Research Statement

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My research mainly focuses on the surface parameterization problem and their applications. A surface parameterization is a bijective mapping that maps a surface to a domain of simple shape. In particular, a conformal parameterization of an arbitrary surface can be carried out by the cohomology groups of the surface. The algorithm based on the holomorphic differentials is first proposed by Gu and Yau [1]. The field of computational geometry becomes more popular thereafter. In practice, large deformations can be easily handled by the surface parameterization while preserving much geometric information. The surface parameterization has been widely applied to tasks of digital image and geometry processing, such as surface registration, surface resampling, surface remeshing and texture mapping. Practical applications of 3D animation and medical image analysis can be smoothly carried out via the one-to-one correspondence between the surface and the domain of simple shape.

In recent years, under the guidance of Professor Yau[1], we have successfully developed an efficient algorithm for the computation of surface conformal parameterizations of simply and multiply connected surfaces of genus zero based on the Koebe's generalized uniformization theorem. Figure 1 shows a multiply connected surface and its conformal parameterization. The efficiency outperforms other state-of-the-art algorithms by more than 5 times. The academic paper [2] has been published in *Journal of Scientific Computing*. Such an encouraging result enables the conformal parameterization to be applied to the real-time applications. Furthermore, a year later, the algorithm is generalized to the computation of area-preserving parameterization based on the modified Laplace-Beltrami operators. The preprint [3] has been submitted to *Journal of Scientific Computing*.

On the other hand, based on our proposed efficient algorithm of the surface conformal parameterizations, we have developed the applications on 3D animation and 3D medical image analysis which is summarized in brief in the following section. The US patent [4] of the applications on 3D animation is currently under revise.

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[1]Professor Shing-Tung Yau, department of mathematics, Harvard University.
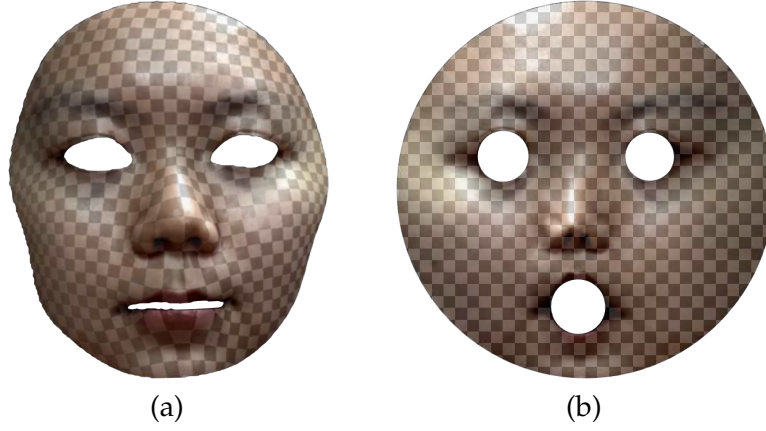
Figure 1: (a) The multiply connected surface of a human face. (b) The conformal parameterization obtained by the proposed algorithm.

Figure 2: The 3D scanner in ST Yau Center, Taiwan.

1 Applications on 3D Animation

In this section, we introduce the following applications of surface parameterizations on 3D animation:

1. 3D video compression,
2. Texture mapping for surfaces in 3D,
3. 3D motion retargeting.

The real data are obtained using the 3D scanner, shown in Figure 2, in ST Yau Center in Taiwan. The demo videos are available at `https://mhyueh.github.io/projects.html`. The patent number of the applications is *US 20170186208 A1* [4].

1.1 3D Video Compression

The 3D video, captured using the 3D scanner, contains 30 surfaces together with texture images per second. Each surface data is of size roughly 30
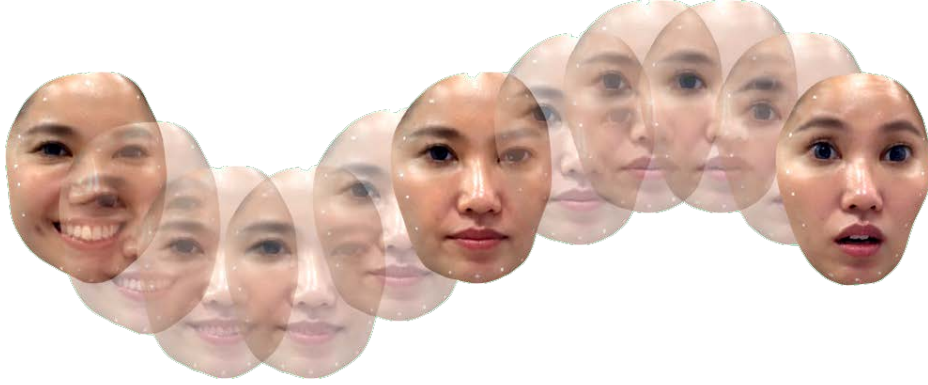
Figure 3: An illustration of the cubic spline homotopy.

megabytes. The goal of the 3D video compression is to use less quantity of the raw data of a 3D video to approximate the original 3D video. Note that each frame of the 3D video is a simply connected open surface, which is conformally equivalent to a unit disk. By using the proposed algorithm, the conformal equivalence can be computed efficiently and accurately. Then the diffeomorphism between each pair of surfaces can be constructed on the disk. Ultimately, the video sequence is reconstructed by the cubic spline homotopy between each pair of diffeomorphisms, as illustrated in Figure 3. As a result, the whole video sequence is reconstructed by merely 1.6% of the original data while the resolution of the video is preserved.

1.2 Texture Mapping for Surfaces in 3D: Virtual Makeup

Texture mapping refers to replace the texture of a surface with another one. It has been widely applied to the movie industries, e.g., the famous movie, *Avatar*. In the movie, actors and actress perform without heavy makeup. Instead, a *virtual makeup* technique is applied to create a lifelike visual effect. The key issue is to find an appropriate diffeomorphism between the surface and the desired texture image that preserves features. The texture image is regarded as a planar surface in $\mathbb{R}^2$, so that the diffeomorphism between the mesh and the texture image can be computed similarly. With the proposed algorithm, the virtual makeup can be performed efficiently and robustly. Figure 4 shows the virtual makeup of a human face.

1.3 3D Motion Retargeting

Motion retargeting is aimed to use the information of a sequence of 3D video to drive a target surface. In other words, the motion of a target surface model is controlled by the motion of a given 3D video. The application already
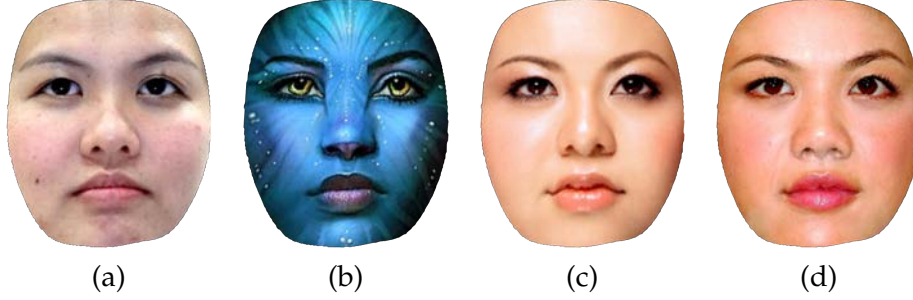
Figure 4: (a) The simply connected open surface of a human face. (b), (c), (d) The human face with virtual makeup.

appears in *iPhone X*. By applying the proposed algorithm, the conformal equivalence can be efficiently computed. The feature-preserving diffeomorphism between each pair of surfaces is constructed on the disk. Next, a vector field is constructed by the difference of two consecutive frames. Then the embedding of the target surface is controlled by the vector field via the topological equivalence.

2 Applications on Medical Image Analysis

Surface parameterizations have been applied to deal with the issues from the medical image analysis [5, 6]. However, the efficiency of the computations is not satisfactory, especially when the data size is huge. Due to this reason, the technology of parameterization cannot be widely used in clinical medicine at present. To overcome this issue, we aim to apply the proposed algorithm to solve the problem efficiently.

In this section, we introduce the following applications of surface parameterizations on medical image analysis:

1. Visualization of brain deformation,
2. Area-preserving intestine flattening,
3. Measurement of hippocampus deformation.

The demo videos are available at https://mhyueh.github.io/projects.html.

2.1 Visualization of Brain Deformation

The deformation of human brain is subtle and not easy to be observed. In order to efficiently detect the deformation, the one-to-one correspondence between two brain surfaces is necessary. With the efficient parameterization algorithms for genus zero closed surfaces, the registration problem between
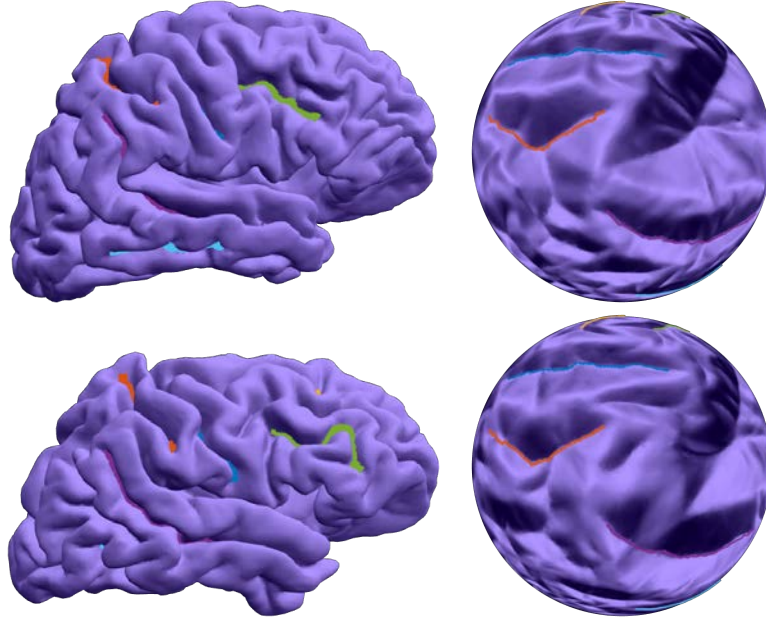
Figure 5: The healthy brain (top) and the depauperated brain (bottom), and their spherical parameterizations.

two brains is reduced into the registration problem on the unit sphere, which is much easier since the shape of domain are identical. Then the deformation from one brain to another can be visualized via the homotopy between the identity mapping and the registration mapping between two brains. Figure 5 shows the surfaces of healthy brain (top) and the slightly depauperated brain (bottom), respectively.

2.2 Area-Preserving Intestine Flattening

Polyps are the predecessor of intestinal cancers. However, polyp detection is usually not an easy task, especially when the polyps are hidden in the folds. In order to efficiently detect the polyps on the intestinal surface, an area-preserving mapping can be applied to flatten the surface to a domain of rectangle, so that the polyps can be easily found in the rectangle domain. Then the exact locations of polyps can be obtained by the one-to-one correspondence between the intestinal surface and the rectangle domain. Figure 6 shows a surface of intestine model (left) and its area-preserving parameterization (right).
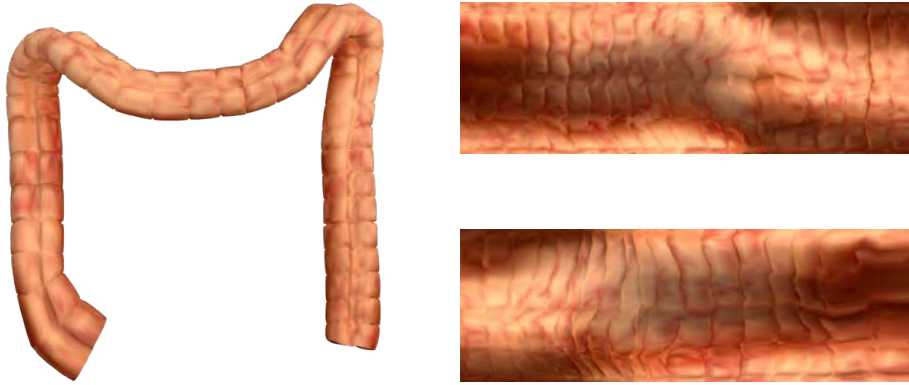
Figure 6: The surface of intestine model (left) and its area-preserving parameterization (right).

2.3 Hippocampus Deformation

It is known that Alzheimer's disease can be detected by the deformation of the hippocampus. The deformation can be measured via the one-to-one correspondence between two surfaces of hippocampi. Similar as the brain mappings, the mapping between two hippocampi can be obtained via the registration mapping on the spherical domain of parameterization mappings. Figure 7 shows the original surface (left) and the deformed surface (right) of a hippocampus. The color represents the quantity of the displacement.
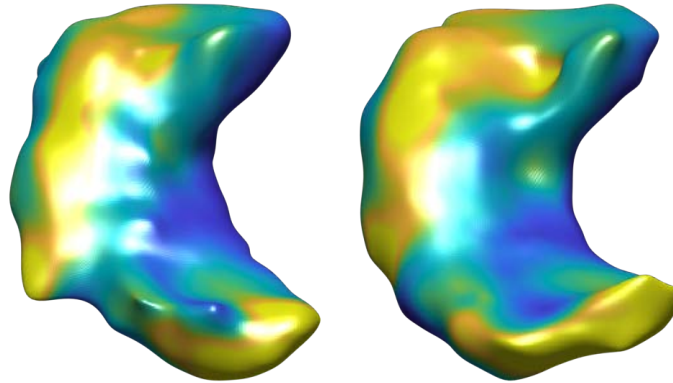

Figure 7: The original surface (left) and the deformed surface (right) of a hippocampus. The color represents the quantity of the displacement.

3 Summary of Selected Research

In this section, I briefly summarize my research on the following topics:

1. Surface parameterizations:

    (a) Algorithm for conformal parameterizations [2],

    (b) Algorithm for area-preserving parameterizations [3],

    (c) Applications of surface parameterizations on 3D animation (US Patent) [4].

2. High dimensional nonlinear signal filtering:

    (a) Algorithm of the Yau-Yau method [7],

    (b) Algorithm for high dimensional nonlinear filtering problems [8],

    (c) Method for solving high dimensional nonlinear filtering problems (US Patent) [9].

3.1 Efficient Algorithms of Surface Parameterizations

A surface parameterization is a bijective mapping that maps a surface to a simply shaped domain. The surface parameterization has been widely applied to tasks of digital image and geometry processing, such as surface registration, surface resampling, surface remeshing and texture mapping. It is usually difficult and time-consuming to carry out a task of geometry processing on a surface of a complicated geometrical structure. An appropriate parameterization for a surface can be applied to simplify the task via the one-to-one correspondence between the surface and the domain of a simple shape. Most commonly used parameterizations are based on *conformal mappings* or *equiareal mappings*. A mapping is said to be conformal if it preserves angles. A mapping is said to be equiareal if it preserves areas. Figure 8 shows the mesh models of *Human Brain*, and its conformal and equiareal parameterizations.

On this topic, we have proposed efficient algorithms for the computation of conformal parameterizations [2] and equiareal parameterizations [3] of simply and multiply connected surfaces of genus zero.

The patent number of the applications of surface parameterizations on 3D animation is *US 20170186208 A1* [4].

3.2 High Dimensional Nonlinear Filtering Algorithms

The nonlinear filter has important applications in military, engineering and commercial industries. The aim of the filtering problem is to accurately
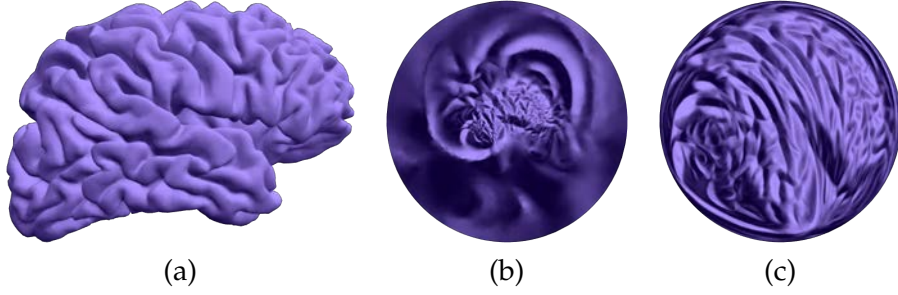
Figure 8: (a) The mesh model of *Human Brain* with a boundary. (b) The conformal parameterization of the human brain surface. (c) The equiareal parameterization of the human brain surface.

estimate the position of particles in an n-dimensional space. The motions of the particles are modeled by a stochastic differential equation

$$\begin{cases} d\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t))\, dt + d\mathbf{v}(t) & \mathbf{x}(0) = \mathbf{x}_0, \\ d\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t))\, dt + d\mathbf{w}(t) & \mathbf{y}(0) = 0, \end{cases}$$

where the drift term $\mathbf{f}$ and the observation term $\mathbf{h}$ are nonlinear. The Yau-Yau method reduces the nonlinear filtering problem into the initial-value problem of Kolmogorov equations [7]. The solutions of the Kolmogorov equations are exactly the probability density functions with respect to the position of the particles. In this research, we propose a nonnegativity-preserving numerical algorithm of the Yau-Yau method for solving high-dimensional nonlinear filtering problems by applying the quasi-implicit Euler method (QIEM) with discrete sine transform (DST). Consequently, our iterative process preserves the probability density functions. Furthermore, our algorithms are directly applicable on the compact difference schemes, so that the number of spatial points can be substantially reduced and retain the same accuracy [8]. Numerical results indicate that the proposed algorithm is capable of solving up to six-dimensional nonlinear filtering problems efficiently and accurately, as shown in Figure 9.

The patent number of the applications on solving high dimensional nonlinear filtering problem is *US 20170124026 A1* [9].

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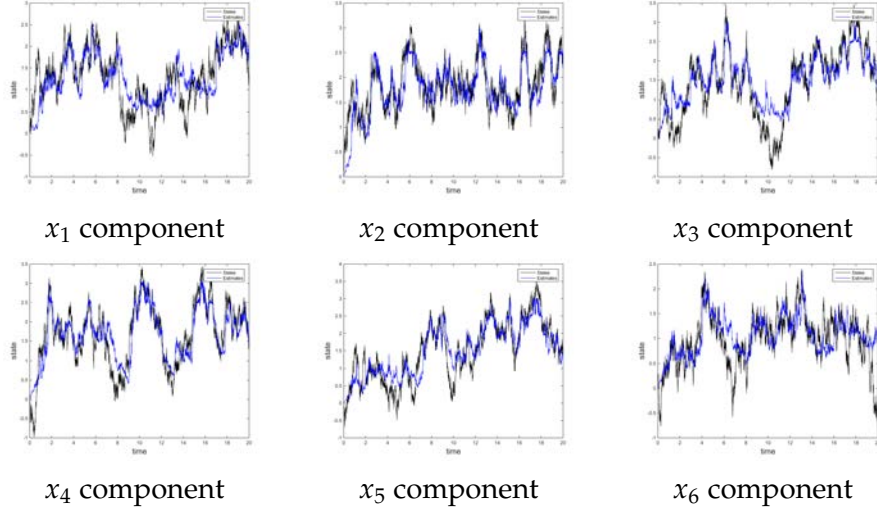
x_1 component	x_2 component	x_3 component
x_4 component	x_5 component	x_6 component

Figure 9: The numerical results of QIEM with DST for 6-D cubic sensor problem

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