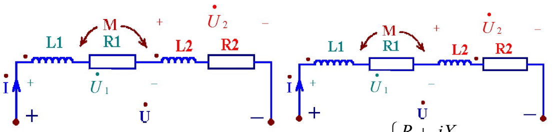
Example 2. Two coupled inductors are connected to a sinusoidal source with an effective value of 220V and the frequency of 50Hz。If the two inductors are connected *in aiding*, I=2.7A,P=218.7W； if they are connected *in opposition*, I=7A。Find the mutual inductance M=?



Solution: $Z = R_1 + R_2 + j\omega(L_1 + L_2 \pm 2M) = \begin{cases} R + jX_1 \\ R + jX_2 \end{cases}$

connection *in aiding*： $\frac{U}{I} = 81.481 = \sqrt{R^2 + X_1^2}$ $\frac{P}{I^2} = 30 = R$ $X_1 = 75.756$

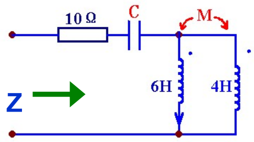
connection *in opposition*: $\frac{U}{I} = 31.429 = \sqrt{R^2 + X_2^2}$ $X_2 = 9.7066$

$X_1 - X_2 = 4\omega M$ $\therefore$ $M = \frac{X_1 - X_2}{4\omega} = 53.07mH$

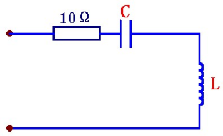
Example 3. ω=4rad/s, C = 5F , M=3H. 1) Find the input impedance Z; 2) What value should the capacitor C be to make Z be a pure resistance?

Solution: The mutual inductance are connected in aiding parallel, so

$$L = \frac{4 \times 6 - 3^2}{4 + 6 - 2 \times 3} = \frac{15}{4} H$$



And the equivalent circuit is as follows



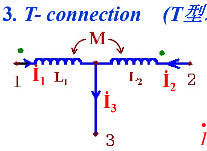
Thus

$$Z = 10 - j0.05 + j15$$
$$= 10 + j14.95(\Omega)$$

To assure Z be a pure resistance,

$\frac{1}{\omega C} = 15$ $\therefore$ $C = \frac{1}{60} F$

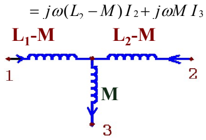
3. *T- connection (T型联接)*

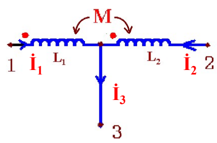


$\dot I_1 + \dot I_2 = \dot I_3$

$\dot U_{13} = j\omega L_1 \dot I_1 + j\omega M \dot I_2$
$= j\omega(L_1 - M)\dot I_1 + j\omega M \dot I_3$

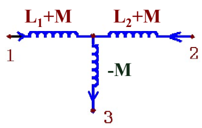
$\dot U_{23} = j\omega L_2 \dot I_2 + j\omega M \dot I_1$
$= j\omega(L_2 - M)\dot I_2 + j\omega M \dot I_3$



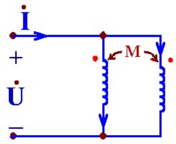


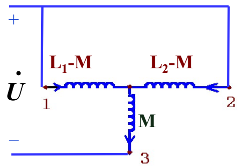
$\dot U_{13} = j\omega(L_1 + M)\dot I_1 - j\omega M \dot I_3$

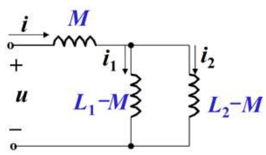
$\dot U_{23} = j\omega(L_2 + M)\dot I_2 - j\omega M \dot I_3$



Decoupling Equivalent Circuit of Parallel-aiding Connectio



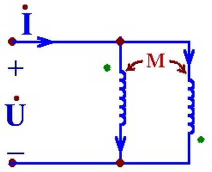


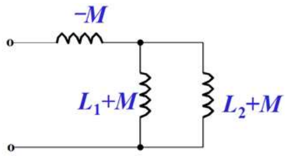


$(L_1 - M)//(L_2 - M) + M$

$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

Decoupling Equivalent Circuit of Parallel-opposing Connection





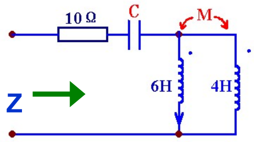
$$(L_1 + M)//(L_2 + M) - M$$

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

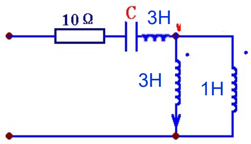
Example 3. ω=4rad/s, C = 5F , M=3H. 1) Find the input impedance Z; 2) What value should the capacitor C be to make Z be a pure resistance?

Solution: The mutual inductance are connected in aiding parallel, so

$$L = \frac{3}{4} + 4 = \frac{15}{4} H$$



And the equivalent circuit is as follows



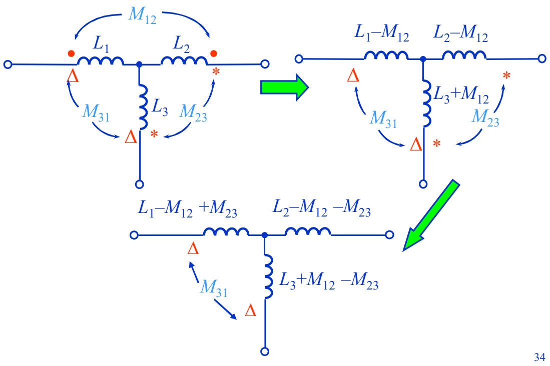
Thus

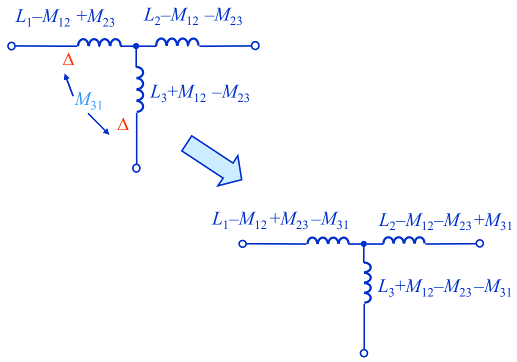
$$Z = 10 - j0.05 + j15$$
$$= 10 + j14.95(\Omega)$$

To assure Z be a pure resistance,

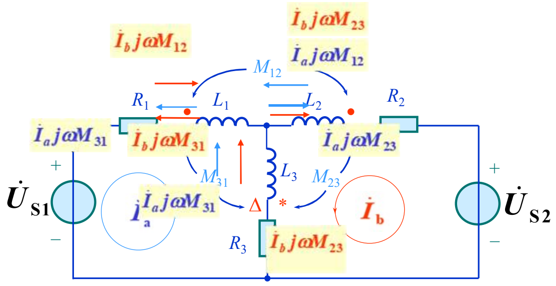
$$\frac{1}{\omega C} = 15 \qquad \therefore\ C = \frac{1}{60} F$$

Exercise: Find the decoupling circuit of the following three-coulpled inductance.





Example 4 Write the mesh-current equation



Loop a

$$(R_1+j\omega L_1+j\omega L_3+R_3)\dot I_a+(R_3+j\omega L_3)\dot I_b$$
$$-2j\omega M_{31}\dot I_a \quad +(j\omega M_{12}-j\omega M_{31}-j\omega M_{23})\dot I_b = \dot U_{S1}$$

Loop b:

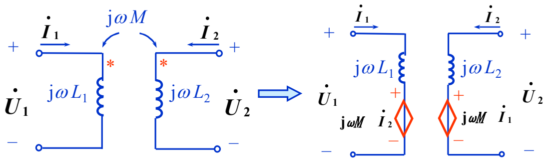
$$(R_2+j\omega L_2+j\omega L_3+R_3)\dot I_b+(R_3+j\omega L_3)\dot I_a$$
$$-2j\omega M_{23}\dot I_b +(j\omega M_{12}-j\omega M_{31}-j\omega M_{23})\dot I_a \quad = \dot U_{S2}$$

Obtained:

$$\begin{cases} (R_1+j\omega L_1-j2\omega M_{31}+j\omega L_3+R_3)\dot I_a + \\ \quad (R_3+j\omega L_3+j\omega M_{12}-j\omega M_{31}-j\omega M_{23})\dot I_b=\dot U_{S1} \\ (R_3+j\omega L_3+j\omega M_{12}-j\omega M_{31}-j\omega M_{23})\dot I_a + \\ \quad (R_2+j\omega L_2-j2\omega M_{23}+j\omega L_3+R_3)\dot I_b=\dot U_{S2} \end{cases}$$

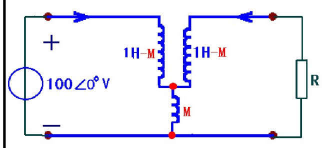
4、Decoupling Equivalent Circuit with CCVS

（受控源等效电路）



$$\begin{cases} \dot U_1 = j\omega L_1\dot I_1 + j\omega M\dot I_2 \longleftarrow \dot I_2 \text{ CCVS} \\ \dot U_2 = j\omega L_2\dot I_2 + j\omega M\dot I_1 \end{cases}$$

Example 5. **Let ω=10rad/s, R=10Ω. Find $\dot I_1$, $\dot I_2$ and the power absorbed by R if 1)k=0.5; 2)k=1.**



Solution: The decoupling circuit is

1）K=0.5，M = 0.5H,so

$$j5\dot I_1 + j5(\dot I_1+\dot I_2)=100\angle 0°$$
$$j5\dot I_2 + j5(\dot I_1+\dot I_2)+10\dot I_2 = 0$$

Thus

$$\dot I_1 = 11.3\angle 81.87°A$$
$$\dot I_2 = 4\angle -216.87°A$$
$$\boxed{P=160W}$$

2）K=1，M = 1H,then

$$j10\dot I_1 + j10\dot I_2 = 100\angle 0°$$
$$j10\dot I_1 + (10+j10)\dot I_2 = 0$$

so $\dot I_1 = 10\sqrt{2}\angle -45°A$

$$\dot I_2 = 10\angle -180°$$
$$\boxed{P=1000W}$$

Example 6. Assume that Z obtained the maximum power, calculate the value of Z. Where

$u(t) = 10\sqrt{2}\cos(10^4 t + 53.1°)V$

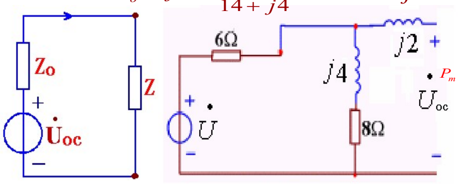
Solution: Labeled the dotted terminals and draw this circuit's diagram

The decoupling equivalent circuit is

Find open-circuit voltage $U_{oc} = \frac{8+j4}{14+j4} 10\angle 0° = 6.11\angle 10.61° V$

Find Z_0 $Z_o = j2 + \frac{6(8+j4)}{14+j4} = 3.6 + j2.67$ $\therefore Z = \overset{*}{Z_o}$

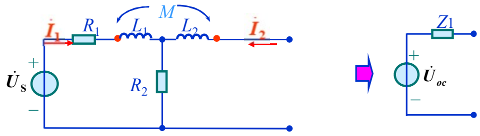
$= 3.6 - j2.67(\Omega)$



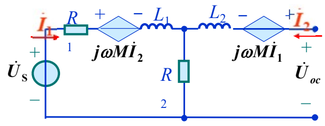
$P_m = \frac{U_{oc}^2}{4R_o} = 2.59W$

Example 7 $\omega L_1 = \omega L_2 = 10\Omega$, $\omega M = 5\Omega$, $R_1 = R_2 = 6\Omega$, $U_S = 6V$,

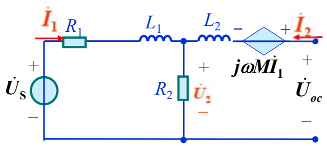
Find Thevenin equivalent circuit.



Solution 1:



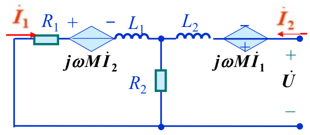
Find $\dot{U}_{oc}$ $\dot{I}_2 = 0$



$$\dot{I}_1 = \frac{\dot{U}_S}{R_1 + j\omega L_1 + R_2} = \frac{6\angle 0°}{12 + j10} = \frac{6\angle 0°}{15.62\angle 39.8°} = 0.384\angle -39.8°\ A$$

$$\dot{U}_{OC} = j\omega M\,\dot{I}_1 + R_2\dot{I}_1 = (6+j5)\times 0.384\angle -39.8° = 3\angle 0°\ V$$

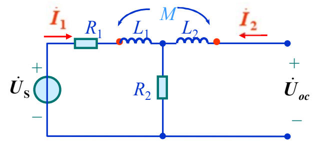
Find Z_i 加压求流



$$\begin{cases}(R_1 + R_2 + j\omega L_1)\dot{I}_1 + R_2\dot{I}_2 + j\omega M\,\dot{I}_2 = 0 \\ (R_2 + j\omega L_2)\dot{I}_2 + R_2\dot{I}_1 + j\omega M\,\dot{I}_1 = \dot{U}\end{cases}$$

$$\dot{I}_2 = \frac{\dot{U}}{3+j7.5}, \quad Z_i = \frac{\dot{U}}{\dot{I}_2} = 3 + j7.5 = 8.08\angle 68.2°\Omega$$

Solution 2:



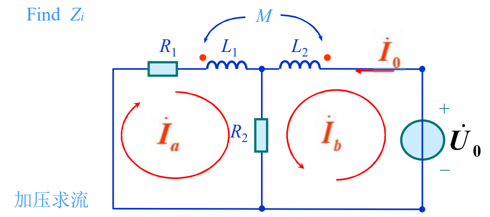
Find $\dot U_{OC}$ $\qquad$ $\dot I_2 = 0$

$$\dot I = \frac{\dot U_S}{R_1 + j\omega L_1 + R_2} = \frac{6\angle 0^\circ}{12 + j10} = \frac{6\angle 0^\circ}{15.62\angle 39.8^\circ} = 0.384\angle -39.8^\circ\ A$$

$$\dot U_{OC} = j\omega M\, \dot I_1 + R_2 \dot I_1$$

$$= (6 + j5) \times 0.384\angle -39.8^\circ = 3\angle 0^\circ\ V$$

Find Z_i

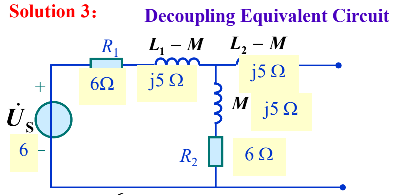


加压求流

$$\begin{cases} (R_1 + R_2 + j\omega L_1)\dot I_a + R_2 \dot I_b + j\omega M\, \dot I_b = 0 \\ (R_2 + j\omega L_2)\dot I_b + R_2 \dot I_a + j\omega M\, \dot I_a = \dot U_0 \end{cases}$$

$$\dot I_0 = \dot I_b = \frac{\dot U_0}{3 + j7.5}, \quad Z_i = \frac{\dot U_0}{\dot I_0} = 3 + j7.5 = 8.08\angle 68.2^\circ\ \Omega$$

Solution 3: $\qquad$ **Decoupling Equivalent Circuit**



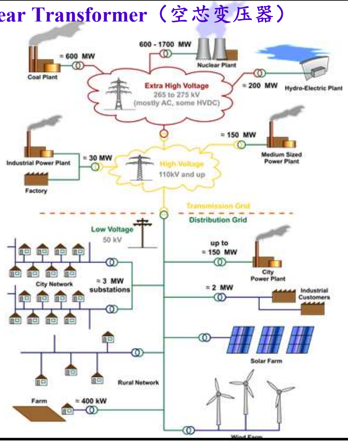
$$\dot U_0 = \frac{6 + j5}{6 + j5 + j5 + 6} = 3\angle 0^\circ$$

$$Z_i = 5 + \frac{(6 + j5)(6 + j5)}{(6 + j5) + (6 + j5)}$$

$$= j5 + \frac{6 + j5}{2} = j5 + 3 + j2.5$$

$$= 3 + j7.5 = 8.08\angle 68.2^\circ\ \Omega$$

13. 4 The Linear Transformer（空芯变压器）



13.4 The Linear Transformer（空芯变压器）

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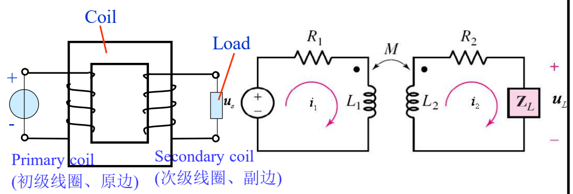
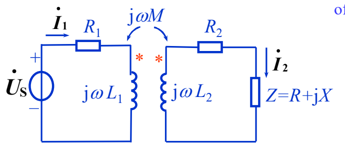
A Transfermer is generally a four-terminal device comprising two (or more) magnetically coupled coils.

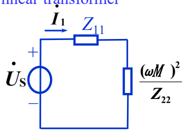


Coil { Air, plastic / silicon steel sheet, amorphous alloy

A linear transformer containing a source in the primary circuit and a load in the secondary circuit. Resistance is also included in both the primary and the secondary.



Primary coil equivalent circuit of linear transformer



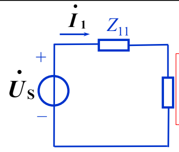
原边等效电路

$$\begin{cases} \underbrace{(R_1 + j\omega L_1)}_{Z_{11}}\dot{I}_1 - j\omega M \dot{I}_2 = \dot{U}_S \\ -j\omega M \dot{I}_1 + \underbrace{(R_2 + j\omega L_2 + Z)}_{Z_{22}}\dot{I}_2 = 0 \end{cases}$$

$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + \dfrac{(\omega M)^2}{Z_{22}}}$$

$$\dot{I}_2 = \frac{j\omega M \dot{I}_1}{Z_{22}}$$

$$Z_{in} = \frac{\dot{U}_S}{\dot{I}_1} = Z_{11} + \frac{(\omega M)^2}{Z_{22}}$$



$\boxed{\dfrac{(\omega M)^2}{Z_{22}}}$ ***reflected impedance*（Z_l）**

副边对原边的引入阻抗、反射阻抗

$$Z_l = \frac{(\omega M)^2}{Z_{22}} = \frac{\omega^2 M^2}{R_{22} + jX_{22}} = \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - \frac{j\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

$$= R_l + jX_l$$

$$R_l = \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2}$$ ***reflected resistance*（R_l）**

$\dot{I}_2 = 0, \quad Z_{in} = Z_{11}$

$\dot{I}_2 \neq 0, \quad Z_i = Z_{11} + Z_l$

$$X_l = -\frac{\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$ ***reflected reactance*（X_l）**

The reactance which the secondary reflects into the primary circuit has a sign which is opposite to that of X_{22}

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Transmission of energy(传送功率)

$$Z_l = \frac{(\omega M)^2}{Z_{22}} = \frac{\omega^2 M^2}{R_{22} + jX_{22}} = \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - \frac{j\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

$$= R_l + jX_l$$

$$R_l = \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2}$$

$R_l > 0$, absorbing power, providing by primary coil

Power developed by source = Power absorbed by load

$= I_1^2 R_1 + I_1^2 R_l$

absorbing by primary coil

absorbing by secondary coil, transfering by mutual inductance

$$I_1^2 R_l^2 = I_2^2 R_{22}$$

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