In [1]:
```
# Install TensorFlow
# !pip install -q tensorflow-gpu==2.0.0-beta1

try:
  %tensorflow_version 2.x  # Colab only.
except Exception:
  pass

import tensorflow as tf
print(tf.__version__)
```

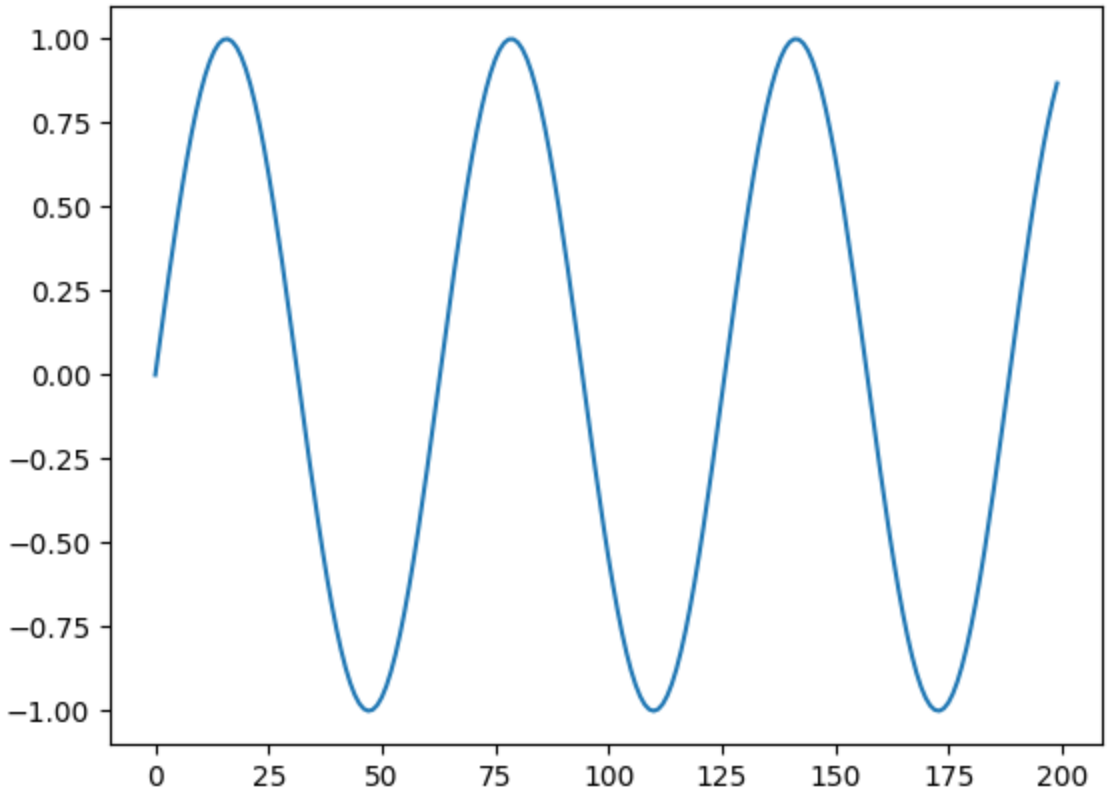
```
Colab only includes TensorFlow 2.x; %tensorflow_version has no effect.
2.12.0
```

In [2]:
```
from tensorflow.keras.layers import Input, Dense
from tensorflow.keras.models import Model
from tensorflow.keras.optimizers import SGD, Adam

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [3]:
```
# make the original data
series = np.sin(0.1*np.arange(200)) #+ np.random.randn(200)*0.1

# plot it
plt.plot(series)
plt.show()
```



In [4]:
```
### build the dataset
# let's see if we can use T past values to predict the next value
T = 10
X = []
Y = []
for t in range(len(series) - T):
  x = series[t:t+T]
  X.append(x)
  y = series[t+T]
  Y.append(y)

X = np.array(X).reshape(-1, T)
Y = np.array(Y)
N = len(X)
print("X.shape", X.shape, "Y.shape", Y.shape)
```

```
X.shape (190, 10) Y.shape (190,)
```

In [5]:
```
### try autoregressive linear model
i = Input(shape=(T,))
x = Dense(1)(i)
model = Model(i, x)
model.compile(
  loss='mse',
  optimizer=Adam(learning_rate=0.1),
)

# train the RNN
r = model.fit(
  X[:-N//2], Y[:-N//2],
  epochs=80,
  validation_data=(X[-N//2:], Y[-N//2:]),
)
```

```
Epoch 1/80
3/3 [==============================] - 2s 307ms/step - loss: 1.0414 - val_loss: 0.
3091
Epoch 2/80
3/3 [==============================] - 0s 60ms/step - loss: 0.7047 - val_loss: 0.6
006
Epoch 3/80
3/3 [==============================] - 0s 46ms/step - loss: 0.3712 - val_loss: 0.1
169
Epoch 4/80
3/3 [==============================] - 0s 67ms/step - loss: 0.1716 - val_loss: 0.2
412
Epoch 5/80
3/3 [==============================] - 0s 61ms/step - loss: 0.3015 - val_loss: 0.0
911
Epoch 6/80
3/3 [==============================] - 0s 61ms/step - loss: 0.0685 - val_loss: 0.0
497
Epoch 7/80
3/3 [==============================] - 0s 44ms/step - loss: 0.0764 - val_loss: 0.1
077
Epoch 8/80
3/3 [==============================] - 0s 46ms/step - loss: 0.0881 - val_loss: 0.0
124
Epoch 9/80
3/3 [==============================] - 0s 60ms/step - loss: 0.0122 - val_loss: 0.0
641
Epoch 10/80
3/3 [==============================] - 0s 33ms/step - loss: 0.0598 - val_loss: 0.0
795
Epoch 11/80
3/3 [==============================] - 0s 53ms/step - loss: 0.0359 - val_loss: 0.0
132
Epoch 12/80
3/3 [==============================] - 0s 50ms/step - loss: 0.0154 - val_loss: 0.0
267
Epoch 13/80
3/3 [==============================] - 0s 39ms/step - loss: 0.0282 - val_loss: 0.0
109
Epoch 14/80
3/3 [==============================] - 0s 30ms/step - loss: 0.0060 - val_loss: 0.0
046
Epoch 15/80
3/3 [==============================] - 0s 41ms/step - loss: 0.0109 - val_loss: 0.0
070
Epoch 16/80
3/3 [==============================] - 0s 30ms/step - loss: 0.0068 - val_loss: 6.0
705e-04
Epoch 17/80
3/3 [==============================] - 0s 79ms/step - loss: 0.0022 - val_loss: 0.0
053
Epoch 18/80
3/3 [==============================] - 0s 30ms/step - loss: 0.0048 - val_loss: 7.6
269e-04
Epoch 19/80
3/3 [==============================] - 0s 32ms/step - loss: 0.0012 - val_loss: 0.0
```

```
036
Epoch 20/80
3/3 [==============================] - 0s 45ms/step - loss: 0.0034 - val_loss: 0.0
022
Epoch 21/80
3/3 [==============================] - 0s 27ms/step - loss: 0.0011 - val_loss: 9.2
565e-04
Epoch 22/80
3/3 [==============================] - 0s 50ms/step - loss: 0.0016 - val_loss: 0.0
016
Epoch 23/80
3/3 [==============================] - 0s 39ms/step - loss: 7.4577e-04 - val_loss:
3.9687e-04
Epoch 24/80
3/3 [==============================] - 0s 46ms/step - loss: 8.8570e-04 - val_loss:
5.8324e-04
Epoch 25/80
3/3 [==============================] - 0s 62ms/step - loss: 5.0554e-04 - val_loss:
1.2148e-04
Epoch 26/80
3/3 [==============================] - 0s 70ms/step - loss: 3.2880e-04 - val_loss:
3.2844e-04
Epoch 27/80
3/3 [==============================] - 0s 42ms/step - loss: 2.5685e-04 - val_loss:
1.8922e-04
Epoch 28/80
3/3 [==============================] - 0s 72ms/step - loss: 2.4662e-04 - val_loss:
4.7960e-04
Epoch 29/80
3/3 [==============================] - 0s 50ms/step - loss: 1.9882e-04 - val_loss:
3.8730e-05
Epoch 30/80
3/3 [==============================] - 0s 46ms/step - loss: 8.5354e-05 - val_loss:
1.6838e-04
Epoch 31/80
3/3 [==============================] - 0s 29ms/step - loss: 1.0823e-04 - val_loss:
2.1532e-05
Epoch 32/80
3/3 [==============================] - 0s 33ms/step - loss: 5.1065e-05 - val_loss:
5.6453e-05
Epoch 33/80
3/3 [==============================] - 0s 63ms/step - loss: 5.6591e-05 - val_loss:
3.6474e-06
Epoch 34/80
3/3 [==============================] - 0s 30ms/step - loss: 2.1166e-05 - val_loss:
3.7548e-05
Epoch 35/80
3/3 [==============================] - 0s 79ms/step - loss: 3.2235e-05 - val_loss:
9.3392e-06
Epoch 36/80
3/3 [==============================] - 0s 50ms/step - loss: 1.8339e-05 - val_loss:
3.7056e-05
Epoch 37/80
3/3 [==============================] - 0s 82ms/step - loss: 2.0156e-05 - val_loss:
2.5060e-06
Epoch 38/80
```

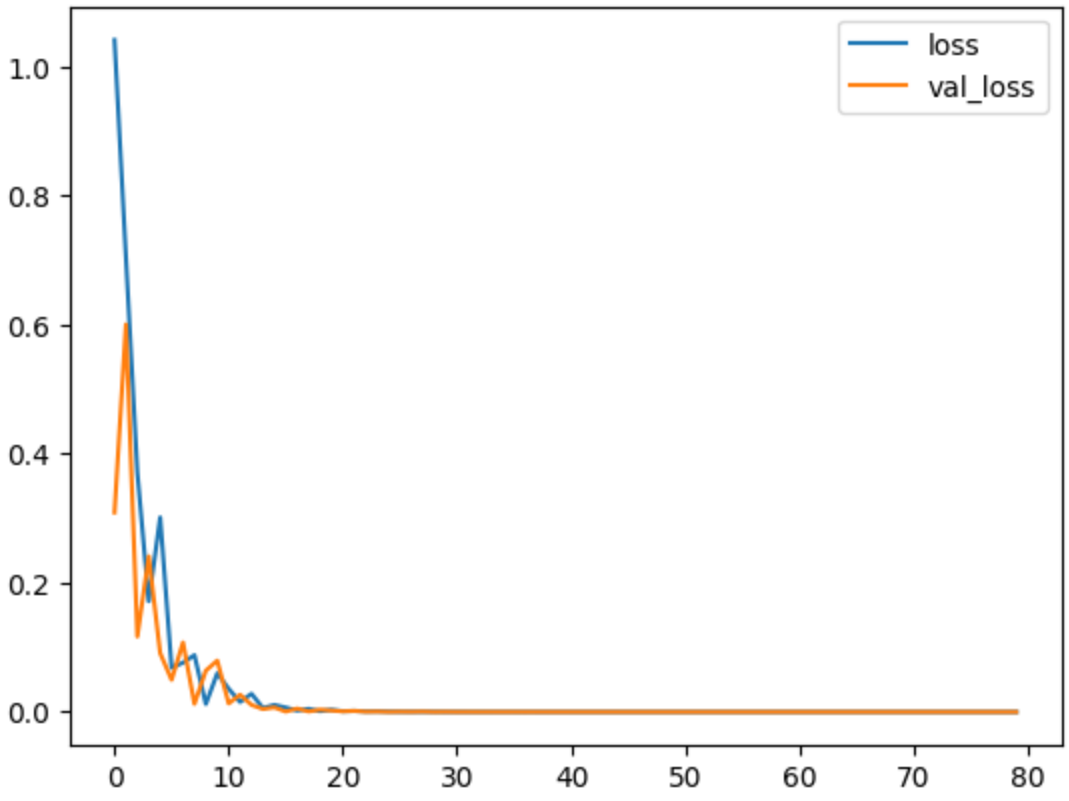
```
3/3 [==============================] - 0s 89ms/step - loss: 1.0181e-05 - val_loss:
1.7263e-05
Epoch 39/80
3/3 [==============================] - 0s 63ms/step - loss: 7.7175e-06 - val_loss:
2.5964e-06
Epoch 40/80
3/3 [==============================] - 0s 38ms/step - loss: 7.5396e-06 - val_loss:
4.0097e-06
Epoch 41/80
3/3 [==============================] - 0s 39ms/step - loss: 2.6210e-06 - val_loss:
2.4908e-06
Epoch 42/80
3/3 [==============================] - 0s 70ms/step - loss: 4.6903e-06 - val_loss:
2.1050e-06
Epoch 43/80
3/3 [==============================] - 0s 46ms/step - loss: 1.9012e-06 - val_loss:
4.7211e-06
Epoch 44/80
3/3 [==============================] - 0s 47ms/step - loss: 3.2062e-06 - val_loss:
8.2927e-07
Epoch 45/80
3/3 [==============================] - 0s 74ms/step - loss: 6.0612e-07 - val_loss:
1.9833e-06
Epoch 46/80
3/3 [==============================] - 0s 40ms/step - loss: 1.3810e-06 - val_loss:
2.4794e-07
Epoch 47/80
3/3 [==============================] - 0s 74ms/step - loss: 5.1100e-07 - val_loss:
5.8506e-07
Epoch 48/80
3/3 [==============================] - 0s 58ms/step - loss: 5.0116e-07 - val_loss:
1.9053e-07
Epoch 49/80
3/3 [==============================] - 0s 31ms/step - loss: 4.6948e-07 - val_loss:
2.0794e-07
Epoch 50/80
3/3 [==============================] - 0s 37ms/step - loss: 1.7509e-07 - val_loss:
4.3521e-07
Epoch 51/80
3/3 [==============================] - 0s 63ms/step - loss: 3.9908e-07 - val_loss:
7.5434e-08
Epoch 52/80
3/3 [==============================] - 0s 34ms/step - loss: 8.9318e-08 - val_loss:
3.1495e-07
Epoch 53/80
3/3 [==============================] - 0s 41ms/step - loss: 1.9043e-07 - val_loss:
2.4548e-08
Epoch 54/80
3/3 [==============================] - 0s 28ms/step - loss: 8.5833e-08 - val_loss:
8.8907e-08
Epoch 55/80
3/3 [==============================] - 0s 45ms/step - loss: 6.2142e-08 - val_loss:
2.7163e-08
Epoch 56/80
3/3 [==============================] - 0s 38ms/step - loss: 6.1695e-08 - val_loss:
3.2408e-08
```

```
Epoch 57/80
3/3 [==============================] - 0s 95ms/step - loss: 3.4882e-08 - val_loss:
7.2736e-08
Epoch 58/80
3/3 [==============================] - 0s 57ms/step - loss: 4.4078e-08 - val_loss:
2.2638e-09
Epoch 59/80
3/3 [==============================] - 0s 39ms/step - loss: 1.4235e-08 - val_loss:
4.1176e-08
Epoch 60/80
3/3 [==============================] - 0s 33ms/step - loss: 2.0542e-08 - val_loss:
4.3448e-09
Epoch 61/80
3/3 [==============================] - 0s 36ms/step - loss: 1.5404e-08 - val_loss:
1.0541e-08
Epoch 62/80
3/3 [==============================] - 0s 36ms/step - loss: 5.6928e-09 - val_loss:
9.1912e-09
Epoch 63/80
3/3 [==============================] - 0s 32ms/step - loss: 1.0733e-08 - val_loss:
7.4715e-10
Epoch 64/80
3/3 [==============================] - 0s 55ms/step - loss: 3.1074e-09 - val_loss:
7.3095e-09
Epoch 65/80
3/3 [==============================] - 0s 37ms/step - loss: 4.0479e-09 - val_loss:
1.2230e-09
Epoch 66/80
3/3 [==============================] - 0s 50ms/step - loss: 3.3763e-09 - val_loss:
2.4913e-09
Epoch 67/80
3/3 [==============================] - 0s 30ms/step - loss: 1.2956e-09 - val_loss:
2.9143e-09
Epoch 68/80
3/3 [==============================] - 0s 45ms/step - loss: 2.7696e-09 - val_loss:
2.0057e-10
Epoch 69/80
3/3 [==============================] - 0s 79ms/step - loss: 1.3130e-09 - val_loss:
1.2503e-09
Epoch 70/80
3/3 [==============================] - 0s 53ms/step - loss: 7.8127e-10 - val_loss:
1.0741e-09
Epoch 71/80
3/3 [==============================] - 0s 43ms/step - loss: 1.1415e-09 - val_loss:
5.3256e-11
Epoch 72/80
3/3 [==============================] - 0s 32ms/step - loss: 3.6900e-10 - val_loss:
1.1946e-09
Epoch 73/80
3/3 [==============================] - 0s 43ms/step - loss: 5.4660e-10 - val_loss:
1.8693e-10
Epoch 74/80
3/3 [==============================] - 0s 93ms/step - loss: 4.8654e-10 - val_loss:
1.3430e-10
Epoch 75/80
3/3 [==============================] - 0s 84ms/step - loss: 1.4643e-10 - val_loss:
```

```
2.5620e-10
Epoch 76/80
3/3 [==============================] - 0s 34ms/step - loss: 2.2484e-10 - val_loss:
7.3435e-11
Epoch 77/80
3/3 [==============================] - 0s 44ms/step - loss: 1.4493e-10 - val_loss:
6.5524e-11
Epoch 78/80
3/3 [==============================] - 0s 47ms/step - loss: 5.0456e-11 - val_loss:
1.3158e-10
Epoch 79/80
3/3 [==============================] - 0s 44ms/step - loss: 7.8791e-11 - val_loss:
9.2106e-12
Epoch 80/80
3/3 [==============================] - 0s 34ms/step - loss: 4.4088e-11 - val_loss:
2.5487e-11
```

In [6]:
```
# Plot loss per iteration
import matplotlib.pyplot as plt
plt.plot(r.history['loss'], label='loss')
plt.plot(r.history['val_loss'], label='val_loss')
plt.legend()
```

Out[6]:
```
<matplotlib.legend.Legend at 0x7f10e705a020>
```



In [7]:
```
# "Wrong" forecast using true targets

validation_target = Y[-N//2:]
validation_predictions = []

# index of first validation input
```

```
i = -N//2

while len(validation_predictions) < len(validation_target):
  p = model.predict(X[i].reshape(1, -1))[0,0] # 1x1 array -> scalar
  i += 1

  # update the predictions list
  validation_predictions.append(p)
```

```
1/1 [==============================] - 0s 93ms/step
1/1 [==============================] - 0s 52ms/step
1/1 [==============================] - 0s 32ms/step
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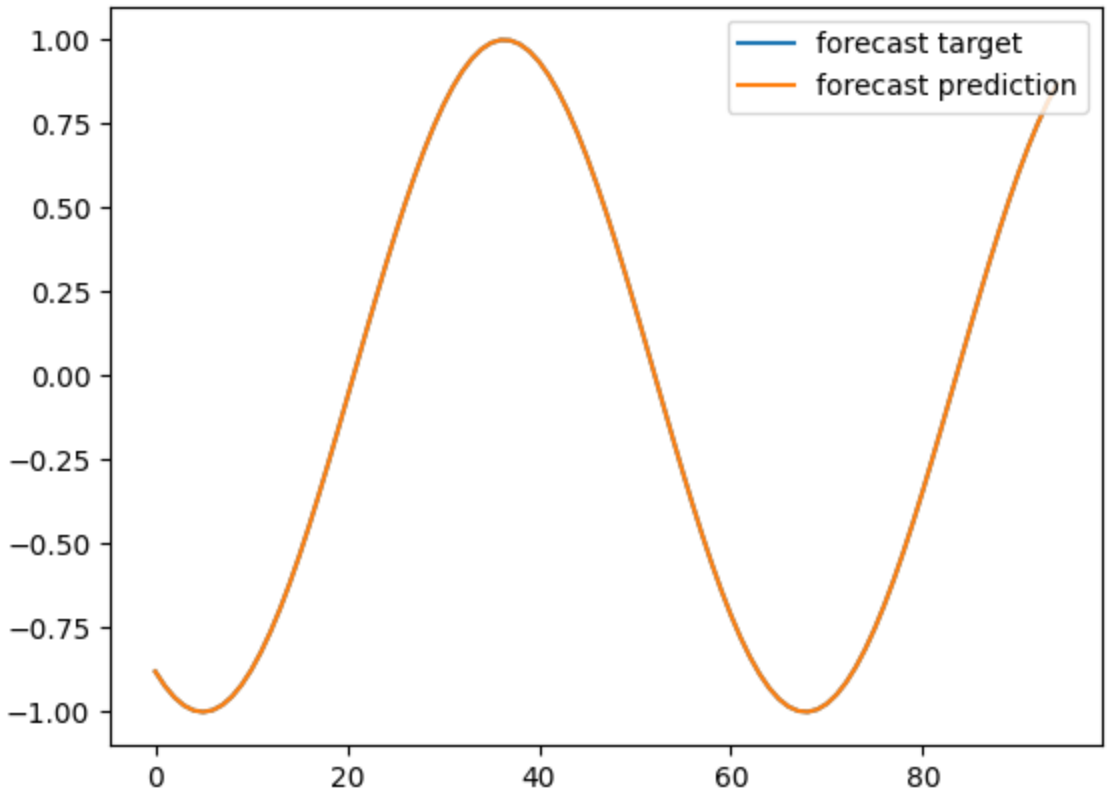
```

In [8]:
```
plt.plot(validation_target, label='forecast target')
plt.plot(validation_predictions, label='forecast prediction')
plt.legend()
```

Out[8]:
```
<matplotlib.legend.Legend at 0x7f105c2f0460>
```



In [9]:
```
# Forecast future values (use only self-predictions for making future predictions)

validation_target = Y[-N//2:]
validation_predictions = []

# first validation input
last_x = X[-N//2] # 1-D array of length T

while len(validation_predictions) < len(validation_target):
  p = model.predict(last_x.reshape(1, -1))[0,0] # 1x1 array -> scalar

  # update the predictions list
  validation_predictions.append(p)

  # make the new input
  last_x = np.roll(last_x, -1)
  last_x[-1] = p
```

```
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```

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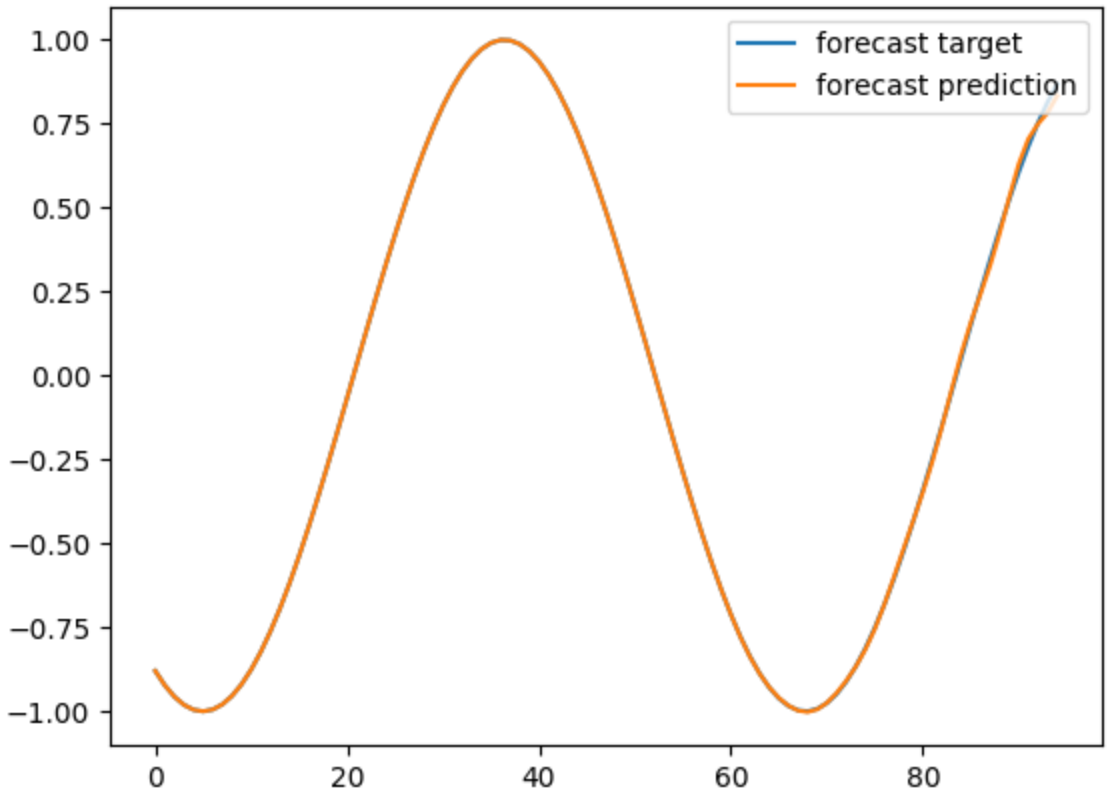
```

In [10]:
```
plt.plot(validation_target, label='forecast target')
plt.plot(validation_predictions, label='forecast prediction')
plt.legend()
```

Out[10]:
```
<matplotlib.legend.Legend at 0x7f105dca6590>
```


forecast target
forecast prediction
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
20
40
60
80