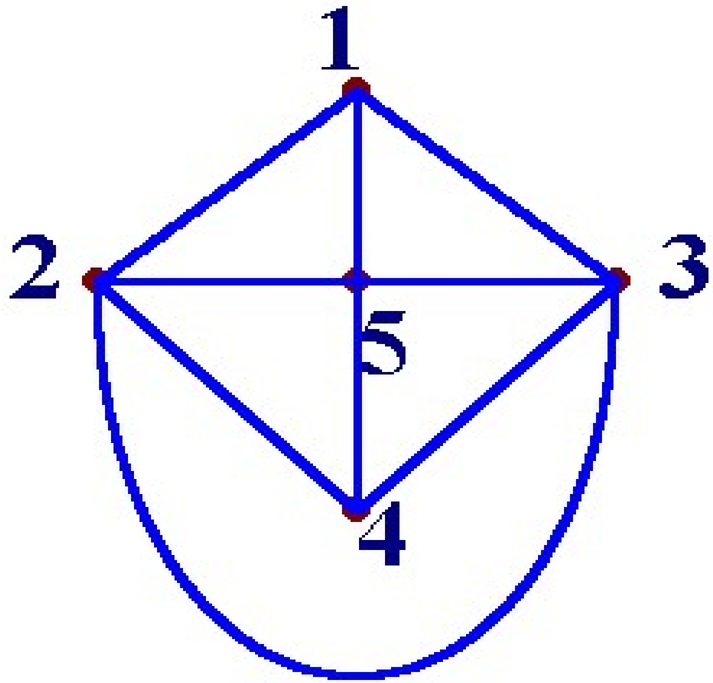
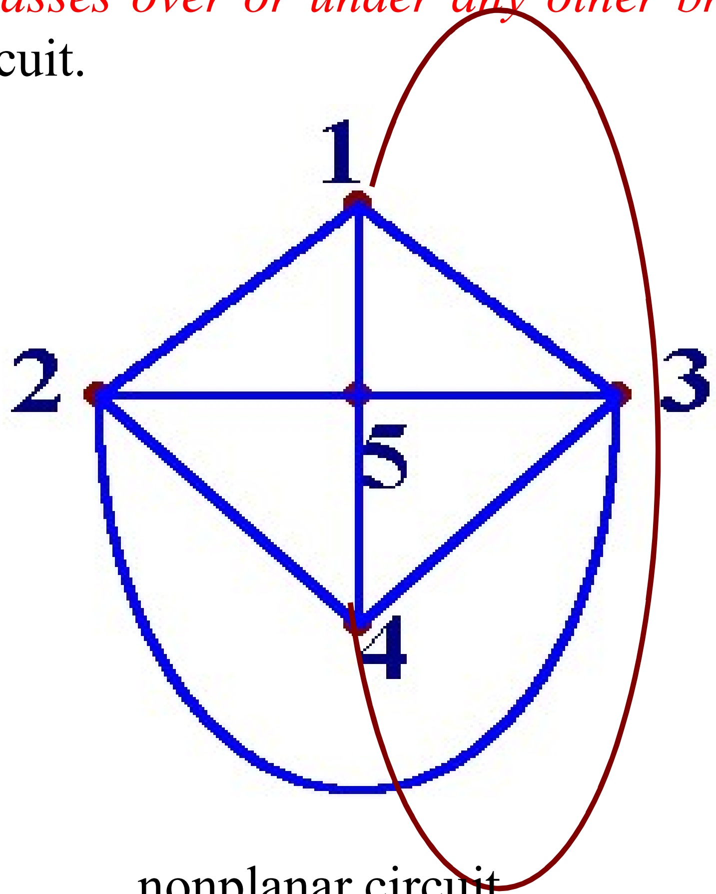
3.4 Mesh Analysis (1)

Planar network: If it is possible to draw the diagram of a circuit on a plane surface in such a way that *no branch passes over or under any other branch*, then that circuit is said to be a planar circuit.



planar circuit



nonplanar circuit

1.Mesh

A *mesh* is a loop which does not contain any other loops within it.

2.Mesh current

We define a mesh current as a current that flows only around the perimeter of a mesh.

$i_1=I_a$

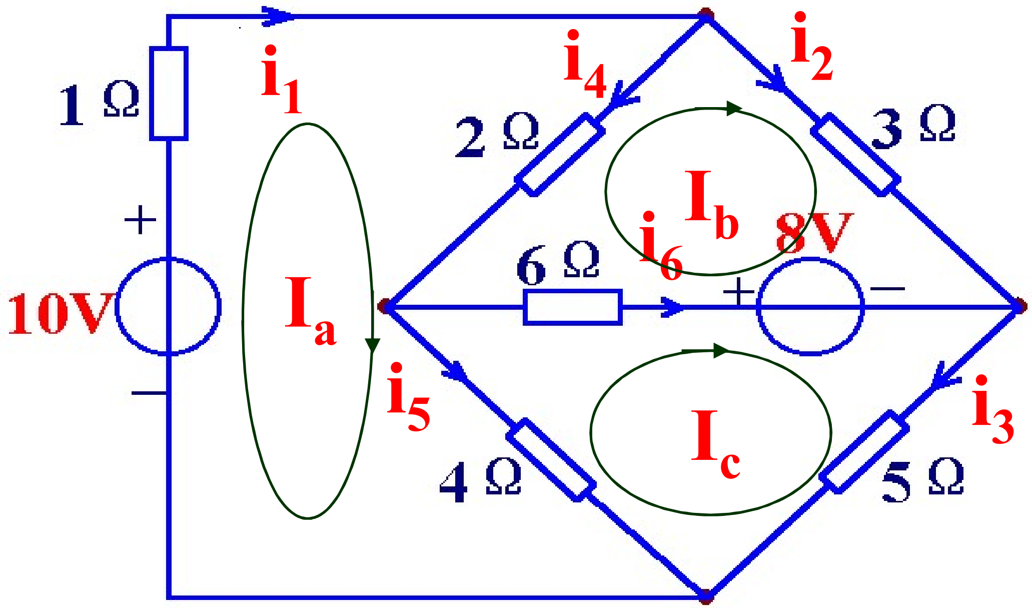
$i_2=I_b$

$i_3=I_c$

$i_4=i_1-i_2=I_a-I_b$

$i_5=i_1-i_3=I_a-I_c$

$i_6=i_3-i_2=I_c-I_b$



- Mesh analysis provides another general procedure for analyzing circuits using ***mesh currents*** as the circuit variables.

- Nodal analysis applies KCL to find unknown voltages in a given circuit, while ***mesh analysis applies KVL*** to find unknown currents.

- **We should notice that the mesh current is imaginative, not measurable directly.**

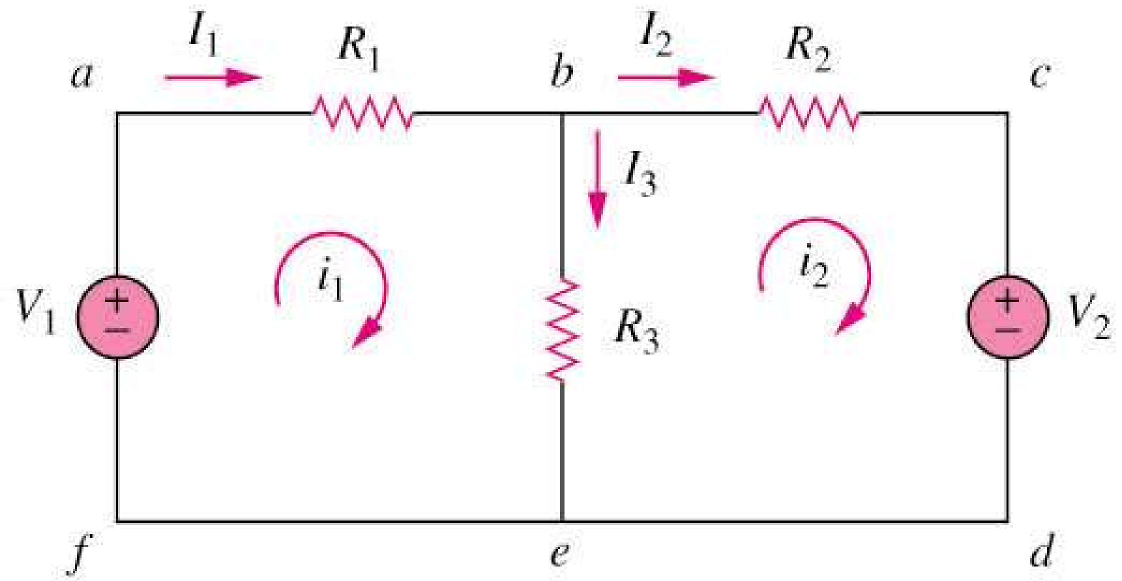
3.4 Mesh Analysis (2)
(circuits containing only independent voltage source)

Steps to determine the mesh currents:

- ***Assign*** **mesh currents i_1, i_2, ..., in to the n meshes.**
- ***Apply*** **KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.**
- ***Solve*** **the resulting n simultaneous equations to get the mesh currents.**

3.4 Mesh Analysis (3)

<u>Example 8</u> – circuit with independent voltage sources



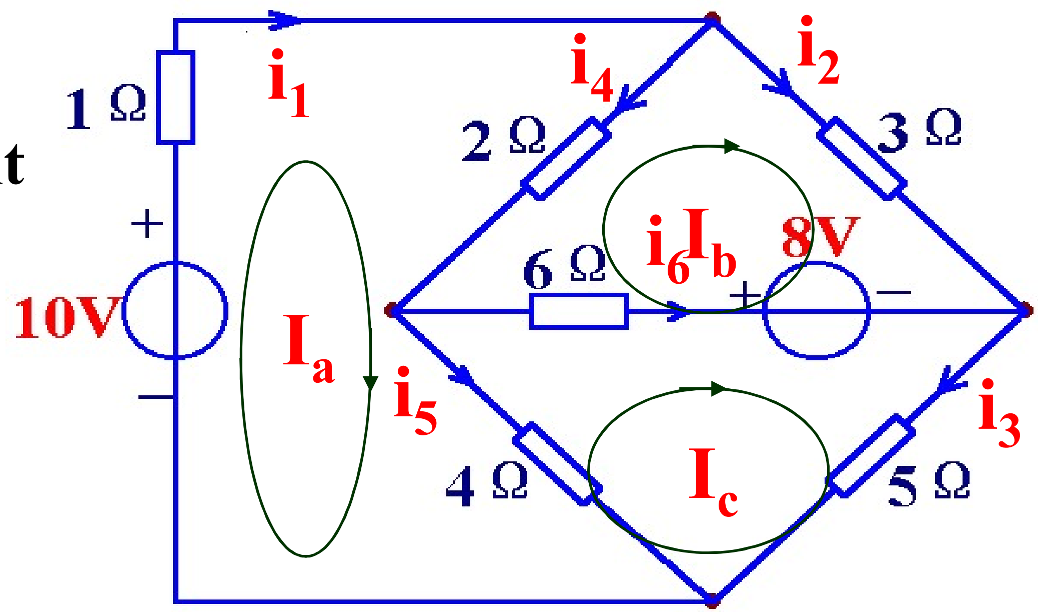
Note:

i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

Derive mesh current equations from simple circuit



Apply KVL to mesh a,

$i_1 + 2i_4 + 4i_5 = 10$

or

$i_a + 2(i_a - i_b) + 4(i_a - i_c) = 10$

or $\boxed{(1+2+4)i_a - 2i_b - 4i_c = 10}$

回路电压源电压升代数和

互电阻

self-
自电

mutual-resistance
互电阻

$\boxed{-2i_a + (3+2+6)i_b - 6i_c = 8}$

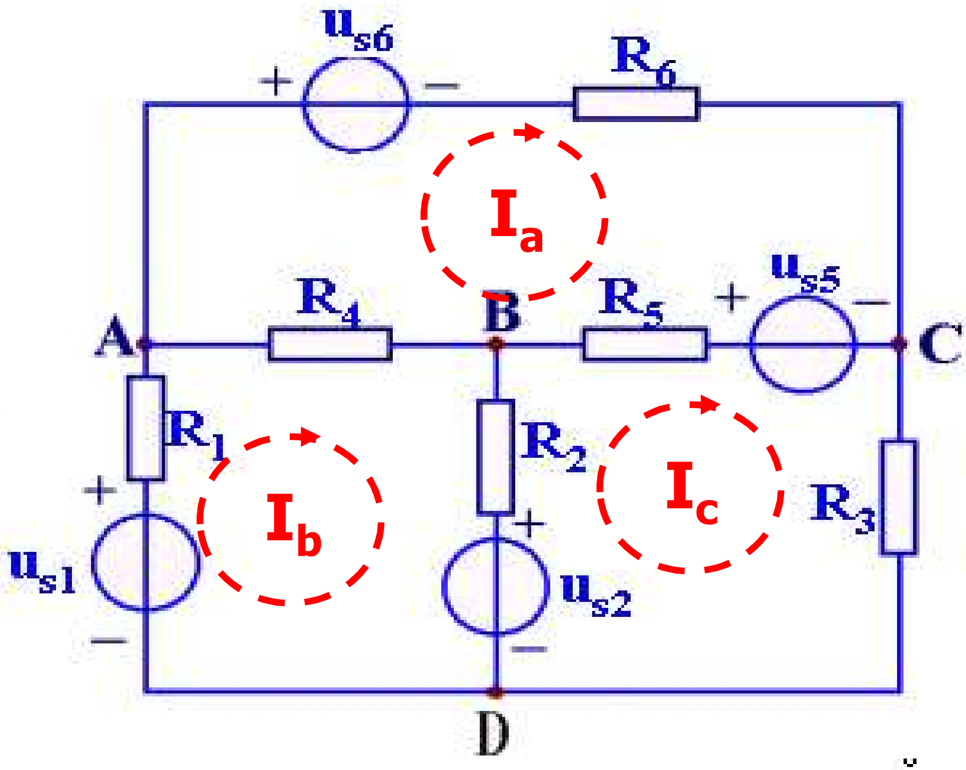
$\boxed{-4i_a - 6i_b + (4+6+5)i_c = -8}$

The self-resistance is the effective resistance of the resistors in series within a mesh, so it is simple the sum of all the resistance in the mesh.

The mutual-resistance(with a minus sign) is the resistance that the mesh has in common with the neighboring mesh.

Example 9

Write the mesh equations of the circuit shown in right Fig.



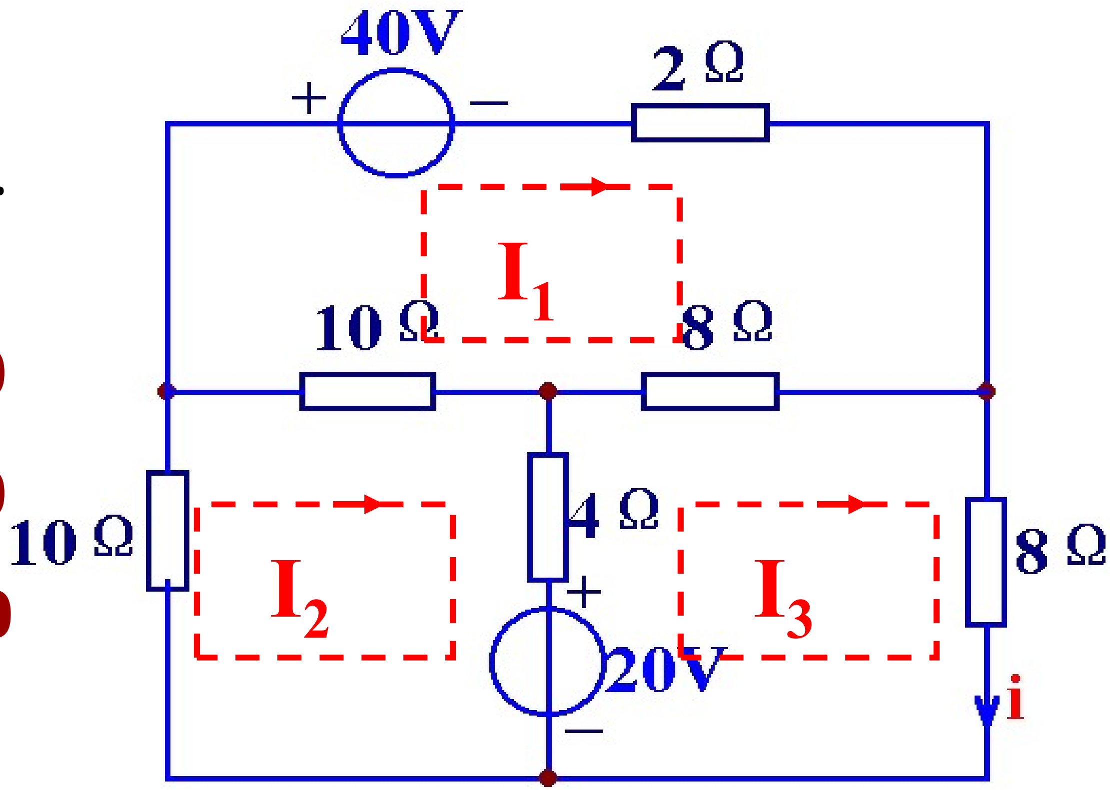
$$(R_4+R_5+R_6)I_a-R_4I_b-R_5I_c=u_{s5}-u_{s6}$$

$$-R_4I_a+(R_4+R_1+R_2)I_b-R_2I_c=u_{s1}-u_{s2}$$

$$-R_5I_a-R_2I_b+(R_5+R_3+R_2)I_c=u_{s2}-u_{s5}$$

Example 11. Find I in the following circuit using mesh analysis.

Solution:



Assign mesh direction and write the KVL equation for each mesh.

$$20I_1 - 10I_2 - 8I_3 = -40$$

$$-10I_1 + 24I_2 - 4I_3 = -20$$

$$-8I_1 - 4I_2 + 20I_3 = 20$$

Solve the equations, we obtain that

$$I_3 = \frac{\begin{vmatrix} 20 & -10 & -40 \\ -10 & 24 & -20 \\ -8 & -4 & 20 \end{vmatrix}}{\begin{vmatrix} 20 & -10 & -8 \\ -10 & 24 & -4 \\ -8 & -4 & 20 \end{vmatrix}} = -0.956\text{A}$$

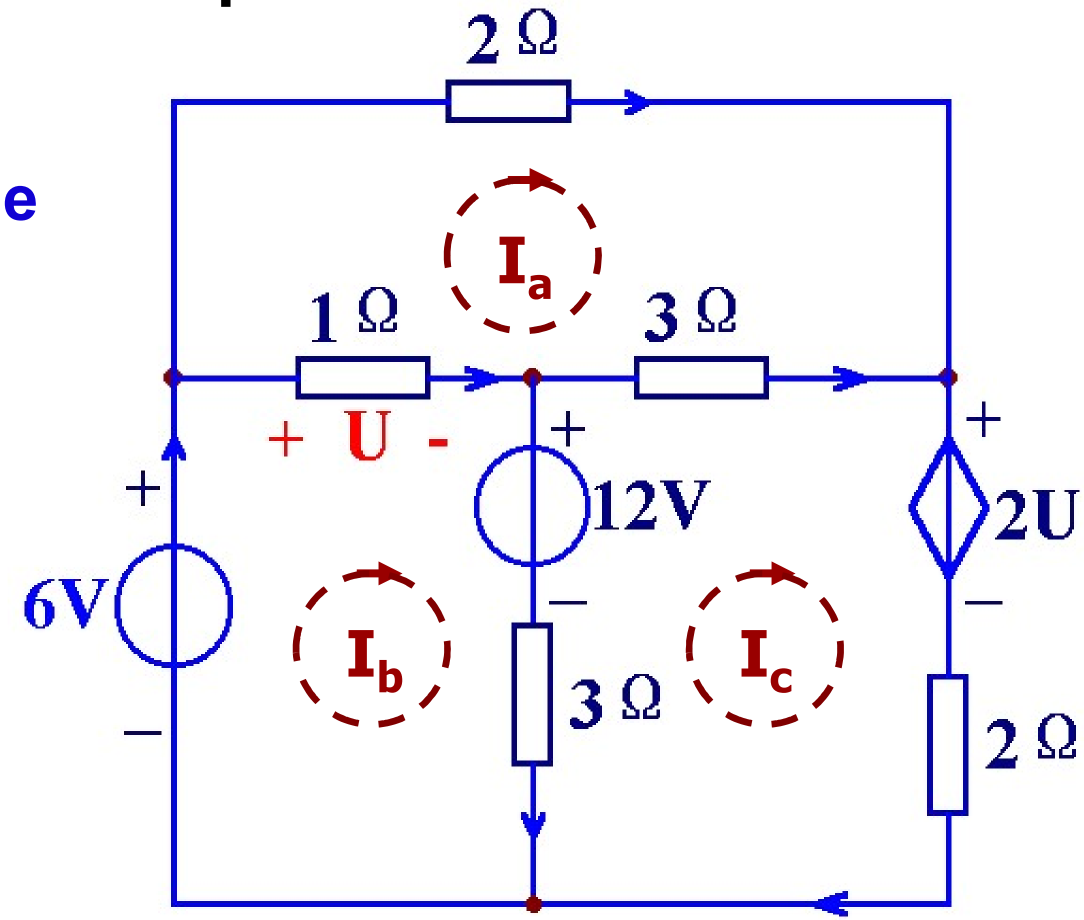
thus $i = I_3 = -0.956\text{A}$

3.4 Mesh Analysis (4)

The mesh-current method and dependent sources

Method: First, treat dependent sources ,just as we have in the past, as though it were an independent source when writing the KVL equations. Then write the controlling equation for the dependent source.

Find all the current in the circuit shown in Fig.



Solution:

Assign the mesh currents and apply KVL to each mesh

We obtain

$$6I_a - I_b - 3I_c = 0$$

$$-I_a + 4I_b - 3I_c = -6$$

$$-3I_a - 3I_b + 8I_c = 12 - 2U$$

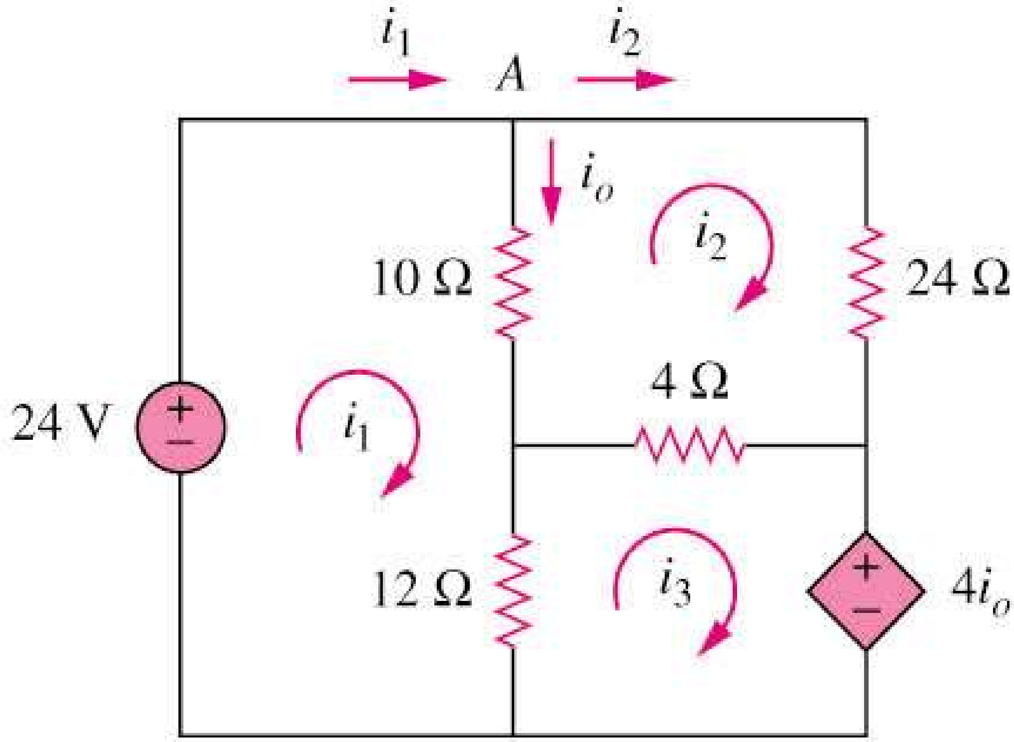
$$U = I_b - I_a$$

Solve for the mesh currents

$I_a = 1.29A$ $I_b = 0.61A$

$I_c = 2.38A$ $U = -0.68V$

<u>Example 12</u> – circuit with dependent voltage source

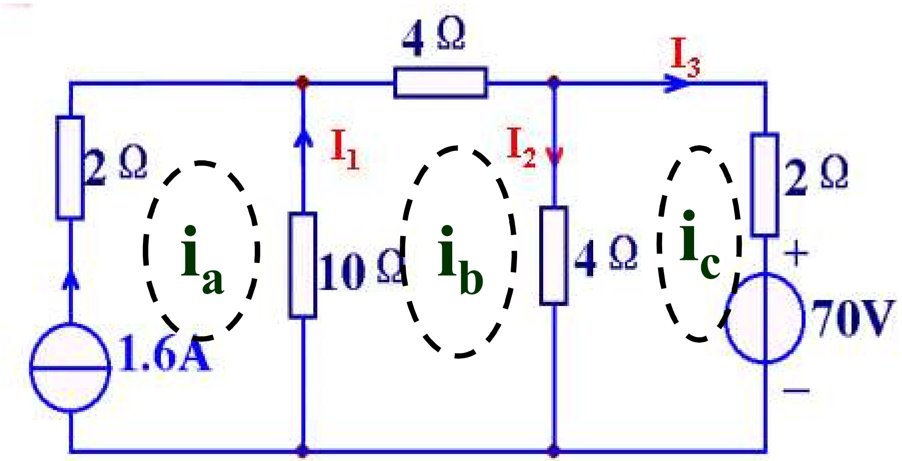


*Answer I_o = 1.5A

3.5 Mesh Analysis with Current Source (1)

Case 1. When a current source exists only in one mesh.

For the mesh containing the current source, its mesh current is known.



For other meshes, write the mesh current equation using KVL.

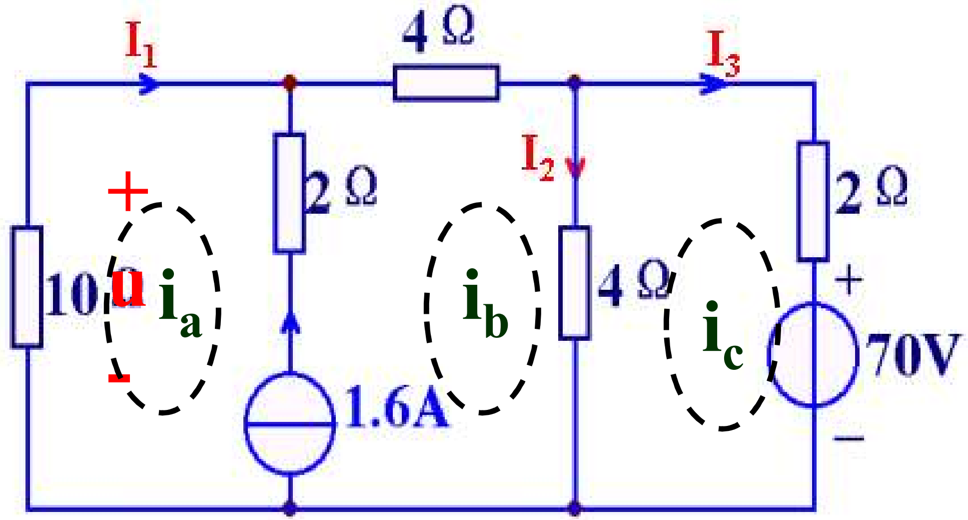
$$i_a = 1.6$$

$$-10i_a+18i_b-4i_c=0$$

$$-4i_b+6i_c=-70$$

Case 2. A current source exists between two meshes .

2 approaches



Solution 1: Set u as the voltage across the current source, then add a constraint equation.

$$12i_a - 2i_b = -u$$

$$-4i_b + 6i_c = -70$$

$$-2i_a + 10i_b - 4i_c = u$$

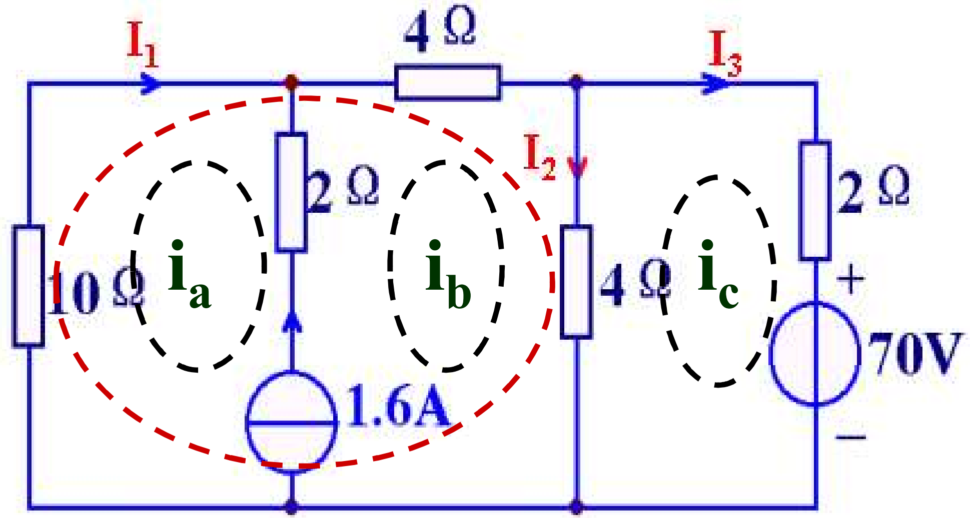
$$i_b - i_a = 1.6$$

Solution 2: Use supermesh(广义网孔) to solve the problem(网孔回路法).

$$i_b - i_a = 1.6$$

$$(4+4+10)i_a+(4+4)i_b-4i_c= 0$$

$$-4i_a-4i_b+6i_c= -70$$

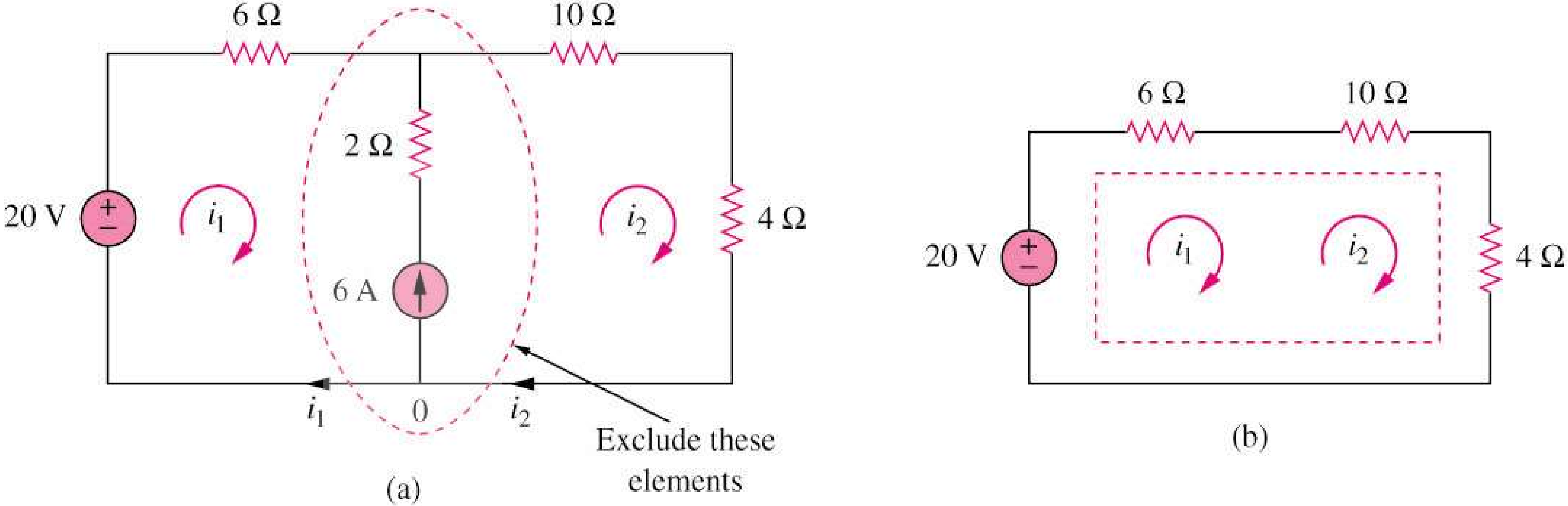


Loop a:
$4(i_a+i_b)+4(i_a+i_b-i_c)+10i_a=0$

Loop c:
$2i_c+4(i_c-i_a-i_b)=-70$

supermesh

Circuit with current source

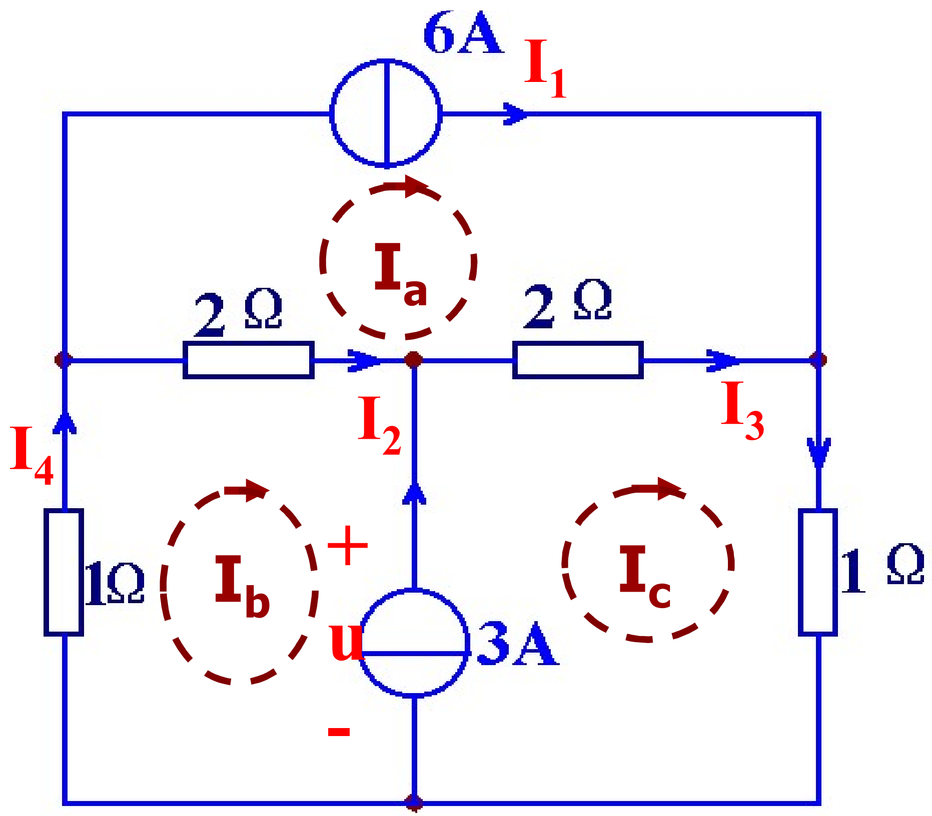


A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

Find all the branch currents.

Solution 1:

Assign mesh currents



And write the mesh current equations using KVL

$$I_a=6$$

$$-2I_a+3I_b = -u$$

$$-2I_a+3I_c= u$$

$$-I_b+I_c=3$$

Thus

$$I_b=2.5A$$

$$I_c=5.5A$$

So

$$I_1=I_a=6A$$

$$I_2=I_b-I_a=-3.5A$$

$$I_3=I_c-I_a=-0.5A$$

Solution 2:

Use supermesh(广义网孔) to solve the problem.

$I_a=6$

$I_c=3$

$(2+2+1+1)\ I_b-(2+2)I_a+(2+1)I_c=0$

thus

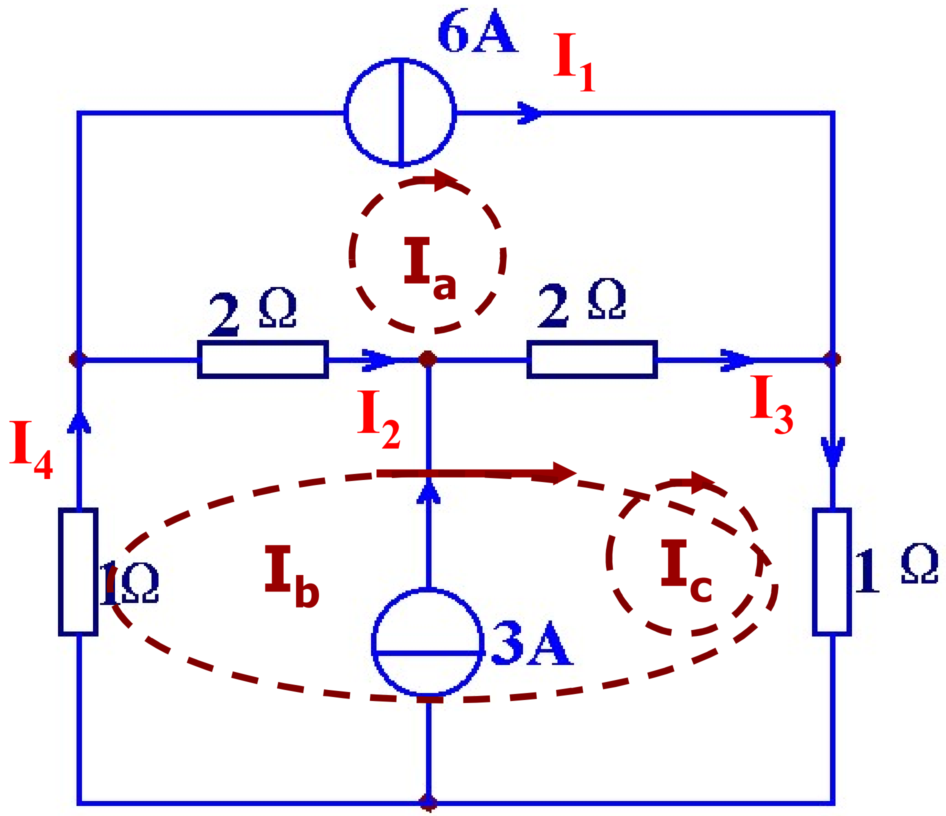
$I_b=2.5A$

So

$I_1=I_a=6A$

$I_2=I_b-I_a=-3.5A$

$I_3=I_b+I_c-I_a=-0.5A$



3.5 Mesh Analysis with Current Source (2)

The properties of a super-mesh:

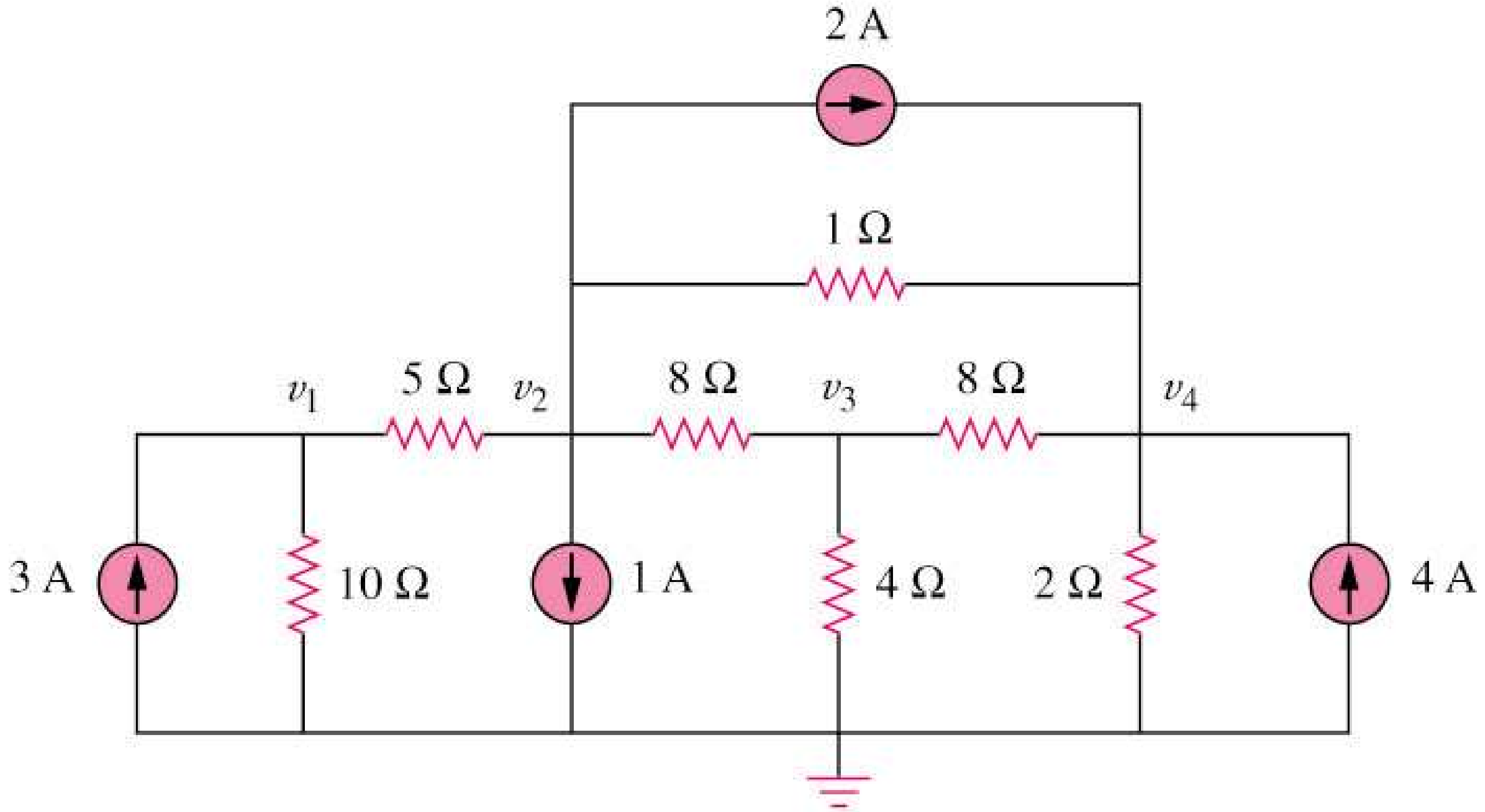
1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

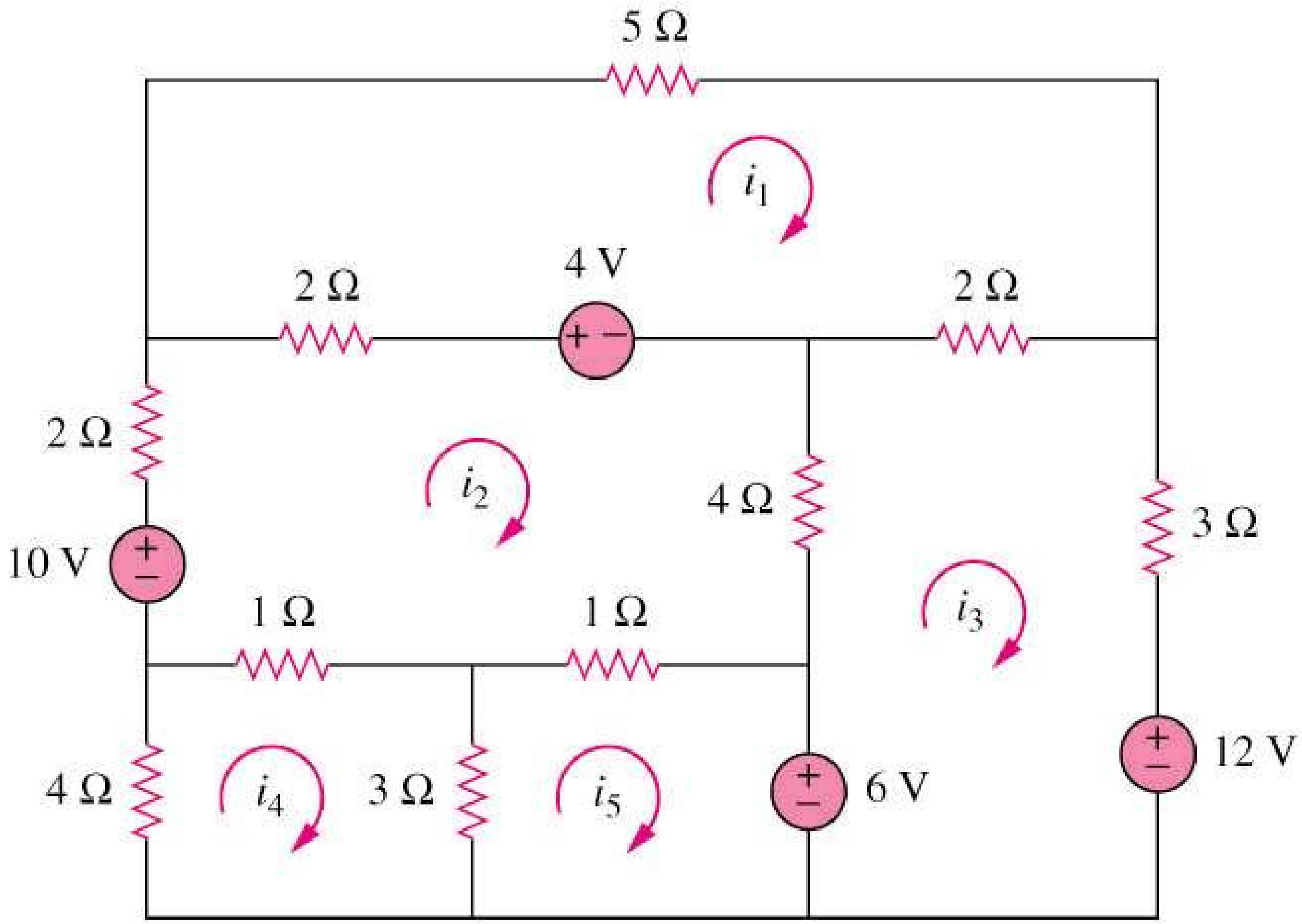
3. A super-mesh requires the application of both KVL and KCL.

3.6 Nodal and Mesh Analysis with Inspection (1)

<u>Example 13</u> – By inspection, write the nodal voltage equations for the circuit



Example 14 – By inspection, write the mesh-current equations for the circuit



3.7 Nodal versus Mesh Analysis (1)

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.

   *Choose mesh analysis for circuit with fewer meshes than nodes.

   *Networks that contain many series-connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.

   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.

2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

3.8 Summary and Review

- Prior to beginning an analysis, make a neat, simple circuit diagram. Indicate all element and source values. Each source should have a reference symbol.
- If nodal analysis is the chosen approach, choose one of the nodes as a reference node. Then label the node voltage v_1, v_2, …, $v_{N\text{-}1}$, remembering that each is understood to be measured with respect to the reference node.
- If the circuit contains only current sources, apply KCL at each nonreference node.

- If the circuit contains voltages sources, form a supernode about each one, and then proceed to apply KCL at all nonreference nodes and supernodes.
- If you are considering mesh analysis, first make certain that the network is a planar network.
- Assign a clockwise mesh current in each mesh: i_1, i_2, …, i_M.
- If the circuit contains only voltage sources, apply KVL around each mesh.
- If the circuit contains current sources, create a supermesh for each one current source that is common to two meshes, and then apply KVL around each mesh and supermesh.

- In deciding whether to use nodal or mesh analysis for a planar circuit, a circuit with fewer nodes/supernodes will result fewer equations using nodal analysis.
- Computer-aided analysis is useful for checking results and analyzing circuits with large numbers of elements. However, common sense must be used to check simulation results.