例3 欲接收载波频率为10MHz的某短波电台的信号，试设计接收机输入谐振电路的电感线圈。要求带宽Δf=100kHz，C=100pF。

解：由 $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

求得：

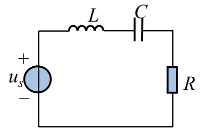
$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \times 10^{14} \times 10^{-10}}\,\mathrm{H} = 2.53\mu\mathrm{H}$$

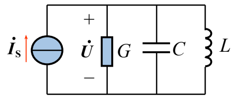
$$Q = \frac{f_0}{\Delta f} = \frac{10\times 10^6}{100\times 10^3} = 100$$

$$R = \frac{1}{Q\omega_0 C} = \frac{1}{100\times 2\pi \times 10^7 \times 10^{-10}}\,\Omega = 1.59\Omega$$

由此得到电感线圈的参数为L=2.53μH和R=1.59Ω。

2. Parallel Resonance （并联谐振 ）





R L C Series Resonance

$$Z = R + \mathrm{j}(\omega L - \frac{1}{\omega C})$$

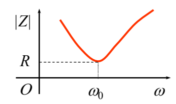
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

G C L Parallel Resonance

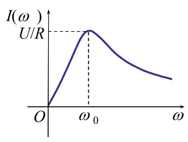
$$Y = G + \mathrm{j}(\omega C - \frac{1}{\omega L})$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

R L C Series Resonance

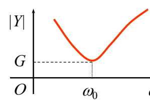


$|Z|_{min}=R$

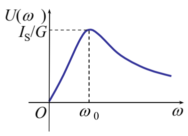


The current is the maximum.

G C L Parallel Resonance

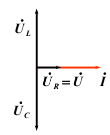


$|Y|_{min}=G$ $|Z|_{max}$



The voltage is the maximum.

R L C Series Resonance

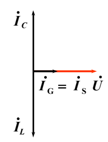


Voltage resonance

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$U_L(\omega_0)=U_C(\omega_0)=QU_S$$

G C L Parallel Resonance



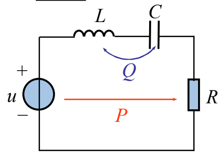
Current resonance

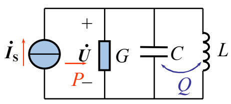
$$Q = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 GL} = \frac{1}{G}\sqrt{\frac{C}{L}}$$

$$I_L(\omega_0)=I_C(\omega_0)=QI_S$$

$$\dot{I}_C = j\omega_0 C\dot{U} = j\omega_0 C\frac{\dot{I}_s}{G} = jQ\dot{I}_s$$

RLC Series Resonance   GCL Parallel Resonance





$P = RI^2 = U^2/R$

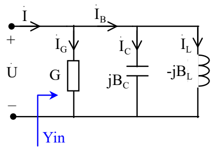
$$Q_L = \omega_0 L I^2 \quad Q_C = -\frac{1}{\omega_0 C} I^2$$

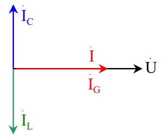
$$Q = Q_L + Q_C = 0$$

$P = UI = I^2/G$

$$Q_L = \frac{U^2}{\omega_0 L} \quad Q_C = -\omega_0 C U^2$$

$$Q = Q_L + Q_C = 0$$





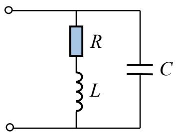
$$w_C(t) + w_L(t) = LQ^2 I_s^2$$

Notice:

The parallel LC combination acts like an open circuit, so that the entire currents flows through R.

The inductor and capacitor current can be much more than the source current at resonance.

3. Resonance Occurs In RLC Circuit



$$Y = j\omega C + \frac{1}{R + j\omega L}$$

$$= \frac{R}{R^2 + (\omega L)^2} + j(\omega C - \frac{\omega L}{R^2 + (\omega L)^2})$$

$$= G + jB$$

Resonant frequency

$$\omega_0 C - \frac{\omega_0 L}{R^2 + (\omega_0 L)^2} = 0$$

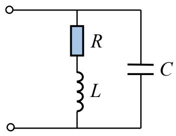
$$\omega_0 = \sqrt{\frac{1}{LC} - (\frac{R}{L})^2}$$

Resonance occurs only when: $\frac{1}{LC} > (\frac{R}{L})^2$ **or** $R < \sqrt{\frac{L}{C}}$

Simplification of RLC Circuit

$$Y = j\omega C + \frac{1}{R + j\omega L}$$

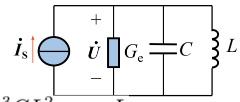
$$= \frac{R}{R^2 + (\omega L)^2} + j(\omega C - \frac{\omega L}{R^2 + (\omega L)^2})$$



When $R << \omega L$

$$Y \approx \frac{R}{(\omega L)^2} + j(\omega C - \frac{1}{\omega L})$$

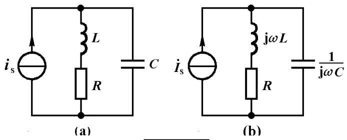
$$G_e = \frac{R}{(\omega_0 L)^2} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$



$$Q = \frac{\omega_0 C}{G_e} = \frac{\omega_0 C}{R/(\omega_0 L)^2} = \frac{\omega_0^3 C L^2}{R} = \frac{\omega_0 L}{R}$$

Quality factor of inductance coil

例 下图是电感线圈和电容器并联的电路模型。已知R=1Ω, L=0.1mH, C=0.01μF。试求电路的谐振角频率和谐振时的阻抗。



解: $$\omega_0=\frac{1}{\sqrt{10^{-4}\times10^{-8}}}\sqrt{1-\frac{10^{-8}}{10^{-4}}}\text{rad/s}=10^6\text{rad/s}$$

$$R<<\omega L\quad Z=\frac{1}{G_e}=\frac{(\omega_0 L)^2}{R}=\frac{10^{-6}\times10^{-4}}{1}\Omega=10k\Omega$$

27

例 ***RLC*并联谐振电路中，已知*R*=10kΩ，*L*=1H, *C*=1μF。试求电路的谐振角频率、品质因数和3dB带宽。**

解:

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{1\times10^{-6}}}\text{rad/s}=10^3\text{rad/s}$$

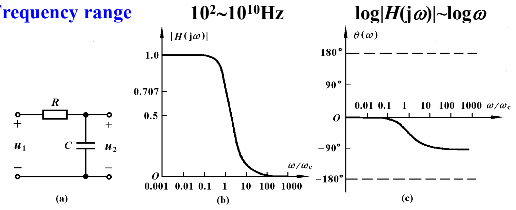
$$Q=R\omega_0 C=\frac{R}{\omega_0 L}=R\sqrt{\frac{C}{L}}=10$$

$$\Delta\omega=\frac{\omega_0}{Q}=100\text{rad/s}\qquad \Delta f=\frac{100}{2\pi}\text{Hz}=15.9\text{Hz}$$

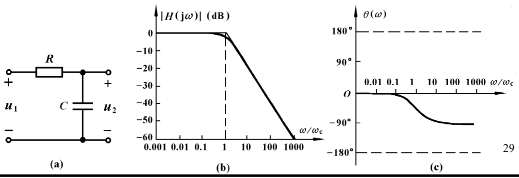
28

Bode Plots (波特图)

Frequency range **10^2~10^{10}Hz** **log|H(jω)|~logω**



20log|H(jω)|~log(ω/ω_C)



29

Bode Plots (波特图)

20log|H(jω)|~log(ω/ω_C)

dB

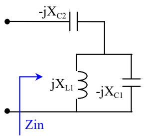
Becibel Scale（dB,分贝）

A	0.01	0.1	0.707	1	2	10	100	1000
20log*A*/dB	-40	-20	-3.0	0	6.0	20	40	60

30

例：L_1 = 100mH，要使$Z_{in}(10^3)$= ∞，确定C_1值；要使$Z_{in}(500)$= 0，在已确定C_1情况下，确定C_2值。

解：



L_1与C_1发生并联谐振，$\omega_0 = 10^3\ rad/s$

$$\because\ \omega_0^2 = \frac{1}{L_1C_1},\quad \Rightarrow C_1 = \frac{1}{\omega_0^2 L_1} = \frac{1}{(10^3)^2\times 100\times 10^{-3}} = 10\mu F$$

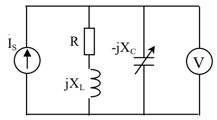
在$\omega_1 = 500\ rad/s$时发生串联谐振

$$Z_{in} = -j\frac{1}{\omega_1 C_2} + \frac{1}{j(\omega_1 C_1 - \frac{1}{\omega_1 L_1})} = 0$$

$$\frac{-1}{\omega_1 C_2} = \frac{1}{\omega_1 C_1 - \frac{1}{\omega_1 L_1}} \quad\Rightarrow\quad C_2 = 30\mu F$$

例：测电感L的电阻与电感，角频率ω=10³ rad/s，I_S=1A，C=50μF时，电压表读数为最大，U=50V。求R、L。

解：



$$Y = \frac{R}{R^2+(\omega L)^2} - j(\frac{\omega L}{R^2+(\omega L)^2} - \omega C)$$

$$= G - j(B_L - B_C) = G - jB = ye^{-j\varphi}$$

电压表：$U = \frac{I_s}{y}$

y最小时U最大，改变C不影响G。

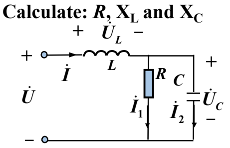
∴ B=0（并联谐振）时U最大。 $\Rightarrow$ $G = \frac{I_S}{U} = \frac{1}{50} = 0.02S$

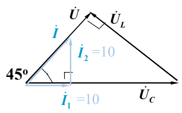
$B_L = B_C = \omega C = 10^3\times 50\times 10^{-6} = 0.05S$

$$\frac{R}{R^2+(\omega L)^2} = G = 0.02 \qquad \frac{\omega L}{R^2+(\omega L)^2} = B_L = 0.05 \quad\Rightarrow\quad \omega L = 17.24\Omega,\ L = 17.2mH \quad R = 6.9\Omega$$

Example **Resonance occurs in the circuit，then U=100V，I_1=I_2=10A，**
Calculate: R, X_L and X_C

Solution：Phasor Diagrams





According to the current triangle **I=14.14 A**

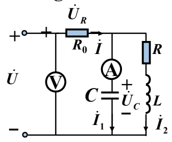
According to the voltage triangle
U_L=U=100 V **U_C=141.4 V**

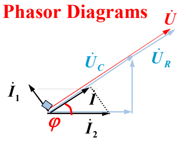
R=U_C/ I_1=14.14Ω ，X_C=－U_C/ I_2=－ 14.14 Ω

X_L=U_L/I=7.07 Ω

Example **Resonance occurs when ω=1000rad/s，we know R_0=25Ω，C=16μF，the reading of voltmeter is 100V，the reading of Ammeter is 1.2A，calculate R and L。**

Solution：Phasor Diagrams





$I_1 = 1.2A$

$U_C = 1.2 / (1000\times 1.6\times 10^{-6}) = 75V$

According to the resonance condition

$U_R = 25V \qquad I = 1A$

$I_2 = \sqrt{I_1^2 + I^2} = 1.562A$

$\varphi = \arctan(I_1 / I) = 50.2^\circ$

$|Z| = U / I_2 = 75 / 1.562 = 48.01\Omega$

$R = 48.01\cos 50.2^\circ = 30.7\Omega$

$L = 48.01\sin 50.2^\circ / 1000 = 36.9mH$