1-4 *Circuit Elements*

- A element is the basic building block of a circuit.
- An electric circuit is simply an interconnection of the elements.
- Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

- There are two types of elements found in electric circuits:
  - ✓Passive elements: An passive element is not capable of generating energy. Example passive element are resistors, capacitors, and inductors.
  - ✓Active elements: An active element is capable of generating energy. Type active element include generators, batteries, and operational amplifiers.

Lumped parameter element:

Electromagnetic process generate inside element

Lumped Condition: $d \ll \lambda$

For example:

- For work frequency： f = 50Hz: $\lambda = vT = \frac{v}{f} = \frac{3\times10^8}{50} = 6000km$
- For audio frequency： f = 25 kHz $\lambda = \frac{v}{f} = \frac{3\times10^8}{25\times10^3} = 12km$

Lumped parameter circuit: It consists of lumped element

Notice:

In lumped parameter circuit, u、i is function of time, but it is independent of space coordinate

An ideal basic circuit element has three attributes:

- ✓ It has **only two terminals**, which are points of connection to other circuit components;
- ✓ It is described mathematically **in terms of current and/or voltage**;
- ✓ It **cannot be subdivided into** other elements.

Note that:

- ✓ We use the word **ideal** to imply that a basic circuit element does not exist as a realizable physical component.
- ✓ We use the word **basic** to imply that the circuit element cannot be further reduced or subdivided into other elements.
- ✓ Thus the basic circuit elements form the building blocks for constructing circuit models, but ***they themselves cannot be modeled with any other type of element***.

- There are five ideal basic circuit elements:
  - voltage sources
  - current sources
  - resistors,
  - inductors
  - capacitors.

- Although this may seem like a small number of elements with which to begin analyzing circuits, many practical systems can be modeled with just sources and resistors. They are also a useful starting point because of their relative simplicity: *the mathematical relationships* between voltage and current in sources and resistors *are algebraic*.

- Thus you will be able to begin learning the basic techniques of circuit analysis with only algebraic manipulations.

  In this section we discuss their characteristics.

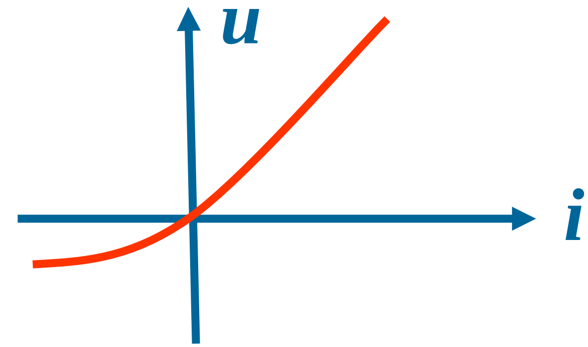
1-5 Resistance

● Resistance

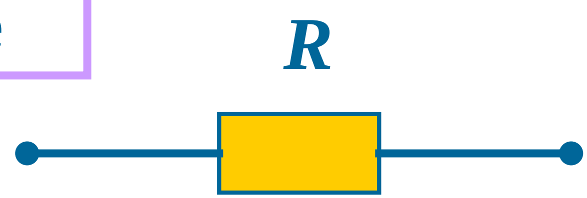
A element which ***dissipates energy*** but stores none is said to consist solely of resistance.

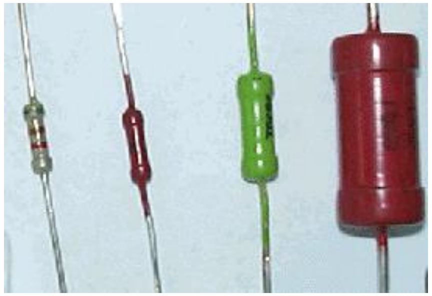
$f(u, i) = 0$

Volt-ampere relationship



● Symbol for resistance





Practical Resistors

金属膜电阻器

Metal Film Fixed Resistor（MF TYPE）

金属氧化物电阻器

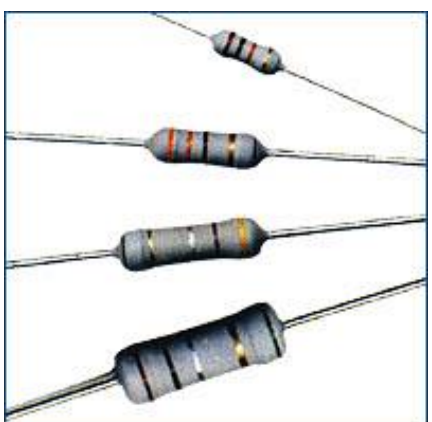
Metal Oxide Film Resistor (MOF TYPE)

碳膜电阻器

Carbon Film Fixed Resistor

熔断涂覆电阻器

Fusible Film Resistor



线绕涂覆电阻器

Wire Wound Resistor (KNP TYPE)

绕涂覆电阻器

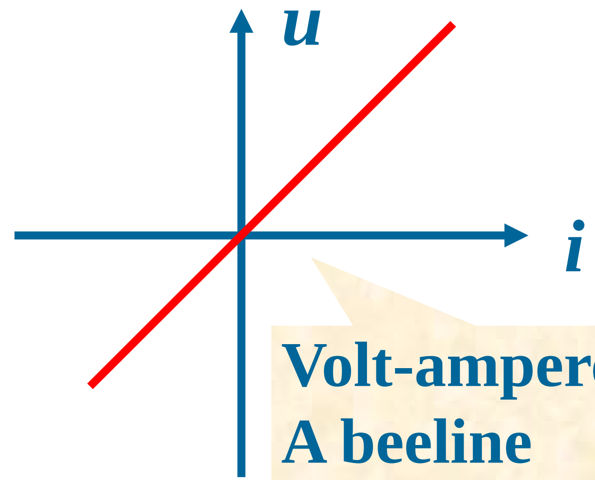
Wire Wound Resistor (KNH TYPE)

● $u \sim i$ relations → **Ohm's Law**

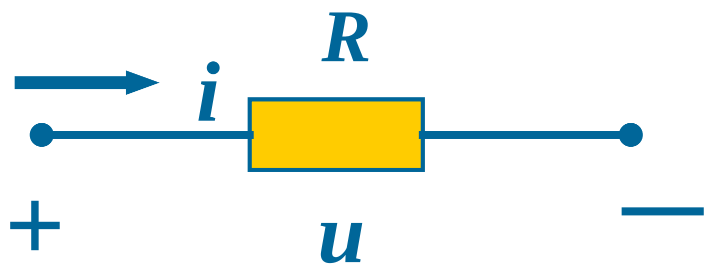
$$u = Ri \qquad R = u/i$$

$$i = u/R = Gu$$

u、i : Associate reference direction



Volt-ampere : A beeline through origin



● **in S.I. units**

***R--Resistance*：Ω (欧) (Ohm，欧姆)**

***G –conductance:* S(西门子) (Siemens，西门子)**

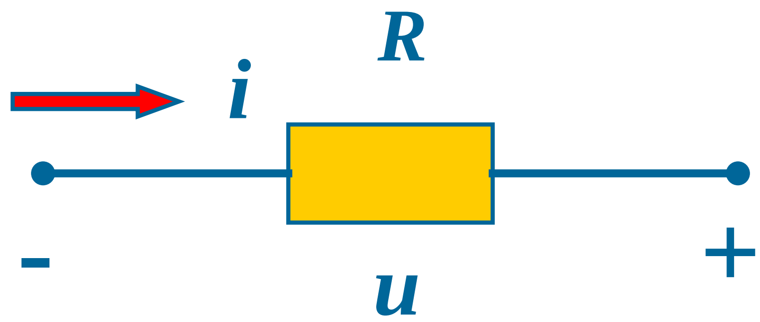
Notice:

Ohm's Law

(1) Be the same with linearity resistance

(2) Imply linearity resistance is nonmemory

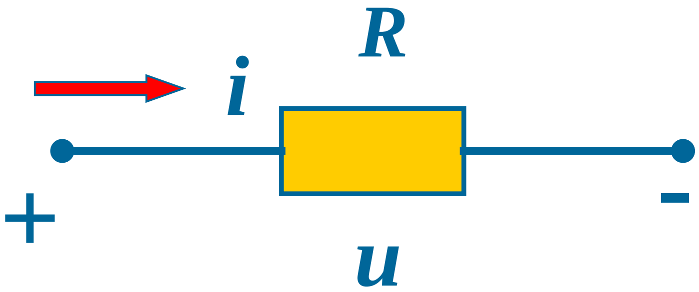
(3) In non- Associate reference direction



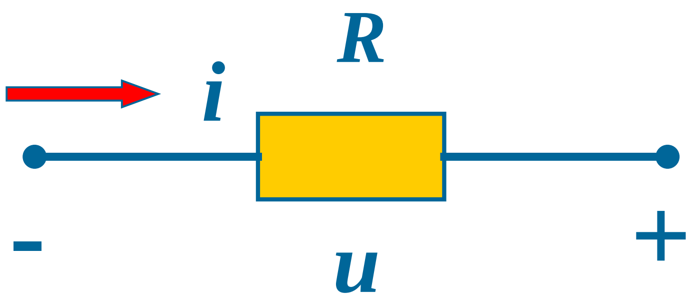
The expression for Ohm's Law is

$$u = -R\,i \qquad i = -G\,u$$

- **Power of Resistance**



$$p = u\,i = i^2 R = u^2 / R$$



$$p = -u\,i = -(-R\,i)\,i = i^2 R$$

$$= -u(-u/R) = u^2/R$$

Resistor dissipate energy

● Energy of Resistance

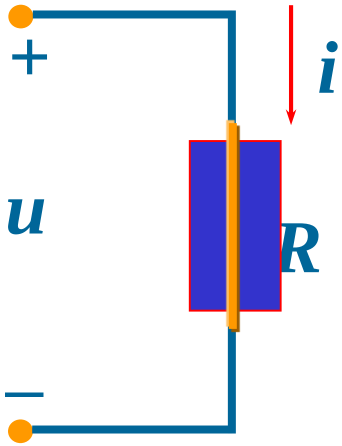
The energy absorbed by resistor from time *t* to time t_0 is :

$$W_R = \int_{t_0}^{t} p \, d\xi = \int_{t_0}^{t} u i \, d\xi$$

Energy is the capacity to do work, measured in joules (J)

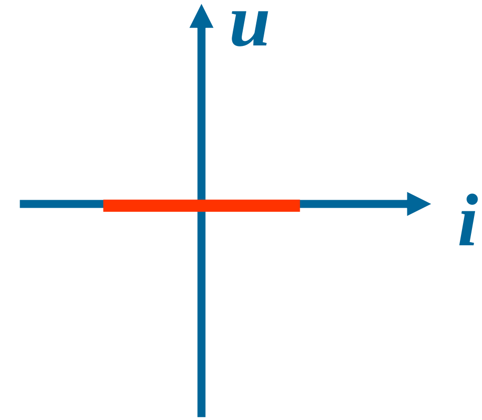
Open circuit and short circuit

✓ short circuit

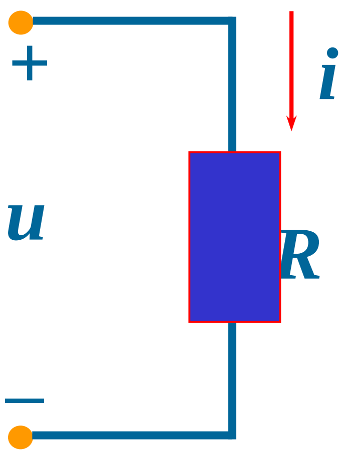


$i \neq 0 \quad u = 0$

$R = 0 \text{ or } G = \infty$

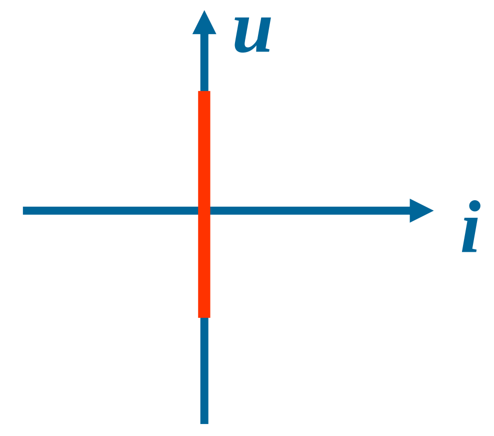


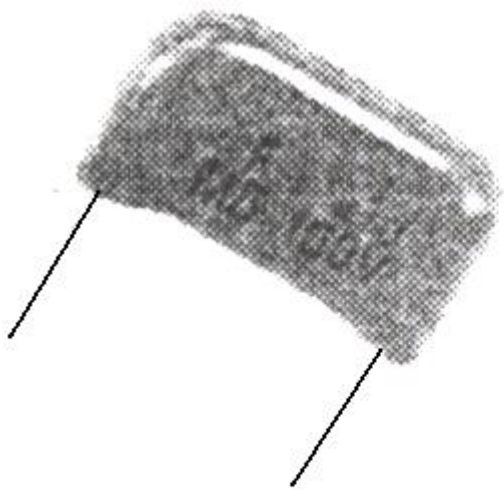
Open circuit

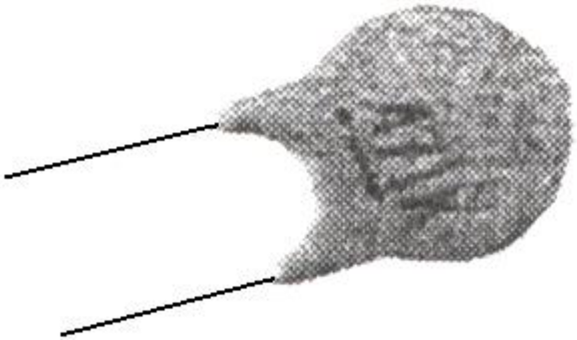


$i = 0 \quad u \neq 0$

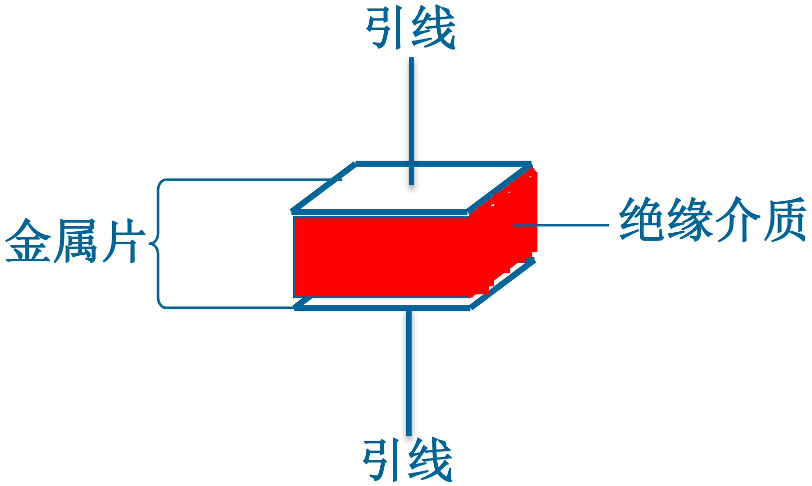
$R = \infty \text{ or } G = 0$







A capacitor consists of two conducting plates separated by an insulator(or dielectric).

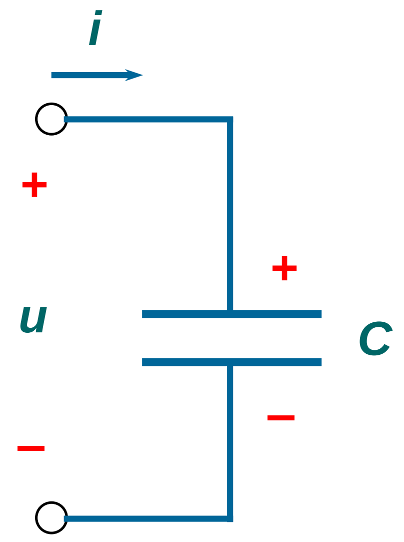


Circuit symbol

6.1、电容器 CAPACITORS

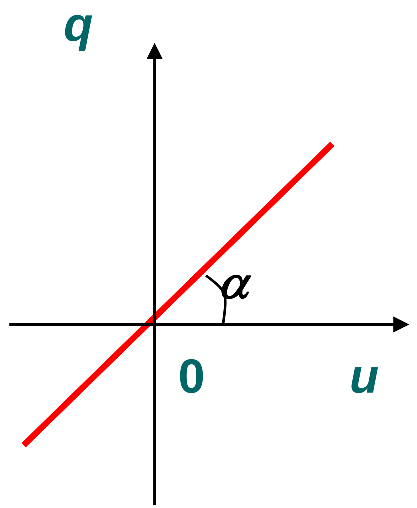
A capacitor is a passive element designed to store energy in its electric field.

Capacitors are widely used in the turning circuits of radio receivers and as dynamic memory elements in computer systems.



1. Circuit theory definition: a two_terminal element will be called a capacitor if at any time t, its charge q(t) and its voltage v(t) satisfy a relation defined by a curve in the q_v Plane.

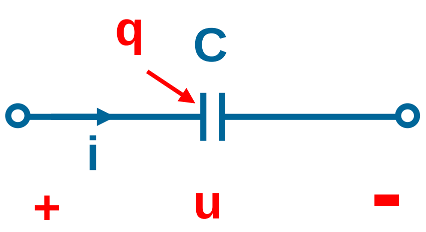
$$C \overset{\text{def}}{=} \frac{q}{u}$$

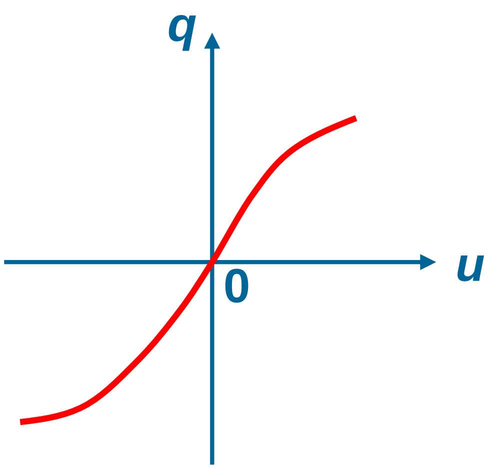


Capacitance: is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads(F).

$$F = C/V = A \cdot s/V = s/\Omega$$

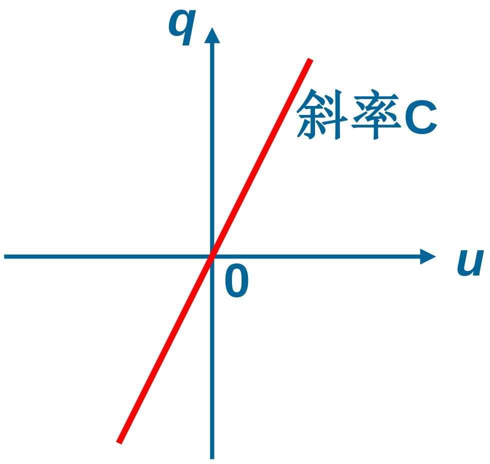
Nonlinear capacitor:



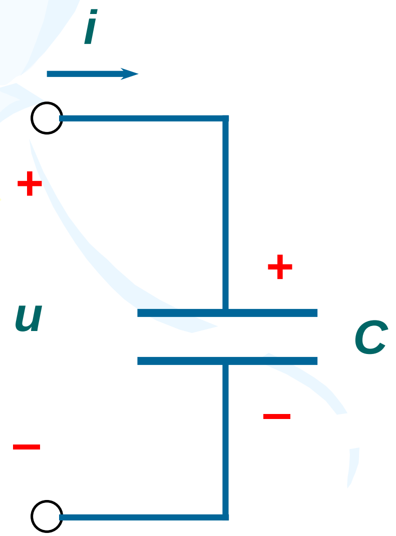


To the linear capacitor:

$$q = Cu$$



2. U-I relationship of the linear capacitor:



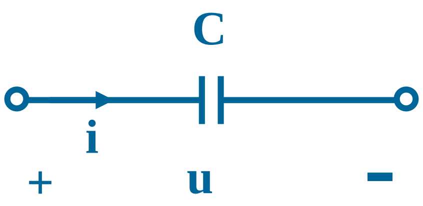
$$i = \frac{\mathrm{d}q}{\mathrm{d}t} = C\frac{\mathrm{d}u}{\mathrm{d}t}$$

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i\,\mathrm{d}\xi = u(t_0) + \frac{1}{C}\int_{t_0}^{t} i\,\mathrm{d}\xi$$

$$q(t) = q(t_0) + \int_{t_0}^{t} i\,\mathrm{d}\xi$$

$$i(t) = C\frac{du(t)}{dt}$$

$$u(t) = u(0) + \frac{1}{C}\int_0^t i(\xi)d\xi$$

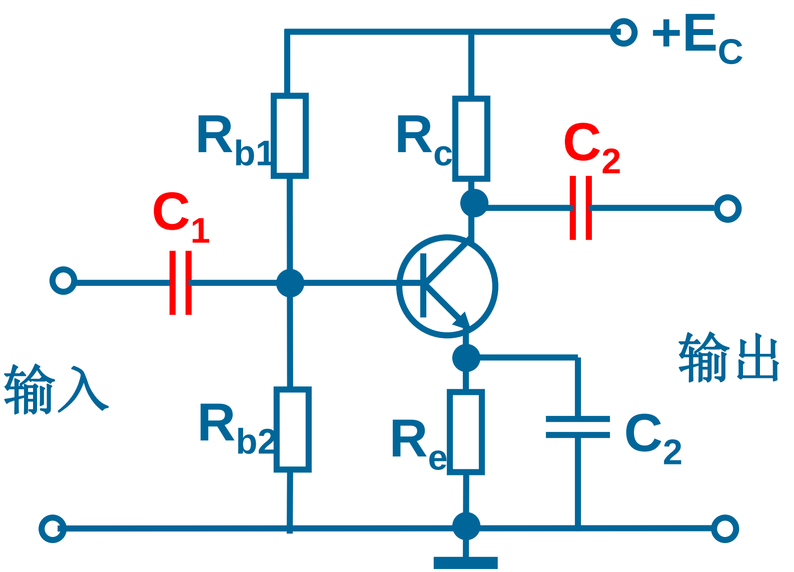


3、Character

(1) dynamic character

Isolate DC

（如放大器电路中电容的耦合作用）



$$i(t) = C\frac{du(t)}{dt} \qquad u(t) = u(0) + \frac{1}{C}\int_0^t i(\xi)d\xi$$

(2) memory

Example: a timer

0≤t ≤1

$u(t)=u(0)+5\times10^5\int_0^t 10^{-6}d\xi \quad = 0.5t$

$u(1)=0.5V$

1≤t ≤2

$u(t)=u(1)+5\times10^5\int_0^t 0d\xi$

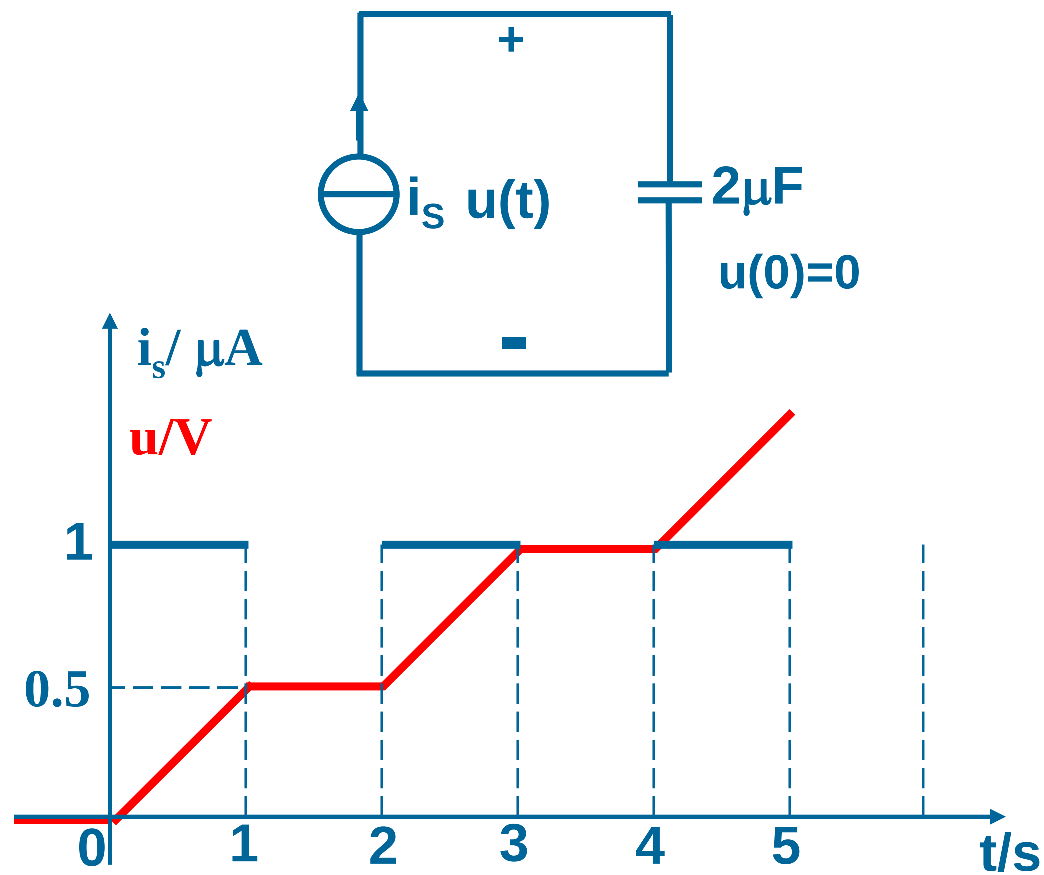
$=0.5V$

2≤t ≤3

$u(t)=u(2)+5\times10^5\int_2^t 10^{-6}d\xi$

$=0.5+0.5(t-2)$

$u(3)=1V$



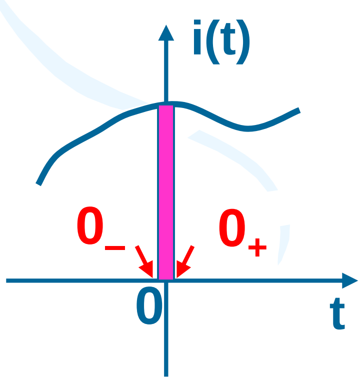
$$i(t) = C\frac{du(t)}{dt} \qquad u(t) = u(0) + \frac{1}{C}\int_0^t i(\xi)d\xi$$

(3) **the voltage across a capacitor is continuous.**

i(t) is bounded

如果i(t)在任一时间都有界，则u(t)在任一时间的变化都是连续的。即在任一时间，电容电压都不可能**即时地**从一个值跃变到另一个值。

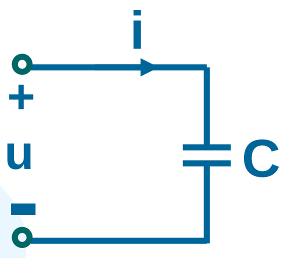
特别，如果在t=0时有界，



则 $u(0_+) = u(0_-) + \frac{1}{C}\int_{0_-}^{0_+} i(t)dt$

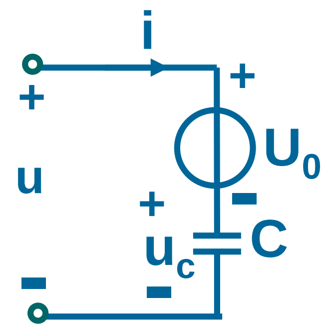
$$u(0_+) = u(0_-)$$

(4) The capacitor with initial voltage



$u(0)=U_0$

$$u(t)=U_0+\frac{1}{C}\int_0^t i(\xi)d\xi$$



$u_c(0)=0$

$$i(t) = C\frac{du(t)}{dt} \qquad u(t) = u(0) + \frac{1}{C}\int_0^t i(\xi)d\xi$$

Summary:

1. Capacitors have memory.

2. A capacitor is an open circuit to dc.

3. The voltage on a capacitor cannot change abruptly as long as the current remains bounded.

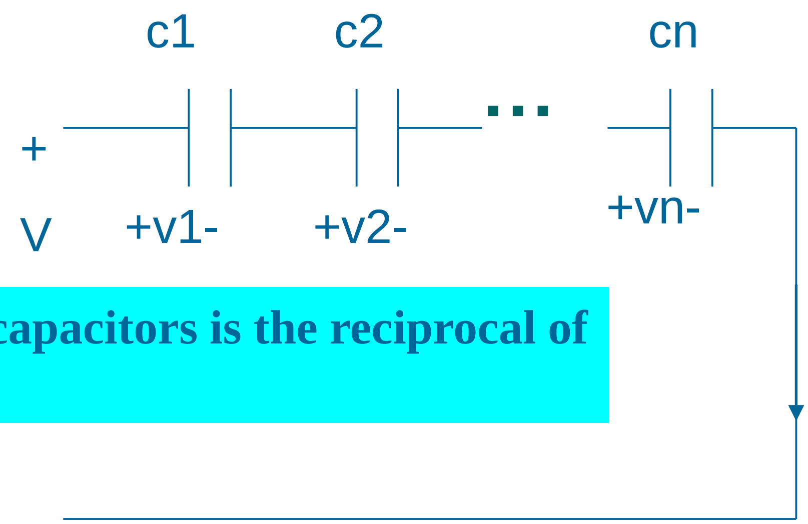
The instantaneous power delivered to the capacitor is:

$$p = vi = cv\frac{dv}{dt}$$

The energy stored in the capacitor is:

$$w = \int_{-\infty}^{t} p\,dt = c\int_{-\infty}^{t} v\frac{dv}{dt}\,dt = c\int_{-\infty}^{t} v\,dv = \frac{1}{2}cv(t)^2\Big|_{t=-\infty}^{t}$$

6. 2 series capacitors



The equivalent capacitance of *n* series_connected capacitors is the reciprocal of the sum of the individual capacitances.

V=v1+v2+…+vn

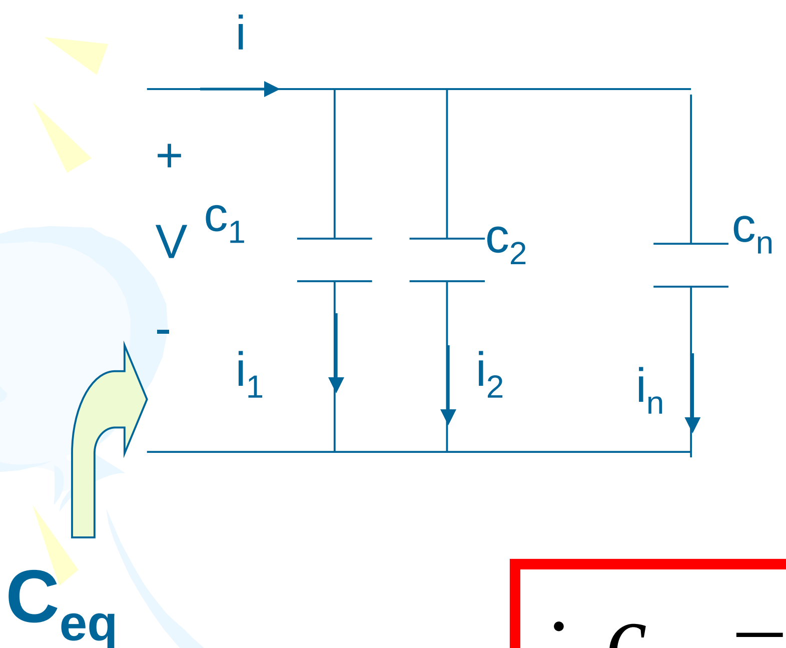
$$v=\frac{1}{c_1}\int_{t_0}^{t} i(t)dt+v_1(t_0)+\frac{1}{c_2}\int_{t_0}^{t} i(t)dt+v_2(t_0)+\cdots+\frac{1}{c_n}\int_{t_0}^{t} i(t)dt+v_n(t_0)$$

$$=(\frac{1}{c_1}+\frac{1}{c_2}+\cdots+\frac{1}{c_n})\int_{t_0}^{t} i(t)dt+v_1(t_0)+v_2(t_0)+\cdots+v_n(t_0)$$

$$=\frac{1}{c_{eq}}\int_{t_0}^{t} i(t)dt+v(t_0)$$

$$\boxed{\frac{1}{c_{eq}}=\frac{1}{c_1}+\frac{1}{c_2}+\cdots+\frac{1}{c_n}}$$

6. 3 parallel capacitors



i=i1+i2+i3+…+in

$$i = c_1 \frac{dv}{dt} + c_2 \frac{dv}{dt} + \cdots + c_n \frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{n} c_k\right) \frac{dv}{dt} = c_{eq} \frac{dv}{dt}$$

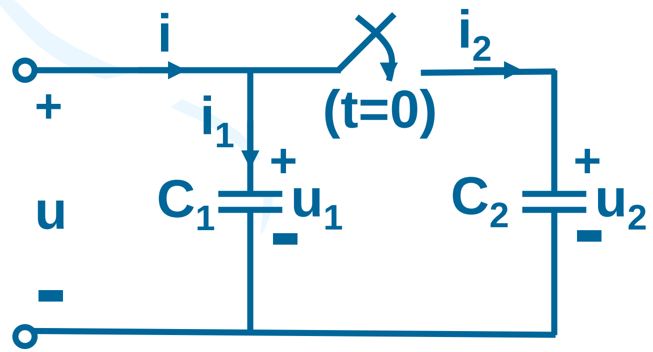
$$\boxed{\therefore c_{eq} = c_1 + c_2 + \cdots + c_n}$$

The equivalent capacitance of *n* parallel_connected capacitors is the sum of the individual capacitances.

Discussion: Parallel capacitors with initial voltages:

情况一　并联前各电容电压相同

情况二　并联前各电容电压不同

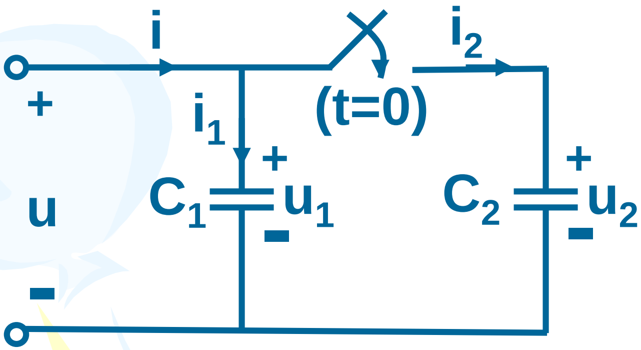


$u_1(0_-) \neq u_2(0_-)$

$u_1(0_+)=u_2(0_+)=u(0_+)$

$$i(t) = C\frac{du(t)}{dt} \qquad u(t) = u(0) + \frac{1}{C}\int_0^t i(\xi)d\xi$$

Conservation of the charge:

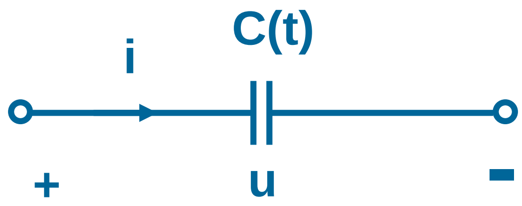


$$\sum_{k=1}^{m} q_k(0_+) = \sum_{k=1}^{m} q_k(0_-)$$

$$(C_1+C_2)u(0_+)=C_1u_1(0_-)+ C_2u_2(0_-)$$

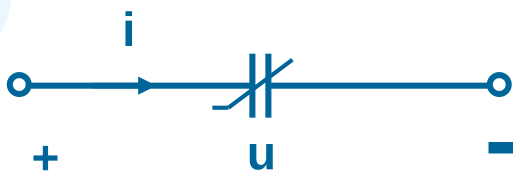
When the switch is closed, some charge is dumped from one capacitor to another instantaneously, this implies that an impulse of current flows from one capacitor to another.

Nonlinear,time-varying capacitors:



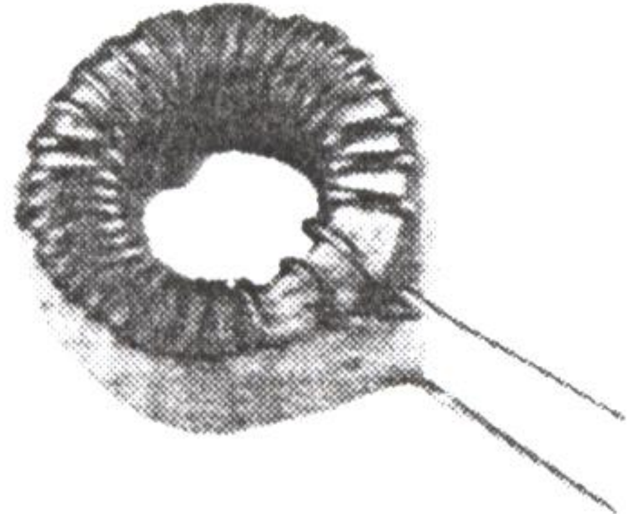
$$q=C(t)u$$

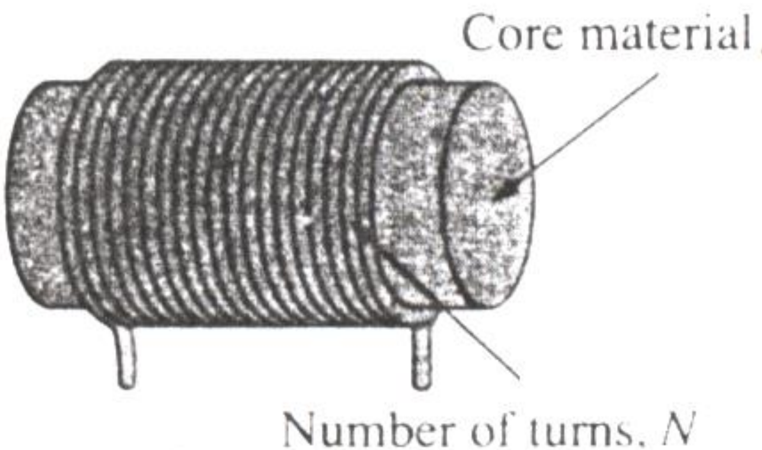
$$i=\frac{dq}{dt}=C(t)\frac{du}{dt}+u\frac{dC(t)}{dt}$$



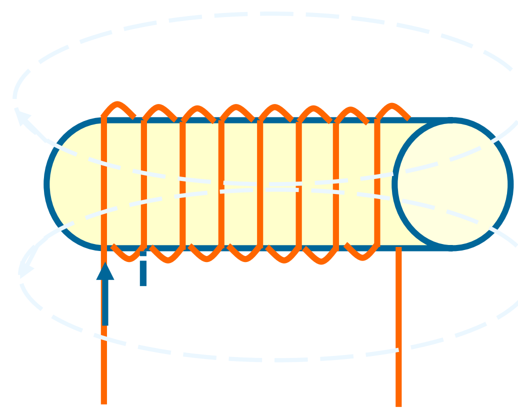
$$q = f(u)$$

$$i=\frac{dq}{dt}=\frac{df(u)}{du}\frac{du}{dt}$$





An practical inductor is usually formed into a cylindrical coil with many turns of conducting wires.



An inductor is a passive element designed to store energy in its magnetic field.

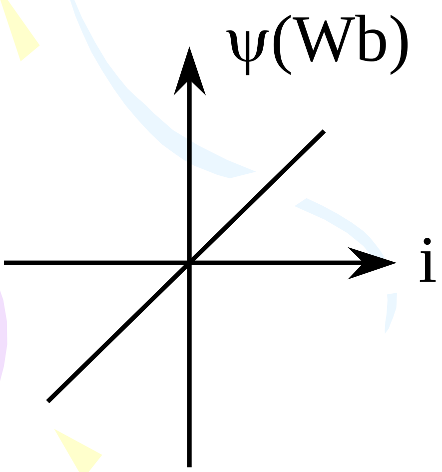
inductors are widely used in power supplies, transformers, radios, TVs, radars, and electric motors.

The element called an inductor is an idealization of the physical inductors.

6.4 INDUCTOR

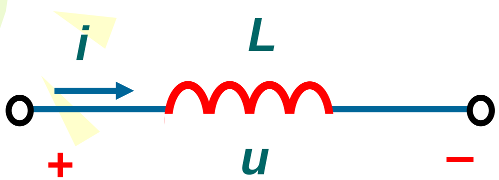
1. Circuit theory definition: a two_terminal element will be called an inductor if at any time t, its flux Ψ(t) and its current I(t) satisfy a relation defined by a curve in the Ψ_I Plane.

2 Characteristics of the linear time_invariant inductors



$\Psi(t)=Li(t)$

(1). Linear inductor



$$L \overset{\text{def}}{=} \frac{\psi}{i}$$

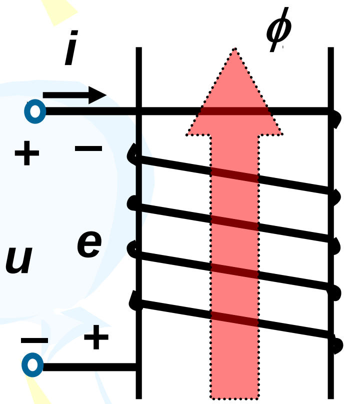
$\psi = N\phi$ *flux*(电感线圈的磁链)

L *inductance*(自感系数)

unit：H (Henry）亨（利）

Joseph Henry (1797-1878)，

(2). VCR of the linear inductors：



Faraday's induction law:

$$u(t) = \frac{d\psi}{dt}$$

$$= L\,\frac{di(t)}{dt}$$

(2)、VCR

$$u(t)=\frac{d\psi}{dt}=L\frac{di(t)}{dt}$$

$$i(t)=i(0)+\frac{1}{L}\int_0^t u(\xi)d\xi$$

(3) 、characters

*dynamic

*memory

* the current is continuous as long as the voltage is bounded.

$$u(t)=L\frac{di(t)}{dt} \qquad i(t)=i(0)+\frac{1}{L}\int_0^t u(\xi)d\xi$$

（3） the current through an inductor is continuous

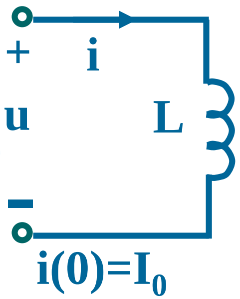
u(t) is bounded

Specially， if u(t)is bounded at t=0，

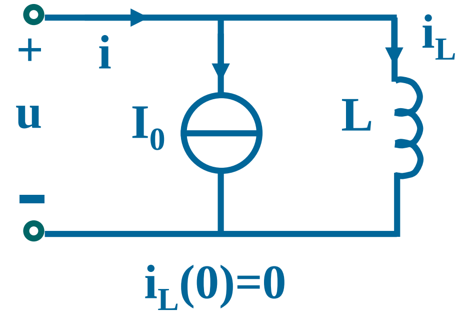
$$\boxed{i(0_+)= i(0_-)}$$

$$u(t)=L\frac{di(t)}{dt} \qquad i(t)=i(0)+\frac{1}{L}\int_0^t u(\xi)d\xi$$

(4) An inductor with initial current



$$i(t)=i(0)+\frac{1}{L}\int_0^t u(\xi)d\xi$$



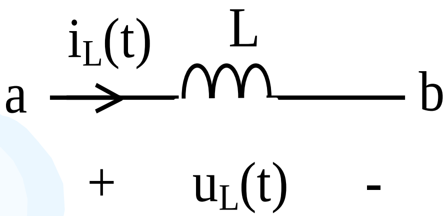
The instantaneous power delivered to the inductor is:

$$p = vi = L\frac{di}{dt}i$$

The energy stored in the capacitor is:

$$w = \int_{-\infty}^{t} p\,dt = L\int_{-\infty}^{t} i\frac{di}{dt}dt = L\int_{-\infty}^{t} i\,di = \frac{1}{2}Li(t)^2\Big|_{t=-\infty}^{t}$$

SUMMARY:



$$\boxed{u_L(t) = L\frac{di_L(t)}{dt}}$$

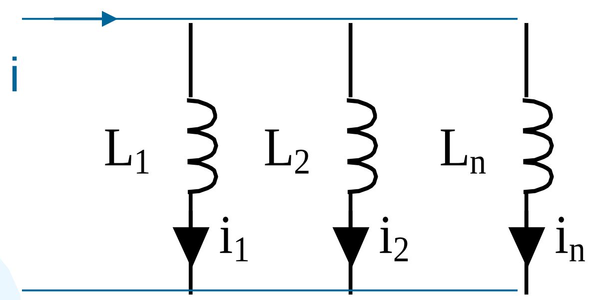
$$i = \frac{1}{L}\int_{-\infty}^{t} v(t) = \frac{1}{L}\int_{t_0}^{t} v(t) + i(t_0)$$

Remark: 1). Inductors have memory.

2). The inductors look like a short circuit to dc.

3) The current in an inductor cannot change abruptly as long as the voltage across it remains bounded.

6.5 parallel inductors



L_{eq}

The equivalent inducance of n parallel_connected inductors is the sum of the reciprocals of the individual capacitances.

i=i1+i2+i3+…+in

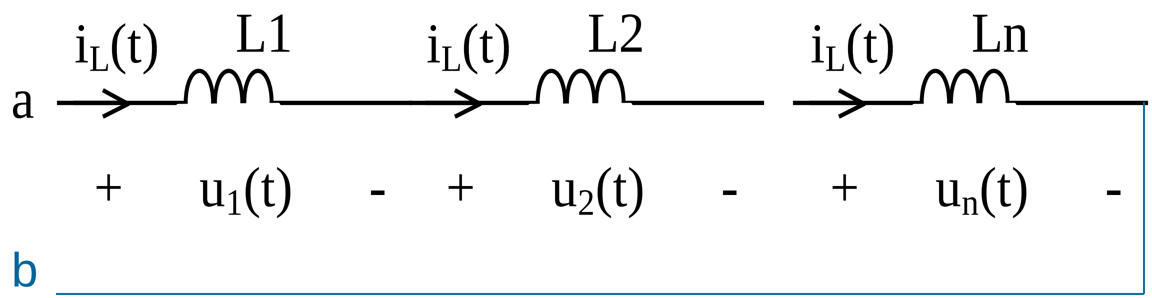
$$i = \frac{1}{L_1}\int_{t_0}^{t} v(t)dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v(t)dt + i_2(t_0) + \cdots + \frac{1}{L_n}\int_{t_0}^{t} v(t)dt + i_n(t_0)$$

$$= (\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n})\int_{t_0}^{t} v(t)dt + i_1(t_0) + i_2(t_0) + \cdots + i_n(t_0)$$

$$= \frac{1}{L_{eq}}\int_{t_0}^{t} i(t)dt + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$

6.6 Series Inductors



The equivalent inductance of n series_connected inductors is the sum of the individual inductances.

$$v = L_1 \frac{di_l}{dt} + L_2 \frac{di_l}{dt} + \cdots + L_n \frac{di_l}{dt}$$

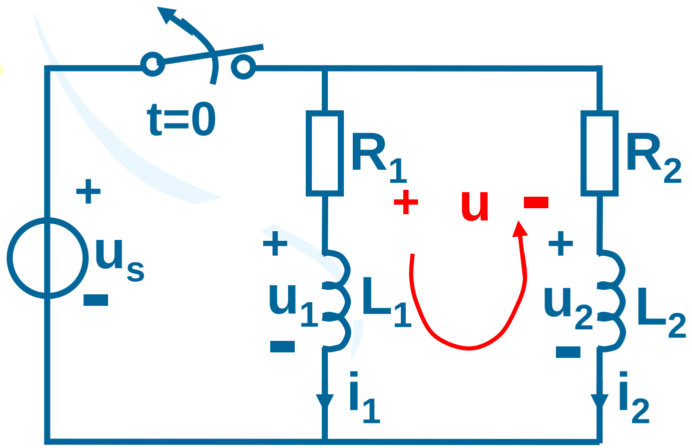
$$= \left(\sum_{k=1}^{n} L_k\right) \frac{di_L}{dt} = L_{eq} \frac{di_L}{dt}$$

$$\boxed{\therefore L_{eq} = L_1 + L_2 + \cdots + L_n}$$

A DISSCUSSION :
SERIES INDUCTORS WITH INITIAL CURRENTS:

Case 1：inductors carry the same initial current

Case 2 ： initial currents are not equal.



$i_1(0\text{-}) \neq i_2(0\text{-})$

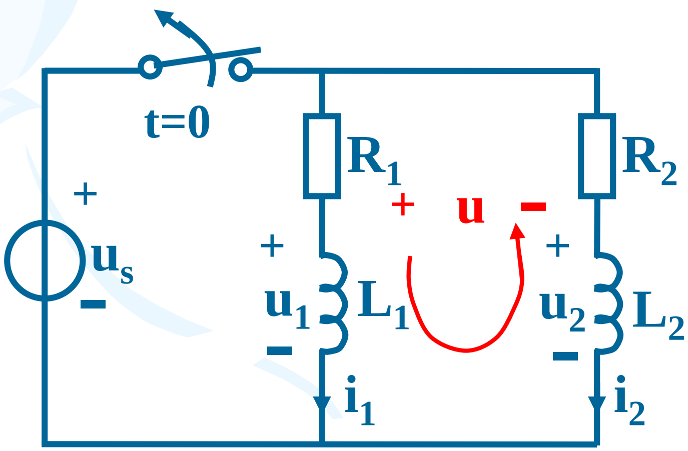
$i_1(0_+) = -\,i_2(0_+)$

Conservation of flux:

irrespective of the manner in which the connection is made for the m inductors, the total flux remains fixed.

$$\sum_{k=1}^{m} \psi_k(0_+) = \sum_{k=1}^{m} \psi_k(0_-)$$

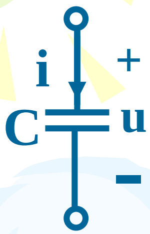
（m为回路包含电感元件的总数）



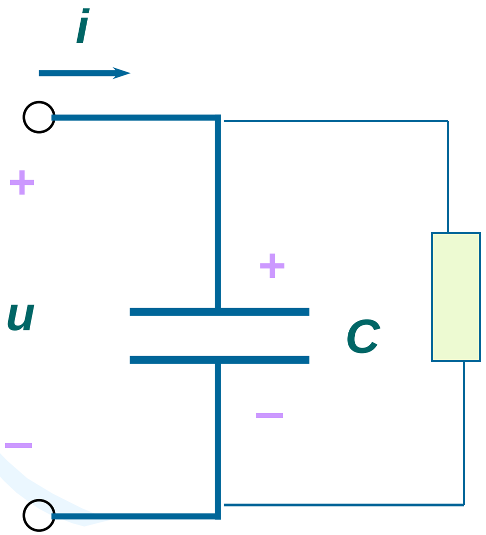
$L_1 i_1(0_+) - L_2 i_2(0_+) =$
$L_1 i_1(0_-) - L_2 i_2(0_-)$

$(L_1+L_2) i_1(0_+) =$
$L_1 i_1(0_-) - L_2 i_2(0_-)$

Comparisons between L,C

元件	约束方程	电压-电流关系		连续性	储存的能量
	$q=C\,u$	$i(t)=C\dfrac{du(t)}{dt}$	$u(t)=u(0)+\dfrac{1}{C}\int_0^t i(\xi)d\xi$	电压	$\varepsilon(t)=0.5Cu^2(t)$
	$\psi=L\,i$	$u(t)=L\dfrac{di(t)}{dt}$	$i(t)=i(0)+\dfrac{1}{L}\int_0^t u(\xi)d\xi$	电流	$\varepsilon(t)=0.5Li^2(t)$

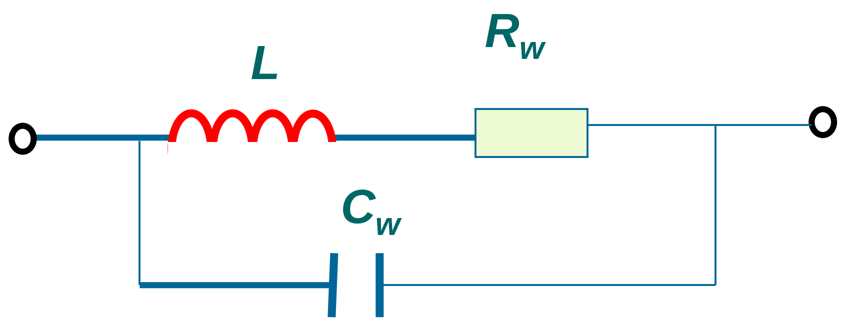
A real, nonideal capacitor:



Leakage resistance

Leakage resistance may be as high as 100MΩ.

A real, nonideal inductor:



Winding resistance *Rw* is usually very small

Winding capacitance *Cw* is usually very small except at high frequencies.