

Task 1: Compute Probabilities

Customer A:

$$z = 0.8(1) + 0.4(4) - 4 = -1.6$$

$$\rightarrow \hat{y} = \frac{1}{1+e^{1.6}} \approx 0.168$$

Customer B:

$$z = 0.8(2) + 0.4(3) - 4 = -1.2$$

$$\rightarrow \hat{y} = \frac{1}{1+e^{1.2}} \approx 0.231$$

Customer C:

$$z = 0.8(3) + 0.4(7) - 4 = 1.2$$

$$\rightarrow \hat{y} = \frac{1}{1+e^{-1.2}} \approx 0.769$$

Customer D:

$$z = 0.8(5) + 0.4(2) - 4 = 0.8$$

$$\rightarrow \hat{y} = \frac{1}{1+e^{-0.8}} \approx 0.690$$

Customer E:

$$z = 0.8(6) + 0.4(6) - 4 = 3.2$$

$$\rightarrow \hat{y} = \frac{1}{1+e^{-3.2}} \approx 0.960$$

Customer	x_1	x_2	y	\hat{y}
A	1	4	0	0.168
B	2	3	0	0.231
C	3	7	1	0.769
D	5	2	1	0.69
E	6	6	1	0.96

Task 2: Compute Average Loss

Binary Cross-Entropy (BCE) Loss: $L = -(y \cdot \ln(\hat{y}) + (1-y) \cdot \ln(1-\hat{y}))$

Customer A ($x_1=1, x_2=4, y=0, \hat{y} \approx 0.168$)

$$L = -[0 \cdot \ln(0.168) + (1-0) \ln(1-0.168)]$$

$$L = -[0 + \ln(0.832)]$$

$$L = -\ln(0.832)$$

$$L \approx 0.1839$$

Customer B ($x_1=2, x_2=3, y=0, \hat{y} \approx 0.231$)

$$L = -[0 \cdot \ln(0.231) + (1-0) \ln(1-0.231)]$$

$$L = -[0 + \ln(0.769)]$$

$$L = -\ln(0.769)$$

$$L \approx 0.2633$$

Customer C ($x_1=3, x_2=7, y=1, \hat{y} \approx 0.769$)

$$L = -[1 \cdot \ln(0.769) + (1-1) \ln(1-0.769)]$$

$$L = -[\ln(0.769) + 0]$$

$$L = -\ln(0.769)$$

$$L \approx 0.2633$$

Customer D ($x_1=5, x_2=2, y=1, \hat{y} \approx 0.690$)

$$L = -[1 \cdot \ln(0.690) + (1-1) \ln(1-0.690)]$$

$$L = -[\ln(0.690) + 0]$$

$$L = -\ln(0.690)$$

$$L \approx 0.3711$$

Customer E ($x_1=6, x_2=6, y=1, \hat{y} \approx 0.961$)

$$L = -[1 \cdot \ln(0.961) + (1-1) \ln(1-0.961)]$$

$$L = -[\ln(0.961) + 0]$$

$$L = -\ln(0.961)$$

$$L \approx 0.03995$$

Customer	x_1	x_2	y	\hat{y}	Loss
A	1	4	0	0.168	0.1839
B	2	3	0	0.231	0.2633
C	3	7	1	0.769	0.2633
D	5	2	1	0.69	0.3711
E	6	6	1	0.96	0.0399

Average Loss = $(0.1839 + 0.2633 + 0.2633 + 0.3711 + 0.0399) / 5 = 0.2243$

Task 3: Gradient Descent Update

$$\begin{aligned}\frac{\partial L}{\partial m_1} &= \frac{1}{5} [(0.168)(1) + (0.231)(2) + (-0.231)(3) + (-0.310)(5) + (-0.040)(6)] \\&= \frac{1}{5} [0.168 + 0.462 - 0.693 - 1.550 - 0.240] \\&= \frac{-1.853}{5} \approx -0.3706\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial m_1} &= \frac{1}{5} [(0.168)(4) + (0.231)(3) + (-0.231)(7) + (-0.310)(2) + (-0.040)(6)] \\&= \frac{1}{5} [0.672 + 0.693 - 1.617 - 0.620 - 0.240] \\&= \frac{-1.112}{5} \approx -0.2224\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial m_1} &= \frac{1}{5} [0.168 + 0.231 - 0.231 - 0.310 - 0.040] \\&= \frac{-0.182}{5} \approx -0.0364\end{aligned}$$

Learning rate $\eta = 0.1$

$$m_1^{new} = m_1 - \eta \frac{\partial L}{\partial m_1} = 0.8 - 0.1(-0.3706) = 0.8 + 0.0371 \approx 0.8371$$

$$m_2^{new} = m_2 - \eta \frac{\partial L}{\partial m_1} = 0.4 - 0.1(-0.2224) = 0.4 + 0.0222 \approx 0.4222$$

$$b^{new} = b - \eta \frac{\partial L}{\partial m_1} = -4 - 0.1(-0.0364) = -4 + 0.0036 \approx -3.9964$$

Task 4: New Probabilities

Customer A ($x_1=1, x_2=4, y=0$)

$$z = 0.8371(1) + 0.4222(4) - 3.9964$$

$$= 0.8371 + 1.6888 - 3.9964 = -1.4705$$

$$= -1.4705$$

$$\hat{y} = \frac{1}{1 + e^{-(-1.4705)}} = \frac{1}{1 + e^{1.4705}} \approx 0.1869$$

Customer B ($x_1=2, x_2=3, y=0$)

$$z = 0.8371(2) + 0.4222(3) - 3.9964$$

$$= 1.6742 + 1.2666 - 3.9964$$

$$= -1.0556$$

$$\hat{y} = \frac{1}{1 + e^{1.0556}} \approx 0.2582$$

Customer C ($x_1=3, x_2=7, y=1$)

$$z = 0.8371(3) + 0.4222(7) - 3.9964$$

$$= 2.5113 + 2.9554 - 3.9964$$

$$= 1.4703$$

$$\hat{y} = \frac{1}{1 + e^{-1.4703}} \approx 0.8131$$

Customer D ($x_1=5, x_2=2, y=1$)

$$z = 0.8371(5) + 0.4222(2) - 3.9964$$

$$= 4.1855 + 0.8444 - 3.9964$$

$$= 1.0335$$

$$\hat{y} = \frac{1}{1 + e^{-1.0335}} \approx \mathbf{0.7376}$$

Customer E ($x_1=6$, $x_2=6$, $y=1$)

$$z = 0.8371(6) + 0.4222(6) - 3.9964$$

$$= 5.0226 + 2.5332 - 3.9964$$

$$= 3.5594$$

$$\hat{y} = \frac{1}{1 + e^{-3.5594}} \approx \mathbf{0.9723}$$

Customer	x_1	x_2	y	z (4 dp)	new \hat{y}
A	1	4	0	-1.4705	0.1869
B	2	3	0	-1.0556	0.2582
C	3	7	1	1.4703	0.8131
D	5	2	1	1.0335	0.7376
E	6	6	1	3.5594	0.9723

Task 5: New Average Loss

Customer A

$$L = -[0 \cdot \ln(0.1869) + (1-0) \ln(1-0.1869)]$$
$$\ln(0.813133371673) \approx -0.206860134091$$

Loss ≈ 0.2069

Customer B

$$L = -[0 \cdot \ln(0.2582) + (1-0) \ln(1-0.2582)]$$
$$\ln(0.741848802098) \approx -0.298609827336$$

Loss ≈ 0.2986

Customer C

$$L = -[1 \cdot \ln(0.813102980271) + (1-1) \ln(1-0.813102980271)]$$
$$\ln(0.813102980271) \approx -0.206897510455$$

Loss ≈ 0.2069

Customer D

$$L = -[1 \cdot \ln(0.737593880956) + 0]$$
$$\ln(0.737593880956) \approx -0.304361902658$$

Loss ≈ 0.3044

Customer E

$$L = -\ln(0.972331440445)$$
$$\ln(0.972331440445) \approx -0.028058544533$$

Loss ≈ 0.0281

Customer	x_1	x_2	y	new \hat{y}	new Loss
A	1	4	0	0.1869	0.2069
B	2	3	0	0.2582	0.2986
C	3	7	1	0.8131	0.2069
D	5	2	1	0.7376	0.3044
E	6	6	1	0.9723	0.0281

New Average Loss = 0.206860134091 + 0.298609827336+ 0.206897510455 + 0.304361902658 + 0.028058544533 / 5

= 1.044788 / 5

New Average Los \approx 0.209