Transmission of energy(传送功率)

$$Z_l = \frac{(\omega M)^2}{Z_{22}} = \frac{\omega^2 M^2}{R_{22}+jX_{22}} = \frac{\omega^2 M^2 R_{22}}{R_{22}^2+X_{22}^2} - \frac{j\omega^2 M^2 X_{22}}{R_{22}^2+X_{22}^2}$$

$$= R_l + jX_l$$

$$R_l = \frac{\omega^2 M^2 R_{22}}{R_{22}^2+X_{22}^2}$$

$R_l > 0$, absorbing power, providing by primary coil

Power developed by source = Power absorbed by load $= I_1^2 R_1 + I_1^2 R_l$

absorbing by primary coil

absorbing by secondary coil, transfering by mutual inductance

$$I_1^2 R_l^2 = I_2^2 R_{22}$$

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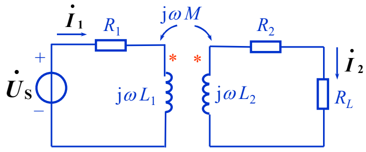
Example 1. L_1=3.6H , L_2=0.06H , M=0.465H , R_1=20Ω , R_2=0.08Ω , R_L=42Ω , ω=314rad/s,



$\dot U = 115\angle 0°$ V

Find $\dot I_1$, $\dot I_2$.

Solution 1：Mesh Analysis

$$\begin{cases} \dot I_1(R_1 + j\omega L_1) - \dot I_2 j\omega M = \dot U_S \\ \dot I_2(R_2 + R_L + j\omega L_2) - \dot I_1 j\omega M = 0 \end{cases}$$

$$\dot I_1 = 110.5\angle(-64.9°)\ \text{A} \qquad \dot I_2 = 0.351\angle 1°\ \text{A}$$

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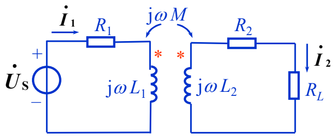
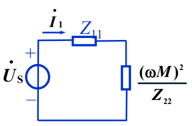
Solution 2：Primary coil equivalent circuit of linear transformer





$$Z_{11} = R_1 + j\omega L_1 = 20 + j1130.4\Omega$$

$$Z_{22} = R_2 + R_L + j\omega L_2 = 42.08 + j18.85\Omega$$

$$Z_l = \frac{X_M^2}{Z_{22}} = 464\angle(-24.1°)\ \Omega$$

$$\dot I_1 = \frac{\dot U_S}{Z_{11} + Z_l} = 110.5\angle(-64.9°)\ \text{A} \qquad \dot I_2 = \frac{j\omega M \dot I_1}{Z_{22}} = 0.351\angle 1°\ \text{A}$$

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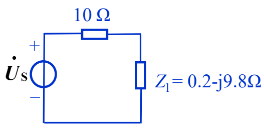
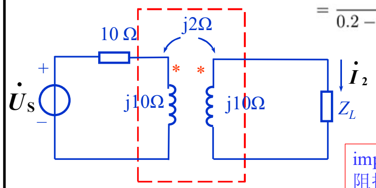
Example 2. U_S= 20V, add a transformer between the source and load, proof that the circuit transfers maximum power transfer, Find P_l.



Solution:

Reflected impedance

$$Z_l = \frac{(\omega M)^2}{Z_{22}} = \frac{4}{0.2 - j9.8 + j10} = 10\text{-}j10$$



$$Z_l = Z_{11}^*$$

$$P_l = \frac{20^2}{4\times 10} = 10W$$

impedance matching
阻抗匹配

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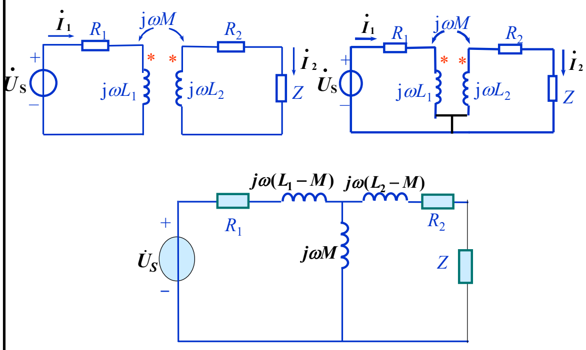
T equivalent circuit of linear transformer



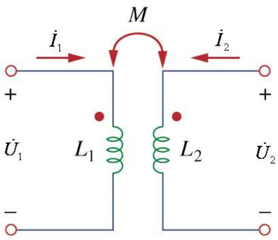
13.5 The Ideal Transformer

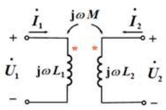
An ideal transformer is the limiting case of two coupled inductors where the inductances approach infinitely and coupling is perfect.

$$\boxed{K=\frac{M}{\sqrt{L_1L_2}}}\qquad M=\sqrt{L_1L_2}$$

$$\dot{U}_1=j\omega L_1\dot{I}_1+j\omega M\dot{I}_2$$

$$\dot{U}_2=j\omega M\dot{I}_1+j\omega L_2\dot{I}_2$$





$$\begin{cases}\dot{U}_1=j\omega L_1\dot{I}_1+j\omega M\dot{I}_2\\ \dot{U}_2=j\omega L_2\dot{I}_2+j\omega M\dot{I}_1\end{cases}$$

$$\dot{I}_1=\frac{\dot{U}_2-j\omega L_2\dot{I}_2}{j\omega M}$$

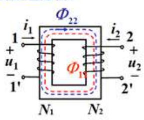
$$\dot{U}_1=\frac{L_1}{M}(\dot{U}_2-j\omega L_2\dot{I}_2)+j\omega M\dot{I}_2=\frac{L_1}{M}\dot{U}_2$$

全耦合时： $M=\sqrt{L_1L_2}$ ， $k=1$

$$\boxed{\frac{\dot{U}_1}{\dot{U}_2}=\frac{L_1}{M}=\frac{M}{L_2}=\sqrt{\frac{L_1}{L_2}}=\frac{N_1}{N_2}=n}$$

n称为变比

从磁通分析:



$$u_1=N_1\frac{d\Phi_1}{dt},\quad u_2=N_2\frac{d\Phi_2}{dt}$$

$$\Phi_1=\Phi_2=\Phi_{11}+\Phi_{22}$$

$$\frac{u_1}{u_2}=\frac{N_1}{N_2}=n$$

$$\dot{U}_1=j\omega L_1\dot{I}_1+j\omega M\dot{I}_2$$

$$\Rightarrow\ \dot{I}_1=\frac{\dot{U}_1-j\omega M\dot{I}_2}{j\omega L_1}=\frac{\dot{U}_1}{j\omega L_1}-\frac{j\omega M}{j\omega L_1}\dot{I}_2=\frac{\dot{U}_1}{j\omega L_1}-\frac{1}{n}\dot{I}_2$$

全耦合变压器的电压、电流关系:

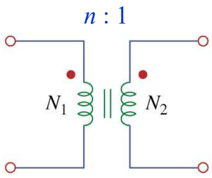
$$\begin{cases}\dot{U}_1=n\dot{U}_2\\ \dot{I}_1=\dfrac{\dot{U}_1}{j\omega L_1}-\dfrac{1}{n}\dot{I}_2\end{cases}$$

A transformer is said to be ideal if it has the following properties:

1. Coils have very large reactances $(L_1 \to \infty, L_2 \to \infty, M \to \infty)$
2. Coupling coefficient is equal to unity (k=1)
3. Primary and secondary coils are lossless $(R_1 = R_2 = 0)$

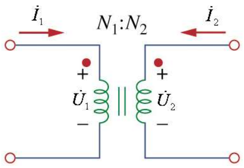
An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances

Circuit symbol for ideal transformer

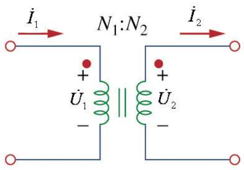


$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$

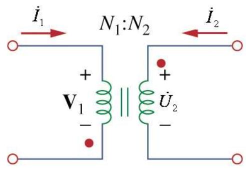
The different relationships between the polarity of the voltages and the direction of the currents for the transformer



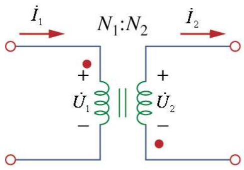
$$\frac{\dot{U}_2}{\dot{U}_1} = \frac{N_2}{N_1} \qquad \frac{\dot{I}_2}{\dot{I}_1} = -\frac{N_1}{N_2}$$



$$\frac{\dot{U}_2}{\dot{U}_1} = \frac{N_2}{N_1} \qquad \frac{\dot{I}_2}{\dot{I}_1} = \frac{N_1}{N_2}$$



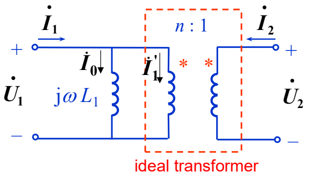
$$\frac{\dot{U}_2}{\dot{U}_1} = -\frac{N_2}{N_1} \qquad \frac{\dot{I}_2}{\dot{I}_1} = \frac{N_1}{N_2}$$



$$\frac{\dot{U}_2}{\dot{U}_1} = -\frac{N_2}{N_1} \qquad \frac{\dot{I}_2}{\dot{I}_1} = -\frac{N_1}{N_2}$$

A unity-coupled transformer by using circuit symbol of ideal transformer

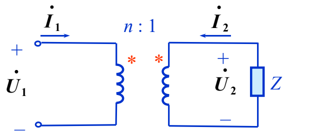
$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = \dfrac{1}{j\omega L_1}\dot{U}_1 - \dfrac{1}{n}\dot{I}_2 \end{cases}$$

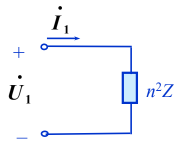


Characteristics of ideal transformer

(1) Impedance transformation

$$\begin{cases} \dot{U}_1 = n\dot{U}_2 \\ \dot{I}_1 = -\dfrac{1}{n}\dot{I}_2 \end{cases}$$



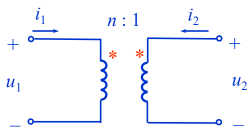


$$\frac{\dot{U}_1}{\dot{I}_1} = \frac{n\dot{U}_2}{-1/n\dot{I}_2} = n^2(-\frac{\dot{U}_2}{\dot{I}_2}) = n^2 Z$$

Characteristics of ideal transformer

(2) Power consumption

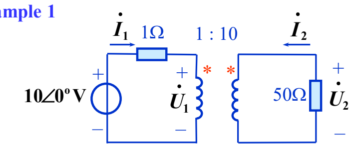
$$\begin{cases} u_1 = nu_2 \\ i_1 = -\dfrac{1}{n}i_2 \end{cases}$$



$$p = u_1 i_1 + u_2 i_2 = u_1 i_1 + \frac{1}{n}u_1 \times (-n i_1) = 0$$

The ideal transformer absorbs no power, which is losslees.

Example 1

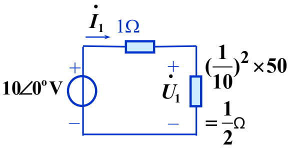


Find: $\dot{U}_2$.

Solution 1: Mesh Analysis

$$\begin{cases} 1 \times \dot{I}_1 + \dot{U}_1 = 10\angle 0^\circ \\ 50\dot{I}_2 + \dot{U}_2 = 0 \\ \dot{U}_1 = \dfrac{1}{10}\dot{U}_2 \\ \dot{I}_1 = -10\dot{I}_2 \end{cases} \xrightarrow{\text{obtained}} \dot{U}_2 = 33.33\angle 0^\circ\ \text{V}$$

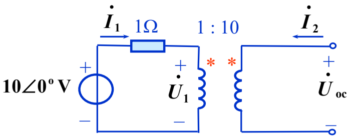
Solution 2：Impedance transformation



$$\dot U_1 = \frac{10\angle 0^\circ}{1+1/2}\times\frac{1}{2} = \frac{10}{3}\angle 0^\circ\ \text{V}$$

$$\dot U_2 = n\dot U_1 = 10\,\dot U_1 = 33.33\angle 0^\circ\ \text{V}$$

Solution 3：Thewenin equivalent circuit

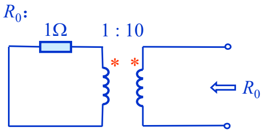


Find $\dot U_{oc}$

$$\because \dot I_2 = 0,\quad \therefore \dot I_1 = 0$$

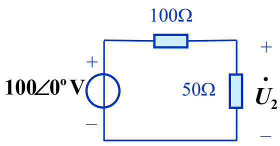
$$\dot U_{oc} = 10\dot U_1 = 10\dot U_S = 100\angle 0^\circ\ \text{V}$$

Find R_0：



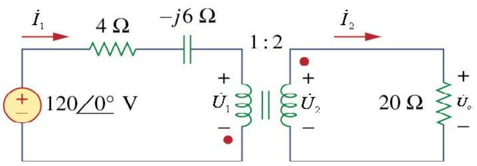
$R_0=10^2\times 1=100\Omega$

Thewenin equivalent circuit



$$\dot U_2 = \frac{100\angle 0^\circ}{100+50}\times 50 = 33.33\angle 0^\circ\ \text{V}$$

Example 2 For the ideal transformer circuit shown in Fig., find : (1) the source current $\dot I_1$,(b) the output $\dot U_o$, and (c) the complex power supplied by the source.



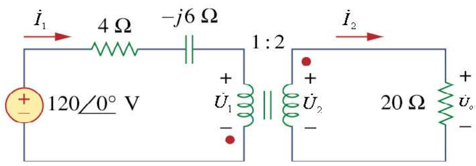
Solution:

(1) The impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

$$\mathbf{Z}_{in} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\underline{/-33.69^\circ}\ \Omega$$

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{\mathbf{Z}_{in}} = \frac{120\underline{/0^\circ}}{10.82\underline{/-33.69^\circ}} = 11.09\underline{/33.69^\circ}\ \text{A}$$



(2) Since both I_1 and I_2 leave the dotted terminals

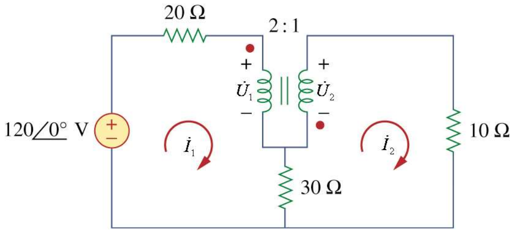
$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\underline{/33.69^\circ}\ \text{A}$$

$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\underline{/213.69^\circ}\ \text{V}$$

(3) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\underline{/0^\circ})(11.09\underline{/-33.69^\circ}) = 1{,}330.8\underline{/-33.69^\circ}\ \text{VA}$$

Example 3 Calculate the power supplied to 10-Ω resistor in the ideal transformer circuit shown in the following Fig.



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Solution:

There is direct connection between the primary and secondary sides due to the 30- resistor. We apply mesh analysis.

For mesh 1,

$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

$$50\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 120$$

For mesh 2.

$$-\mathbf{V}_2 + (10 + 30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

$$-30\mathbf{I}_1 + 40\mathbf{I}_2 - \mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1$$

$$\mathbf{I}_2 = -2\mathbf{I}_1$$

$$\mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$

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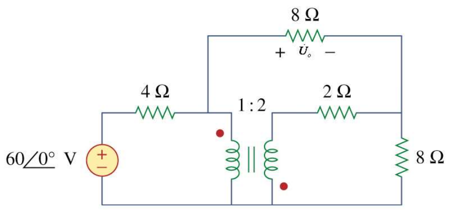
Practice Find $\dot U_o$ in the circuit shown in following Fig.



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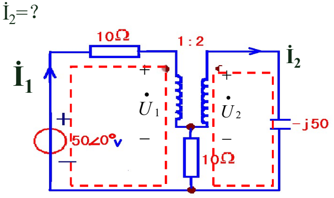
Examle 4 In the circuit shown in following Fig., find $\dot I_1$=? $\dot I_2$=?



Solution **Using mesh analysis:**

$$20\dot I_1 - 10\dot I_2 = 50\angle 0° - \dot U_1 \qquad \dot U_2 = 2\dot U_1 \quad \dot I_2 = \frac{1}{2}\dot I_1$$

$$-10\dot I_1 + (10 - j50)\dot I_2 = \dot U_2 \qquad \therefore \quad \dot I_1 = 2 + j2(A)$$

$$\dot I_2 = 1 + j(A)$$

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Summary

- **Dot Convention.**
- **Decoupling Equivalent Circuit**
  - **series connection** $L = L_1 + L_2 + 2M$
  - ***parallel connection***
  - ***T- connection***
- **Linear Transformer** $Z_l = \frac{(\omega M)^2}{Z_{22}}$
- **Ideal transformer**

$$\begin{cases} u_1 = nu_2 \\ i_1 = -\frac{1}{n} i_2 \end{cases}$$

$n^2 Z$

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