Automatic Tuning Algorithm of the PID Controller Using Two Nyquist Points Identification

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Abstract: This paper deals with an advanced automatic tuning algorithm of the PID controller using two Nyquist points identification. The critical point on the Nyquist plot and its neighboring point are identified from the modified relay feedback experiment and used to make the formulas to determine the parameters of the PID controller. The formulas are derived from the dominant poles approximation and placement method. We can consider the time domain response of the process with damping ratio(ζ) as a design specification.

1. Introduction

The Proportional-Integral-Derivative(PID) controllers are still widely used in the process industries even though more advanced control techniques have been developed. The PID controller has many merits of the simple configuration and operation. However, it is difficult to tune the parameters of the PID controller correctly with less effort. The procedure to tune the parameters requires difficult and time-consuming routines for testing, comparing and analyzing the process control loop. The process engineers have to retune the parameters whenever the process dynamics have changed due to aging or equipment change as well as initial process operation. So, many control engineers have recognized the importance of the automatic tuning of the PID controller and studied the problem with eager.

Generally, the automatic tuning methods are composed of the procedures to identify the process dynamics and to determine the parameters of the PID controller, and divided into two main categories according to the identification method of the process. The continuous cycling methods[1-4] use the relay feedback experiment which generates a limit oscillation in the process response. The critical point on the Nyquist plot of the process is identified from the experiment and directly applied to the formulas to figure out the parameters of the PID controller such as Ziegler-Nichols formulas[5,6]. The System identification methods[7-9] have the procedure to make the process model using various system identification algorithms based on the data set acquired from the process input and output. The parameters of the PID controller are computed directly from the identified process model.

This paper presents an automatic tuning algorithm of the PID controller. As an identification method of the given process dynamics, the modified relay feedback configuration is suggested to identify the Nyquist critical point and its neighboring point on the complex plane of the process. The formulas to determine the parameters of PID controller are derived from the dominant poles approximation and placement method. The proposed algorithm provides more design flexibility considering the time response of the process than the existing algorithm.

2. Automatic Tuning Algorithm

2.1 Nyquist points identification

The modified relay feedback configuration shown in Figure 1 is introduced to proceed to the identification step of the presented tuning algorithm. It has the combination of a relay and a designated time delay(T_d). The two Nyquist points (the critical point and its neighboring point) are identified from the results of this experiment.

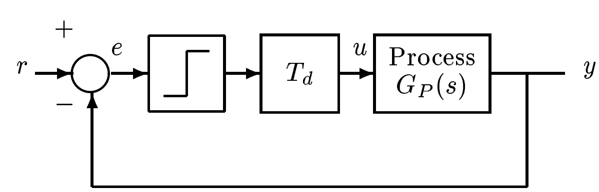


Figure 1. The modified relay feedback configuration

In case of $T_d = 0$, Figure 1 represents the ordinary relay feedback configuration[1]. As the result of the experiment, a limit oscillation having its original amplitude and period is generated. The critical gain(K_c) and the critical frequency(ω_c) are computed by the describing function analysis as given

$$K_c = \frac{4d}{\pi h}\ , \qquad \omega_c = 2\pi/T_c\ , \tag{1}$$

where d is the amplitude of the relay output, h is the amplitude of the oscillation and T_c is the period of the oscillation. The Nyquist equation of the process gives the critical point on the Nyquist Plot.

$$G_P(\omega_c) = -1/K_c \ , \tag{2}$$

that is

$$|G_P(\omega_c)| = 1/K_c \ , \qquad \arg|G_P(\omega_c)| = -\pi \ .$$

Now, this point should be converted to the point on the orthogonal complex plane for the future formulation.

$$G_P(j\omega_c) = a_1 + jb_1 \ , \tag{3}$$

where

$$a_1 = -1/K_c \ , \qquad b_1 = 0 \ .$$

In case of $T_d = \delta > 0$, the modified relay feedback experiment gives a new limit oscillation having different amplitude and period. The modified critical gain($\hat{K_c}$) and the modified critical frequency ($\hat{\omega_c}$) are computed as given

$$\hat{K_c} = \frac{4\hat{d}}{\pi\hat{h}} \ , \qquad \hat{\omega_c} = 2\pi/\hat{T_c} \ . \tag{4}$$

They gives the neighboring point of the Nyquist critical point.

$$G_P(\hat{\omega_c}) = -1/\hat{K_c} \tag{5}$$

Since the designated time delay T_d effects on the argument of $G_P(j\hat{\omega_c})$, this point gets a slightly lower position than that of the critical point on the Nyquist plane, i.e.,

$$|G_P(\hat{\omega_c})| = 1/\hat{K_c} \ , \qquad \arg|G_P(\hat{\omega_c})| = -\pi + T_d\hat{\omega_c} \ .$$

This point is converted to the point on the orthogonal complex plane

$$G_P(j\hat{\omega_c}) = a_2 + jb_2 \ , \tag{6}$$

where

$$\begin{aligned} a_2 &= \frac{1}{\hat{K_c}} \cos\arg|G_P(j\hat{\omega_c})| \\ b_2 &= \frac{1}{\hat{K_c}} \sin\arg|G_P(j\hat{\omega_c})| \ . \end{aligned}$$

2.2 Dominant poles approximation

The dominant poles located near the imaginary axis of the complex plane are determined from the knowledge of the two Nyquist points and the dominant poles approximation. The poles of the closed loop transfer function are given by the characteristic equation

$$1 + G_O(s) = 0 \ , \tag{7}$$

where $G_O(s)$ is the open loop transfer function. A Taylor series expansion around $s = j\omega$ gives

$$\begin{aligned} &1 + G_O(\sigma + j\omega) \\ &\quad = 1 + G_O(j\omega) + j\sigma G_O{}'(j\omega) + \ldots = 0 \ , \end{aligned} \tag{8}$$

where

$$G_O{}'(j\omega) = \frac{dG_O(j\omega)}{d\omega} \ .$$

Neglecting the terms of the second and higher orders, we can make the equation.

$$\sigma = j\frac{1 + G_O(j\omega)}{G_O{}'(j\omega)} \tag{9}$$

Then, both σ and ω of the dominant poles are determined. If we replace the differential term of the transfer function for the difference term between ω_c and $\hat{\omega_c}$, (9) is reproduced to

$$\sigma = \frac{G_O(j\omega_c) + 1}{G_O(j\omega_c) - G_O(j\hat{\omega_c})} j(\omega_c - \hat{\omega_c}) \ . \tag{10}$$

2.3 Determining PID parameters

The typical configuration of the PID control system is represented in Figure 2, where $G_C(s)$ is the PID controller transfer function and $G_P(s)$ is the process transfer function.

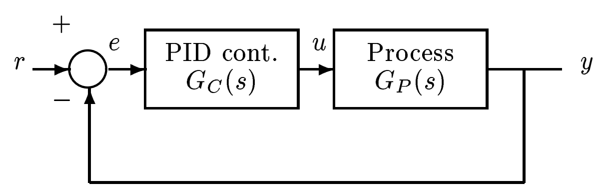


Figure 2. The typical PID control system

The PID controller is designed from the dominant poles placement method. The dominant poles are placed to the specific positions of the complex plane by the action of the PID controller. We can locate the dominant poles which are represented in terms of the damping ratio.

The values of the process transfer function at the two frequencies are

$$\begin{aligned} G_P(j\omega_c) &= a_1 + jb_1 \\ G_P(j\hat{\omega_c}) &= a_2 + jb_2 \ . \end{aligned} \tag{11}$$

The PID controller transfer function is

$$G_C(s) = K_p\left[1 + \frac{1}{T_i s} + T_d s\right]$$

and let

$$T_d = \alpha T_i \ , \tag{12}$$

then

$$G_C(s) = K_p\left[1 + \frac{1}{T_i s} + \alpha T_i s\right] \ . \tag{13}$$

The values of the open loop transfer function at ω_c and $\hat{\omega}_c$ are

$$\begin{aligned} G_O(j\omega_c) &= G_C(j\omega_c)G_P(j\omega_c) = c_1 + jd_1 \\ G_O(j\hat{\omega}_c) &= G_C(j\hat{\omega}_c)G_P(j\hat{\omega}_c) = c_2 + jd_2. \end{aligned} \tag{14}$$

Now, The design problem is to determine the parameters of PID controller(K_p, T_i and T_d) so that the points of (11) are mapped to the points of (14). The relationship between the damping ratio(ζ) and the natural frequency(σ) is given by

$$\sigma = \frac{\zeta\omega_c}{\sqrt{(1-\zeta^2)}} \ . \tag{15}$$

The equations (10) and (15) make

$$\frac{G_O(j\omega_c - j\hat{\omega}_c)}{G_O(j\omega_c)+1} = \frac{\sqrt{1-\zeta^2}}{\zeta}\frac{j(\omega_c - \hat{\omega}_c)}{\omega_c} = j\rho \ . \tag{16}$$

Putting the values of (14) into (16) gives

$$\begin{aligned} c_1 - c_2 + \rho_1 &= 0 \\ d_1 - d_2 - \rho(c_1+1) &= 0 \ . \end{aligned} \tag{17}$$

The integral time(T_i) and the proportional gain(K_p) are computed as follows

$$T_i = \frac{-\mathcal{B} + \sqrt{\mathcal{B}^2 - 4\mathcal{AC}}}{2\mathcal{A}} \ , \tag{18}$$

where

$$\begin{aligned} \mathcal{A} &= \alpha\rho a_1\omega_c + \alpha b_2\hat{\omega}_c \\ \mathcal{B} &= a_1 - a_2 \\ \mathcal{C} &= -(\frac{\rho a_1}{\omega_c} + \frac{b_2}{\hat{\omega}}) \ , \end{aligned}$$

and

$$K_p = \frac{\rho}{m_1 - m_2 - \rho a_2} \ , \tag{19}$$

where

$$\begin{aligned} m_1 &= a_1(\alpha\omega_c T_i - \frac{1}{\omega_c T_i}) \\ m_2 &= a_2(\alpha\hat{\omega}_c T_i - \frac{1}{\hat{\omega}_c T_i}) + b_2 \ . \end{aligned}$$

The derivative time(T_d) is directly given from (12).

3. Simulations

Simulations were proceeded on the two typical process models.

$$G_1(s) = \frac{1}{(s+1)(2s+1)(5s+1)} \tag{20}$$

$$G_2(s) = \frac{1.5}{(20s+1)^2}e^{-2s} \tag{21}$$

The Nyquist critical point and its neighboring point of the each process were identified from the modified relay feedback experiment shown in Figure 1. The each designated time delay(T_d) was chosen as 0.1 and 0.2 second. The results of the each experiment were computed from (1), (3), (4) and (6), and summarized in Table 1.

Table 1. The identified Nyquist points

	K_c	ω_c	$\hat{K}_c$	$\hat{\omega}_c$
$G_1(s)$	5.536	0.571	5.093	0.546
$G_2(s)$	11.575	0.209	9.794	0.190

	Critical point	Neighbor point
$G_1(s)$	$-0.181 + j0.000$	$-0.196 - j0.011$
$G_2(s)$	$-0.086 + j0.000$	$-0.102 - j0.004$

The parameters of the PID controllers for each process were computed from (12), (18) and (19). The ratio(α) between T_d and T_i was fixed to 0.1 so that T_d had relatively lower value than that of T_i. The damping ratio(ζ) was selected as 0.3 and 0.7 for each process model from the range of $0 < \zeta < 1$. The results are summarized in Table 2, and the step responses of each process are shown in Figure 3 and Figure 4 respectively.

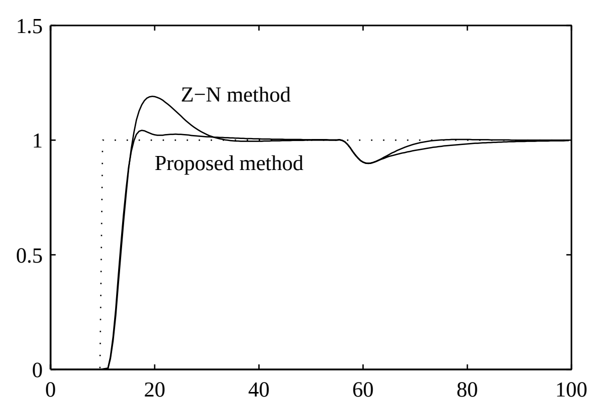
Table 2. The parameters of PID controller

(a) Proposed method

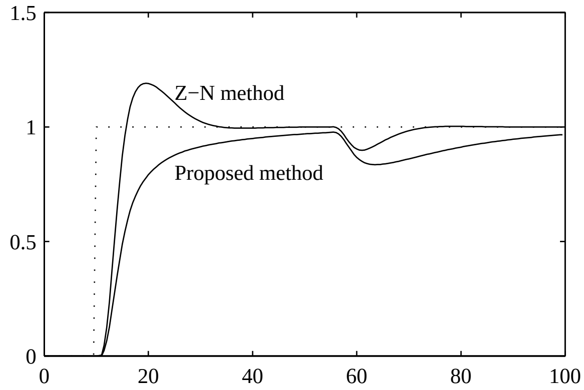
	α	ζ	K_p	T_i	T_d
$G_1(s)$	0.1	0.3	3.854	10.623	1.062
	0.1	0.7	1.949	16.703	1.670
$G_2(s)$	0.1	0.3	9.775	33.225	3.323
	0.1	0.7	4.837	68.207	6.821

(b) Ziegler-Nichols method

	K_p	T_i	T_d
$G_1(s)$	3.322	5.500	1.320
$G_2(s)$	6.945	15.000	3.600

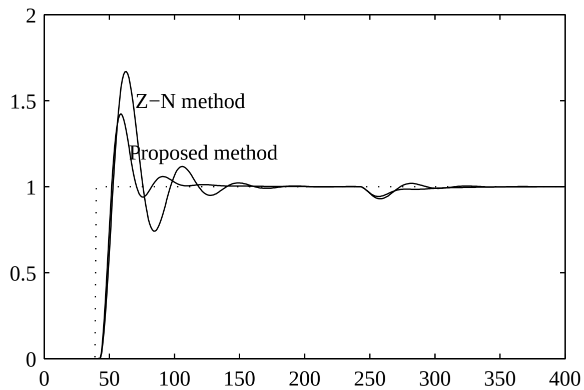


(a) $\zeta = 0.3, \quad \alpha = 0.1$

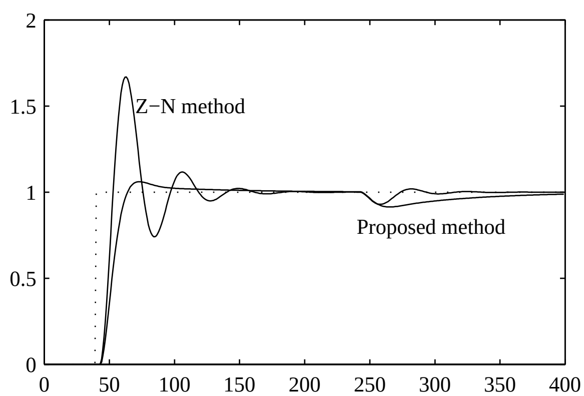


(b) $\zeta = 0.7, \qquad \alpha = 0.1$

Figure 3. The step responses of $G_1(s)$



(a) $\zeta = 0.3, \qquad \alpha = 0.1$



(b) $\zeta = 0.7, \qquad \alpha = 0.1$

Figure 4. The step responses of $G_2(s)$

The PID controller designed from proposed algorithm showed various setpoint tracking and disturbance rejection performance according to the value of ζ. The smaller ζ gives the faster response but the bigger overshoot, while the larger ζ gives the slower response but the smaller overshoot. So, we can design the PID controller considering the desired response of the process by selecting the value of damping ratio(ζ).

4. Conclusions

An automatic tuning algorithm of the PID controller was discused in this paper. The modified relay feedback configuration was suggested to identify the Nyquist critical point and its neighboring point on the complex plane of the process. The formulas to determine the parameters of the PID controller were derived from the dominant poles approximation and the dominant poles placement method. We can make use of the damping ratio(ζ) as a time domain specification so that the design flexibility is considered for the stability and performance of the process. The proposed algorithm can be implemented to the PID controllers as an automatic tuning feature.

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