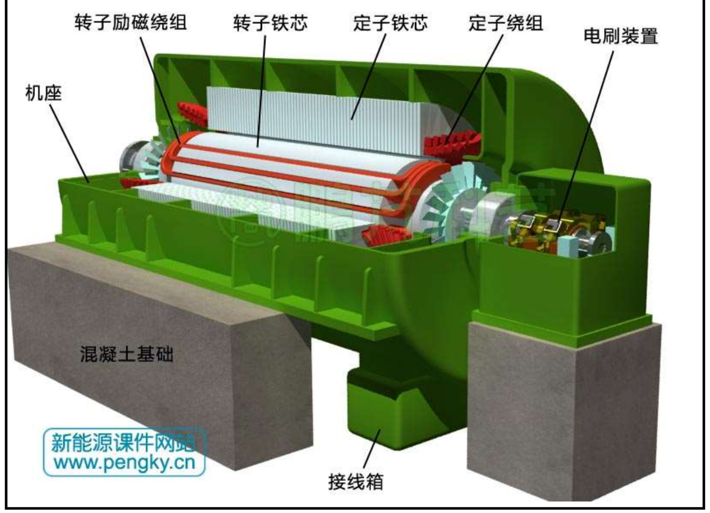
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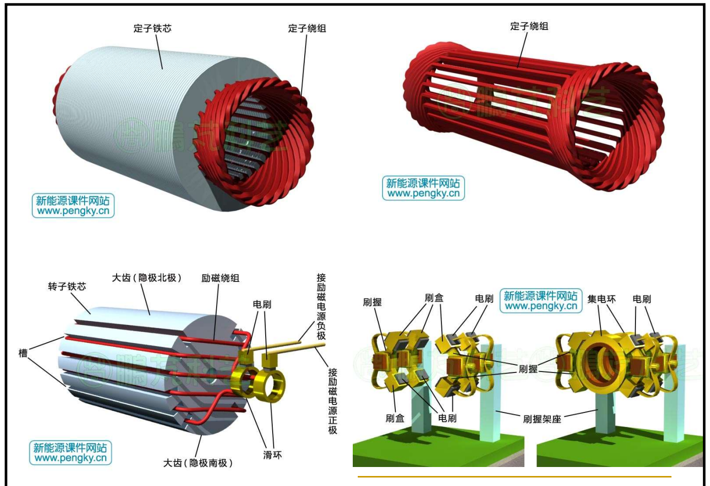
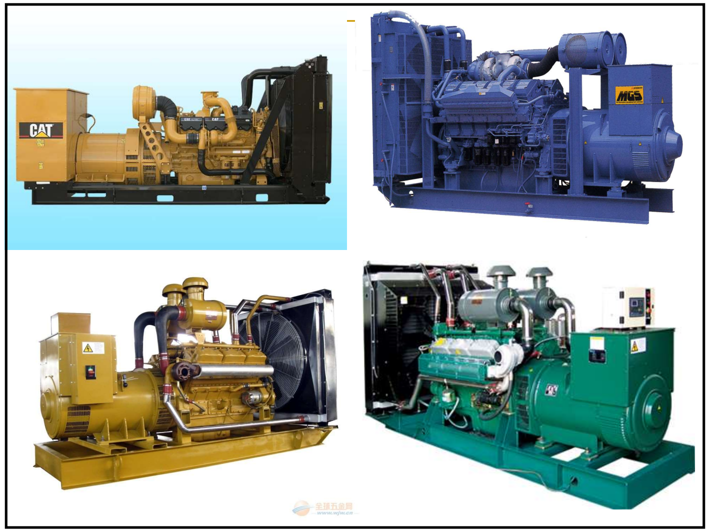
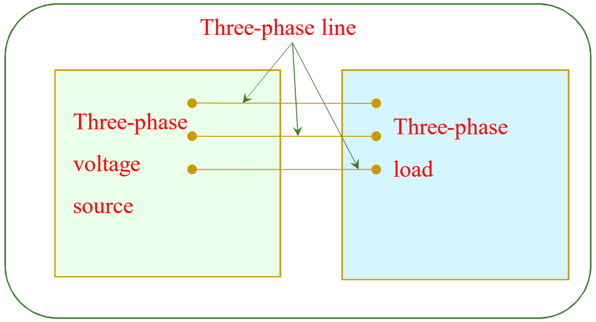




A basic three-phase circuit



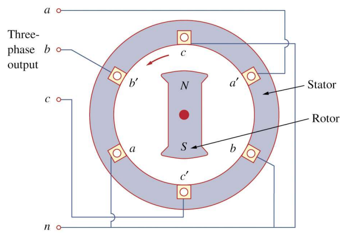


Fig. 10.1 A three-phase generator

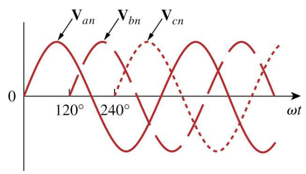


Fig. 10.2 The generated voltages are 120° apart from each other

Chapter 12
Three-Phase Circuits

Chapter Contents

1. **Basic concepts of three-phase circuit**
2. **Analysis of the Wye-Wye (Y-Y) circuit**
3. **Analysis of the Wye-Delta (Y-△) circuit**

Chapter Objective

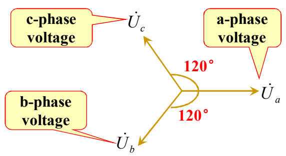
- Learning the distinction between single-phase and polyphase systems
- Becoming familiar with working with both Y- and △-connected three-phase sources
- Becoming familiar with working with both Y- and △-connected networks
- Mastering the technique of per-phase analysis of three-phase systems

12.1 Basic concepts of three-phase circuit

1. Balanced three-phase voltages

$$\boxed{u_a + u_b + u_c = 0}$$ $\dot{U}_a$ $\dot{U}_b$ $\dot{U}_c$

$$\boxed{\dot{U}_a + \dot{U}_b + \dot{U}_c = 0}$$



Have equal amplitudes and frequencies
Each of the three voltages is 120° out of phase with each of the other two.

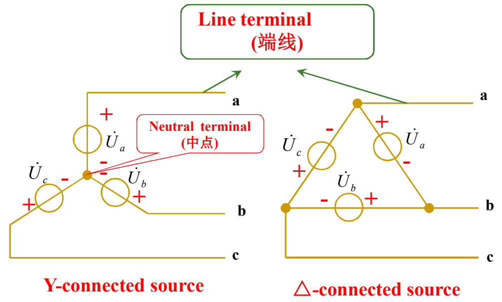
Phase sequence

Choose a-phase as the reference phase. $\dot{U}_a = U_a\angle 0^o V$

- **abc (or positive) phase sequence(正序): the b-phase voltage lags the a-phase voltage by 120º, and the c-phase voltage lags the b-phase voltage by 120º.** $\dot{U}_b = \dot{U}_a\angle -120^o V$ $\dot{U}_c = \dot{U}_a\angle +120^o V$
- **acb (or negative) phase sequence (反序): the b-phase voltage leads the a-phase voltage by 120º, and the c-phase voltage leads the b-phase voltage by 120º.** $\dot{U}_b = \dot{U}_a\angle 120^o V$ $\dot{U}_c = \dot{U}_a\angle -120^o V$

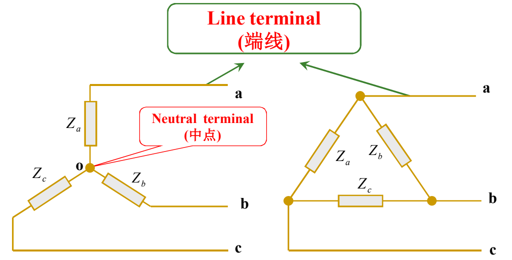
2. Three-phase voltage sources

Two basic connection of an ideal three-phase source



3. Three-phase loads

Two basic connection of three-phase loads

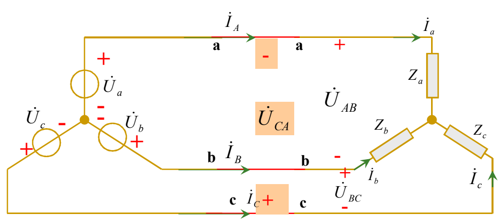


Y-connected source **△-connected source**

4. Three-phase circuit

Source	Load
Y	Y
Y	△
△	Y
△	△

Balanced Three-Phase Circuits: **balanced three-phase voltages, balanced three-phase loads**



Y-Y connection

Line voltage（线电压）: the voltage across any pair of lines（端线）. $\dot{U}_{AB}$ $\dot{U}_{BC}$ $\dot{U}_{CA}$

Phasor voltage(相电压): voltage across a single phase 每相元件两端电压。 $\dot{U}_a$ $\dot{U}_b$ $\dot{U}_c$

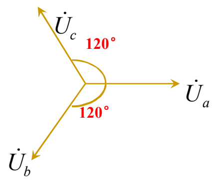
Line current（线电流）: the current in a single line (端线）. $\dot{I}_A$ $\dot{I}_B$ $\dot{I}_C$

Phasor current （相电流）: the current in a single phase 。 $\dot{I}_a$ $\dot{I}_b$ $\dot{I}_c$

Some terms

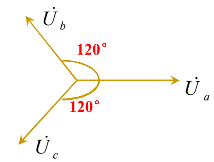
three-phase circuits 三相电路
balanced three-phase circuits 平衡三相电路
three-phase source 三相电源
three-phase load 三相负载

The phrase sequence is the time order in which the voltages pass through their respective maximum values



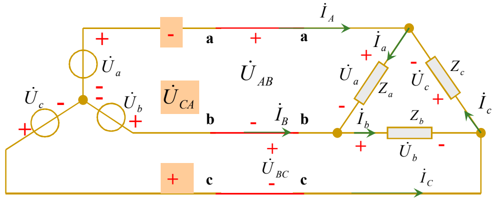
abc (or positive) phase sequence 正序

the b-phase voltage lags the a-phase voltage by 120º, and the c-phase voltage lags the b-phase voltage by 120º



acb (or negative) phase sequence 反序

the b-phase voltage leads the a-phase voltage by 120º, and the c-phase voltage leads the b-phase voltage by 120º



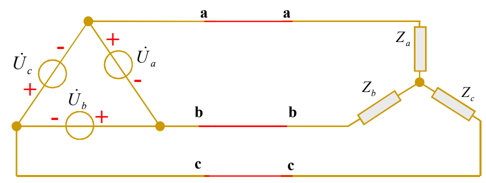
Y-△ connection

Phase current $\dot{I}_a \quad \dot{I}_b \quad \dot{I}_c$

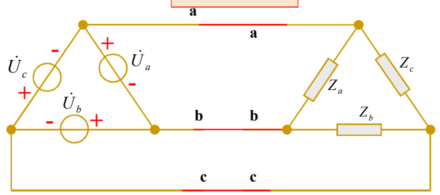
Line current $\dot{I}_A \quad \dot{I}_B \quad \dot{I}_C$

Phase voltage $\dot{U}_a \quad \dot{U}_b \quad \dot{U}_c$

Line voltage $\dot{U}_{AB} \quad \dot{U}_{BC} \quad \dot{U}_{CA}$



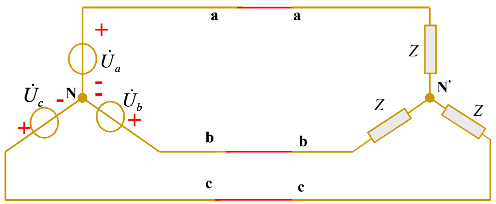
△ -Y connection



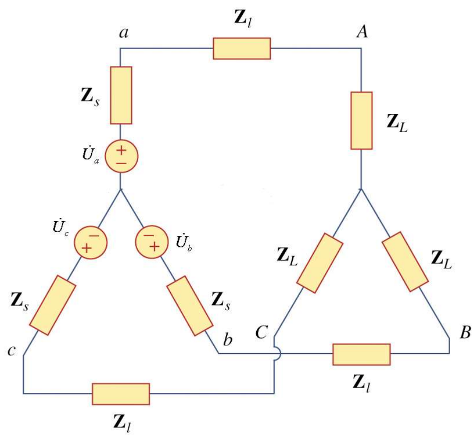
△ - △ connection

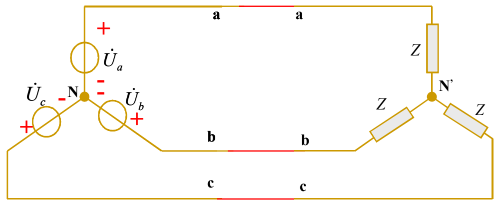
12.2 Analysis of the Wye-Wye (Y-Y) Circuit

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



Y-Y connection (three-phase three-wire circuit)



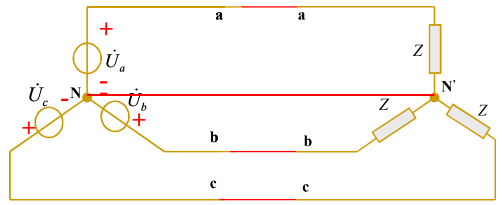


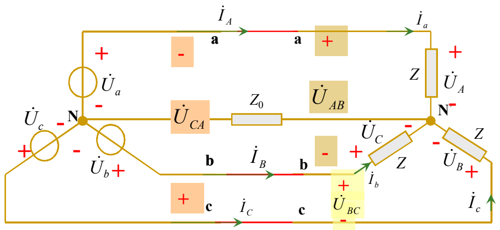
Apply the nodal analysis and select the node N' as reference node, we have

$$\dot U_{NN'} = \frac{\dot U_a/Z+\dot U_b/Z+\dot U_c/Z}{1/Z+1/Z+1/Z} = \frac{1}{3}(\dot U_a+\dot U_b+\dot U_c)=0$$

Thus we can also get the following equivalent circuit

Thus we can get the following equivalent circuit



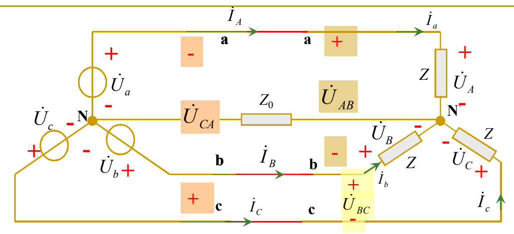


Y-Y connection (three-phase four-wire circuit)

Suppose $\dot U_a$ $\dot U_b$ $\dot U_c$ are known and in the abc sequences, try to calculate

(1) Phase currents $\dot I_a$ $\dot I_b$ $\dot I_c$ **(2)** Line currents $\dot I_A$ $\dot I_B$ $\dot I_C$

(3) Phase voltage $\dot U_A$ $\dot U_B$ $\dot U_C$ **(4)** Line voltage $\dot U_{AB}$ $\dot U_{BC}$ $\dot U_{CA}$



Apply the nodal analysis, we have

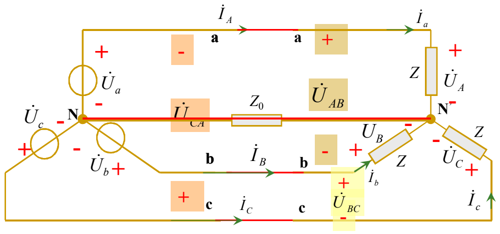
$$\dot U_{NN'} = \frac{\dot U_a/Z+\dot U_b/Z+\dot U_c/Z}{1/Z+1/Z+1/Z+1/Z_0} = \frac{\dot U_a+\dot U_b+\dot U_c}{1+1+1+Z/Z_0}$$

Since

$$\dot U_a+\dot U_b+\dot U_c=0 \qquad \dot U_{NN'}=0$$

Thus we can get the following equivalent circuit

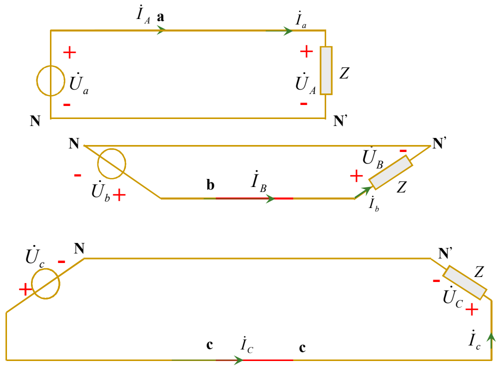
Thus we can get the following equivalent circuit

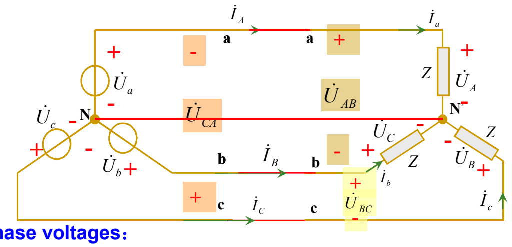


A three-phase circuit changes into three single phase circuits now.

(一个三相电路→三个单相电路)

It is easy to find phase currents and phase voltage.





phase voltages:

$$\dot{U}_A = \dot{U}_a$$

$$\dot{U}_B = \dot{U}_b = \dot{U}_a\angle -120^0 = \dot{U}_A\angle -120^0$$

$$\dot{U}_C = \dot{U}_c = \dot{U}_a\angle +120^0 = \dot{U}_A\angle +120^0$$

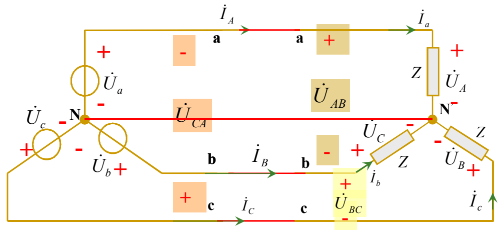
phase currents:

$$\dot{I}_a = \frac{\dot{U}_A}{Z}$$

$$\dot{I}_b = \frac{\dot{U}_B}{Z} = \frac{\dot{U}_A\angle -120^0}{Z} = \dot{I}_a\angle -120^0$$

$$\dot{I}_c = \frac{\dot{U}_C}{Z} = \frac{\dot{U}_A\angle +120^0}{Z} = \dot{I}_a\angle +120^0$$

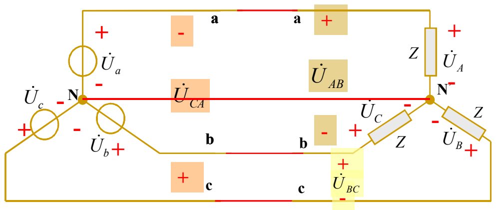
To get phase voltage and phase currents: ***just calculate one phase voltage and phase current***, then we can write other phase voltages and phase currents directly.

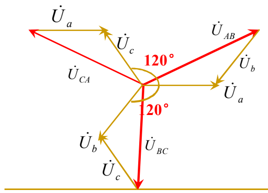


line currents: $\dot{I}_A = \dot{I}_a \quad \dot{I}_B = \dot{I}_b = \dot{I}_A\angle -120^0 \quad \dot{I}_C = \dot{I}_c = \dot{I}_A\angle +120^0$

To get line currents: ***just calculate one phase current***, then we can write all of the line currents directly.

line voltages:





$$\dot U_{AB} = \dot U_a - \dot U_b \Rightarrow \dot U_a = \dot U_b + \dot U_{AB}$$

So $\dot U_{AB} = \sqrt{3}\dot U_a \angle 30^0 = \sqrt{3}\dot U_A \angle 30^0$

We can also obtain the following result in the similar way

$$\dot U_{BC} = \sqrt{3}\dot U_b \angle 30^0 = \sqrt{3}\dot U_A \angle -120^0 + 30^0$$

$$\dot U_{CA} = \sqrt{3}\dot U_c \angle 30^0 = \sqrt{3}\dot U_A \angle 120^0 + 30^0$$

Conclusion:

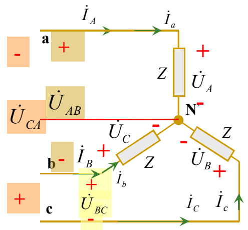
- **In the Y-Y circuit, $\dot I_l = \dot I_p$.**

- **In the balanced Y-Y circuit,** $U_l = \sqrt{3} U_p$

  $\varphi_{AB} = \varphi_A + 30^0$

  $\varphi_{BC} = \varphi_B + 30^0$

  $\varphi_{CA} = \varphi_C + 30^0$



Steps for analysis the balanced Y-Y circuit

- **Draw the single-phase equivalent circuit (usually a-phase circuit)**
- **Calculate its phase voltage and phase current.**
- **Write other phase voltages and phase currents.**
- **Write line current and line voltage.**

Example 1. In one balanced three-phase circuits, $Z=6+j8\Omega$,

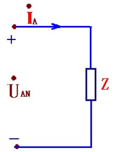
$u_{AB}(t) = 380\sqrt{2}\cos(\omega t + 30°)V$ **Calculate the currents.**

Solution:

Since

$\dot U_{AB} = 380\angle 30° V$

$\dot U_A = \dfrac{380\angle 30°}{\sqrt{3}}\angle -30° = 220\angle 0° V$



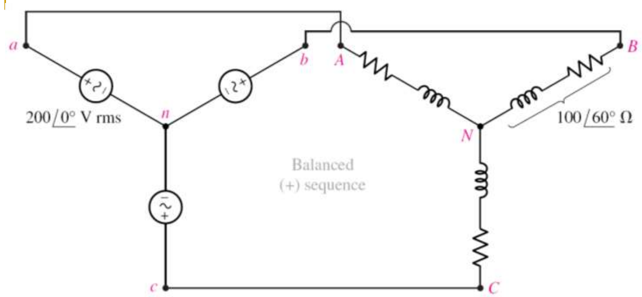
Draw the a-phase circuit

$\therefore\ \dot I_A = \dfrac{220\angle 0°}{6+j8} = 22\angle -53.1° A$

$\dot I_B = 22\angle -173.1° A \qquad \dot I_C = 22\angle 66.9° A$

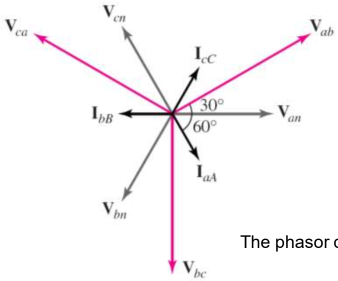
How to get the voltage?

EXAMPLE 12.2



A balanced three-phase three-wire Y-Y connected system.

The phasor diagram for this circuit is shown in following Fig.

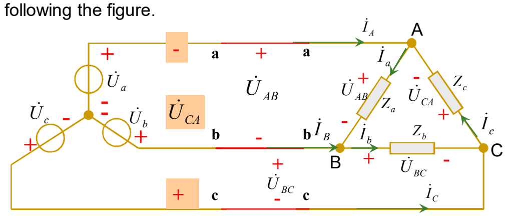


The phasor diagram for Example 2

Once we knew any of the line voltage magnitudes and any of the line current magnitudes, the angles for all three voltages and all three currents could have been easily obtained by reading the diagram.

12.3 Analysis of the Wye-Delta (Y-△) Circuit

The balanced Y-Δsystem consists of a balanced Y-connected source feeding a balanced Δ-connected load. The balanced Y-Δsystem is shown following the figure.



Assuming the positive phase sequence , the phase voltages are again

$$\dot{U}_a = U_p\angle 0^o \qquad \dot{U}_b = \dot{U}_a\angle -120^0 \qquad \dot{U}_c = \dot{U}_a\angle +120^0$$

The line voltages are

$$\dot{U}_{ab} = \sqrt{3}\dot{U}_a\angle 30^o = \dot{U}_{AB} \quad \dot{U}_{bc} = \sqrt{3}\dot{U}_b\angle 30^o = \dot{U}_{BC} \quad \dot{U}_{ca} = \sqrt{3}\dot{U}_c\angle 30^o = \dot{U}_{CA}$$

showing that the line voltages are equal to the voltages across the load impedances for this system configuration. From these voltages, we can obtain th phase currents as

$$\dot{I}_a = \frac{\dot{U}_{AB}}{Z_a} = \frac{\dot{U}_{AB}}{Z} \qquad \dot{I}_b = \frac{\dot{U}_{BC}}{Z_c} = \frac{\dot{U}_{AB}\angle -120^O}{Z} \qquad \dot{I}_c = \frac{\dot{U}_{CA}}{Z_b} = \frac{\dot{U}_{AB}\angle 120^O}{Z}$$

These currents have the same magnitude but are out of phase with each other by 120°.

The line currents are obtained from the phase currents by applying KCL at nodes A,B, and C. Thus

$$\dot{I}_A = \dot{I}_a - \dot{I}_c \qquad \dot{I}_B = \dot{I}_b - \dot{I}_a \qquad \dot{I}_C = \dot{I}_c - \dot{I}_b$$

$$= \sqrt{3}\dot{I}_a\angle -30^o \qquad = \sqrt{3}\dot{I}_b\angle -30^o \qquad = \dot{I}_c\angle -30^O$$

Showing that the magnitude of I_L of the line current is $\sqrt{3}$

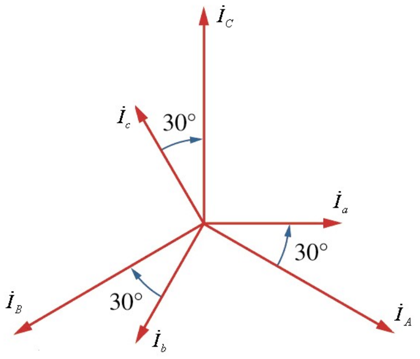
Times the magnitude I_p of the phase current, or

$$I_L = \sqrt{3}I_p$$

where

$$I_L = |\dot{I}_A| = |\dot{I}_B| = |\dot{I}_C| \qquad I_P = |\dot{I}_a| = |\dot{I}_b| = |\dot{I}_c|$$

Also, the line currents lag the corresponding phase currents by 30°.



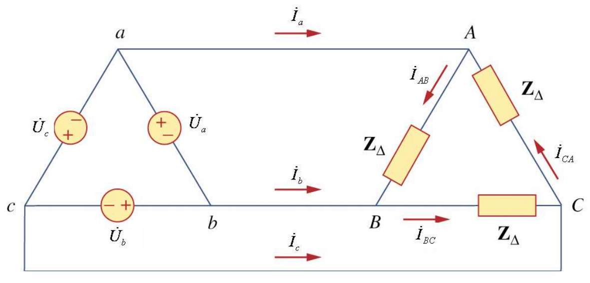
Phasor diagram illustrating the relationship between phase and line currents.

EXAMPLE 12.3

A balanced abc-sequence Y-connected source with $\dot{U}_a = 100\angle 10^o$ is connected to Δ-connected balanced load $(8 + j4)\Omega$ per phase. Calculate the phase and line currents.

12.4 Balance Delta-delta Connection

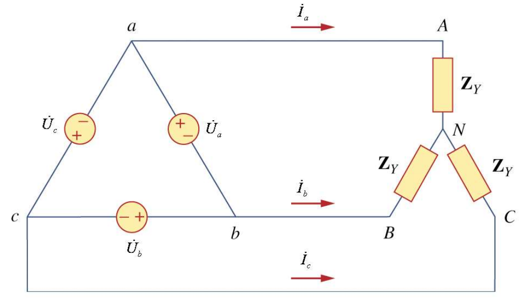
A balanced Δ- Δsystem is one in which both the balanced source and balanced load are Δ-connected



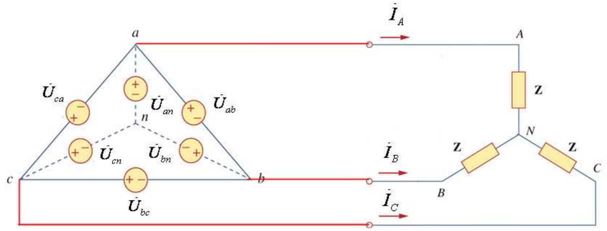
A balanced Δ- Δ connection

12.5 Balance Delta-Wye Connection

A balanced Δ- Y system consists of a balanced Δ-connected source feeding a balanced Y-connected load.



A balanced Δ- Y connection



$$\dot{U}_{an}=\frac{\dot{U}_{ab}}{\sqrt{3}}\angle-30^o=\frac{U_{ab}}{\sqrt{3}}\angle-30^o$$ (choose $\dot{U}_{ab}$ as a reference phasor)

$$\dot{U}_{bn}=\frac{\dot{U}_{bc}}{\sqrt{3}}\angle-30^o=\frac{U_{ab}}{\sqrt{3}}\angle-150^o$$

$$\dot{U}_{cn}=\frac{\dot{U}_{ca}}{\sqrt{3}}\angle-30^o=\frac{U_{ab}}{\sqrt{3}}\angle 90^o$$

$$\dot{I}_A=\frac{U_{ab}/\sqrt{3}\angle-30^\circ}{Z}$$

$$\dot{U}_{AN}=\frac{\dot{U}_{ab}}{\sqrt{3}}\angle-30^o=\frac{U_{ab}}{\sqrt{3}}\angle-30^o$$

$$\dot{U}_{BN}=\dot{U}_{AN}\angle-120^o$$

$$\dot{U}_{CN}=\dot{U}_{AN}\angle 120^o$$

12.6 Summary of Balance Connection

Summary of phase and line voltages/ currents for balanced three-phase systems

Connection	Phase voltages/ Currents	Line voltages / Currents
Δ-Δ	$\dot{U}_{ab}=U_p\angle 0^\circ$ $\dot{U}_{bc}=U_p\angle-120^\circ$ $\dot{U}_{ca}=U_p\angle 120^\circ$ $\dot{I}_{AB}=\dot{U}_{ab}/Z_\Delta$ $\dot{I}_{BC}=\dot{U}_{bc}/Z_\Delta$ $\dot{I}_{CA}=\dot{U}_{ca}/Z_\Delta$	Line voltages are the same as phase voltages $\dot{I}_A=\sqrt{3}\dot{I}_{AB}\angle-30^\circ$ $\dot{I}_B=\dot{I}_A\angle-120^\circ$ $\dot{I}_C=\dot{I}_A\angle 120^\circ$

Summary of phase and line voltages/ currents for balanced three-phase systems

Connection	Phase voltages/ Currents	Line voltages / Currents
Y-Δ	$\dot{U}_{an}=U_p\angle 0^\circ$ $\dot{U}_{bn}=U_p\angle-120^\circ$ $\dot{U}_{cn}=U_p\angle 120^\circ$ $\dot{I}_{AB}=\dot{U}_{AB}/Z_\Delta$ $\dot{I}_{BC}=\dot{U}_{BC}/Z_\Delta$ $\dot{I}_{CA}=\dot{U}_{CA}/Z_\Delta$	$\dot{U}_{ab}=\dot{U}_{AB}=\sqrt{3}U_p\angle 30^\circ$ $\dot{U}_{bc}=\dot{U}_{BC}=\dot{U}_{ab}\angle-120^\circ$ $\dot{U}_{ca}=\dot{U}_{CA}=\dot{U}_{ab}\angle-120^\circ$ $\dot{I}_A=\sqrt{3}\dot{I}_{AB}\angle-30^\circ$ $\dot{I}_B=\dot{I}_A\angle-120^\circ$ $\dot{I}_C=\dot{I}_A\angle 120^\circ$

Summary of phase and line voltages/ currents for balanced three-phase systems

Connection	Phase voltages/ Currents	Line voltages / Currents
Δ-Δ	$\dot{U}_{ab}=U_p\angle 0^\circ$ $\dot{U}_{bc}=U_p\angle-120^\circ$ $\dot{U}_{ca}=U_p\angle 120^\circ$ $\dot{I}_{AB}=\dot{U}_{ab}/Z_\Delta$ $\dot{I}_{BC}=\dot{U}_{bc}/Z_\Delta$ $\dot{I}_{CA}=\dot{U}_{ca}/Z_\Delta$	Line voltages are the same as phase voltages $\dot{I}_A=\sqrt{3}\dot{I}_{AB}\angle-30^\circ$ $\dot{I}_B=\dot{I}_A\angle-120^\circ$ $\dot{I}_C=\dot{I}_A\angle 120^\circ$

Summary of phase and line voltages/ currents for balanced three-phase systems		
Connection	Phase voltages/ Currents	Line voltages / Currents
Δ-Y	$\dot{U}_{ab} = U_p\angle 0^\circ$ $\dot{U}_{bc} = U_p\angle -120^\circ$ $\dot{U}_{ca} = U_p\angle 120^\circ$ Phase currents are the same as line currents	Line voltages are the same as phase voltages $\dot{I}_A = \sqrt{3}\dot{I}_{AB}\angle 30^\circ$ $\dot{I}_B = \dot{I}_A\angle -120^\circ$ $\dot{I}_C = \dot{I}_A\angle 120^\circ$

12.7 Power in Balance System

Let us now consider the power in a balanced three-phase system. Examine the instantaneous power absorbed by the load.

For a Y-connected load, the phase voltages are

$$u_{AN}(t) = \sqrt{2}U_p\cos\omega t \qquad u_{BN}(t) = \sqrt{2}U_p\cos(\omega t - 120^\circ)$$

$$u_{CN}(t) = \sqrt{2}U_p\cos(\omega t + 120^\circ)$$

If $Z_Y = Z\angle\theta$, the phase currents lag behind their corresponding phase voltages by θ. Thus

$$i_A(t) = \sqrt{2}I_p\cos(\omega t - \theta) \quad i_B(t) = \sqrt{2}I_p\cos(\omega t - \theta - 120^\circ)$$
$$i_B(t) = \sqrt{2}I_p\cos(\omega t - \theta + 120^\circ)$$

Where I_p is the effective value of the phase current.

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$p = p_a + p_b + p_c = u_{AN}i_A + u_{BN}i_B + u_{CN}i_C$$

$$p = 2U_pI_p[\cos\omega t\cos(\omega t - \theta) + \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ)$$
$$+\cos(\omega t + 120^\circ)\cos(\omega t - \theta + 120^\circ)]$$

Applying the trigonometric identity

$$\cos A\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

gives

$$p = U_pI_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) + \cos(2\omega t + 240^\circ)]$$
$$= U_pI_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta)\cos 240^\circ + \sin(2\omega t - \theta)\sin 240^\circ$$
$$+\cos(2\omega t - \theta)\cos 240^\circ - \sin(2\omega t - \theta)\sin 240^\circ]$$

$$p = U_pI_p[3\cos\theta + \cos(2\omega t - \theta) + 2\cos(2\omega t - \theta)\cos 240^\circ]$$
$$= U_pI_p[3\cos\theta + \cos(2\omega t - \theta) - 2(-\frac{1}{2})\cos(2\omega t - \theta)]$$
$$= 3U_pI_p\cos\theta = \sqrt{3}U_LI_L\cos\theta$$

1、average power

$$P = P_A + P_B + P_C \quad = U_AI_A\cos\theta_A + U_BI_B\cos\theta_B + U_CI_C\cos\theta_C$$

Balanced three phase system:

$$P = 3U_pI_p\cos\varphi = \sqrt{3}U_lI_l\cos\varphi$$

2、reactive power

$$Q = Q_A + Q_B + Q_C = U_AI_A\sin\varphi_A + U_BI_B\sin\varphi_B + U_CI_C\sin\varphi_C$$

Balanced three phase system:

$$Q = 3U_pI_p\sin\varphi = \sqrt{3}U_lI_l\sin\varphi$$

3、apparent power $S=\sqrt{P^2+Q^2}$

Balanced three phase system:

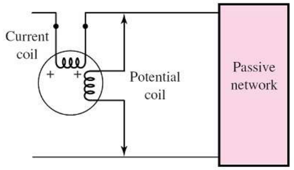
$$S = 3U_p I_p = \sqrt{3}U_l I_l$$

4、complex power

$$\tilde{S} = 3\dot{U}_p \dot{I}_p^* = P + jQ = \sqrt{3}U_L I_L \angle\theta$$

5、Measurement of a power

A. Use of the Wattmeter



(*) A wattmeter connection that will ensure an upscale reading for the power absorbed by the passive network.

Use of the Wattmeter

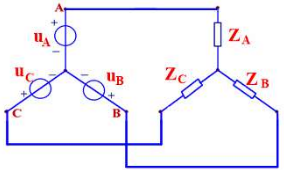
- The wattmeter is used by connecting it into a network in such a way that the current flowing in the current coil is the current flowing into the network and the voltage across the potential coil is the voltage across the two terminals of the network.
- The current in the potential coil is thus the input voltage divided by the resistance of the potential coil.
- It is apparent that the wattmeter has four available terminals, and correct connections must be made to these terminals in order to obtain an upscale reading on the meter.

- To be specific, let us assume that we are measuring the power absorbed by a passive network.
- The current coil is inserted in series with one of the two conductors connected to the load, and the potential coil is installed between the two conductors, usually on the “load side” of the current coil.
- The potential coil terminals are often indicated by arrows. Each coil has two terminals, and the proper relationship between the sense of the current and voltage must be observed.

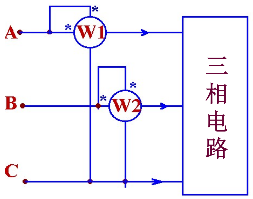
- One end of each coil is usually marked (+), and an upscale reading is obtained if a positive current is flowing into the (+) end of the current coil while the (+) terminal of the potential coil is positive with respect to the unmarked end.
- The wattmeter shown in the network of the Fig.(*), therefore gives an upscale deflection when the network to the right is absorbing power. A reversal of either coil, but not both, will cause the meter to try to deflect downscale; a reversal of both coils will never affect the reading.

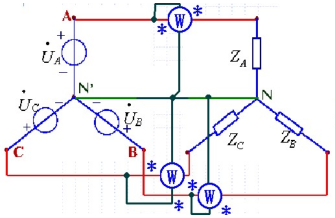
B. The Three-Wattmeter Method in a Three-Phase System

$$P = P_A + P_B + P_C$$



C.The Two-Wattmeter Method in a Three-Phase System





$$P = P_1 + P_2$$

Example: Known the Power of the motor M P=2.5kW,cosφ=0.866,and the motor is connected to the balanced three-phase source with 380V line voltage. Find the readings of the Wattmeter

solution:

$$I_l = \frac{P}{\sqrt{3}U_l\cos\varphi} = 4.386A$$

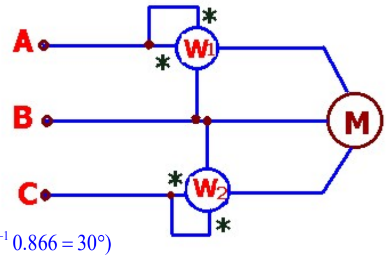
let $\dot{U}_{AB} = 380\angle 0°$

$\dot{U}_{AN} = 220\angle -30°$

$\dot{I}_A = 4.386\angle -60° A \quad (\cos^{-1}0.866 = 30°)$

$\dot{U}_{CB} = -\dot{U}_{BC} = 380\angle 60°$

$\dot{I}_C = 4.386\angle 60° A$



$P_1 = 833.3W$

$P_2 = 1666.7W$

$P = P_1 + P_2 = 2.5kW$

12.8 UnBalance three-phase System

An unbalanced system is due to unbalanced voltage sources or an unbalanced load. Generally, the source voltage is balanced, but the load is unbalanced.

Unbalanced three-phase system are solved by direct application of mesh and nodal analysis.

Example 12.4

In the circuit shown in the figure . $U_L = 380V$. Find line currents.

Solution

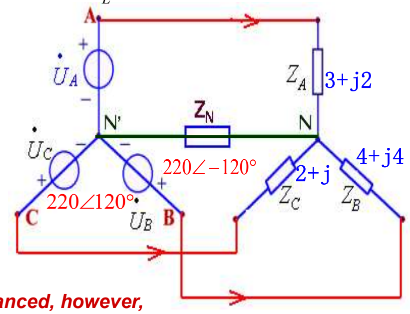
U_p=220V

1) when Z_N=0:

$\dot{I}_A = 61\angle -33.7°$

$\dot{I}_B = 38.9\angle -165°$

$\dot{I}_C = 98.4\angle 93.4°$



Load currents are unbalanced, however, they are independent , and do not impact each other.

2）when Z_N=4+j3:

$$\dot{U}_{NN'} = \frac{\dot{U}_A/Z_A + \dot{U}_B/Z_B + \dot{U}_C/Z_C}{1/Z_A + 1/Z_B + 1/Z_C + 1/Z_N}$$

$\dot{U}_{NN'} = 54.16\angle 120° V$

$\dot{U}_{AN} = \dot{U}_A - \dot{U}_{NN'} \approx 232\angle -12°$

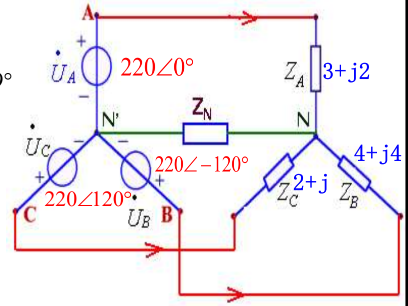
$\dot{U}_{BN} = \dot{U}_B - \dot{U}_{NN'} \approx 257\angle -109°$

$\dot{U}_{CN} = \dot{U}_C - \dot{U}_{NN'} \approx 165\angle 120°$

$\dot{I}_A = \dfrac{\dot{U}_{AN}}{Z_A} \approx 64.4\angle -45.7°$

$\dot{I}_B = \dfrac{\dot{U}_{BN}}{Z_B} \approx 45.4\angle -154°$

$\dot{I}_C = \dfrac{\dot{U}_{CN}}{Z_C} \approx 73.8\angle 93.3°$



Load currents and voltages are unbalanced, and they are dependent and impact each other.

3）when Z_N=∞:

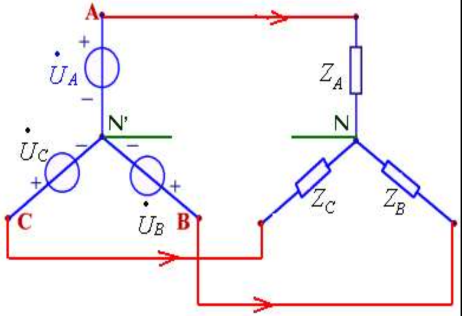
$$\dot{U}_{NN'} = \frac{\dot{U}_A/Z_A + \dot{U}_B/Z_B + \dot{U}_C/Z_C}{1/Z_A + 1/Z_B + 1/Z_C}$$

$\dot{U}_{NN'} = 61.27\angle 115.76° V$

$\dot{U}_{AN} = \dot{U}_A - \dot{U}_{NN'}$
$\approx 253\angle -13°$

$\dot{U}_{BN} = \dot{U}_B - \dot{U}_{NN'}$
$\approx 260\angle -109°$

$\dot{U}_{CN} = \dot{U}_C - \dot{U}_{NN'}$
$\approx 159\angle 122°$



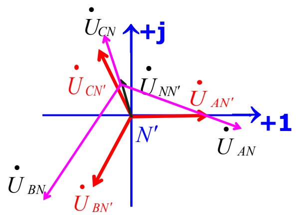
Phasor diagram

$\dot{U}_{NN'} = 61.27\angle 115.76° V$

$\dot{U}_{AN} \approx 253\angle -13°$

$\dot{U}_{BN} \approx 260\angle -109°$

$\dot{U}_{CN} \approx 159\angle 122°$



- From the phasor diagram, we know that the voltage between the neutral point of the source and the neutral of the load does not equal zero , because of the absence of neutral line, and results in unbalanced phase voltages.
- Therefore, in the three-phase four-wire system, the neutral line has large diameter, and it does not connect to the switch and fuse-element

Summary

- The phase sequence is the order in which the phase voltages of a three-phase generator occur with respect to time. In an *abc* sequence of balanced source voltages, $\dot{U}_{an}$ leads $\dot{U}_{bn}$ by 120°,which in turn leads $\dot{U}_{cn}$ by 120°. In an *acb* sequence of balanced source voltages, $\dot{U}_{an}$ leads $\dot{U}_{cn}$ by 120°,which in turn leads $\dot{U}_{bn}$ by 120°.
- A balanced wye-or delta-connected load is one in which the three-phase impedances are equal.
- The easiest way to analyze a balanced three-phase circuit is to transform both the source and the load to Y-Y system and then analyze the single-phase equivalent circuit.
- The line current I_L is the current flowing from the generator to the load in each transmission line in a three-phase system. The line voltage U_L is the voltage between each pair of lines, excluding the neutral line if it exists. The phase voltage U_p is the voltage of each phase. For a wye-connected load,

$U_L = \sqrt{3}U_P, \quad I_L = I_P$

For a delta-connected load

$U_L = U_P, \quad I_L = \sqrt{3}I_P$

- The total instantaneous power in a balanced three-phase system is constant and equal to the average power.
- The total complex power absorbed by a balanced three-phase Y-connected or Δ-connected load is $\tilde{S} = P + jQ = \sqrt{3}U_L I_L \angle\theta$ where θ is the angle of the load impedances.
- An unbalanced three-phase system can be analyzed using nodal or mesh analysis.
- The total real powr is measured in three-phases using either the three-wattmeter method or the two wattmeter method