1-8 Kirchhoff's Laws

The techniques we will learn are based on two relatively simple laws: Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

KCL is based on the principle of conservation of charge, and KVL is based on the principle of conservation of energy—both fundamental physical laws.

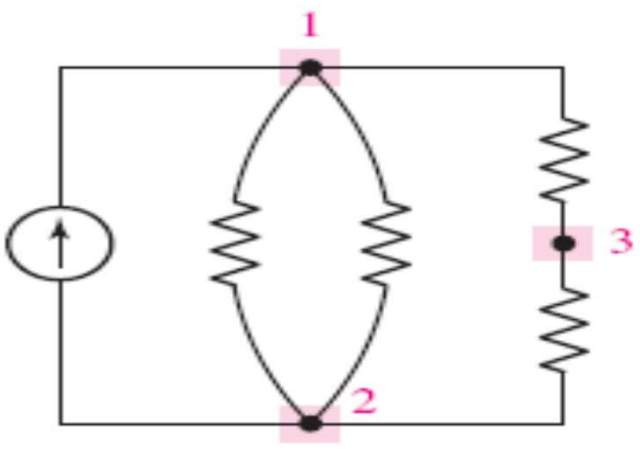
Once familiar with basic analysis, we make further use of KCL and KVL to reduce series and parallel combinations of resistors, voltage sources, or current sources, and we develop the important concepts of voltage and current division.

In subsequent chapters, we learn additional techniques that allow us to efficiently analyze even more complex networks.

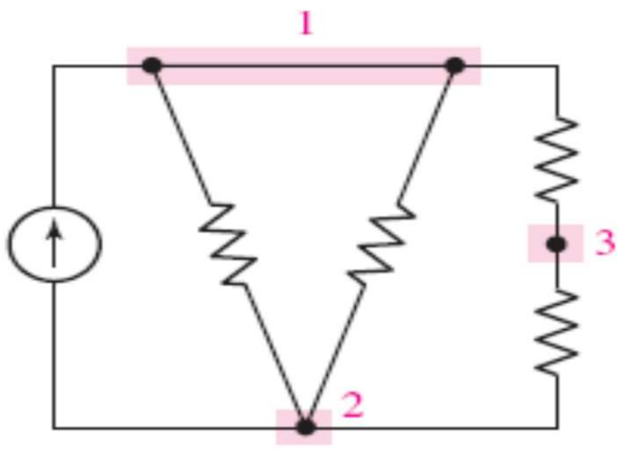
1. Several term

(1) Node

A point at which two or more elements have a common connection is called a *node*.



(*a*)



(*b*)

For example, Fig.a shows a circuit containing three nodes.

（2） Path , Loop and Mesh

Suppose that we start at one node in a network and move through a simple element to the node at the other end. We then continue from that node through a different element to the next node, and continue this movement until we have gone through as many elements as we wish.

If no node was encountered more than once, then the set of nodes and elements that we have passed through is defined as a *path*.

If the node at which we started is the same as the node on which we ended, then the path is, by definition, a closed path or a *loop*.

We define a *branch* as a single path in a network, composed of one simple element and the node at each end of that element.

对平面电路，其内部不含任何支路的回路称网孔。

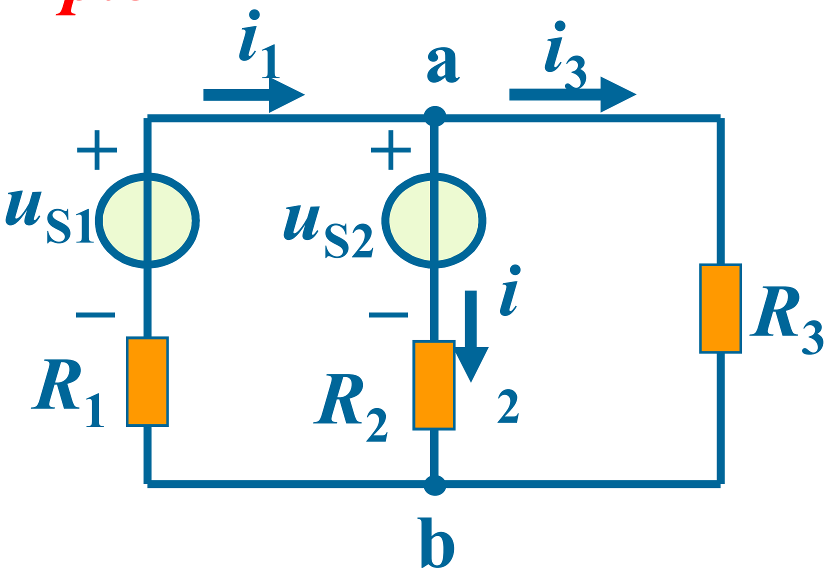
网孔是回路，但回路不一定是网孔

- A network with b branches, n nodes, and *l* independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1 \qquad (2.1)$$

- **Circuit topology is of great value to the study of voltages and currents in an electric circuit.**
- **Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.**
- **Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.**

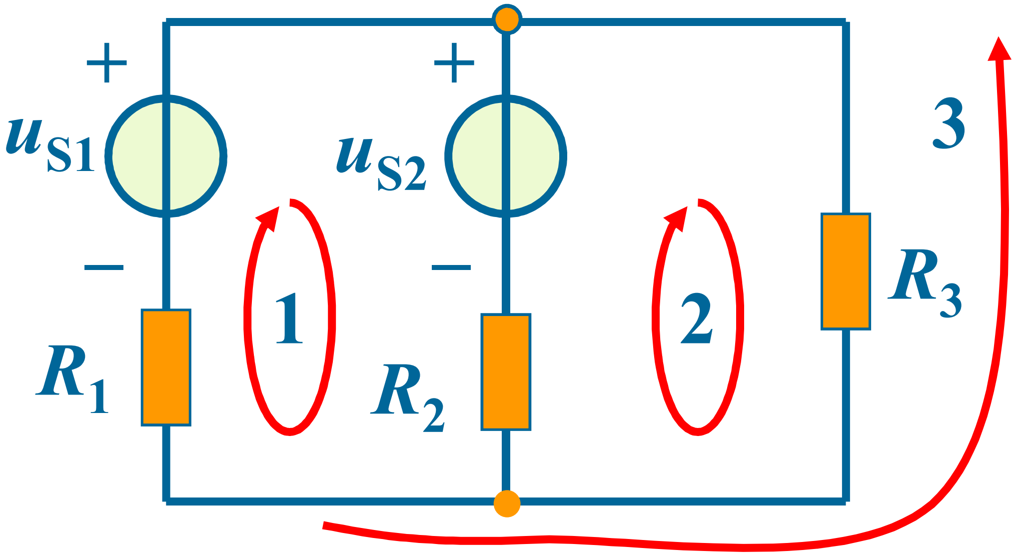
Example



b=5

b=3

n=2



l=3

2. Kirchhoff's current law (KCL)

The algebraic sum of the currents entering any node is zero.

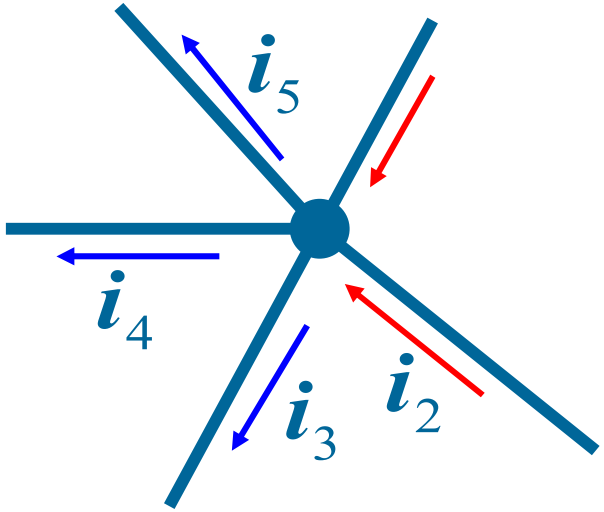
$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.

By the way, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

To prove KCL, assume a set of currents $i_k(t)=1,2,...,$ flow into a node. The algebraic sum of currents at the node is

$$i_T = i_1 + i_2 + i_3 + \ldots$$



Integrating both sides of Eq. gives

$$q_T = q_1 + q_2 + q_3 + \ldots$$

But the law of conservation of electric charge requires that the algebraic sum of electric charges at the node must not charge; that is, the node stores no net charge.

Thus $q_T=0, \longrightarrow i_T=0,$ confirming the validity of KCL

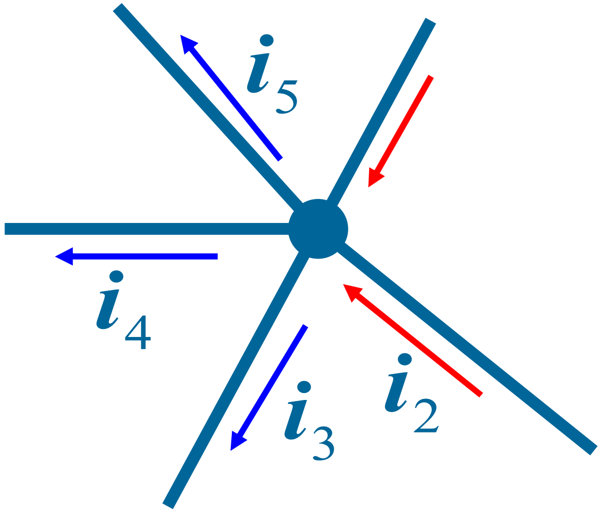
Example

Entering node:"+"

Leaving node:"-"

$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

$$i_1 + i_2 = i_3 + i_4 + i_5$$



Conclusion:

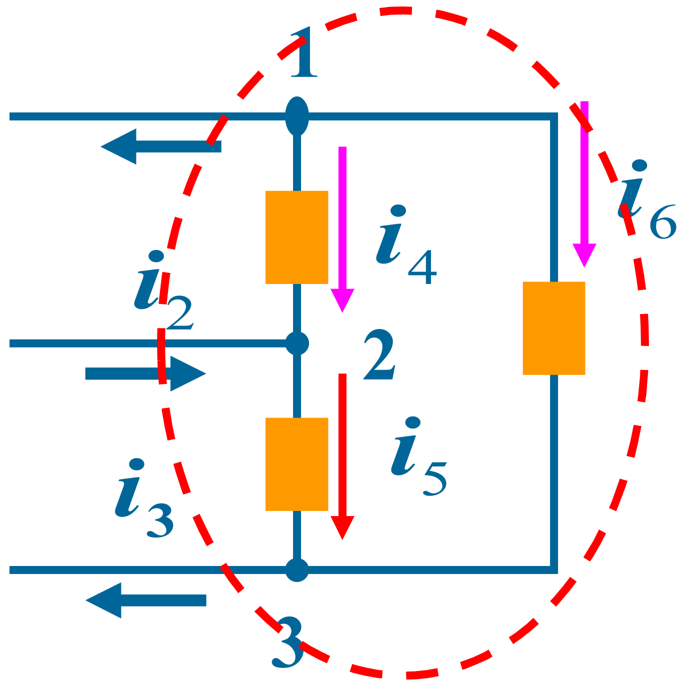
The sum of the currents entering a node is equal to the sum of the current leaving the node

Example

$$i_1 + i_4 + i_6 = 0$$
$$-i_2 - i_4 + i_5 = 0$$
$$i_3 - i_5 - i_6 = 0$$

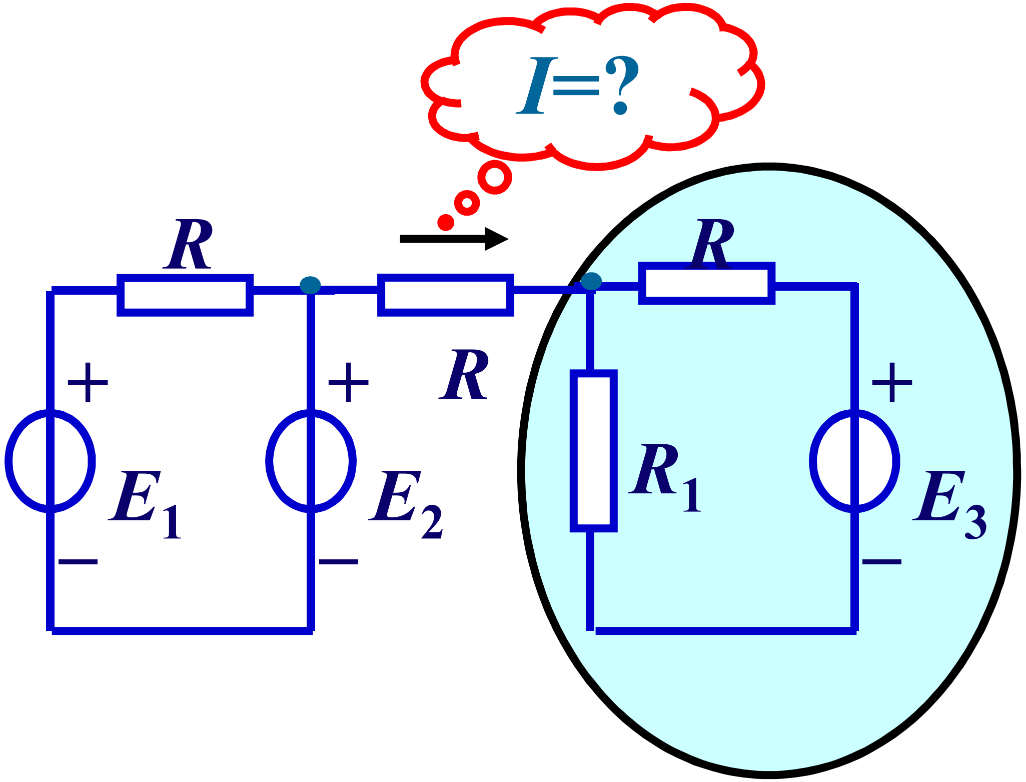
adding:

$$i_1 - i_2 + i_3 = 0$$



KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point.

Quiz



$I=0$

3. Kirchhoff's voltage law（KVL）

The algebraic sum of the voltages around any closed path is zero.

$$\sum_{m=1}^{M} u_m = 0 \qquad \textbf{\textit{or}} \qquad \sum u_{降} = u_{升}$$

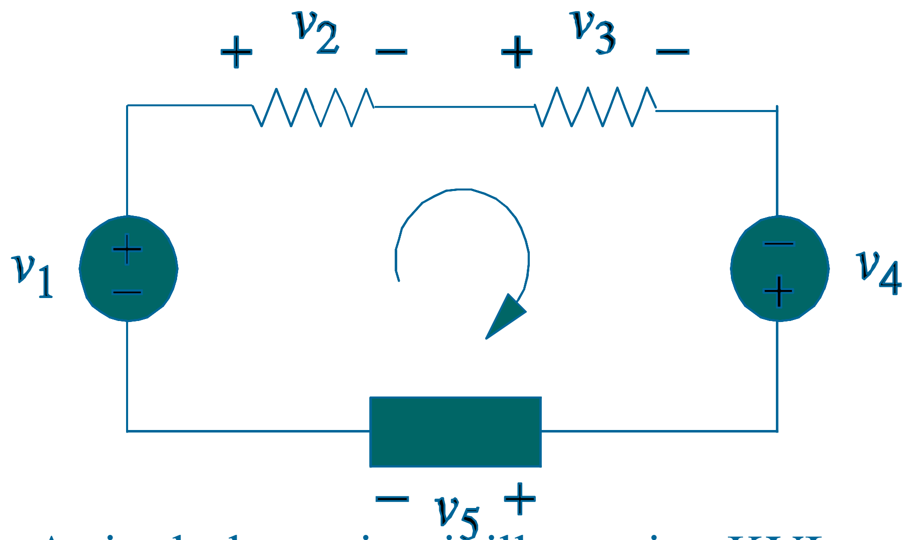
where M is the number of voltages in the loop (or the number of branches in the loop) and u_m is the mth voltage

Kirchhoff's voltage law is based on the principle of conservation of energy

To illustrate KVL, consider the current in Fig. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise.

Suppose we start with the voltage source and go clockwise around the loop as shown; then the voltage would be

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$



A single-loop circuit illustrating KVL

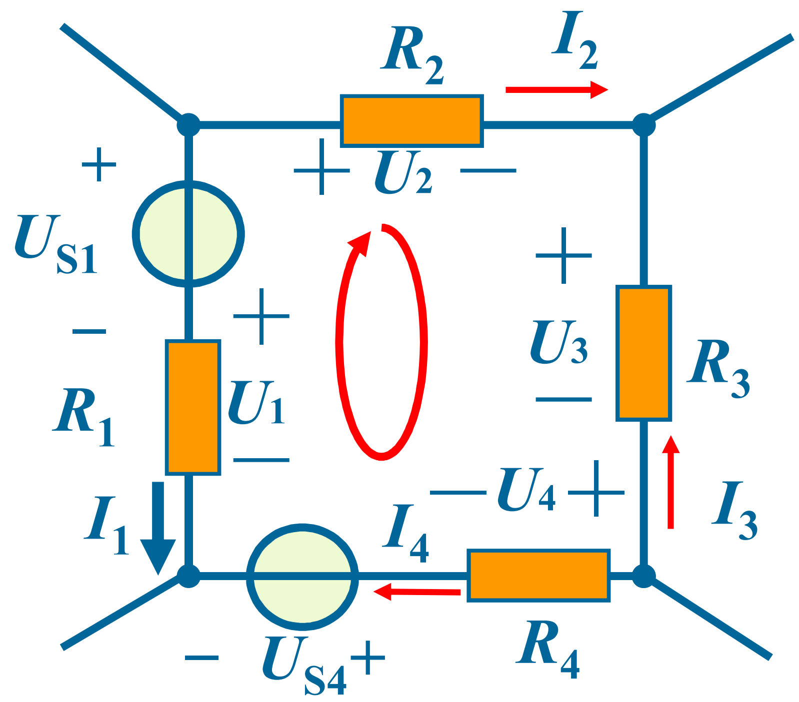
Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4$$

Which may be interpreted as

Sum of voltage drops =Sum of voltage rises

Example



（1）标定各元件电压参考方向

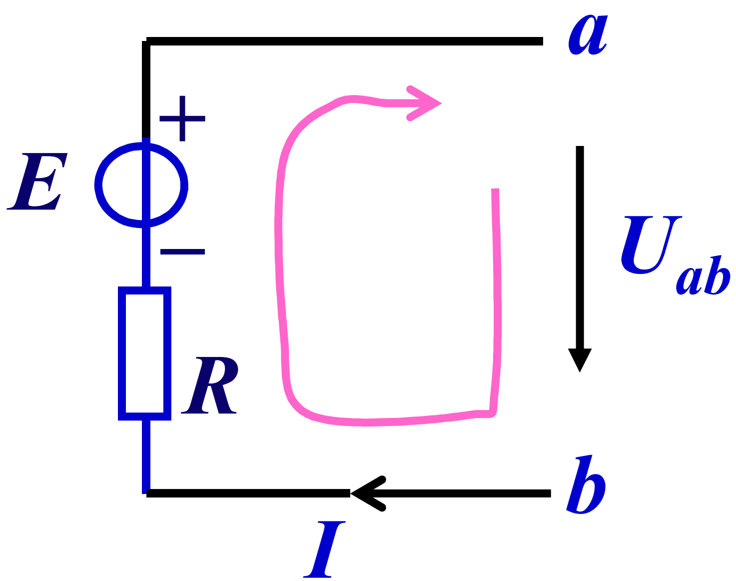
（2）选定回路绕行方向，
顺时针或逆时针.

$$-U_1-U_{S1}+U_2+U_3+U_4+U_{S4}=0$$

Or： $U_2+U_3+U_4+U_{S4}=U_1+U_{S1}$

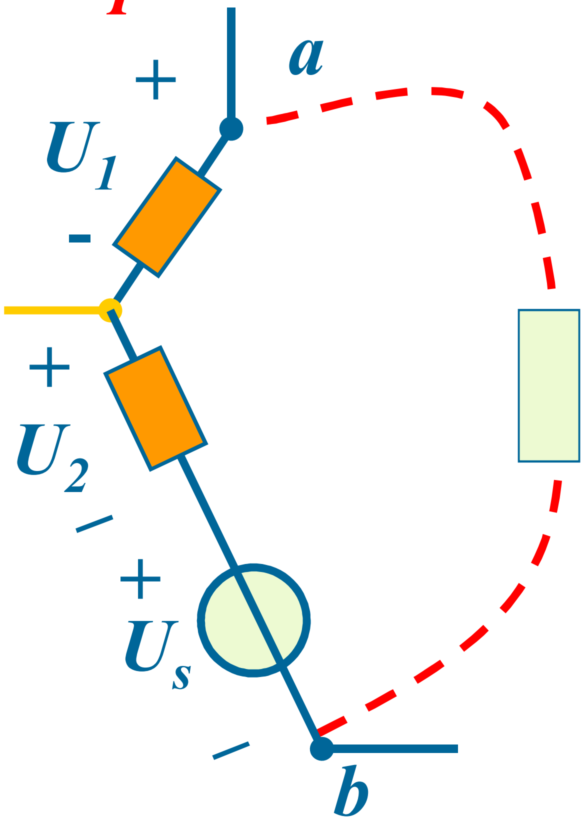
$$-R_1I_1+R_2I_2-R_3I_3+R_4I_4=U_{S1}-U_{S4}$$

Quiz



$$\underbrace{E}_{\text{rise}} = \underbrace{U_{ab} + I \cdot R}_{\text{drop}}$$

Example



$$U_{ab} = U_1 + U_2 + U_S$$

KCL also applies to a closed boundary

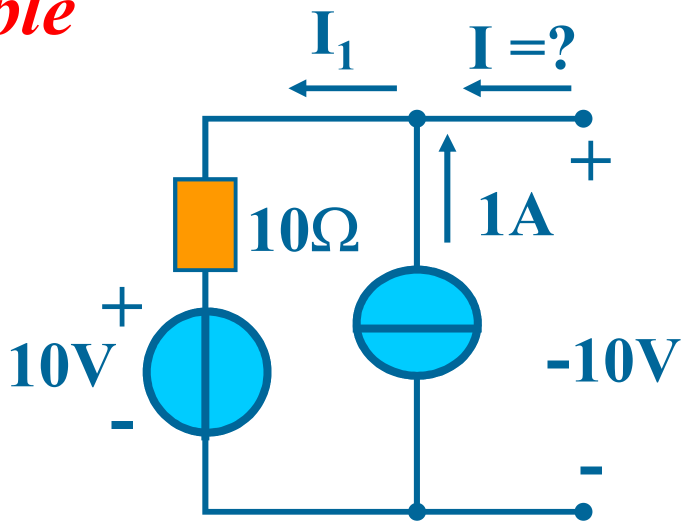
4. Summary for KCL、KVL：

(1) KCL states that the currents at any node algebraically sum to zero. In other words, the sum of currents entering a node equals the sum of the currents leaving the node.

(2) KVL states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.

(3) KCL is based on *the principle of conservation of charge*, and KVL is based one *the principle of conservation of energy*

Example



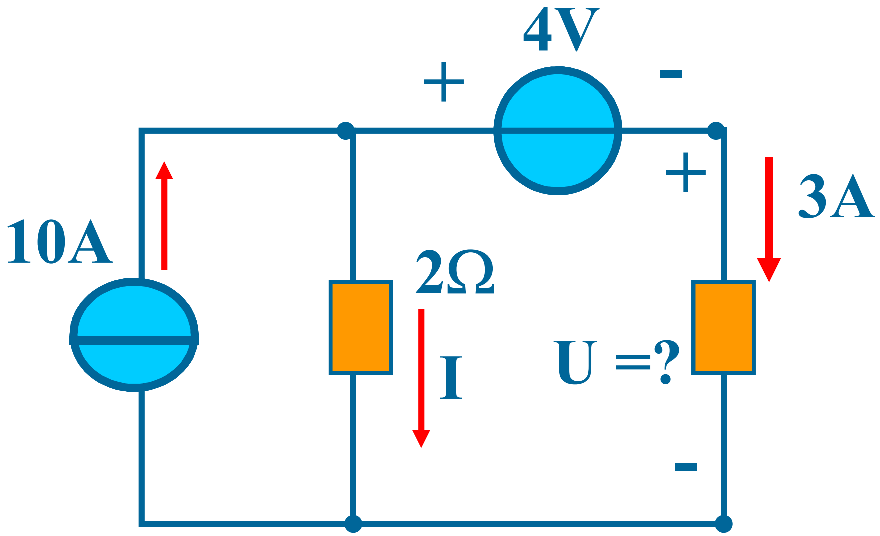
Solution

$$10I_1 + 10 - (-10) = 0$$

$$I_1 = -2A$$

$$I = I_1 - 1 = -2 - 1 = -3A$$

Example



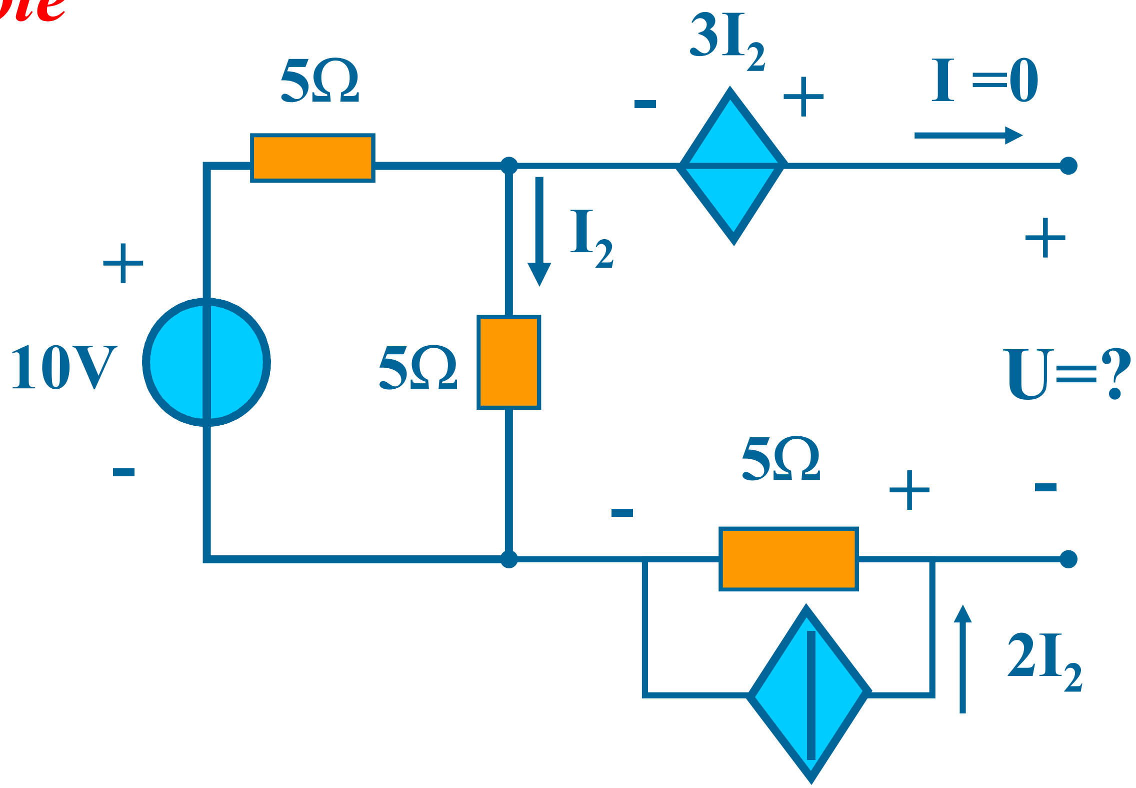
Solution

$$I = 10 - 3 = 7A$$

$$4 + U - 2I = 0$$

$$U = 2I - 4 = 14 - 4 = 10V$$

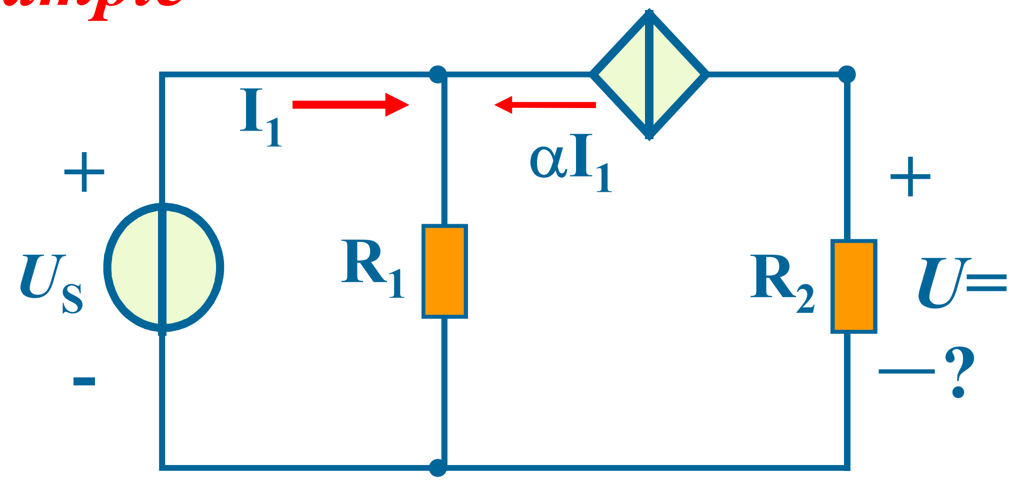
Example



Solution

$$I_2 = \frac{10}{5+5} = 1A$$

Example



Solution

$$U = -R_2 \alpha I_1$$

$$I_1 + \alpha I_1 = U_S / R_1$$

$$\Rightarrow I_1 = \frac{U_S}{R_1(1+\alpha)}$$

$$\Rightarrow U = -\frac{\alpha R_2 U_S}{R_1(1+\alpha)}$$

$$\left|\frac{U}{U_S}\right| = \frac{R_2}{R_1}\frac{\alpha}{(1+\alpha)}$$

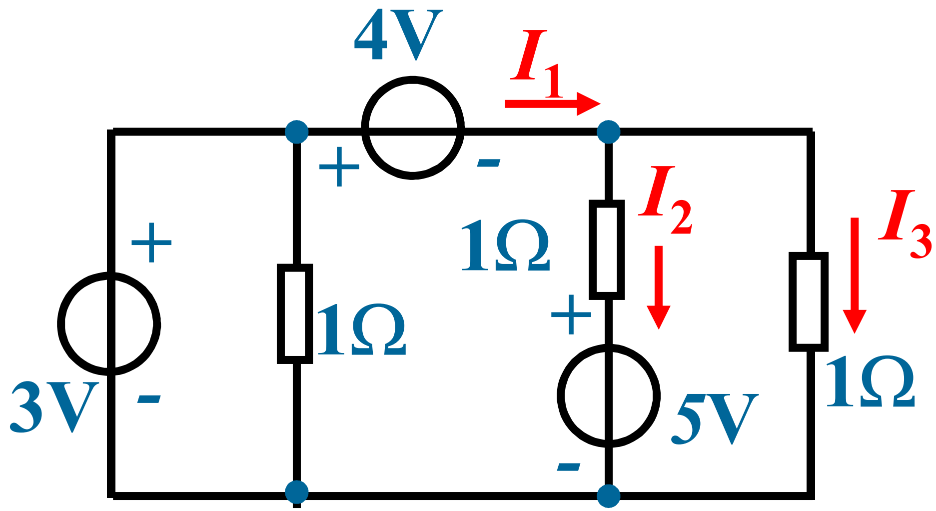
$$P_S = U_S I_1 = \frac{U_S^2}{R_1(1+\alpha)}$$

$$P_o = R_2 \alpha^2 \frac{U_S^2}{R_1^2(1+\alpha)^2}$$

$$\left|\frac{P_0}{P_S}\right| = \frac{R_2}{R_1}\frac{\alpha^2}{(1+\alpha)}$$

选择参数可以得到电压和功率放大

Quiz



Find: I_1、I_2、I_3

$$I_3 = \frac{3-4}{1} = -1\ \mathrm{A}$$

$$I_2 = \frac{3-4-5}{1} = -6\ \mathrm{A}$$

$$I_1 = I_2 + I_3 = -7\ \mathrm{A}$$