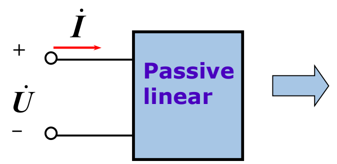
## Impedance（阻抗） and Admittance（导纳）

### 1. Impedance（阻抗）

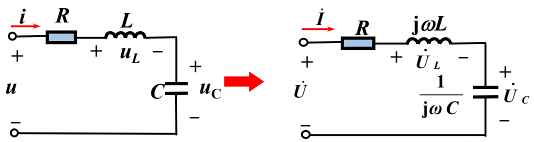



$$Z \overset{\text{def}}{=} \frac{\dot{U}}{\dot{I}} = |Z| \angle \varphi = R + jX$$

$$(\varphi = \psi_u - \psi_i)$$

- Resistor $Z=R$
- Inductor $Z=j\omega L=jX_L$ $\quad X_L=\omega L$
- Capacitor $Z=1/j\omega C=jX_C$ $\quad X_C=-\frac{1}{\omega C}$






$$Z = \frac{\dot{U}}{\dot{I}} = \frac{\dot{U}_R + \dot{U}_C + \dot{U}_L}{\dot{I}} \quad \text{KVL}$$

$$= R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j(\omega L - \frac{1}{\omega C}) \quad X_L = \omega L \quad X_C = -\frac{1}{\omega C}$$

$$= R + j(X_L + X_C)$$

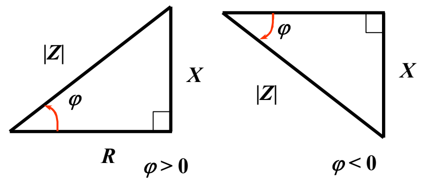
$$= R + jX$$

Resistance(电阻) Reactance(电抗)

$$Z = \frac{\dot{U}}{\dot{I}} = R + jX = |Z| \angle \varphi$$

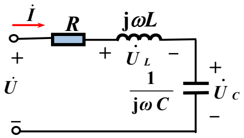
$$\begin{cases} |Z| = \sqrt{R^2 + X^2} = \frac{U}{I} \quad \Omega \\ \varphi = \text{arctg}\frac{X}{R} = \varphi_u - \varphi_i \end{cases}$$

impedance angle(阻抗角)

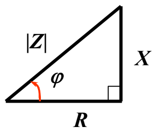



impedance triangle(阻抗三角形)





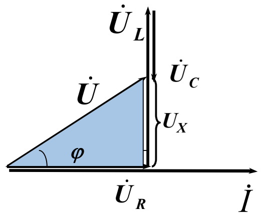

$(\omega L > 1/\omega C)$



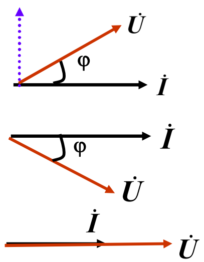

$\varphi > 0$

**For $R$, $L$, and $C$ series circuits**

$Z=R+j(\omega L-1/\omega C)=|Z|\angle\varphi$




similar triangles

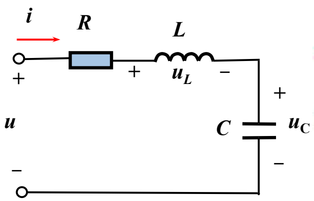



(1) $\omega L > 1/\omega C$, $\varphi > 0$ inductive circuit

(2) $\omega L < 1/\omega C$, $\varphi < 0$ capacitive circuit

(3) $\omega L = 1/\omega C$, $\varphi = 0$ resistance circuit

**Example 5**

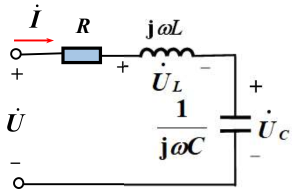



$R=15\ \Omega$, $L=12$ mH, $C=5\ \mu$F,

$u(t) = 100\sqrt{2}\cos(5000t)V$

Find $Z, \dot{I}, \dot{U}_R, \dot{U}_L, \dot{U}_C$, and draw the phase

**Solution:** $\dot{U} = 100\angle 0° V$ $\quad j\omega L = j60\Omega \quad -j\frac{1}{\omega C} = -j40\Omega$



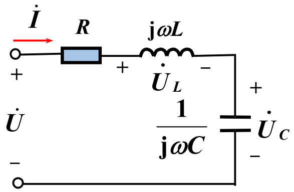

$$Z = R + j\omega L - j\frac{1}{\omega C} = 25\angle 53.1°\Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = 4\angle -53.13° A$$

$$\dot{U}_R = \dot{I}R = 60\angle -53.13° V$$

$$\dot{U}_L = j\omega L\dot{I} = 240\angle 36.87° V$$

$$\dot{U}_C = -j\frac{1}{\omega C}\dot{I} = 160\angle -143.13° V$$



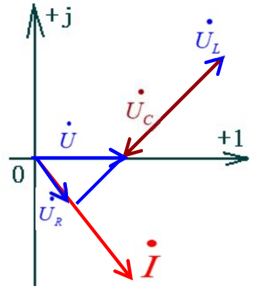

$\dot U = 100\angle 0°V$

$\dot I = \frac{\dot U}{Z} = 4\angle -53.13°A$

$\dot U_R = \dot I R = 60\angle -53.13°V$

$\dot U_L = j\omega L \dot I = 240\angle 36.87°V$

$\dot U_C = -j\frac{1}{\omega C}\dot I = 160\angle -143.13°V$

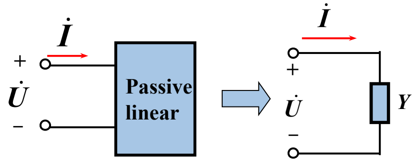



$U_L > U$



## Impedance（阻抗） and Admittance（导纳）

### 2. Admittance（导纳）

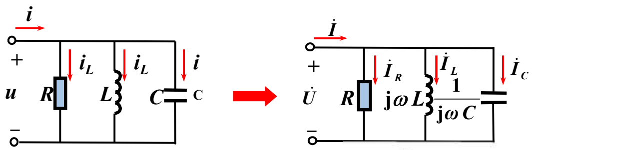



$$Y \overset{\text{def}}{=} \frac{\dot I}{\dot U} = G + jB = |Y|\angle\varphi' \qquad (\varphi' = \psi_i - \psi_u)$$

- Resistor $Y_R = 1/R$
- Inductor $Y_L = \frac{1}{j\omega L} = jB_L$ $\quad B_L = -\frac{1}{\omega L}$
- Capacitor $Y_C = j\omega C = jB_C$ $\quad B_C = \omega C$

$Y = \frac{1}{Z}$






$$\dot Y = \frac{\dot I}{\dot U} = \frac{\dot I_R + \dot I_L + \dot I_C}{\dot U}$$ KCL

$$= G + \frac{1}{j\omega L} + j\omega C \qquad B_L = -\frac{1}{\omega L} \qquad B_C = \omega C$$

$$= G + j(B_L + B_C)$$

$$= G + jB$$

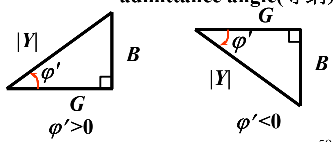
Conductance(电导) Suspectance(电纳

$$Y = \frac{\dot I}{\dot U} = \frac{I\angle\psi_i}{U\angle\psi_u} = \frac{I}{U}\angle\psi_i - \psi_u$$

$$= G + jB = |Y|\angle\varphi'$$

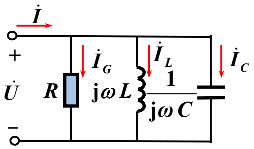
$$\begin{cases} |Y| = \sqrt{G^2 + B^2} = \frac{I}{U} \quad \text{S} \\ \varphi' = \text{arctg}\frac{B}{G} = \varphi_i - \varphi_u \end{cases}$$

admittance angle(导纳角)

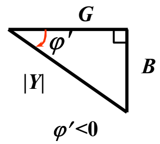


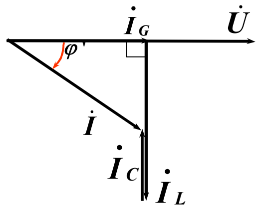

admittance triangle(导纳三角形)






($\omega C < 1/\omega L$, $\varphi'<0$)




similar triangles

$$I = \sqrt{I_G^2 + I_B^2} = \sqrt{I_G^2 + (I_L - I_C)^2}$$

**For *R*, *L*, and *C* parallel circuits**

$Y = G + j(\omega C - 1/\omega L) = |Y|\angle\varphi'$

(1) $\omega C > 1/\omega L$, $B > 0$, $\varphi' > 0$,

capacitive circuit, *i* leads *u*;

(2) $\omega C < 1/\omega L$, $B < 0$, $\varphi' < 0$,

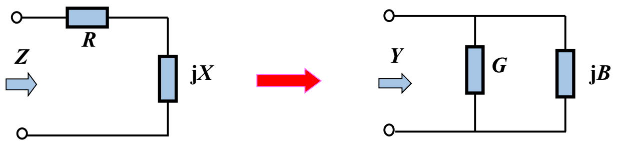
inductive circuit, *i* lags *u*;

(3) $\omega C = 1/\omega L$, $B = 0$, $\varphi' = 0$,

resistance circuit, *i* and *u* in phase.

## Impedance（阻抗） and Admittance（导纳）

### 3. Equivalent relation between Impedance and Admittance




$$Z = R + \mathrm{j}X = |Z|\angle\varphi \quad \Rightarrow \quad Y = G + \mathrm{j}B = |Y|\angle\varphi'$$

$$Y = \frac{1}{Z} = \frac{1}{R+\mathrm{j}X} = \frac{R-\mathrm{j}X}{R^2+X^2} = G + \mathrm{j}B$$

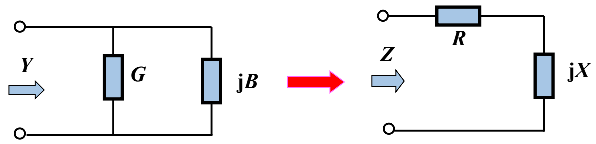
$$\therefore \quad G = \frac{R}{R^2+X^2}, \quad B = \frac{-X}{R^2+X^2}$$

$$|Y| = \frac{1}{|Z|}, \quad \varphi' = -\varphi$$

(1) $G \neq 1/R \quad B \neq 1/X$

(2) inductive circuit，$X>0$，$B<0$






$$Y = G + \mathrm{j}B = |Y|\angle\varphi', \quad Z = R + \mathrm{j}X = |Z|\angle\varphi$$

$$Z = \frac{1}{Y} = \frac{1}{G+\mathrm{j}B} = \frac{G-\mathrm{j}B}{G^2+B^2} = R + \mathrm{j}X$$

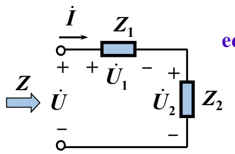
$$\therefore \quad R = \frac{G}{G^2+B^2}, \quad X = \frac{-B}{G^2+B^2}$$

$$|Y| = \frac{1}{|Z|}, \quad \varphi = -\varphi'$$



## Impedance（阻抗） and Admittance（导纳）

### 4. Series and parallel impedances

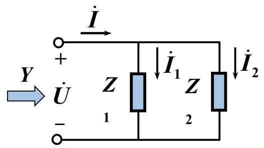



equivalent impedance

$$Z = Z_1 + Z_2$$

voltage divider

$$\dot{U}_1 = \frac{Z_1}{Z_1+Z_2}\dot{U}, \quad \dot{U}_2 = \frac{Z_2}{Z_1+Z_2}\dot{U}$$




equivalent admittance

$$Y = Y_1 + Y_2 = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{Z_1+Z_2}{Z_1 Z_2}$$

equivalent impedance

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

current divider

$$\dot{I}_1 = \frac{Z_2}{Z_1+Z_2}\dot{I}, \quad \dot{I}_2 = \frac{Z_1}{Z_1+Z_2}\dot{I}$$



## Impedance（阻抗） and Admittance（导纳）

### 4. Series and parallel impedances

(1) A series combination of N impedances

equivalent impedance

$$Z = \sum_{k=1}^{n} Z_k$$

voltage divider

$$\dot{U}_k = \frac{Z_k}{\sum_{k=1}^{n} Z_k}\dot{U} \quad (k = 1,2,\cdots n)$$

(2) A parallel combination of N impedances

equivalent admittance

$$Y = \sum_{k=1}^{n} Y_k$$

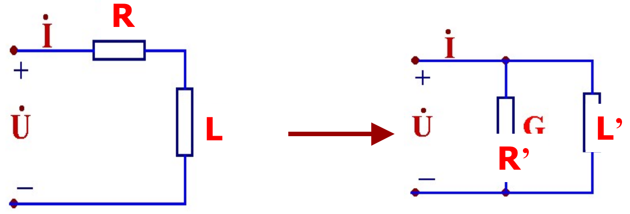
current divider

$$\dot{I}_k = \frac{Y_k}{\sum_{k=1}^{n} Y_k}\dot{I} \quad (k = 1,2,\cdots n)$$

Example 6. $R=6\Omega$, $X=8\Omega$, $f=50\text{Hz}$.

(1) Find $G$ and $B$.

(2) Find element parameters in series and parallel structures.




**Solution:** $Z=6+j8=10\angle 53.13^\circ$ $\quad R=6\Omega \quad L=25.48mH$

$$Y=\frac{1}{Z}=0.1\angle -53.13^\circ \qquad R'=\frac{1}{G}=16.67\Omega$$

$$=0.06-j0.08 \qquad L'=39.8mH$$

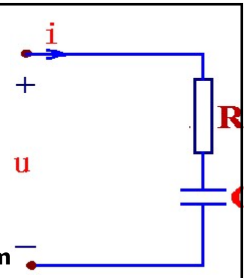
$$Y=G+jB \quad G=0.06S \quad B=-0.08S$$



## Exercise.

$$u(t)=2\sqrt{2}\cos(10^4 t+30^\circ)V$$

$$i(t)=100\sqrt{2}\cos(10^4 t+60^\circ)mA$$




**Find Z、Y and equivalent element param**

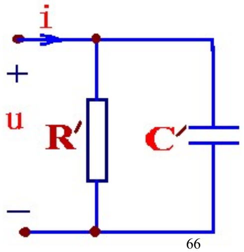
**Solution:** $\dot{U}=2\angle 30^\circ V \quad \dot{I}=100\angle 60^\circ mA$

$$Z=\frac{\dot{U}}{\dot{I}}=20\angle -30^\circ=17.32-j10(\Omega)$$

$$R=17.32\Omega \quad C=10\mu F$$

$$Y=\frac{\dot{I}}{\dot{U}}=0.05\angle 30^\circ \qquad R'=\frac{1}{G'}=23.1\Omega$$

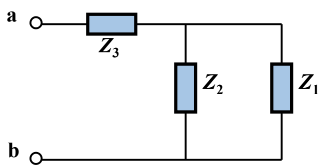
$$=0.0433+j0.025(S) \qquad C'=\frac{B'}{\omega}=2.5\mu F$$






**Example** $Z_1=10+j6.28\Omega$, $Z_2=20-j31.9\,\Omega$, $Z_3=15+j15.7\,\Omega$

**Find $Z_{ab}$**




**Solution:**

$$Z=\frac{Z_1 Z_2}{Z_1+Z_2}=\frac{(10+j6.28)(20-j31.9)}{10+j6.28+20-j31.9}$$

$$=\frac{11.81\angle 32.13^\circ\times 37.65\angle -57.61^\circ}{39.45\angle -40.5^\circ}$$

$$=10.89+j2.86\Omega$$

答题规范：
写出表达式
→代入数值
→得到结果
→写上单位

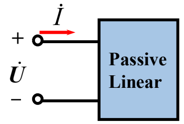
$$\therefore Z_{ab}=Z_3+Z=15+j15.7+10.89+j2.86$$

$$=25.89+j18.56=31.9\angle 35.6^\circ\ \Omega$$



# Brief summary

（1）




$$Z=\frac{\dot{U}}{\dot{I}} \quad \dot{U}=\dot{I}Z \qquad 相量形式$$

$$Y=\frac{\dot{I}}{\dot{U}} \quad \dot{I}=Y\dot{U} \qquad 欧姆定理$$

**（2）Sinusoidal Steady-State Analysis Steps**

① **Transform the circuit to the phasor or frequency domain.**

**Transform all independent sources to their phasor equivalent.**

$u\ i \longrightarrow \dot{U}\ \dot{I}$

**Calculate the impedance (Z) of all passive circuit elements.**

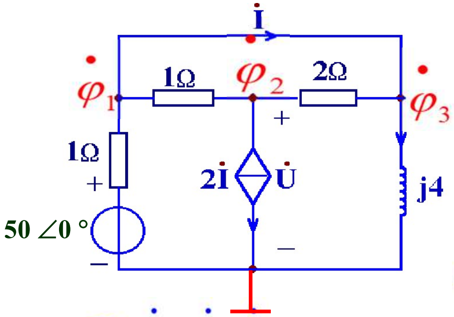
$R, L, C \longrightarrow Z, Y$

② **Apply analysis method in the frequency domain.**

③ **Transform the resulting phasor to the time domain.**

# 9.6 Nodal and Mesh Analysis

**Example 6: Find I in the following circuit.**




**Solution 1**

$$2\dot{\varphi}_1-\dot{\varphi}_2=50\angle 0°-\dot{I}$$

$$-\dot{\varphi}_1+\frac{3}{2}\dot{\varphi}_2-\frac{1}{2}\dot{\varphi}_3=-2\dot{I}$$

$$-\frac{1}{2}\dot{\varphi}_2+\left(\frac{1}{2}-j\frac{1}{4}\right)\dot{\varphi}_3=\dot{I}$$

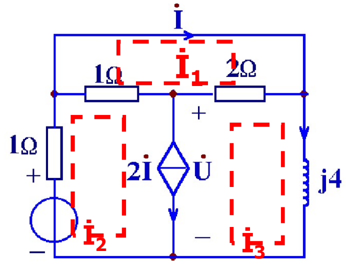
$$\dot{\varphi}_1=\dot{\varphi}_3$$

$$\dot{\varphi}_1=12.308+j21.539$$

$$\dot{\varphi}_2=-9.231+j33.847$$

$$\dot{I}=\frac{50-\dot{\varphi}_1}{1}+\frac{\dot{\varphi}_2-\dot{\varphi}_1}{1}=18.61\angle-29.75°A$$

*




**Solution 2**

$$3\dot{I}_1-\dot{I}_2-2\dot{I}_3=0$$

$$-\dot{I}_1+2\dot{I}_2=50\angle 0°-\dot{U}$$

$$-2\dot{I}_1+(2+j4)\dot{I}_3=\dot{U}$$

$$\dot{I}_2-\dot{I}_3=2\dot{I}$$

$$\dot{I}=\dot{I}_1$$

$$\dot{I}=18.61\angle-29.75°A$$

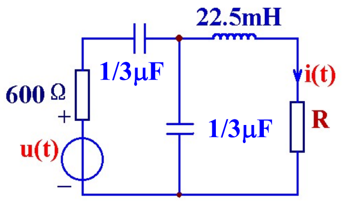
*

## 9.7 Superposition, Source Transformations, and Thevenin's theorem

Circuits containing inductors and capacitors were still linear, so the benefits of linearity were again available. Included among these were all above theorems.

Example 7. In the following circuit, $u(t)=60\sqrt{2}\cos(10^4t)V$

Calculate $i(t)$ when (1) $R$= 75Ω (2) $R$= 25 Ω.

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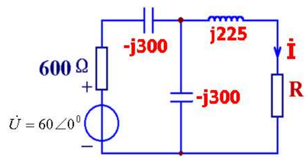
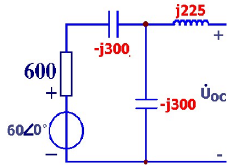
**Solution 1. Thevenin's theorem**

**The phasor equivalent circuit is**

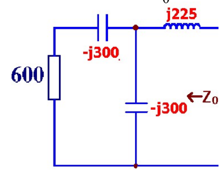



**Calculate** $\dot{U}_{oc}$



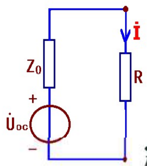

$$\dot{U}_{oc}=\frac{30}{\sqrt{2}}\angle-45°V$$

**Calculate** $Z_0$




$$Z_o=75\Omega$$

**The equivalent circuit is**




**if R=75V**

$$\dot{I}=\frac{0.2}{\sqrt{2}}\angle-45°A$$

$$i(t)=0.2\cos(10^4t-45°)A$$

**if R=25V**

$$\dot{I}=\frac{0.3}{\sqrt{2}}\angle-45°A$$

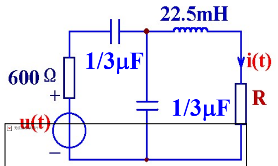
$$i(t)=0.3\cos(10^4t-45°)A$$

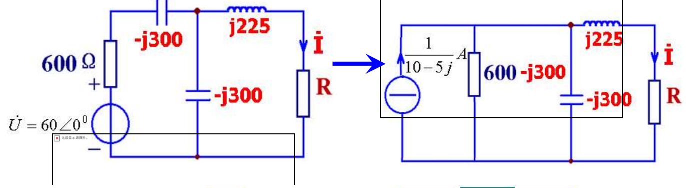
*

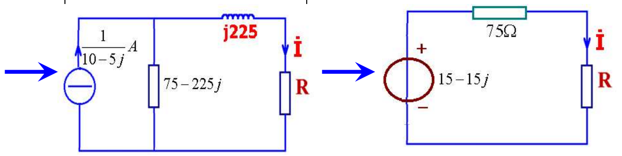
Solution 2.

Source Transformations

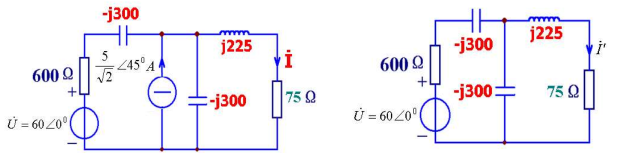
The phasor equivalent circuit is




Example 8. In the following circuit, use superposition to find I.



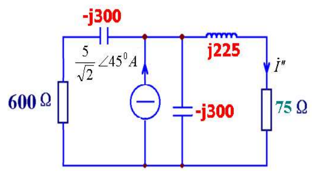

**Solution:** Let voltage source acts alone

$$\dot{I}'=0.2-0.2j\ A$$

Let current source acts alone

$$\dot{I}''=-2.5+5j\ A$$

$$\dot{I}=\dot{I}'+\dot{I}''=-2.3+4.8j\ A$$




## Brief summary

**Sinusoidal Steady-State Analysis Steps**

① Transform the circuit to the phasor or frequency domain.

Transform all independent sources to their phasor equivalent.

$u\ i \longrightarrow \dot{U}\ \dot{I}$

Calculate the impedance (Z) of all passive circuit elements.

$R, L, C \longrightarrow Z, Y$

② Apply analysis method in the frequency domain.

③ Transform the resulting phasor to the time domain.

Nodal and Mesh Analysis, Superposition, Source Transformations, and Thevenin's theorem...



$$\dot{U}=50e^{j15^\circ} \neq 50\sqrt{2}\cos(\omega t+15^\circ)$$

复数

瞬时值

已知: $i = 10\cos(\omega t + 45^\circ)$

$I \cancel{=} \dfrac{10}{\sqrt{2}} \angle 45^\circ$

有效值

$\dot{I} \cancel{=} 10e^{45^\circ}$

j45°



已知: $u = 10\sqrt{2}\cos(\omega t - 15^\circ)$

则: $U = 10$

$\dot{U} \cancel{=} 10e^{j15^\circ}$

$-15^\circ$



已知: $\dot{I} = 100\angle 50^\circ$

则: $i \cancel{=} 100\cos(\omega t + 50^\circ)$

最大值

$I_m = \sqrt{2}I = 100\sqrt{2}$