Fundamentals of Electric Circuits

Chapter 3

Methods of Analysis

(Basic Nodal and Loop Analysis)

Chapter3 Methods of Analysis

3.1 Motivation

3.2 Nodal analysis.

3.3 Nodal analysis with voltage sources.

3.4 Loop (Mesh)analysis.

3.5 Loop (Mesh) analysis with current sources.

3.6 Nodal and mesh analysis by inspection.

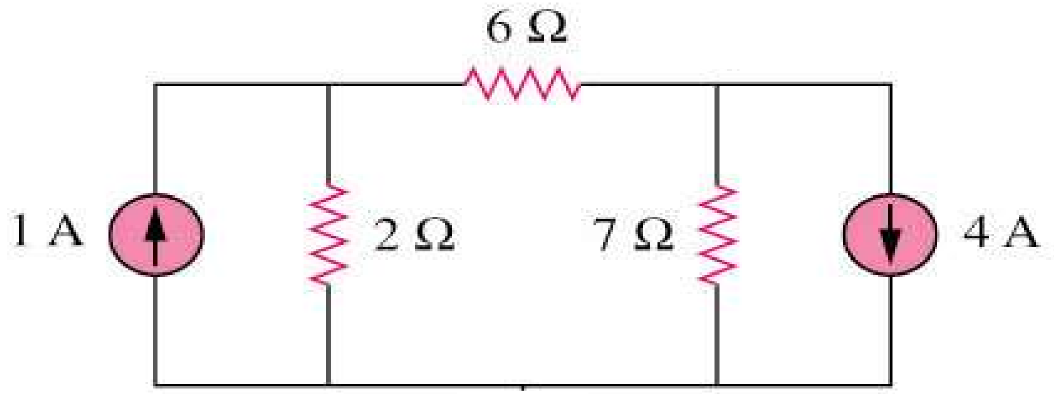
3.7 Nodal versus mesh analysis.

3.8 Applications: DC Transistor Circuits

3.9 Summary

3.1 Motivation (1)

If you are given the following circuit, how can we determine (1) the voltage across each resistor, (2) current through each resistor. (3) power generated by each current source, etc.



What are the things which we need to know in order to determine the answers?

3.1 Motivation (2)

Things we need to know in solving any resistive circuit with current and voltage sources only:

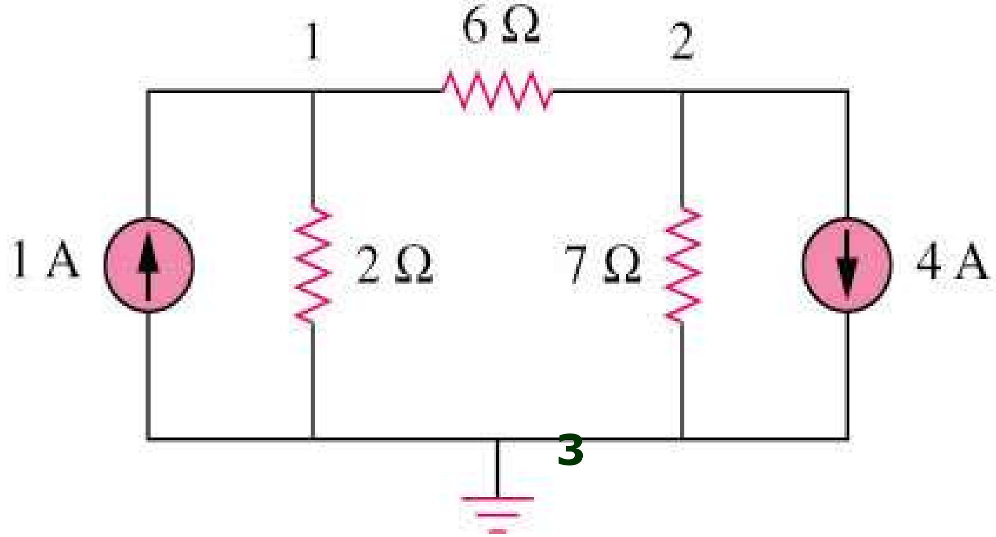
- Kirchhoff's Current Laws (KCL)
- Kirchhoff's Voltage Laws (KVL)
- Ohm's Law

How should we apply these laws to determine the answers?

3.2 Nodal Analysis (1)

It provides a general procedure for analyzing circuits using <u>node voltages</u> as the circuit variables.

<u>Example 1</u>

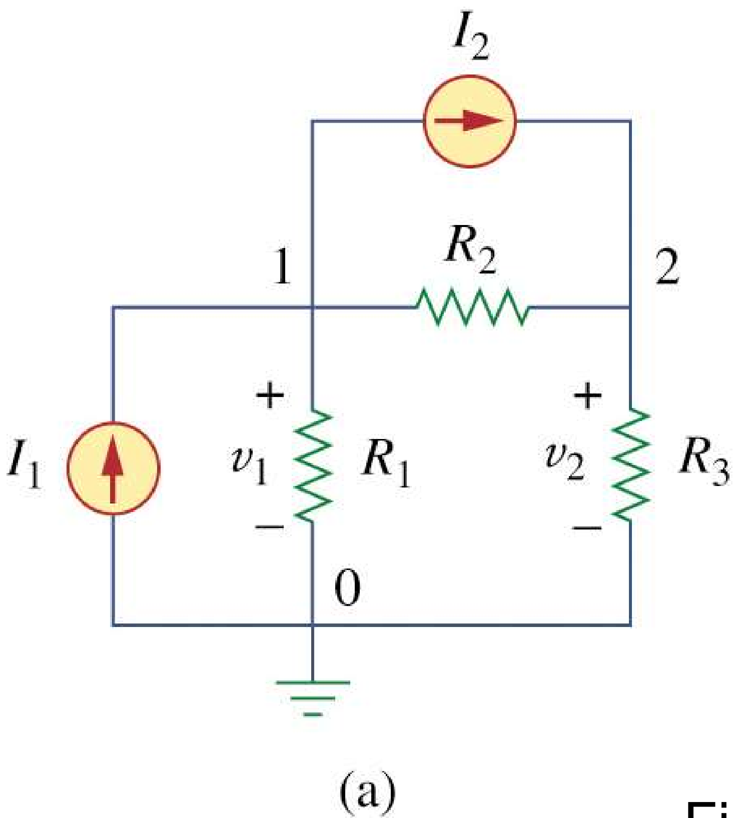


3.2 Nodal Analysis (2)

Steps to determine the node voltages:

1. Select a node as the reference node.
2. Assign voltages v1,v2,…,vn-1 to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm’s law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Consider the circuit in Fig.3.1(a)



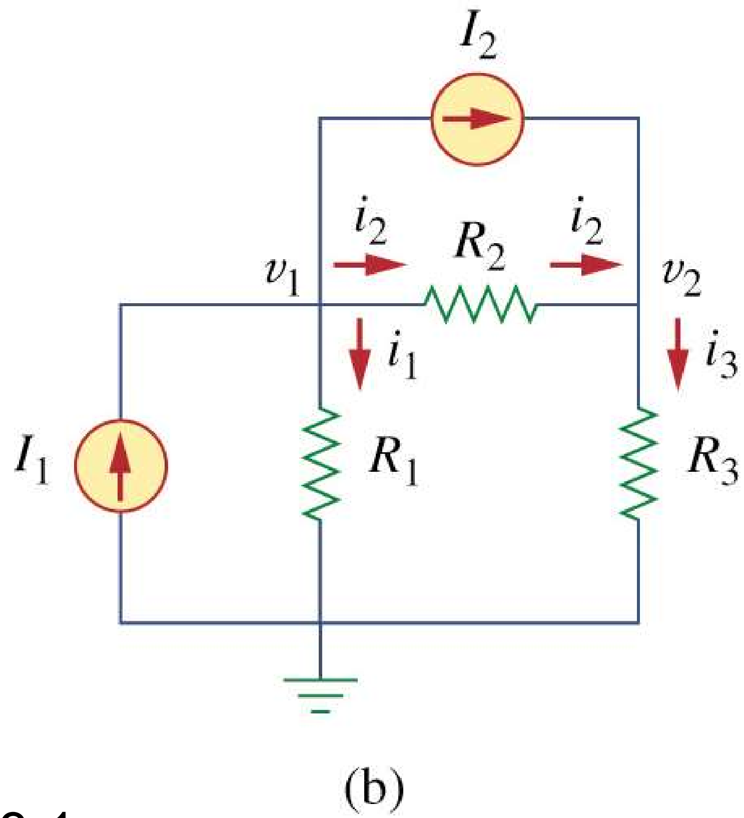


Fig.3.1

Select node 0 as reference node, while nodes 1 and 2 are assigned voltages v_1 and v_2 ,respectively.

Applying KCL to each nonreference node in the circuit. For convenience, we redraw Fig. 3.1(a) in Fig. 3.1(b),where we now label i_1,i_2 and i_3 as the currents through resistors R_1,R_2 and R_3 , respectively.

At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2 \qquad (3.2.1)$$

At node 2

$$I_2 + i_2 = i_3 \qquad (3.2.2)$$

Applying Ohm's law to express the unknown currents i_1, i_2, and i_3 in terms of node voltage. The key idea to bear in mind is that, by the passive sign convention, current must always flow from a higher potential to lower potential. With this in mind, we obtain from Fig.3.2.1(b),

$$i_1 = \frac{v_1}{R_1} \quad or \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad or \quad i_2 = G_2 (v_1 - v_2) \qquad (3.2.3)$$

$$i_3 = \frac{v_2}{R_3} \quad or \quad i_2 = G_3 v_2$$

Substituting Eq(3.2.3) into Eqs(3.2.1) and (3.2.2) and rearranging terms, we obtain

$$(G_1 + G_2) v_1 - G_2 v_2 = I_1 - I_2 \qquad (3.2.4)$$

$$-G_2 + (G_2 + G_3) v_2 = -I_2$$

Consider the circuit in Fig.3.2

1.Select node D as reference node

2.To generate the node-voltage equations, we apply KCL and Ohm's law

3.The node-voltage equation derived at node A is

$$\frac{\varphi_A-\varphi_B}{R_4}+$$

The algebraic sum of the values of all current sources connected to the node(与节点相连的电流源电流的代数和)

$$\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_6}\right)\varphi_A-\frac{1}{R_4}\varphi_B-\frac{1}{R_6}\varphi_C=\frac{u_{s1}}{R_1}+\frac{u_{s6}}{R_6}$$

At node B and C we obtain

$$-\frac{1}{R_4}\varphi_A+\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)\varphi_B-\frac{1}{R_5}\varphi_C=\frac{u_{s2}}{R_2}+\frac{u_{s5}}{R_5}$$

$$-\frac{1}{R_6}\varphi_A-\frac{1}{R_5}\varphi_B+\left(\frac{1}{R_3}+\frac{1}{R_5}+\frac{1}{R_6}\right)\varphi_C=-\frac{u_{s6}}{R_6}-\frac{u_{s5}}{R_5}$$

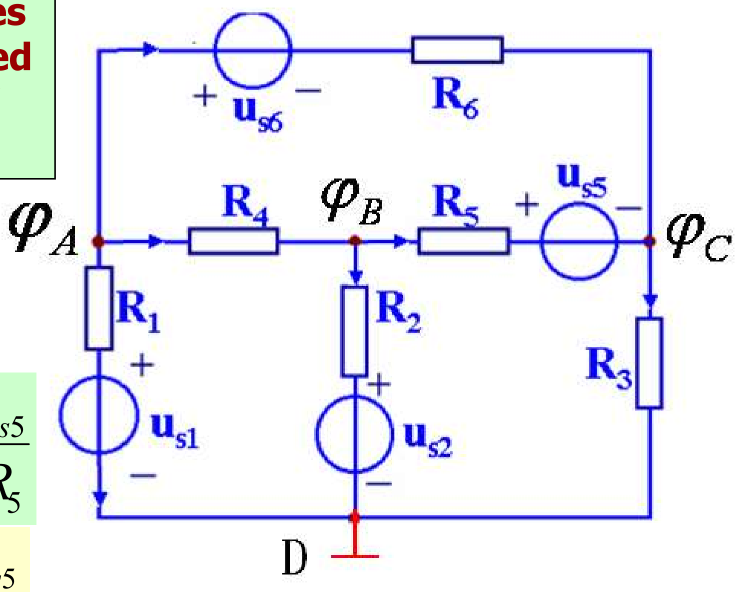


Fig.3.2

Self-conductance is equal to the sum of all conductances at the node.

Mutual conductance is equal to the sum of conductance directly connecting two nodes.

The values of those sources whose current flows towards the node are taken positive while, in the opposite case, they are taken as negative.

Review :Steps in node-voltage analysis

Step 1: One node of the circuit is defined as the reference node. Usually choose the node to which most branches are connected as the reference node.

Step 2: For the remaining *N*-1 nodes, after number them, the node voltages with respect to the reference node are defined.

Step 3: The node-voltage equations are written in the form as in the previous case.

Step 4: The equations are solved and the node voltage are known.

Step 5: The voltage of all branches are calculated combining the node voltage and as a consequence the currents of all circuit elements are known.

observation

When applying nodal voltage analysis to solve electric analysis problems, for convenience we can transfer the series combination of an independent voltage source and a resistor into the parallel combination of an independent current source and a resistor.

An equivalent condition is

$$i_s = \frac{v_s}{R} \qquad or \qquad v_s = Ri_s$$

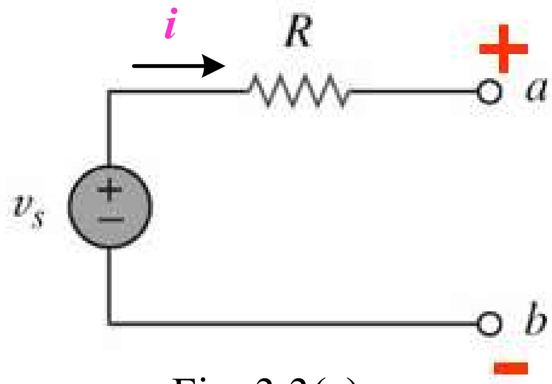


Fig. 3.3(a)

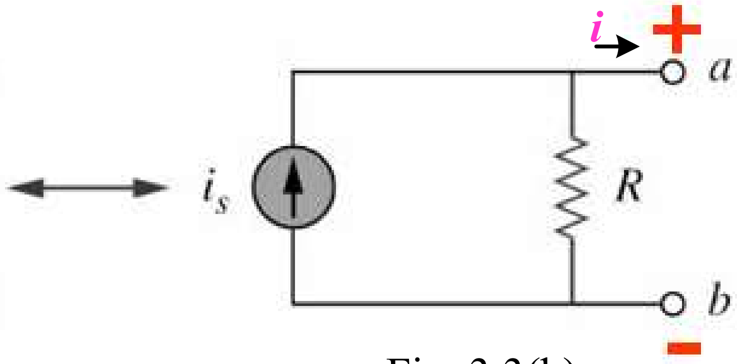
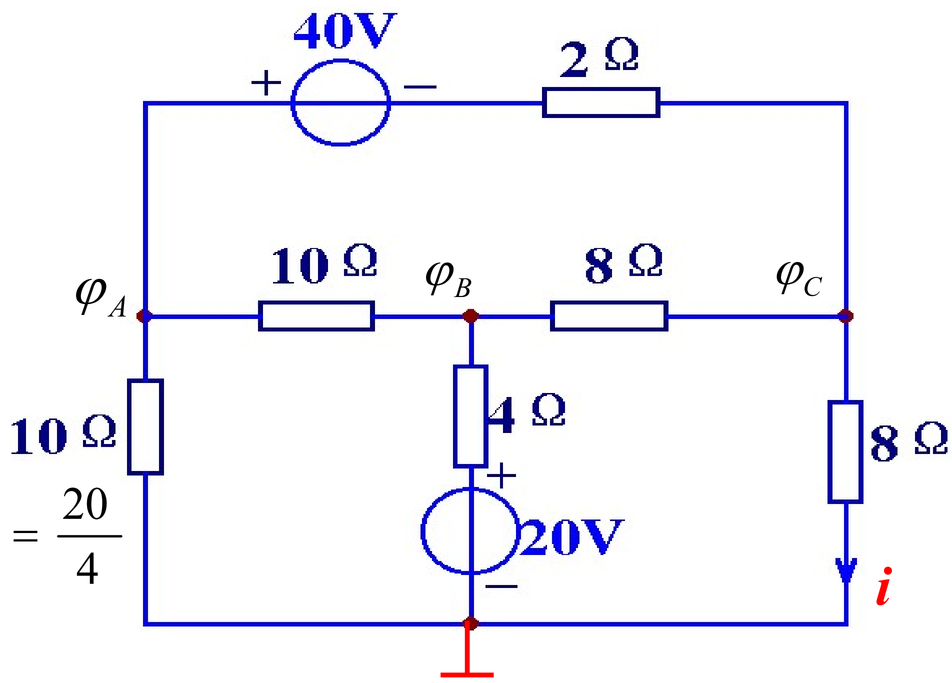


Fig. 3.3(b)

Example 1. Find i in the following circuit.

Solution:

The reference node is defined and the remaining nodes are labeled.



The node equations are

$$\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{2}\right)\varphi_A-\frac{1}{10}\varphi_B-\frac{1}{2}\varphi_C=\frac{40}{2}$$

$$-\frac{1}{10}\varphi_A+\left(\frac{1}{10}+\frac{1}{4}+\frac{1}{8}\right)\varphi_B-\frac{1}{8}\varphi_C=\frac{20}{4}$$

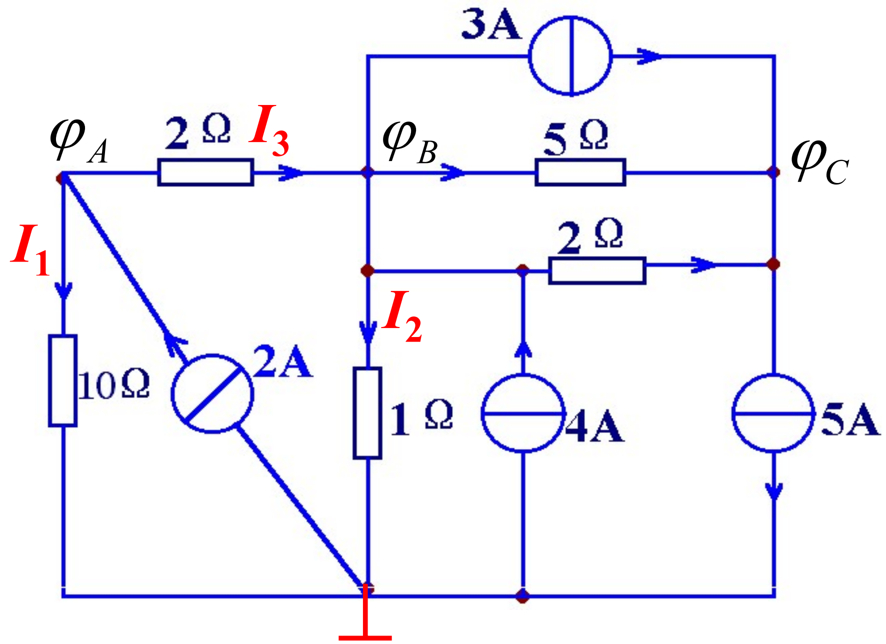
$$-\frac{1}{2}\varphi_A-\frac{1}{8}\varphi_B+\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{8}\right)\varphi_C=-\frac{40}{2}$$

Solving for φ_C gives $\quad \varphi_C=-4.21416V$

Hence $\quad i=-0.527A$

Example 2. Determine I_1, I_2 and I_3 in the following circuit.

Solution:



The reference node is defined and the remaining nodes are labeled.

The node equations are

$$0.6\varphi_A - 0.5\varphi_B = 2$$

$$-0.5\varphi_A + 2.2\varphi_B - 0.7\varphi_C = 1$$

$$-0.7\varphi_B + 0.7\varphi_C = -2$$

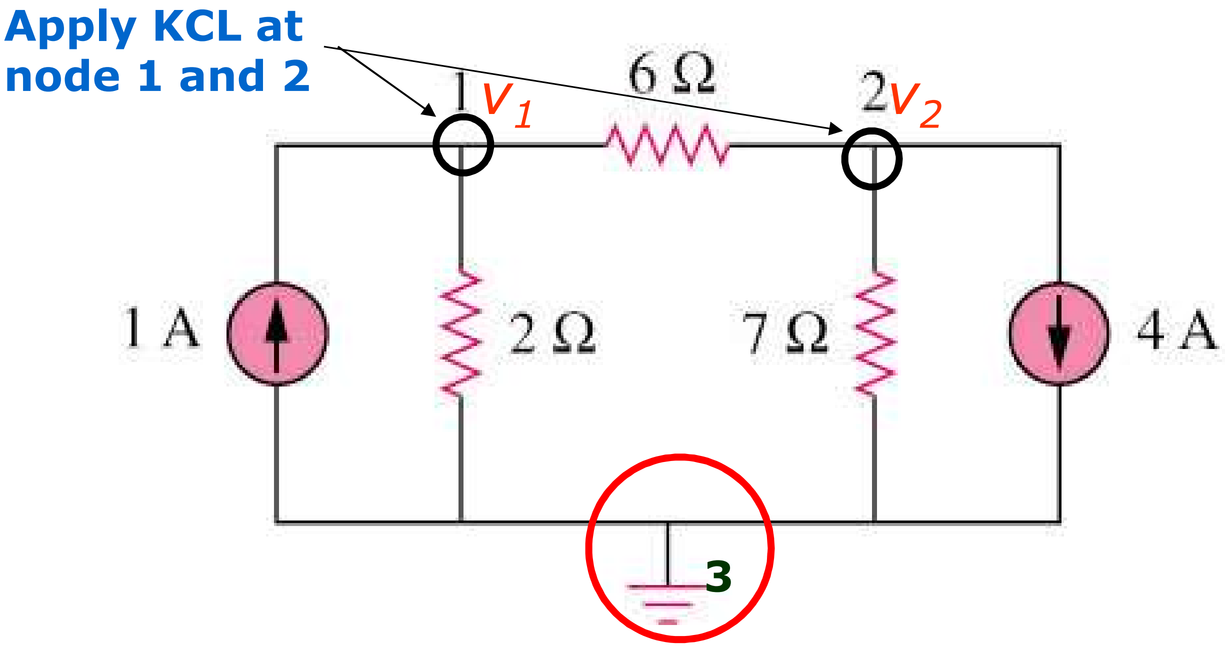
Thus $\varphi_A = 3.864V$ $\varphi_B = 0.615V$ $\varphi_C = -2.242V$

Solving current yields

$$I_1 = 0.3864A \quad I_2 = 0.615A \quad I_3 = 1.4285A$$

3.2 Nodal Analysis (3)

<u>Example 3</u> – circuit independent current source only



Solution

Applying KCL to node 1, we get

$$-1+\frac{v_1}{2}+\frac{v_1-v_2}{6}=0 \qquad (3.2.5)$$

The KCL equation node 2 is

$$\frac{v_2-v_1}{6}+\frac{v_2}{7}+4=0 \qquad (3.2.6)$$

Rearrange Eqs(3.2.5)and (3.2.6), we obtain

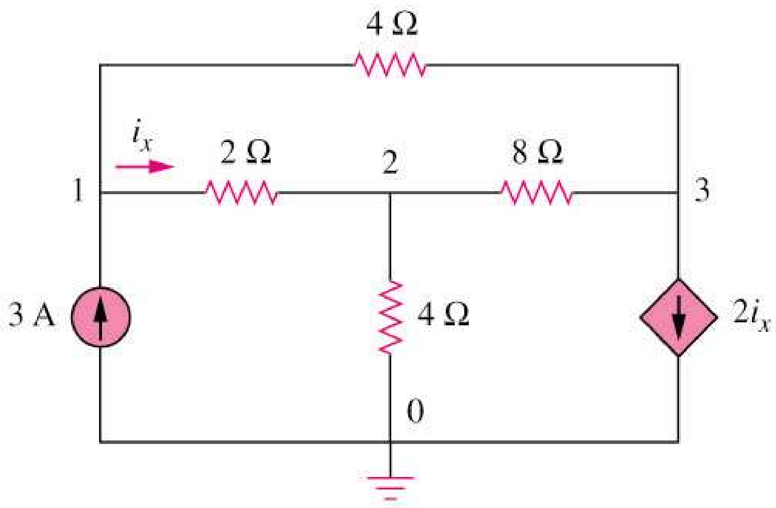
$$\left(\frac{1}{2}+\frac{1}{6}\right)v_1-\frac{1}{6}v_2=1 \qquad (3.2.7)$$

$$-\frac{1}{6}v_1+\left(\frac{1}{6}+\frac{1}{7}\right)v_2=-4 \qquad (3.2.8)$$

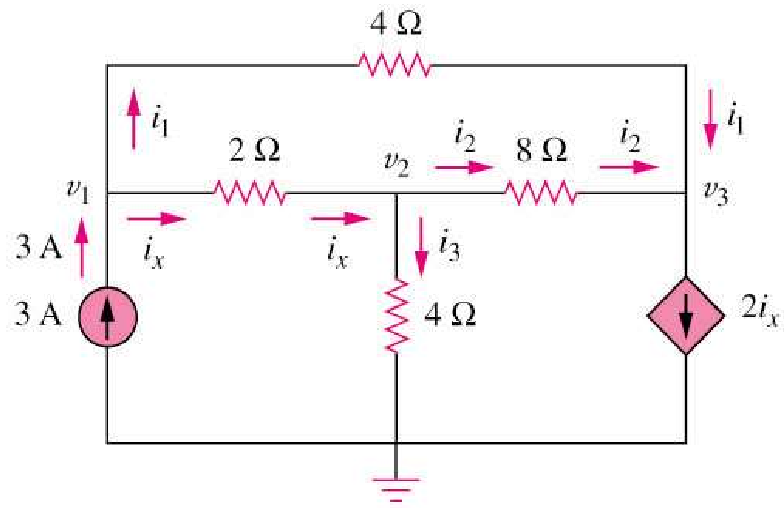
Answer: v_1 **= -2V,** v_2 **= -14V**

3.2 Nodal Analysis (4)

<u>Example 3</u> – current with dependant current source



(a)



(b)

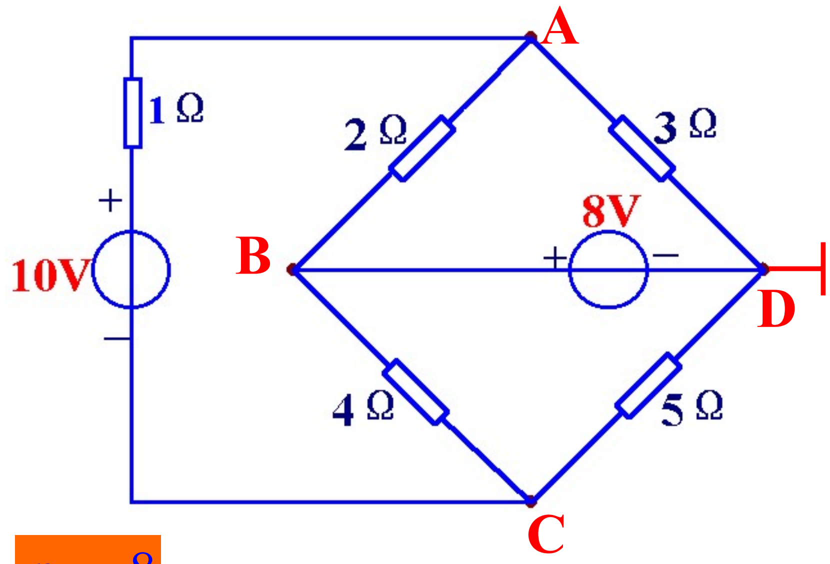
Answer $v_1 = 4.8V$, $v_2 = 2.4V$, $v_3 = -2.4V$

3.3 Nodal Analysis with Voltage Source (1)

Approach 1

A voltage source is the only element between two nodes

First, the reference node is defined at the negative terminal of the voltage source.



Then, the voltage of its positive terminal is known.

$$\varphi_B = 8$$

Finally, other node-voltage equations are written as previous way.

$$(1+\frac{1}{2}+\frac{1}{3})\varphi_A - \frac{1}{2}\varphi_B - \varphi_C = \frac{10}{1}$$

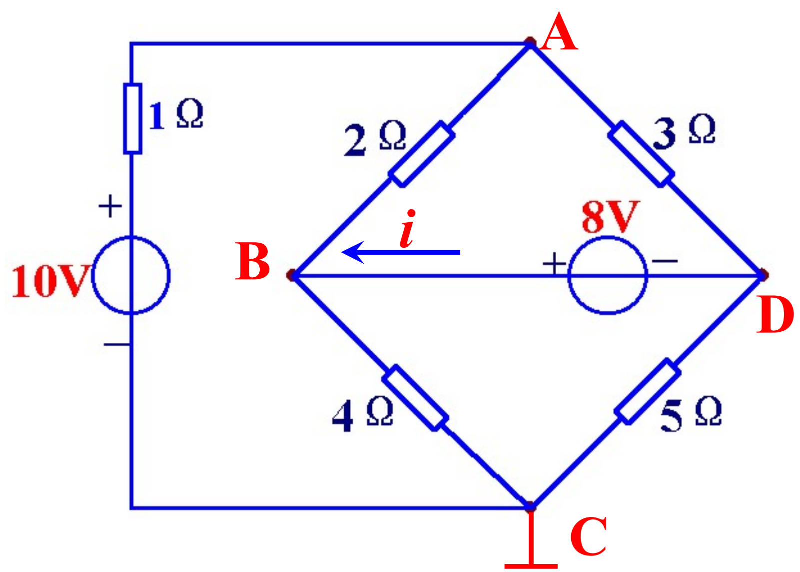
$$-\varphi_A - \frac{1}{4}\varphi_B + (1+\frac{1}{4}+\frac{1}{5})\varphi_C = -\frac{10}{1}$$

Approach 2:

First, Introduce the current of the voltage source because we cannot express the current with node

Then, other node-voltage equations are written as previous way.

Then, the equations are supplemented by the voltage across the considered voltage source.



$$(1+\frac{1}{2}+\frac{1}{3})\varphi_A-\frac{1}{2}\varphi_B-\frac{1}{3}\varphi_D=\frac{10}{1}$$

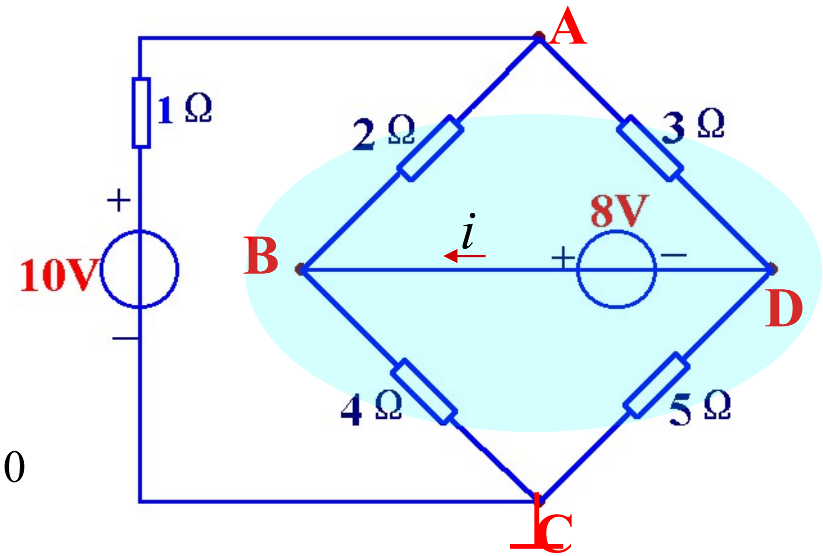
$$-\frac{1}{2}\varphi_A+(\frac{1}{2}+\frac{1}{4})\varphi_B=i$$

$$-\frac{1}{3}\varphi_A+(\frac{1}{3}+\frac{1}{5})\varphi_D=-i$$

$$\boxed{\varphi_B-\varphi_D=8}$$

Approach 3:

Choose a supernode containing nodes connected with voltage sources and apply KCL to the supernode.



$$\frac{\varphi_A-\varphi_B}{2}+\frac{\varphi_A-\varphi_D}{3}+\frac{0-\varphi_B}{4}+\frac{0-\varphi_D}{5}=0$$

or $$(\frac{1}{2}+\frac{1}{4})\varphi_B+(\frac{1}{3}+\frac{1}{5})\varphi_D-(\frac{1}{2}+\frac{1}{3})\varphi_A=0$$

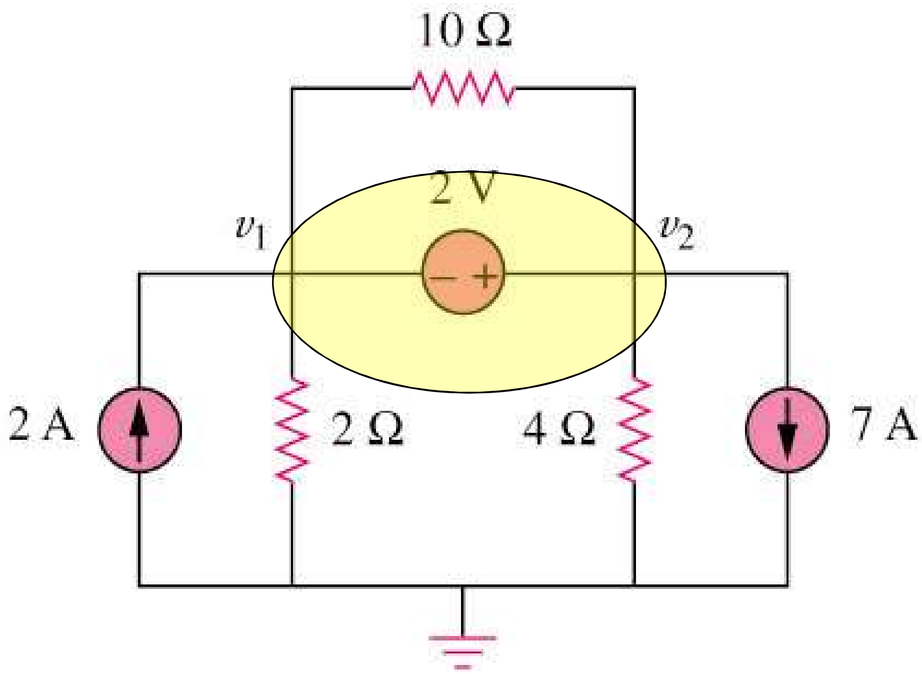
$$-\frac{1}{2}\varphi_A+(\frac{1}{2}+\frac{1}{4})\varphi_B=i$$

$$-\frac{1}{3}\varphi_A+(\frac{1}{3}+\frac{1}{5})\varphi_D=-i$$

$$(1+\frac{1}{2}+\frac{1}{3})\varphi_A-\frac{1}{2}\varphi_B-\frac{1}{3}\varphi_D=\frac{10}{1}$$

$$\varphi_B-\varphi_D=8$$

<u>Example 4</u> –circuit with independent voltage source



How to handle the 2V voltage source?

supernode

- A super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

- *Note: We analyze a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently

The supernode technique

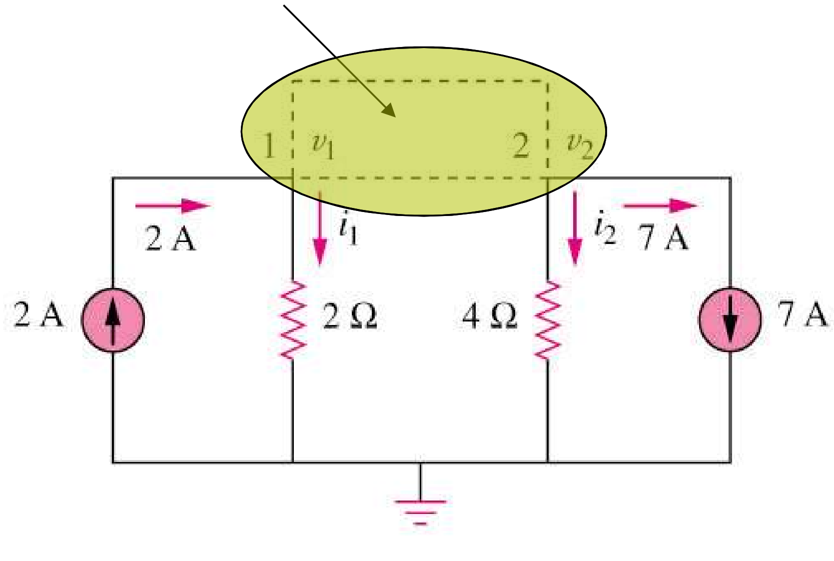
Basic steps:

1. Use it when a branch between two nonreference nodes contains a voltage source
2. First encircle the voltage source and the two connecting nodes to form the supernode
3. Write the equation that defines the voltage relationship between the two nonreference nodes as a result of the presences of the voltage source.
4. Write the KCL equations for the supernode.
   - Take off all voltage sources in super-nodes and apply KCL to super-nodes.
   - Put voltage sources back to the nodes and apply KVL to relative loops.
5. If the voltage source is dependent, then the controlling equation for the dependent source is also needed.

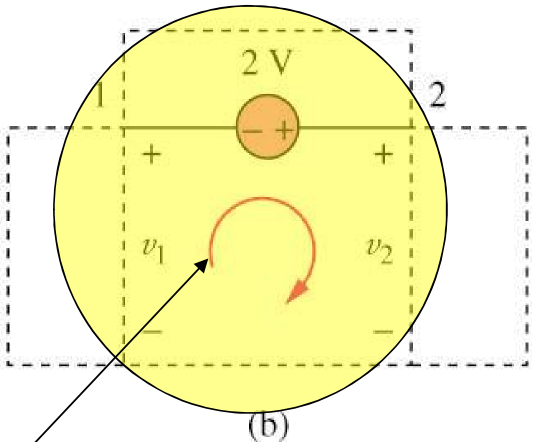
3.3 Nodal Analysis with Voltage Source (2)

<u>Example 5</u> – circuit with independent voltage source

Super-node => $-2+i_1+i_2+7 = 0$



(a)



(b)

Apply KVL => $v_1+2-v_2 = 0$

3.3 Nodal analysis with Voltage sources(3)

- A general rule:
  - When writing KCL,KVL equations for the network, treat the dependent source as though it were an independent source.
  - Write the equation that specifies the relationship of the dependent source to the controlling parameter
  - Solve the equations for the unknowns. Be sure that the number of linearly independent equations matches the number of unknowns.

Practice 1. Find *u* and *i*.

$$\varphi_A = -1$$

$$-2\varphi_A + 3\varphi_B - \varphi_C \qquad = 2$$

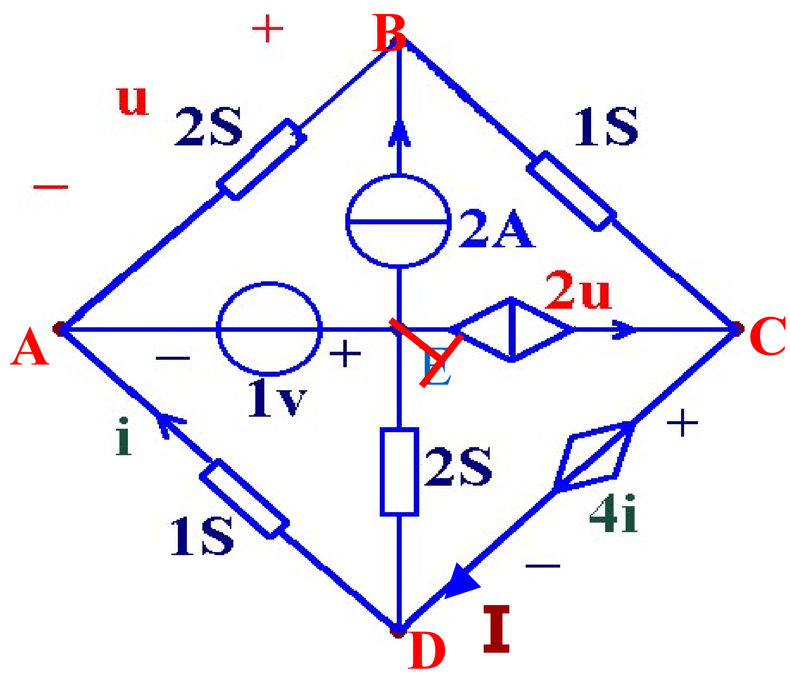
$$-\varphi_B + \varphi_C = 2u - I$$

$$-\varphi_A \qquad + 3\varphi_D = I$$

$$\varphi_C - \varphi_D = 4i$$

$$u = \varphi_B - \varphi_A$$

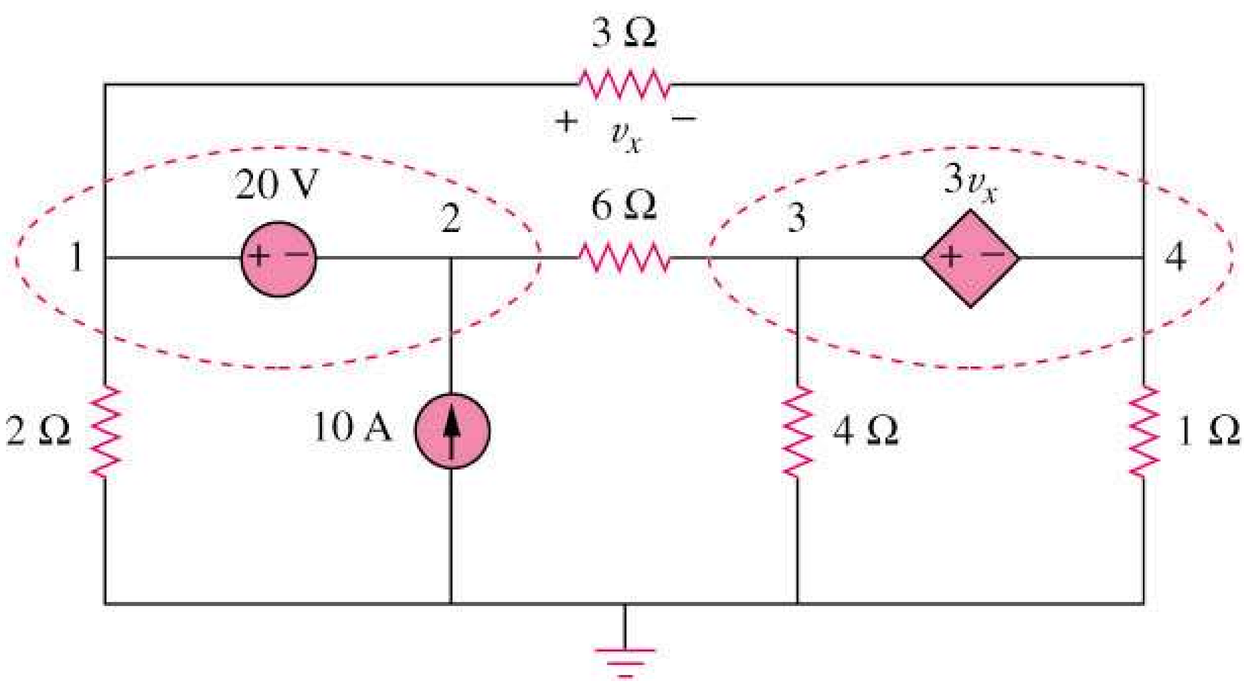
$$i = \varphi_D - \varphi_A$$

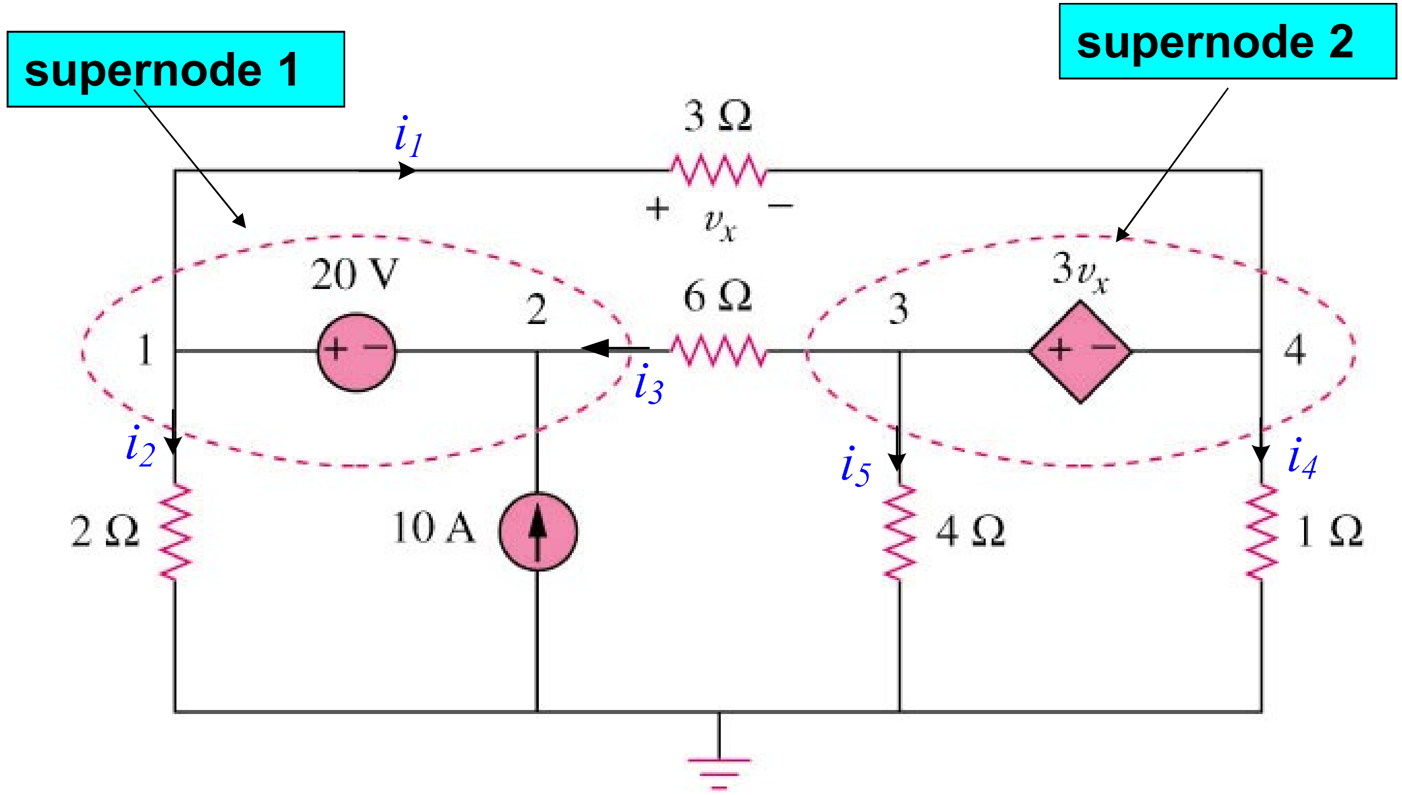


$$\varphi_A = -1V \qquad \varphi_B = \frac{17}{9}V \qquad \varphi_C = \frac{17}{3}V \qquad \varphi_C = \frac{1}{3}V$$

3.3 Nodal Analysis with Voltage Source (4)

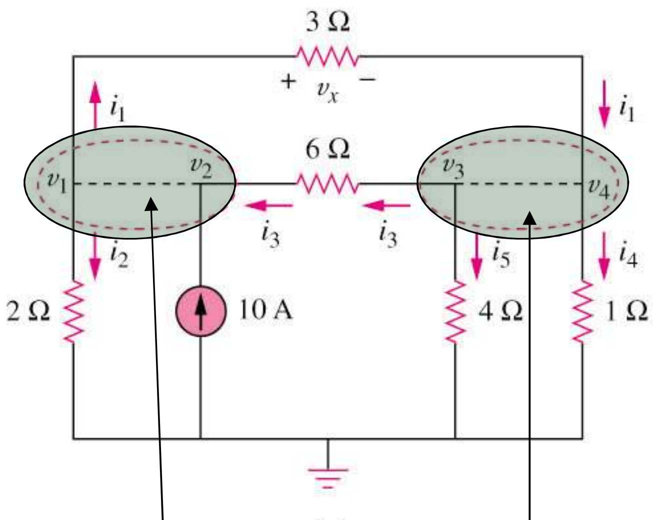
<u>Example 7</u> – circuit with one independent voltage source and one dependent voltage source



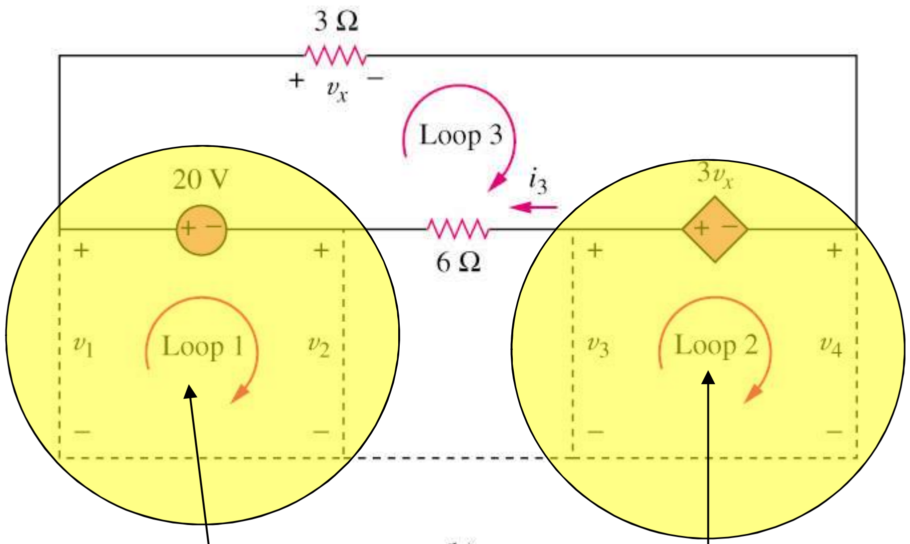


supernode 1
supernode 2
3 Ω
i1
+ vx −
20 V
6 Ω
3vx
1
2
3
4
i3
i2
i5
i4
2 Ω
10 A
4 Ω
1 Ω

Example 7 – circuit with two independent voltage sources



(a)



(b)

$i_1+i_2-i_3=10$ $\quad$ $i_3+i_5+i_4-i_1=0$ $\quad$ $-v_1+20+v_2=0$ $\quad$ $-v_3+3v_x+v_4=0$

Express currents using node voltages, then we obtain

$$i_1 = \frac{v_1 - v_4}{3} \qquad i_2 = \frac{v_1}{2} \qquad i_3 = \frac{v_1 - v_2}{6} \qquad i_4 = \frac{v_4}{1} \qquad i_5 = \frac{v_3}{4}$$

$$\frac{2}{3}v_1 + \frac{1}{6}v_2 - \frac{1}{3}v_4 = 10$$

$$-\frac{1}{6}v_1 - \frac{1}{6}v_2 + \frac{1}{4}v_4 + \frac{4}{3}v_4 = 0$$

$$-v_1 + v_2 = -20$$

$$-v_3 + 3v_x + v_4 = 0$$

$$v_x = v_1 - v_4$$

- **One or more branch is the connection of current sources (independent, dependent) and resistors (or conductors) in series**

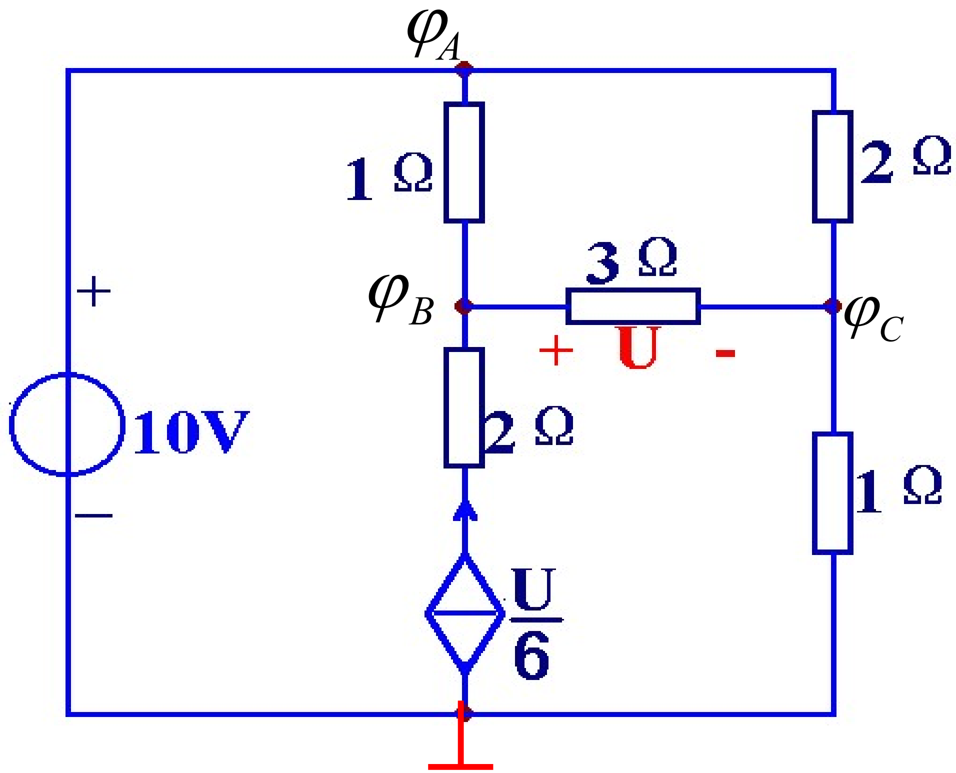
Method: In this case, the resistor or conductance which are connected to current sources in series are not considered in KCL equations.

$$\varphi_A = 10$$

$$-\varphi_A + (1 + \frac{1}{2} + \frac{1}{3})\varphi_B - \frac{1}{3}\varphi_C = \frac{U}{6}$$

$$-\frac{1}{2}\varphi_A - \frac{1}{3}\varphi_B + (1 + \frac{1}{2} + \frac{1}{3})\varphi_C = 0$$

$$U = \varphi_B - \varphi_C$$



Problem-solving Strategy
Nodal Analysis

- Select one node in N-node circuit as the reference node. Assume that the node voltage is zero and measure all nodes voltages with respect to this node.
- If only independent current sources are present in the network, write the KCL equations at the N-1 nonreference nodes. If dependent current sources are present, write the KCL equations as is done for networks with only independent current sources; then write the controlling equations for the dependent sources.

- If voltage sources are present in the network, they may be connected

  (1) between the reference node and a nonreference node;

  (2) between two nonreference nodes.

  In the former case, if the voltage source is an independent source, then the voltage at one of the nonreference nodes is known. If the source is dependent, it is treated as an independent source when writing the KCL equation, but an additional constraint equation is necessary, as described previously.

- In the latter case, if the source is independent, the voltage between the two nodes is constrained by the value of the voltage source, and an equation describing the this constraint represents one of the N-1 linearly independent equations required to determine the N-1 node voltages. The surface of the network described by the constraint equation (i.e., the source and two connecting nodes) is called a supernode.

One of the remaining N-1 linearly independent equations is obtained by applying KCL at this supernode. If the voltage source is dependent, it is treated as an independent source when writing the KVL equations, but an additional constraint equation is necessary, as described previously.