The navier stokes equation is the following:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} \tag{1}$$

For now, we will consider it in two dimensions as in $\mathbf{v} = (u, v)$. We write this as the following set of differential equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) \tag{3}$$

$$\frac{\partial^2 p}{dx^2} + \frac{\partial^2 p}{\partial y^2} = -\rho(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}]\frac{\partial v}{\partial y}) + \rho\frac{\partial}{\partial t}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \tag{4}$$

We use finite difference methods to discretize each formula, using a combination of forward-difference, backward-difference and central-difference estimators.

$$\frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} + u_{i,j}^n \frac{u_{i,j}^n - u_{i-1,j}^n}{\Delta x} + v_{i,j}^n \frac{u_{i,j}^n - u_{i,j-1}^n}{\Delta y} = \tag{5}$$

$$-\frac{1}{\rho}(\frac{p_{i+1,j}^n - p_{i-1,j}^n}{2\Delta x}) + \nu(\frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{\Delta x^2} + \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j}^n}{\Delta y^2}) \tag{6}$$

A similar one can be done for the v momentum equation. We also write out the estimator for the Poisson pressure equation:

$$\frac{p_{i+1,j}^n - 2p_{i,j}^n + p_{i-1,j}^n}{\Delta x^2} + \frac{p_{i,j+1}^n - 2p_{i,j}^n + p_{i,j-1}^n}{\Delta y^2} = \tag{7}$$

$$-\rho((\frac{u_{i+1,j}^n - u_{i-1,j}^n}{2\Delta x})^2 + 2(\frac{u_{i,j+1}^n - u_{i,j-1}^n}{2\Delta y})(\frac{v_{i+1,j}^n - v_{i-1,j}^n}{2\Delta x}) + (\frac{v_{i,j+1}^n - v_{i,j-1}^n}{2\Delta y})^2) \tag{8}$$

$$+\frac{\rho}{\Delta t}(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y}) \tag{9}$$

Notice in the first finite difference equation, there is only one unknown: $u_{i,j}^{n+1}$, everything else is known. For pressure, it is an iterative algorithm so we need to run it many times.