9.4 Kirchhoff's Laws in the Frequency Domain (频域中的基尔霍夫定律)

9.4.1 KCL

time domain $\sum_{k=1}^{n} i_k(t) = 0$

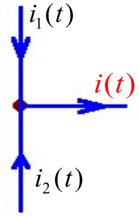
$$\sum_{k=1}^{n} \mathrm{Re}\{\dot{I}_{km} e^{j\omega t}\} = 0$$

frequency domain $\sum_{k=1}^{n} \dot{I}_{km} = 0$ $\quad \sum_{k=1}^{n} \dot{I}_{k} = 0$

*

Example 1. $i_1(t) = 5\sqrt{2}\cos(\omega t + 53.1°)$ $\quad i_2(t) = 10\sqrt{2}\cos(\omega t - 36.9°)$

Find $i(t) = i_1(t) + i_2(t)$



Solution:

$$\dot{I}_1 = 5\angle 53.1° = 3 + j4$$

$$\dot{I}_2 = 10\angle -36.9° = 8 - j6$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = 11 - j2 = 11.18\angle -10.3°$$

$$\therefore \quad i(t) = 11.18\sqrt{2}\cos(\omega t - 10.3°)$$

*

9.4.2 KVL

time domain $\sum_{k=1}^{n} u_k(t) = 0$

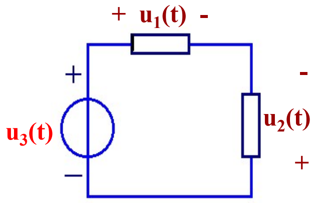
$$\sum_{k=1}^{n} \mathrm{Re}\{\dot{U}_{km} e^{j\omega t}\} = 0$$

frequency domain $\sum_{k=1}^{n} \dot{U}_{km} = 0$ $\quad \sum_{k=1}^{n} \dot{U}_{k} = 0$

Amplitude phasor **Effective value phasor**

*

Example 2. In the right circuit,



$$u_1(t) = 6\sqrt{2}\cos(\omega t + 30°)$$

$$u_2(t) = 4\sqrt{2}\cos(\omega t + 60°)$$

Find $u_3(t)$

Solution: $\dot{U}_1 = 6\angle 30°$

$$\dot{U}_2 = 4\angle 60°$$

$$\dot{U}_3 = \dot{U}_1 - \dot{U}_2 = (5.19 + j3) - (2 + j3.45)$$

$$= 3.19 - j0.45 = 3.22\angle -8.03°$$

$$\therefore \quad u_3(t) = 3.22\sqrt{2}\cos(\omega t - 8.03°)$$

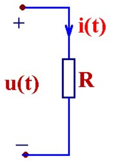
*

9.5 The Passive Circuit Elements in the Frequency Domain

Establish the relationship between the phasor current and voltage at the terminals of the resistor, inductor, and capacitor.

9.5.1 The V-I Characteristic for a Resistor

time domain

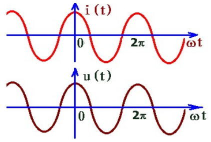


$$i(t)=\sqrt{2}I\cos(\omega t+\varphi_i)$$

$$u(t)=Ri(t)$$

$$=\sqrt{2}IR\cos(\omega t+\varphi_i)$$

$$=\sqrt{2}U\cos(\omega t+\varphi_u)$$



$U=IR \quad \varphi_u=\varphi_i$

in phase

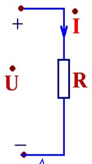
frequency domain

$$\dot{I}=I\angle\varphi_i$$

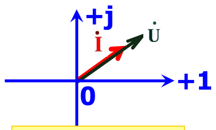
$$\dot{U}=U\angle\varphi_u$$

$$=RI\angle\varphi_i$$

$$\dot{U}=R\dot{I}$$



The frequency-domain equivalent circuit of a resistor



Phasor Diagram

*

Instantaneous power(瞬时功率）

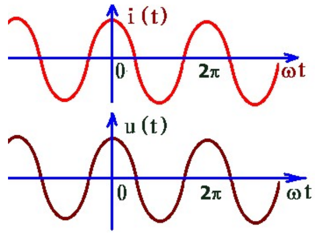
$$p(t)=u(t)i(t)$$

$$i(t)=\sqrt{2}I\cos(\omega t+\varphi_i)$$

$$u(t)=\sqrt{2}U\cos(\omega t+\varphi_u)$$

$$p(t)=\sqrt{2}U\cos(\omega t+\varphi)\sqrt{2}I\cos(\omega t+\varphi)$$

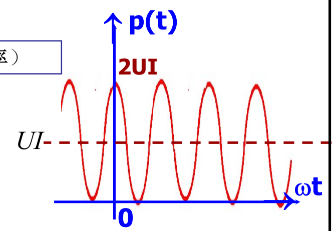
$$=UI+UI\cos[2(\omega t+\varphi)] \qquad p(t)\geq 0$$



Average power（平均功率、有功功率）

$$P=\frac{1}{T}\int_0^T p(t)dt \quad =UI$$

$$=RI^2=\frac{U^2}{R}(W)$$



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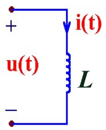
9.5.2 The V-I Characteristic for a Inductor

time domain



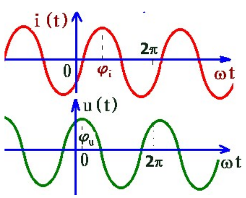
$$i(t)=\sqrt{2}I\cos(\omega t+\varphi_i)$$

$$u(t)=L\frac{di(t)}{dt}$$

$$=-\sqrt{2}I\omega L\sin(\omega t+\varphi_i)$$

$$=\sqrt{2}I\omega L\cos(\omega t+\varphi_i+90°)$$

$$=\sqrt{2}U\cos(\omega t+\varphi_u)$$



(waveform)

$\therefore\ U=\omega L I$

$\varphi_u=\varphi_i+90°$

phase difference

The voltage leads (超前) the current by 90°

$X_L=\omega L$ (inductive reactance) 感抗 Ω

$U=X_L I$

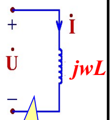
frequency domain

$$\dot{I}=I\angle\varphi_i$$

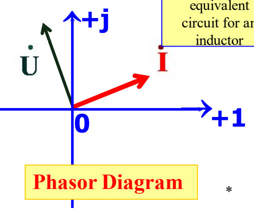
$$\dot{U}=U\angle\varphi_u$$

$$=I\omega L\angle(\varphi_i+90°)$$

$$\dot{U}=j\omega L\dot{I}=jX_L\dot{I}$$



The frequency-domain equivalent circuit for an inductor



Phasor Diagram

*

Instantaneous power

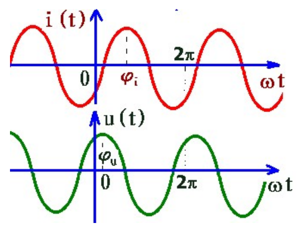
$$p(t)=u(t)i(t)$$

$$i(t)=\sqrt{2}I\cos(\omega t+\varphi_i)$$

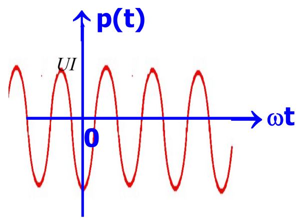
$$u(t)=\sqrt{2}U\cos(\omega t+\varphi_u)$$

$$p(t)=\sqrt{2}U\cos(\omega t+\varphi_u)\sqrt{2}I\cos(\omega t+\varphi_i)$$

$$=UI\cos(2\omega t+2\varphi_i+90°)$$



Average power $\quad P=\frac{1}{T}\int_0^T p(t)dt=0$

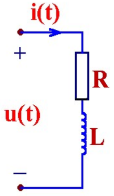


*

Example 3. In the model of a real inductor, R=10 Ω, L=50 mH ,

$$i(t) = 10\sqrt{2}\cos(314t + 36.9°)A$$

Find $u_R(t)$, $u_L(t)$ and $u(t)$。



Solution

$\dot{I} = 10\angle 36.9°$ $\dot{U}_R = \dot{I}R = 100\angle 36.9° = 80 + j60$

$\dot{U}_L = jX_L\dot{I} = j\omega L\dot{I} = 157\angle 126.9° = -94.27 + j125.55$

$\dot{U} = \dot{U}_R + \dot{U}_L = -14.27 + j185.55 = 186.1\angle 94.4°$

$\therefore \quad u(t) = 186.1\sqrt{2}\cos(\omega t + 94.4°)V$

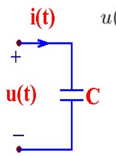
$u_R(t) = 100\sqrt{2}\cos(\omega t + 36.9°)V$

$u_L(t) = 157\sqrt{2}\cos(\omega t + 126.9°)V$

*

9.5.3 The V-I Characteristic for a Capacitor

time domain



$u(t) = \sqrt{2}U\cos(\omega t + \varphi_u)$

$i(t) = C\dfrac{du(t)}{dt}$

$= -\sqrt{2}U\omega C\sin(\omega t + \varphi_u)$

$= \sqrt{2}U\omega C\cos(\omega t + \varphi_u + 90°)$

$= \sqrt{2}I\cos(\omega t + \varphi_i)$

frequency domain

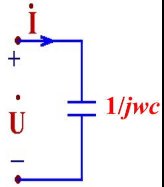
$\dot{U} = U\angle\varphi_u$

$\dot{I} = I\angle\varphi_i$

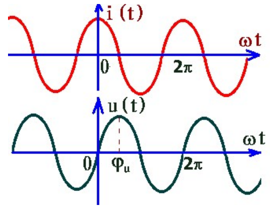
$= U\omega C\angle\varphi_u + 90°$

$= j\omega CU\angle\varphi_u$

$\dot{I} = j\omega C\dot{U} = jB_C\dot{U}$



$\boxed{\dot{U} = \dfrac{1}{j\omega C}\dot{I} = -jX_C\dot{I}}$

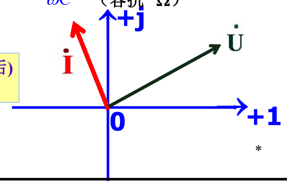


∴ I=UωC

$\varphi_i = \varphi_u + 90°$

The voltage lags (滞后) the current by 90º

$X_C = \dfrac{1}{\omega C}$ Capacitive reactance （容抗 Ω）



*

Instantaneous power

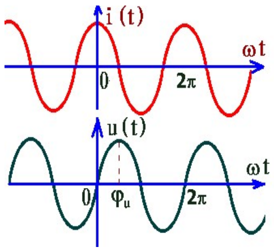
$p(t) = u(t)i(t)$

$i(t) = \sqrt{2}I\cos(\omega t + \varphi_i)$

$u(t) = \sqrt{2}U\cos(\omega t + \varphi_u)$

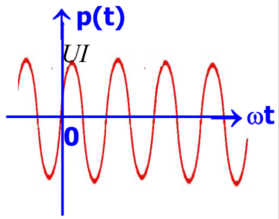
$p(t) = \sqrt{2}U\cos(\omega t + \varphi_u)\sqrt{2}I\cos(\omega t + \varphi_i)$

$= UI\cos(2\omega t + 2\varphi_u + 90°)$



Average power

$P = \dfrac{1}{T}\int_0^T p(t)dt = 0$



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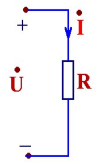
Kirchhoff's Laws in the Frequency Domain (频域中的基尔霍夫定律)

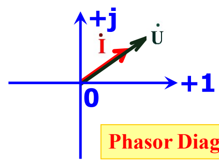
$\sum_{k=1}^{n}\dot{I}_{km} = 0$ $\sum_{k=1}^{n}\dot{I}_{k} = 0$

$\sum_{k=1}^{n}\dot{U}_{km} = 0$ $\sum_{k=1}^{n}\dot{U}_{k} = 0$

The V-I Characteristic for a Resistor



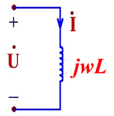
$\dot{U} = R\dot{I}$



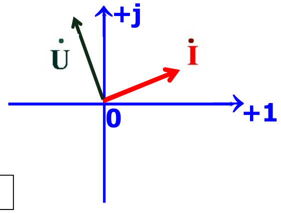
Phasor Diagram

*

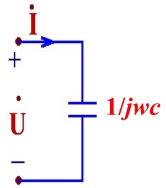
The V-I Characteristic for a Inductor



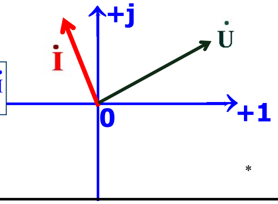
$\dot{U}=j\omega L\dot{I}=jX_L\dot{I}$



The V-I Characteristic for a Capacitor



$\dot{U}=\frac{1}{j\omega C}\dot{I}=-jX_C\dot{I}$



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Exercise 3. In the following circuit, U_R= 6 V, U_L= 18 V, U_C=10 V。

Find U =?

解：设 $\dot{I}=I\angle 0°$ （reference phasor）

$\dot{U}_R=6\angle 0°$ $\dot{U}_L=18\angle 90°$ $\dot{U}_C=10\angle -90°$

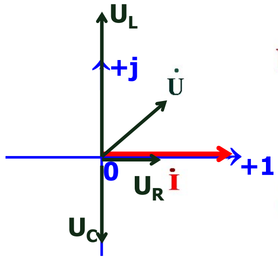
$\dot{U}=\dot{U}_R+\dot{U}_L+\dot{U}_C$

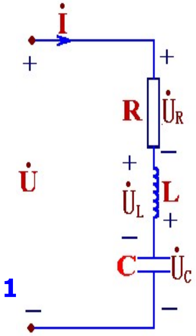
$=6+j18-j10$

$=6+j8$

$=10\angle 53.1°V$

$\therefore\ U=10V$





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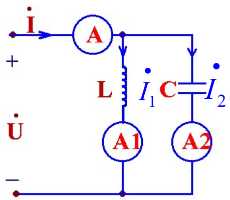
Example 4. In the circuit, the value of A1、A2 are both 10A。 Find the value of the I.

Solution

Assume $\dot{U}=U\angle 0°$ $\dot{I}_2=10\angle 90°$

$\dot{I}_1=10\angle -90°$

$\dot{I}=\dot{I}_1+\dot{I}_2=0$



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Brief summary

元件	u, i 关系	相量关系	大小关系	相位	P(W)
$i(t)$, $+$, $u_R(t)$, $-$, R	$u=Ri$	$\dot{U}=R\dot{I}$	$U=RI$	$\dot{U}$, $\dot{I}$	I^2R
$i(t)$, $+$, $u_L(t)$, $-$, L	$u=L\frac{di}{dt}$	$\dot{U}=j\omega L\dot{I}$	$U=\omega LI$	$\dot{U}$, $\dot{I}$	0
$i_C(t)$, $+$, $u(t)$, $-$, C	$i=C\frac{du}{dt}$	$\dot{U}=-j\frac{1}{\omega C}\dot{I}$	$U=\frac{1}{\omega C}I$	$\dot{I}$, $\dot{U}$	0

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Impedance（阻抗） and Admittance（导纳）

$\dot{U} = R\dot{I}$

$\dot{U} = \frac{1}{j\omega C}\dot{I} = -jX_C\dot{I}$

$\dot{U} = j\omega L\dot{I} = jX_L\dot{I}$

Z--- Impedance $\dot{U} = Z\dot{I}$

$Z = \frac{\dot{U}}{\dot{I}} = R + jX = |Z|\angle\varphi_z$

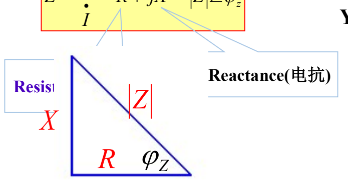
Y--- Admittance

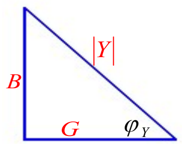
$Y = \frac{\dot{I}}{\dot{U}} = G + jB = |Y|\angle\varphi_Y$

Conductance(电导)

Susceptance(电纳)

Y=1/Z





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符号约定

瞬时值 --- 小写 u、i

有效值 --- 大写 U、I

最大值 --- 大写+下标 U_m

复数、相量 --- 大写 + “.” $\dot{U}$

51

$u = 100\cos\omega t \cancel{=} \dot{U}$

瞬时值

复数

52