In [1]:
```
# Install TensorFlow
# !pip install -q tensorflow-gpu==2.0.0-beta1

try:
  %tensorflow_version 2.x  # Colab only.
except Exception:
  pass

import tensorflow as tf
print(tf.__version__)
```

```
Colab only includes TensorFlow 2.x; %tensorflow_version has no effect.
2.12.0
```

In [2]:
```
from tensorflow.keras.layers import Input, SimpleRNN, GRU, LSTM, Dense, Flatten
from tensorflow.keras.models import Model
from tensorflow.keras.optimizers import SGD, Adam

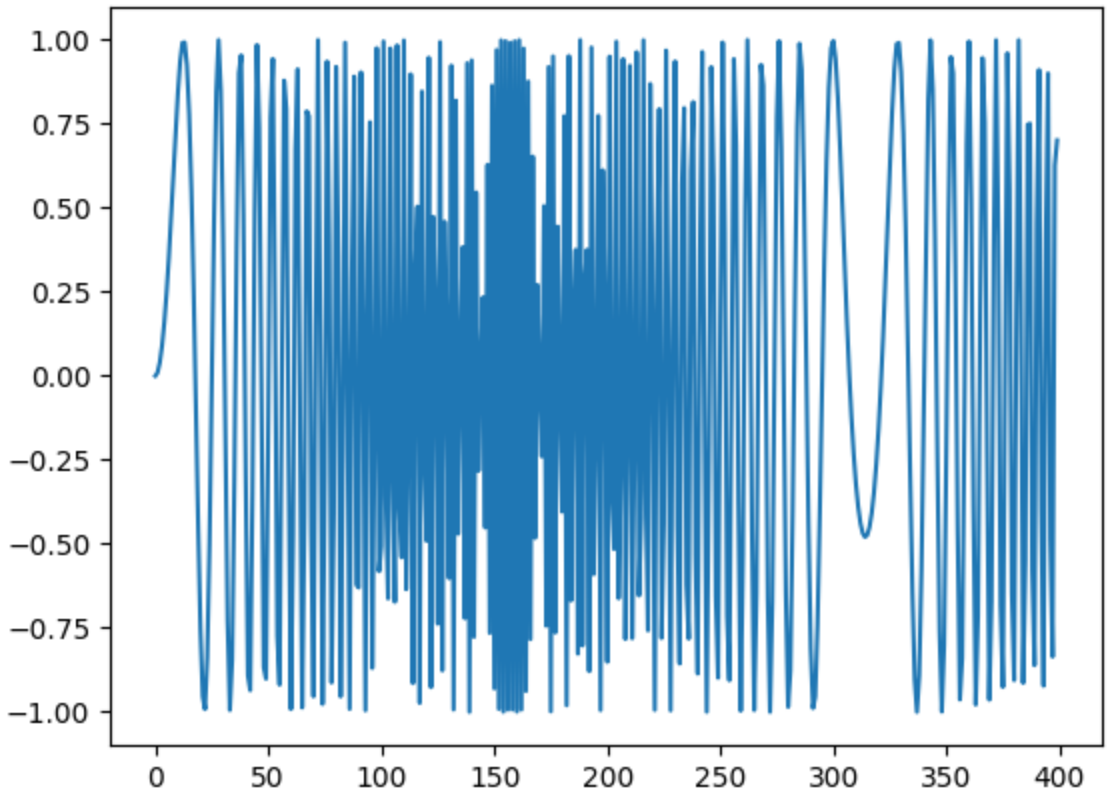
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [3]:
```
# make the original data
series = np.sin((0.1*np.arange(400))**2)
```

This is a time series of the form:

$$x(t) = \sin(\omega t^2)$$

In [4]:
```
# plot it
plt.plot(series)
plt.show()
```



In [5]:
```
### build the dataset
# let's see if we can use T past values to predict the next value
T = 10
D = 1
X = []
Y = []
for t in range(len(series) - T):
  x = series[t:t+T]
  X.append(x)
  y = series[t+T]
  Y.append(y)

X = np.array(X).reshape(-1, T) # make it N x T
Y = np.array(Y)
N = len(X)
print("X.shape", X.shape, "Y.shape", Y.shape)
```

```
X.shape (390, 10) Y.shape (390,)
```

In [6]:
```
### try autoregressive linear model
i = Input(shape=(T,))
x = Dense(1)(i)
model = Model(i, x)
model.compile(
  loss='mse',
  optimizer=Adam(learning_rate=0.01),
)

# train the RNN
r = model.fit(
  X[:-N//2], Y[:-N//2],
```

```
  epochs=80,
  validation_data=(X[-N//2:], Y[-N//2:]),
)
```

```
Epoch 1/80
7/7 [==============================] - 7s 28ms/step - loss: 1.2428 - val_loss: 0.9
801
Epoch 2/80
7/7 [==============================] - 0s 8ms/step - loss: 1.0419 - val_loss: 0.88
91
Epoch 3/80
7/7 [==============================] - 0s 9ms/step - loss: 0.9057 - val_loss: 0.81
07
Epoch 4/80
7/7 [==============================] - 0s 9ms/step - loss: 0.8055 - val_loss: 0.72
34
Epoch 5/80
7/7 [==============================] - 0s 8ms/step - loss: 0.7274 - val_loss: 0.66
30
Epoch 6/80
7/7 [==============================] - 0s 8ms/step - loss: 0.6700 - val_loss: 0.62
36
Epoch 7/80
7/7 [==============================] - 0s 9ms/step - loss: 0.6256 - val_loss: 0.59
23
Epoch 8/80
7/7 [==============================] - 0s 9ms/step - loss: 0.5909 - val_loss: 0.56
51
Epoch 9/80
7/7 [==============================] - 0s 9ms/step - loss: 0.5639 - val_loss: 0.54
88
Epoch 10/80
7/7 [==============================] - 0s 9ms/step - loss: 0.5447 - val_loss: 0.54
19
Epoch 11/80
7/7 [==============================] - 0s 9ms/step - loss: 0.5307 - val_loss: 0.54
21
Epoch 12/80
7/7 [==============================] - 0s 8ms/step - loss: 0.5191 - val_loss: 0.54
66
Epoch 13/80
7/7 [==============================] - 0s 8ms/step - loss: 0.5109 - val_loss: 0.55
05
Epoch 14/80
7/7 [==============================] - 0s 9ms/step - loss: 0.5047 - val_loss: 0.54
80
Epoch 15/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4982 - val_loss: 0.55
35
Epoch 16/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4943 - val_loss: 0.55
95
Epoch 17/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4913 - val_loss: 0.56
37
Epoch 18/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4906 - val_loss: 0.57
06
Epoch 19/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4861 - val_loss: 0.57
```

```
20
Epoch 20/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4885 - val_loss: 0.5
742
Epoch 21/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4897 - val_loss: 0.58
21
Epoch 22/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4900 - val_loss: 0.58
83
Epoch 23/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4892 - val_loss: 0.58
70
Epoch 24/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4869 - val_loss: 0.5
887
Epoch 25/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4884 - val_loss: 0.59
00
Epoch 26/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4879 - val_loss: 0.58
80
Epoch 27/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4877 - val_loss: 0.59
12
Epoch 28/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4891 - val_loss: 0.5
981
Epoch 29/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4880 - val_loss: 0.59
43
Epoch 30/80
7/7 [==============================] - 0s 16ms/step - loss: 0.4902 - val_loss: 0.5
950
Epoch 31/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4881 - val_loss: 0.59
65
Epoch 32/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4877 - val_loss: 0.6
006
Epoch 33/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4874 - val_loss: 0.60
05
Epoch 34/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4888 - val_loss: 0.59
99
Epoch 35/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4904 - val_loss: 0.6
037
Epoch 36/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4925 - val_loss: 0.60
47
Epoch 37/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4938 - val_loss: 0.59
23
Epoch 38/80
```

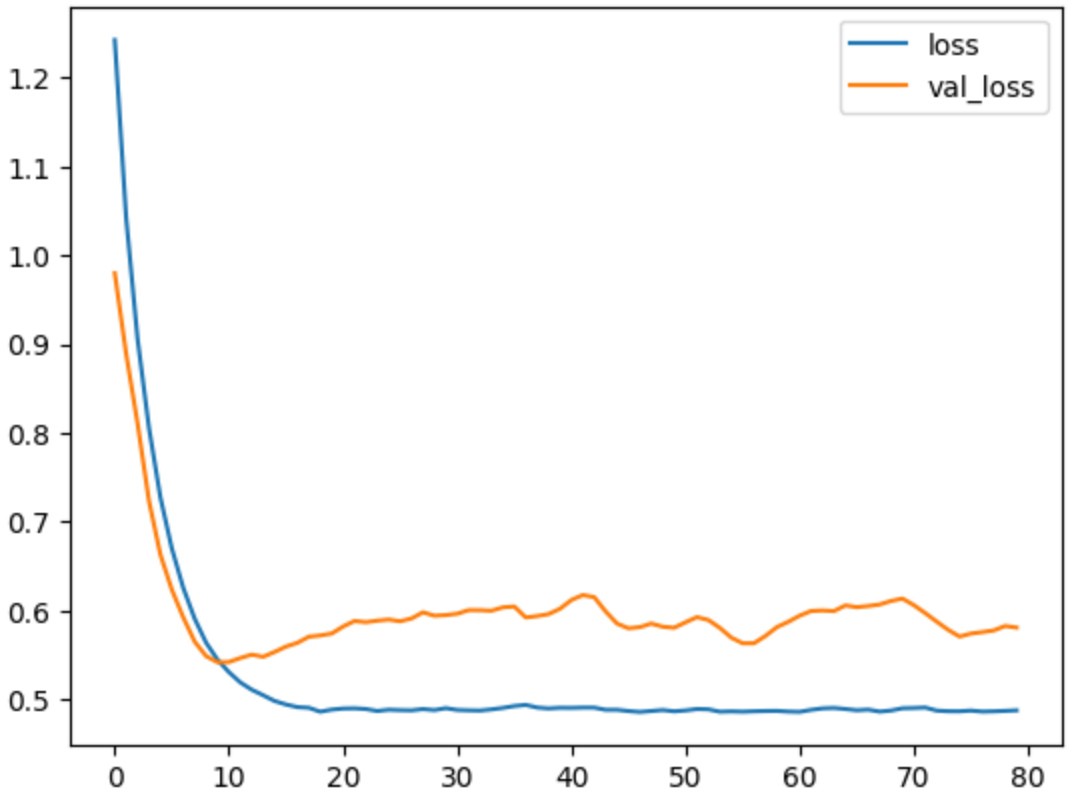
```
7/7 [==============================] - 0s 8ms/step - loss: 0.4908 - val_loss: 0.59
39
Epoch 39/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4898 - val_loss: 0.59
61
Epoch 40/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4905 - val_loss: 0.60
21
Epoch 41/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4904 - val_loss: 0.6
120
Epoch 42/80
7/7 [==============================] - 0s 15ms/step - loss: 0.4908 - val_loss: 0.6
176
Epoch 43/80
7/7 [==============================] - 0s 14ms/step - loss: 0.4908 - val_loss: 0.6
152
Epoch 44/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4883 - val_loss: 0.5
991
Epoch 45/80
7/7 [==============================] - 0s 14ms/step - loss: 0.4883 - val_loss: 0.5
854
Epoch 46/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4869 - val_loss: 0.5
801
Epoch 47/80
7/7 [==============================] - 0s 13ms/step - loss: 0.4859 - val_loss: 0.5
814
Epoch 48/80
7/7 [==============================] - 0s 15ms/step - loss: 0.4869 - val_loss: 0.5
855
Epoch 49/80
7/7 [==============================] - 0s 19ms/step - loss: 0.4880 - val_loss: 0.5
819
Epoch 50/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4866 - val_loss: 0.5
807
Epoch 51/80
7/7 [==============================] - 0s 13ms/step - loss: 0.4876 - val_loss: 0.5
868
Epoch 52/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4892 - val_loss: 0.5
927
Epoch 53/80
7/7 [==============================] - 0s 14ms/step - loss: 0.4889 - val_loss: 0.5
894
Epoch 54/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4861 - val_loss: 0.5
806
Epoch 55/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4865 - val_loss: 0.5
697
Epoch 56/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4862 - val_loss: 0.5
632
```

```
Epoch 57/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4867 - val_loss: 0.5
633
Epoch 58/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4869 - val_loss: 0.5
715
Epoch 59/80
7/7 [==============================] - 0s 13ms/step - loss: 0.4870 - val_loss: 0.5
814
Epoch 60/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4863 - val_loss: 0.5
874
Epoch 61/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4860 - val_loss: 0.5
943
Epoch 62/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4882 - val_loss: 0.5
996
Epoch 63/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4899 - val_loss: 0.6
002
Epoch 64/80
7/7 [==============================] - 0s 19ms/step - loss: 0.4904 - val_loss: 0.5
995
Epoch 65/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4892 - val_loss: 0.6
059
Epoch 66/80
7/7 [==============================] - 0s 19ms/step - loss: 0.4878 - val_loss: 0.6
039
Epoch 67/80
7/7 [==============================] - 0s 12ms/step - loss: 0.4885 - val_loss: 0.6
052
Epoch 68/80
7/7 [==============================] - 0s 11ms/step - loss: 0.4863 - val_loss: 0.6
069
Epoch 69/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4875 - val_loss: 0.6
109
Epoch 70/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4900 - val_loss: 0.61
36
Epoch 71/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4903 - val_loss: 0.60
62
Epoch 72/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4908 - val_loss: 0.59
72
Epoch 73/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4874 - val_loss: 0.5
877
Epoch 74/80
7/7 [==============================] - 0s 7ms/step - loss: 0.4867 - val_loss: 0.57
84
Epoch 75/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4866 - val_loss: 0.57
```

```
08
Epoch 76/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4874 - val_loss: 0.57
41
Epoch 77/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4863 - val_loss: 0.57
58
Epoch 78/80
7/7 [==============================] - 0s 9ms/step - loss: 0.4866 - val_loss: 0.57
77
Epoch 79/80
7/7 [==============================] - 0s 10ms/step - loss: 0.4871 - val_loss: 0.5
827
Epoch 80/80
7/7 [==============================] - 0s 8ms/step - loss: 0.4877 - val_loss: 0.58
09
```

In [7]:
```
# Plot loss per iteration
import matplotlib.pyplot as plt
plt.plot(r.history['loss'], label='loss')
plt.plot(r.history['val_loss'], label='val_loss')
plt.legend()
```

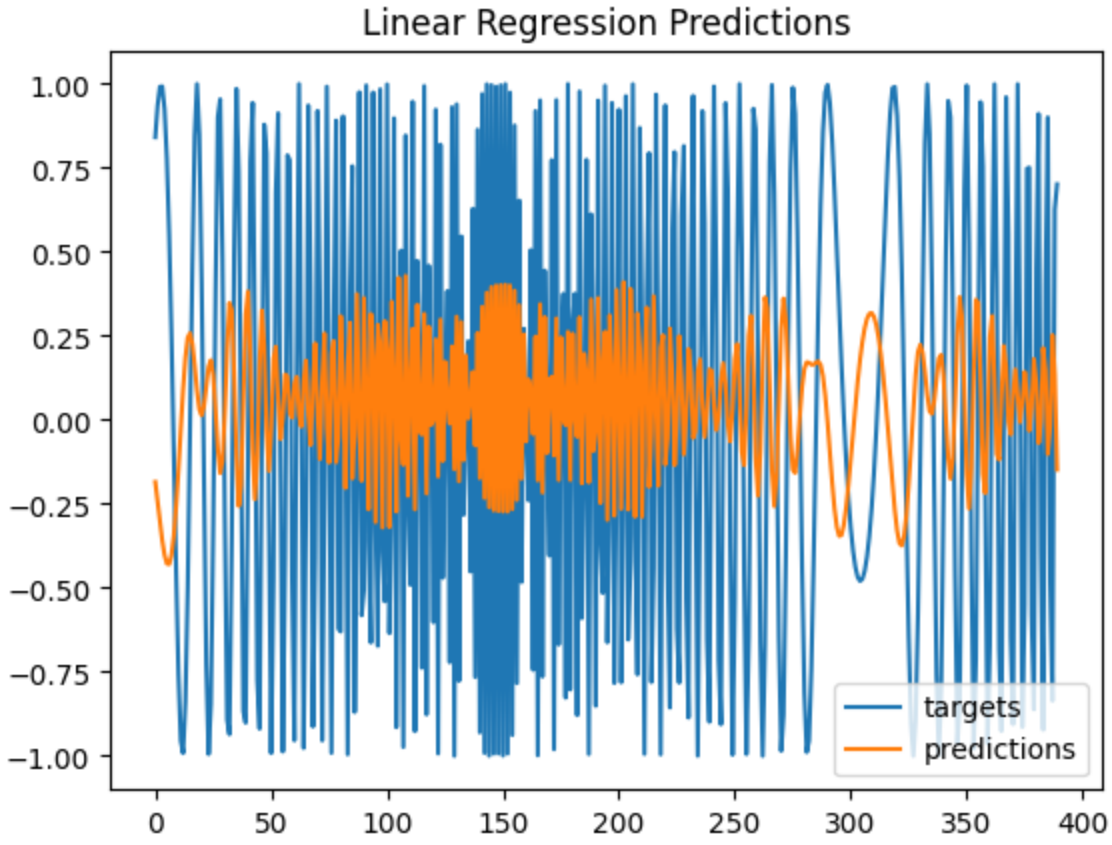
Out[7]: <matplotlib.legend.Legend at 0x7fceb2dfdea0>



In [8]:
```
# One-step forecast using true targets
# Note: even the one-step forecast fails badly
outputs = model.predict(X)
print(outputs.shape)
predictions = outputs[:,0]
```

```
plt.plot(Y, label='targets')
plt.plot(predictions, label='predictions')
plt.title("Linear Regression Predictions")
plt.legend()
plt.show()
```

```
13/13 [==============================] - 0s 2ms/step
(390, 1)
```



In [9]:

```
# This is the code we had before - it does the same thing

# One-step forecast using true targets

validation_target = Y[-N//2:]
validation_predictions = []

# index of first validation input
i = -N//2

while len(validation_predictions) < len(validation_target):
  p = model.predict(X[i].reshape(1, -1))[0,0] # 1x1 array -> scalar
  i += 1

  # update the predictions list
  validation_predictions.append(p)

plt.plot(validation_target, label='forecast target')
plt.plot(validation_predictions, label='forecast prediction')
plt.legend()
```

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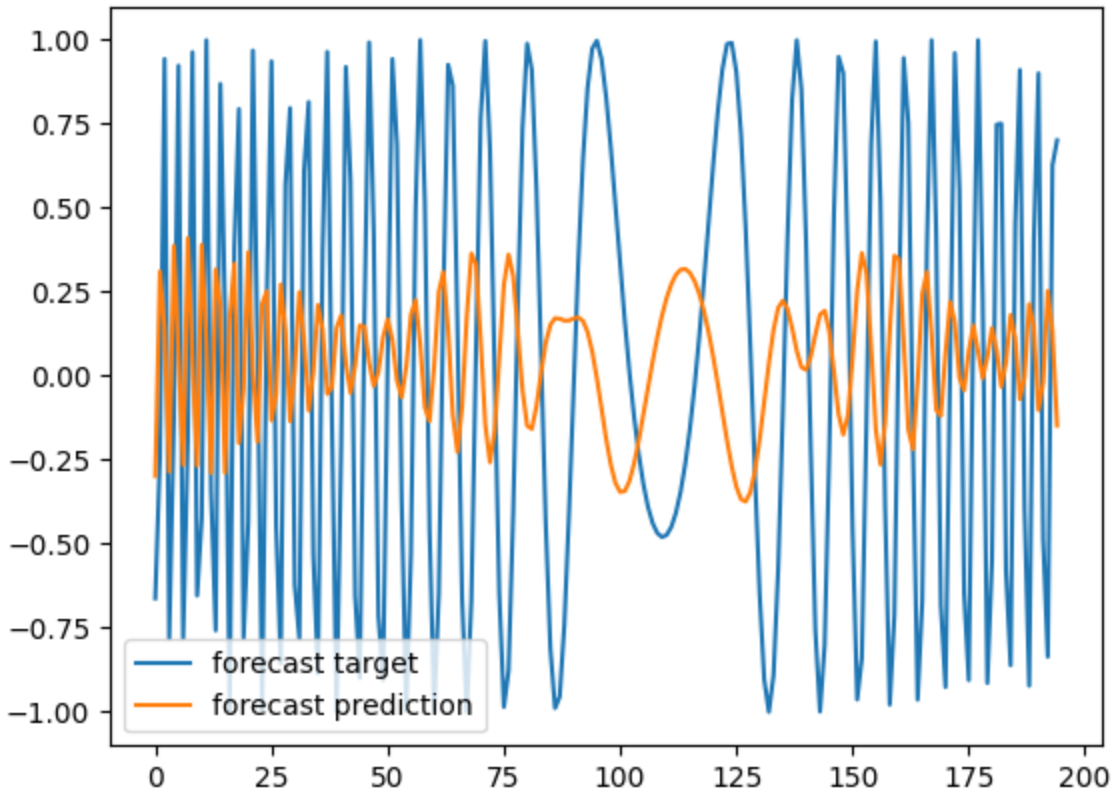
```

Out[9]: `<matplotlib.legend.Legend at 0x7fce8a1e3b20>`



In [10]:
```
# Multi-step forecast
validation_target = Y[-N//2:]
```

```
validation_predictions = []

# first validation input
last_x = X[-N//2] # 1-D array of length T

while len(validation_predictions) < len(validation_target):
  p = model.predict(last_x.reshape(1, -1))[0,0] # 1x1 array -> scalar

  # update the predictions list
  validation_predictions.append(p)

  # make the new input
  last_x = np.roll(last_x, -1)
  last_x[-1] = p

plt.plot(validation_target, label='forecast target')
plt.plot(validation_predictions, label='forecast prediction')
plt.legend()
```

```
1/1 [==============================] - 0s 20ms/step
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1/1 [==============================] - 0s 39ms/step
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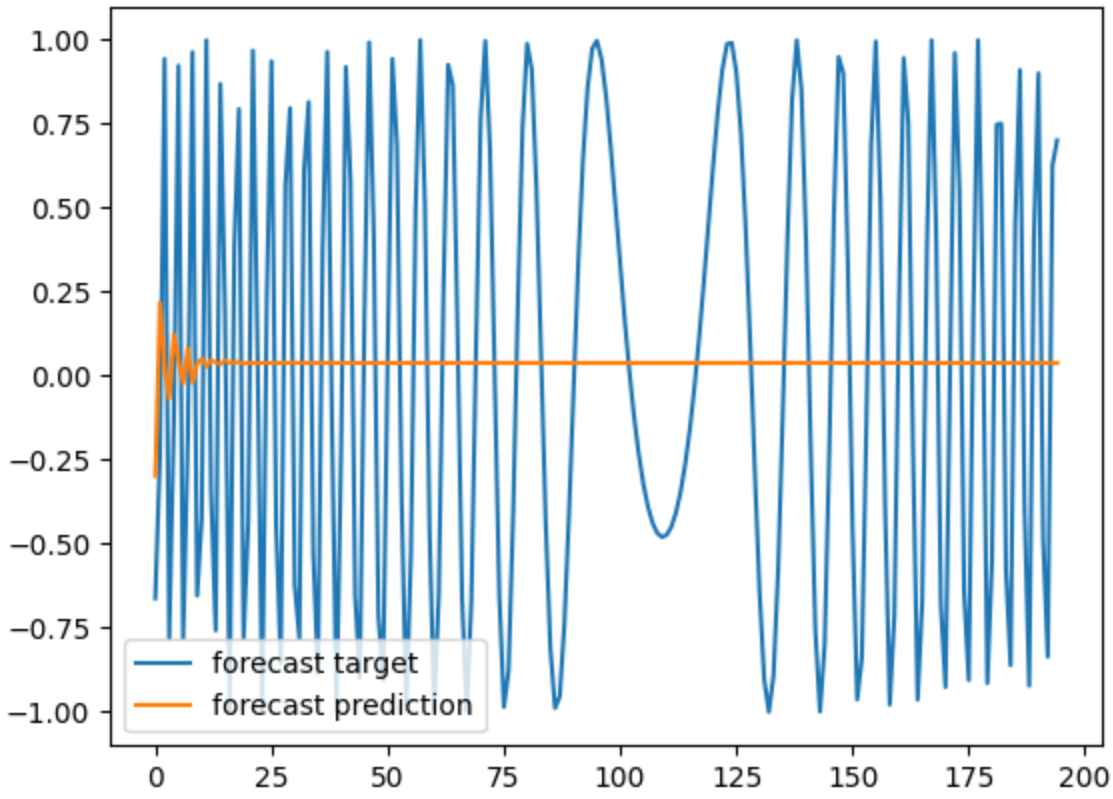
```

Out[10]: `<matplotlib.legend.Legend at 0x7fce8a0ee320>`



In [11]:
```
### Now try RNN/LSTM model
X = X.reshape(-1, T, 1) # make it N x T x D
```

```
# make the RNN
i = Input(shape=(T, D))
x = LSTM(10)(i)
x = Dense(1)(x)
model = Model(i, x)
model.compile(
  loss='mse',
  optimizer=Adam(learning_rate=0.05),
)

# train the RNN
r = model.fit(
  X[:-N//2], Y[:-N//2],
  batch_size=32,
  epochs=200,
  validation_data=(X[-N//2:], Y[-N//2:]),
)
```

```
Epoch 1/200
7/7 [==============================] - 3s 71ms/step - loss: 0.5327 - val_loss: 0.5
294
Epoch 2/200
7/7 [==============================] - 0s 10ms/step - loss: 0.4724 - val_loss: 0.5
237
Epoch 3/200
7/7 [==============================] - 0s 12ms/step - loss: 0.4564 - val_loss: 0.5
494
Epoch 4/200
7/7 [==============================] - 0s 10ms/step - loss: 0.3699 - val_loss: 0.4
568
Epoch 5/200
7/7 [==============================] - 0s 11ms/step - loss: 0.2757 - val_loss: 0.3
712
Epoch 6/200
7/7 [==============================] - 0s 11ms/step - loss: 0.2479 - val_loss: 0.4
499
Epoch 7/200
7/7 [==============================] - 0s 10ms/step - loss: 0.3016 - val_loss: 0.4
334
Epoch 8/200
7/7 [==============================] - 0s 11ms/step - loss: 0.2171 - val_loss: 0.2
587
Epoch 9/200
7/7 [==============================] - 0s 10ms/step - loss: 0.1384 - val_loss: 0.2
250
Epoch 10/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0804 - val_loss: 0.1
739
Epoch 11/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0775 - val_loss: 0.2
020
Epoch 12/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0471 - val_loss: 0.1
023
Epoch 13/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0316 - val_loss: 0.1
114
Epoch 14/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0291 - val_loss: 0.1
033
Epoch 15/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0197 - val_loss: 0.1
056
Epoch 16/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0151 - val_loss: 0.0
937
Epoch 17/200
7/7 [==============================] - 0s 14ms/step - loss: 0.0110 - val_loss: 0.1
114
Epoch 18/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0092 - val_loss: 0.1
104
Epoch 19/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0068 - val_loss: 0.1
```

```
182
Epoch 20/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0067 - val_loss: 0.1
089
Epoch 21/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0064 - val_loss: 0.1
114
Epoch 22/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0058 - val_loss: 0.1
218
Epoch 23/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0092 - val_loss: 0.1
514
Epoch 24/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0156 - val_loss: 0.1
356
Epoch 25/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0098 - val_loss: 0.1
220
Epoch 26/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0049 - val_loss: 0.1
153
Epoch 27/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0094 - val_loss: 0.1
001
Epoch 28/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0083 - val_loss: 0.0
965
Epoch 29/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0048 - val_loss: 0.0
982
Epoch 30/200
7/7 [==============================] - 0s 18ms/step - loss: 0.0047 - val_loss: 0.0
977
Epoch 31/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0037 - val_loss: 0.0
986
Epoch 32/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0031 - val_loss: 0.1
065
Epoch 33/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0043 - val_loss: 0.0
968
Epoch 34/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0053 - val_loss: 0.1
081
Epoch 35/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0041 - val_loss: 0.0
995
Epoch 36/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0040 - val_loss: 0.0
973
Epoch 37/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0036 - val_loss: 0.1
094
Epoch 38/200
```

```
7/7 [==============================] - 0s 10ms/step - loss: 0.0041 - val_loss: 0.1
061
Epoch 39/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0051 - val_loss: 0.1
063
Epoch 40/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0047 - val_loss: 0.1
117
Epoch 41/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0032 - val_loss: 0.0
932
Epoch 42/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0053 - val_loss: 0.1
129
Epoch 43/200
7/7 [==============================] - 0s 14ms/step - loss: 0.0048 - val_loss: 0.1
109
Epoch 44/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0029 - val_loss: 0.0
986
Epoch 45/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0027 - val_loss: 0.1
179
Epoch 46/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0026 - val_loss: 0.1
074
Epoch 47/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0020 - val_loss: 0.1
151
Epoch 48/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0022 - val_loss: 0.1
092
Epoch 49/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0022 - val_loss: 0.1
021
Epoch 50/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0024 - val_loss: 0.1
092
Epoch 51/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0017 - val_loss: 0.1
136
Epoch 52/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0014 - val_loss: 0.1
130
Epoch 53/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0013 - val_loss: 0.1
135
Epoch 54/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0012 - val_loss: 0.1
111
Epoch 55/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0022 - val_loss: 0.1
081
Epoch 56/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0022 - val_loss: 0.1
125
```

```
Epoch 57/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0017 - val_loss: 0.1
124
Epoch 58/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0021 - val_loss: 0.1
010
Epoch 59/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0022 - val_loss: 0.1
005
Epoch 60/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0018 - val_loss: 0.1
004
Epoch 61/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0021 - val_loss: 0.1
086
Epoch 62/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0024 - val_loss: 0.0
970
Epoch 63/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0026 - val_loss: 0.1
003
Epoch 64/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0040 - val_loss: 0.0
946
Epoch 65/200
7/7 [==============================] - 0s 20ms/step - loss: 0.0021 - val_loss: 0.0
949
Epoch 66/200
7/7 [==============================] - 0s 15ms/step - loss: 0.0030 - val_loss: 0.0
827
Epoch 67/200
7/7 [==============================] - 0s 16ms/step - loss: 0.0033 - val_loss: 0.1
079
Epoch 68/200
7/7 [==============================] - 0s 15ms/step - loss: 0.0035 - val_loss: 0.0
826
Epoch 69/200
7/7 [==============================] - 0s 21ms/step - loss: 0.0049 - val_loss: 0.0
771
Epoch 70/200
7/7 [==============================] - 0s 21ms/step - loss: 0.0049 - val_loss: 0.1
023
Epoch 71/200
7/7 [==============================] - 0s 22ms/step - loss: 0.0069 - val_loss: 0.0
793
Epoch 72/200
7/7 [==============================] - 0s 21ms/step - loss: 0.0063 - val_loss: 0.1
062
Epoch 73/200
7/7 [==============================] - 0s 21ms/step - loss: 0.0055 - val_loss: 0.0
875
Epoch 74/200
7/7 [==============================] - 0s 22ms/step - loss: 0.0034 - val_loss: 0.1
043
Epoch 75/200
7/7 [==============================] - 0s 19ms/step - loss: 0.0029 - val_loss: 0.0
```

```
875
Epoch 76/200
7/7 [==============================] - 0s 22ms/step - loss: 0.0037 - val_loss: 0.1
045
Epoch 77/200
7/7 [==============================] - 0s 15ms/step - loss: 0.0036 - val_loss: 0.1
018
Epoch 78/200
7/7 [==============================] - 0s 22ms/step - loss: 0.0024 - val_loss: 0.0
929
Epoch 79/200
7/7 [==============================] - 0s 17ms/step - loss: 0.0019 - val_loss: 0.0
943
Epoch 80/200
7/7 [==============================] - 0s 21ms/step - loss: 0.0022 - val_loss: 0.0
960
Epoch 81/200
7/7 [==============================] - 0s 22ms/step - loss: 0.0031 - val_loss: 0.0
946
Epoch 82/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0018 - val_loss: 0.0
940
Epoch 83/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0017 - val_loss: 0.1
111
Epoch 84/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0014 - val_loss: 0.0
982
Epoch 85/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0015 - val_loss: 0.1
042
Epoch 86/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0013 - val_loss: 0.1
128
Epoch 87/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0016 - val_loss: 0.1
041
Epoch 88/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0012 - val_loss: 0.1
004
Epoch 89/200
7/7 [==============================] - 0s 11ms/step - loss: 8.9212e-04 - val_loss:
0.1052
Epoch 90/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0013 - val_loss: 0.1
023
Epoch 91/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0013 - val_loss: 0.1
065
Epoch 92/200
7/7 [==============================] - 0s 11ms/step - loss: 9.7914e-04 - val_loss:
0.1097
Epoch 93/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0012 - val_loss: 0.1
126
Epoch 94/200
```

```
7/7 [==============================] - 0s 11ms/step - loss: 0.0023 - val_loss: 0.0
920
Epoch 95/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0019 - val_loss: 0.1
006
Epoch 96/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0019 - val_loss: 0.1
146
Epoch 97/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0021 - val_loss: 0.1
045
Epoch 98/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0014 - val_loss: 0.1
138
Epoch 99/200
7/7 [==============================] - 0s 11ms/step - loss: 7.7199e-04 - val_loss:
0.1061
Epoch 100/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0011 - val_loss: 0.1
172
Epoch 101/200
7/7 [==============================] - 0s 11ms/step - loss: 9.3456e-04 - val_loss:
0.1126
Epoch 102/200
7/7 [==============================] - 0s 9ms/step - loss: 0.0014 - val_loss: 0.10
96
Epoch 103/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0022 - val_loss: 0.1
119
Epoch 104/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0015 - val_loss: 0.1
014
Epoch 105/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0024 - val_loss: 0.1
119
Epoch 106/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0021 - val_loss: 0.1
019
Epoch 107/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0023 - val_loss: 0.1
183
Epoch 108/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0016 - val_loss: 0.1
251
Epoch 109/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0014 - val_loss: 0.1
149
Epoch 110/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0011 - val_loss: 0.1
118
Epoch 111/200
7/7 [==============================] - 0s 12ms/step - loss: 8.7829e-04 - val_loss:
0.1201
Epoch 112/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0012 - val_loss: 0.1
109
```

```
Epoch 113/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0019 - val_loss: 0.1
109
Epoch 114/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0030 - val_loss: 0.1
020
Epoch 115/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0040 - val_loss: 0.1
020
Epoch 116/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0095 - val_loss: 0.0
875
Epoch 117/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0107 - val_loss: 0.0
844
Epoch 118/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0124 - val_loss: 0.0
953
Epoch 119/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0120 - val_loss: 0.0
695
Epoch 120/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0088 - val_loss: 0.0
670
Epoch 121/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0069 - val_loss: 0.0
937
Epoch 122/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0065 - val_loss: 0.0
697
Epoch 123/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0067 - val_loss: 0.1
121
Epoch 124/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0086 - val_loss: 0.0
955
Epoch 125/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0059 - val_loss: 0.0
973
Epoch 126/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0041 - val_loss: 0.0
871
Epoch 127/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0033 - val_loss: 0.0
989
Epoch 128/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0025 - val_loss: 0.0
926
Epoch 129/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0018 - val_loss: 0.0
850
Epoch 130/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0026 - val_loss: 0.1
061
Epoch 131/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0044 - val_loss: 0.0
```

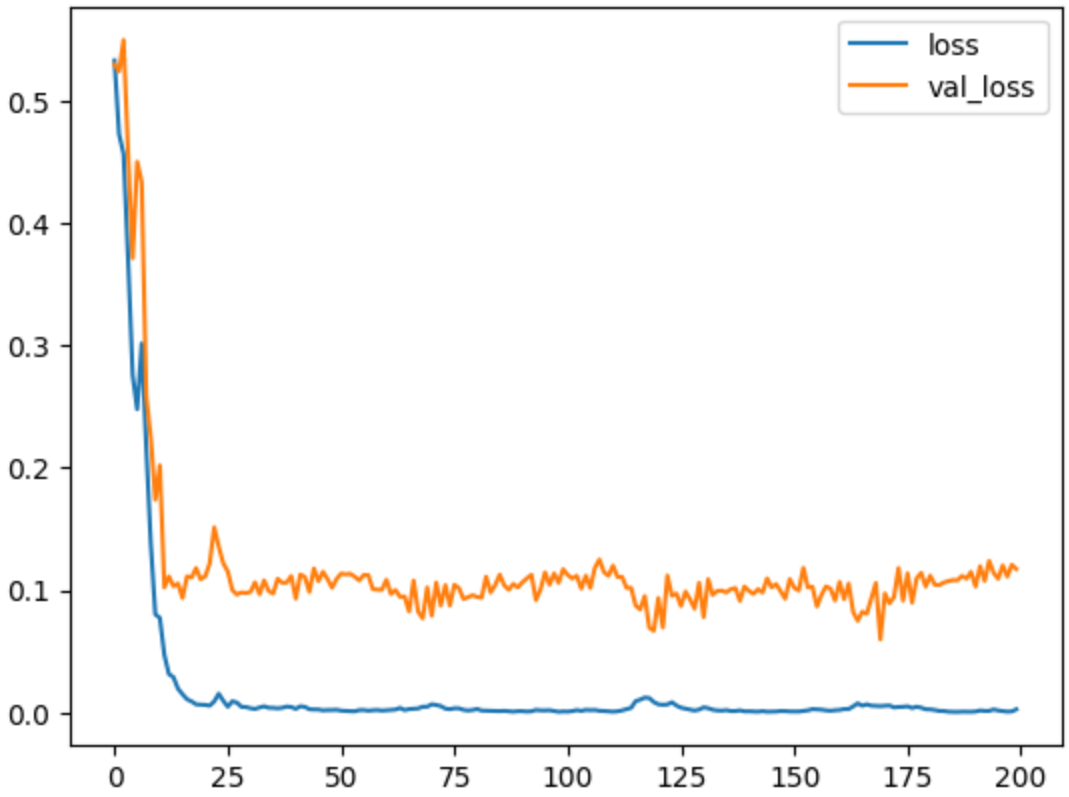
```
781
Epoch 132/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0037 - val_loss: 0.1
091
Epoch 133/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0024 - val_loss: 0.0
962
Epoch 134/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0019 - val_loss: 0.0
995
Epoch 135/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0017 - val_loss: 0.0
997
Epoch 136/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0021 - val_loss: 0.0
983
Epoch 137/200
7/7 [==============================] - 0s 13ms/step - loss: 0.0014 - val_loss: 0.1
009
Epoch 138/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0014 - val_loss: 0.1
019
Epoch 139/200
7/7 [==============================] - 0s 18ms/step - loss: 0.0020 - val_loss: 0.0
914
Epoch 140/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0011 - val_loss: 0.1
031
Epoch 141/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0012 - val_loss: 0.0
995
Epoch 142/200
7/7 [==============================] - 0s 12ms/step - loss: 9.6197e-04 - val_loss:
0.0970
Epoch 143/200
7/7 [==============================] - 0s 11ms/step - loss: 8.8874e-04 - val_loss:
0.1009
Epoch 144/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0013 - val_loss: 0.0
982
Epoch 145/200
7/7 [==============================] - 0s 11ms/step - loss: 8.4202e-04 - val_loss:
0.1097
Epoch 146/200
7/7 [==============================] - 0s 12ms/step - loss: 9.1971e-04 - val_loss:
0.1024
Epoch 147/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0010 - val_loss: 0.1
051
Epoch 148/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0014 - val_loss: 0.0
993
Epoch 149/200
7/7 [==============================] - 0s 14ms/step - loss: 0.0013 - val_loss: 0.0
931
Epoch 150/200
```

```
7/7 [==============================] - 0s 11ms/step - loss: 0.0011 - val_loss: 0.1
091
Epoch 151/200
7/7 [==============================] - 0s 10ms/step - loss: 9.7763e-04 - val_loss:
0.1021
Epoch 152/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0010 - val_loss: 0.0
999
Epoch 153/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0014 - val_loss: 0.1
184
Epoch 154/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0020 - val_loss: 0.1
023
Epoch 155/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0030 - val_loss: 0.1
029
Epoch 156/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0028 - val_loss: 0.0
868
Epoch 157/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0025 - val_loss: 0.0
963
Epoch 158/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0019 - val_loss: 0.1
032
Epoch 159/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0016 - val_loss: 0.1
013
Epoch 160/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0021 - val_loss: 0.0
917
Epoch 161/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0023 - val_loss: 0.1
069
Epoch 162/200
7/7 [==============================] - 0s 13ms/step - loss: 0.0030 - val_loss: 0.0
928
Epoch 163/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0030 - val_loss: 0.1
055
Epoch 164/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0052 - val_loss: 0.0
822
Epoch 165/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0077 - val_loss: 0.0
751
Epoch 166/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0057 - val_loss: 0.0
825
Epoch 167/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0069 - val_loss: 0.0
808
Epoch 168/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0058 - val_loss: 0.0
932
```

```
Epoch 169/200
7/7 [==============================] - 0s 18ms/step - loss: 0.0056 - val_loss: 0.1
060
Epoch 170/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0054 - val_loss: 0.0
599
Epoch 171/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0056 - val_loss: 0.0
974
Epoch 172/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0059 - val_loss: 0.0
893
Epoch 173/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0044 - val_loss: 0.0
944
Epoch 174/200
7/7 [==============================] - 0s 12ms/step - loss: 0.0047 - val_loss: 0.1
182
Epoch 175/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0049 - val_loss: 0.0
916
Epoch 176/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0053 - val_loss: 0.1
139
Epoch 177/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0039 - val_loss: 0.0
899
Epoch 178/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0049 - val_loss: 0.1
097
Epoch 179/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0041 - val_loss: 0.1
142
Epoch 180/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0028 - val_loss: 0.1
030
Epoch 181/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0028 - val_loss: 0.1
119
Epoch 182/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0022 - val_loss: 0.1
046
Epoch 183/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0015 - val_loss: 0.1
043
Epoch 184/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0014 - val_loss: 0.1
063
Epoch 185/200
7/7 [==============================] - 0s 10ms/step - loss: 8.7073e-04 - val_loss:
0.1075
Epoch 186/200
7/7 [==============================] - 0s 12ms/step - loss: 7.5564e-04 - val_loss:
0.1084
Epoch 187/200
7/7 [==============================] - 0s 11ms/step - loss: 6.6525e-04 - val_loss:
```

```
0.1081
Epoch 188/200
7/7 [==============================] - 0s 13ms/step - loss: 8.9494e-04 - val_loss:
0.1117
Epoch 189/200
7/7 [==============================] - 0s 10ms/step - loss: 8.1483e-04 - val_loss:
0.1095
Epoch 190/200
7/7 [==============================] - 0s 11ms/step - loss: 7.9022e-04 - val_loss:
0.1151
Epoch 191/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0010 - val_loss: 0.1
030
Epoch 192/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0019 - val_loss: 0.1
197
Epoch 193/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0015 - val_loss: 0.1
077
Epoch 194/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0015 - val_loss: 0.1
240
Epoch 195/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0027 - val_loss: 0.1
141
Epoch 196/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0018 - val_loss: 0.1
094
Epoch 197/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0013 - val_loss: 0.1
203
Epoch 198/200
7/7 [==============================] - 0s 10ms/step - loss: 9.0915e-04 - val_loss:
0.1112
Epoch 199/200
7/7 [==============================] - 0s 10ms/step - loss: 0.0011 - val_loss: 0.1
208
Epoch 200/200
7/7 [==============================] - 0s 11ms/step - loss: 0.0028 - val_loss: 0.1
174
```

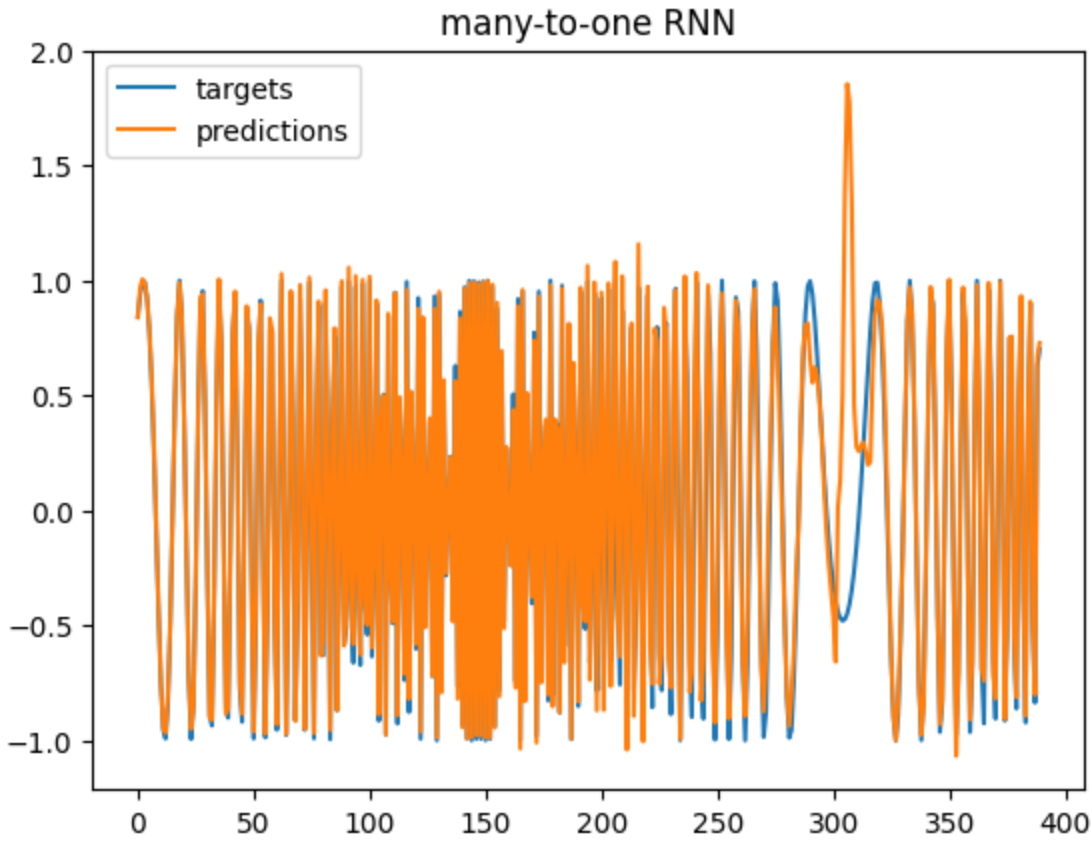
In [12]:
```
# plot some data
plt.plot(r.history['loss'], label='loss')
plt.plot(r.history['val_loss'], label='val_loss')
plt.legend()
plt.show()
```



In [13]:
```
# One-step forecast using true targets
outputs = model.predict(X)
print(outputs.shape)
predictions = outputs[:,0]

plt.plot(Y, label='targets')
plt.plot(predictions, label='predictions')
plt.title("many-to-one RNN")
plt.legend()
plt.show()
```

```
13/13 [==============================] - 0s 2ms/step
(390, 1)
```



In [14]:
```
# Multi-step forecast
forecast = []
input_ = X[-N//2]
while len(forecast) < len(Y[-N//2:]):
  # Reshape the input_ to N x T x D
  f = model.predict(input_.reshape(1, T, 1))[0,0]
  forecast.append(f)

  # make a new input with the latest forecast
  input_ = np.roll(input_, -1)
  input_[-1] = f

plt.plot(Y[-N//2:], label='targets')
plt.plot(forecast, label='forecast')
plt.title("RNN Forecast")
plt.legend()
plt.show()
```

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