What's next?

（1）Frequency characteristics; Resonance

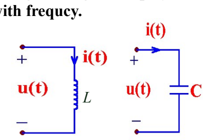
（2）Magnetically coupled circuits; Transformers

（3）Three-Phase circuits;

（4）Periodic, nonsinusoidal excitations

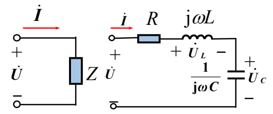
Frequency response(频率响应)

The frequency response of a circuit is the variation in its behavior with change in singnal frequcy, usally the variation of amplitude and phase with frequcy.



$X_L = \omega L \quad X_C = -\frac{1}{\omega C}$

Amplitude and phase of impedance variate with frequcy.



$Z = R + j(\omega L - \frac{1}{\omega C})$

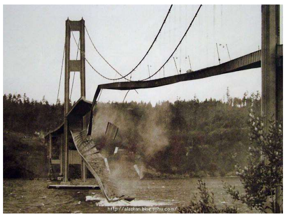
$|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

amplitude response 幅频特性

$\varphi = \arctan \frac{\omega L - \frac{1}{\omega C}}{R}$

phase response 相频特性

RESONANCE (谐振)



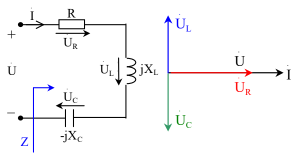
虎门大桥 2020

Tacoma Narrows Bridge, USA, 1940

RESONANCE (谐振)

Resonance is a condition in an RLC circuit in which the capacitive and inductive reactance are equal in magnitude, thereby resulting in a purely resistive impedance.

1. Series Resonance（串联谐振）



$Z = R + j(X_L - X_C) = R + jX$

(1) $\omega L > 1/\omega C$ inductive circuit

(2) $\omega L < 1/\omega C$ capacitive circuit

(3) $\omega L = 1/\omega C$ resistance circuit

$X = X_L - X_C = 0$

Resonance results when the imaginary part of impedance Z is zero.

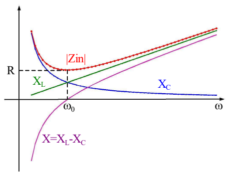
Resonant frequency

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

Notice the characteristics at Series Resonance:

(1) $\dot{U}$ **and** $\dot{I}$ **in phase, cos φ = 1.**

(2) $Z = R$**,** $|Z|$ **reaches its minimum.**



Characteristic impedance (特征阻抗):

$$\rho = \omega_0 L = \frac{1}{\omega_0 C}\ , \quad or \quad \rho = \sqrt{\frac{L}{C}}\ , \quad \Omega$$

Quality factor of a series resonant circuit (品质因数):

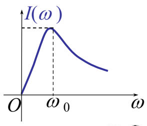
$$Q = \frac{\omega_0 L}{R} = \frac{\rho}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \text{dimensionless(无量纲)}$$

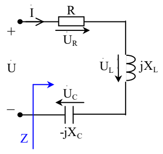
Notice the characteristics at Series Resonance:

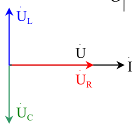
(3) ***I*** **reaches its maximum.**

(4) The LC series combination acts like a short circuit, and the entire voltage is across R.

The inductor voltage and capacitor voltage can be much more than the source voltage.







$$Q = \frac{\omega_0 L}{R}$$

$$Q = \frac{1}{\omega_0 CR}$$

$$\dot{U}_L = j\omega_0 L\dot{I}_0\ , \quad \dot{U}_C = -j(\frac{1}{\omega_0 C})\dot{I}_0 \qquad \dot{U}_L = j\omega_0 L\dot{I}_0 = j\omega_0 L\frac{\dot{U}}{R} = jQ\dot{U}$$

$$\dot{U}_X = \dot{U}_L + \dot{U}_C = 0 \qquad \dot{U}_C = \frac{\dot{I}_0}{j\omega_0 C} = -j\frac{\dot{U}_0}{\omega_0 CR} = -jQ\dot{U}$$

$$\therefore \dot{U} = \dot{U}_R$$

Voltage resonance

Notice the characteristics at Series Resonance:

(5) Power at Series Resonance

Power absorbed by load

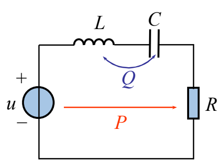
$$P = RI^2 = U^2/R$$

$$\boldsymbol{Q_L = \omega_0 L I^2} \qquad \boldsymbol{Q_C = -\frac{1}{\omega_0 C}I^2} \qquad \boldsymbol{Q = Q_L + Q_C = 0}$$

Power developed by source

$$\boldsymbol{P = UI\cos\varphi = RI^2}$$

$$\boldsymbol{Q = UI\sin\varphi = 0}$$



Energy change between inductor and capacitor,but NOT between the source and the reactive loads.

Notice the characteristics at Series Resonance:

(6) Energy Storage at Series Resonance

$$u_S(t) = U_{sm}\sin(\omega_0 t) \qquad I_0(t) = I_m\sin(\omega_0 t) = \frac{U_{sm}}{R}\sin(\omega_0 t)$$

Electric-field energy

$$u_C(t) = Qu_S(t\text{-}90^o) = QU_{sm}\sin(\omega_0 t\text{-}90^o)$$

$$= -QRI_m\cos(\omega_0 t)$$

$$w_C(t) = \frac{1}{2}Cu_C^2(t)$$

$$= \frac{1}{2}C(\sqrt{\frac{L}{C}})^2 I_m^2\cos^2(\omega_0 t) \qquad Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{= \frac{1}{2}LI_m^2\cos^2(\omega_0 t)}$$

Magnetic-field energy

$$w_L(t) = \frac{1}{2}LI_0^2(t)\ \boxed{= \frac{1}{2}LI_m^2\sin^2(\omega_0 t)} \qquad \therefore W_{Lm} = W_{Cm}$$

$$w_C(t) + w_L(t) = \frac{1}{2}LI_m^2 = \frac{1}{2}CU_{Cm}^2 = CQ^2U^2$$

Notice the characteristics at Series Resonance：

（7）Physical significance of *Q*

Rate of inductor voltage or capacitor voltage to the source

$$\dot{U}_L = j\omega_0 L\dot{I}_0 = jQ\dot{U}$$

Rate of Electric-field and Magnetic-field energy to the energy dissipated by the circuit in one period

$$Q = \frac{\omega_0 L}{R} = \omega_0 \cdot \frac{LI_0^2}{RI_0^2} = 2\pi\frac{LI_0^2}{RI_0^2 T_0}$$

The third one...

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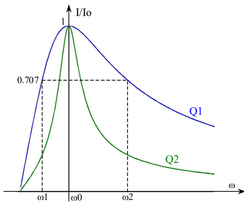
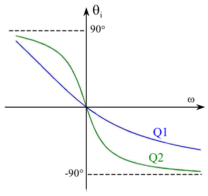
Frequency response of Series Resonance Current

Selectivity（选择性）

The bandwidth（带宽） B is defined as the difference between the two half-power frequencies. $B = \omega_2 - \omega_1$ $\boxed{I/I_0 = 1/\sqrt{2}}$



amplitude response($Q_2 > Q_1$)



phase response($Q_2 > Q_1$)

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Frequency response of Series Resonance Current

Normalization（归一化处理）：universal resonance curve（通用谐振曲线 ）

$$\omega \to \frac{\omega}{\omega_0} = \eta, \quad I(\omega) \to \frac{I(\omega)}{I(\omega_0)} = \frac{I}{I_0}$$

$$\frac{I(\omega)}{I(\omega_0)} = \frac{U/|Z|}{U/R} = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{1}{\sqrt{1 + (\frac{\omega L}{R} - \frac{1}{\omega RC})^2}}$$

$$= \frac{1}{\sqrt{1 + (\frac{\omega_0 L}{R}\cdot\frac{\omega}{\omega_0} - \frac{1}{\omega_0 RC}\cdot\frac{\omega_0}{\omega})^2}} = \frac{1}{\sqrt{1 + (Q\cdot\frac{\omega}{\omega_0} - Q\cdot\frac{\omega_0}{\omega})^2}}$$

$$\frac{I}{I_0} = \frac{1}{\sqrt{1 + Q^2(\eta - \frac{1}{\eta})^2}}$$

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Frequency response of Series Resonance Current

Normalization（归一化处理）：universal resonance curve（通用谐振曲线 ）

$$\eta_1 = \frac{\omega_1}{\omega_0}, \quad \eta_2 = \frac{\omega_2}{\omega_0}, \quad \omega_2 > \omega_1.$$

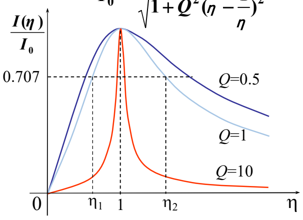
$$\frac{I}{I_0} = \frac{1}{\sqrt{1 + Q^2(\eta - \frac{1}{\eta})^2}}$$

$$\frac{1}{\sqrt{1 + Q^2(\eta - \frac{1}{\eta})^2}} = \frac{1}{\sqrt{2}}$$

$$\eta_1 = \frac{-1 + \sqrt{1 + 4Q^2}}{2Q}$$

$$\eta_2 = \frac{1 + \sqrt{1 + 4Q^2}}{2Q}$$

$$\Delta\eta = \eta_2 - \eta_1 = \frac{1}{Q} \quad \boxed{Q = \frac{\omega_0}{\Delta\omega}}$$



universal resonance curve

The third aspect of Physical significance of ***Q***

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Frequency response of Series Resonance Voltage

Normalization（归一化处理）

$$U_C(\omega)=\frac{I}{\omega C}=\frac{U}{\omega C\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}$$

$$=\frac{QU}{\sqrt{\eta^2+Q^2(\eta^2-1)^2}}$$

$$\frac{d}{d\eta}[\eta^2+Q^2(\eta^2-1)^2]=0$$

$$\eta_{C1}=0$$

$$\eta_{C2}=\sqrt{1-\frac{1}{2Q^2}}<1$$

$$U_{Cm}(\omega_{cm})=\frac{QU}{\sqrt{1-\frac{1}{4Q^2}}}>QU$$

$$U_L(\omega)=\omega LI=\omega L\cdot\frac{U}{|Z|}=\frac{\omega LU}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}$$

$$=\frac{QU}{\sqrt{\frac{1}{\eta^2}+Q^2(1-\frac{1}{\eta^2})^2}}$$

$Q>\frac{1}{\sqrt{2}}$ ω_{cm}存在

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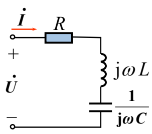
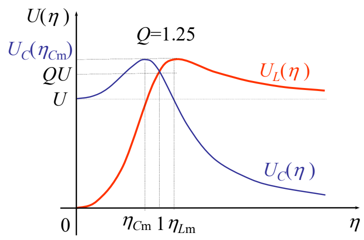
Frequency response of Series Resonance Voltage





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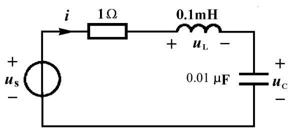
例 1 电路如图12-18所示。已知 $u_S(t)=10\sqrt{2}\cos\omega t$ V

求: (1) 频率ω为何值时，电路发生谐振。

(2)电路谐振时, U_L和U_C为何值。



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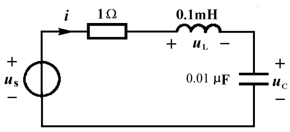

解: (1)电压源的角频率应为

$$\omega=\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{10^{-4}\times10^{-8}}}\text{rad/s}=10^6\text{rad/s}$$

(2)电路的品质因数为

$$Q=\frac{\omega_0 L}{R}=100$$

则

$$U_L=U_C=QU_S=100\times10\text{V}=1000\text{V}$$

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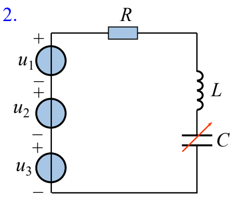
例2.



一接收器的电路参数为:

L=250μH, R=20Ω,

$U_1=U_2=U_3$=10μV,

f_0=820 kHz.

$$820\times10^3=\frac{1}{2\pi\sqrt{LC}}$$

请问当电容调至C=150pF时谐振，接收到那个电台的信号？

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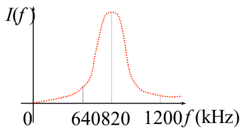
	电台1	电台2	电台3
f(kHz)	820	640	1026
ωL（Ω）	1290	1000	1612
$\frac{1}{\omega C}$（Ω）	1290	–1660	1034
X	0	– 660	577
$I=U/\|Z\|$ (μA)	I_0=0.5	I_1=0.015	I_2=0.017



∴收到台820kHz的节目。

* 选择性与Q有关

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例3 欲接收载波频率为10MHz的某短波电台的信号，试设计接收机输入谐振电路的电感线圈。要求带宽Δf=100kHz，C=100pF。

解：由

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