UNIVERSITY OF COPENHAGEN
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Master's Thesis

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Aging and r*: A Rebound in Sight?

A quantitative simulation study of the natural real interest rate in the USA

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Abstract

This thesis investigates the relationship between aging and the natural real interest rate. Since the 1980s, real interest rates in advanced economies have steadily declined, reaching historically low levels before the onset of the COVID-19 pandemic. This prolonged downward trend coincided with significant demographic shifts, including rising life expectancy and declining fertility rates, contributing to an aging population and an increasing old-age dependency ratio. Extensive literature highlights a negative correlation between aging and the natural real interest rate. This thesis seeks to examine this relationship further by employing a large-scale overlapping generations (OLG) model calibrated to the U.S. economy. In particular, the thesis attempts to quantify and verify the hypothesis put forward by Goodhart and Pradhan who predict a rebound in the natural real interest rate due to dissaving among large elderly cohorts and mounting pressures on public expenditures.

The thesis finds that aging, composed of rising life expectancy and declining fertility rates, have contributed to a decline in the natural real interest rate of more than 1.5 percentage points since 1970. The analysis further reveals that the baby boomer generation born after World War II has placed a distinct downward pressure on the natural interest rate as they have accumulated savings for retirement and since gradually exited the labor force. The model predicts a minor rebound in the natural interest rate after 2025 as the downward pressure from the baby boomer cohorts subsides, generating a temporary increase in the growth rate of the labor force. Meanwhile, a rising dependency ratio is expected to increase expenditures on pensions and healthcare, which will exert upward pressure on the natural interest rate, whether financed through higher taxes or public debt. Depending on the extent to which public expenditures will rise, it is more likely that the natural real interest rate will settle at a higher level than pre-pandemic norms. In the short to medium term, the thesis thus supports Goodhart and Pradhan's prediction that the natural real interest rate is set for a rebound. Over the longer term, however, it finds that potential continued increases in life expectancy and further declines in fertility will reintroduce downward pressure on the natural real interest rate, consistent with the existing literature. It thus rejects the notion that a permanently elevated natural real interest rate is imminent due to dissaving by retirees. Instead, it confirms that rising public expenditures will remain a critical driver of upward pressure on the natural real interest rate.

This thesis, by examining the specific effects of the baby boomer generation and the impact of aging on public expenditures, thereby provides a more nuanced perspective on the narrative that demographic trends are inevitably linked with falling interest rates and secular stagnation.

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Preface

I would like to express my gratitude to the people who have supported me throughout the thesis process. First and foremost, I wish to thank my supervisor, Søren Hove Ravn, for his valuable and qualified guidance. Thanks also to Rasmus Klitte Andersen, Oliver Á Rógvi, and Thomas Bernt Henriksen for their helpful advice and input. Finally, a special thanks to Andrea Papetti for his willingness to engage in academic discussions.[1]

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[1] This thesis was originally written in Danish. It has been translated into English by the author with assistance from AI tools. It also contains some minor revisions.

Contents

1 Introduction

Since the Industrial Revolution, an increasing number of countries have undergone a demographic transition, characterized by significant declines in fertility and mortality rates. In all advanced economies, birth rates are now below the level required to maintain the population without immigration. Combined with rising life expectancy, this has led to an increasing old-age dependency ratio. Since the mid-1980s, real interest rates have been declining across all developed economies. During the 2010s and up to 2022, several economies experienced negative interest rates, sparking discussions about secular stagnation and liquidity traps. However, this trend, which most economists attribute to a persistent decline in the natural rate of interest, was reversed after the COVID-19 pandemic. Since 2022, real interest rates have risen significantly as central banks responded to the post-pandemic surge in inflation. Whether this rise is a short-term phenomenon or real interest rates will stabilize at a permanently higher level than before the pandemic remains an open question. If interest rates remain high, it would indicate that the underlying structural interest rate, also referred to as the *natural* rate of interest, has shifted. An increase in the natural rate of interest would, among other things, raise borrowing costs for governments far into the future, thereby impacting the sustainability of public finances.

Several factors influence the development of the natural rate of interest, including demographics. Demographic processes can be viewed as an exogenous factor affecting economic variables, and demographic data and projections are generally associated with a low degree of uncertainty. This makes demographic variables a natural starting point in efforts to explain long-term macroeconomic trends.[2]

This thesis aims to explore the relationship between aging and the natural rate of interest. By simulating a quantitative overlapping generations (OLG) model calibrated to the U.S. economy, the thesis finds that demographic developments have exerted downward pressure on the natural rate of interest since 1970. This confirms much of the related literature, which shows that increasing life expectancy and declining labor force growth rates collectively reduce the natural rate of interest. At the same time, the thesis makes a unique contribution by testing, within the framework of a quantitative model, whether future demographic trends could lead to a turning point in the natural rate of interest, as suggested by Goodhart and Pradhan (2020). The analysis reveals that the large cohorts born after 1945, known as the baby boomer generation, have applied particularly strong downward pressure on the natural rate of interest since the 1990s. Since this negative pressure is expected to diminish, the thesis concludes that a turning point in the natural rate of interest is likely to occur after 2025. Moreover, as public spending on pensions and healthcare for the elderly increases, this will exert additional upward pressure on the natural rate of interest.

The thesis, therefore, suggests that real interest rates are unlikely to return to the low levels seen before 2022 in the medium term. Consequently, it rejects the notion that the U.S.—and

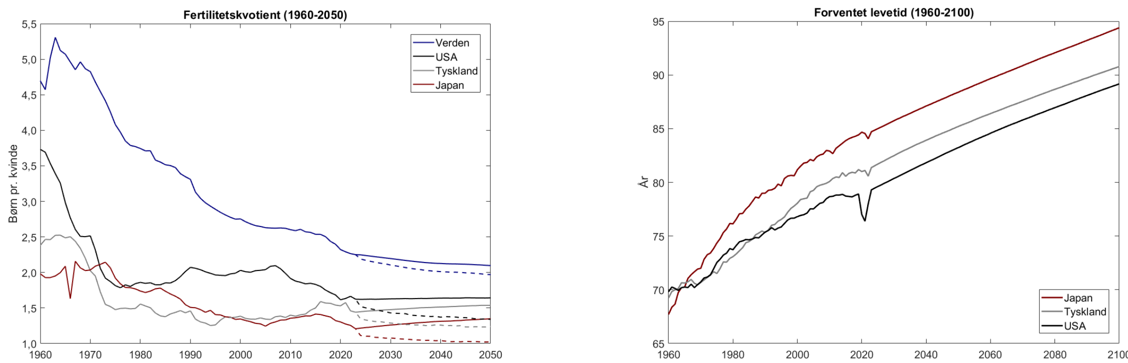
[2] Other factors include among others productivity growth and global capital flows (Barrett et al., 2023).

likely other advanced economies—is inevitably trapped in secular stagnation due to demographics.

This thesis focuses on the United States, the world's largest economy and bond market, making it particularly relevant for interest rate movements globally. Additionally, the nature of the demographic transition in the U.S. is comparable to that of other developed economies, implying the thesis findings are also relevant in a European.

Fertility rates have been declining in both the U.S. and the rest of the world since the 1960s (see Figure 1a). Historically, fertility in the U.S. has been higher than in other developed economies, but it has declined further since 2007, reaching 1.62 children per woman in 2023. The UN's population projections for the U.S. assume in their medium scenario that fertility rates will abruptly stop falling and stabilize at 1.65. However, if the downward trend continues, the lower bound of the UN's 80% confidence interval seems more plausible, with fertility rates expected to stabilize below 1.30 by 2100.

Figure 1: Fertility (a) and life expectancy (b).



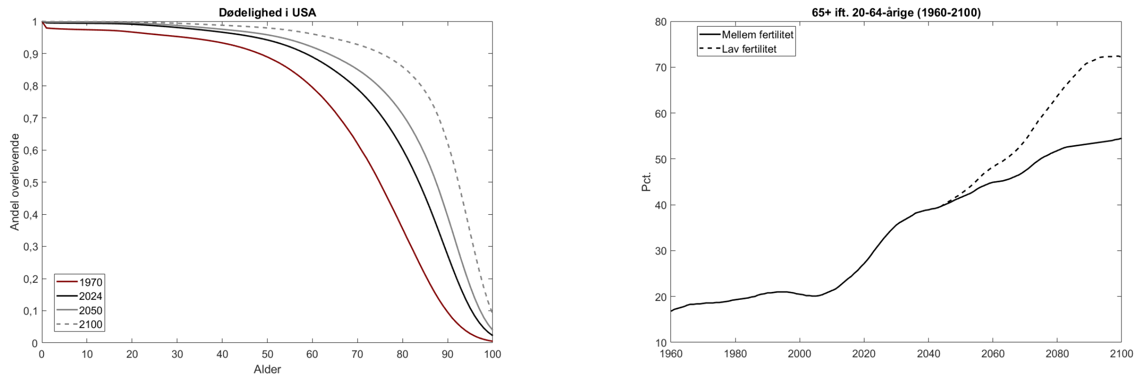
(a) Total fertility rate (children per woman), 1960–2050. Projections from the medium scenario after 2023. The dashed lines represent projections from UN's lower 80% confidence interval. *Verden*: World, *Tyskland*: Germany. Source: UN (2024).

(b) Life expectancy, 1960–2100. Defined as the average lifespan of a newborn, given the age-specific mortality rates in a given year. Weighted average for both sexes. Projections based on the medium scenario after 2023. Tyskland: *Germany*. Source: UN (2024).

At the same time, mortality rates have declined (see Figure 2a), resulting in higher life expectancy. In the U.S., life expectancy increased from just under 71 years in 1970 to 79.3 years in 2023. This trend has been almost uninterrupted, except during the COVID-19 pandemic, which hit the U.S. particularly hard (see Figure 1b). Life expectancy is expected to continue rising, reaching 83.2 years by 2050 and 89.2 years by 2100. In other developed economies, the trend is even more pronounced.

The combination of declining fertility and rising life expectancy has resulted in a higher dependency ratio between the elderly and the working-age population. In 2023, the U.S. had 30% of individuals aged 65 or older relative to the population aged 20 to 64. By 2050, this ratio is projected to exceed 40%, and by 2100, it will range between 54% and 72%, depending on future fertility developments (see Figure 2b).

Figure 2: Mortality (a) and dependency ratio (b)



(a) Unconditional survival curves in the USA. Age in years on the primary axis. 2024, 2050 and 2100 are based on UN's projections. Source: UN (2024) and own contribution.

(b) The number of individuals aged 65 and older relative to those aged 20–64 in the USA, 1960–2100. Projections after 2023. The medium scenario assumes a fertility rate stabilizing at 1.65, while the low scenario assumes a rate of 1.15 children per woman. Source: UN (2024).

Definition and identification of the natural real interest rate

The natural rate of interest is defined as the risk-free real interest rate that ensures a long-run equilibrium between the supply of and demand for savings.[3] A distinction is made between the concepts of the *natural* and the *neutral* rate of interest.[4] The distinction arises from the following relation, cf. Platzer et al. (2022); the nominal interest rate i_t set by the central bank can be decomposed into three components, as shown in (1):[5] the neutral real rate of interest $r_t^{n\star}$, expected inflation $E_t[\pi_{t+1}]$, and a monetary policy component MP_t, which is determined by the central bank based on how restrictive monetary policy is intended to be. The central bank has direct control only over the monetary policy component and no influence on the neutral real rate of interest.

$$i_t = r_t^{n\star} + E_t[\pi_{t+1}] + MP_t. \tag{1}$$

The neutral real interest rate can be divided into a long-term structural component and a short-term cyclical component, such that $r_t^{n\star} = r_t^{s\star} + \zeta_t$, where $r_t^{s\star}$ is the structural component and ζ_t is the cyclical component, with $\zeta_t = 0$ in a cyclically neutral situation. The neutral rate is the level of the real interest rate that ensures actual economic activity equals potential activity,

[3] The concept dates back to Wicksell (1936), and in the literature, the terms *equilibrium rate* or the *Wicksellian* rate are also used to refer to the natural rate of interest.

[4] This distinction is not necessarily made in the broader literature, where the two terms are often conflated.

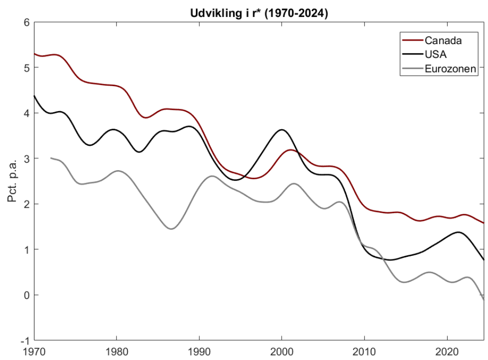
[5] Unlike the central bank rate, market interest rates additionally include a risk premium, which depends on factors such as maturity and the borrower's credibility.

where potential activity is defined as the level consistent with stable prices and wages in the absence of transitory shocks and nominal rigidities (Papetti, 2021a).

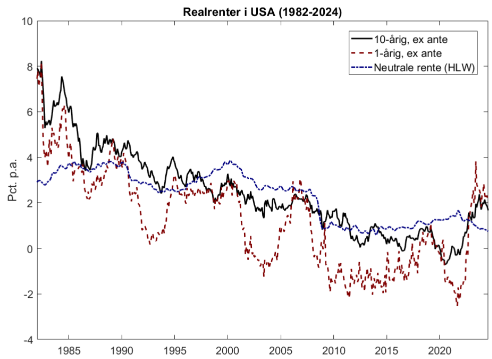
The natural real rate of interest represents the long-run level of the neutral real rate and, by definition, moves slowly and is unaffected by cyclical fluctuations (Platzer et al., 2022). In this thesis, the term *natural real rate of interest* refers to the structural component of the neutral real interest rate, which is the primary focus of the analysis. Furthermore, the definition $r_t^\star \equiv r_t^{s\star}$ is used to denote the natural real rate of interest. The terms *natural real rate of interest*, *natural rate of interest*, $r^\star$, and, unless otherwise specified, *the interest rate* are used synonymously.

The natural rate of interest is a theoretical concept and cannot be measured precisely. Any estimate of the natural rate of interest is therefore associated with uncertainty. It is beyond the scope of this thesis to estimate the natural interest rate econometrically. Instead, reference is made to the estimates provided by Holston, Laubach, and Williams (HLW) as an empirical measure of the neutral interest rate in the U.S. Figure 3a shows the evolution of the natural rate of interest in the U.S., Canada, and the Eurozone. The HLW series are smoothed using an HP filter to remove the cyclical component (Hodrick and Prescott, 1997). According to HLW, the natural real rate of interest in the U.S. has declined by more than 3 percentage points (pp) between 1970 and 2024.

Figure 3: Evolution of $r^\star$ (a) and real interest rates in the USA (b)



(a) Evolution of $r^\star$ 1970-2024 (July) in Canada, USA and the Eurozone. Smoothed with a HP filter, $\lambda = 100$. Source: Holston et al. (2024) and own contribution.



(b) Real interest rates (ex ante) in the USA 1982-2024 (July). The 10-year and 1-year real interest rates are calculated by dividing the market yield on government bonds with constant maturity with the expected inflation rate. Monthly averages. Source: FRED (2024c), Holston et al. (2024), and own contribution.

In Figure 3b, it appears that both short- and long-term real interest rates in the U.S., defined here as the effective ex-ante real rates on 10-year and 1-year government bonds, have been declining since the early 1980s until 2021, after which they rose sharply. The same trend has been observed in other advanced economies (see Figure A14). The neutral interest rate estimated by HLW has also been declining over this period and closely tracks the 10-year real rate. A notable exception is the early 1980s, when the neutral real interest rate was significantly lower than actual real interest rates. After 2018, the neutral real rate also diverges markedly from actual real rates. HLW finds that the neutral interest rate has been declining since late 2021, coinciding with the period when actual real rates began to rise. This suggests that the increase in real interest rates following the COVID-19 pandemic may not be persistent. However, estimates of the neutral real interest rate are subject to uncertainty—particularly during turning points and periods of economic shocks (Ingholt et al., 2023).

1.1 The four mechanisms

Aging is a consequence of declining fertility, which leads to lower population growth, and declining mortality, which results in higher life expectancy. The effect of aging on interest rates can be divided into three mechanisms (Carvalho et al., 2016). If the pension system is funded through taxes on labor, a fourth mechanism also arises (Papetti, 2021a):

1. *Labor supply.* Lower population growth reduces the growth of the labor force, which, for a given stock of capital, leads to a higher capital-to-labor ratio. This lowers the marginal product of capital, exerting downward pressure on the natural interest rate.

2. *Life expectancy.* An increase in life expectancy, in the form of a higher probability of survival, increases households' incentives to save more, as they anticipate a longer retirement period. This increases the supply of savings, exerting downward pressure on the natural interest rate.

3. *Dependency ratio.* Lower population growth and higher life expectancy lead to a higher dependency ratio, as the number of retirees relative to the working-age population increases. Since retirees are more likely to dissave, this reduces the stock of capital, all else equal, exerting upward pressure on the natural interest rate.

4. *Pension expenditures.* If the pension system is funded through income taxes, aging will, for a given replacement rate, increase pressure on public finances. This, all else equal, requires higher taxes. A higher tax burden will crowd out the stock of private capital, as households' ability to save diminishes. This exerts upward pressure on the natural interest rate.

Thus, there are two mechanisms through which aging lowers the natural interest rate and two mechanisms that suggest the opposite. The overall effect of aging on the natural interest rate depends on which of these mechanisms dominate.

To examine how demographic developments have affected the natural interest rate in the United States, I construct a quantitative OLG model[6] for a closed economy, inspired by Eggertsson et al. (2019a). The term *quantitative* OLG model refers to the specification introduced

[6]OLG stands for *overlapping generations.* The terms OLG model and lifecycle model are used synonymously.

by Auerbach and Kotlikoff (1988), which includes all one-year cohorts within a given age interval. This allows the model to be calibrated to the empirical age distribution. The model differs from Eggertsson et al. (2019a), as I extend the framework to include 74 generations and introduce a pension system financed by income taxes inspired by Papetti (2021a).

2 Literature Review

Interest in the relationship between demographics and the natural interest rate was heightened when real interest rates reached historically low levels in the 2010s, including negative rates in many countries, while aggregate demand remained low. This led to a resurgence of the debate on secular stagnation,[7] which describes a situation where investment demand is chronically too low to absorb the supply of savings, with $r^\star < 0$ (Summers, 2013, 2015).[8] Since 2022, however, real interest rates have risen again, and among others, Lawrence Summers, who himself was an advocate for the reintroduction of the concept in 2013, has since stated that he no longer expects secular stagnation, citing, i.a., the growing global government debt and the slowdown in life expectancy growth (Summers, 2023). On the other hand, among others, Olivier Blanchard maintains that advanced economies remain in secular stagnation, as the underlying structural factors, including demographic developments, will persist (Blanchard, 2023).

Eggertsson et al. (2019a) apply a quantitative life-cycle model for a closed economy with 56 generations to analyze how aging in the USA have influenced the real interest rate, with particular focus on secular stagnation. Their main findings show that the natural interest rate fell by a total of 4 pp between 1970 and 2015. Of this, demographic developments alone caused a decline of 3.76 pp, of which 1.84 pp are attributed to falling fertility, and 1.92 pp are due to lower mortality. Lower productivity growth is estimated to have contributed to a decline of 1.9 pp, while higher government debt, all else equal, has increased the natural interest rate by 2.11 pp. They find that the baby boomer generation put upward pressure on the natural interest rate between 1970 and 2000, while subsequently having exerted downward pressure on the rate up to 2024. Their model shows a turning point in the interest rate between 2025 and 2040. They also find that a decline in the wage share and the relative price of capital have contributed to the fall in the natural interest rate to a lesser extent. Since the model predicts a negative real interest rate in the long run, they conclude that secular stagnation is likely to remain a persistent phenomenon due to demographic developments. They link the declining wage share to the higher profit rate observed in the USA during the period, which they believe reflects that the economy has become more concentrated. This development has caused a decoupling between the interest rate and the marginal product of capital. While both have fallen due to

[7]The concept originally dates back to Hansen (1939) in relation to the Great Depression of the 1930s.

[8]The definition $r^\star < 0$ was proposed by Pigou (1943) as a formalization of Alvin Hansen's theory. Olivier Blanchard uses the definition $r < g$, where r is the risk-free real interest rate, and g is the growth rate of the economy (Blanchard, 2023).

the decline in the natural interest rate, the decoupling between them has allowed the average return on capital to remain constant (Eggertsson et al., 2021). Empirically, this is seen in the fact that bond yields have fallen, while returns on stocks and productive capital have remained constant, if not rising (Gomme et al., 2015). Others point to the emergence of non-physical capital and increased aggregate risk (Farhi and Gourio, 2018) as well as an increase in demand for safe assets (Del Negro et al., 2017; Del Negro et al., 2019) as explanations for the decoupling between the interest rate and the return on capital.

Papetti (2021a) also employs a quantitative life-cycle model for a closed economy to analyze the effect of aging on the natural interest rate in the Eurozone. He finds that demographic factors have caused a decline in the natural interest rate of 1.4 pp, measured from the average of the 1980s to 2030. Without the presence of tax-financed pensions, the real interest rate would be 50 basis points (bp) lower. The contribution from the increase in life expectancy is estimated at -90 bp, while the contribution from lower fertility is estimated at -50 bp. He also examines the effect of raising the retirement age, which he finds has a negligible effect on the real interest rate. He rejects Goodhart and Pradhan's prediction that aging will lead to a turning point in real interest rates due to increased pension expenditures. Nevertheless, his model shows that the natural real interest rate is set to reverse its downward trend after 2030, after which it will stabilize at a marginally higher level — although still 1 pp lower than in the 1980s. Papetti, like Eggertsson et al. (2019a) and Gagnon et al. (2016), finds that the natural interest rate dropped sharply around 2007 due to demographics. This decline can be explained by the relatively large cohorts from the baby boomer generation retiring at that time, which lowered the growth rate of the labor force. This reduced investment demand, which thereby exerted downward pressure on the natural interest rate, cf. mechanism 1. The fact that this drop in real interest rates coincided with the financial crisis contributed to interest rates being particularly low in the 2010s, which according to Papetti (2021a) and Gagnon et al. (2016) was misunderstood at the time as being purely a cyclical phenomenon.

Gagnon et al. (2016) simulate, like Eggertsson et al. (2019a), a quantitative OLG model for the USA as a closed economy. They estimate that the natural real interest rate in the USA fell by nearly 2 pp between 1980-2015, of which 1.25 pp were due to demographic factors. They also examine the effect on the growth rate of GDP per capita, which they likewise estimate has fallen by 1.25 pp compared to 1980 due to demographics. They point out that the large cohorts born after 1945 pushed up interest rates, wages, and economic growth from 1960-1980 when they entered the labor force. Since they themselves had fewer children, a subsequent demographic dividend occurred in the form of a low dependency ratio, which supported economic growth. Their model shows that the decline in the natural interest rate intensified after the year 2000, when the cohorts from the baby boomer generation began to retire. The drop in the interest rate here was driven by an increase in capital per worker (cf. mechanism 1), which occurred despite the fact that the savings rate fell during the same period (cf. mechanism 3). They find that fertility, primarily due to the baby boomer generation, has had a greater impact on the natural interest rate than the increase in life expectancy. This result partially contradicts Eggertsson et al. (2019a), Papetti (2021a), and Bielecki et al. (2020), who find that the increase in life expectancy, cf. mechanism 2, has been the dominant factor behind the decline in the natural interest rate, although lower fertility is still an important factor.

Carvalho et al. (2016), Carvalho et al. (2023), and Acedański and Włodarczyk (2018) find, however, that the increase in life expectancy far outweighs the significance of declining population growth. This result can partly be traced back to the fact that their models are based on Gertler (1999), which uses the specification introduced by Blanchard (1985) and Yaari (1965). This type of OLG model cannot capture the effect of the baby boomer generation and its descendants, as it only contains a homogeneous group of workers and retirees. Carvalho et al. (2016) analyzes a representative OECD country as a closed economy and finds that the natural interest rate fell by over 1.5 pp due to aging between 1990-2014.

Carvalho et al. (2023) set up an OLG model that includes three countries at different stages of the demographic transition: a small economy with a young population, a small economy with an old population, and the rest of the world. The three countries have different degrees of financial integration, so there is imperfect capital mobility. Their model is thus a hybrid between the completely closed economy of Carvalho et al. (2016) and the completely open economy of Ferrero (2010). They find that a higher degree of financial integration makes the real interest rate more sensitive to developments abroad, while the significance of domestic factors diminishes. Their results show that the real interest rate fell by more than 2 pp in the global economy between 1990-2020 due to aging. In the young economy, the fall is just under 3 pp, while the real interest rate fell by 1.5 pp in the old economy. Without financial integration, the real interest rate in the old economy would be lower, while it would be higher in the young economy. More openness thus leads to more convergence between the natural interest rates of different economies. Furthermore, they find that higher productivity growth, higher government debt, or an increase in public spending results in a higher natural interest rate. These results align with Rachel and Summers (2019), who find that the natural interest rate in the OECD would have fallen much more between 1980-2018 had it not been for the increase in government debt and public spending on elderly during the same period.

Krueger and Ludwig (2007) use a quantitative OLG model for an open economy, applying it to the USA and the rest of the OECD. They predict that the real interest rate in the USA will fall by 86 bp between 2005 and 2080 due to aging. If the USA were a closed economy, the real interest rate would fall less (79 bp), since the aging process in other OECD countries is more advanced. This result is confirmed by Rachel and Summers (2019), who also find that the natural interest rate would be higher in the USA if it were a closed economy. However, the modest difference between the two estimates demonstrates that treating the USA as a closed economy does not make a significant difference in the outcome for the natural interest rate.

Bielecki et al. (2020) analyze the Eurozone as an open economy using a quantitative OLG model and find that the natural real interest rate fell by 2 pp between 1985 and 2030. Of this, aging explains two-thirds or over 1.3 pp, with lower mortality accounting for about two-thirds of this decline. They assess that the development would have been roughly unchanged if the Eurozone had been a closed economy, since the rest of the world has undergone a similar aging process. They find that a higher retirement age has a modest but positive effect on the natural interest rate, while lower pension expenditures have a negative effect.

The literature applying structural life-cycle models unanimously finds that aging puts downward pressure on the natural interest rate. This holds true whether a closed or open economy is considered. In light of the fact that real interest rates were negative in several advanced economies after the financial crisis, many argue that secular stagnation is a persistent and structural phenomenon due to demographic trends (Gagnon et al., 2016; Eggertsson et al., 2019a). However, opinions are more divided in other parts of the literature, where it is rejected that the U.S. and global economy were in secular stagnation (Hamilton et al., 2016; Gomme et al., 2015; Rogoff and Lo, 2015), and the connection between aging and the natural interest rate is challenged (Hamilton et al., 2016; Goodhart and Pradhan, 2017).

Goodhart and Pradhan (2017, 2020) offer an alternative hypothesis, arguing that aging will lead to a *higher* natural real interest rate in the future. They reject the idea that life expectancy is a driving factor behind the the natural interest rate, explaining the past decline in real interest rates as an extraordinary phenomenon that occurred following the baby boomer generation saving for retirement. Instead, they argue that the increasing number of elderly individuals will lead to a lower savings rate, which will push the natural interest rate upward. Additionally, they point out that the decline in real interest rates between 1980-2015 was amplified by the inclusion of China and other emerging economies in the global production chain, which acted as a positive supply shock by releasing labor and capital. In particular, China's high savings rate contributed to a global savings surplus, part of which was invested in U.S. Treasury bonds.[9]

According to Goodhart and Pradhan (G&P), however, this positive supply shock is now over, as developing countries like China have also become older and wealthier. At the same time, the population in advanced economies is older yet, which will further increase the dependency ratio. They predict that long-term real interest rates, inflation, and real wages will rise, and inequality will fall.[10] They express concern that the period of low interest rates has led to a high accumulation of public debt, which will become more difficult to service when interest rates rise and when spending on the elderly grows. A key premise of their argument is that health and pension benefits provided by the government will not be reduced due to political pressure from the elderly. This will increase public spending and lead to higher taxes and more public debt, putting upward pressure on the natural interest rate. Similarly, they reject the

[9]This was also described by the U.S. Federal Reserve Chairman Ben Bernanke as a global "savings glut" (Bernanke, 2005).

[10]While they maintain that aging will put upward pressure on the natural real interest rate, they are ambivalent about whether monetary policy rates and short-term real rates will rise. Their argument is that, due to political constraints, central banks will not be able to fully combat the inflationary pressure stemming from increased public spending and a shrinking global workforce (Goodhart and Pradhan, 2020).

idea that the retirement age will rise, as they believe this will not be politically feasible. They acknowledge that aging will also dampen investment demand, but argue that the supply of savings will fall relatively more, causing the natural interest rate to rise. They dismiss much of the OLG literature, including Gagnon et al. (2016), which they believe do not provide an accurate depiction of household behavior and places too little emphasis on global factors such as the surplus savings from China. Nevertheless, G&P's hypothesis within the framework of a neoclassical OLG model can be interpreted as mechanisms 3 and 4 overshadowing mechanisms 1 and 2, so that aging ultimately leads to a higher natural interest rate. In the analysis, I will test G&P's hypothesis based on these four mechanisms as the theoretical foundation, as well as quantify the effect on the natural rate of interest of increasing government spending.

3 A Simple OLG Model

In this section, a simple OLG model with two periods and two generations is used to illustrate the effect of aging on the interest rate and the investment ratio. The model described here can therefore serve as a starting point to understand the intuition behind the results generated by the quantitative model used in the rest of the thesis.[11]

In the simple OLG model, the economy consists of a generation of young ($j = 0$) workers and a generation of retirees ($j = 1$). Members of the young generation in the first period will become retirees in the next period if they survive. The young work, receive wages, pay taxes, consume, and save for retirement. The elderly do not work, receive pensions, and consume the savings generated by themselves in the previous period. The number of young in period t is given by $N_{t,0}$, and the probability of surviving to the next period, where they will become retirees, is given by s_t. The number of retirees in the subsequent period is therefore given by $N_{t+1,1} = s_t N_{t,0}$. A young household in period t maximizes the utility function:

$$U_t = \frac{(c_{t,0})^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}} + s_t\beta \cdot \frac{(c_{t+1,1})^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}},$$

where $c_{t,0}$ is consumption in the first period when the household is young, and $c_{t+1,1}$ is consumption in the next period when the household is retired. The parameters β and ρ represent the discount factor and the intertemporal elasticity of substitution for consumption, respectively. Households supply their labor inelastically in the first period and receive wage income w_t, from which they are taxed at the rate τ_t. Therefore, the young households in the first period receive the after-tax income $w_t(1-\tau_t)$. In the second period, when households are retired, they receive a pension benefit d_{t+1}, which is financed through taxes on labor income of the young in the second period. The pensioner's savings are denoted by $a_{t+1,1}$, which generates a return r_{t+1} equal to the real interest rate in that period. The assets left by those who do not survive to the

[11] The model in this section is a modified version of the one used by Papetti (2021a, s. 5-8). The derivations are shown in Appendix A.

second period are evenly distributed among the surviving retirees. The budget constraints are given by:

$$c_{t,0} + a_{t+1,1} = (1 - \tau_t) w_t$$
$$c_{t+1,1} = \frac{a_{t+1,1}(1 + r_{t+1})}{s_t} + d_{t+1}.$$

The aggregate labor supply is given by the number of young individuals,

$$L_t = N_{t,0},$$

and production is given by

$$Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha},$$

where technology is given by $A_t = (1+g)^t A_0$, which grows with the rate g, and the depreciation rate is given by $0 < \delta < 1$. Factor prices are thereby:

$$r_t = \alpha \tilde{k}_t^{\alpha - 1} - \delta$$

$$w_t = (1 - \alpha) A_t \tilde{k}_t^{\alpha},$$

where $\tilde{k}_t = \frac{K_t}{A_t L_t}$ denotes capital per effective worker.

The budget constraint of the government implies that the pension benefit is given by:

$$d_{t+1} = \tau_{t+1} w_{t+1} \frac{(1 + n_{t+1})}{s_t},$$

where $(1 + n_t) = \frac{N_{t+1,0}}{N_{t,0}}$ denotes the growth rate of the number of young people and thereby the labor force. By assuming a fixed replacement rate of $\bar{d}$, it follows that the pension benefit is given by:

$$d_t = \bar{d} w_t (1 - \tau_t).$$

Finally, it applies that capital accumulation is given by:

$$K_{t+1} = N_{t,0} \cdot a_{t+1,1}$$

$$K_{t+1} = (1 - \delta) K_t + I_t.$$

These can be rewritten into

$$(1 + n_t) A_{t+1} \tilde{k}_t = a_{t+1,1}$$

$$(1 + g)(1 + n_t) \tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + \tilde{i}_t,$$

where $\tilde{i}_t = \frac{I_t}{A_t L_t}$ denotes gross investments per effective worker.

Balanced growth path

On a balanced growth path where capital per effective worker is constant, so $\tilde{k}_t = \tilde{k}$ for all t, firms' investment demand (ι^D) and households' supply of savings (ι^S) as a function of the interest rate r are expressed by:

$$\iota^D = \alpha \frac{(1+g)(1+n) - (1-\delta)}{r+\delta} \tag{2}$$

$$\iota^S = \left(1 - \frac{1-\delta}{(1+g)(1+n)}\right) \left[\frac{(1-\tau)(1-\alpha)\vartheta}{1+\vartheta} - \frac{(1+g)(1+n)(1-\alpha)\tau}{(1+r)(1+\vartheta)}\right], \tag{3}$$

where $\iota = \frac{I}{Y}$ denotes the gross investment ratio and where $\vartheta \equiv s\beta^\rho(1+r)^{\rho-1}$. The tax rate is given by:

$$\tau = \frac{\bar{d}}{\bar{d} + \frac{1+n}{s}}. \tag{4}$$

The effect of aging

Aging is characterized by declining fertility and increasing life expectancy. In the model, a decline in fertility can be expressed as a decrease in the growth rate of the labor force, n, while increasing life expectancy can be represented by a rise in the survival probability, s. On the balanced growth path, these variables remain constant. The effect of aging can therefore be examined by comparing an economy transitioning from an initial steady state, where the population is relatively young, to a subsequent steady state, where the population has aged.

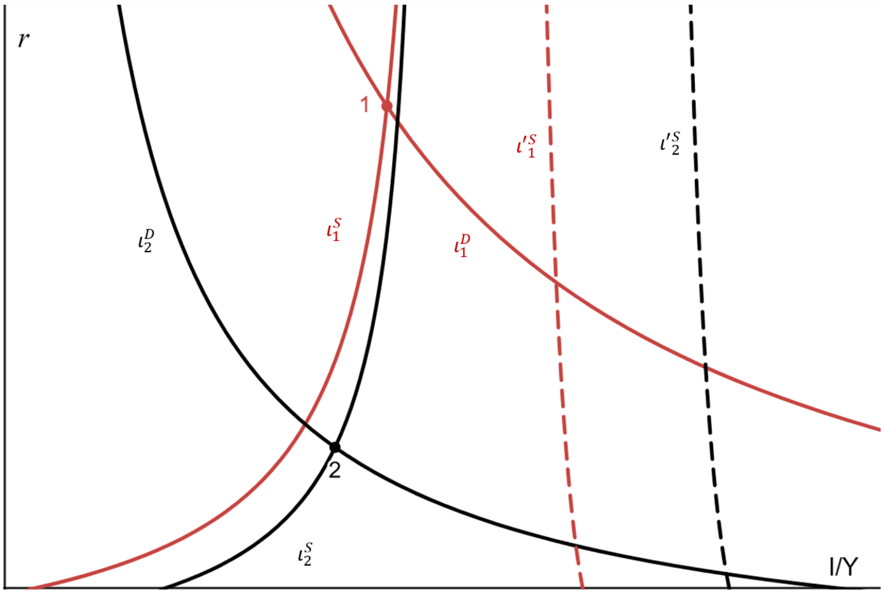
Figure 4 illustrates the demand and supply curves expressed in (2) and (3) under a scenario where the economy experiences an aging shock, involving a decline in the labor force growth rate and an increase in life expectancy. It is assumed that one period spans 30 years, corresponding to one generation. Therefore, the assumed growth rates are scaled to reflect the accumulated growth over a 30-year period. The growth rate of the labor force decreases from 1.4 percent to -0.8 percent annually, while the survival probability increases from 0.55 to 0.84.[12] It is further assumed that productivity growth declines from 2 percent to 0.85 percent annually. Additionally, the following parameter values are used $\alpha = 0.33$, $\beta = 0.975$, $\rho = 0.95$, $\delta = 0.957$, and $\bar{d} = 0.5$.

In the example shown in Figure 4 and Table 1, it appears that both the interest rate and the investment rate decline. However, this does not hold if the tax rate is set to zero, as illustrated by ι'^S, where the investment rate increases. The decline in population growth and productivity growth unambiguously reduces the demand for investments as given in (2). This is reflected in Figure 4 by the leftward shift of the demand curve, indicating that firms' demand for capital decreases at any given interest rate. This, all else equal, results in a lower equilibrium interest rate, as described under mechanism 1.

On the other hand, the supply of savings increases for a given interest rate, which is illustrated by a rightward shift of the supply curve. This reflects the effect of households increasing

[12] These values are chosen for illustrative purposes and can be considered a compromise between the rise in life expectancy and the dependency ratio in the U.S. from 1970 to 2024.

Figure 4: The effect of aging on the capital market



Note: The investment-output ratio is shown on the primary axis, and the interest rate on the secondary axis. The curves represent the demand for investments and the supply curve for savings as given in (2) and (3). The red curves represent the initial balanced growth path before the economy is hit by the aging shock, while the black curves represent the same after the shock has occurred. Point 1 indicates the initial steady state equilibrium, while Point 2 indicates the steady state equilibrium after the aging shock. The dashed curves represent the supply curves without pensions, where $\tau = 0$. The parameters used are listed in Table 1. Source: Own contribution.

their preference for saving when life expectancy increases, cf. mechanism 2. In practice, this corresponds to households discounting future consumption to a lesser extent. When the supply curve shifts to the right overall, it indicates that mechanism 2 dominates mechanisms 3 and 4. In the expression for the supply of savings in (3), it appears that an increase in the probability of survival has opposing effects on the supply of savings, as a higher dependency ratio increases the tax given in (4), which, all else equal, displaces the young's ability to save (cf. mechanism 4). An increase in the tax rate can also be caused by a higher replacement rate, which reduces the young's incentive to save, as the need to save decreases (cf. mechanism 2). Nevertheless, it can be shown that an increase in s, for a given tax rate, has a uniquely positive effect on ι^S, so that mechanism 2 will always dominate mechanism 3 on a balanced growth path when life expectancy increases.[13] An increase in life expectancy will therefore, all else equal, lead to a lower interest rate and a higher investment ratio in equilibrium.[14]

A lower population growth rate has an overall negative impact on the supply of savings. Firstly, a decrease in n implies that there are fewer young people to save, which lowers the savings rate, cf. mechanism 3. A lower population growth rate also implies that taxes increase, as there are fewer young people to finance pensions. This has a negative effect on the supply

[13] See Appendix A.

[14] This still holds when the tax rate is not fixed, as shown in Figure 4.

Table 1: Aging shock in the simple OLG model

Parameter	Equilibrium 1 (young)	Equilibrium 2 (old)
g	0.82	0.28
n	0.52	-0.22
s	0.55	0.84
τ	0.15	0.35
r	4.78	1.43
I/Y	0.16	0.13

Note: g and n denote the accumulated growth rates over a 30-year period. The interest rate r is shown as the annual rate. Source: Own contribution.

of savings, cf. mechanism 4. If there is no pension system, so that taxes do not increase, the negative effect on the supply of savings from a decrease in n will be minimal, if not close to zero, since $\frac{1-\delta}{(1+g)(1+n)} \ll 1$.[15] This suggests that mechanism 3 is not expected to be dominant on a balanced growth path, as mechanism 1 will pull the equilibrium interest rate in the opposite direction.

Equilibrium

An equilibrium between investment demand, as given in (2), and savings supply, as given in (3), can be found by equating the two. If $\rho = 1$ and $\delta = 1$, a unique solution is given by:[16]

$$\iota = \frac{s\beta\alpha(1-\alpha)(1-\tau)}{(1+s\beta)\alpha + \tau(1-\alpha)}$$
$$r = \frac{\alpha}{\iota}(1+g)(1+n) - 1.$$

It follows from the example illustrated by the simple OLG model that aging, in the form of a decrease in population growth n and an increase in the probability of survival given by s, overall negatively affects the interest rate in the stationary equilibrium. The main reason is that mechanisms 1 and 2 are dominant, while mechanisms 3 and 4 at most have a dampening effect on the fall in the interest rate. The explanation is, on the one hand, that a lower population growth rate reduces investment demand, as the marginal product of capital falls.[17] On the other

[15]This will always be the case for realistic parameter values, since an annual depreciation rate of, e.g., 10% implies that $\delta = 1 - (1-0,1)^{30} = 0,96 \approx 1$ over a generation of 30 years.

[16]When $\rho = 1$, the substitution and income effects will cancel each other out. This yields a perfectly inelastic supply curve for savings if the wealth effect is zero. The latter will be the case in the simple OLG model without pensions. See Appendix A.

[17]The decrease in investment demand is also due to the decrease in g, which in the model is independent of demographic factors. However, it should be considered plausible that aging has a negative effect on productivity growth (Papetti, 2021a; Aksoy et al., 2019).

hand, a higher life expectancy leads households to save more, as they have a longer expected retirement period. This overall leads to a lower interest rate in equilibrium.

However, the effect on the investment share is ambiguous, as it can both increase or decrease, depending on whether investment demand falls relatively more or less than savings supply increases. A necessary condition for the investment rate to fall is that the supply curve for savings has a positive slope in the ι, r-diagram, such that an increase in the interest rate is associated with an increase in the investment rate. This will be the case if the income effect is dominated by the substitution and wealth effects. Moreover, it turns out that productivity growth g does not affect the investment rate in equilibrium. Thus, all else equal, lower productivity growth will lead to a lower interest rate but an unchanged investment rate on the balanced growth path.[18]

Furthermore, the example used in this section only shows the effect of aging when the economy is on a balanced growth path. The interaction between the four mechanisms may therefore deviate from this when the economy is in transition between two steady states, such that mechanisms 3 and 4 may temporarily be dominant.

[18] This will not necessarily hold in the general case where ρ and δ are not equal to one.

4 The Model

In this section, the model used in the thesis is described. The model is a modified version of the one used by Eggertsson et al. (2019a), which is based on Auerbach and Kotlikoff (1988) and Ríos-Rull (1996).

4.1 Population and labor supply

The economy in the model consists of a large number of households with identical utility functions. Households enter the economy at the age of 25 ($j = 1$), after which they work, have children, consume, and participate in financial markets. They die no later than the age of J. The terminal age is set to 98 years, corresponding to $J = 74$ in the model. Population growth is determined by the total fertility rate (Γ). Individuals face a stochastic risk of dying before reaching the age of J. The conditional probability of surviving from age j to $j+1$ is given by s_j.[19] The unconditional probability of surviving to age j is given by su_j.[20] The total population alive in period t is given by N_t, which is the sum of the number of households at each age, $N_{j,t}$. The population in any given single-year cohort, $N_{j,t}$, is equal to the number of survivors from the previous year.[21] The population of 25-year-olds is given by the product of the population of their parents' generation and their parents' fertility rate in period $t-25$. The population in the model will thus evolve according to:

$$N_t = \sum_{j=1}^{J} N_{j,t} \tag{5}$$

$$N_{j+1,t+1} = s_j N_{j,t}, \quad \text{for } j \in \{1, J-1\} \tag{6}$$

$$N_{1,t} = N_{1,t-25} \cdot \Gamma_{t-25}. \tag{7}$$

Population dynamics are thus determined by fertility and the age at which households have children. In steady state, for a given fertility rate Γ, the growth rate n of the population will be given by:

$$n = \Gamma^{\frac{1}{25}} - 1. \tag{8}$$

In steady state, each generation will thus be $(1+n)$ times larger than the previous one. Consequently, in steady state, the total population will be given by:

$$N = \sum_{j=1}^{J} N_j, \tag{9}$$

[19] Age-specific survival probabilities may vary over time, t. However, I omit adding a time subscript for t for simplicity in notation.

[20] This can be calculated as the product of the conditional probabilities: $su_j = \Pi_{m=1}^{j-1} s_m$.

[21] Immigration is not included in the model.

where

$$N_{j+1} = s_j \frac{N_j}{1+n}, \quad \text{for } j \in \{1, J-1\}. \tag{10}$$

By normalizing the population of 25-year-olds in period 1 to 1, the total population in steady state in period 1 will be given by:

$$\bar{N}_1 = \frac{1}{\sum_{j=1}^{J} \frac{su_j}{(1+n)^{j-1}}}. \tag{11}$$

Each household has an identical labor productivity for a given age. Labor productivity, or human capital, is denoted by hc_j and depends on age. Households receive no wage income after retirement. The retirement age is assumed to be 67 years, corresponding to $j = 43$ in the model. After-tax wage income is thus given by the wage rate multiplied by the individual's labor productivity hc_j and $(1 - \tau_t)$, where τ_t is the income tax rate. The human capital profile for 25-year-olds is normalized to one, while it is assumed to be zero for retirees, such that $hc_{j=1} = 1$ and $hc_{j>42} = 0$. Households supply their labor inelastically,[22] and the aggregate labor supply in equilibrium is given by:

$$L_t = \sum_{j=1}^{J} N_{j,t} hc_j. \tag{12}$$

The labor supply is thus given by the number of individuals of working age multiplied by their respective human capital profile.

4.2 Households

Households derive utility from consumption (c) and from leaving bequests (x). The utility function for consumption, $u(\cdot)$, is characterized by constant elasticity of substitution (CES), with the intertemporal elasticity of substitution determined by the parameter ρ. The same applies to the utility function for bequests, $v(\cdot)$, where the argument x represents the bequest left per descendant.

$$u(c_{j,t+j-1}) = \frac{(c_{j,t+j-1})^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}} \tag{13}$$

$$v(x_{j,t+j-1}) = \frac{(x_{j,t+j-1})^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}} \tag{14}$$

A household aged $j = 1$, entering the economy in period t, thus maximizes its total expected lifetime utility, given by:

$$U_t = \sum_{j=1}^{J} su_j \cdot \beta^{j-1} \cdot u(c_{j,t+j-1}) + su_J \cdot \beta^{J-1} \cdot \mu \cdot v(x_{J,t+J-1}), \tag{15}$$

[22]Gertler (1999) demonstrates that in a model with a declining labor force growth rate and increasing life expectancy, agents respond by increasing labor supply. However, this behavior contradicts empirical evidence for most developed economies, including the U.S., as shown in Figure A10b (Carvalho et al., 2023; OECD, 2024a).

where β is the household's discount factor for consumption, and μ is a discounting parameter for the utility derived from bequests left when the household dies at age J. The constant su_j denotes the unconditional survival probability. The inclusion of bequests in the utility function reflects that households have a bequest motive, which influences their savings behavior.[23]

Households receive bequests, denoted by $q_{j,t}$, from their grandparents. Bequests are assumed to be zero in all periods except at age $k \equiv J - 49 = 25$ (49 years), which corresponds to one year after their grandparents reach the terminal age $J = 74$ (98 years) and pass away. Likewise, bequests are only left if a household reaches age J. The assumption that grandchildren, rather than children, inherit is based on the fact that children would themselves have retired by the time their parents survive to age 98. To make the age of inheritance more realistic, I therefore let the grandchildren inherit instead.

Since mortality is stochastic, not all households will survive to age J to pass on a bequest. To avoid stochastic inequality within each generation—where some households receive bequests from their grandparents while others do not—it is assumed that all generations participate in a market where they insure themselves against missing out on an inheritance. At the maximum age J, all surviving members of each cohort pool the assets they plan to leave as bequests and distribute them equally among their surviving grandchildren. Thus, the relationship between bequests given (at age J) and bequests received by grandchildren (at age k) in period t is given by:

$$q_{k,t} = \frac{N_{J,t-1} \cdot x_{J,t-1} \cdot \Gamma_{t-J+26} \cdot \Gamma_{t-J+1}}{N_{k,t}}, \tag{16}$$

where $q_{k,t}$ denotes the bequests received by the cohort that is aged $k = 25$ in period t. The bequests are received the year after the grandparents have passed away. The assumption about the distribution of bequests helps make the model more realistic, as households would otherwise not be able to inherit if their parents passed away before reaching the age of 98, which would be the case for the majority of households.

A household of age j can purchase or borrow real assets $a_{j,t}$, which can be used for consumption or saving. In period $t+1$, these assets will yield a return of r^k_{t+1}, corresponding to the rental rate of capital, and the asset will have a value after depreciation of $(1-\delta)$. All households participate in annuity markets with perfect competition. If a household dies before reaching age J, the remaining assets will be distributed equally among the surviving households within the same cohort, such that each surviving household receives $\frac{1-s_{j-1}}{s_{j-1}} a_{j,t}$.

Households of working age receive a share of the firms' profits given by $\Pi_{j,t}$. Pensioners receive a pension benefit each period, given by $d_t \cdot w_t$, where d_t is the replacement rate relative to the pre-tax wage rate. The pension benefit constitutes a constant proportion of the after-tax wage level:

[23] Without a bequest motive, households would consume all their savings before reaching the terminal age. This is unrealistic, as individuals in reality cannot know with certainty when they will die.

$$d_t w_t = \bar{d}(1-\tau_t)w_t. \tag{17}$$

The budget constraint for a household ages j in period t is thereby:

$$\begin{aligned} c_{j,t} &+ a_{j+1,t+1} + \Gamma_{t-j+26} \cdot \Gamma_{t-j+1} \cdot x_{j,t} \\ &= (1-\tau_t)w_t hc_j + (1-\tau_p)\Pi_{j,t} + d_t w_t \cdot \mathbb{1}\{j > 42\} \\ &+ (1+r_t)\left(a_{j,t} + q_{j,t} + \frac{1-s_{j-1}}{s_{j-1}} a_{j,t}\right). \end{aligned} \tag{18}$$

Households of working age have the option to borrow assets with their future income as collateral. The borrowing constraint is given by:

$$a_{j,t} \geq -D_{j,t} \cdot w_{t+1} \cdot hc_{j+1}, \tag{19}$$

where $0 < D < 1$ indicates the cap on how much of one's income can be borrowed. This constraint will only be binding if households wish to consume more than they earn in the given period. The household's first-order conditions are given by:

$$\frac{1}{\beta} = \left(\frac{c_{j+1,t+1}}{c_{j,t}}\right)^{-\frac{1}{\rho}} \cdot (1+r_{t+1}) + \lambda_{j+1,t+1} \cdot \frac{(c_{j,t})^{\frac{1}{\rho}}}{su_j \beta^j} \tag{20}$$

and

$$x_{J,t+J-1} = \left(\frac{\Gamma_{t-J+1} \cdot \Gamma_{t-J+26}}{\mu}\right)^{-\rho} \cdot c_{J,t+J-1} \quad \text{for } J \in \{74\}, \tag{21}$$

where (20) is the Euler equation for consumption, and (21) is the optimality condition for the transfer of bequests.[24] It follows that the households' allocations are characterized by these transitional dynamics:

$$a_{j+1,t+1} = \frac{(1+r_t) \cdot a_{j,t}}{s_{j-1}} + (1-\tau_t) \cdot w_t \cdot hc_j + (1-\tau_p)\Pi_{j,t} - c_{j,t} \tag{22}$$
$$\text{for } j \in \{1, \ldots, 42\} \setminus \{24\},$$

$$a_{j+1,t+1+1} = \frac{(1+r_t) \cdot a_{j,t}}{s_{j-1}} + (1-\tau_t) \cdot w_t \cdot hc_j + (1-\tau_p)\Pi_{j,t} + q_{j+1,t+1} - c_{j,t} \tag{23}$$
$$\text{for } j \in \{24\},$$

$$a_{j+1,t+1} = \frac{(1+r_t) \cdot a_{j,t}}{s_{j-1}} - c_{j,t} + d_t w_t \tag{24}$$
$$\text{for } j \in \{43, \ldots, 73\},$$

$$c_{j,t} = \frac{(1+r_t) \cdot a_{j,t}}{s_{j-1}} - \Gamma_{t-j+26} \cdot \Gamma_{t-j+1} \cdot x_{j,t} + d_t w_t \tag{25}$$
$$\text{for } j \in \{74\}.$$

[24] See Appendix B for derivation.

4.3 Production

Production in the model is carried out by firms that produce intermediate goods and operate under perfect competition. These products are purchased and differentiated by a sector of monopoly firms, which compete monopolistically and sell the final goods to consumers at a markup.

Intermediate goods

There exists a sector of firms operating under perfect competition that produces intermediate goods. These firms sell their goods at a price of p_t^{int}/P_t. The firms operate with a CES production function, hiring labor at the wage rate w_t and renting capital at the rental rate r_t^k. The elasticity of substitution between labor and capital is denoted by σ. The representative firm maximizes its profit:

$$\max_{\{K_t,L_t\}} \Pi_t^{int} = \frac{p_t^{int}}{P_t} Y_t - w_t L_t - r_t^k K_t \tag{26}$$

subject to the budget constraint:

$$Y_t = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{27}$$

Technological growth is assumed to affect only labor productivity, denoted by A_t.[25] Labor productivity grows exogenously at the rate g_t^A in each period, such that $A_{t+1} = (1+g_t^A)A_t$. The first-order conditions determining the demand for labor and capital are given by:[26]

$$w_t = \frac{p_t^{int}}{P_t}(1-\alpha)A_t^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \tag{28}$$

$$r_t^k = \frac{p_t^{int}}{P_t} \alpha \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}. \tag{29}$$

The rental rate r_t^k is thus given by the marginal product of capital multiplied with the price, which the intermediate goods firms can sell their products for. It follows from (36) that

$$r_t^k = \frac{\theta_t - 1}{\theta_t} \cdot MPK,$$

where

$$MPK \equiv \frac{\partial Y_t}{\partial K_t} = \alpha \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}.$$

Likewise, it applies for the wage rate that

$$w_t = \frac{\theta_t - 1}{\theta_t} \cdot MPL,$$

[25] This implies that technological growth is Harrod-neutral, which ensures that the model converges to a balanced growth path (Romer, 2019, p. 10). Harrod-neutrality is required for the general CES production function unless $\sigma = 1$ (de la Fonteijne, 2018).

[26] See Appendix B for derivation.

where MPL is the marginal product of labor given by

$$MPL \equiv \frac{\partial Y_t}{\partial L_t} = (1-\alpha) A_t^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}}.$$

Moreover, it follows that

$$\lim_{\theta_t \to \infty} r_t^k = MPK \tag{30}$$

and

$$\lim_{\theta_t \to \infty} w_t = MPL. \tag{31}$$

The risk-free real interest rate is given by the rental rate of capital after depreciation:

$$1 + r_t = 1 + r_t^k - \delta \Leftrightarrow r_t = r_t^k - \delta. \tag{32}$$

For simplicity, the model does not differentiate between the nominal and real interest rate. Instead, only the real interest rate is used. This aligns with the purpose of the thesis, which is to examine the natural real interest rate. Since the model does not incorporate aggregate risk or business cycle fluctuations, the real interest rate given in (32) corresponds to the natural real interest rate, as shown in (1). It follows from (29) and (36) that the rental rate of capital is given by:

$$r_t^k = \frac{\theta_t - 1}{\theta_t} \alpha \left(\frac{\tilde{y}_t}{\tilde{k}_t}\right)^{\frac{1}{\sigma}},$$

where $\tilde{y}_t \equiv \frac{Y_t}{A_t L_t}$ and $\tilde{k}_t \equiv \frac{K_t}{A_t L_t}$ denote output and capital per effective worker respectively. This expression can be rewritten into a function of $\tilde{k}_t$ only:

$$r_t^k = \frac{\theta_t - 1}{\theta_t} \alpha \left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right)^{\frac{1}{\sigma-1}} \cdot \tilde{k}_t^{\frac{-1}{\sigma}}. \tag{33}$$

This applies for all $\tilde{k}_t > 0$, so

$$\frac{\partial r_t^k}{\partial \tilde{k}_t} < 0.$$

Thus, the rental rate of capital r_t^k, and consequently the interest rate r_t, negatively depends on the level of capital per effective worker. It follows that, for a given level of technology A_t, an increase in the stock of capital per worker leads to a decrease in the rental rate of capital and, therefore, the interest rate. The intuition is that an increase in the capital stock for a given labor force reduces the marginal product of capital, resulting in a lower equilibrium interest rate. The opposite holds if capital per (effective) worker decreases.[27]

Final goods

There exists a continuum of monopoly firms indexed by i, each producing a differentiated final good. The total measure of these firms is normalized to one. These final goods firms purchase intermediate goods, differentiate them at zero cost, and sell the resulting final goods to households

[27] See Appendix B.

at a mark-up. The quantity of final goods is represented by an aggregate of the differentiated goods, characterized by a constant elasticity of substitution θ_t:

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\theta_t - 1}{\theta_t}} di\right]^{\frac{\theta_t}{\theta_t - 1}}.$$

These firms operate under monopolistic competition and set prices in each period. The demand for final goods is given by:

$$y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta_t}, \tag{34}$$

where θ_t varies over time and serves as a parameter for the firm's market power. An increase in θ_t reduces the firm's market power and lowers its equilibrium mark-up, which is given by $\theta_t/(\theta_t - 1)$. Each monopoly firm uses y_t^m intermediate goods to produce its output and follows a linear production technology given by $y_t^f = y_t^m$. The monopolist chooses the real price $p_t(i)/P_t$ and indirectly the quantity y_t^f to maximize its real profit, subject to (34):

$$\max_{\frac{p_t(i)}{P_t}} \Pi_t^f = \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i),$$

where p_t^{int}/P_t is the price of intermediate goods, which the final goods firm takes as given. The optimality condition for the real price of the final good is given by the time-varying mark-up applied to the price of intermediate goods:

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}}{P_t}. \tag{35}$$

Since all final goods firms pay the same price for intermediate goods and follow an identical production technology, and given the absence of price rigidities, it applies that $p_t(i) = P_t$, which gives

$$\frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t}. \tag{36}$$

In equilibrium the aggregate profit equals:

$$\Pi_t = \frac{Y_t}{\theta_t}. \tag{37}$$

The profit from the final goods firms is distributed according to the households' labor income so that the individual household receives before taxes:

$$\Pi_{j,t} = hc_j \frac{\Pi_t}{L_t}. \tag{38}$$

In equilibrium the aggregate distribution equals the aggregate profit:

$$\frac{Y_t}{\theta_t} = \sum_{j=1}^{J} N_{j,t} \Pi_{j,t}. \tag{39}$$

Capital accumulation

The aggregate capital stock K_t evolves according to:

$$K_{t+1} = I_t + (1-\delta)K_t, \tag{40}$$

where δ is the depreciation rate, and I_t represents gross investments. The capital stock in period t is given by the sum of household assets minus government debt B_t, which households hold in the form of bonds:[28]

$$K_t = \sum_{j=1}^{J} N_{j,t} \frac{a_{j,t}}{s_{j-1}} - B_t. \tag{41}$$

It follows from (40) and (41) that gross investments are given by:

$$\begin{aligned} I_t &= K_{t+1} - (1-\delta)K_t \\ &= \sum_{j=1}^{J} N_{j,t} a_{j+1,t+1} - B_{t+1} - (1-\delta)\left(\sum_{j=1}^{J}(N_{j,t}a_{j,t}/s_{j-1}) - B_t\right). \end{aligned} \tag{42}$$

4.4 Government

The government consumes a constant share of output g_t and can accumulate debt. The government's budget constraint is given by:

$$B_{t+1} = b_{t+1}Y_{t+1} = g_t Y_t + (1+r_t)b_t Y_t + E_t - T_t, \tag{43}$$

where b_t denotes the government debt-to-GDP ratio. Tax revenues are given by $T_t \equiv \tau_t w_t L_t + \tau_p \Pi_t$, such that total tax collection comes from income tax on wages and profits. E_t represents total pension expenditures and is given by:

$$E_t \equiv \sum_{j=43}^{J} d_t w_t \cdot N_{j,t}. \tag{44}$$

To simulate the model, fiscal policy is assumed to be specified by an exogenous process for government consumption (g_t) and the debt-to-GDP ratio (b_t). The income tax rate τ_t is then determined endogenously to satisfy the government budget constraint. The following definitions are applied:

$$G_t \equiv g_t Y_t + r_t b_t Y_t + E_t, \tag{45}$$

$$F_t \equiv \frac{b_{t+1}Y_{t+1} - b_t Y_t}{G_t},$$

[28]Since households allocate their assets one period in advance, the total stock of assets in period t is given by the individual assets $a_{j,t}$ multiplied by the population of the corresponding cohort that was alive in period $t-1$, represented by $N_{j-1,t-1} = N_{j,t}/s_{j-1}$. Government debt is subtracted because it does not contribute to the productive capital stock.

$$B_t \equiv b_t Y_t, \tag{46}$$

where G_t is the government's total expenditures in period t, F_t denotes the public deficit measured as a share of expenditures in period t, and B_t denotes the public debt in period t. The government's budget constraint is satisfied, so that

$$\begin{aligned} T_t \equiv \tau_t \cdot w_t L_t + \tau_p \Pi_t = G_t(1 - F_t) = G_t - \Delta B_{t+1} \\ \Leftrightarrow \tau_t = \frac{1}{w_t L_t} \left(G_t - b_{t+1} Y_{t+1} + b_t Y_t - \tau_p \Pi_t \right). \end{aligned} \tag{47}$$

This means that the income tax rate on wages, τ_t, is adjusted to cover all government expenditures after accounting for the issuance of government debt and the collection of profit taxes.

Aggregate definitions

The aggregate resource constraint is given by

$$Y_t = L_t w_t + r_t^k K_t + \Pi_t = C_t + I_t + g_t Y_t. \tag{48}$$

The labor share is given by the aggregate wage income divided by the gross income or GDP:

$$LS_t \equiv \frac{L_t w_t}{L_t w_t + r_t^k K_t + \Pi_t}. \tag{49}$$

The consumption share is given by

$$\frac{C_t}{Y_t} = \frac{\sum_{j=1}^{J} N_{j,t} c_{j,t}}{L_t w_t + r_t^k K_t + \Pi_t}. \tag{50}$$

The households' disposable net income is given by

$$Y_t^{disp} \equiv Y_t - T_t - \delta K_t + E_t + r_t B_t, \tag{51}$$

and the households' net savings rate is given by

$$\mathcal{O}_t \equiv \frac{Y_t^{disp} - C_t}{Y_t^{disp}}. \tag{52}$$

The total consumer debt is given by:

$$\mathcal{F}_t \equiv \sum_{j=1}^{j=41} N_{j,t} \cdot a_{j,t} \cdot \mathbb{1}\{a_{j,t} < 0\}. \tag{53}$$

4.5 Dynamic efficiency

The economy is dynamically inefficient if investments exceed the return on capital, corresponding to $I > MPK \cdot K$ (Abel et al., 1989). The criterion for dynamic efficiency is thus $I < MPK \cdot K$, where I is gross investments, MPK is the marginal product of capital, and K is the capital stock. The rental rate of capital in the model is given by $r_t^k = \frac{\theta_t - 1}{\theta_t} \cdot MPK$, cf. (29), where $\frac{\theta_t - 1}{\theta_t}$ is the inverse of the monopolist's markup. This relationship reflects an assumption of imperfect competition in production, which causes the rental rate of capital to consistently fall below the marginal product of capital.

Since the real interest rate in the model is derived from the rental rate of capital, it is possible for a situation to arise where the interest rate is negative while the marginal product of capital (net of depreciation) remains positive. The economy can therefore still be dynamically efficient even if the interest rate is negative. The assumption of imperfect competition thus enables the model to simulate an economy in secular stagnation (i.e., $r^\star < 0$) without simultaneously risking dynamic inefficiency (Eggertsson et al., 2019a).

4.6 Solving the model

A competitive general equilibrium is defined by the sequence of household allocations: $\{\{c_{j,t}, a_{j,t}, \Pi_{j,t}, q_{j,t}, x_{j,t}\}_{j=1}^{J}\}_{t=0}^{\infty}$, a sequence of aggregate variables $\{Y_t, K_t, L_t, \Pi_t, E_t\}_{t=0}^{\infty}$, a sequence of price variables $\{w_t, r_t^k, r_t, \frac{p_t^{int}}{P_t}\}_{t=0}^{\infty}$, a sequence of government variables $\{\tau_t\}_{t=0}^{\infty}$, and a sequence of exogenous processes $\{\{N_{j,t}, g_t^A, g_t, b_t, d_t\}_{j=0}^{J}\}_{t=0}^{\infty}$, which satisfy the following conditions:

1. Households maximize their consumption and bequests as defined in (15), subject to the constraints (18) and (19). The solution to the households' optimization problem is given by (20) and (21).
2. The allocation of assets satisfies the budget constraint (18), with $a_1 = 0$.
3. Profits from firms are distributed among households proportionally to their labor income, as specified in (38).
4. Bequests received are equal to bequests left, as defined in (16).
5. Gross domestic product is determined by the production function (27).
6. The total population is given by the sum of individual cohorts, as stated in (5).
7. The aggregate labor supply is determined by (12).
8. Aggregate profit is given by (37).
9. Perfect factor markets ensure that (28), (29), and (32) hold.
10. The price of intermediate goods is given by (36).
11. Pension benefits are defined by (17).
12. Pension expenditures are determined by (44).

13. The government satisfies its budget constraint, as defined in (47).

14. Capital markets clear, ensuring that (41) holds.

4.7 Steady state

The model starts and ends in steady state, which represents a stationary equilibrium where exogenous variables remain constant. The starting year is set to 1970, while the endpoint is 200 years later, in 2170. During the transition period, the equations are solved for all endogenous variables, including the optimality conditions for consumption and bequests for all generations. Since the model starts in steady state, households experience a shock in period 2, where the expected trajectory of exogenous variables changes, including fertility, mortality, productivity growth, and others. In any given period, households first decide how many assets to allocate to the next period and subsequently adjust their consumption and savings.

In steady state, the model adheres to the same definitions as before, but all variables remain constant over time, adjusted for the growth rate of technology (g^A) and population growth (n). Along the balanced growth path, aggregate variables grow at a rate of $(1+g^A)(1+n)$, while economic variables per capita, such as the wage rate, grow at $(1+g^A)$. In the stationary equilibrium, there are $4 \cdot J + 13$ unknowns: J cohorts' consumption c_j, J net assets a_j, J shares of firms' profits Π_j, J cohorts N_j, one variable for bequests given x, one variable for bequests received q, aggregate output Y, aggregate capital K, aggregate labor supply L, aggregate profit Π, wage rate w, rental rate of capital r^k, interest rate r, the price of intermediate goods $\frac{p^{int}}{P}$, the income tax rate τ, the replacement rate d, and pension expenditures E. This corresponds to $4 \cdot J + 13$ equations:[29]

1. $(J-1)$ Euler equations, (A.55)

2. (1) equation for bequests given, (A.56)

3. (1) equation for bequests received, (A.60)

4. (J) equations for intertemporal budget constraints, (A.58), (A.59), (A.61), and (A.62)

5. (J) equations for the distribution of profits, (A.65)

6. (1) equation for the initial asset holdings of households, (A.57)

7. (J) equations for the population, (10) and (A.54)

8. (1) equation for aggregate output, (A.70)

9. (1) equation for the capital stock, (A.80)

10. (1) equation for aggregate labor supply, (A.78)

[29]The equations are provided in Appendix B.

11. (1) equation for aggregate profits, (A.72)
12. (1) equation for the wage rate, (A.68)
13. (1) equation for the markup, (A.66)
14. (1) equation for the rental rate of capital, (A.69)
15. (1) equation for the real interest rate, (A.71)
16. (1) equation for the government budget constraint, (A.76)
17. (1) equation for the replacement rate, (A.81)
18. (1) equation for total pension expenditures, (A.82)

The model is solved in MATLAB using the Dynare extension. Through numerical optimization, the program calculates the steady state for both the initial and final periods, as well as the transition in between. Perfect foresight is assumed, meaning agents have full knowledge of the development of exogenous processes.[30] Additionally, the model cannot be solved analytically due to the borrowing constraint given in (19) (Eggertsson et al., 2019b).

5 Data and Calibration

The starting point is 1970, as a series of structural shifts occurred in the economy around this time. First, it marked the entry of the large cohorts of the baby boom generation, born after World War II, into the labor force. Second, the demographic transition accelerated as fertility and mortality rates declined further due to social changes and medical advancements. Third, the 1970s also saw a slowdown in productivity growth, settling at a permanently lower level compared to previous decades. Fourth, the growth in real wages diminished, while the labor share declined, and the profit rate increased (Eggertsson et al., 2019a).

5.1 Demographic data

5.1.1 Fertility

The total fertility rate (TFR), which measures the number of children born per woman in a given year, is used.[31] A 5-year moving average is computed to smooth fluctuations in fertility, ensuring that population dynamics in the model are less sensitive to the assumption that households have children at age 25, as per (7). The smoothed series is then divided by two, so the fertility rate in

[30] The assumption of perfect foresight can be debated but is required to solve the model. An alternative assumption, where agents are surprised by developments in exogenous processes such as demographic factors each period, would arguably not be more realistic.

[31] The total fertility rate is calculated as the sum of live births per 1,000 women in 5-year age intervals, multiplied by 5 (CDC, 2023).

the model (Γ_t) represents the number of children born per person in period t. Data for 1950–2100 is sourced from the UN database, with post-2023 data based on the UN's July 2024 projections (UN, 2024). Data for 1943–1950 is sourced from the Centers for Disease Control (CDC, 2023). Although the simulation begins in 1970, knowledge of prior fertility rates is required because agents only enter the economy at age 25.

In the baseline scenario, fertility is assumed to converge to 1.64 children per woman ($\Gamma = 0.82$), which corresponds to the average for 2019–2023. This is close to the most recent observed value of 1.62 children per woman in 2023. According to the UN's medium scenario, fertility in the U.S. is expected to remain around 1.65 until 2100, a scenario deemed most likely by the UN. The UN's medium scenario is nearly identical to assuming a constant fertility rate for the rest of the century. However, considering the long-term decline in fertility rates in the U.S. and globally, as shown in Figure 1a, it seems plausible that this trend could continue. Alternative simulations will use projections from the UN's lower 80% confidence interval and the low scenario.[32] In these cases, fertility rates are expected to stabilize at 1.30 and 1.15 children per woman, respectively, by 2100.

Given the model's assumption that each cohort has children at a single point in time, the size of each newborn cohort is directly proportional to the birth rate and the size of their parent cohort. This results in persistent "echo effects" in population dynamics, where the sizes of successive cohorts remain proportional. In reality, such echo effects gradually dissipate because women within a given cohort have children at different ages. To avoid excessive persistence of these demographic echo effects, fertility rates are adjusted for the period 2004–2043, with these adjustments reflected in the model 25 years later. This entails fertility rates being assumed lower than actual rates for 2004–2017, and higher than actual rates for 2018–2043.[33]

5.1.2 Mortality

Mortality data is sourced from the UN's life tables for both genders combined. Survival probabilities are calculated based on the number of survivors in a hypothetical cohort of 100,000 individuals who have reached age j, denoted $I(j)$.[34] The conditional probability of surviving from age j to $j+1$ is then calculated as $s_j = I(j+1)/I(j)$. The unconditional probability of surviving to age j is given by $su_j = I(j)/100000$, which is also equal to the product of the conditional probabilities: $su_j = \prod_{m=1}^{j-1} s_m$. Survival probabilities are indexed such that $su_1 = s_0 = 1$, meaning no one dies before reaching age 25. Mortality data after 2023 is based on the UN's latest projections (UN, 2024). Survival curves for selected years are shown in Figure 2a.

Mortality rates for 2020–2022 are set to 2019 levels to account for excess deaths during the

[32] The UN's low scenario is given by the medium scenario's fertility rate minus 0.5.

[33] This method reflects Eggertsson et al. (2019a). See Figure A9 in Appendix D.

[34] The hypothetical cohorts are calculated by the UN and follow the equation $I(j+1) = I(j) \cdot \exp(-m(j))$, where $m(j)$ is the empirically observed mortality rate for age j.

COVID-19 pandemic.[35] In the model, life expectancy is assumed to remain constant after 2024. This assumption is partly necessary to achieve a transition to the balanced growth path by 2100. Additionally, it helps ensure a more realistic development of the dependency ratio, as the absence of immigration in the model would otherwise overestimate the long-term dependency ratio, as shown in Figure 5. Moreover, there is ongoing debate about whether life expectancy will continue to increase at the same rate as it has historically (Olshansky et al., 2024). In Section 6.8, the model is simulated under alternative projections for fertility and life expectancy.

5.2 Economic data

Productivity: Data on total factor productivity in the U.S., adjusted for capacity utilization, are sourced from Fernald (2024). Since the data exhibit cyclical fluctuations, the series is smoothed using a Hodrick-Prescott filter ($\lambda = 10$) for the period 1948-2023. The value is set to 0.85% after 2020, corresponding to the average for 2000-2020. The productivity growth in steady state for 1970 is set to 2%, roughly corresponding to the average for 1960-1970. The evolution of productivity growth is shown in Figure A11.[36]

Government debt: Data for government debt as a percentage of GDP is sourced from the Federal Reserve (FRED, 2024d). The data series includes all debt owed by the U.S. federal government to individuals, corporations, states, municipalities, central banks, and foreign governments (Fiscal Data, 2024) and is shown in Figure A13a. In the model, the government debt ratio b_t is set to the 5-year moving average between 1972-2021. For 1970 and 1971, the value is set to 35%, and after 2022, government debt is assumed to remain constant at 120% of GDP.

Public consumption: Data for public consumption expenditures are sourced from the Bureau of Economic Analysis and cover consumption expenditures which excludes gross investments, transfers, and interest payments (BEA, 2024b). The public consumption share has been slightly declining over the period, decreasing from 18% in 1970 to 13.4% of GDP in 2023, as shown in Figure A13b.[37] In the model, the government consumption ratio g_t is set to the average for 1970-2022, which is 15.54%. Nominal GDP data is sourced from the Federal Reserve (FRED, 2024d).

Human capital: Data on age-specific labor income is sourced from the National Transfer Accounts (NTA, 2017). The human capital profile hc_j for households in the working-age population in the model is set to the indexed data from NTA, where $hc_1 = 1$, so income for 25-year-olds is normalized to 1. The human capital profile is assumed to be constant over time and is shown in Figure A12.

[35] This adjustment does not significantly affect the results.

[36] Using data on total factor productivity growth instead of labor productivity growth is a methodological choice that can be discussed. However, this choice is consistent with similar studies, including Eggertsson et al., 2019a. See de la Fonteijne (2018) for a discussion on how productivity growth in CES functions should be treated.

[37] While the public consumption share in the U.S. has been slightly declining, total public expenditures have been slightly increasing as a share of GDP, as shown in Figure A13b.

Retirement age: The retirement age is set to 67 years ($j = 43$ in the model), as this is the age at which one is entitled to full pension benefits from Social Security in the U.S. (SSA, 2024a).[38] Thus, $hc_j = 0$ for $j > 42$, meaning retired households do not receive any labor income.

5.3 Calibration of parameters

The parameters α, β, θ, μ, and D are calibrated to match a series of empirical moments in steady state. This is achieved by minimizing the squared deviations between the simulated and empirical moments.[39]

The capital share α is calibrated to recreate a ratio of gross investments to GDP of 21.7% in steady state in period 1, corresponding to the 5-year moving average in 1970 (FRED, 2024d).

The discount factor β is calibrated so that the interest rate in steady state in period 1 is approximately 4.3%, corresponding to the natural real interest rate in the U.S. in 1970—calculated as the smoothed value of HLW's estimates, as shown in Figure 3a.

The market power parameter θ is calibrated to match a wage share of 63% in the first period, where the simulation starts in steady state, and 56% in the last period, where the economy ends in steady state. These shares correspond to the 5-year moving averages for the wage share in the U.S. in 1970 and 2023 (BLS, 2024). This provides two calibrated parameter values for θ: one for steady state in period 1 and one for steady state in the terminal period. For the period 1971-2022, θ_t is interpolated linearly between these two values, making it time-varying during this period. After 2023, it is held constant at the terminal steady state value.

The credit limit D is calibrated relative to the 5-year moving average for the amount of private consumer debt as a percentage of GDP in 1970 (2.2%) and 2015 (12.3%) (FRED, 2024a).[40] This results in two calibrated parameter values: D_1 and D_T for steady state in period 1 and the terminal period T. It is further assumed that $D_{j,t}$ increases over time but remains constant throughout each cohort's life. This means that older cohorts in a given year are less able to borrow against their future labor income than younger cohorts. $D_{j,t+1}$ is set to the linearly interpolated values between the calibrated values D_1 and D_T. This implies that $D_{j,t+1} = \min\left(D_{j,t} + \frac{D_T - D_1}{42}, D_T\right)$ and $D_{j,t} = D_{j-1,t-1}$. This helps create a smoother consumption cycle for households in the model, which aligns better with the empirical data. In steady state, the credit limit will be the same for all individuals in the working-age population, so $D_j = D_{j+1}$ for $j \in \{1, \ldots 41\}$.

The bequest parameter μ is calibrated to match a ratio of inherited wealth to GDP of 0.6% in the terminal steady state.[41]

[38] The average retirement age in the U.S. is lower, as many Americans retire before reaching 67 years (Munnell, 2015). However, this is already reflected in the income profile, as income declines after age 50 (see Figure A12).

[39] The calibration is performed in Dynare using the optimization procedure `fmincon`, which finds the best parameter values to match empirical moments. The function iteratively adjusts the parameters under given constraints and stops when the difference between the model's results and the targets is sufficiently minimized.

[40] The choice of 2015 instead of 2023 is made because it better aligns with the rest of the calibration.

[41] There is no official statistic on inheritance transfers in the U.S. Alvaredo et al. (2015) estimates that the share

The intertemporal elasticity of substitution for consumption is set to $\rho = 0.95$. This parameter value is chosen based on what creates the most realistic consumption cycle for individuals in the model and is consistent with the existing literature. I choose $\rho < 1$, as microeconometric studies find that $0 < \rho < 1$ (Havranek et al., 2013; Hall, 1981).[42]

The elasticity of substitution between labor and capital, represented by the parameter σ, is set to 0.6. This value reflects the assumption that capital and labor are more complementary than substitutes. Gechert et al. (2022) finds in a meta-study that $0.3 < \sigma < 0.9$, so the midpoint is $\sigma = 0.6$.[43]

The depreciation rate is set to $\delta = 0.10$, which is a standard value for annual frequencies.[44]

The post-tax replacement rate is set to $\bar{d} = 0.505$, which corresponds to the post-tax replacement rate in the U.S. in 2022 (OECD, 2022). The pre-tax replacement rate is then given by $d_t = \bar{d}(1 - \tau_t)$.

6 Analysis

In this section, the main results from the model simulation are presented, along with a series of different experiments and robustness checks. Unless otherwise specified, the results presented come from the model as described in section 5.

First, the model's ability to emulate the empirical development of the dependency ratio, wage share, and the amount of consumer debt relative to GDP is illustrated. In figure 5, the development of the dependency ratio in the model is shown compared to the data. Both the ratio of people aged 65+ to those aged 20-64 and the ratio of people aged 67+ to those aged 25-66 are compared, with the latter corresponding to the definition used in the model. The dependency ratio in the model deviates from the data, as the demographic process in the model, as given by equations (5) and (7), assumes a constant age of childbearing and no immigration. Before 2002, the dependency ratio in the model is lower, due to the age distribution in the first period being given by (10). Nonetheless, the model follows the data closely. In the period 2003-2024, the dependency ratio in the model is almost identical to the empirical data.

Figure 6 shows the development of the labor share (left) given in (49) and private consumption debt (right) given in (53) as a percentage of GDP. It appears that the model is able to recreate a labor share that is decreasing over the period, as well as a private debt ratio that is increasing. After 2002, the debt ratio in the model is lower than in the data, which follows from the calibration of the parameter D_T described in section 5.3. However, households' access to consumer loans has only a minor effect on the economic variables, including the interest rate, as shown in section 6.2.

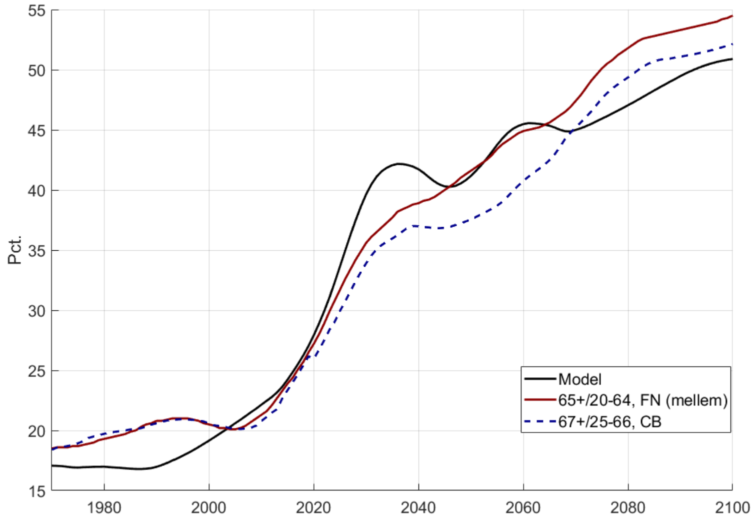
lies between 5% and 10%. However, the model is unable to replicate this without disturbing the other results.

[42] In the existing literature, $\rho = 1$ is often used. See Appendix A.

[43] In the literature, $\sigma = 1$ is often used (Eggertsson et al., 2019b; Papetti, 2021a).

[44] Similar values are used, among others, by Bielecki et al. (2020), Papetti (2021a), and Carvalho et al. (2023).

Figure 5: Dependency ratio: Model vs. Data (1970-2100)



Note: The figure shows the development of the old-age dependency ratio in the U.S. compared to the data (forecast after 2023). In the model, the dependency ratio is defined as the population aged 67+ relative to the population aged 25-66. Source: UN (2024), CB (2023), and own contribution.

6.1 Main results

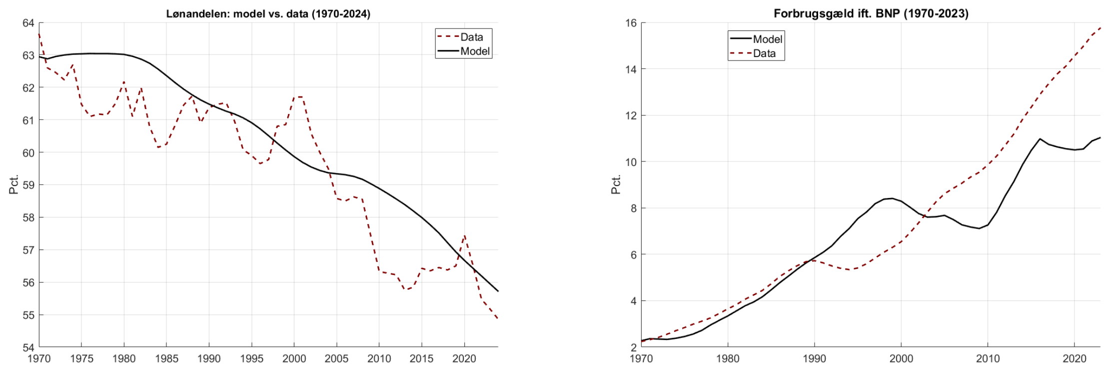
In Figure 7 on the left, the development of the simulated interest rate in the baseline model is shown with and without tax-financed pensions between 1970-2024. In both simulations, the parameters are calibrated to match the same empirical moments described in section 5.3, including the natural real interest rate in 1970 being 4.3%. The parameter values are shown in Table A1 in Appendix C.

In the simulation without pensions, the interest rate falls more (3.01 pp) than when pensions are included (2.73 pp). This result is expected, as described by mechanism 4, since tax-financed pensions crowd out private savings. In both cases, the interest rate falls more toward 1990 than in HLW.[45] After 2000 and 2008, the simulated interest rates are higher than those in HLW. In 2024, the simulated $r^\star$ is 1.57% with pensions and 1.26% without pensions. The HLW estimate is 0.82% in 2024. The simulated interest rates being higher than HLW's estimate can possibly be attributed to global factors not included in the model, such as the USA being a destination for savings surpluses from other economies (see Section 7.3 and Figure A17). Nonetheless, the simulated interest rates closely follow HLW, both in terms of the overall decline over the period and the fluctuations along the way.

In Figure 7 on the right, the development of the simulated $r^\star$ is shown toward 2100. The significance of publicly funded pensions gradually increases as the population ages. In the model

[45]This is partly due to the assumption of perfect foresight, which initially creates *front-loading* effects in the model.

Figure 6: Labor share and consumer debt



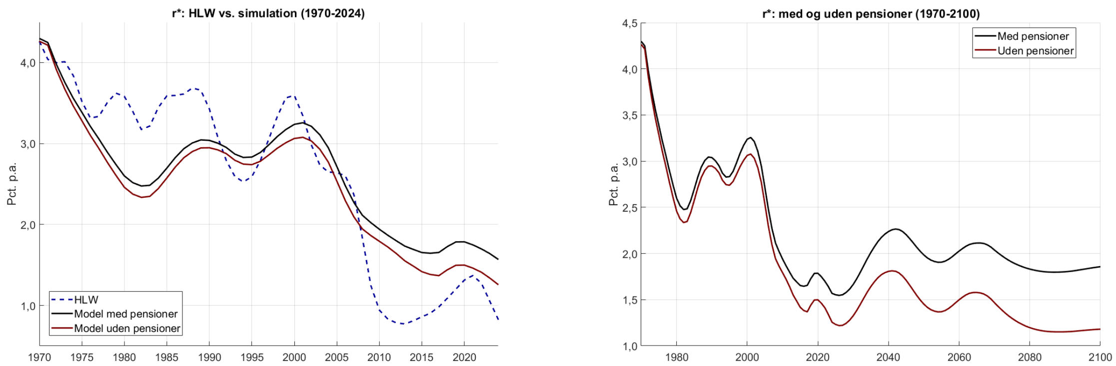
Note: The figure shows the development of the labor share (left) and the consumer debt as a share of GDP (right) compared to the data. Data for consumer debt is smoothed with a 5-year moving average. Source: BLS (2024), FRED (2024a), and own contribution.

with pensions, the tax rate τ_t increases from 25.3% in 1970 to 30.3% in 2024, which is comparable to the effective tax rate on labor income in the USA in 2023 of 29.9% (OECD, 2023). In the long run, τ_t rises to 34.5%. In the model *without* pensions, the tax rate stabilizes at 23.4%. In both cases, a turning point in the natural interest rate occurs after 2025 and 2055, respectively, due to demographic developments. Here, the natural interest rate increases by 68 bp (with pensions) and 54 bp (without pensions) between 2024 and 2042. After 2080, the natural interest rate stabilizes at 1.86% (with pensions) and 1.18% (without pensions), where it will remain in the long-run stationary equilibrium. In the simulation *without* pensions, this corresponds to roughly the same level as in 2024 (1.26%), while the natural interest rate in the simulation *with* pensions will be higher than in 2024 (1.57%) in the stationary equilibrium. Since the turning point occurs both with and without a tax-financed pension system, and since all exogenous variables are constant after 2023 (except for fertility, cf. Section 5.1.1), it can be concluded that the turning points are due to demographic factors through mechanisms 1 and 3. However, pensions contribute to amplifying the turning point, which is due to mechanism 4.

The development of the interest rate is reflected in the growth rate of the labor supply and capital per worker, as shown in Figure (8). It appears that the growth rate of the labor force increases after 2025, while the growth rate of capital per worker decreases. When an increase in the growth rate of the labor force leads to an increase in the interest rate, it reflects that labor supply affects investment demand and thereby the interest rate, as described in mechanism 1. When the turning point after 2055 is smaller than the turning point after 2025, it is due to the phasing out of echo effects from the baby boomer generation and their descendants (see Section 5.1.1). In the model, these echo effects are fully present until 2029. Therefore, it is likely that the turning point after 2025 is more pronounced in the model than it will be in reality.

At no point does the natural real interest rate become negative. The model therefore immediately rejects the possibility that there has been or will be secular stagnation in the U.S.

Figure 7: $r^\star$ in 1970-2024 and 1970-2100.



Note: The left panel shows the evolution in the natural real interest rate ($r^\star$) in 1970-2024 in the model with (black) and without (red) social security compared to the estimates from HLW (blue, dashed). The right panel shows the simulated interest rate with and without social security to 2100. Source: Holston et al. (2024) and own contribution.

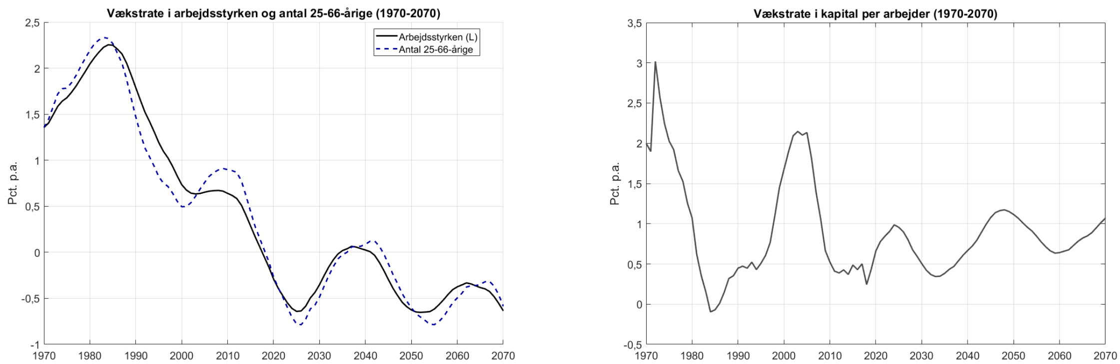
economy. However, this result may be sensitive to the assumed parameter values (cf. Section 6.7), developments in the demographic variables (cf. Section 6.8), and the assumption of a closed economy (cf. Section 7).

6.2 Isolating the drivers

It is not feasible to completely isolate the four mechanisms, as fertility and mortality simultaneously influence savings behavior and investment demand through the four mechanisms mentioned in Section 1.1. Lower mortality has two opposing effects on the natural real interest rate. On the one hand, higher life expectancy leads to a higher savings rate among workers (mechanism 2), while on the other hand, it increases the dependency ratio (mechanism 3). Similarly, a lower birth rate also has opposing effects on the natural interest rate, as it reduces the labor force (mechanism 1) and increases the dependency ratio (mechanism 3). Fertility does not affect life expectancy, and mortality among the elderly does not impact labor supply unless the retirement age increases. Both fertility and mortality can influence the tax burden and pension expenditures (mechanism 4). Nevertheless, it can be deduced that if lower mortality leads to a lower real interest rate in equilibrium, mechanism 2 must dominate mechanisms 3 and 4. Similarly, if a decline in fertility results in a decrease in the natural real interest rate, this must reflect that mechanism 1 dominates mechanisms 3 and 4.

In Figure 9, the evolution of the natural real interest rate is shown when mortality, fertility, and productivity growth are each held constant at their 1970 levels. Keeping fertility constant after 1970 implies that the baby boom from 1946–1964 is still included. It appears that the interest rate declines the least when either mortality or productivity growth is held constant, reflecting that these factors have contributed the most to the decline in the natural real interest rate since 1970.

Figure 8: Growth rate in the labor force (L_t) and capital per worker (k_t)



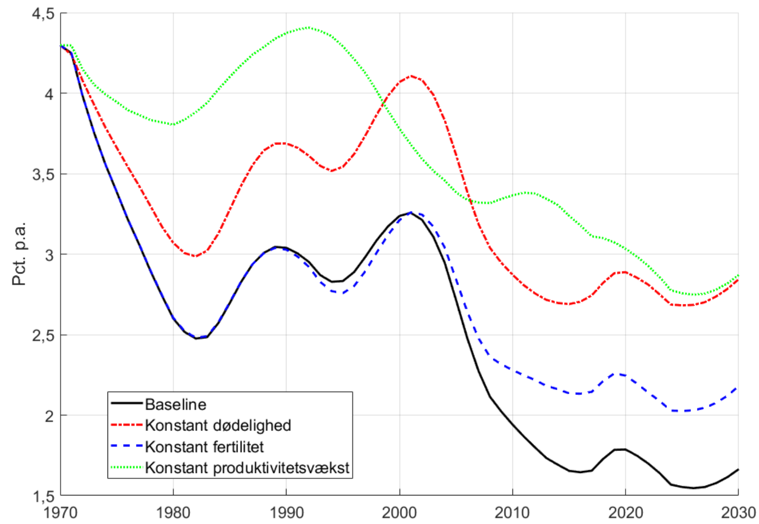
Note: The left panel shows the annual growth rate in the labor force as given in (12) as well as the population in the working age. The right panel shows the annual growth rate in the stock of capital per worker calculated by dividing (41) with (12). Notice that capital per worker is distinguished from capital per *effective* worker, which is given by $\tilde{k}_t = k_t/A_t$. Source: Own contribution.

The scenario with constant fertility diverges from the baseline only in the 1990s, as it takes 25 years for agents to enter the labor force. Nevertheless, it is evident that the natural interest rate would have been higher after 2000 if fertility had not declined since 1970, reflecting that mechanism 1 is stronger than mechanism 3. Similarly, the fact that the increase in life expectancy, in the form of declining mortality, has exerted downward pressure on the natural rate indicates that mechanism 2 overall dominates mechanisms 3 and 4. In other words, the effect of a declining labor force growth rate, combined with households' increased preference for saving more in anticipation of living longer, has outweighed the effect of rising dependency ratios and tax burdens. This result is as expected, cf. Section 3.

In all four scenarios, a minor turning point in the natural interest rate occurs after 2025. This is due to the final large cohorts from the baby boom having retired, while their grandchildren enter the labor force. This results in a decline in capital per effective worker, as the growth rate of the labor force increases (cf. Mechanism 1), as shown in Figure 8. At the same time, there is an isolated downward pressure on the savings rate as large cohorts of retirees decumulate their savings (cf. mechanism 3). In Section 6.2.1, a scenario is simulated where the baby boom did not occur to isolate its effect, as the extraordinarily high fertility prior to 1970 has had a significant impact on subsequent economic developments. It is also noted that the labor force, as given by (12), depends on both the number of individuals of working age and the age composition of the workforce, as the human capital profile varies with age. Consequently, the evolution of the productive labor force, as defined in (12), can deviate in the short term from the number of individuals of working age. In Section 6.7, a scenario is simulated where human capital is constant across all ages within the working-age population.

Table 2 shows the growth contributions from all the exogenous processes in the model to the change in the natural interest rate between 1970 and 2024, amounting to 2.73 pp (cf. Figure

Figure 9: $r^{\star}$: Isolating the drivers (1970-2030)



Note: The figure shows the development in the simulated interest rate when mortality (red), fertility (blue dashed), and productivity growth (green) respectively are held constant at their 1970 level. Source: Own contribution.

7, with pensions). The growth contributions are calculated by holding one exogenous variable constant at a time. The findings indicate that lower mortality and fertility, reduced productivity growth, and increased market concentration have exerted downward pressure on the natural interest rate, while increases in both public and private debt have exerted upward pressure. The combined growth contribution from demographic factors is -1.58 pp. Additionally, the decline in the competition parameter θ_t causes the interest rate to fall more than the marginal product of capital (see Figure A6). This results in a decoupling between the rental rate and the return on capital, as the profit rate rises, reflecting the empirical evidence (Gomme et al., 2015).

The growth contributions do not sum to the total decline of 2.73 pp for two reasons. First, it is not possible to perfectly isolate the effect of each exogenous variable due to equilibrium interaction effects. Second, the baby boom, which occurred before 1970, continues to influence the economy afterward, even if the exogenous variables are held constant at their 1970 levels. If all exogenous processes are held constant from and after 1970, the natural interest rate still declines between 1970 and 2024 due to the baby boom, cf. Section 6.2.1.

Table 2: Contribution from exogenous variables (1970-2024)

Exogenous variable	$r^\star$ i 2024 (%)	Contribution (% points)
Mortality (s_j)	2.69	-1.12
Fertility (Γ_t)*	2.03	-0.46
Productivity growth (g_t^A)	2.78	-1.21
Government debt (b_t)	0.79	0.77
Market concentration (θ_t)	2.04	-0.48
Private credit (D_t)	1.37	0.20

Note: Column 2 shows the simulated interest rate in 2024 when the relevant exogenous variable is held constant at its 1970 level. The growth contributions in Column 3 are calculated by comparing these scenarios to the baseline, where the interest rate is 1.57% in 2024, corresponding to a total decline of 2.73 pp since 1970. The sum of the contributions does not equal the total decline due to interaction effects and the influence of the baby boom. *Excluding the effect of the baby boom. With the baby boom, the contribution from fertility is 1.11 pp, cf. Section 6.2.1. Source: Own contribution.

6.2.1 Effect of the baby boom

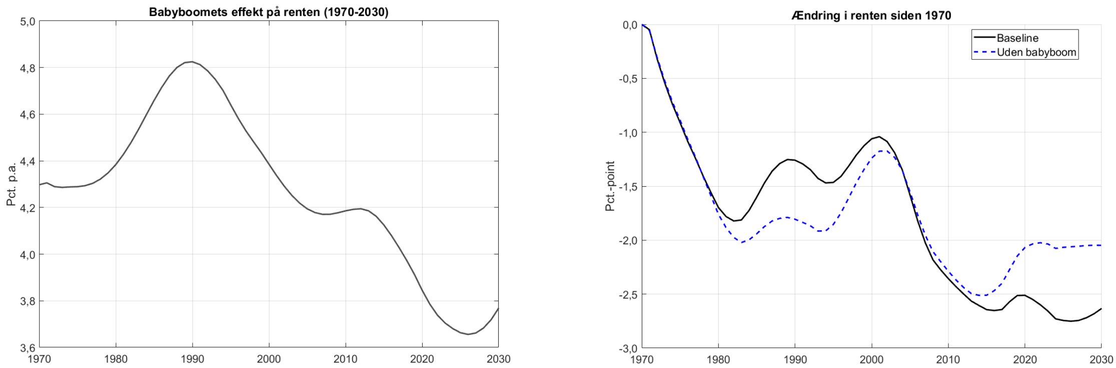
The large cohorts born during the period 1946–1964 have had a significant impact on the economy since 1970. In the left panel of Figure 10, the development of the natural interest rate is shown under a hypothetical scenario where all exogenous processes are held constant from 1970 onward, so the only exogenous variation is the high fertility before 1970.[46]

The decline in the interest rate between 1970–2024 is 62 bp. Between 1990–2024, it exceeds 1 pp. It appears that the baby boom exerted upward pressure on the natural interest rate between 1970–1990, after which it exerted downward pressure until 2025. After 2025, the natural interest rate rises again as the downward pressure diminishes. At this point, the last large cohorts from the boomer generation will have retired, while their grandchildren will be entering the labor force.

Another way to isolate the effect of the baby boom is to simulate a scenario where the baby boom did not occur, but the exogenous variables still follow the same trajectory. In the right panel of Figure 10, the decline in the real interest rate is shown for a scenario where all exogenous processes develop as in the baseline, but fertility before 1970 is set equal to the fertility rate in 1970. Here, it appears that the natural real interest rate would have declined by 65 bp less than in the baseline by 2024. Since the only difference here is the absence of the baby boom, 65 bp is thus the most precise estimate of the effect of the baby boom, all else equal, on the development of the natural interest rate between 1970–2024. This estimate is also very close to the 62 bp by which the interest rate declines in the simulation where the baby boom is the only exogenous variation. Without the baby boom, no turning point occurs after 2025.

[46]Here, it is disregarded that the baby boom officially ended in 1964.

Figure 10: Effect of the baby boom



Note: The left panel shows the development of the interest rate when all exogenous variables are held constant throughout the period, except for fertility before 1970, so that the baby boom is the only exogenous variation. The right panel shows the decline in the interest rate both with (*baseline*) and without (*uden*) the baby boom, where the exogenous variables are *not* held constant. Therefore, the two figures are not directly comparable. Source: Own contribution.

The reason why the sum of growth contributions in Table 2 is less than the total decline is, therefore, that the presence of the baby boomer generation itself exerted downward pressure on the natural interest rate during the period. When this is taken into account, the total growth contribution from fertility becomes -0.46 - 0.65 = -1.11 pp instead of -0.46 pp. In that case, the contribution from fertility is exactly as large as the contribution from changes in mortality, which is consistent with Eggertsson et al. (2019a). The total growth contribution from demographic factors thus becomes -2.23 pp when accounting for the baby boom.

In Table A2, the growth contributions from each of the exogenous variables are shown under the scenario where the baby boom did not occur. It turns out that the individual growth contributions are almost identical to the results in Table 2. This indicates that the isolated contributions from the development of the exogenous processes are not affected by interaction effects from the baby boom. This underscores that higher life expectancy generally has a stronger effect than lower fertility on the natural interest rate. In other words, I find that the importance of mechanism 2 has been the greatest among the four mechanisms. This result is consistent with the related literature, including Papetti (2021a), Bielecki et al. (2020), and Carvalho et al. (2016).

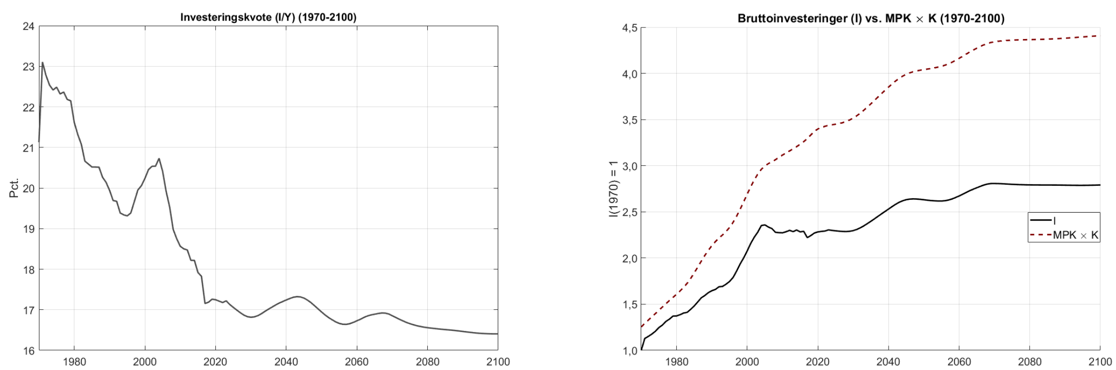
6.3 Investments and savings

Figure 11 shows the development of investments in the model. The left panel illustrates gross investments, as given in (42), as a share of GDP. The right panel depicts the development of gross investments and capital multiplied by its marginal product. If $I_t < MPK_t \cdot K_t$, which holds true throughout the period, the criterion for dynamic efficiency is satisfied, cf. Section 4.5.

It is evident that the investment share declines over the period, albeit with temporary fluctuations along the way. In 1970-1971, the investment share rises sharply. This follows from the model's structure, as agents receive a shock in the first period when they become aware of the future trajectory of exogenous processes, including the increase in expected lifespan. When households learn that life expectancy will rise, they increase their savings rate. This leads to a decline in the interest rate, which raises the demand for capital in equilibrium. However, the high investment share does not persist. After 1982, the investment share drops below its 1970 level and continues to decline until 1995.

The long-term decline in the investment share, despite households' increased preference for saving, is due to a simultaneous decrease in investment demand, driven by lower productivity growth (see Figure A11) and a slowing growth of the labor force (see Figure 8), the latter of which is described in mechanism 1. While the rise in life expectancy, all else equal, should increase the equilibrium investment share, the decline in population growth pulls in the opposite direction (see Figure A4).[47]

Figure 11: Gross investments



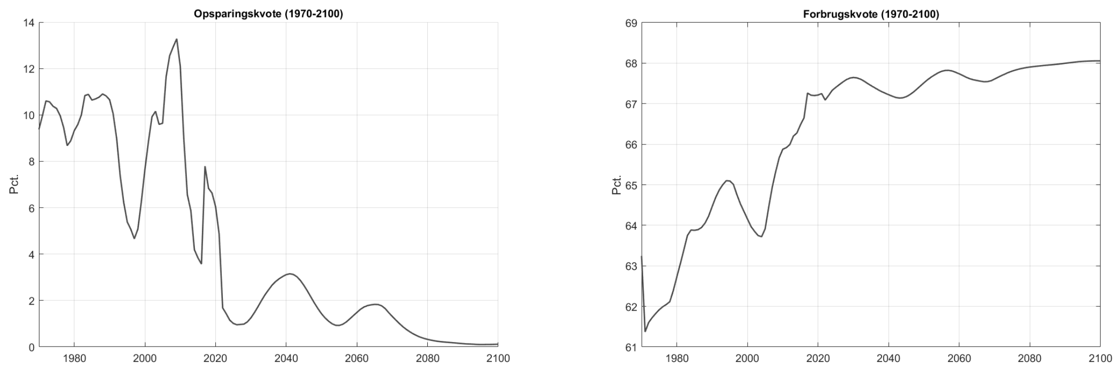
Note: The left panel shows gross investments as a share of GDP. The right panel shows the development of investments and $MPK_t \cdot K_t$, with investments in 1970 indexed to one. Source: Own contribution.

The declining investment rate is reflected in a decreasing aggregate private savings rate. Population aging itself causes the savings rate to decline, as retired households tend to have a low or even negative savings rate. The aggregate household savings rate is shown in the left panel of Figure 12. Between 1997 and 2009, the savings rate increases significantly as the baby boomer generation approaches retirement age.[48] Additionally, productivity growth rises between 1993 and 2000 (cf. Figure A11), which also has a positive effect on the equilibrium savings rate, as it increases investment demand (cf. Figure 11). After 2009, the savings rate declines again as the first cohorts of the baby boomer generation retire. The savings rate rises after 2025, despite

[47]The investment share also declines in the model without pensions (see Figure A4 (left) in Appendix C). This is because the wealth effect remains present in the extended model, unlike in the simple two-generation model described in Section 3.

[48]Fluctuations in the private savings rate are amplified in the model due to sharp variations in the tax rate between 1970 and 2020. See Figure A7 in Appendix C.

Figure 12: Aggregate savings rate and consumption share



Note: The left panel shows households' aggregate net savings rate as a percentage of disposable income, given by (52). The right panel shows the consumption share, given by (50). Source: Own contribution.

the growing share of retirees. This indicates that mechanism 3 is not a significant driver of the turning point in the interest rate during this period.

In the long term, the net savings rate converges toward a value slightly above zero, reflecting the fact that the aggregate capital stock grows at $(1+g^A)(1+n)-1 \approx 0^+$ on the balanced growth path (see Section 4.7).[49] Correspondingly, the aggregate consumption rate increases in the long run, as shown in the right panel of Figure 12. The declining savings rate aligns with empirical evidence, which similarly indicates that household savings rates in developed economies have been declining since the 1970s. However, this trend is only partially reflected in the investment rate, which in the U.S. has remained at roughly the same level over the period 1970–2023 (see Figure A16). This discrepancy is partly due to the fact that the U.S. is a net recipient of foreign capital, a factor not captured by the model. Section 7.3 discusses the implications of the U.S.'s status as an open economy for the natural interest rate.

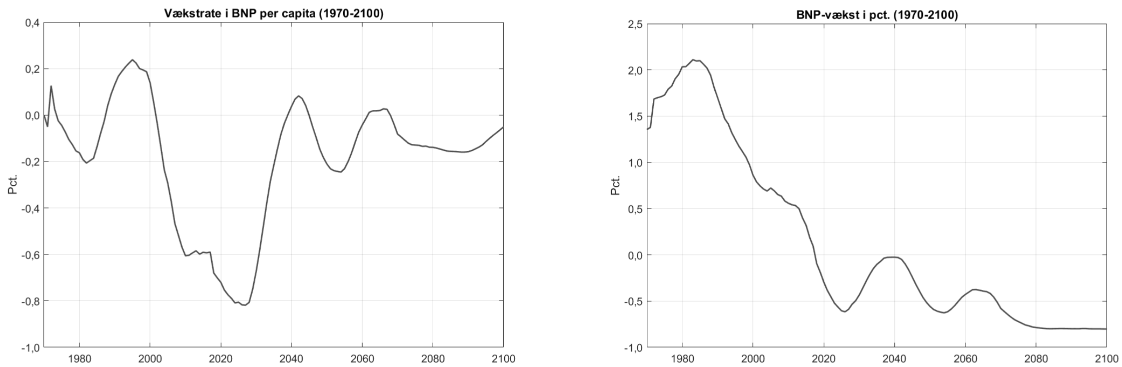
6.4 Economic growth

In the model, economic growth is primarily determined by the development of productivity and population. In the long run, aggregate output will grow at a rate of $(1+g^A)(1+n)$, while capital, consumption, and output per capita will grow at $(1 + g^A)$, cf. Section 4.7. Since productivity growth is exogenously given in the model, it is not suited to analyze the long-term effects of aging on economic growth. However, this does not change the fact that changes in population growth and demographic composition can have a transitory effect on economic growth.

Figure 13 illustrates the development of GDP per capita growth and total GDP growth in the model, where productivity growth is set to zero to isolate the effects of demographic changes on economic growth. The left panel shows that the growth rate of GDP per capita is negative for most of the period, converging toward zero in the long term. Between 1982-1995, the growth

[49]The growth rate is slightly positive because $|g^A| = 0.0085 > |n| = 0.008$ in steady state.

Figure 13: Aging and economic growth



Note: The left panel shows the annual growth rate of GDP per capita. The right panel shows the annual growth rate of GDP. In both simulations, productivity growth is set to zero. Source: Own contribution.

rate increases, becoming positive from 1987-2001. In 2024, the growth rate is -0.8%. Overall, GDP per capita decreases by 11.25% between 1970 and 2024, corresponding to an annual growth contribution of -0.22 percentage points. This indicates that demographic changes have exerted downward pressure on economic growth.

The right panel of Figure 13 shows that the growth rate of total GDP has also declined due to population aging. However, the growth rate only turns negative after 2018, subsequently converging toward the population growth rate of $n = -0.8\%$ in the long run. Both GDP and GDP per capita growth rates exhibit temporary fluctuations during the period due to demographic echo effects. Similar to the turning point in the real interest rate after 2025 (cf. Figure 7), the model also predicts a turning point in economic growth at the same time, driven by demography.

6.5 Behavior of the households

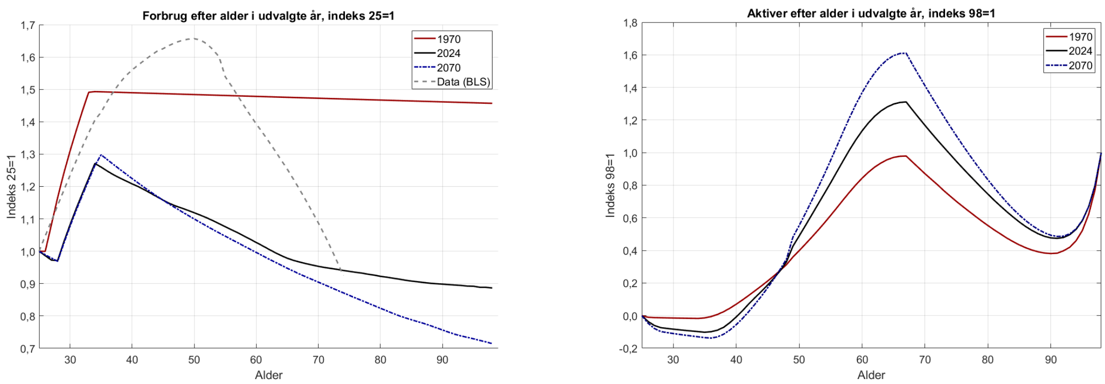
This section examines household behavior regarding consumption and savings. Figure 14 shows households' consumption patterns (left) and asset holdings (right) distributed by age in selected years. It is evident that households exhibit increasing individual consumption until the age of 34, after which they gradually reduce their consumption to save for retirement. This pattern partially aligns with empirical evidence, which also shows households first experiencing rising and then falling consumption. However, agents in the model smooth their consumption to a greater extent than observed empirically.[50]

It also appears that households of a given age smooth their consumption more over time relative to their consumption at age 25. This result is primarily driven by older households

[50] A higher ρ creates a better match with data. However, this requires $\rho > 1$, which is rejected by empirical evidence (cf. Section 5.3). See Figure A3. When compared to the estimates from Gourinchas and Parker (1999), the consumption profile in the model aligns more closely with the data.

increasing their savings rate as life expectancy rises. Another factor is the increase in the credit limit $D_{j,t}$ between 1970 and 2024, which allows younger households to shift their consumption forward.[51] This is also reflected in the fact that net assets are negative for younger households in 2024 and 2070 but not in 1970. Between ages 35 and 66, households build up savings, which they begin to decumulate after retirement. After the age of 90, the assets of surviving households increase again due to the insurance market, where the assets of deceased individuals are evenly distributed among the surviving members of the same cohort. Additionally, the bequest motive strengthens as individuals approach the terminal age.

Figure 14: Consumption and assets by age



Note: The left panel shows households' consumption by age in selected years, with consumption for 25-year-olds indexed to 1. It is compared to interpolated data from BLS. The right panel shows households' assets by age in selected years, with assets for 98-year-olds indexed to 1. Source: BLS (2023) and own contribution.

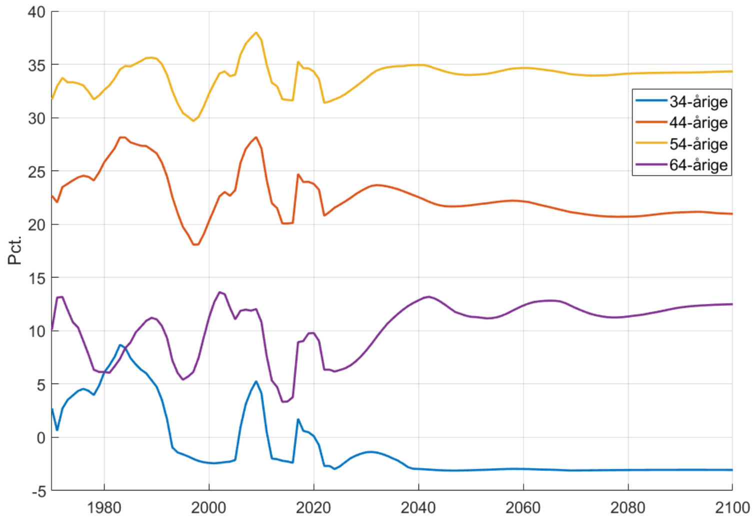
In Figure 15, the savings rate for selected age groups within the working-age population over time is shown. It appears that the savings rate for individuals aged 34–54 increases during the first part of the period. This reflects their response to the rise in expected life expectancy by saving more. In the long run, however, the savings rate for those aged 34 and 44 becomes lower than in 1970, while the opposite is true for those aged 54 and 64. Thus, in the long run, older households within the working-age population increase their savings rate, while the savings rate for younger households remains unchanged or even decreases. The latter can be attributed to younger households becoming more creditworthy over time, prompting them to bring forward their consumption, as also seen in Figure 14.

The continued advancement of consumption by younger households even after 2024, when the credit limit $D_{j,t}$ is no longer increased, is due to the continued rise in the real wage w_t, which relaxes the borrowing constraint, cf. (19). If the model is simulated without an increase in the credit limit between 1970 and 2024, the households' consumption cycle becomes more realistic compared to the data, as consumption more closely follows the income profile (see Figure A2). However, excluding access to credit would prevent the model from replicating the rise in the

[51] If the credit limit $D_{j,t}$ is held constant, the consumption curves align better with data. The curves also better match data in the model without pensions. See Figure A2.

private debt-to-consumption ratio, cf. Figure 6. Furthermore, the impact of the increase in the credit limit on the natural interest rate is modest (20 bp), cf. Table 2.

Figure 15: Savings rates for selected age groups (1970-2100)



Note: The (net) savings rate is denoted as a percentage of disposable income. Source: Own contribution.

In Section 6.2, it was shown that the increasing probability of survival was one of the primary factors behind the decline in the natural real interest rate between 1970 and 2024, as rising life expectancy enhances households' incentive to save, cf. mechanism 2. However, despite this, the households' savings rate does not increase significantly, and for some age groups, it decreases in the long run. This outcome is due to opposing effects in equilibrium.

When the interest rate, which the individual household takes as given, changes, their allocation between consumption and savings is influenced by the substitution effect, income effect, and wealth effect. When $\rho < 1$, the income effect dominates the substitution effect, meaning that a decline in the interest rate increases the agent's incentive to save. Since the model assumes $\rho = 0.95 < 1$, the income effect should therefore dominate, and the households' savings rate should increase in response to a lower interest rate. However, this is not the case due to the presence of the wealth effect, which aligns with the substitution effect. The intuition here is that a lower interest rate, all else equal, increases the present value of the individual's future income, thereby reducing the incentive to save.[52] Additionally, lower investment demand itself causes the equilibrium interest rate to decline for a given investment rate (cf. Section 3).

[52]See Appendix A for a detailed discussion of the importance of the intertemporal elasticity of substitution.

6.6 Experiments

In this section, a series of experiments are conducted in an attempt to quantify Goodhart and Pradhan's hypothesis that the natural real interest rate is facing a turning point, leaving it a persistently higher level. Beyond the fact that the equilibrium interest rate in the savings market itself will change as a result of demographic change, they argue that a range of related factors will contribute to an increase in the real interest rate. First, they assess that public expenditures on pensions and healthcare will rise due to an increasing number of elderly people. This will increase the tax burden and public debt, leading to higher real interest rates. Second, they suggest, also based on demographic developments, that the labor share will rise as labor becomes a scarcer resource, strengthening workers' bargaining power vis-à-vis employers (Goodhart and Pradhan, 2020).

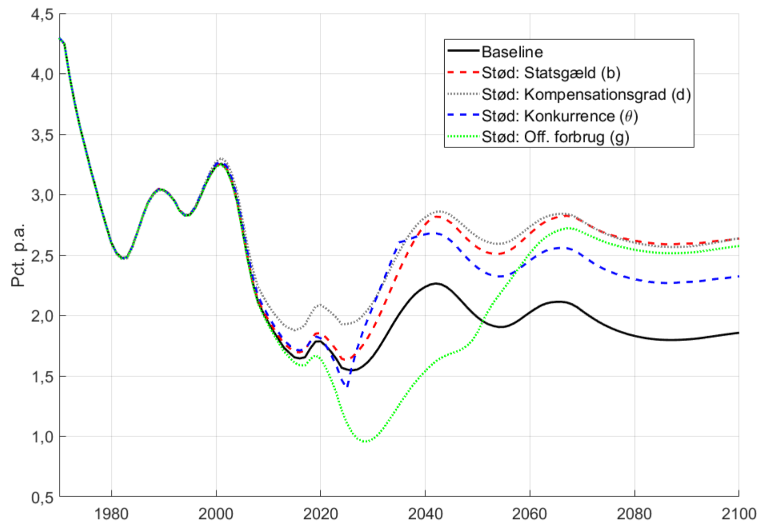
The first part of this hypothesis can be translated in the model to increases in pension expenditures given in (44), public expenditures given in (45), and public debt given in (46). In the model, pension expenditures automatically rise in line with the increase in the dependency ratio. This part of the hypothesis is thus already incorporated, as a higher number of retirees will lead to an increase in government pension expenditures, thereby also increasing total public expenditures.

Figure 7 shows that there is a turning point in the interest rate after 2025. However, this is not driven by higher pension expenditures, as the turning point also appears in the simulation without pensions. Nevertheless, the interest rate remains permanently higher in the long term due to increased pension expenditures. In the model, growing pension expenditures result in a higher tax rate, as the government's budget constraint given in (47) must be met. This puts upward pressure on the natural real interest rate, cf. mechanism 4.

In the model, public debt and public consumption expenditures as a share of GDP are, however, exogenously determined. To test the effect of higher public debt and increased healthcare expenditures, I introduce a permanent shock to the exogenous variables b_t and g_t. The hypothesis that the labor share will rise can be directly translated in the model to an increase in the competition parameter θ_t. This follows from the fact that a higher value of θ_t reduces firms' profits as wage costs rise, cf. (31). I also perform a simulation where the replacement rate increases, as this represents a scenario in which government expenditures on transfers rise. Although it is not part of G&P's hypothesis that the replacement rate will increase, they point out that it is unlikely to decrease, as such a change would be politically difficult to implement. In the model, however, an increased replacement rate can be used as a proxy for rising healthcare expenditures or other forms of transfers to retirees.

The results are shown in Figure 16, which compares the development of the natural real interest rate under the different shocks. The shock to public debt is implemented by gradually increasing the debt-to-GDP ratio from 120% in 2024 to 200% of GDP in 2040. By 2050, the real interest rate is 58 bp higher, and by 2100, it is 78 bp higher than in the baseline.

Figure 16: $r^{\star}$ under different shocks



Note: The figure shows the development of $r^{\star}$ in 1970-2100 under different shocks to selected exogenous variables. Source: Own contribution.

The shock to the public consumption share is phased in between 2025 and 2050, where the consumption share gradually doubles from 15.5% to 31% of GDP. In the long term, the interest rate rises and settles at a permanently higher level, but in the period before the shock is introduced, the interest rate falls sharply. This result follows from the fact that agents in the model have perfect foresight about the development of exogenous processes. Thus, they anticipate well in advance that government consumption will grow, leading to higher taxes. In response, they increase their savings before the shock occurs to maximize utility over their lifetime. The magnitude of this response is admittedly unrealistic, as households in reality have limited knowledge of or interest in the long-term development of public finances. Nevertheless, a permanently higher public consumption share still results in a permanently higher interest rate in the long run. By 2100, the natural real interest rate is thus 72 bp higher. The intuition, as described in mechanism 4, is that higher taxes crowd out private savings.

The shock to the competition parameter θ_t is implemented by gradually increasing it from 3.99 to 6.95 between 2025 and 2035, so that θ_t in 2035 equals its parameter value in 1970. It is evident that the real interest rate rises and settles at a permanently higher level, 47 bp above the baseline. An increase in θ_t directly affects the interest rate, as a higher θ_t brings the rental rate closer to the marginal product of capital, cf. (30).

The shock to the replacement rate is phased in between 2025 and 2040, where it gradually doubles to $\bar{d} = 1$, making pension benefits equal to post-tax labor income. It is evident that the interest rate rises and settles at a permanently higher level. Due to households' foresight, the interest rate increases even before the shock occurs, as the prospect of higher pension benefits reduces the need to save for retirement. This increase cannot therefore be attributed to mechanism 4, but rather to mechanism 2, which is dampened by access to publicly funded pensions. The effect of an increase in the replacement rate thus differs from the effect of an increase in public consumption, as households react differently despite the tax burden rising in both cases.

All four experiments show that the natural interest rate will rise in the long run. Had the experiments been conducted in a model where the shocks were unforeseen by the agents, the increase in the interest rate would likely have been more abrupt, without the large fluctuations beforehand, as is particularly seen in the shock to public consumption, where the interest rate only exceeds the baseline level after the shock is fully phased in. Nevertheless, the long-term results provide a valid estimate of how changes in the relevant variables will affect the interest rate in a long-run equilibrium.

Stress test: raising $r^{\star}$ by 1 percentage point

Table 3 shows how much the relevant exogenous variables must change for the natural real interest rate to increase by 1 pp in steady state. In all cases, it requires a significant and seemingly unrealistic change. For example, the replacement rate must exceed the wage rate, and the competition parameter must approach a level that approximates perfect competition. One exception may be the government's debt-to-GDP ratio. It appears that an increase from 120% to 221% of GDP would raise the natural interest rate by 1 pp. This is roughly equivalent to a 100 bp increase in the debt-to-GDP ratio raising the natural interest rate by 1 bp. The same effect can be derived from Table 2. This result is consistent with related studies, which also find that increased government debt puts upward pressure on the natural interest rate (Rachel and Summers, 2019; Carvalho et al., 2023; Eggertsson et al., 2019a). However, the effect is likely underestimated here, as there is no risk premium in the model. Given that the U.S. national debt has grown rapidly in recent decades (see Figure A13a), and that countries like Japan already have a government debt-to-GDP ratio of over 200% (TE, 2024), it cannot be ruled out that this scenario will unfold within this century. Furthermore, it cannot be ruled out that a combination of increases in several of the exogenous variables will occur. In such a case, it seems less unlikely that any economic shocks, including those resulting from demographic developments, will lead to a persistently higher natural real interest rate.

Table 3: Required change to raise $r^\star$ by 1 pp

Exogenous variable	Baseline	Hypothetical value
Fertility (Γ)	1.64	2.68
Replacement rate ($\bar{d}$)	0.505	1.16
Competition (θ)	3.99	45
Productivity growth (g^A)	0.0085	0.018
Government consumption (g)	0.1554	0.358
Government debt (b)	1.20	2.21

Note: The table shows how much each exogenous variable has to change to raise $r^\star$ by 1 pp in steady state. Fertility is denoted as children born per woman. Source: Own contribution.

6.6.1 Simulating G&P's hypothesis

The results in Figure 16 and Table 3 show that a range of changes in the exogenous variables will lead to an increase in the natural interest rate, which is consistent with Goodhart and Pradhan's theory. However, individually, these changes seem unlikely. In this section, an attempt is made to simulate a series of more complex and potentially realistic scenarios of how G&P's hypothesis might be modeled. This can be done by allowing several of the exogenous variables to change simultaneously and by incorporating public health expenditures into the model. In total, three scenarios are presented for how G&P's hypothesis can be simulated. The results are shown in Figure 17 and Table 4.

The United States differs from several other developed economies in having a high degree of private financing of healthcare. Nevertheless, since 1965, the country has had a government health program, *Medicare*, which subsidizes healthcare expenditures for the elderly (Wikipedia, 2024). In all three scenarios, the pension system in the model is expanded to also include public expenditures on Medicare. This is done by raising the replacement rate so that the pension benefit includes both Social Security payments and healthcare expenditures covered by Medicare. The replacement rate $\bar{d}$ is multiplied each year by a factor $(1 + m_t)$, where m_t is the ratio between government expenditures on Medicare and Social Security in the relevant year. Thus, the benefit to pensioners given in (17) changes to $d_t w_t = \bar{d}(1 + m_t)(1 - \tau_t)w_t$.

Between 1970-2007, Medicare expenditures grew faster than Social Security expenditures (see Figure A19). Since 2019, however, the ratio of these expenditures has stabilized. Therefore, it is assumed that m_t remains at 0.768 after 2022, which is the average for 2021-2023.[53] This implies that $\bar{d}(1 + m_t) = 0.615$ in 1970, gradually increasing to $\bar{d}(1 + m_t) = 0.893$ in 2023. In Scenario 1, both public expenditures on pensions and healthcare will grow as the population

[53] The ratio is smoothed with a 5-year moving average. Data on government expenditures on Social Security and Medicare are sourced from the Federal Reserve (FRED, 2024e).

ages, while the expenditures for each pensioner, measured relative to the after-tax wage rate, are assumed to remain constant after 2023.

Table 4: G&P's hypothesis: Three scenarios

	Baseline	Scenario 1	Scenario 2	Scenario 3
$\Delta r^{\star}$ 2024-2040	0.67	0.85	1.08	0.97
$\Delta r^{\star}$ 2024-2100	0.29	0.68	1.02	1.29
Government debt (b)	1.20	1.50	1.50	1.50
Competition (θ)	3.99	3.99	5.50	5.50
Replacement rate ($\bar{d}(1+m)$)	0.51	0.89	0.89	0.89
Replacement rate, gross (d)	0.33	0.51	0.52	0.46
Government consumption (g)	0.16	0.16	0.16	0.20
Tax rate (τ) in steady state	0.34	0.42	0.42	0.49
Expenditure on elderly in % of GDP (E/Y)	5.17	7.98	8.80	7.84

Note: The parameter values are from 2050 and forward. The change in $r^{\star}$ is denoted in percentage points. The tax rate and expenditure on social security are in steady state, which is roughly equal to the same as in 2100. In Scenarios 1-3 social security expenditure E_t also include expenditure on health care. Numbers are rounded to two decimal places. Source: Own contribution.

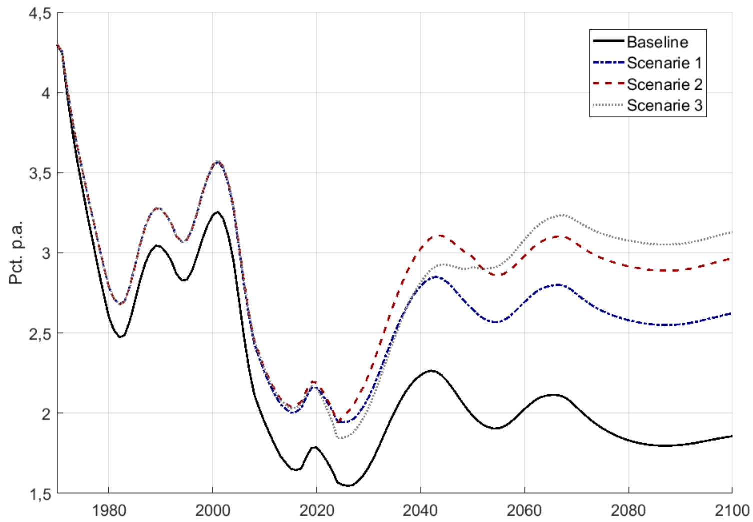
In the second scenario, it is further assumed, in line with G&P's hypothesis, that the labor share rises. This is modeled by increasing the parameter θ_t to 5.5—the midpoint between its value in 1970 and 2024—which results in the labor share rising from 55.5% in 2024 to 61% in 2050, approximately 2 percentage points lower than in 1970, as shown in Figure 6.

In the third scenario, it is further assumed that the public consumption share increases to 20% of GDP between 2025-2050, equivalent to a rise in public consumption expenditures of 4.5% of GDP. In this scenario, total public expenditures thus grow more than in the first two scenarios. This could represent a situation where healthcare costs per pensioner also increase. Such a development could stem from rising expectations of state-provided services or increased labor costs for healthcare professionals due to labor shortages.

In all three scenarios, the interest rate is higher in the long run compared to the baseline. This result is expected, as shown in Figure 16. Throughout almost the entire period, the interest rate in the three scenarios remains above the baseline, as the higher replacement rate increases the tax burden (cf. mechanism 4) and reduces households' incentive to save (cf. mechanism 2). However, in none of the scenarios does the interest rate return to its 1970 level.

In Scenario 3, the interest rate rises less than in Scenario 2 before 2050, owing to households' response to the anticipated future increase in the tax burden. Nonetheless, the interest rate rises the most in Scenario 3 in the long run, as both the replacement rate and the public consumption share increase in this scenario. The difference compared to Scenario 2 is modest, as the higher

Figure 17: $r^{\star}$ under G&P's three scenarios



Note: The figure shows the development of $r^{\star}$ under the three scenarios described in Table 4. In all simulations, β is recalibrated so the interest rate equals 4.3% i 1970. Source: Own contribution.

tax rate, all else equal, reduces expenditures on pensions and healthcare, which in the model are linked to the after-tax wage rate. If pensions and healthcare benefits had instead been modeled as a fixed proportion of the pre-tax wage rate, expenditures and the tax burden would rise further, thereby amplifying the upward pressure on the interest rate via mechanism 4. Due to the higher tax rate in Scenario 3, the ratio between E and Y reaches its highest level in Scenario 2.

However, total public expenditures are still largest in Scenario 3. Assuming that the increase in g from 0.1554 to 0.20 in Scenario 3 represents growing healthcare expenditures for the elderly, total expenditures on the elderly in Scenario 3 would amount to 12.3% of GDP in steady state or after 2100. This aligns precisely with the U.S. authorities' own estimate for 2098 (SSA, 2024b). In 2023, expenditures on Social Security and Medicare accounted for 4.9% and 3.6% of GDP, respectively, or 8.5% in total (see Figure A19). In the baseline model, pension expenditures constitute 3.4% of GDP in 2023, which is below the actual 4.9%. This discrepancy can partly be attributed to the fact that Social Security expenditures in reality cover more than just retirees over 66 years of age (SSA, 2024a). Nevertheless, this indicates that the baseline model underestimates the extent of healthcare and pension expenditures, while Scenario 3 cannot be dismissed as a realistic projection of their long-term development.

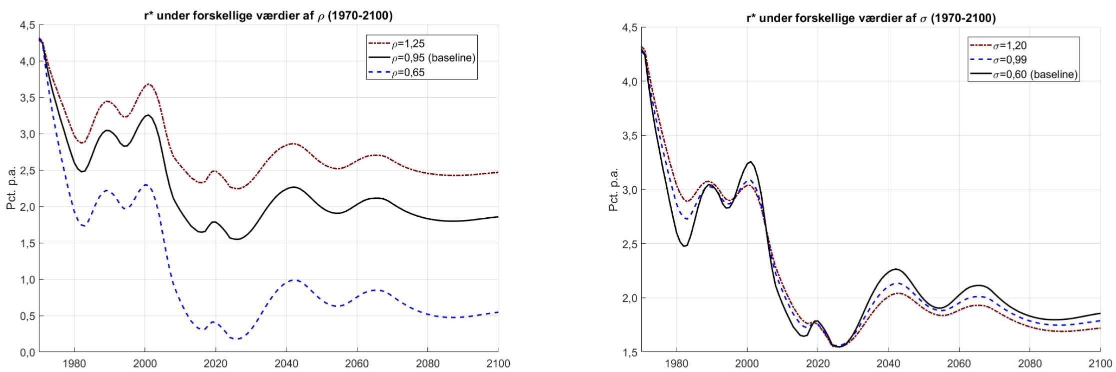
6.7 Robustness

In this section, the sensitivity of the simulated interest rate's development to the assumed values of the elasticity of substitution for consumption (ρ) and production (σ), the human capital profile

(hc_j), and the retirement age is examined.

Figure 18 shows the simulated interest rate under different parameter values for the intertemporal elasticity of substitution for consumption, given by ρ, and the elasticity of substitution between capital and labor in production, given by σ. In all cases, the model is recalibrated to match the same moments as described in Section 5.3. The natural real interest rate declines regardless of the values of the parameters ρ and σ. However, the degree of the decline in the interest rate is sensitive to the value of ρ, as a lower intertemporal elasticity of substitution intensifies the decline, whereas the opposite is true for a higher parameter value of ρ. This occurs because a lower intertemporal elasticity of substitution, all else equal, strengthens the income effect relative to the substitution effect when the interest rate changes.[54] Even when $\rho = 0.65$, the interest rate does not become negative. However, it cannot be ruled out that an even lower parameter value of ρ would result in a negative natural real interest rate between 2010-2025 or in steady state.[55]

Figure 18: $r^{\star}$: Different values of ρ and σ



Note: The left panel shows the simulated interest rate under different values of ρ. The right panel illustrates the development of the interest rate under different values of σ. In all simulations, the model is recalibrated to match the same moments. Source: Own contribution.

In contrast, the development of the simulated interest rate is relatively robust to different values of σ. However, a higher value of σ appears to have a smoothing effect on fluctuations in the interest rate, including the magnitude of the turning points in 2025 and 2055. This can be explained by the fact that a higher elasticity of substitution between production factors makes it easier to substitute between labor and capital when one input becomes relatively more expensive or cheaper than the other (Papetti, 2021a). This means that changes in the labor supply have a smaller effect on the marginal product of capital, thereby making the interest rate less sensitive to fluctuations in the growth rate of the labor force.[56]

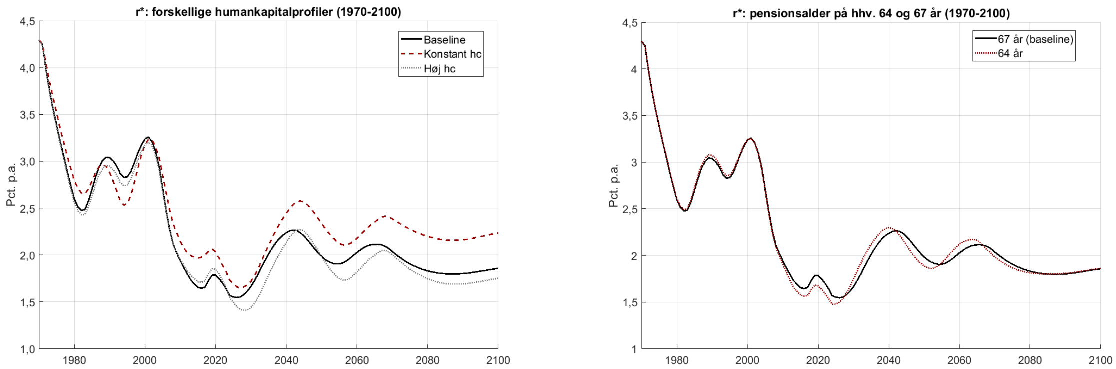
[54] See Appendix A.

[55] The model could not be solved when $\rho < 0.60$.

[56] When the interest rate becomes marginally lower in the long run in Figure 18 (r.) with a higher value of σ, it is because the parameter α is recalibrated to a lower value.

Figure 19 shows the development of the natural interest rate under different assumptions about the human capital profile (left) and the retirement age (right). Since the labor supply is influenced both by the number of individuals of working age and by the human capital profile (cf. (12) and Figure 8), the latter can be expected to have an independent effect on the development of the natural interest rate. Two different scenarios for the human capital profile are tested and compared with the baseline: one where human capital remains constant across all ages, and another where human capital does not decline after the age of 55. In the baseline, human capital increases up to the age of 40, then stagnates, and declines after the age of 55 (see Figure A12).

Figure 19: $r^{\star}$: Human capital and retirement age



Note: The left panel shows the development of $r^{\star}$ under different distributions of the human capital profile. The right panel shows the development of $r^{\star}$ with a retirement age of 64 years and 67 years (baseline). In all simulations, the model is recalibrated to match the same moments. Source: Own contribution.

In the scenario where human capital is constant across all ages, such that $hc_j = 1$ for all $j < 43$, the interest rate remains higher than the baseline for most of the period, particularly in the long term. By 2100, the interest rate is 38 bp higher. When the human capital profile is constant, the downward pressure on the interest rate in the long term is reduced. As the labor force ages, a decline in human capital for older households in the labor force corresponds to a lower growth rate of the labor force, which, via mechanism 1, puts downward pressure on the interest rate. This effect does not occur when human capital is constant across all ages. The opposite effect arises in the 1990s, when the labor force temporarily becomes younger as the first cohorts born to the baby boomers enter the workforce. A constant human capital profile also means that younger households have less ability to save since they do not experience any income growth. Similarly, older households have less incentive to save as their labor income does not decline.

In the second scenario, human capital remains constant after age 55 ("høj hc" in the figure), meaning that labor income does not decline for older households. It is evident that the interest rate is marginally lower in the long run (11 bp). This is because older households are able to save more before retiring. This increases the stock of savings, leading to a lower interest rate as capital per worker rises. The limited magnitude of this effect is due to the fact that higher human capital simultaneously increases labor supply, which, via mechanism 1, raises the marginal product of capital and thus the interest rate. This demonstrates that any economic policies aimed at increasing productivity or labor force participation among older individuals will have only a modest, but still negative, effect on the natural interest rate. However, this might not hold if the retirement age is also increased. If a rising retirement age is combined with increasing productivity among older individuals—e.g., in the form of "healthy aging"—it effectively neutralizes the impact of increased life expectancy on agents' saving behavior. In such a case, the natural interest rate would rise as mechanism 2 is muted. A higher retirement age combined with constant or increasing human capital for older individuals would also raise the interest rate via mechanism 1, as it increases labor supply (Papetti, 2021a).

The right panel of Figure 19 shows the simulated interest rate when the retirement age is assumed to be 64 years instead of 67 years. The human capital profile for working households remains unchanged. Given that many Americans retire before the age of 67 (Munnell, 2015), and that it is possible to claim reduced Social Security benefits starting at age 62 in the U.S. (SSA, 2024a), a lower retirement age might be more representative. The findings show that a lower retirement age has a negligible effect on the interest rate in the model. In the long term, there is no difference. However, the turning points around 2025 and 2055 occur a few years earlier. The limited impact on the interest rate is due to the presence of offsetting effects associated with changes in the retirement age. On the one hand, a lower retirement age increases pension expenditures, which, via mechanism 4, puts upward pressure on the natural interest rate. On the other hand, agents spend less time in the labor market and more time in retirement, which, via mechanism 2, exerts downward pressure on the interest rate. A lower retirement age also reduces the labor force, which puts downward pressure on the interest rate via mechanism 1. The opposite effects would occur if the retirement age were increased. If the retirement age is changed without recalibrating the model, I find that an increase in the retirement age by one year raises the natural interest rate by approximately 7 bp in steady state (see Figure A5).

6.8 Alternative projections

This section examines the development of the natural interest rate under different assumptions regarding fertility and life expectancy trends. Three alternative scenarios are presented. In the first alternative scenario, life expectancy is assumed to continue rising toward 2100, based on the UN's latest projections. The fertility rate is assumed to remain at 1.64, as in the baseline, which roughly corresponds to the UN's medium scenario, cf. Section 5.

In the second scenario, mortality is assumed to remain constant at its 2024 level, while the fertility rate is assumed to decline to 1.15 children per woman by 2045, corresponding to the UN's low scenario.

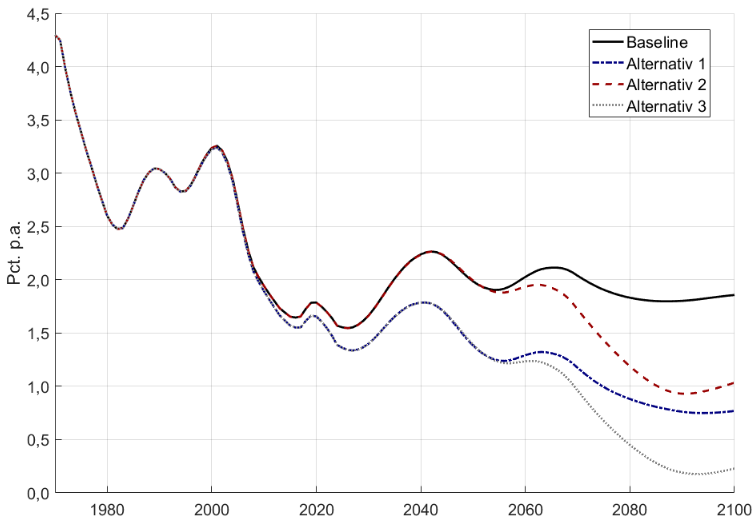
In the third scenario, both mortality and fertility are assumed to decline. Fertility is assumed to decrease to 1.30 children per woman by 2045, roughly corresponding to the lower bound of the UN's 80% confidence interval for projected fertility. No scenario is tested where fertility increases, as this seems implausible given recent historical trends.[57] The UN has also received criticism for overestimating fertility trends in several countries (Bricker and Ibbitson, 2019, pp. 44-54).[58] The results are shown in Table 5 and Figure 20.

Table 5: Alternative projections

	Baseline	Alternative 1	Alternative 2	Alternative 3
$\Delta r^\star$ 2024-2100	0.29	-0.62	-0.53	-1.16
Fertility after 2045	1.64	1.64	1.15	1.30
Life expectancy in 2100	79.5	89.2	79.5	89.2

Note: Changes in $r^\star$ are expressed in percentage points. Fertility is expressed in children per woman. Life expectancy is expressed in years for newborns. Figures are rounded to two decimal places. Source: UN (2024) and own contribution.

Figure 20: $r^\star$: Alternative projections



Note: The figure shows the development of $r^\star$ under the alternative demographic projections described in Table 5. Source: Own contribution.

In all three alternative scenarios, the natural rate of interest declines between 2024 and 2100. This is a noteworthy result, as the baseline model predicts a marginal increase in the natural rate of interest in the long run due to rising pension expenditures, cf. Section 6.1. In the third scenario, the rate drops by 1.16 pp, nearing zero by 2100. Consistent with the findings in Section

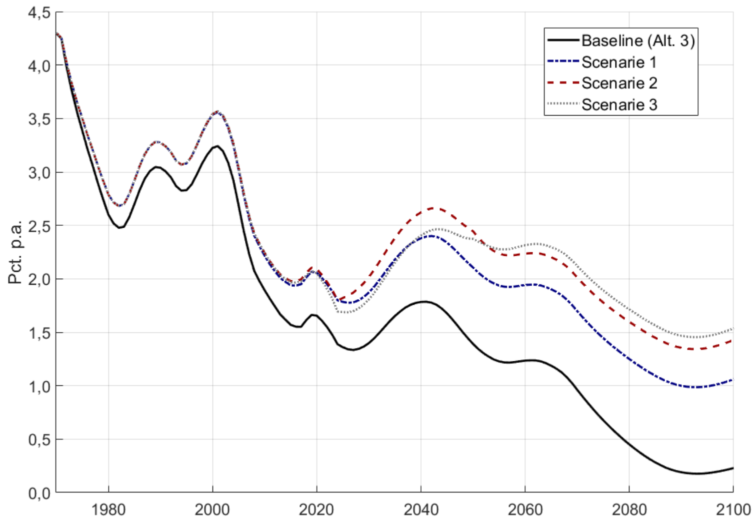
[57]See Figure 1a

[58]In the case of the U.S., this is evident from the UN revising its long-term fertility estimate from 1.72 in its 2022 projections to 1.65 in its 2024 projections (UN, 2024).

6.2, it is evident that an increasing life expectancy or declining fertility leads to a lower natural rate of interest, as mechanisms 1 and 2 outweigh mechanisms 3 and 4. It is also notable that the rate falls below the baseline after 2010 in the alternative scenarios where life expectancy continues to rise. This result stems from the assumption that households have perfect foresight regarding the evolution of exogenous processes, including life expectancy. A temporary rebound in the rate still occurs after 2025 in all three alternative scenarios.

It follows that the rise in the rate observed in the scenarios for G&P's hypothesis presented in Section 6.6.1 might be less pronounced if life expectancy increases or fertility declines further. Therefore, G&P's hypothesis is re-evaluated by simulating the interest rate under the three scenarios described in Section 6.6.1, using the alternative demographic projection where both mortality and fertility decline, as outlined in Table 5 (Alternative 3). The results are presented in Figure 21.

Figure 21: $r^\star$: G&P's scenarios under Alternative 3



Note: The figure shows the development in $r^\star$ in Alternative 3 described in Table 5 under the scenarios described in Table 4. Change in $r^\star$ in basis points between 2024-2040: 40 (Baseline), 58 (Scenario 1), 83 (Scenario 2), 71 (Scenario 3). Source: Own contribution.

It is evident that the natural rate of interest declines in all scenarios over the long term. Even in the third scenario, the rate in 2100 is lower than in 2024 (by 16 bp). G&P's hypothesis—that the natural real interest rate will rise and settle at a permanently higher level—is thus rejected under the assumptions that life expectancy continues to increase and fertility maintains its downward trend. This outcome arises despite the fact that a further rise in life expectancy would, all else equal, increase expenditures for the elderly.

However, the alternative projections do not change the fact that the turning point in the interest rate between 2025 and 2042 becomes more pronounced under all three scenarios. Under the alternative projection, the interest rate increases by 40 bp between 2024 and 2040 in the baseline scenario, while it rises by 71 bp in Scenario 3—a difference of 31 bp. Therefore, in the medium term, it cannot be ruled out that G&P's hypothesis will materialize.

7 Discussion

This section discusses the implications and validity of the results from the analysis. Additionally, the limitations of the model are addressed, along with aspects that could be of research interest moving forward.

7.1 Is a turning point imminent?

In the model, a turning point in the natural real interest rate occurs after 2025, continuing until 2042. A similar pattern is observed, to a lesser extent, after 2055. This result is consistent with parts of the related literature that employ similar models (cf. Papetti, 2021a; Eggertsson et al., 2019a; Bielecki et al., 2020). These fluctuations in the interest rate arise in the model due to demographic echo effects of the baby boomer generation and their descendants. When these echo effects influence the natural real interest rate, it is primarily because they cause fluctuations in the growth rate of the labor force, which affects investment demand, cf. mechanism 1. The effect of retired households dissaving, cf. mechanism 3, cannot be the dominant driver behind the turning points since both the investment and savings rates increase after 2025 and 2055, cf. Figure 11 and Figure 12.

In the model, the first cohorts of the baby boomer generation exit the labor market in 2012, putting downward pressure on the labor force growth rate between 2012 and 2025, cf. Figure 8. This reflects that cohorts born after 1945 entered the economy after 1970 and retired at the age of 67, 42 years later. In reality, this downward pressure likely occurred a few years earlier, as many retire before reaching 67. If so, the development in several model variables, including the real interest rate, labor supply growth rate, and GDP growth rate, aligns more closely with the empirical trends during the financial crisis. Empirically, the U.S. labor force growth rate fell sharply in 2007 (see Figure A20).

It is plausible that older cohorts nearing retirement chose to exit the labor force earlier than planned due to the financial crisis. If so, it is likely that the U.S. recession, and the global downturn, after 2008 was exacerbated by the large cohorts born after World War II leaving the labor market, reducing aggregate demand. This put downward pressure on the natural interest rate during the 2010s, possibly contributing to the low real interest rates observed during this period. Similarly, the labor force growth rate dropped sharply in 2020 during the COVID-19 pandemic. It is also plausible that older cohorts close to retirement exited the labor force early. Thus, it is likely that the dynamics between changes in the labor force and the natural interest rate observed in the model have also occurred in practice.

The turning point in the model after 2025 follows from most of the baby boomer households retiring, easing the downward pressure on the labor force growth rate—and, by extension, the natural interest rate. Simultaneously, their descendants will reach working age, contributing to an increase in the labor force growth rate. Section 6.2.1 showed that the baby boom alone reduced the natural interest rate by 65 basis points between 1970 and 2024. This reflects the size of the rebound between 2025 and 2042, which is 68 bp in the baseline.

However, the model's turning points are subject to uncertainty, depending on how well the model captures actual demographic developments. For instance, the model assumes a fixed childbearing age and excludes migration.[59] The assumption that agents enter the labor market at age 25 may also not be accurate. However, this must be weighed against the fact that the model does not include the dependency burden of children and youth.

Figure 5 shows that fluctuations in the dependency ratio are more pronounced in the model than in reality. Consequently, the turning points resulting from demographic developments in the model are likely to be less pronounced in reality. Nevertheless, this suggests that over the next 20 years, there is at least a prospect of a slowdown in the downward trend in the natural interest rate observed since the 1970s. It is thus of interest whether this trend will continue or abate. The main results indicate that the natural interest rate will stabilize around current levels in the long term, despite the rising dependency ratio (see Figure 7). However, this result is sensitive to the assumed demographic projections. If fertility rates continue to decline or life expectancy rises further, the natural interest rate will, according to the model, decline further in the long term (see Figure 20).

Nonetheless, the model shows that rising public expenditures increase the natural interest rate, thereby potentially intensifying the turning point after 2025. It seems likely that public expenditures will grow due to rising costs associated with an aging population. Overall, the analysis suggests that a modest rebound in the natural real interest rate in the U.S. is expected toward the 2040s. However, the increase is unlikely to exceed 50 bp. In any case, this implies that U.S. monetary policy rates, which were raised in 2022–2023 to combat inflation, are unlikely to return to the low levels observed before the COVID-19 pandemic. A similar trend is expected in the Eurozone, where the trajectory of the natural interest rate closely follows that of the U.S. (see Figure 3a).

7.2 Evaluation of G&P's hypothesis

The hypothesis by Goodhart and Pradhan, which suggests that the natural real interest rate is approaching a turning point, can, within the framework of the model, be translated into mechanisms 3 and 4 exerting upward pressure on the interest rate. However, this is initially dismissed, as mechanisms 1 and 2 generally prove to be dominant. This result corroborates related literature and confirms the theoretical insights presented in section 3. Similarly, the turning point after 2025 is driven by mechanism 1 temporarily reversing direction as the growth rate of the labor force increases, rather than by older households dissaving. G&P's argument

[59] In 1970, the median childbearing age in the U.S. was 26 years. In 2023, it was 30 years (UN, 2024).

that a decline in the savings rate will prompt an increase in the natural interest rate, as per mechanism 3, is generally rejected. However, the analysis does align with G&P in observing that the baby boomer generation has exerted a unique downward pressure on the natural interest rate since 1990 and that this pressure has diminished. In that sense, their hypothesis can also be described through mechanism 1. Nevertheless, this alone is insufficient to generate a sustained increase in the natural real interest rate.

G&P's hypothesis becomes more plausible if it is assumed that public expenditures will rise. Here, the model shows that higher public expenditures increase the natural interest rate, especially if healthcare costs are included in the model. This holds regardless of whether the expenditures are financed through income taxes or public debt issuance, as both place upward pressure on the natural interest rate by crowding out the private sector's ability to accumulate productive capital. In Scenario 3, where public expenditures rise the most—and which aligns most closely with projections for these expenditures—the natural interest rate is over 1 pp higher in 2100 compared to 2024.

However, if both mortality and fertility rates decline further, the natural interest rate will still decrease in the long run according to the model, even under the most generous scenario for increasing public expenditures (see figure 21). This is primarily because agents respond to higher life expectancy by saving more (cf. mechanism 2), and also because demand declines as the labor force shrinks (cf. mechanism 1). Regarding the former, G&P are critical of the theoretical construction of conventional life-cycle models, which they argue fail to adequately capture household behavior (Goodhart and Pradhan, 2020, pp. 77-79). To the extent that their critique is valid, their hypothesis appears more plausible, as the analysis finds—consistent with related studies—that life expectancy is the strongest channel through which aging affects the natural interest rate. Nonetheless, the turning point in the natural interest rate after 2025 is intensified in simulations of G&P's hypothesis, regardless of the demographic projection used. If both mortality and fertility rates decline further, the interest rate still rises by 18–43 bp more between 2024 and 2040 compared to the baseline (see figure 21). This suggests that upward pressure on the natural real interest rate is likely in the medium term. To the extent that the turning point in the interest rate after 2025 in the baseline has a positive bias, 20–40 bp may thus be a more conservative estimate of the rebound's magnitude.

The analysis also confirms that an increase in the labor share raises the natural interest rate. In the model, a higher labor share is achieved by assuming a reduction in market concentration and thereby the profit rate. Here, G&P's prediction appears more speculative and cannot be verified within the model's framework, as the parameter for market concentration is given exogenously. As of now, there are no immediate signs that the labor share in the U.S. will reverse its downward trend, as it was historically low in 2024 (see figure 6).

Overall, the model shows that a lower growth rate of the labor force places downward pressure on the natural interest rate by reducing investment demand (cf. mechanism 1), while the effect on savings supply is negligible (cf. mechanism 3). The model therefore unequivocally rejects the notion that a shrinking labor force alone could increase the natural interest rate.

7.3 Open economy considerations

The model used in this thesis is limited by its assumption of the United States as a closed economy, which prevents it from capturing the influence of global factors. However, it is doubtful that extending the model to an open economy would have produced substantially different results. The reason is that demographic trends in the U.S. have been comparable to those in other advanced economies, which have undergone a similar aging process (see Figure 1b and Figure A15). This has contributed to the decline in the natural real interest rate in other developed countries as well (see Figure 3a and Figure A14).

Studies employing life-cycle models to analyze developments in the natural interest rate generally find that whether the economy is assumed to be closed or open has only a limited effect on the results (Krueger and Ludwig, 2007; Bielecki et al., 2020). For the U.S., however, there is evidence that the natural interest rate would have been higher if the economy were closed, as the American population is younger than in other developed countries (Krueger and Ludwig, 2007; Rachel and Summers, 2019). This may help explain why the thesis's baseline estimate of the natural real interest rate in 2024 is higher than that of HLW (cf. Figure 7).

In theory, an aging and open economy should increase its net foreign assets as higher savings are invested in younger economies with higher natural interest rates. However, this has not been the case for the U.S., which has run a negative current account balance since the 1980s (see Figure A17) alongside a stable gross investment rate (see Figure A16). This does not contradict the idea that the natural interest rate in the U.S. has declined due to aging, as other developed countries have also increased their savings due to demographic trends. However, the scale of the U.S.'s decline in net foreign assets is such that it is unlikely to be explained solely by other advanced economies investing their savings surplus in the U.S.

Instead, it is necessary to consider that emerging economies, most notably China, have also accumulated large savings surpluses, which have been primarily invested in the U.S. since the 1990s. This phenomenon was described as a global "savings glut" by the Chair of the U.S. Federal Reserve in 2005 (Bernanke, 2005) and represents a key pillar of G&P's hypothesis (Goodhart and Pradhan, 2020). This global savings surplus has exerted downward pressure on the natural interest rate—both in the U.S. and globally—which cannot be explained by the aging populations of advanced economies. In the U.S., this has resulted in a negative current account balance while supporting stable domestic gross investment since 1970, even as household savings rates have declined (see Figure A16).

The thesis finds that aging exerts downward pressure on equilibrium investment and savings rates (see Section 6.3) due to a decline in investment demand caused by a lower growth rate of the labor force. The fact that this is only partially reflected in empirical data can be attributed to the U.S.'s status as a debtor nation, with foreign investment in the U.S. exceeding American investment abroad. In an open economy, investments do not need to be entirely financed by domestic savings.

Since quantitative life-cycle models extended to the open economy often treat the rest of the world as an average of other advanced economies, such as the OECD (Krueger and Ludwig, 2007; Bielecki et al., 2020), they fail to capture the effect of this global savings surplus, which primarily originated from China and other non-advanced economies. A promising direction for future research would be to develop a quantitative model with heterogeneous economies characterized by different stages of economic and demographic development.[60]

G&P argue that the savings surplus from emerging economies is likely to cease, which would push the natural interest rate upward in the West and globally. Even if this were to occur, however, it is unclear whether this would exert upward or downward pressure on the global natural interest rate. The aging populations in emerging economies would, according to this thesis's results, also lead to a decline in their natural interest rates.[61] If so, this could potentially alleviate the domestic upward pressure on the natural interest rate in the U.S. predicted by this thesis.

7.4 Future perspectives

To the extent that rising public expenditures can increase the natural interest rate, developments in this area are a highly relevant factor. Given that debt-to-GDP ratios and budget deficits are historically high in many advanced economies, particularly in the United States, this supports the outlook that the natural interest rate will rise or, at the very least, not decline further in the medium term.[62]

If there are also prospects for productivity increases, for instance, driven by artificial intelligence, this would further strengthen the case for a rising natural interest rate. Conversely, the natural interest rate would decline if productivity growth were to slow. It is thus left to other studies to investigate in greater depth the relationship between aging, productivity, and the natural interest rate. While aging in advanced economies has historically coincided with declining productivity growth (Aksoy et al., 2019; Papetti, 2021a), it is not a given that this relationship will persist in the future. This could change, for example, if labor shortages incentivize increased investment in automation (Acemoglu and Restrepo, 2017). Furthermore, there is ongoing debate about whether productivity growth affects the natural interest rate at all (Hamilton et al., 2016). In a neoclassical model, however, as employed in this thesis, this relationship is intrinsic (see Section 3).

This thesis also finds that the natural real interest rate remains positive across all specifica-

[60]See, e.g., Auclert et al. (2024).

[61]Papetti (2021b) estimates that China's current account balance will turn negative after 2030, though its net foreign assets will remain positive. He also argues that aging in India will eventually exert downward pressure on the global natural interest rate.

[62]See Figure A18 in Appendix D.

tions and scenarios, consistently rejecting the notion that the U.S. economy has been, or is likely to be, in a state of structural secular stagnation. This finding has significant monetary policy implications, as it challenges the theoretical basis for unconventional monetary policy measures such as negative interest rates and quantitative easing, which were justified by claims that the natural real interest rate was negative—such as in the Eurozone and Denmark (Adolfsen and Pedersen, 2019). However, the interest rate level in the model should be interpreted with caution, as it is sensitive to the calibration of parameters. The interest rate level was calibrated based on HLW, which is the Federal Reserve's preferred source. However, these estimates are associated with wide confidence intervals and have been subject to criticism (Buncic, 2020; Goodhart and Pradhan, 2020).

Since the analysis is limited to the U.S., this thesis cannot rule out the possibility that other advanced economies have experienced secular stagnation. Nonetheless, the uncertainty associated with estimating the level of the natural interest rate should, in itself, prompt debate, as conducting monetary policy interventions based on an uncertain and potentially flawed foundation could have significant economic consequences. While the natural interest rate as a concept has been the primary focus of this thesis, it is nevertheless acknowledged that the practical applicability of this concept may be limited.

8 Conclusion

This thesis investigated how aging affects the natural real interest rate. By simulating a quantitative overlapping generations (OLG) model with 74 generations, the thesis finds that the natural real interest rate in the U.S. has declined by over 1.5 percentage points between 1970 and 2024 due to aging. The primary drivers are an increase in life expectancy, which incentivizes households to save more for retirement, and a decrease in population growth, which reduces investment demand.

The analysis also finds that the baby boomer generation has exerted a distinct downward pressure on the natural interest rate since the 1990s—amounting to over 60 basis points since 1970—due to large cohorts saving for retirement and gradually exiting the labor force. As this downward pressure is expected to diminish, the thesis projects a transitory rebound for the natural interest rate after 2025, extending toward 2042, as the growth rate of labor supply temporarily increases, boosting investment demand. At the same time, expenditures on pensions and healthcare for the elderly will continue to rise, exerting upward pressure on the natural interest rate, regardless of whether these costs are financed through tax increases or government debt issuance. While the magnitude of this turning point is likely to be limited, it will nonetheless have implications for both fiscal and monetary policy, as the prospect of a period with slightly rising rather than declining real interest rates is itself significant. Given the size of the U.S. economy, an increase in its natural interest rate will also have consequences for the rest of the world.

In the long term, however, the analysis finds that the natural interest rate will resume its downward trend if life expectancy continues to rise or if fertility rates remain in decline. Increasing public expenditures could counteract this downward pressure from aging, making the management of these fiscal pressures a crucial determinant of future interest rate developments. The findings thus partially support Goodhart and Pradhan's hypothesis that the natural interest rate will rise but reject the notion that this upward pressure is driven by a decline in the savings rate. It is also rejected that demographic factors alone could lead to a sustained increase in the natural interest rate in the long run.

While the thesis confirms the existing literature's conclusion that aging generally leads to a lower natural interest rate, it also emphasizes the importance of accounting for heterogeneous demographic structures and endogenous effects on public finances. In doing so, the thesis contributes to a more nuanced understanding of the paradigm that demographic developments necessarily lead to declining interest rates and offers a more differentiated perspective on the macroeconomic implications of aging.

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Appendix A: Derivations for simple OLG model

A young household born in period t maximized the utility function:

$$U_t = \frac{(c_{t,0})^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}} + s_t\beta \cdot \frac{(c_{t+1,1})^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}},$$

under the budget constraints:

$$c_{t,0} + a_{t+1,1} = (1-\tau_t)w_t \tag{A.1}$$

$$c_{t+1,1} = \frac{a_{t+1,1}(1+r_{t+1})}{s_t} + d_{t+1}. \tag{A.2}$$

The household's optimization problem can then be derived by inserting the budget constraints in $c_{t,0}$ and $c_{t+1,1}$ and in the utility function:

$$U_t = \frac{1}{1-\frac{1}{\rho}}\left[((1-\tau_t)w_t - a_{t+1,1})^{1-\frac{1}{\rho}} + s_t\beta\left(\frac{a_{t+1,1}(1+r_{t+1})}{s_t} + d_{t+1}\right)^{1-\frac{1}{\rho}}\right].$$

The first-order condition is found by taking the derivative w.r.t. $a_{t+1,1}$:

$$\frac{\partial U_t}{\partial a_{t+1,1}} = -((1-\tau_t)w_t - a_{t+1,1})^{\frac{-1}{\rho}} + s_t\beta\left(\frac{a_{t+1,1}(1+r_{t+1})}{s_t} + d_{t+1}\right)^{\frac{-1}{\rho}} \cdot \frac{1+r_{t+1}}{s_t}.$$

The expressions for $c_{t,0}$ and $c_{t+1,1}$ are reinserted:

$$\frac{\partial U_t}{\partial a_{t+1,1}} = -(c_{t,0})^{\frac{-1}{\rho}} + s_t\beta\,(c_{t+1,1}))^{\frac{-1}{\rho}} \cdot \frac{1+r_{t+1}}{s_t}.$$

The first-order condition is set equal to zero to derive the Euler equation:

$$\begin{aligned}
\frac{\partial U_t}{\partial a_{t+1,1}} &= 0 \\
&\Leftrightarrow (c_{t,0})^{\frac{-1}{\rho}} = s_t\beta\,(c_{t+1,1})^{\frac{-1}{\rho}} \cdot \frac{1+r_{t+1}}{s_t} \\
&\Leftrightarrow \left(\frac{c_{t+1,1}}{c_{t,0}}\right)^{\frac{1}{\rho}} = \beta(1+r_{t+1}) \\
&\Leftrightarrow (c_{t+1,1})^{\frac{1}{\rho}} = \beta(1+r_{t+1})(c_{t,0})^{\frac{1}{\rho}} \\
&\Leftrightarrow c_{t+1,1} = \beta^{\rho}(1+r_{t+1})^{\rho}c_{t,0}.
\end{aligned} \tag{A.3}$$

The production is Cobb-Douglas and is given by:

$$Y_t = K_t^{\alpha}(A_tL_t)^{1-\alpha},$$

whereby the factor prices are given by the first-order conditions w.r.t. capital and labor:

$$\begin{aligned}
r_t + \delta &= \frac{\partial Y_t}{\partial K_t} \\
&\Leftrightarrow r_t + \delta = \alpha K_t^{\alpha-1}(A_tL_t)^{1-\alpha} \\
&\Leftrightarrow r_t + \delta = \alpha\tilde{k}_t^{\alpha-1} \\
&\Leftrightarrow r_t = \alpha\tilde{k}_t^{\alpha-1} - \delta,
\end{aligned} \tag{A.4}$$

and

$$
\begin{aligned}
w_t &= \frac{\partial Y_t}{\partial L_t} \\
\Leftrightarrow w_t &= K_t^{\alpha}(1-\alpha)A_t^{1-\alpha}L_t^{-\alpha} \\
\Leftrightarrow w_t &= (1-\alpha)A_t\left(\frac{K_t}{A_tL_t}\right)^{\alpha} \\
\Leftrightarrow w_t &= (1-\alpha)A_t\tilde{k}_t^{\alpha}, \qquad \text{(A.5)}
\end{aligned}
$$

where $\tilde{k}_t = \frac{K_t}{A_tL_t}$.

The pension benefit is given by:

$$
d_{t+1} = \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{s_t} \qquad \text{(A.6)}
$$

$$
d_t = \bar{d}w_t(1-\tau_t). \qquad \text{(A.7)}
$$

Capital accumulation is given by:

$$
I_t = K_{t+1} - (1-\delta)K_t
$$

$$
K_{t+1} = N_{t,0}\cdot a_{t+1,1},
$$

where

$$
\begin{aligned}
I_t &= K_{t+1} - (1-\delta)K_t \\
\Leftrightarrow \frac{I_t}{A_tL_t} &= (1+g)(1+n_t)\frac{K_{t+1}}{A_t(1+g)L_t(1+n_t)} - (1-\delta)\frac{K_t}{A_tL_t} \\
\Leftrightarrow \frac{I_t}{A_tL_t} &= (1+g)(1+n_t)\frac{K_{t+1}}{A_{t+1}L_{t+1}} - (1-\delta)\frac{K_t}{A_tL_t} \\
\Leftrightarrow \tilde{i}_t + (1-\delta)\tilde{k}_t &= (1+g)(1+n_t)\tilde{k}_{t+1} \qquad \text{(A.8)} \\
\Leftrightarrow \tilde{i}_t &= (1+g)(1+n_t)\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t, \qquad \text{(A.9)}
\end{aligned}
$$

where $\tilde{i}_t = \frac{I_t}{A_tL_t}$, and

$$
\begin{aligned}
K_{t+1} &= N_{t,0}\cdot a_{t+1,1} = L_t\cdot a_{t+1,1} \\
\Leftrightarrow a_{t+1,1} &= \frac{K_{t+1}}{L_t} \\
\Leftrightarrow \frac{a_{t+1,1}}{A_{t+1}L_{t+1}} &= \frac{\tilde{k}_{t+1}}{L_t} \\
\Leftrightarrow a_{t+1,1} &= \tilde{k}_{t+1}A_{t+1}\frac{L_{t+1}}{L_t} \\
\Leftrightarrow a_{t+1,1} &= (1+n_t)A_{t+1}\tilde{k}_{t+1}. \qquad \text{(A.10)}
\end{aligned}
$$

Households' savings supply

The supply curve of the households' savings can be derived by first solving for $a_{t+1,1}$ in (A.1), inserting this in (A.2) and then employ (A.3):

$$c_{t+1,1} = \frac{((1-\tau_t)w_t - c_{t,0})(1+r_{t+1})}{s_t} + d_{t+1}$$
$$\Leftrightarrow \beta^\rho(1+r_{t+1})^\rho c_{t,0} = \frac{((1-\tau_t)w_t - c_{t,0})(1+r_{t+1})}{s_t} + d_{t+1}.$$

Inserting (A.6) in d_{t+1} and solving for $c_{t,0}$, yields:

$$\begin{aligned}
\beta^\rho(1+r_{t+1})^\rho c_{t,0} &= \frac{((1-\tau_t)w_t - c_{t,0})(1+r_{t+1})}{s_t} + \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{s_t} \\
\Leftrightarrow c_{t,0}\left(\beta^\rho(1+r_{t+1})^\rho + \frac{1+r_{t+1}}{s_t}\right) &= \frac{(1-\tau_t)w_t(1+r_{t+1})}{s_t} + \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{s_t} \\
\Leftrightarrow c_{t,0}\left(\frac{s_t\beta^\rho(1+r_{t+1})^\rho + (1+r_{t+1})}{s_t}\right) &= \frac{(1-\tau_t)w_t(1+r_{t+1})}{s_t} + \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{s_t} \\
\Leftrightarrow c_{t,0}(s_t\beta^\rho(1+r_{t+1})^\rho + (1+r_{t+1})) &= (1-\tau_t)w_t(1+r_{t+1}) + \tau_{t+1}w_{t+1}(1+n_t) \\
\Leftrightarrow c_{t,0}(s_t\beta^\rho(1+r_{t+1})^{\rho-1} + 1) &= (1-\tau_t)w_t + \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{1+r_{t+1}} \\
\Leftrightarrow c_{t,0} &= \frac{(1-\tau_t)w_t}{1+s_t\beta^\rho(1+r_{t+1})^{\rho-1}} + \frac{\tau_{t+1}w_{t+1}(1+n_t)}{(1+r_{t+1})(1+s_t\beta^\rho(1+r_{t+1})^{\rho-1})} \\
\Leftrightarrow c_{t,0} &= \frac{1}{1+s_t\beta^\rho(1+r_{t+1})^{\rho-1}}\left[(1-\tau_t)w_t + \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{1+r_{t+1}}\right]. \qquad \text{(A.11)}
\end{aligned}$$

This expression is inserted in (A.1):

$$\begin{aligned}
a_{t+1,1} &= (1-\tau_t)w_t - c_{t,0} && \text{(A.12)} \\
&= (1-\tau_t)w_t - \frac{1}{1+s_t\beta^\rho(1+r_{t+1})^{\rho-1}}\left[(1-\tau_t)w_t + \tau_{t+1}w_{t+1}\cdot\frac{1+n_t}{1+r_{t+1}}\right] && \text{(A.13)} \\
&= (1-\tau_t)w_t\frac{s_t\beta^\rho(1+r_{t+1})^{\rho-1}}{1+s_t\beta^\rho(1+r_{t+1})^{\rho-1}} - \frac{\tau_{t+1}w_{t+1}(1+n_t)}{(1+r_{t+1})(1+s_t\beta^\rho(1+r_{t+1})^{\rho-1})}. && \text{(A.14)}
\end{aligned}$$

The expression for the wage rate given in (A.5) is inserted:

$$a_{t+1,1} = (1-\tau_t)(1-\alpha)A_t\tilde{k}_t^\alpha \cdot \frac{s_t\beta^\rho(1+r_{t+1})^{\rho-1}}{1+s_t\beta^\rho(1+r_{t+1})^{\rho-1}} - \frac{\tau_{t+1}(1-\alpha)A_{t+1}\tilde{k}_{t+1}^\alpha(1+n_t)}{(1+r_{t+1})(1+s_t\beta^\rho(1+r_{t+1})^{\rho-1})}.$$

From here, (A.10) is inserted on the left hand side, so $\tilde{k}_{t+1}$ can be isolated. To simplify notation, the definition $\vartheta_t \equiv s_t\beta^\rho(1+r_{t+1})^{\rho-1}$ is applied:

$$\begin{aligned}
(1+n_t)A_{t+1}\tilde{k}_{t+1} &= (1-\tau_t)(1-\alpha)A_t\tilde{k}_t^\alpha\cdot\frac{\vartheta}{1+\vartheta} - \frac{\tau_{t+1}(1-\alpha)A_{t+1}\tilde{k}_{t+1}^\alpha(1+n_t)}{(1+r_{t+1})(1+\vartheta)} \\
\Leftrightarrow \tilde{k}_{t+1} &= \frac{(1-\tau_t)(1-\alpha)\tilde{k}_t^\alpha\cdot\vartheta_t}{(1+g)(1+n_t)(1+\vartheta_t)} - \frac{\tau_{t+1}(1-\alpha)\tilde{k}_{t+1}^\alpha}{(1+r_{t+1})(1+\vartheta_t)}. \qquad \text{(A.15)}
\end{aligned}$$

The investment rate, which due to the assumption of a closed economy equals the gross savings rate, is given by (A.8) divided by $\tilde{y}_t = \tilde{k}_t^\alpha$:

$$\iota_t \equiv \frac{I_t}{Y_t} = \frac{\tilde{i}_t}{\tilde{y}_t} = \frac{(1+g)(1+n_t)\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t}{\tilde{k}_t^\alpha}. \qquad \text{(A.16)}$$

Investments, measured in units per effective worker, are expressed by multiplying with $(1+g)(1+n_t)$ and subtracting $(1-\delta)\tilde{k}_t$ from both sides of (A.10). This is then divided by $\tilde{y}_t = \tilde{k}_t^{\alpha}$ to express the investment rate:

$$\begin{aligned}
\tilde{i}_t &= (1+g)(1+n_t)\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t \\
&= \frac{(1-\tau_t)(1-\alpha)\tilde{k}_t^{\alpha}\cdot\vartheta_t}{1+\vartheta_t} - \frac{\tau_{t+1}(1-\alpha)\tilde{k}_{t+1}^{\alpha}(1+g)(1+n_t)}{(1+r_{t+1})(1+\vartheta_t)} - (1-\delta)\tilde{k}_t \\
\Leftrightarrow & \frac{(1+g)(1+n_t)\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t}{\tilde{k}_t^{\alpha}} \\
&= \frac{(1-\tau_t)(1-\alpha)\tilde{k}_t^{\alpha}\cdot\vartheta_t}{\tilde{k}_t^{\alpha}(1+\vartheta_t)} - \frac{\tau_{t+1}(1-\alpha)\tilde{k}_{t+1}^{\alpha}(1+g)(1+n_t)}{\tilde{k}_t^{\alpha}(1+r_{t+1})(1+\vartheta_t)} - \frac{(1-\delta)\tilde{k}_t}{\tilde{k}_t^{\alpha}} \\
\Leftrightarrow \iota_t &= \frac{(1-\tau_t)(1-\alpha)\cdot\vartheta_t}{1+\vartheta_t} - \frac{\tau_{t+1}(1-\alpha)\tilde{k}_{t+1}^{\alpha}(1+g)(1+n_t)}{\tilde{k}_t^{\alpha}(1+r_{t+1})(1+\vartheta_t)} - (1-\delta)\tilde{k}_t^{1-\alpha}. \qquad \text{(A.17)}
\end{aligned}$$

On a balanced growth path, where $\tilde{k}_t = \tilde{k}$ for all t, is the investment rate is given by:

$$\iota = [(1+g)(1+n) - (1-\delta)]\tilde{k}^{1-\alpha}. \qquad \text{(A.18)}$$

By evaluating (A.17) on a balanced growth path, the following is obtained via (A.18):

$$\iota = \frac{(1+g)(1+n)\tilde{k} - (1-\delta)\tilde{k}}{\tilde{k}^{\alpha}} = \frac{(1-\tau)(1-\alpha)\cdot\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)} - (1-\delta)\tilde{k}^{1-\alpha}.$$

From here, first $\tilde{k}$ and then ι are isolated:

$$\begin{aligned}
&\frac{(1+g)(1+n)\tilde{k} - (1-\delta)\tilde{k}}{\tilde{k}^{\alpha}} + (1-\delta)\tilde{k}^{1-\alpha} = \frac{(1-\tau)(1-\alpha)\cdot\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)} \\
&\Leftrightarrow \iota\left(1 + (1-\delta)\tilde{k}^{1-\alpha}\cdot\frac{\tilde{k}^{\alpha}}{(1+g)(1+n)\tilde{k} - (1-\delta)\tilde{k}}\right) = \frac{(1-\tau)(1-\alpha)\cdot\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)} \\
&\Leftrightarrow \iota\left(1 + \frac{(1-\delta)\tilde{k}}{(1+g)(1+n)\tilde{k} - (1-\delta)\tilde{k}}\right) = \frac{(1-\tau)(1-\alpha)\cdot\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)} \\
&\Leftrightarrow \iota\left(1 + \frac{1-\delta}{(1+g)(1+n) - (1-\delta)}\right) = \frac{(1-\tau)(1-\alpha)\cdot\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)} \\
&\Leftrightarrow \iota\left(\frac{(1+g)(1+n)}{(1+g)(1+n) - (1-\delta)}\right) = \frac{(1-\tau)(1-\alpha)\cdot\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)} \\
&\Leftrightarrow \iota = \left(1 - \frac{1-\delta}{(1+g)(1+n)}\right)\left[\frac{(1-\tau)(1-\alpha)\vartheta}{1+\vartheta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)}\right].
\end{aligned}$$

By inserting $\vartheta = s\beta^{\rho}(1+r)^{\rho-1}$, the savings supply curve as function of the interest rate r is given by:

$$\iota^S = \left(1 - \frac{1-\delta}{(1+g)(1+n)}\right)\left[\frac{(1-\tau)(1-\alpha)s\beta^{\rho}(1+r)^{\rho-1}}{1+s\beta^{\rho}(1+r)^{\rho-1}} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+s\beta^{\rho}(1+r)^{\rho-1})}\right]. \qquad \text{(A.19)}$$

It appears from (A.19) that the supply of savings depends positively on an increase in s when the tax rate τ is fixed:

$$\begin{aligned}
\left.\frac{\partial \iota^S}{\partial s}\right|_{\tau=\bar{\tau}} &= \left.\frac{\partial \iota^S}{\partial \vartheta}\right|_{\tau=\bar{\tau}} \\
&= \mathcal{M}\left[\frac{(1-\tau)(1-\alpha)(1+\vartheta)-(1-\tau)(1-\alpha)\vartheta}{(1+\vartheta)^2} - \frac{-(1+r)(1-\alpha)(1+g)(1+n)\tau}{((1+r)(1+\vartheta))^2}\right] \\
&= \mathcal{M}\left[\frac{(1-\tau)(1-\alpha)(1+\vartheta)-(1-\tau)(1-\alpha)\vartheta}{(1+\vartheta)^2} + \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+\vartheta)^2}\right] \\
&= \mathcal{M}\left[\frac{(1+r)(1-\tau)(1+\vartheta)-(1-\tau)\vartheta(1+r)+\tau(1+g)(1+n)}{(1+r)(1+\vartheta)^2}\right](1-\alpha), \qquad \text{(A.20)}
\end{aligned}$$

where $\mathcal{M} \equiv \left(1 - \frac{1-\delta}{(1+g)(1+n)}\right)$. It follows that (A.20) is positive if:

$$\begin{aligned}
&(1-\tau)(1+\vartheta)(1+r)+\tau(1+g)(1+n) > (1-\tau)\vartheta(1+r) \\
&\Leftrightarrow (1-\tau)(1+r)+\tau(1+g)(1+n) > 0,
\end{aligned}$$

which will be the case for realistic parameter values.

Investment demand

The demand curve on the balanced growth path is derived by inserting (A.18) in (A.4). First $\tilde{k}$ is isolated in (A.18):

$$\begin{aligned}
\iota &= [(1+g)(1+n)-(1-\delta)]\tilde{k}^{1-\alpha} \\
\Leftrightarrow \tilde{k}^{1-\alpha} &= \frac{\iota}{(1+g)(1+n)-(1-\delta)} \\
\Leftrightarrow \tilde{k}^{\alpha-1} &= \frac{(1+g)(1+n)-(1-\delta)}{\iota}.
\end{aligned}$$

This is inserted in (A.4) evaluated on the balanced growth path:

$$\begin{aligned}
r = \alpha\tilde{k}^{\alpha-1} - \delta &= \alpha\frac{(1+g)(1+n)-(1-\delta)}{\iota} - \delta \\
\Leftrightarrow r+\delta &= \alpha\frac{(1+g)(1+n)-(1-\delta)}{\iota} \\
\Leftrightarrow \iota^D &= \alpha\frac{(1+g)(1+n)-(1-\delta)}{r+\delta}, \qquad \text{(A.21)}
\end{aligned}$$

where (A.21) is the investment demand expressed as a function of the interest rate.

Government budget constraint

In the simple model, all government revenues and expenditures consist solely of pension benefit payments. The equilibrium tax rate is derived by substituting (A.6) into (A.7). First, τ_{t+1} is isolated in (A.6):

$$\begin{aligned}
d_{t+1} &= \tau_{t+1}w_{t+1}\frac{1+n_t}{s_t} \\
\Leftrightarrow \tau_{t+1} &= \frac{d_{t+1}}{w_{t+1}} \cdot \frac{s_t}{1+n_t}. \qquad \text{(A.22)}
\end{aligned}$$

This expression is then inserted in (A.7) iterated one period forward:

$$\begin{aligned} d_{t+1} &= \bar{d} w_{t+1}(1-\tau_{t+1}) \\ &= \bar{d} w_{t+1}\left(1-\frac{d_{t+1}}{w_{t+1}}\cdot\frac{s_t}{1+n_t}\right) \\ \Leftrightarrow d_{t+1} &= \frac{1+n_t}{1+n_t+\bar{d}s_t}\cdot\bar{d}w_{t+1}. \end{aligned}$$

This expression is inserted in (A.22):

$$\begin{aligned} \tau_{t+1} &= \frac{1+n_t}{1+n_t+\bar{d}s_t}\cdot\bar{d}w_{t+1}\cdot\frac{1}{w_{t+1}}\cdot\frac{s_t}{1+n_t} \\ \Leftrightarrow \tau_{t+1} &= \frac{s_t\bar{d}}{1+n_t+s_t\bar{d}} \\ \Leftrightarrow \tau_{t+1} &= \frac{\bar{d}}{\bar{d}+\frac{1+n_t}{s_t}}. \end{aligned}$$

On the balanced growth path, the tax rate will be equal to

$$\tau = \frac{\bar{d}}{\bar{d}+\frac{1+n}{s}}. \tag{A.23}$$

Equilibrium

An equilibrium in the capital market along the balanced growth path is found by equating (A.21) with (A.19). A unique solution is provided if $\delta = 1$ and $\rho = 1$. In that case, the demand is given by:

$$\iota^D = \alpha\frac{(1+g)(1+n)}{r+1}, \tag{A.24}$$

and the supply curve

$$\iota^S = \frac{(1-\tau)(1-\alpha)s\beta}{1+s\beta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{(1+r)(1+s\beta)}. \tag{A.25}$$

An equilibrium expressed by r is then obtained by setting (A.24) equal to (A.25). First, $(1+r)$ is isolated in (A.24):

$$1+r = \alpha\frac{(1+g)(1+n)}{\iota}.$$

This expression is inserted in (A.25):

$$\begin{aligned} \iota &= \frac{(1-\tau)(1-\alpha)s\beta}{1+s\beta} - \frac{\tau(1-\alpha)(1+g)(1+n)}{\alpha(1+g)(1+n)(1+s\beta)}\cdot\iota \\ &= \frac{(1-\tau)(1-\alpha)s\beta}{1+s\beta} - \frac{\tau(1-\alpha)}{\alpha(1+s\beta)}\cdot\iota \\ &\Leftrightarrow \iota(1+s\beta) = (1-\tau)(1-\alpha)s\beta - \tau\frac{(1-\alpha)}{\alpha}\cdot\iota \\ &\Leftrightarrow \iota\left(1+s\beta+\tau\frac{(1-\alpha)}{\alpha}\right) = (1-\tau)(1-\alpha)s\beta \\ &\Leftrightarrow \iota = \frac{\alpha(1-\tau)(1-\alpha)s\beta}{\alpha+\alpha s\beta+(1-\alpha)\tau} = \frac{s\beta\alpha(1-\alpha)(1-\tau)}{(1+s\beta)\alpha+\tau(1-\alpha)}. \end{aligned}$$

In equilibrium, the interest rate and the investment rate are thereby given by

$$r = \alpha \frac{(1+g)(1+n)}{\iota} - 1 \tag{A.26}$$

$$\iota = \frac{s\beta\alpha(1-\alpha)(1-\tau)}{(1+s\beta)\alpha + \tau(1-\alpha)}. \tag{A.27}$$

It can be shown that the equilibrium investment rate ι in (A.27) depends unambiguously positively on the survival rate s for a given tax rate τ:

$$\left.\frac{\partial \iota}{\partial s}\right|_{\tau=\bar{\tau}} = \frac{\beta\alpha(1-\alpha)(1-\tau)[(1+s\beta)\alpha + \tau(1-\alpha)] - s\beta\alpha(1-\alpha)(1-\tau)\alpha\beta}{[(1+s\beta)\alpha + \tau(1-\alpha)]^2}. \tag{A.28}$$

It holds that (A.28) is positive if:

$$\begin{aligned} \left.\frac{\partial \iota}{\partial s}\right|_{\tau=\bar{\tau}} > 0 &\Leftrightarrow \beta(1-\alpha)(1-\tau)[(1+s\beta)\alpha + \tau(1-\alpha)] > s\beta(1-\alpha)(1-\tau)\alpha\beta \\ &\Leftrightarrow (1+s\beta)\alpha + \tau(1-\alpha) > s\beta\alpha \\ &\Leftrightarrow \alpha + \tau(1-\alpha) > 0, \end{aligned}$$

which always holds for $0 < \alpha < 1$ and $\tau > 0$. It follows that the equilibrium interest rate in (A.26) depends negatively on an increase in s. Furthermore, as shown in (A.27), the equilibrium investment rate does not depend on population growth n for a given tax rate, due to $\delta = 1$ (which is not unrealistic over a 30-year period or longer). This reflects the absence of the effect of lower population growth through mechanism 3 in the stationary equilibrium. Since it has also been demonstrated that the investment rate depends unambiguously positively on an increase in life expectancy (for a given tax rate), it can be concluded that the effect of a higher dependency ratio, as described in mechanism 3, cannot dominate along the balanced growth path.

Intertemporal elasticity of substitution

The intertemporal elasticity of substitution (IES) affects how consumers allocate consumption between periods. The intertemporal elasticity of substitution is determined by the curvature of the utility function (Hall, 1981):

$$IES = -\frac{u'(c_t)}{u''(c_t)c_t}.$$

Given the utility function $u(c_t) = \frac{c_t^{1-\frac{1}{\rho}}}{1-\frac{1}{\rho}}$ it follows that

$$IES = -\frac{c_t^{-\frac{1}{\rho}}}{-\frac{1}{\rho}c_t^{-\frac{1}{\rho}-1}c_t} = \rho. \tag{A.29}$$

The intertemporal elasticity of substitution is thus constant here and determined by the parameter ρ.[63] It follows that the coefficient of relative risk aversion is given by $\frac{1}{\rho}$. A high IES,

[63] Depending on the specification of the utility function, the literature sometimes defines $IES \equiv \sigma = \frac{1}{\rho}$. The utility function specification in this thesis implies that $IES = \rho$, not $\frac{1}{\rho}$. The symbol σ is reserved for the elasticity of substitution in the production function.

corresponding here to a high value of ρ, which reflects low relative risk aversion, indicates that the consumer has a high willingness to postpone consumption if it appears profitable. This leads to the consumer's saving behavior being more sensitive to changes in the interest rate. The opposite holds for a low value of ρ. When $\rho > 1$, the substitution effect will dominate, meaning that an increase in the interest rate will, all else equal, lead the consumer to save more and reduce consumption. Conversely, the income effect will dominate when $\rho < 1$. When $\rho = 1$, the substitution and income effects will offset each other.

In equation (A.11), it is shown that the consumption of a young household in period t is given by:

$$c_{t,0} = \frac{1}{1 + s_t \beta^{\rho}(1 + r_{t+1})^{\rho-1}} \left[(1 - \tau_t) w_t + \tau_{t+1} w_{t+1} \cdot \frac{1 + n_t}{1 + r_{t+1}} \right].$$

By defining the income in period t as $y_t \equiv (1 - \tau_t) w_t$ and the income in period $t + 1$ as $y_{t+1} \equiv \tau_{t+1} w_{t+1}(1 + n_t)$ as well as setting $s_t = 1$, this expression can be rewritten into:

$$c_{t,0} = \frac{1}{1 + \beta^{\rho}(1 + r_{t+1})^{\rho-1}} \left[y_1 + \frac{y_{t+1}}{1 + r_{t+1}} \right]. \tag{A.30}$$

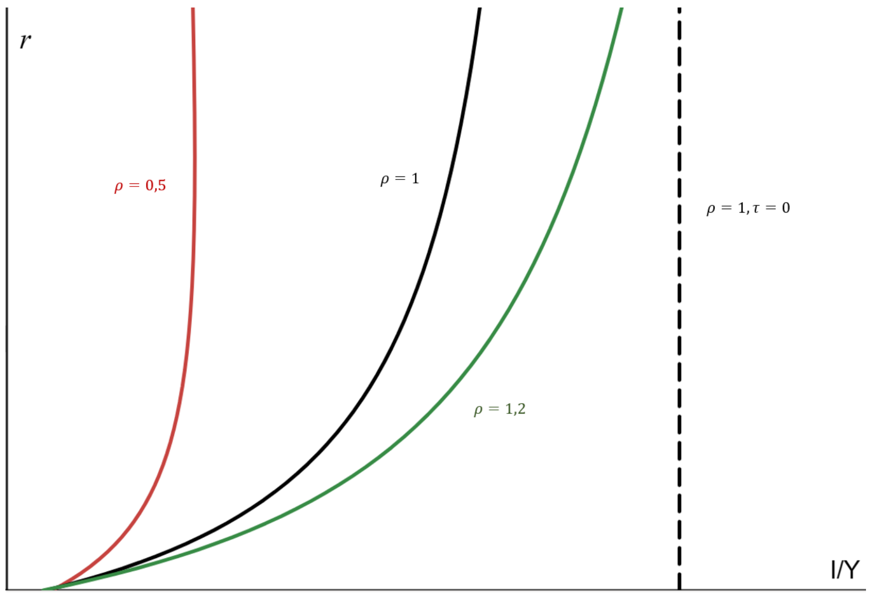
It follows that a change in the interest rate can affect consumption—and thus the savings rate—ambiguously, depending on whether $\rho \gtrless 1$ and the magnitude of future income y_{t+1}. The wealth effect is represented by y_{t+1}, which is discounted by $1 + r_{t+1}$. If $y_{t+1} = 0$, there will be no wealth effect. In this case, an increase in the interest rate will lead to an increase in consumption $c_{t,0}$ when $\rho < 1$, corresponding to the dominance of the income effect. As a result, a rise in the interest rate will lead to a lower savings rate, while a decline in the interest rate will result in a higher savings rate. Conversely, if $\rho > 1$, the substitution effect will dominate. When $\rho = 1$, the substitution and income effects will cancel each other out. In this case, consumption is affected only through the wealth effect, and the expression for $c_{t,0}$ simplifies to:

$$c_{t,0} = \frac{1}{1 + \beta} \left[y_t + \frac{y_{t+1}}{1 + r_{t+1}} \right].$$

It is evident here that an increase in the interest rate leads to a decrease in consumption in period t, as the present value of future income decreases. Conversely, a decline in the interest rate will result in the household choosing to increase its consumption, thereby reducing its savings rate. The wealth effect thus implies that a drop in the interest rate lowers households' savings rate. Since the wealth effect aligns with the substitution effect, it is plausible that a decline in the interest rate will, overall, lead to a lower savings rate if $y_{t+1} > 0$, even when $\rho < 1$. This can be illustrated graphically by showing that the supply curve for households' gross savings rate has a positive slope in an ι, r-diagram, cf. Figure A1.

Figure A1 shows that the slope of the supply curve is positive even when $\rho < 1$, which is due to the wealth effect. In the simple OLG model, the supply becomes perfectly inelastic if $\tau = 0$ and $\rho = 1$, as this implies households do not receive pensions, thereby neutralizing the wealth effect. In that case, an increase in the supply of savings would inevitably lead to a higher equilibrium investment ratio, even if investment demand decreases. However, this would not be

Figure A1: Savings supply curves for different values of ρ



Note: The figure shows households' supply of gross savings as given in (A.19) for different values of ρ. The parameters are set to the same values as in the simple model at Equilibrium 1 (see Table 1). Source: Own contribution.

the case in the extended model with multiple generations, as a wealth effect would still exist without tax-funded pensions. This can be exemplified in the simple OLG model by noting that future income y_{t+1} in (A.30) encompasses not only pension payments but also future labor and capital income. This helps explain why the investment ratio also decreases in the model without pensions (see Figure A4).

Appendix B: Derivations for quantitative OLG model

Households

A household aged $j = 1$ entering the economy in period t maximizes the utility function:

$$\max_{\{c_{j,t+j-1}, x_{j,t+j-1}\}} U_t = \left(\sum_{j=1}^{J} su_j \beta^{j-1} u\left(c_{j,t+j-1}\right) \right) + su_J \beta^{J-1} \mu \cdot v(x_{j,t+j-1}) \tag{A.31}$$

subject to the budget constraint

$$\begin{aligned} & c_{j,t} + a_{j+1,t+1} + \Gamma_{t-j+26} \cdot \Gamma_{t-j+1} \cdot x_{j,t} \\ & \quad = (1-\tau_t) w_t hc_j + (1-\tau_p) \Pi_{j,t} + d_t w_t \cdot \mathbb{1}\{j > 42\} \\ & \quad + (1+r_t) \left(a_{j,t} + q_{j,t} + \frac{1-s_{j-1}}{s_{j-1}} a_{j,t} \right), \end{aligned} \tag{A.32}$$

and the following conditions:

$$a_{j,t} \geq -D_{j,t} \cdot w_{t+1} \cdot hc_{j+1} \tag{A.33}$$

$$c_{j,t} \geq 0 \tag{A.34}$$

$$a_{1,t} = 0 \tag{A.35}$$

$$a_{J+1,t+1} = 0 \tag{A.36}$$

$$q_{j,t} = \frac{N_{J,t-1} x_{J,t-1} \cdot \Gamma_{t-J+1} \cdot \Gamma_{t-J+26}}{N_{j,t}}, \tag{A.37}$$

where:

$$su_j = \prod_{m=1}^{j-1} s_m$$

$$D_{j,t} \geq 0 \quad \text{for } j \leq 42$$

$$D_{j,t} = 0, \quad hc_j = 0, \quad \Pi_{j,t} = 0 \quad \text{for } j > 42$$

$$q_{j,t} = 0 \quad \text{for } j \neq J-49$$

$$x_{j,t} = 0 \quad \text{for } j \neq J.$$

The condition in (A.34) is not binding and can therefore be excluded. By substituting the utility functions given in (13) and (14) as well as the budget constraint in (A.32), the maximization problem under all constraints can be expressed with the Lagrangian:[64]

$$
\begin{aligned}
\max_{\{a_{j+1,t+j},x_{j,t+j-1},\lambda_{j,t+j-1}\}} \mathcal{L}_t = \frac{1}{1-\frac{1}{\rho}}\Bigg\{ & \sum_{j=1}^{J} su_j \cdot \beta^{j-1} \\
& \cdot \Bigg[- a_{j+1,t+j} - \Gamma_{t-j+1} \cdot \Gamma_{t-j+26} \cdot x_{j,t+j-1} + d_t w_t + (1-\tau) w_t h c_j \\
& + (1-\tau_p)\Pi_{j,t+j-1} + (1+r_t) \cdot \left(a_{j,t+j-1} + q_{j,t} + \frac{1-s_{j-1}}{s_{j-1}} a_{j,t+j-1} \right) \Bigg]^{1-\frac{1}{\rho}} \Bigg\} \\
& + \frac{1}{\left(1-\frac{1}{\rho}\right)} \left\{ su_J \cdot \beta^{J-1} \mu \left[x_{J,t+J-1} \right]^{1-\frac{1}{\rho}} \right\} \\
& + \sum_{j=1}^{J} \lambda_{j,t+j-1} \left(a_{j,t+j-1} + D_{j,t+j-1} w_{t+1} h c_{j+1} \right).
\end{aligned}
$$

By taking the derivative with respect to $a_{j+1,t+j}$ and $x_{j,t+j-1}$, while accounting for the KKT conditions (Karush-Kuhn-Tucker), the first-order conditions are obtained:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial a_{j+1,t+j}} = & - su_j \beta^{j-1} \left(c_{j,t+j-1} \right)^{-\frac{1}{\rho}} \\
& + su_{j+1} \beta^{j} \left(c_{j+1,t+j} \right)^{-\frac{1}{\rho}} \frac{(1+r_{t+1})}{s_j} \\
& + \lambda_{j+1,t+j} = 0
\end{aligned} \tag{A.38}
$$

for $\quad j \in \{1, \ldots, J-1\}$,

$$
su_j \beta^{j-1} \left(c_{j,t+j-1} \right)^{-\frac{1}{\rho}} \cdot 0 = 0
$$

for $\quad j \in \{J\}$ and

$$
\frac{\partial \mathcal{L}}{\partial x_{j,t+j-1}} = su_j \beta^{j-1} \left(c_{j,t+j-1} \right)^{-\frac{1}{\rho}} \cdot \left(-\Gamma_{t-j+1} \cdot \Gamma_{t-j+26} \right) + su_j \beta^{j-1} \mu \left(x_{j,t+j-1} \right)^{-\frac{1}{\rho}} = 0 \tag{A.39}
$$

[64] The equation is rewritten so that t becomes $t+j-1$ for $a_{j,t}$ and $x_{j,t}$. It is abstracted from the fact that $\lambda_{j,t+j-1}$ may need to be multiplied by s_{j-1}. This has no implications for the results.

for $j \in \{J\}$. The Euler equation is then obtained from (A.38):

$$\frac{\partial \mathcal{L}}{\partial a_{j+1,t+j}} = -su_j\beta^{j-1}\left(c_{j,t+j-1}\right)^{-\frac{1}{\rho}} + su_{j+1}\beta^{j}\left(c_{j+1,t+j}\right)^{-\frac{1}{\rho}}\frac{(1+r_{t+1})}{s_j}$$

$$+\lambda_{j+1,t+j} = 0$$

$$\Leftrightarrow su_j\beta^{j-1}(c_{j,t+j-1})^{-\frac{1}{\rho}} = su_{j+1}\beta^{j}(c_{j+1,t+j})^{-\frac{1}{\rho}}\frac{(1+r_{t+1})}{s_j}$$

$$+\lambda_{j+1,t+j}$$

$$\Leftrightarrow \beta^{j-1}(c_{j,t+j-1})^{-\frac{1}{\rho}} = \frac{su_{j+1}}{su_j s_j}\beta^{j}(c_{j+1,t+j})^{-\frac{1}{\rho}}(1+r_{t+1})$$

$$+\lambda_{j+1,t+j}\frac{1}{su_j}$$

$$\Leftrightarrow \beta^{j-1}(c_{j,t+j-1})^{-\frac{1}{\rho}} = \frac{s_j}{s_j}\beta^{j}(c_{j+1,t+j})^{-\frac{1}{\rho}}(1+r_{t+1})$$

$$+\lambda_{j+1,t+j}\frac{1}{su_j}$$

$$\Leftrightarrow \frac{1}{\beta} = \left(\frac{c_{j+1,t+j}}{c_{j,t+j-1}}\right)^{-\frac{1}{\rho}}(1+r_{t+1}) + \lambda_{j+1,t+j}\frac{(c_{j,t+j-1})^{\frac{1}{\rho}}}{su_j\beta^j} \tag{A.40}$$

for $j \in \{1,\ldots,J-1\}$. The first-order condition for bequests follows from (A.39) and is given by:

$$\frac{\partial \mathcal{L}}{\partial x_{j,t+j-1}} = 0$$

$$\Leftrightarrow su_j\beta^{j-1}\left(c_{j,t+j-1}\right)^{-\frac{1}{\rho}}\cdot\left(-\Gamma_{t-j+1}\cdot\Gamma_{t-j+26}\right) + su_j\beta^{j-1}\mu\left(x_{j,t+j-1}\right)^{-\frac{1}{\rho}} = 0$$

$$\Leftrightarrow su_j\beta^{j-1}\mu(x_{j,t+j-1})^{-\frac{1}{\rho}} = su_j\beta^{j-1}(c_{j,t+j-1})^{-\frac{1}{\rho}}\cdot\Gamma_{t-j+1}\cdot\Gamma_{t-j+26}$$

$$\Leftrightarrow \mu(x_{j,t+j-1})^{-\frac{1}{\rho}} = (c_{j,t+j-1})^{-\frac{1}{\rho}}\cdot\Gamma_{t-j+1}\cdot\Gamma_{t-j+26}$$

$$\Leftrightarrow x_{J,t+J-1} = \left(\frac{\Gamma_{t-J+1}\cdot\Gamma_{t-J+26}}{\mu}\right)^{-\rho}\cdot c_{J,t+J-1} \tag{A.41}$$

for $J \in \{74\}$. The KKT conditions are given by:

$$\lambda_{j,t+j-1}\left(a_{j,t+j-1} + D_{j,t+j-1}\cdot w_{t+j}hc_{j+1}\right) = 0 \tag{A.42}$$

for $j \in \{1,\ldots,42\}$ and

$$\lambda_{j,t+j-1}\left(a_{j,t+j-1}\right) = 0 \tag{A.43}$$

for $j \in \{43,\ldots,J\}$.

The conditions from (A.33), (A.42) and (A.43) can be summarized in the minimization problems (cf. Crescentini and Giri, 2023; Swarbrick, 2021):

$$\min\left(\lambda_{j,t+j-1}a_{j,t+j-1} + D_{j,t+j-1}\cdot w_{j+1,t+j}hc_{j+1}\right) = 0 \tag{A.44}$$

for $j \in \{1,\ldots,42\}$ and

$$\min\left(\lambda_{j,t+j-1}, a_{j,t+j-1}\right) = 0 \tag{A.45}$$

for $j \in \{43,\ldots,J\}$.

Firms

The firms producing intermediate goods operate under perfect competition. They rent capital K_t from the capital market at the price r_t^k, hire labor L_t at the wage w_t, and sell their output Y_t to monopolistic firms at the real price p_t^{int}/P_t, which they take as given. They maximize the following profit function:

$$\max_{\{K_t,L_t\}} \Pi_t^{\text{int}} = \frac{p_t^{\text{int}}}{P_t} Y_t - w_t L_t - r_t^k K_t$$

subject to the constraint of the production function:

$$Y_t = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

The Lagrangian for the intermediate goods firm is thereby:

$$\max_{\{K_t,L_t\}} \mathcal{L}_t = \frac{p_t^{\text{int}}}{P_t} \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - w_t L_t - r_t^k K_t.$$

By taking the derivative with respect to L_t and K_t, the first-order conditions are obtained:

$$\begin{aligned}
&\frac{\partial \mathcal{L}_t}{\partial L_t} = 0 \\
&\qquad \Leftrightarrow \frac{p_t^{int}}{P_t} \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} (1-\alpha) A_t^{\frac{\sigma-1}{\sigma}} L_t^{\frac{\sigma-1}{\sigma}-1} - w_t = 0 \\
&\qquad \Leftrightarrow w_t = \frac{p_t^{int}}{P_t} \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1-\alpha) A_t^{\frac{\sigma-1}{\sigma}} L_t^{\frac{-1}{\sigma}} \\
&\qquad \Leftrightarrow w_t = \frac{p_t^{int}}{P_t} (1-\alpha)(A_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}}.
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \mathcal{L}_t}{\partial K_t} = 0 \\
&\qquad \Leftrightarrow \frac{p_t^{int}}{P_t} \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \alpha K_t^{\frac{\sigma-1}{\sigma}-1} - r_t^k = 0 \\
&\qquad \Leftrightarrow r_t^k = \frac{p_t^{int}}{P_t} \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \alpha K_t^{\frac{-1}{\sigma}} \\
&\qquad \Leftrightarrow r_t^k = \frac{p_t^{int}}{P_t} \alpha \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}.
\end{aligned}$$

The final goods firm's profit maximization problem is given by:

$$\max \Pi_t^f = \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i), \tag{A.46}$$

subject to the constraint of the the demand function

$$y_t^f = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}. \tag{A.47}$$

Substituting (A.47) into (A.46) yields:

$$\Pi_t^f = \frac{p_t(i)}{P_t} Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta_t} - \frac{p_t^{int}}{P_t} Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta_t}. \tag{A.48}$$

The optimality condition is then derived by taking the derivative with respect to the price:

$$\begin{aligned}
\frac{\partial \Pi_t^f}{\partial \frac{p_t(i)}{P_t}} &= 0 \\
\Leftrightarrow (1-\theta_t)\left(\frac{p_t(i)}{P_t}\right)^{-\theta_t} Y_t + Y_t \theta_t \frac{p_t^{int}}{P_t}\left(\frac{p_t(i)}{P_t}\right)^{-\theta_t - 1} &= 0 \\
\Leftrightarrow \frac{\theta_t - 1}{\theta_t} = \frac{p_t^{int}}{P_t}\frac{P_t}{p_t(i)} & \\
\Leftrightarrow \frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1}\frac{p_t^{int}}{P_t}. &
\end{aligned} \tag{A.49}$$

The nominal price index is given by the weighted geometric mean of the nominal prices of final goods, defined over a continuum of differentiated products in the interval $[0,1]$:

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta_t} di\right)^{\frac{1}{1-\theta_t}}.$$

Since all final goods firms are symmetric and purchase intermediate goods at the same prices, they set identical prices, so that $p_t(i) = P_t$:

$$\begin{aligned}
P_t = \left(\int_0^1 p_t(i)^{1-\theta_t} di\right)^{\frac{1}{1-\theta_t}} & \\
&= \left(p_t(i)^{1-\theta_t}\int_0^1 di\right)^{\frac{1}{1-\theta_t}} \\
&= \left(p_t(i)^{1-\theta_t} \cdot 1\right)^{\frac{1}{1-\theta_t}} \\
&= p_t(i) \\
&\Leftrightarrow P_t = p_t(i).
\end{aligned}$$

It thereby follows from (A.49) that

$$\begin{aligned}
\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1}\frac{p_t^{int}}{P_t} & \\
&\Leftrightarrow 1 = \frac{\theta_t}{\theta_t - 1}\frac{p_t^{int}}{P_t} \\
&\Leftrightarrow \frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t}.
\end{aligned} \tag{A.50}$$

Inserting $\frac{p_t(i)}{P_t} = 1$ and (A.50) into (A.48) yields the aggregate profit:

$$\begin{aligned}\Pi_t = Y_t - \frac{\theta_t - 1}{\theta_t} Y_t \\ &= Y_t \left(1 - \frac{\theta_t - 1}{\theta_t}\right) \\ &= Y_t \left(\frac{1}{\theta_t}\right) \\ &\Leftrightarrow \Pi_t = \frac{Y_t}{\theta_t}. \end{aligned} \tag{A.51}$$

Deriving the interest rate as a function of $\tilde{k}_t$

The rental rate of capital is given by the marginal product of capital multiplied by the inverse of the monopolist's mark-up:

$$r_t^k = \frac{p_t^{int}}{P_t} \frac{\partial Y_t}{\partial K_t} = \frac{\theta_t - 1}{\theta_t} \frac{\partial Y_t}{\partial K_t} = \frac{\theta_t - 1}{\theta_t} \alpha \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}}. \tag{A.52}$$

Since the production function $Y_t = F(K_t, A_t L_t)$ is homogeneous of degree 1, it applies that $c \cdot Y_t = c \cdot F(K_t, A_t L_t) = F(c \cdot K_t, c \cdot A_t L_t)$. This rule is exploited to divide Y_t and K_t by $A_t L_t$, so $MPK = \alpha \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}}$ is expressed as a function of output and capital per effective worker:

$$\begin{aligned} MPK &= \alpha \left(\frac{Y_t / A_t L_t}{K_t / A_t L_t}\right)^{\frac{1}{\sigma}} \\ &= \alpha \left(\frac{\tilde{y}_t}{\tilde{k}_t}\right)^{\frac{1}{\sigma}} \\ &= \alpha \left(\frac{\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(1)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{\tilde{k}_t}\right)^{\frac{1}{\sigma}} \\ &= \alpha \left(\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right)^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1}{\sigma}} \cdot \tilde{k}_t^{\frac{-1}{\sigma}} \\ &= \alpha \left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right)^{\frac{1}{\sigma-1}} \cdot \tilde{k}_t^{\frac{-1}{\sigma}}. \end{aligned}$$

It follows that

$$r_t^k = \frac{\theta_t - 1}{\theta_t} \alpha \left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right)^{\frac{1}{\sigma-1}} \cdot \tilde{k}_t^{\frac{-1}{\sigma}}.$$

The expression for MPK can then be differentiated w.r.t. $\tilde{k}_t$:

$$
\begin{aligned}
\frac{\partial MPK}{\partial \tilde{k}_t} &= \frac{\alpha}{\sigma-1}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{1}{\sigma-1}-1} \cdot \alpha \frac{\sigma-1}{\sigma} \tilde{k}_t^{\frac{\sigma-1}{\sigma}-1} \cdot \tilde{k}_t^{\frac{-1}{\sigma}} \\
&\quad + \alpha \left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{1}{\sigma-1}} \cdot \left(-\frac{1}{\sigma}\tilde{k}_t^{\frac{-1}{\sigma}-1}\right) \\
&= \frac{\alpha^2}{\sigma}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{1-(\sigma-1)}{\sigma-1}} \cdot \tilde{k}_t^{\frac{\sigma-1-\sigma-1}{\sigma}} \\
&\quad - \frac{\alpha}{\sigma}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{1}{\sigma-1}} \cdot \tilde{k}_t^{-\frac{1-\sigma}{\sigma}} \\
&= \frac{\alpha^2}{\sigma}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{2-\sigma}{\sigma-1}} \tilde{k}_t^{\frac{-2}{\sigma}} \\
&\quad - \frac{\alpha}{\sigma}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{1}{\sigma-1}} \tilde{k}_t^{\frac{-1-\sigma}{\sigma}}.
\end{aligned}
$$

It follows from $r_t^k = \frac{\theta_t - 1}{\theta_t} \cdot MPK$, that

$$
\frac{\partial r_t^k}{\partial \tilde{k}_t} = \frac{\theta_t - 1}{\theta_t} \cdot \left[\frac{\alpha^2}{\sigma}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{2-\sigma}{\sigma-1}} \tilde{k}_t^{\frac{-2}{\sigma}} - \frac{\alpha}{\sigma}\left(\alpha \tilde{k}_t^{\frac{\sigma-1}{\sigma}} + 1-\alpha\right)^{\frac{1}{\sigma-1}} \tilde{k}_t^{\frac{-1-\sigma}{\sigma}}\right].
$$

It applies that $\frac{\partial r_t^k}{\partial \tilde{k}_t} < 0$ for all $\tilde{k}_t > 0$ when $0 < \alpha < 1$ and $\sigma > 0$.

Steady state equations

A balanced growth path is defined as a stationary equilibrium where the exogenous processes, including productivity growth, fertility, and mortality, are constant. As a result, effective capital, consumption, and income per capita will remain constant. In the first and last periods, where the economy in the model is in steady state, the following system of equations applies:[65]

$$\bar{N}_{1,1} = 1 \tag{A.53}$$

$$N_{j+1} = \frac{s_j N_j}{1+n} \quad \text{for } j \in \{1, J-1\} \tag{A.54}$$

$$\frac{1}{\beta} = \left(\frac{c_{j+1}}{c_j}\right)^{-\frac{1}{\rho}} \cdot (1+r) + \lambda_{j+1}\frac{(c_j)^{\frac{1}{\rho}}}{su_j \beta^j} \quad \text{for } j \in \{1, J-1\} \tag{A.55}$$

$$x_j = \left(\frac{\Gamma^2}{\mu}\right)^{-\rho} \cdot c_j \quad \text{for } j \in \{J = 74\} \tag{A.56}$$

$$a_j = 0 \quad \text{for } j \in \{1\} \tag{A.57}$$

$$a_{j+1} = \frac{(1+r)a_j}{s_{j-1}} + ((1-\tau)w \cdot hc_j + (1-\tau_p)\Pi_j)\,(1+g^A)^j - c_j \tag{A.58}$$

$$\text{for } j \in \{1, \ldots, 42\} \setminus \{24\}$$

[65] Equation (10) applies only in the first steady state in 1970 ($t = 1$). In the terminal steady state in 2170 ($T = 201$), N_1 is set to a lower value to match the preceding population development. The size of the other cohorts will then be determined based on equation (A.54).

$$a_{j+1} = \frac{(1+r)a_j}{s_{j-1}} + \left((1-\tau)w \cdot hc_j + (1-\tau_p)\Pi_j + q_j\right)(1+g^A)^j - c_j \quad \text{for } j \in \{24\} \tag{A.59}$$

$$q_j = \frac{x_J \cdot \Gamma^2 \cdot N_J}{N_j} \quad \text{for } j \in \{25\} \tag{A.60}$$

$$a_{j+1} = \frac{(1+r)\cdot a_j}{s_{j-1}} + dw(1+g^A)^j - c_j, \quad \text{for } j \in \{43,\ldots,J-1\} \tag{A.61}$$

$$c_j = \frac{(1+r)\cdot a_j}{s_{j-1}} - \Gamma^2 \cdot x_j + dw(1+g^A)^j, \quad \text{for } j \in \{J=74\} \tag{A.62}$$

$$\min\left(\lambda_j, a_j + (D_j \cdot w \cdot hc_j)\cdot(1+g^A)^j\right) = 0, \quad \text{for } j \in \{1,\ldots,42\} \tag{A.63}$$

$$\min(\lambda_j, a_j) = 0 \quad \text{for } j \in \{43,\ldots,J\} \tag{A.64}$$

$$\Pi_j = \frac{hc_j \cdot \Pi}{L} \quad \text{for } j \in \{1,\ldots,42\} \tag{A.65}$$

$$\frac{p^{int}}{P} = \frac{\theta-1}{\theta}. \tag{A.66}$$

According to Eggertsson et al. (2019b), the wage w in period 1, where the model starts in steady state, is used as the numeraire A_{adj}. This allows the variables in the model to be compared with each other, as they are measured in terms of the real wage in steady state in period 1.[66]

$$A_{adj} = \frac{p^{int}}{P} \cdot \left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\cdot(A\cdot L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \cdot (1-\alpha)A^{\frac{\sigma-1}{\sigma}} \cdot L^{\frac{-1}{\sigma}} \tag{A.67}$$

$$w = \frac{\frac{p^{int}}{P} \cdot \left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\cdot(A\cdot L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \cdot (1-\alpha)A^{\frac{\sigma-1}{\sigma}} \cdot L^{\frac{-1}{\sigma}}}{A_{adj}} \tag{A.68}$$

$$r^k = \frac{\frac{p^{int}}{P} \cdot \left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(AL)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \cdot \alpha K^{\frac{-1}{\sigma}}}{A_{adj}} \tag{A.69}$$

$$Y = \frac{\left(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(AL)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \tag{A.70}$$

$$r = r^k - \delta \tag{A.71}$$

$$\Pi = \frac{Y}{\theta} \tag{A.72}$$

$$F \cdot G = \left((1+g^A)\cdot(1+n) - 1\right)\cdot B \tag{A.73}$$

$$B = b \cdot Y \tag{A.74}$$

$$G = g \cdot Y + r \cdot B + E \tag{A.75}$$

$$\tau = \frac{G\cdot(1-F) - \tau_p \Pi}{w \cdot L} \tag{A.76}$$

[66] It follows that $w \equiv 1$ in 1970.

$$N = \sum_{j=1}^{J} N_j \tag{A.77}$$

$$L = \sum_{j=1}^{J} N_j \cdot hc_j \tag{A.78}$$

$$C = \sum_{j=1}^{J} N_j c_j / (1 + g^A)^j \tag{A.79}$$

$$K = \sum_{j=1}^{J} \frac{N_j a_j}{s_{j-1}(1 + g^A)^j} - B \tag{A.80}$$

$$d = \bar{d} \cdot (1 - \tau) w \tag{A.81}$$

$$E = \sum_{j=43}^{J} dw \cdot N_j \tag{A.82}$$

Appendix C: Supplementary results

This section contains results from the analysis that were not prioritized for inclusion in the thesis due to relevance and space considerations.

Table A1: Parameter values in the baseline

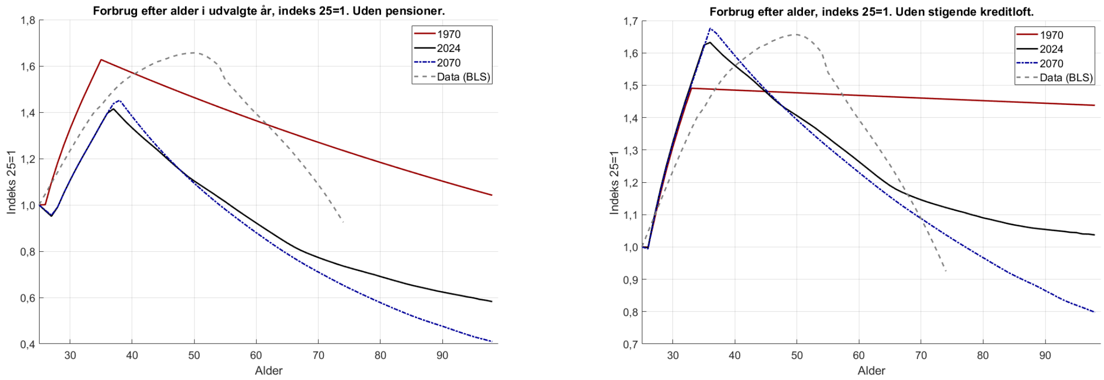
Parameter	With pensions	Without pensions
α	0.3999	0.3999
β	0.9786	0.9720
μ	13.400	23.449
θ_1	6.9474	6.9476
θ_T	3.9999	4.0768
D_1	0.1370	0.1164
D_T	0.7000	0.6086
$\bar{d}$	0.505	0

Table A2: Scenario without the baby boom: Isolating the drivers (1970-2024)

Exogenous variable	$r^\star$ in 2024 (%)	Contribution (% points)
Mortality (s_j)	3.05	-1.12
Fertility (Γ_t)	2.36	-0.44
Productivity growth (g_t^A)	3.16	-1.23
Government debt (b_t)	1.13	0.80
Competition (θ_t)	2.43	-0.50
Private credit (D_t)	1.71	0.22

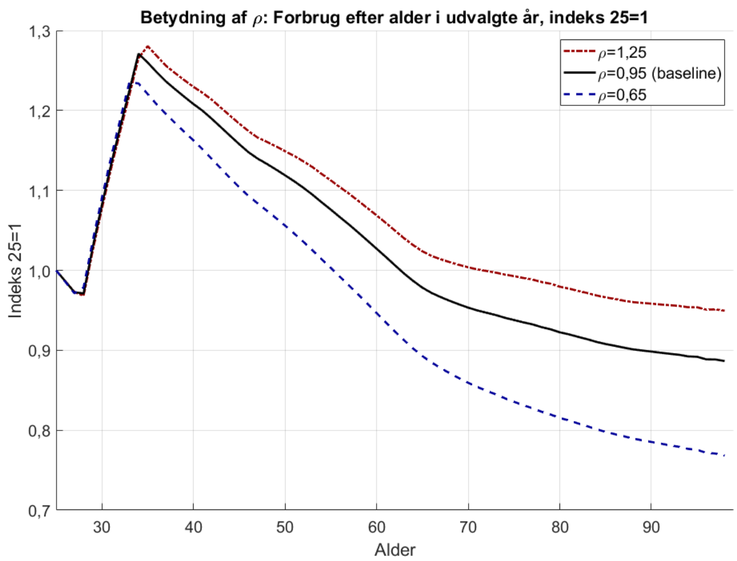
Note: The table shows the simulated interest rate in 2024 when the given exogenous variable is held constant at its 1970 value. The growth contribution is calculated by comparing it to the baseline, *without* the baby boom, where the interest rate equals 1.93% in 2024, corresponding to a total decline of 2.08 pp since 1970 (when the interest rate was 4%). The sum of the growth contributions does not equal the total decline due to interaction effects. Source: Own calculations.

Figure A2: Consumption profile without pensions (l.) and without increase in $D_{j,t}$ (r.)



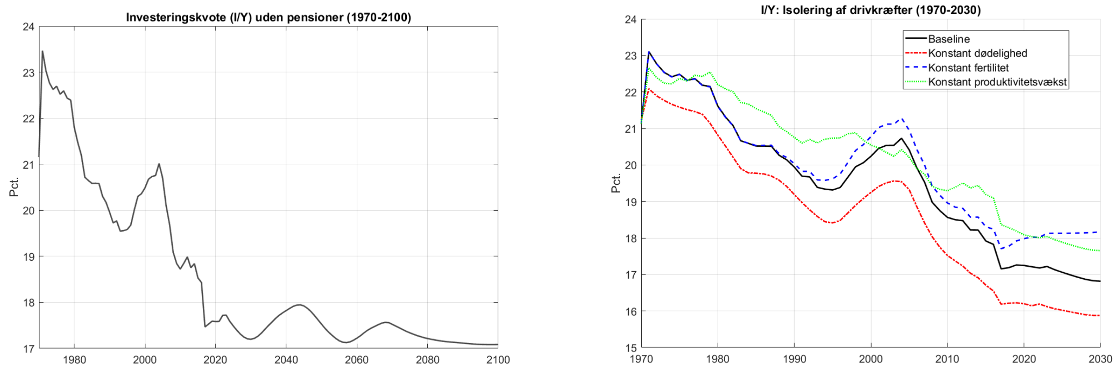
Note: The figure on the left shows consumption curves for selected years in the model without pensions. The figure on the right shows the consumption curves (with pensions), where the credit limit $D_{j,t}$ is held constant at its 1970 level. In all simulations, the model is recalibrated to match the same moments. Data from the BLS are calculated using cubic interpolation. Source: BLS (2023) and own calculations.

Figure A3: Consumption profile for different values of ρ



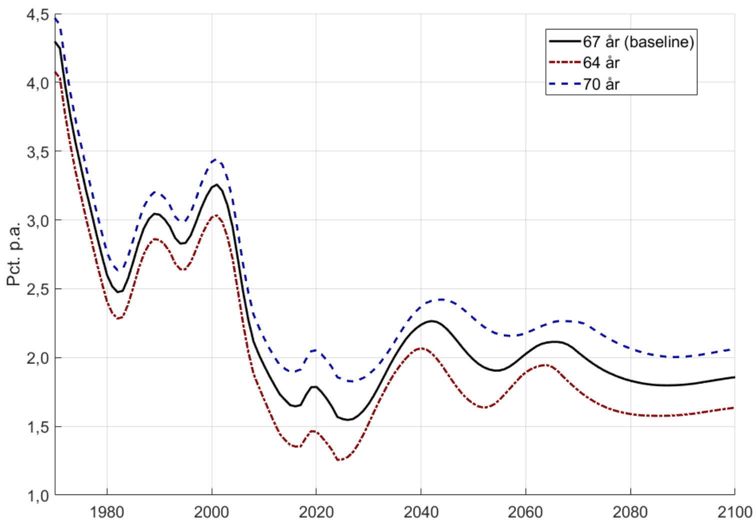
Note: The figure shows the consumption profile in the model (with pensions) in 2024 for different values of the IES, ρ. In all simulations, the model is recalibrated to match the same moments. Source: Own contribution.

Figure A4: Investment rate without pensions (l.) and constant exogenous variation (r.)



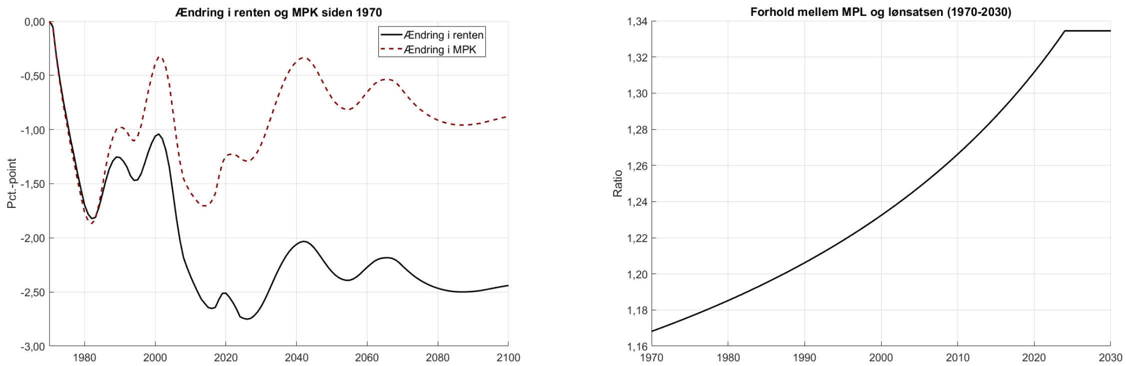
Note: The figure on the left shows the investment share in the model *without* pensions as a percentage of GDP. The figure on the right shows the investment share in the model *with* pensions, where individual exogenous variables are held constant at their 1970 levels. Red: constant mortality (*dødelighed*). Source: Own contribution.

Figure A5: $r^{\star}$ under different retirement ages (1970-2100)



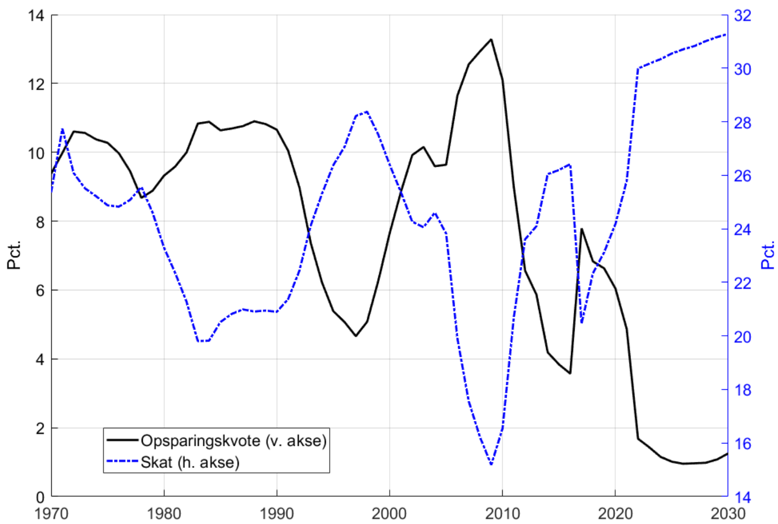
Note: The figure shows the development of the simulated interest rate (with pensions) when the retirement age is either three years lower or higher than in the baseline. In all three simulations, the same calibration is used, which is why the interest rate does not start at the same point in 1970. By 2100, $r^{\star}$ is approximately 21 bp lower/higher than in the baseline. In the simulation where the retirement age is 70 years, hc_j for $j \in \{43, 44, 45\}$ is set equal to hc_{42}. Source: Own contribution.

Figure A6: Factor prices and the marginal products of capital (l.) and labor (r.)



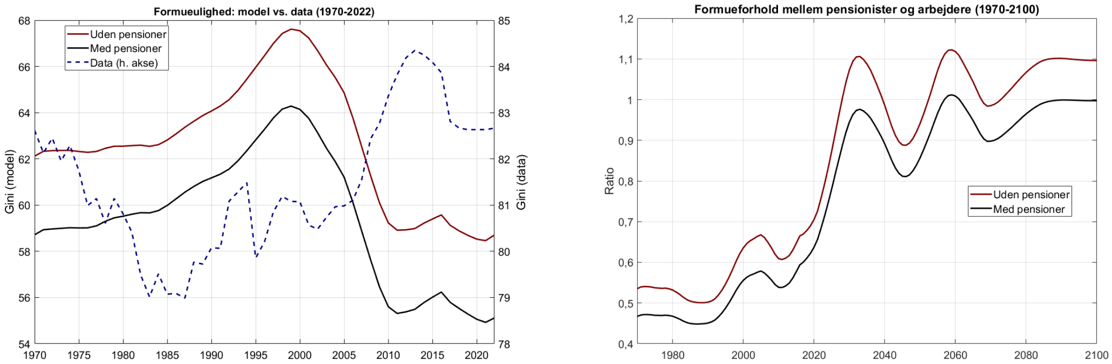
Note: The figure on the left shows the accumulated change in the interest rate (black) and the marginal product of capital (red dashed) in the baseline. The figure on the right shows the marginal product of labor divided by the wage rate. Source: Own contribution.

Figure A7: Household savings rate and the tax rate (1970-2030)



Note: The figure shows the net savings rate of the households as given in (52) (left axis) and the tax rate τ_t (right axis) in the model. Source: Own contribution.

Figure A8: Wealth inequality



Note: The figure on the left compares the development of the Gini coefficient (multiplied by 100) for wealth distribution in the model (left) with data for the USA (right) from 1970-2022. The figure on the right shows the development of the total wealth of retirees in relation to workers' wealth. *Uden/med pensioner*: without/with pensions. Source: WID (2024) and own calculations.

Figure A8 (left) shows the development of the Gini coefficient for wealth inequality between 1970-2022.[67] It appears that wealth inequality rises until 2000, after which it falls and stabilizes at a level lower than in 1970. In the model with pensions, the Gini coefficient reaches 55 in 2024 and 52 in 2100. In the model without pensions, inequality is higher, as there is no redistribution between workers and retirees. The development of inequality in the model partially aligns with the empirical data, which shows increasing inequality between 1985-2012. It cannot, therefore, be ruled out that some of the increase in inequality during this period could be attributed to demographic factors. In the long run, the model predicts that inequality will fall. This is because a larger share of the population will be made up of retirees, who have a more homogeneous income profile. At the same time, the share of younger households, characterized by small or negative net wealth, will decline. The long-term decrease in inequality aligns with G&P, who also predict that income inequality will decrease due to aging. Their explanation, however, is based on the premise that the labor share will rise as labor becomes more scarce (Goodhart and Pradhan, 2020).[68] Nevertheless, the model shows that the demographic development itself will exert downward pressure on wealth inequality. It is also worth noting that the level of inequality in the model is significantly lower than in the empirical data, which follows from the assumption of identical households.

In figure A8 on the right, it is shown that retirees' wealth, measured relative to workers' wealth, increases throughout most of the period. In the long run, retirees will own half of the total wealth in the economy. This is not in contrast to the fact that inequality is also falling, as a larger portion of the population will consist of retirees.

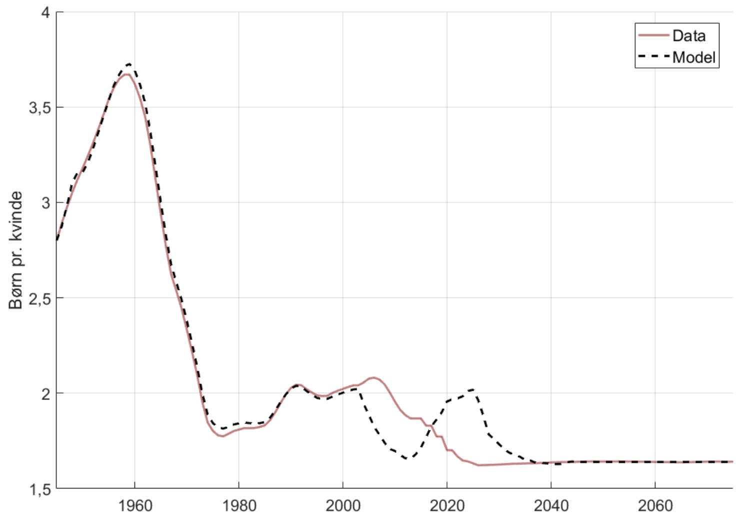
[67]The Gini coefficient is calculated using the formula described in Hasell (2023).

[68]In the model, the declining labor share between 1970-2024 has no effect on inequality among workers, as company profits are distributed according to wage income, see (38).

Appendix D: Supplementary data

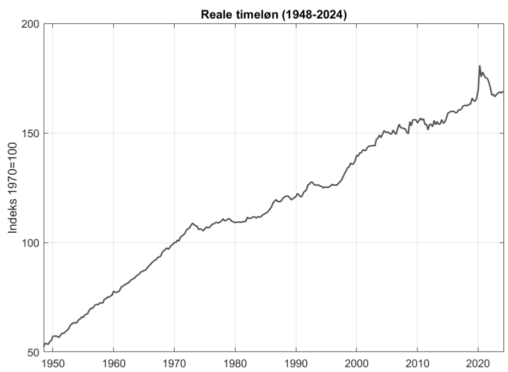
In this section, various data and empirical evidence are presented, some of which have been used in the thesis, while others are included for reference purposes.

Figure A9: Fertility

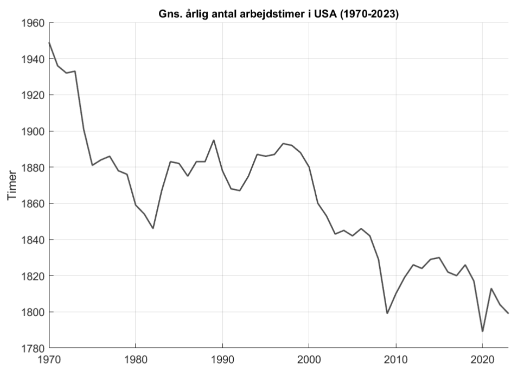


Note: The fertility rate in the model (baseline) and compared with U.S. data 1945-2075. Forecast from the UN's medium scenario after 2023. Source: CDC (2023), UN (2024), and own contribution.

Figure A10: Real wage rate (a) and annual hours worked in the U.S. (b).

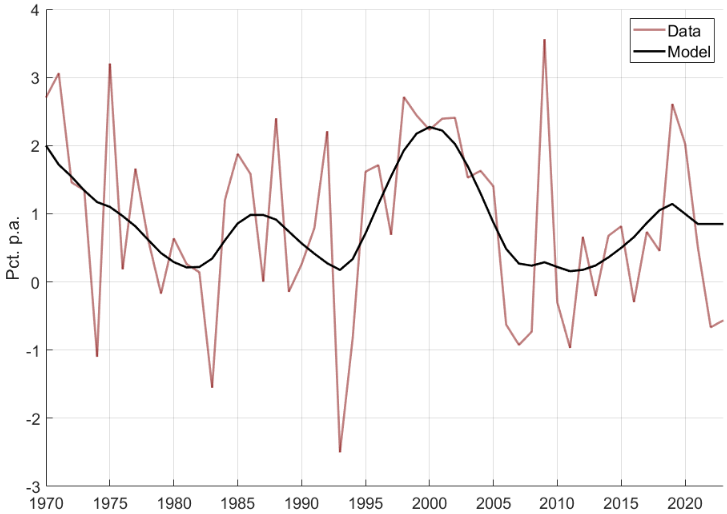


(a) Development of the real hourly wage rate in the U.S. 1948-2024Q2. Index 1970=100. Source: BLS (2024).



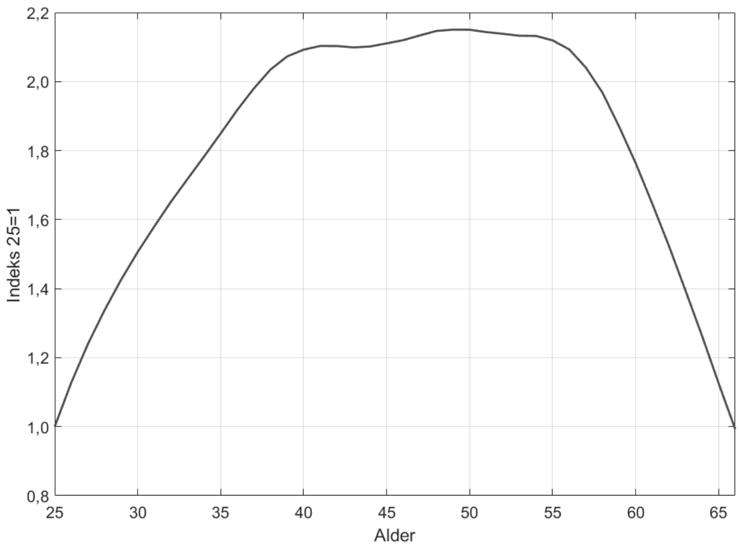
(b) Average annual hours worked per worker in the U.S. in 1970-2023. Source: OECD (2024a).

Figure A11: Productivity growth



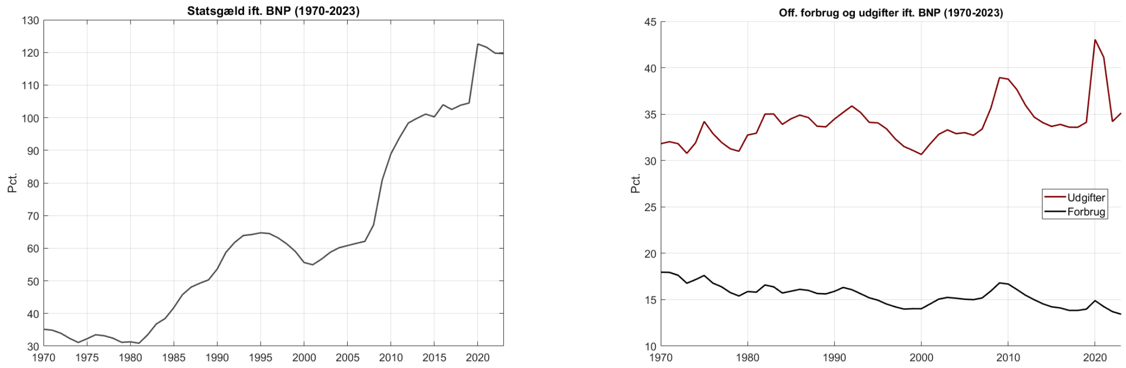
Note: The figure shows the annual productivity growth (TFP) in the USA adjusted for capacity utilization. In the model, this is smoothed using an HP filter with $\lambda = 10$. The value in 1970 is set to 2%, while it is set to 0.85% after 2020. Source: Fernald (2024) and own calculations.

Figure A12: Income profile by age



Note: The figure shows the human capital profile hc_j. The data is based on the average labor income (both employees and self-employed) by age in the USA in 2011. Source: NTA (2017).

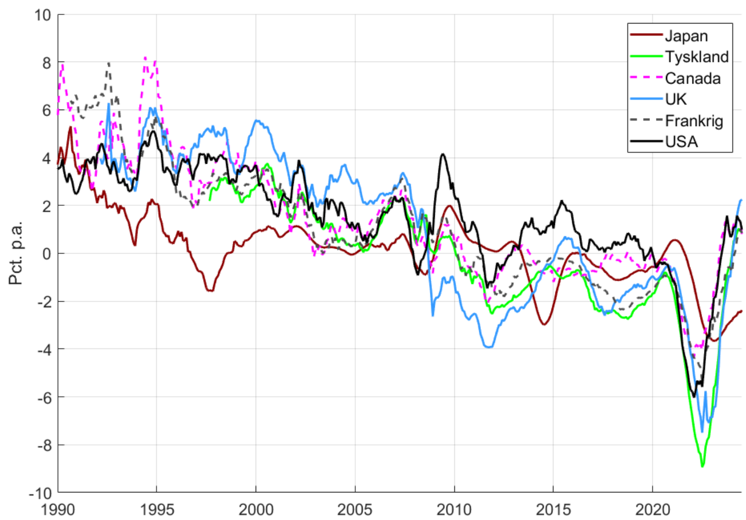
Figure A13: Government debt (a) and expenditure (b) in the U.S. (1970-2023)



(a) Note: U.S. government debt as a percentage of GDP. It includes all debt owed by the federal government to the public. Source: FRED (2024d), series: GFDEGDQ188S.

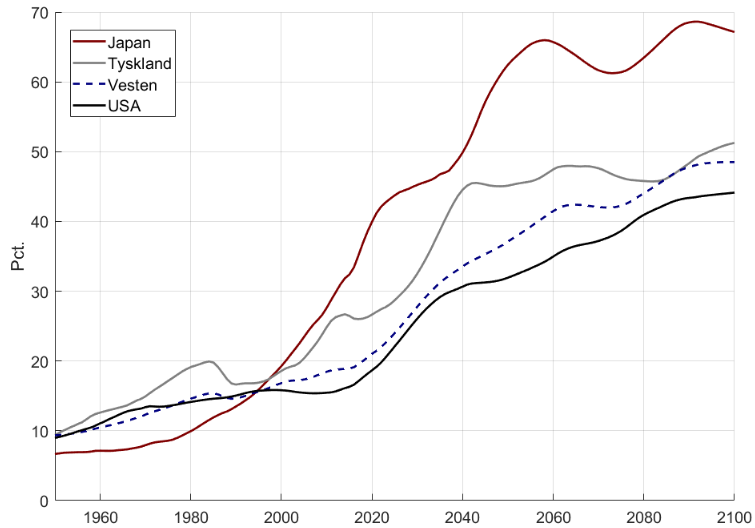
(b) Note: Total public expenditures and consumption expenditures excluding investments, transfers, and interest expenses as a percentage of GDP. Source: FRED (2024d), BEA (2024b), and own calculations.

Figure A14: Real interest rates in advanced economies (1990-2024)



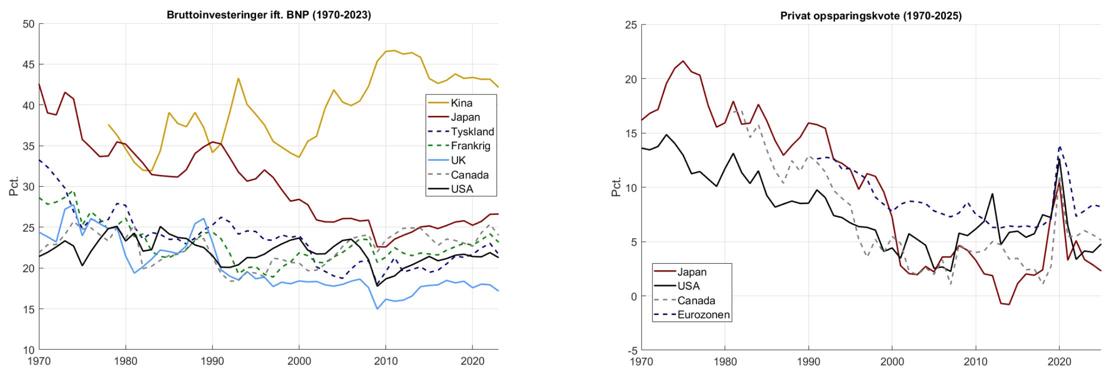
Note: 1-year real interest rates on government bonds for Japan, Germany, and the USA. 2-year real interest rates on government bonds for Canada, France, and the United Kingdom. The real interest rates are calculated by subtracting the smoothed inflation (HP filter, $\lambda = 100$) from the nominal interest rate. The latest observation is from July 2024. Source: Bloomberg (2024) and own contribution.

Figure A15: 70+ year-olds relative to 25-69 year-olds in advanced economies (1950-2100)



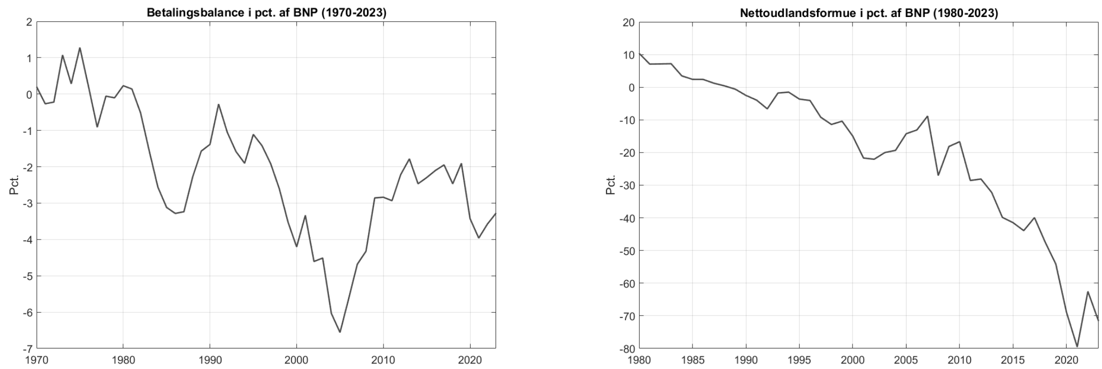
Note: The figure shows the dependency ratio between the number of individuals aged 70+ and the number of individuals aged 25-69. Projections from the UN's medium scenario after 2023. *Vesten* (The West) includes North America, Europe, Australia, and New Zealand. Source: UN (2024).

Figure A16: Gross investments and net savings rate in advanced economies



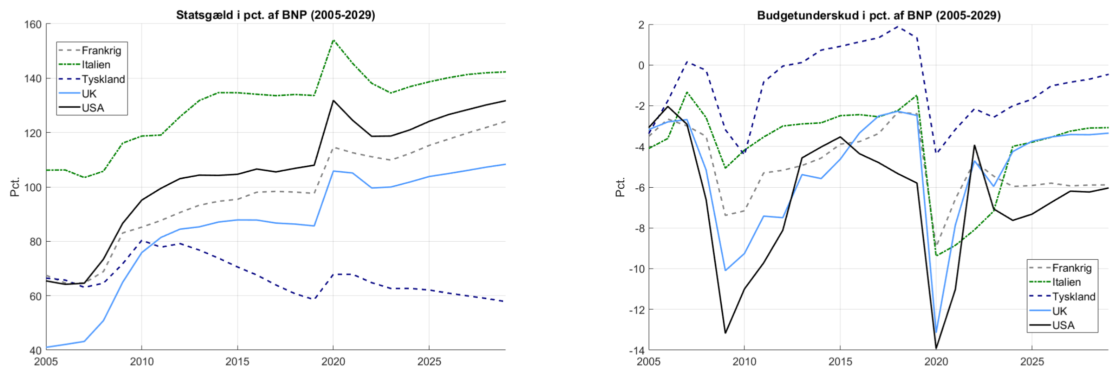
Note: The figure on the left shows gross capital formation relative to GDP. The figure on the right shows the household savings rate relative to their disposable income. Forecast after 2024. Source: OECD (2024b) and OECD (2024c).

Figure A17: U.S. current account and net international investment position



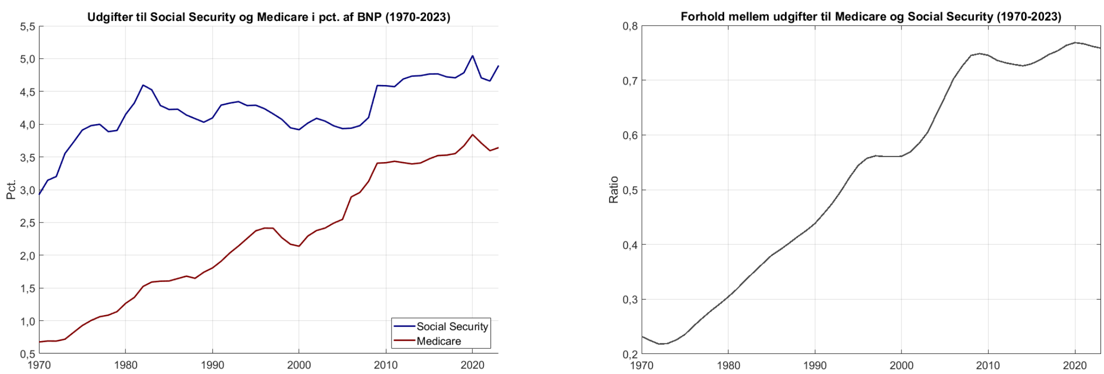
Note: The left panel shows the development of the U.S. current account balance. The right panel shows the U.S. net foreign asset position. Percentage of GDP. Source: BEA (2024a) and own calculations.

Figure A18: Government debt and deficits in advanced economies



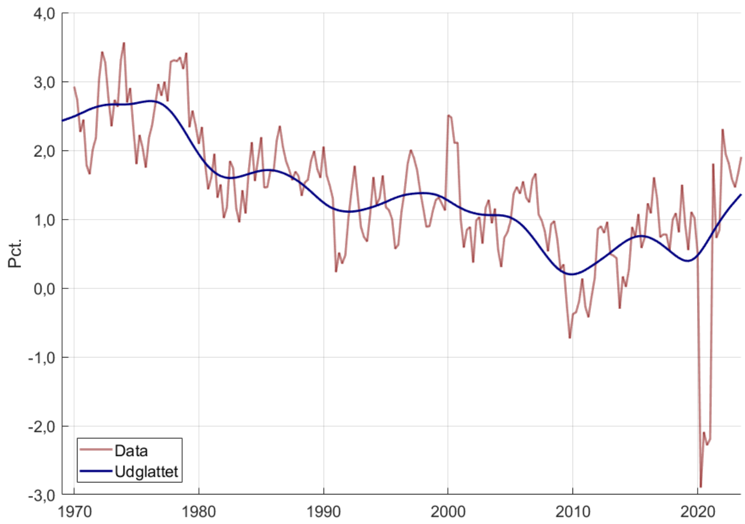
Note: The figure on the left shows gross government debt as a percentage of GDP. The figure on the right shows the government budget deficit (net lending) as a percentage of GDP. Forecasts after 2023. Source: IMF (2024).

Figure A19: Expenditure on Social Security and Medicare relative to GDP



Note: The figure on the left shows public expenditures on Social Security and Medicare as a percentage of GDP. The figure on the right shows Medicare expenditures divided by Social Security expenditures (5-year moving average). Source: FRED (2024e) and own calculations.

Figure A20: Annual growth rate in the U.S. labor force (1970Q1-2024Q3)



Note: The figure shows the annual growth rate of the labor force in the U.S., both in its original form and smoothed with an HP filter ($\lambda = 100$). Source: FRED (2024b) and own calculations.

Appendix E: Code and contact

The model employed in this thesis is simulated in MATLAB with the extension Dynare 6.0. The code to simulate the baseline model is available on Github.[69] The code is developed based on Crescentini and Giri (2023). Questions regarding the thesis can be sent to the e-mail address valder@fredens.net.

[69]See link: https://github.com/mhz379/Speciale.git