



Introduction into Deep Learning

Neural Networks and Multi-Layered Perceptron.
Backpropagation. Tips and tricks for training NNs.

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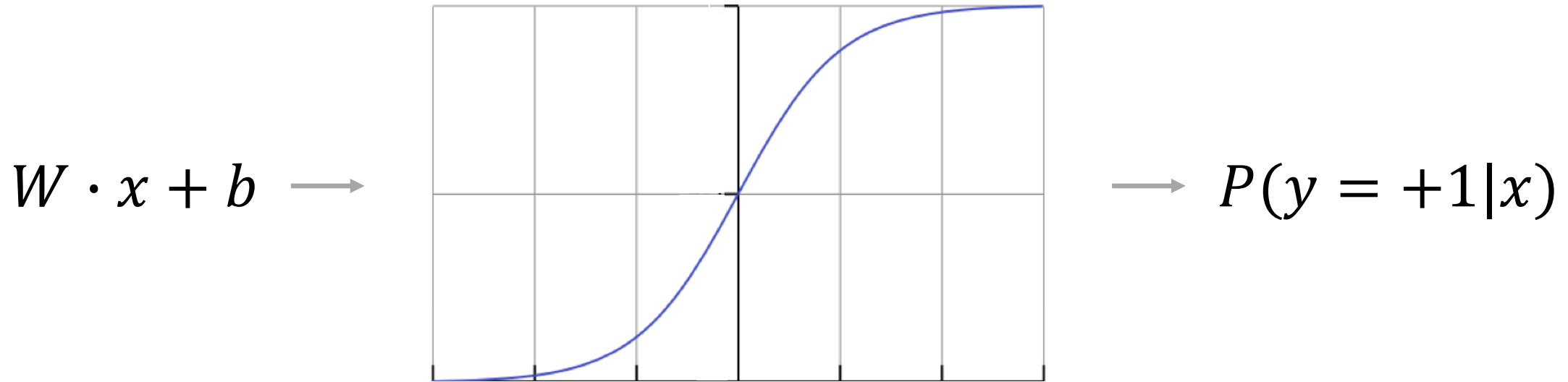
Lecture overview

- The basic principle of *deep learning*
- The one you will be applying to all problems hereinafter
- Absolutely essential for all future material
- Step-by-step example of training a neural network via backpropagation
 - You'll need the knowledge when using the advanced architectures

A thick yellow arrow pointing to the right, with a white background inside its shaft.

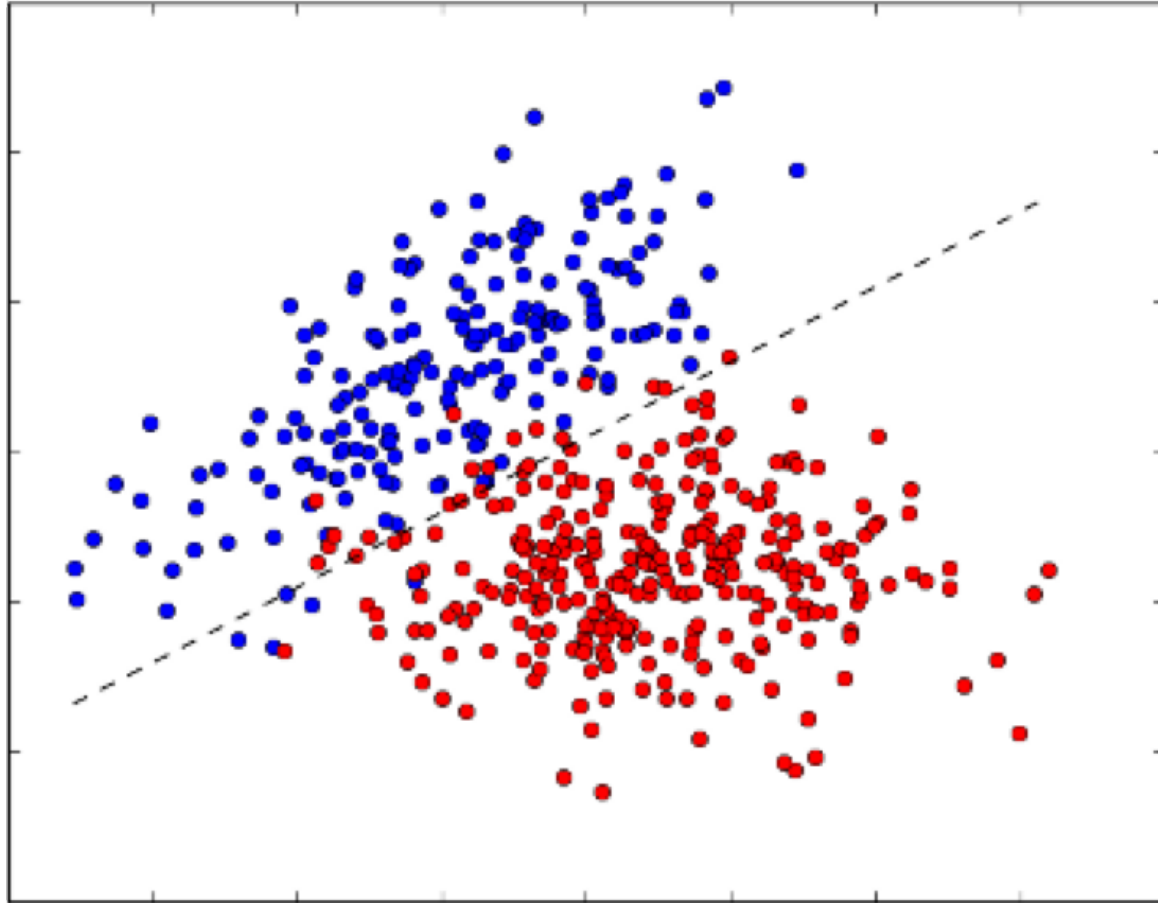
Principles of linear vs. nonlinear models

Recap: linear models



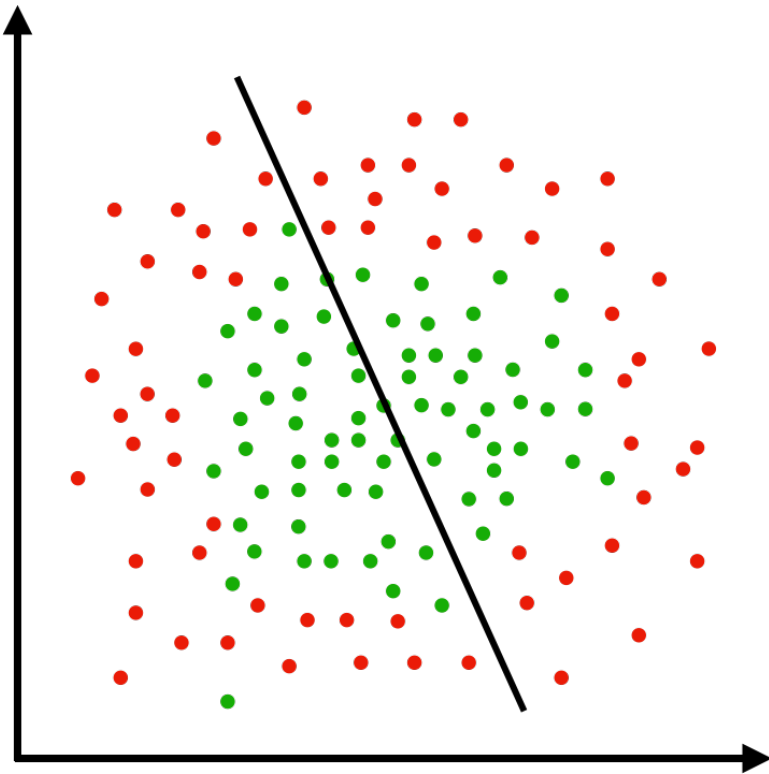
- x : features vector
- W, b : model slope and intercept

Linear dependency

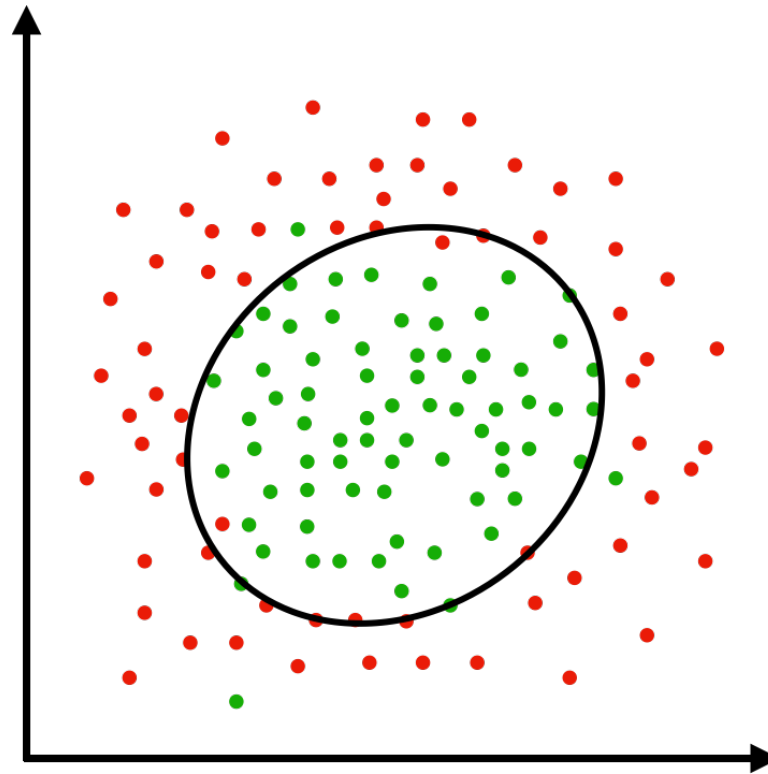


Nonlinear dependencies

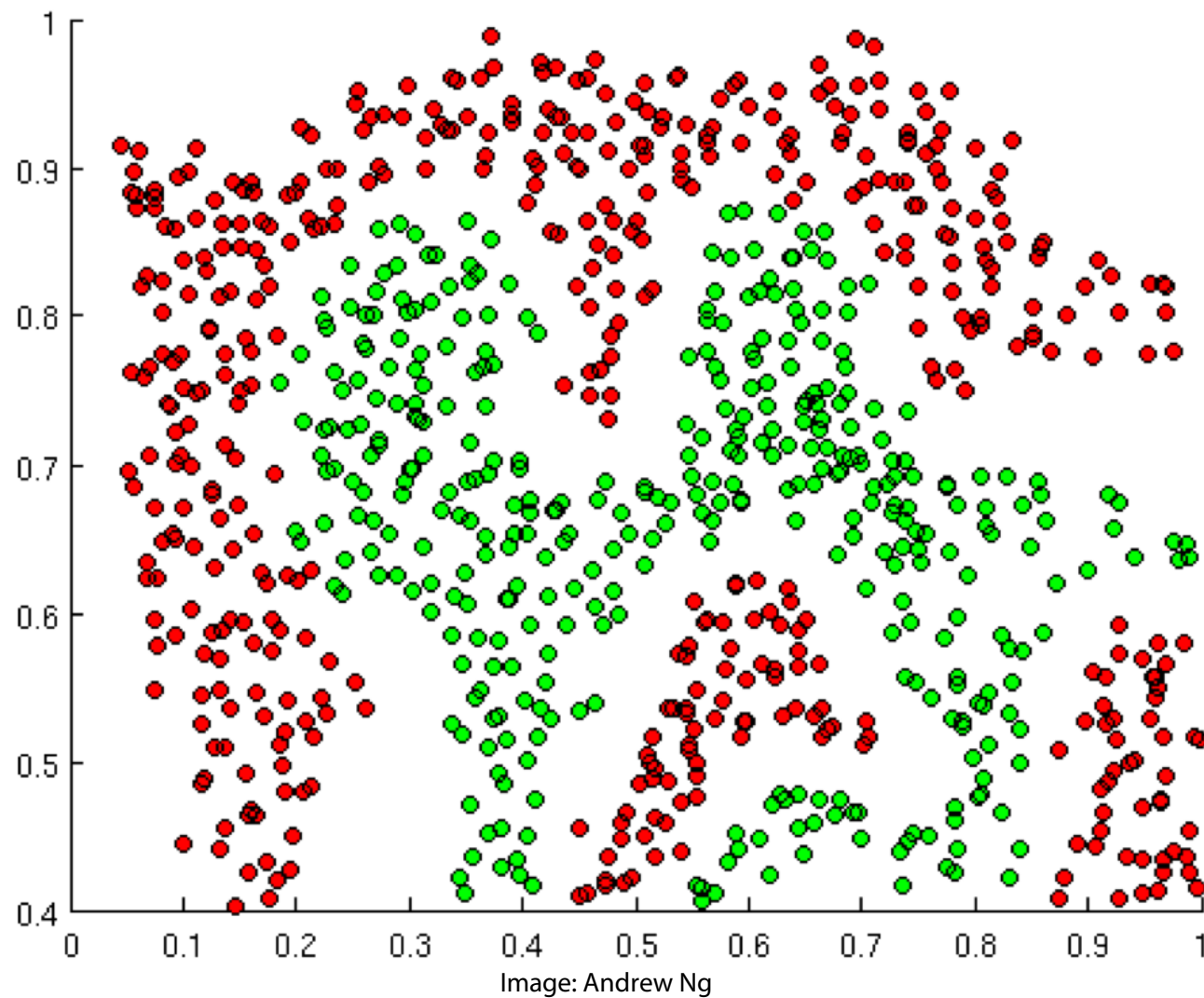
What we have



What we want

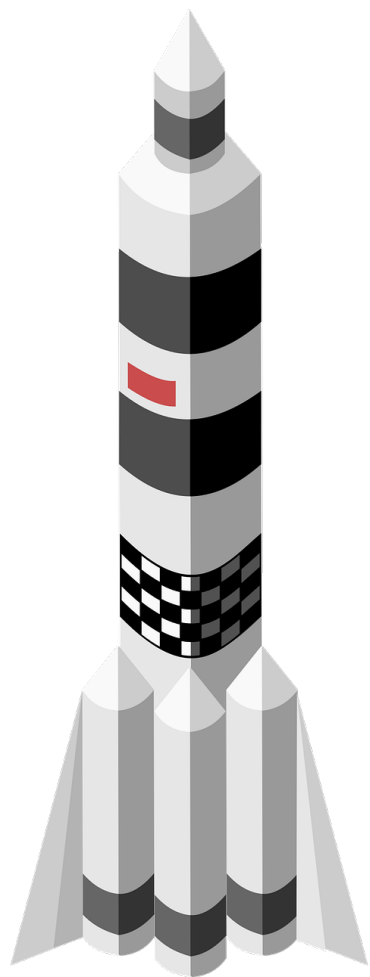


Somewhat nonlinear dependencies



Extremely nonlinear dependencies

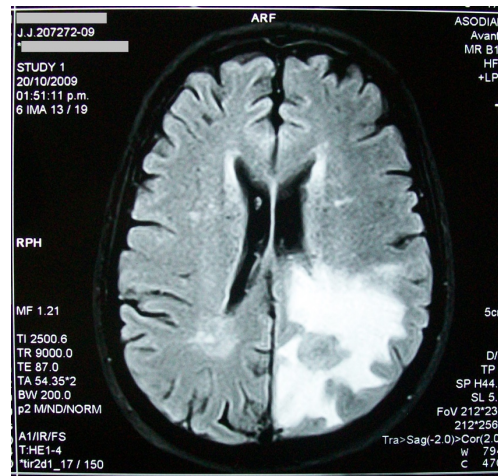
Most of the dependencies in this world!



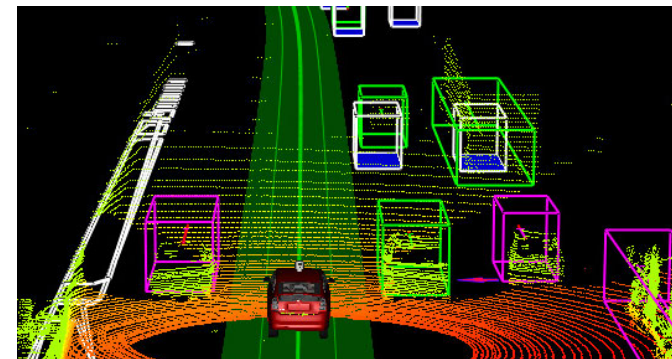
rocket



cat

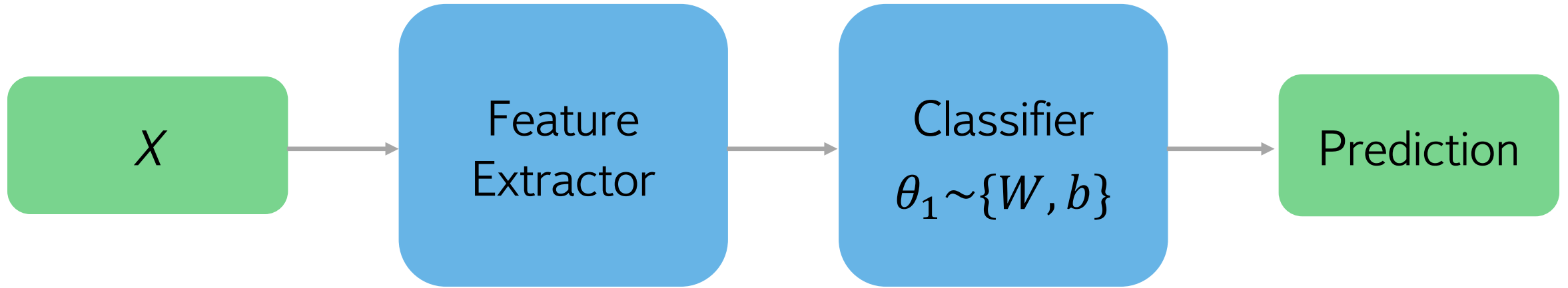


Brain tumors
as seen on MRI



Self-driving LiDAR₈

Extremely nonlinear dependencies

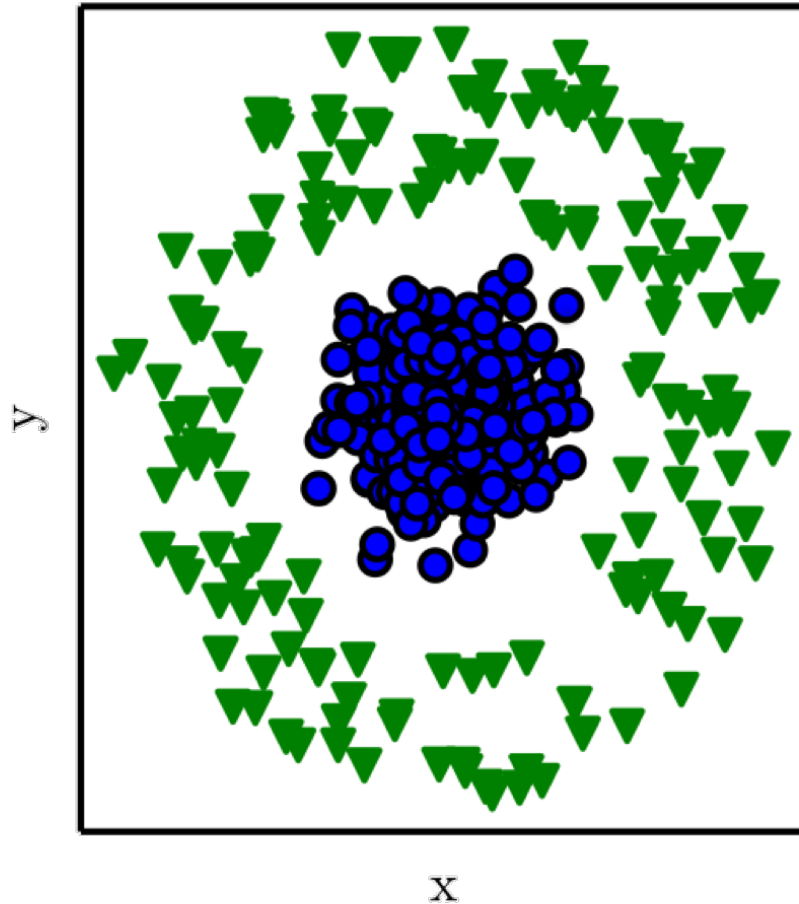


- Decouple feature extractor from the classifier
- Training and inference really can have multiple stages!

Feature *extraction*?

Data that is linearly non-separable in cartesian are l

Cartesian coordinates



Polar coordinates

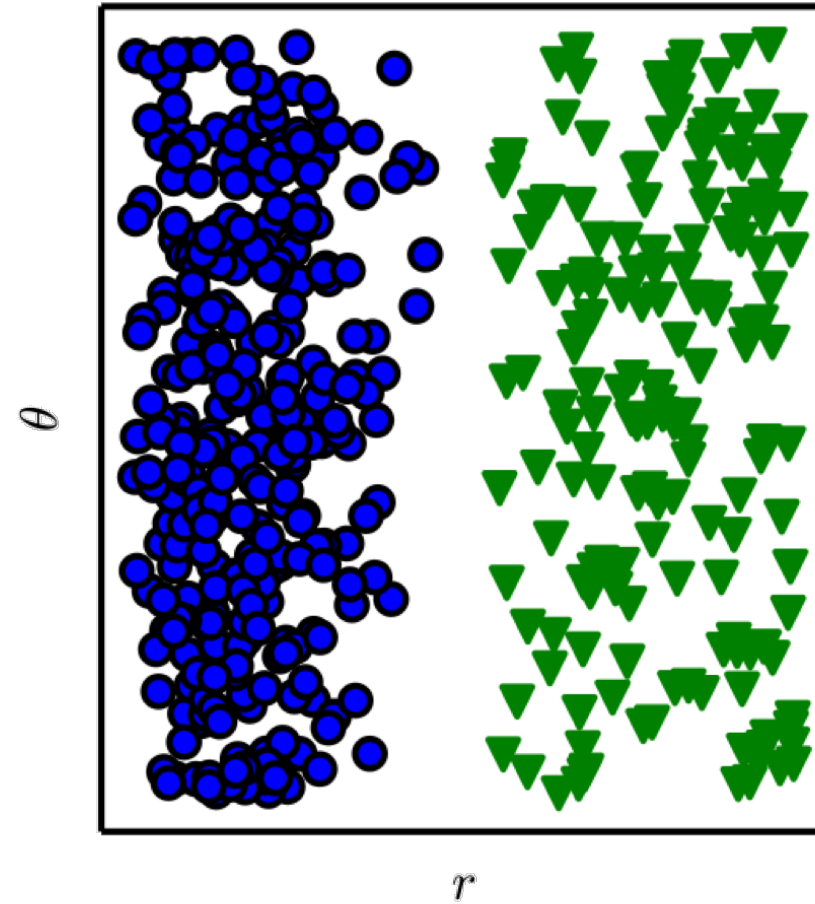
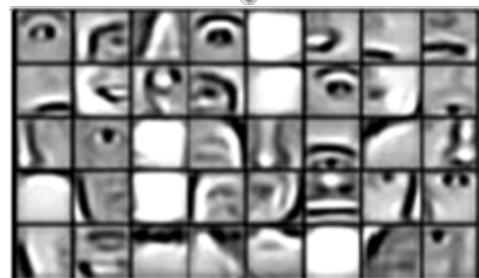


Image: Ian Goodfellow et al.

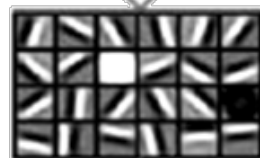


Discrete Choices

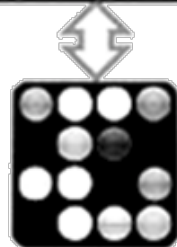
⋮



Layer 2 Features

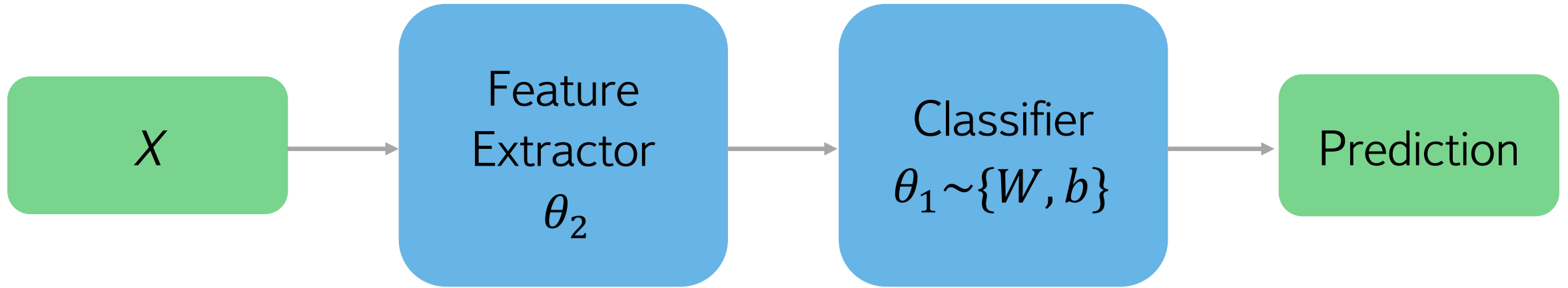


Layer 1 Features



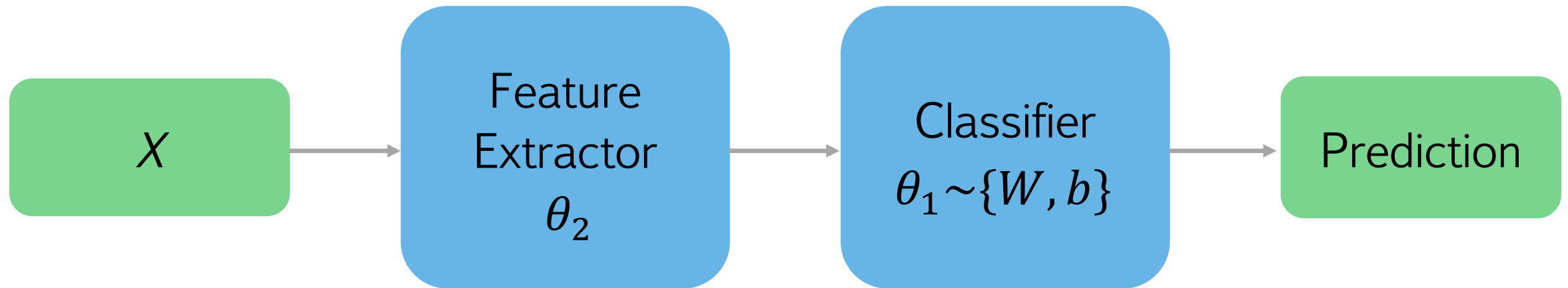
Original Data

Feature extraction



- *Manually* extracted features
- Training is left with finding $\underset{\theta_1}{\operatorname{argmin}} L(y, P(y | x))$

Can it be done automatically?

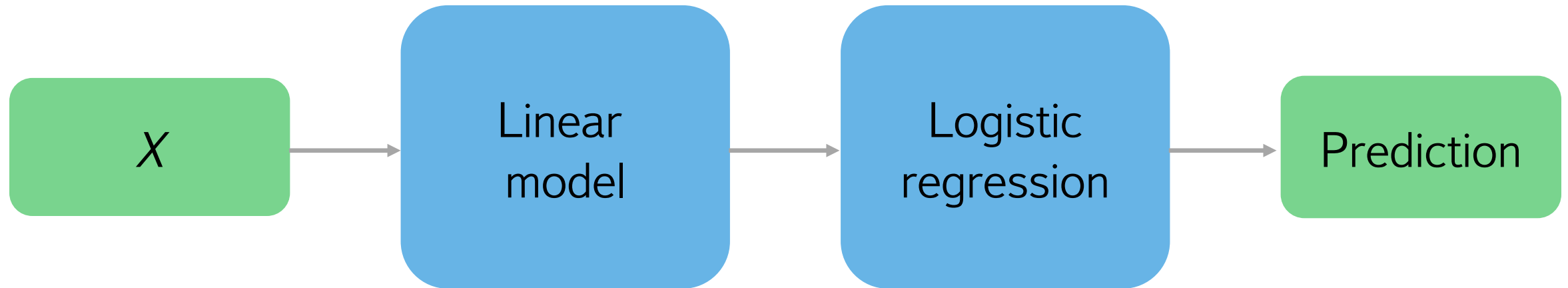


- *Automatically* extracted features
- Training still needs to find $\underset{\theta_1}{\operatorname{argmin}} L(y, P(y | x))$
- Yet, we face a different challenge of finding θ_2

Automatically - can feature ext

Try stacked linear models

Spoiler: Doesn't Work! Still a



- Compute features $h_j = \sum_i w_{ij}^h x_i + b_j^h \quad j \in \{1, 2, \dots, n\}$
- Eventual output of the model $y_{pred} = \sigma \left(\sum_j w_j^o h_j + b^o \right)$
- Train by jointly minimizing loss to search for $\operatorname{argmin}_{w^h, w^o, b^h, b^o} L(y, P(y | x))$

A question

- Will stacking linear functions improve quality?

Answer: no

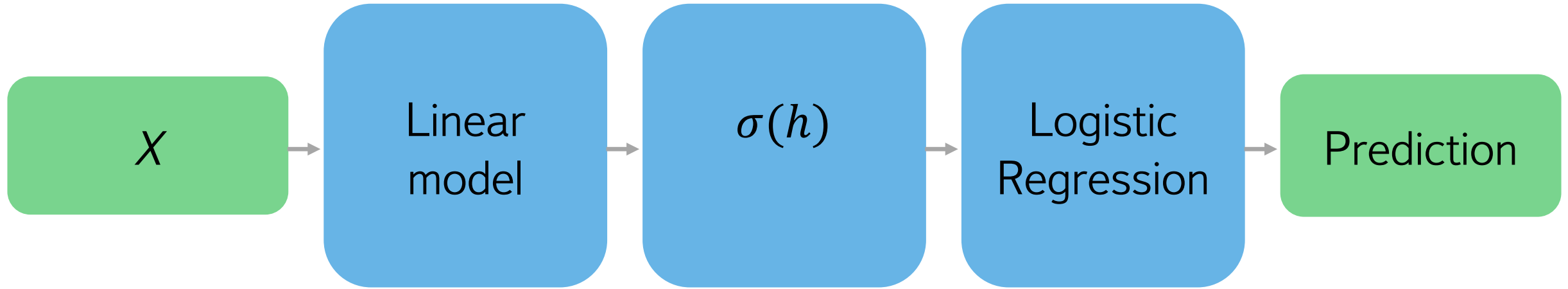
- Why?
- A combination of linear models is a linear model:

$$P(y | x) = \sigma \left(\sum_j w_j^o \left(\sum_i w_{ij}^h x_i + b_j^h \right) + b^o \right)$$

$$w'_i = \sum_j w_j^o w_{ij}^h \quad b' = \sum_j w_j^o b_j^h + b^o$$

$$P(y | x) = \sigma \left(\sum_i w'_i x_i + b' \right)$$

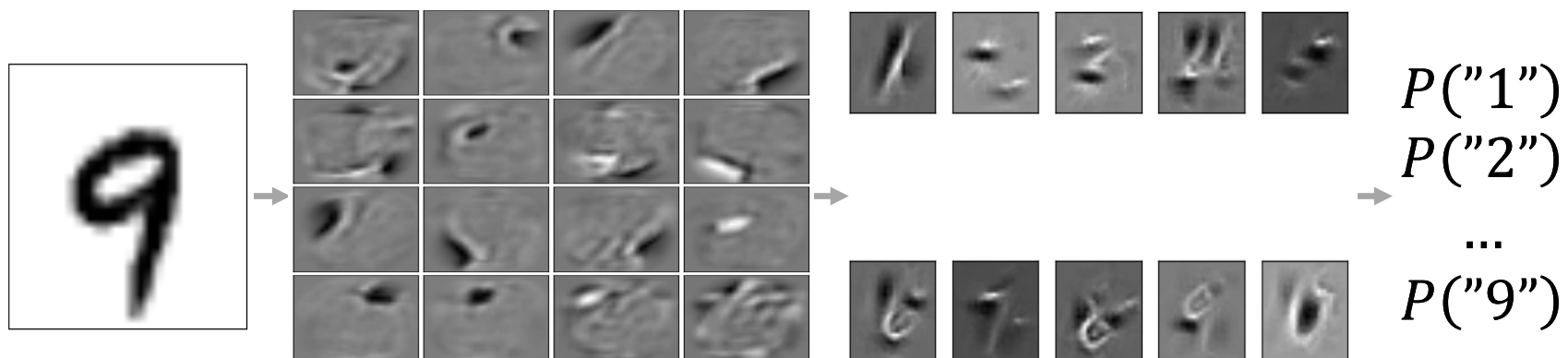
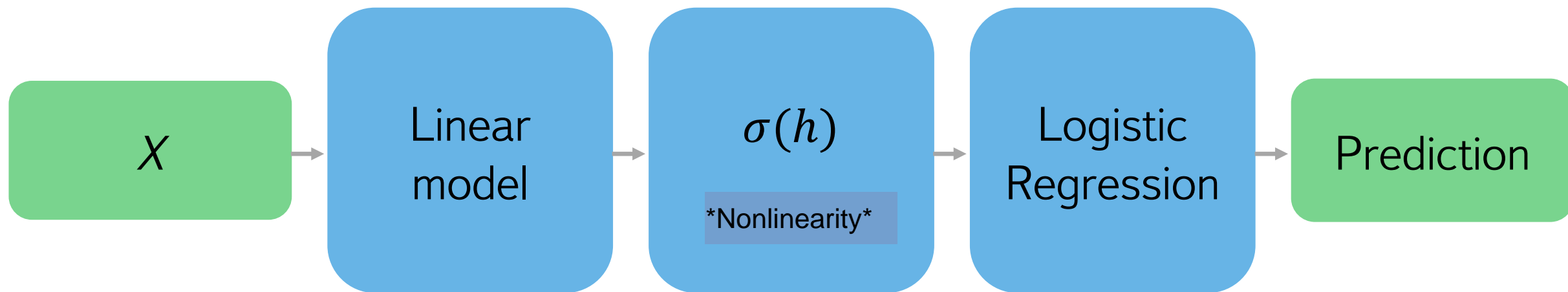
The nonlinearity



- Compute features $h_j = \sigma \left(\sum_i w_{ij}^h x_i + b_j^h \right) \quad j \in \{1, 2, \dots, n\}$
- Eventual output of the model $y_{pred} = \sigma \left(\sum_j w_j^o h_j + b^o \right)$
- Compositionality: $P(y | x) = \sigma \left(\sum_j w_j^o \sigma \left(\sum_i w_{ij}^h x_i + b_j^h \right) + b^o \right)$

Effect of the nonlinearity

lly we combine a bunch of non-lin



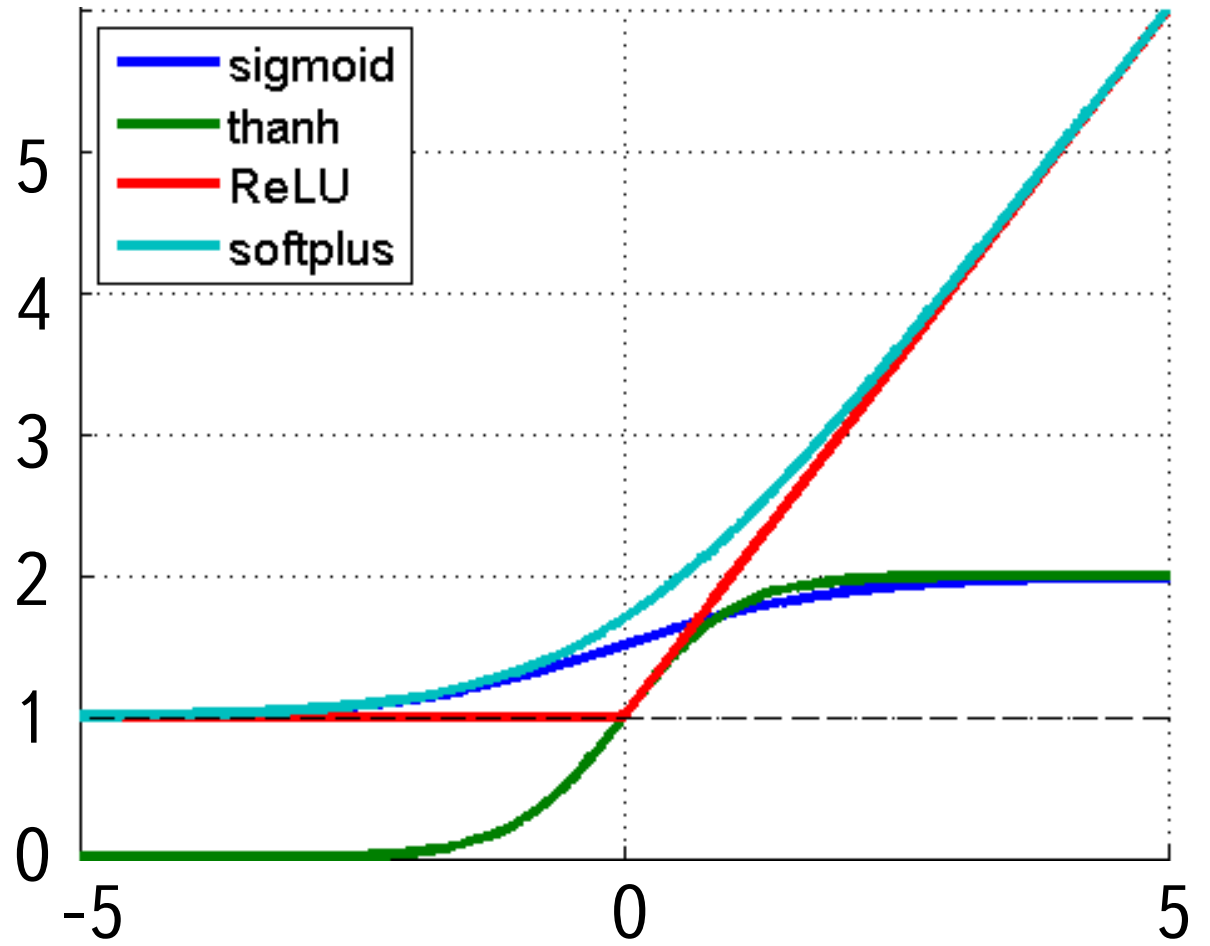
Types of nonlinearity

$$f(a) = \frac{1}{1 + e^a}$$

$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

$$f(a) = \log(1 + e^a)$$



Wei Di, <https://imiloinf.wordpress.com/2013/11/06/rectifier-nonlinearities/>

Recap and terminology

networks - combinations of these

- Layer is a building block for neural network:
 - Input layer
 - Dense layer: $f(x) = Wx + b$
 - Nonlinearity layer: $f(x) = \sigma(x)$
 - A few more: we will cover later
- Output layer
- Activation is layer output
 - i. e. some intermediate signal in the neural network

} Hidden layers

Activation is output of the output

process layers in series of the

Potential caveats?

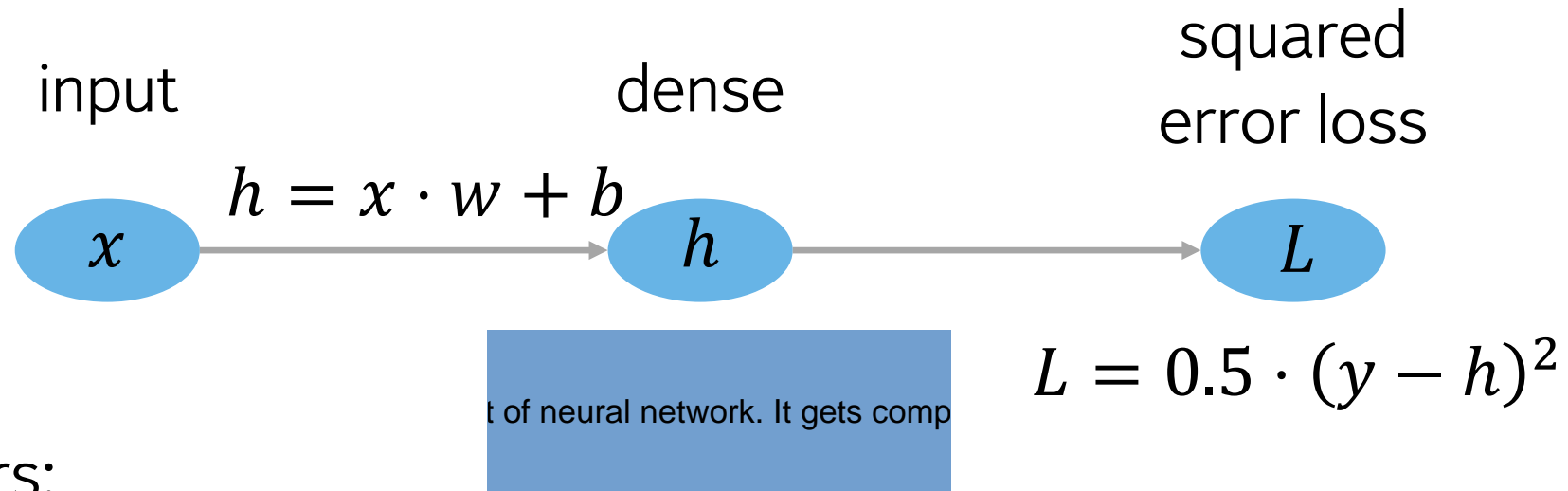
- Hardcore overfitting
- No “golden standard” for architecture
- Computationally heavy



The backpropagation algorithm

The simplest NN ever

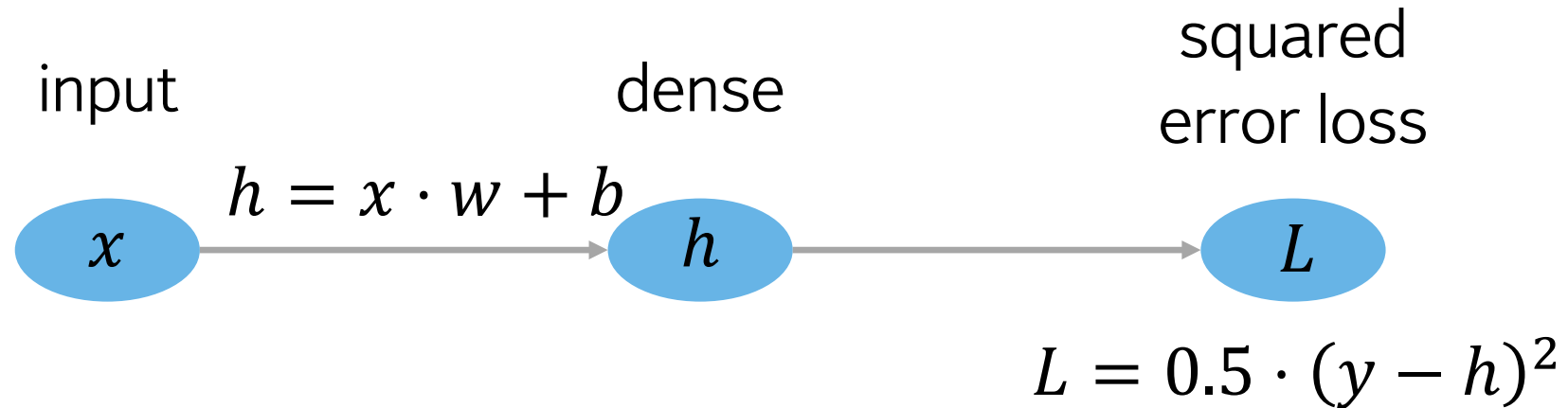
<https://data-science-group-iitr.com/data-science-group-iitr/loss-functions-and-optimization-algorithms/>



- Parameters:
 - Weight w and bias b
- Input: x
- Target: y

Also known as
least squares linear regression

The simplest NN ever

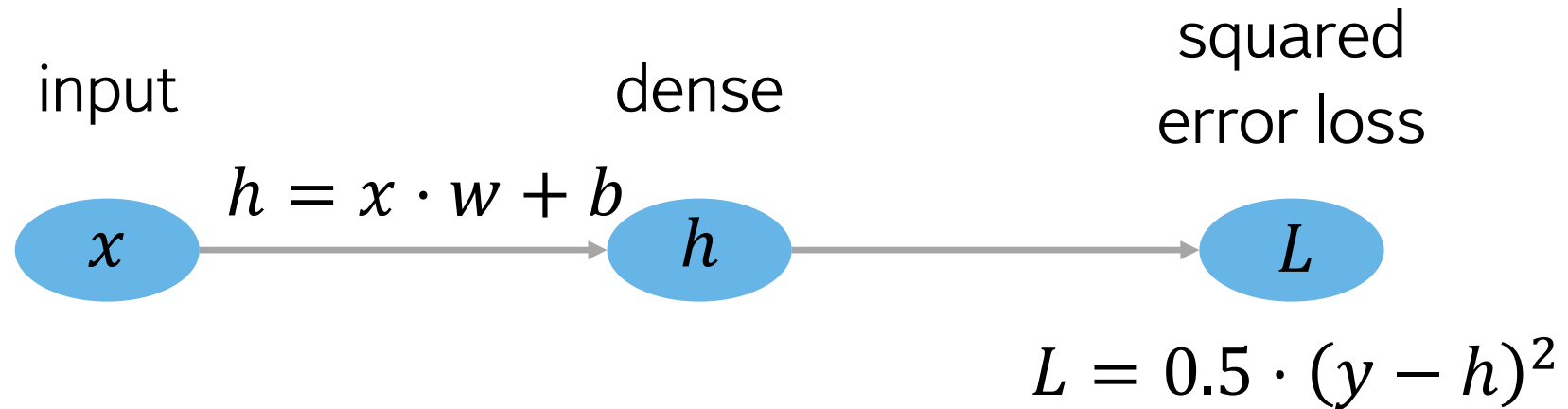


- Parameters:
 - Weight w and bias b
- Input: x
- Target: y

L is just a function of parameters, features and target:

$$L = f(y, g(x, w, b))$$

The simplest NN ever

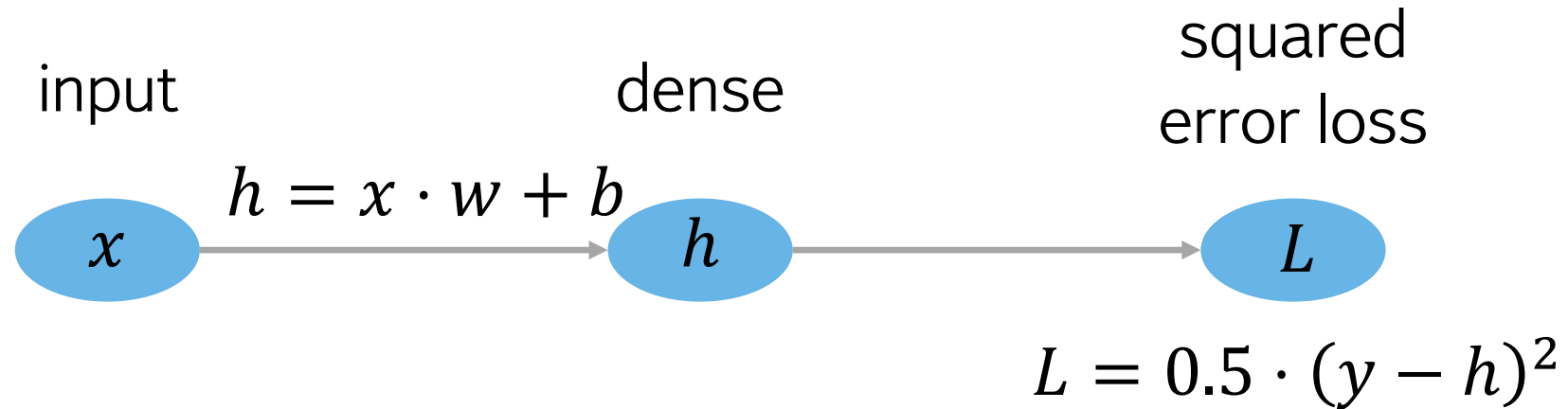


- Gradient? Gradient - one way to do this

- $\frac{\partial L}{\partial b} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial b}$

is a function that maps an event or values of one or m

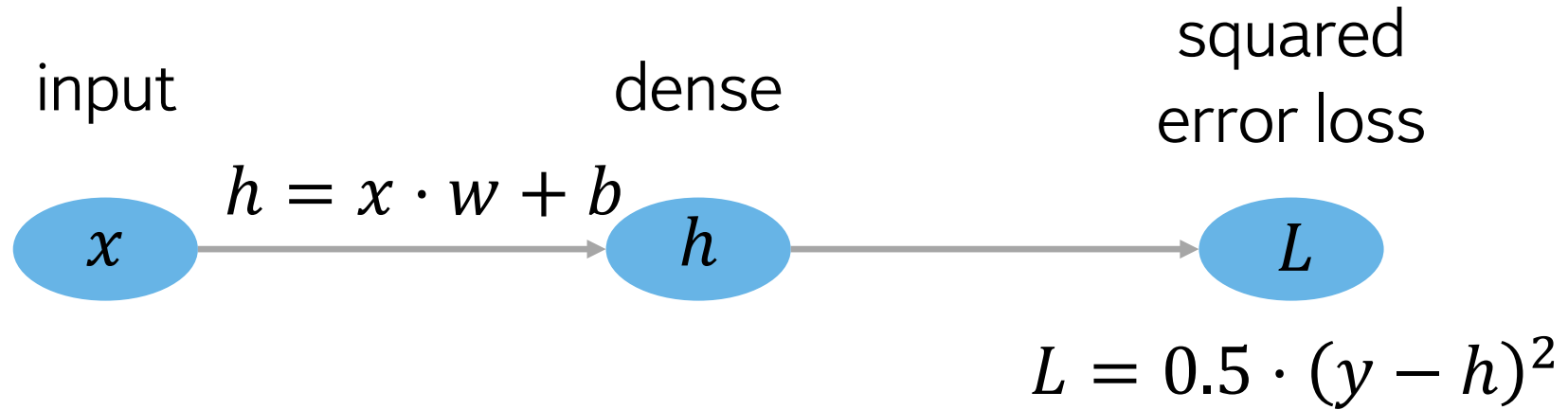
The simplest NN ever



- Let's fit
 - $y = 3, x = 1$
- Initial
 - $w = 0.1, b = 1$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b

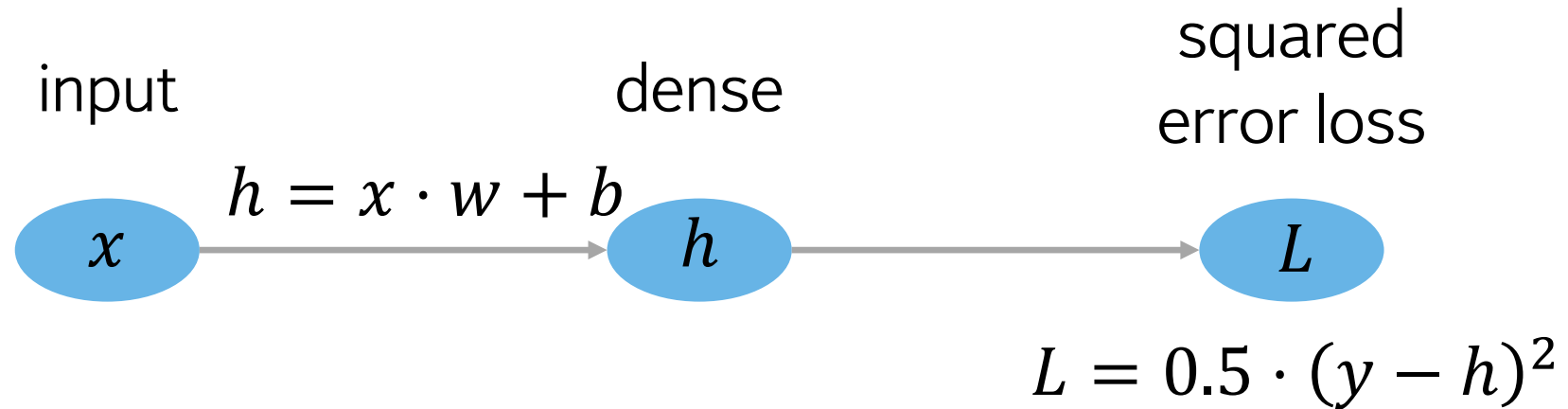
Forward pass



- Let's fit
 - $y = 3, x = 1$
- Initial
 - $w = 0.1, b = 1$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80					

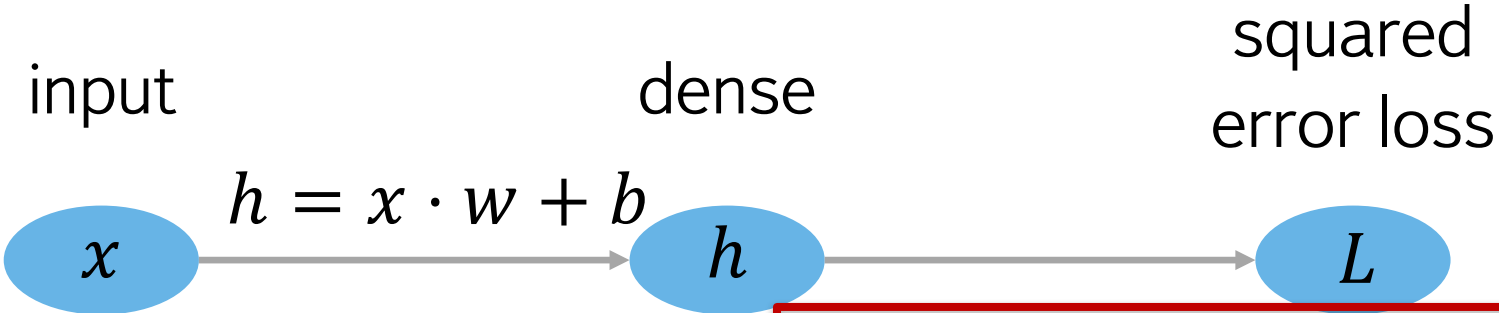
Backward pass



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1.1	1.80	-1.9				

Backward pass

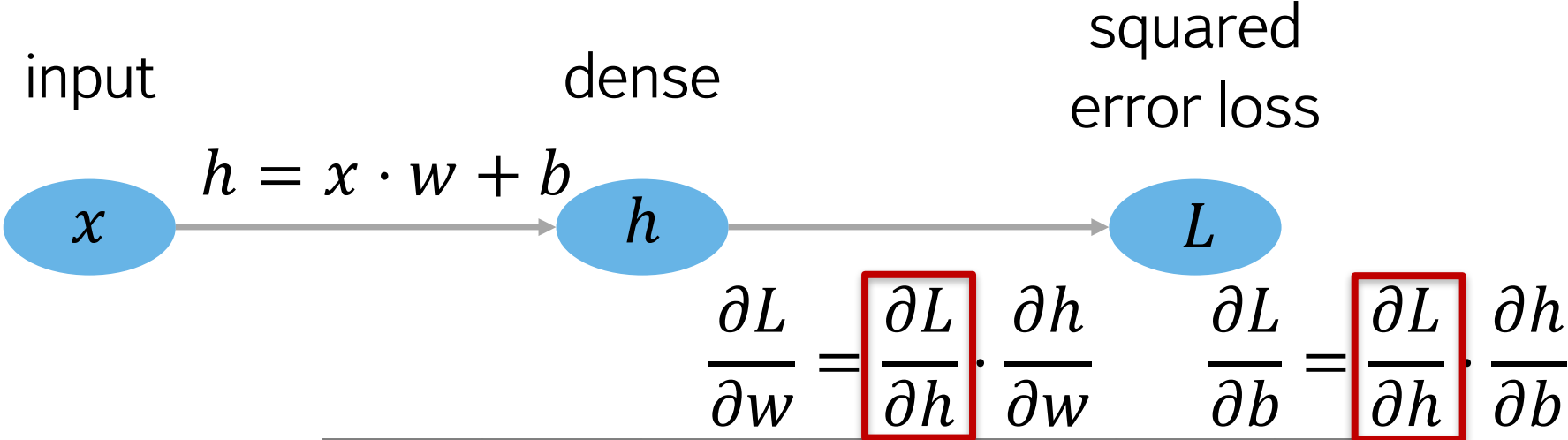


$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial w} \quad \frac{\partial L}{\partial b} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial b}$$

- Let's fit
 - $y = 3, x = 1$
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h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
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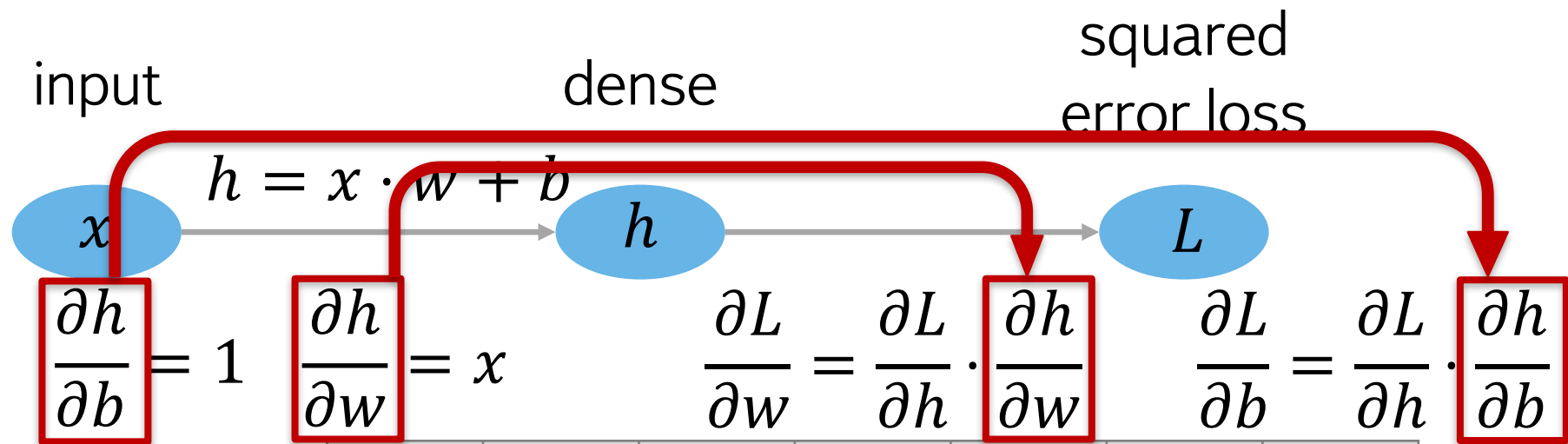
Backward pass



- Let's fit
 - $y = 3, x = 1$
- Initial
 - $w = 0.1, b = 1$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80	-1.9				

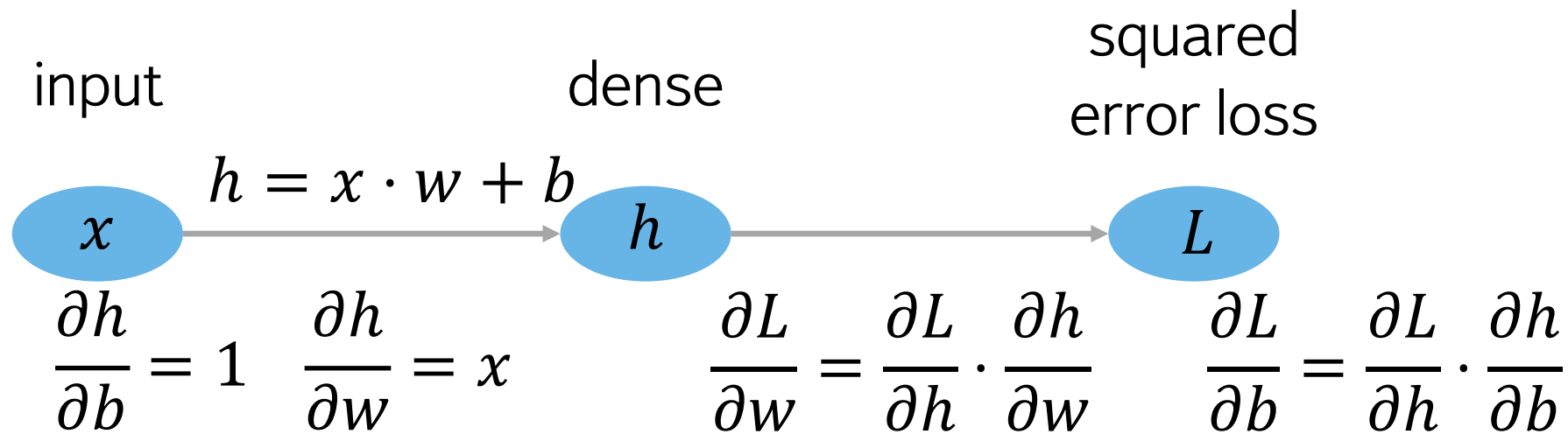
Backward pass



- Let's fit
 - $y = 3, x = 1$ single point
- Initial
 - $w = 0.1, b = 1$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80	-1.9				

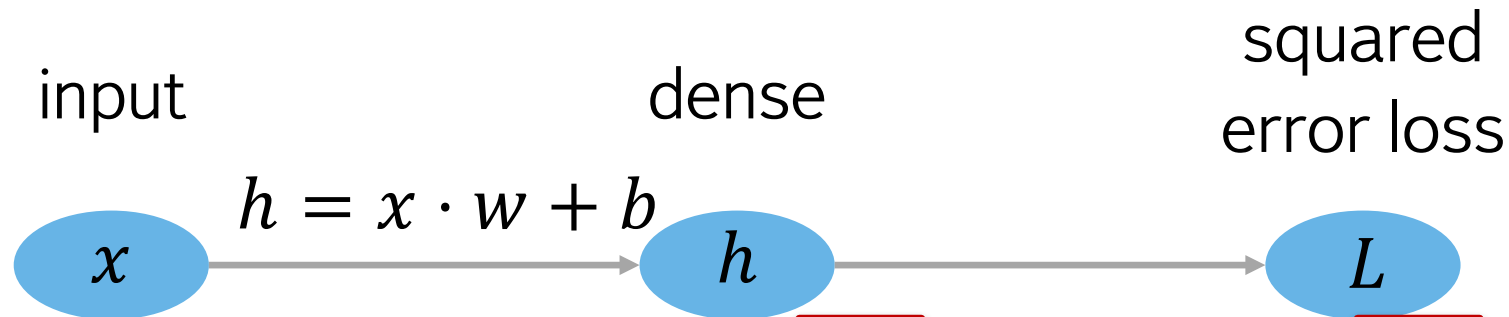
Backward pass



- Let's fit
 - $y = 3, x = 1$
- Initial
 - $w = 0.1, b = 1$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80	-1.9	-1.9	-1.9		

Backward pass



Parameters are updated as it progresses through *backprop

- Update parameters:

$$w -= \eta \frac{\partial L}{\partial w}$$

$$b -= \eta \frac{\partial L}{\partial b}$$

$$\eta = 0.2$$

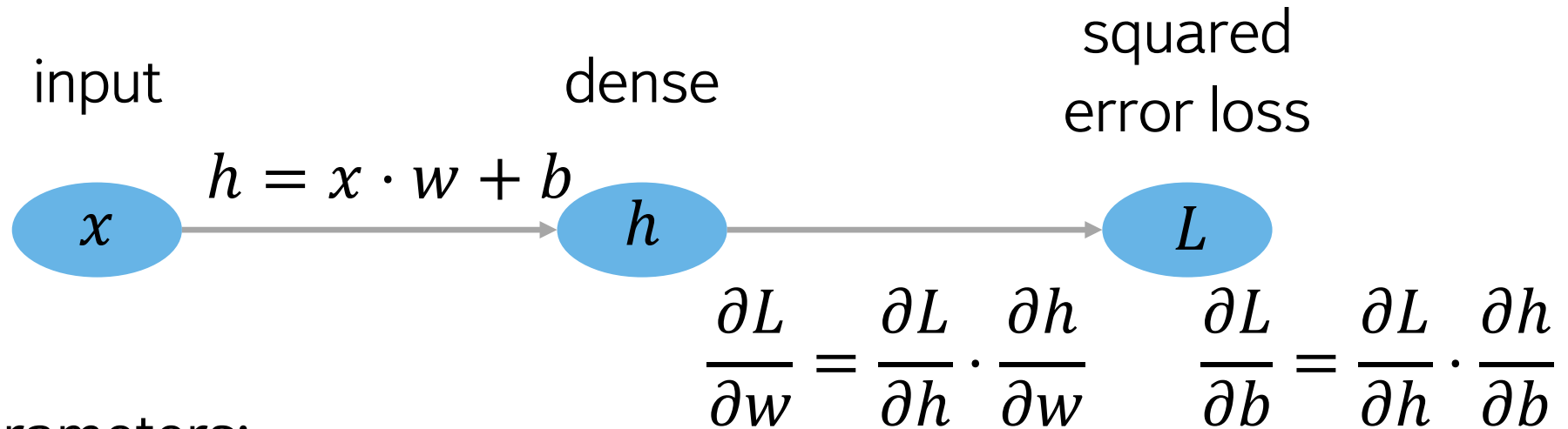
$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial w} \qquad \frac{\partial L}{\partial b} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial b}$$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80	-1.9	-1.9	-1.9		

We choose eta ourselves - design feature.

variable to change. How many percents of your weight do you

Backward pass



- Update parameters:

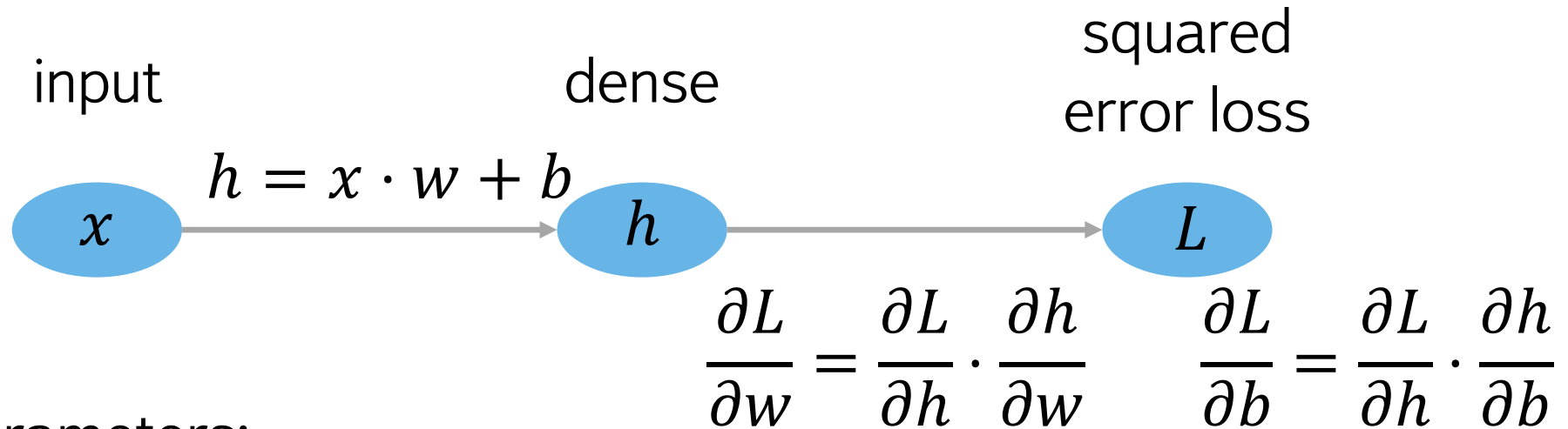
$$w -= \eta \frac{\partial L}{\partial w}$$

$$b -= \eta \frac{\partial L}{\partial b}$$

$$\eta = 0.2$$

h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80	-1.9	-1.9	-1.9	0.48	1.38

After a few more updates...



- Update parameters:

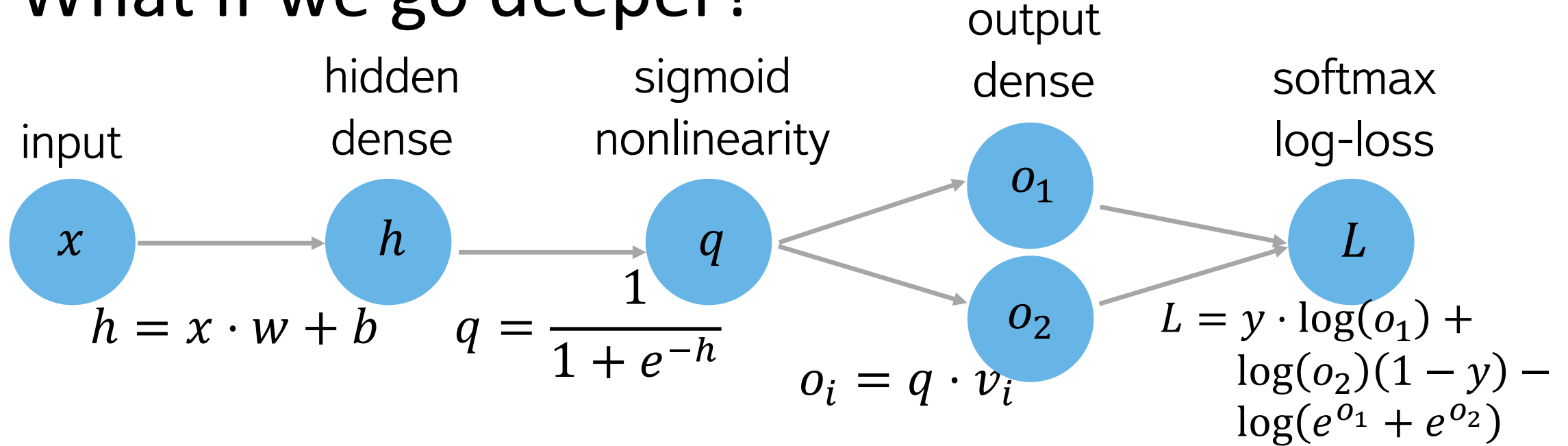
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$$b -= \eta \frac{\partial L}{\partial b}$$

$$\eta = 0.2$$

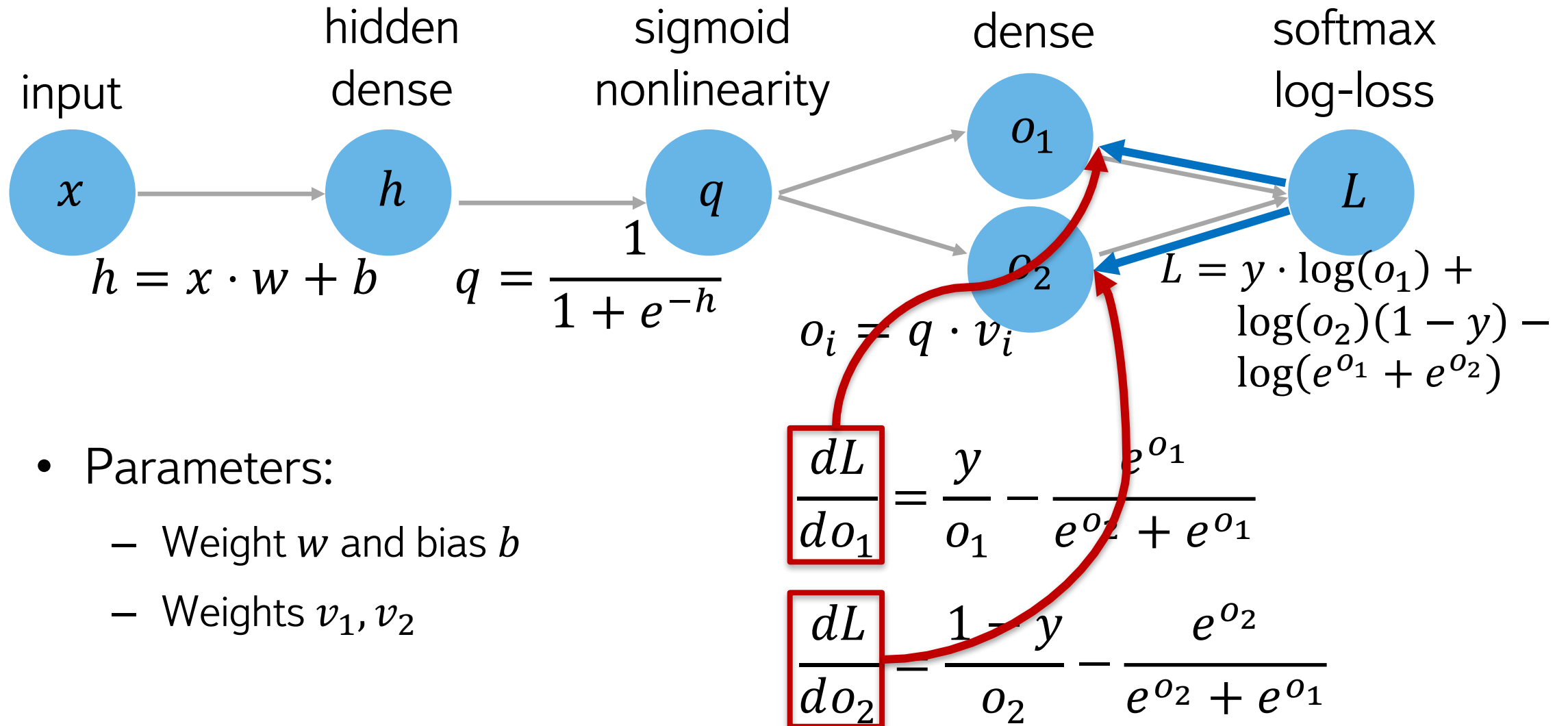
h	L	$\frac{\partial L}{\partial h}$	$\frac{\partial L}{\partial w}$	$\frac{\partial L}{\partial b}$	w	b
1.1	1.80	-1.9	-1.9	-1.9	0.48	1.38
1.86	0.65	-1.14	-1.14	-1.14	0.71	1.61
2.32	0.23	-0.68	-0.68	-0.68	0.84	1.75

What if we go deeper?

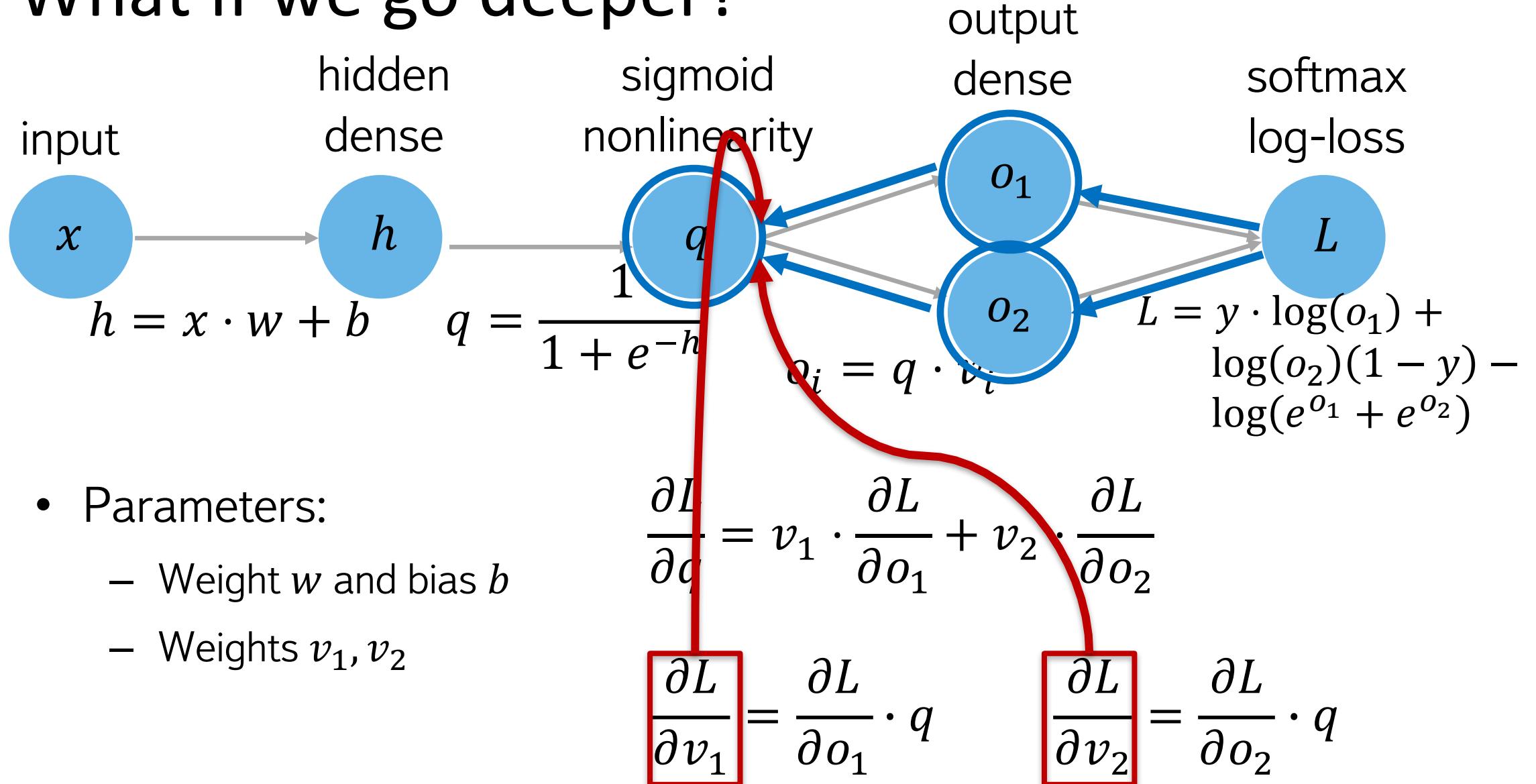


- Parameters:
 - Weight w and bias b
 - Weights v_1, v_2

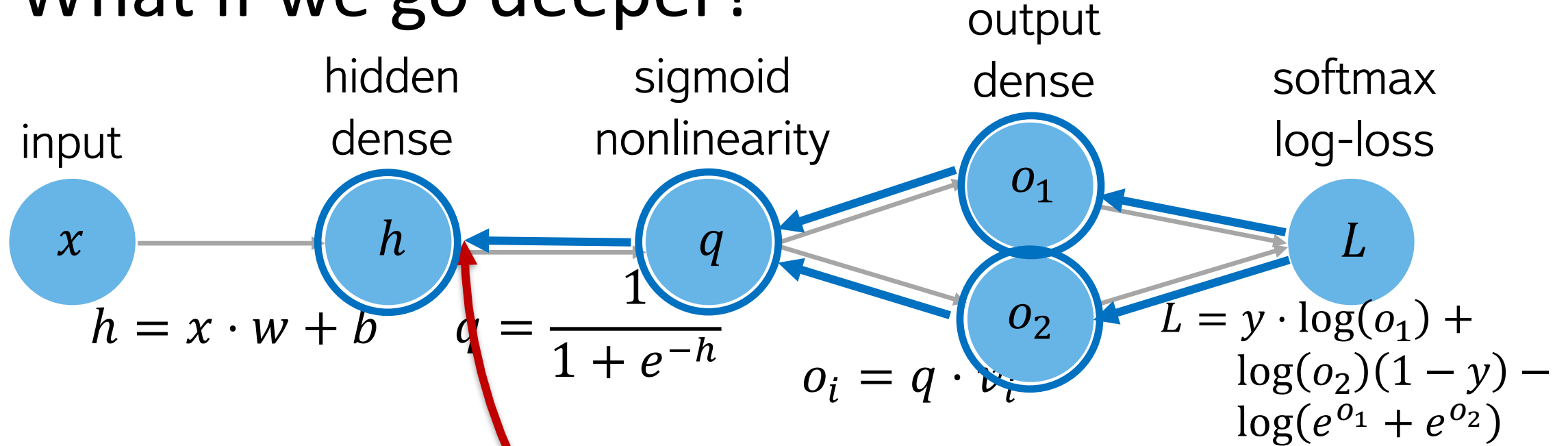
What if we go deeper?



What if we go deeper?



What if we go deeper?



- Parameters:

- Weight w and bias b
- Weights v_1, v_2

$$\boxed{\frac{\partial L}{\partial h}} = \frac{\partial L}{\partial q} \frac{e^{-q}}{(1 + e^{-q})^2}$$

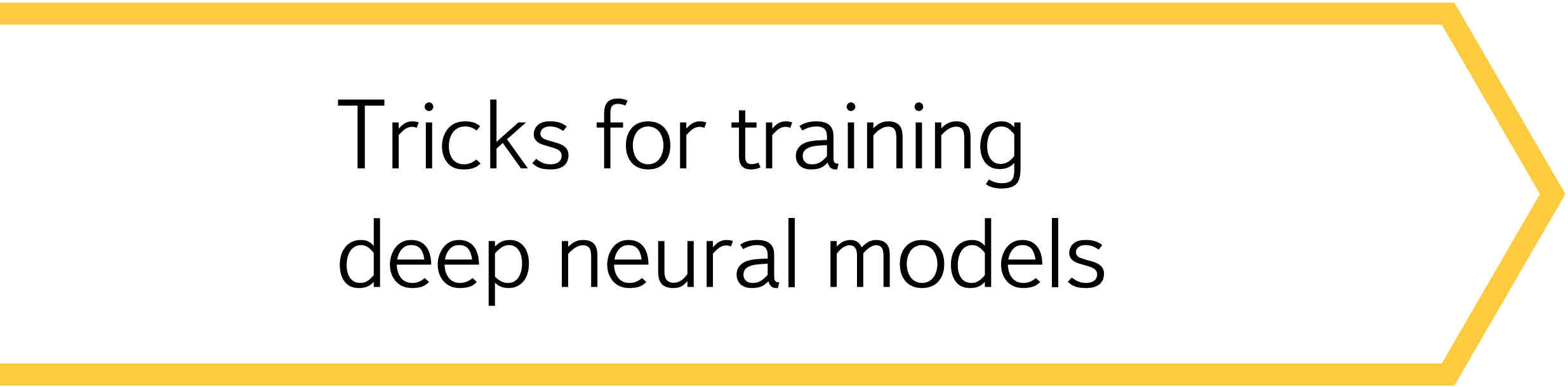
Backpropagation: the algorithm

- Chain rule can be evaluated numerically!
- Compute the network output and the loss value
- Compute "dLoss" / "dActivation_of_output_layer"
- For each layer, starting from the last:
 - Compute "dActivation" / "dLayer_parameters",
"dActivation" / "dLayer_input"
 - Multiply it by "dLoss" / "dActivation", get "dLoss" / ...
- Make optimization step for the parameters

technique called automatic differentiation. In the c

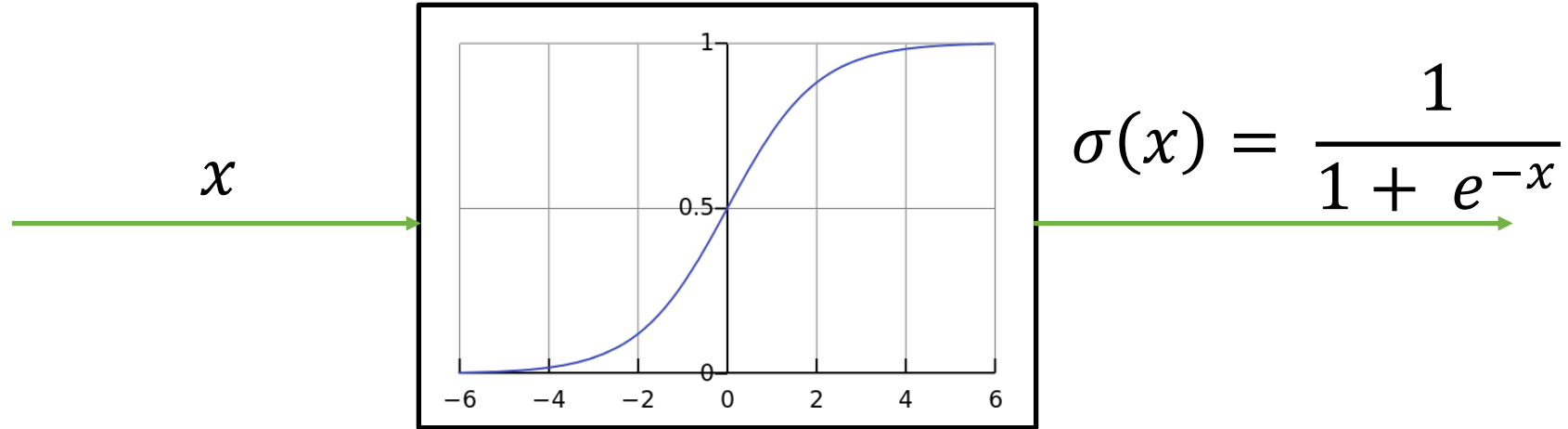
Intermediate conclusion

- You can have any crazy layer as long as you can compute its gradient
- **In fact:** no need to compute the gradients by hand!
 - There are frameworks for that (e.g. theano, tensorflow, and pytorch)

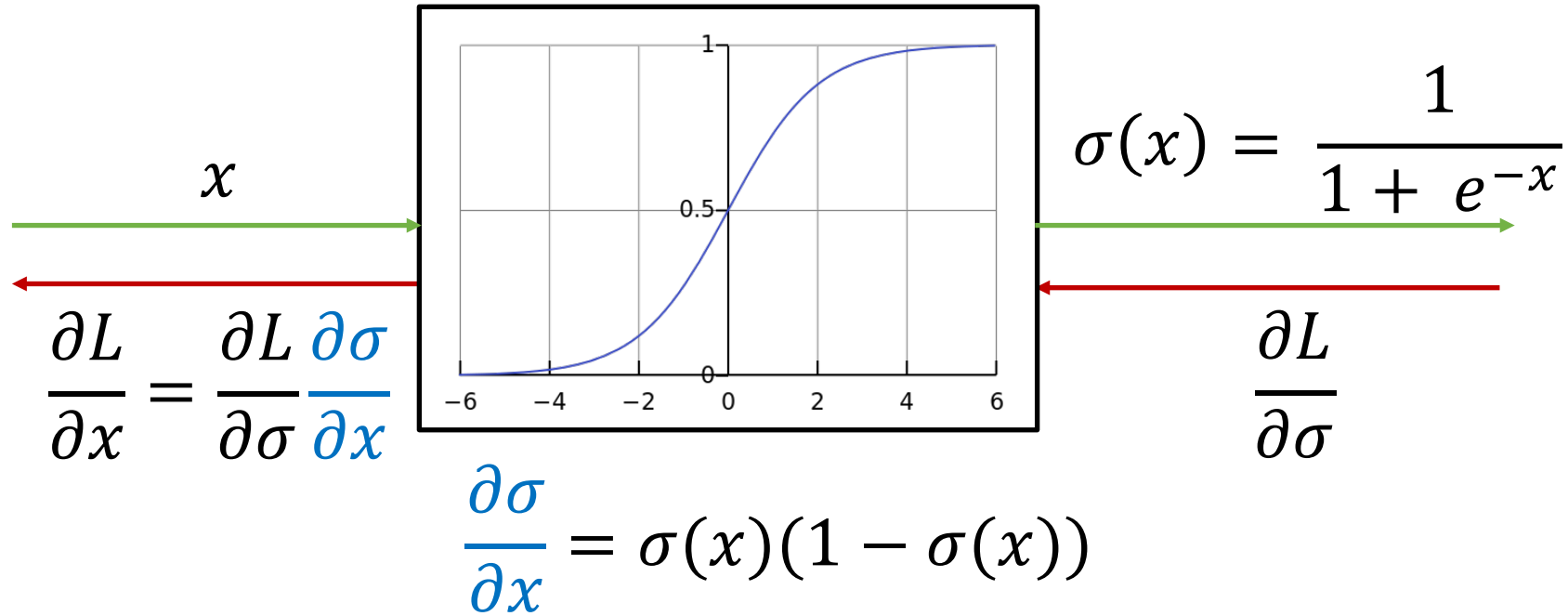
A thick yellow arrow pointing to the right, with a white rectangular area in the center containing the text.

Tricks for training deep neural models

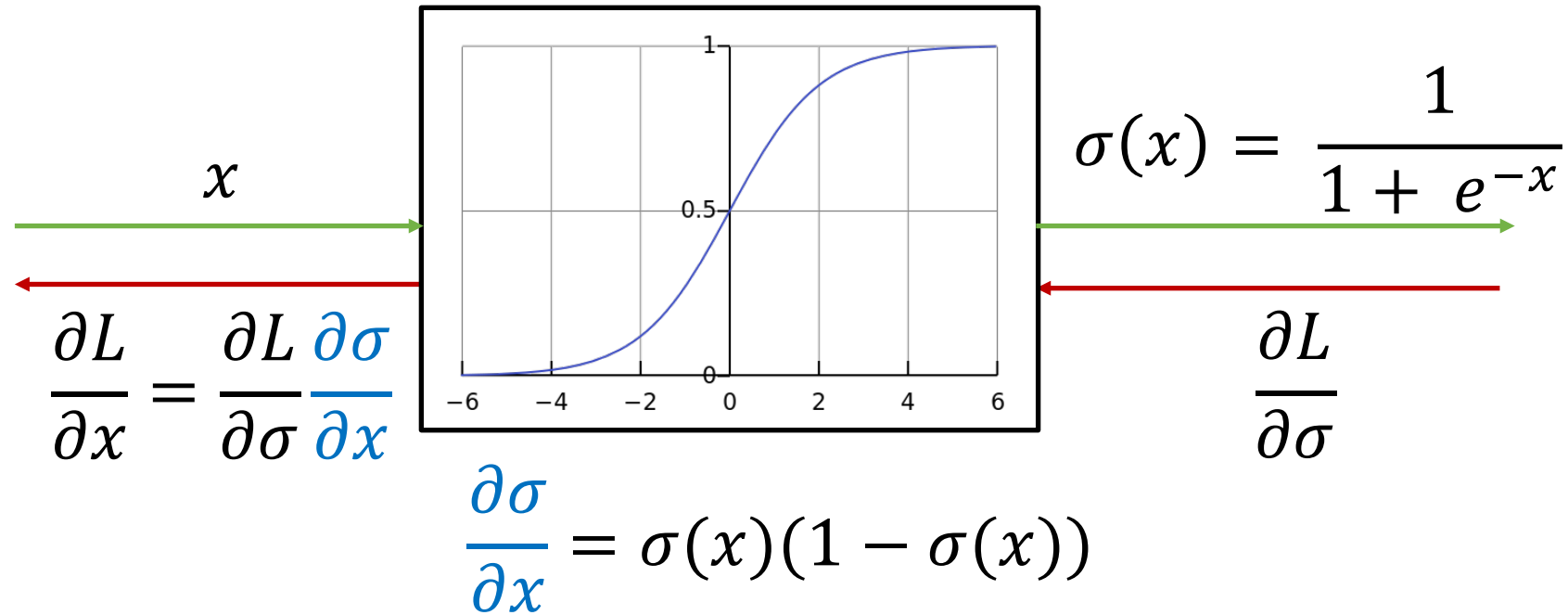
Sigmoid activation



Sigmoid activation



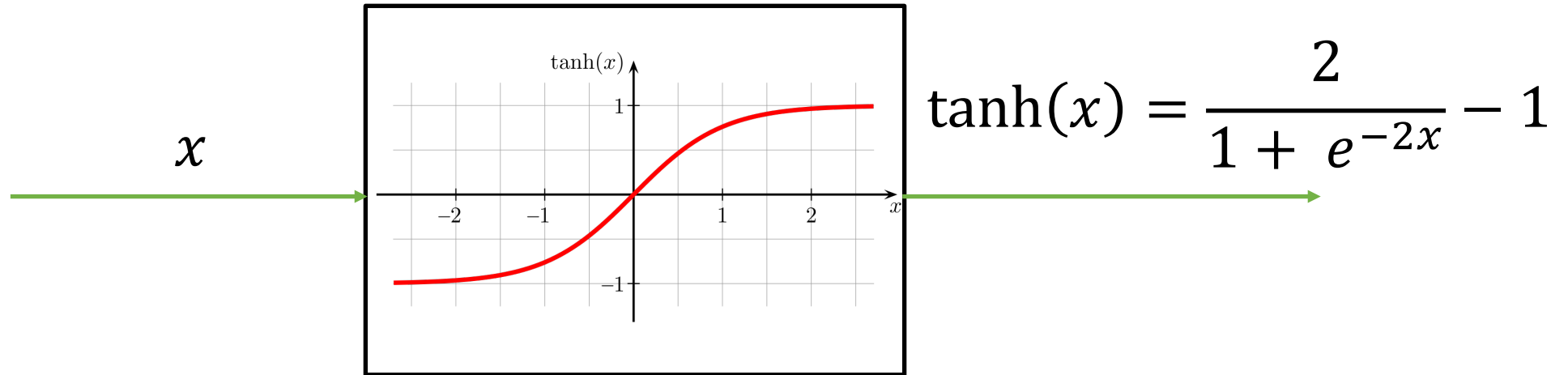
Sigmoid activation



- Sigmoid neurons can saturate and lead to vanishing gradients
- Not zero-centered
- e^x is computationally expensive

any values - no meaningful solution

Tanh activation

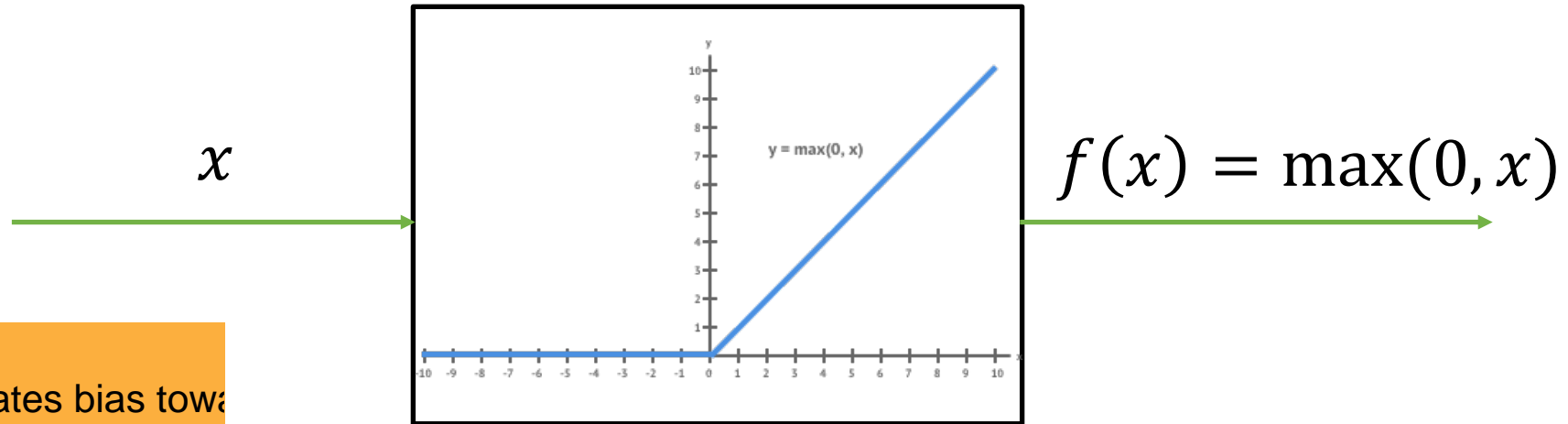


- Zero-centered
- But still pretty much like sigmoid

saturates and has vanishing gra

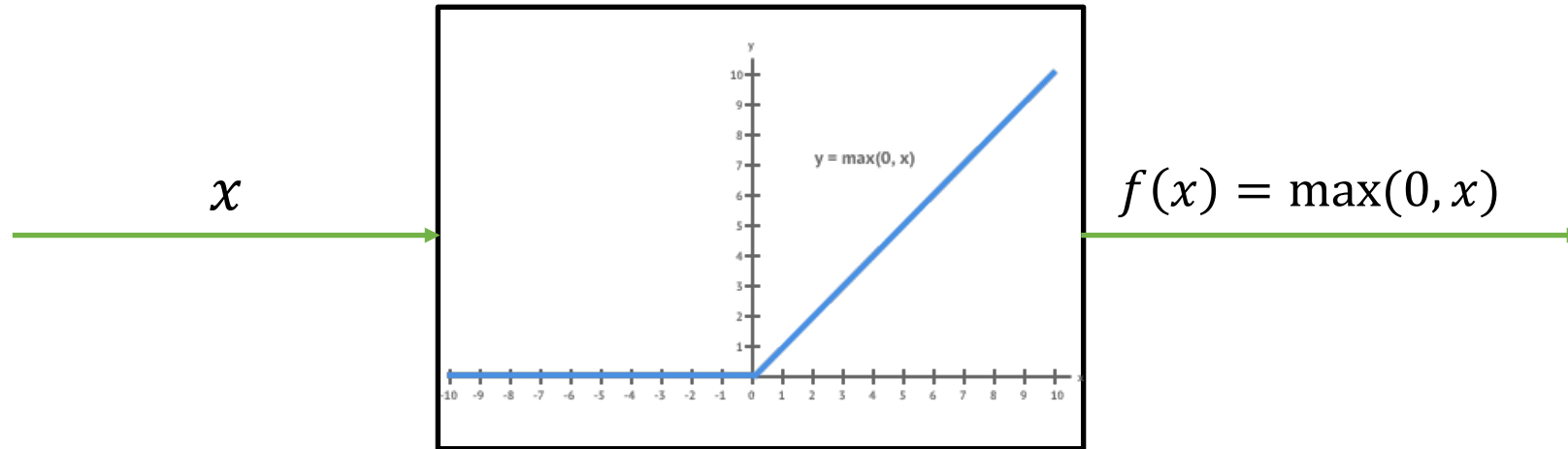
ReLU activation

e property. Eliminates bias toward



- Fast to compute
- Gradients do not vanish for $x > 0$
- Provides faster convergence in practice!

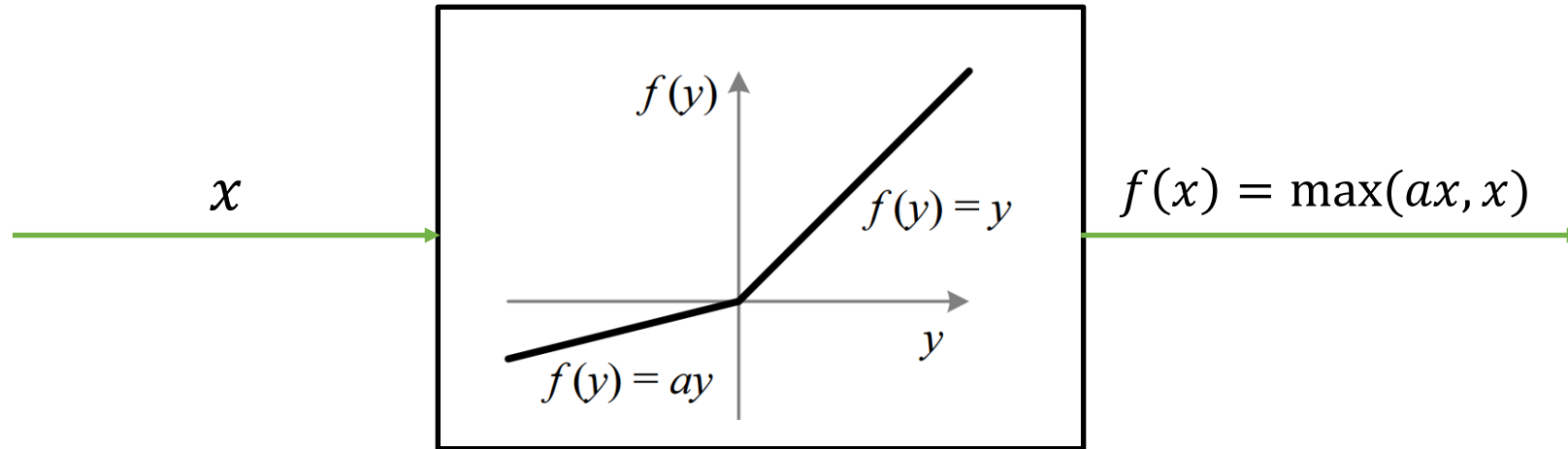
ReLU activation



- Fast to compute.
- Gradients do not vanish for $x > 0$.
- Provides faster convergence in practice!
- Not zero-centered.
- Can die: if not activated, never updates!

inactive values will basically turn-off the

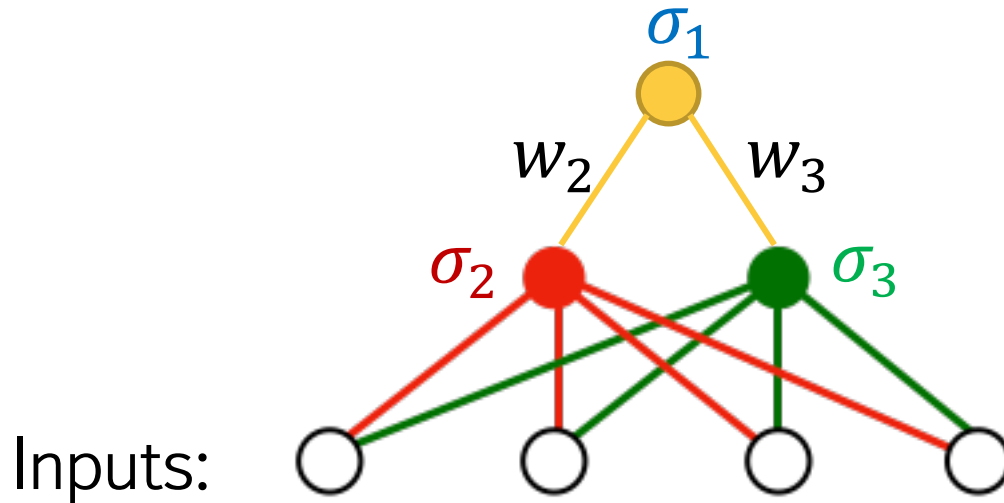
Leaky ReLU activation



- Will not die!
- $a \neq 1$

***Usually 10^{-2}

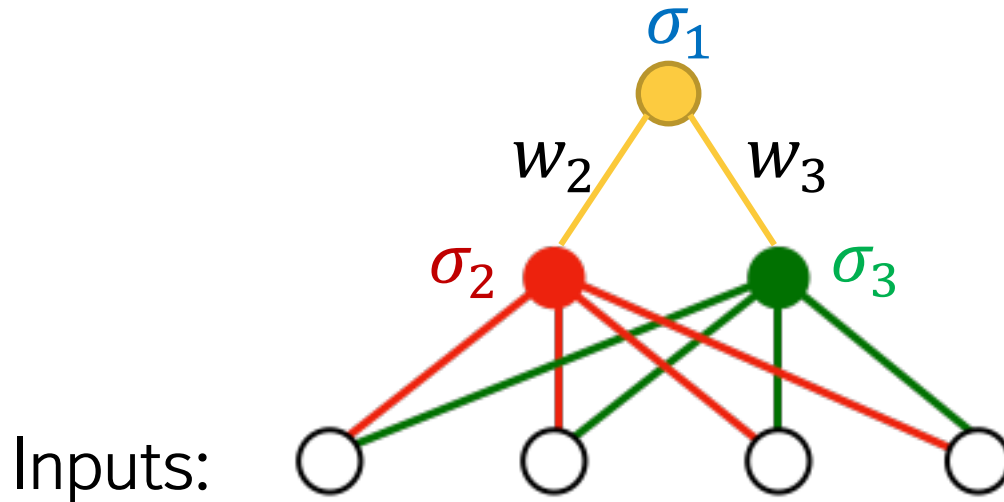
Weights initializations



$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \sigma_1} \sigma_1 (1 - \sigma_1) \sigma_2$$
$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial \sigma_1} \sigma_1 (1 - \sigma_1) \sigma_3$$

- Maybe start with all zeros?

Weights initializations



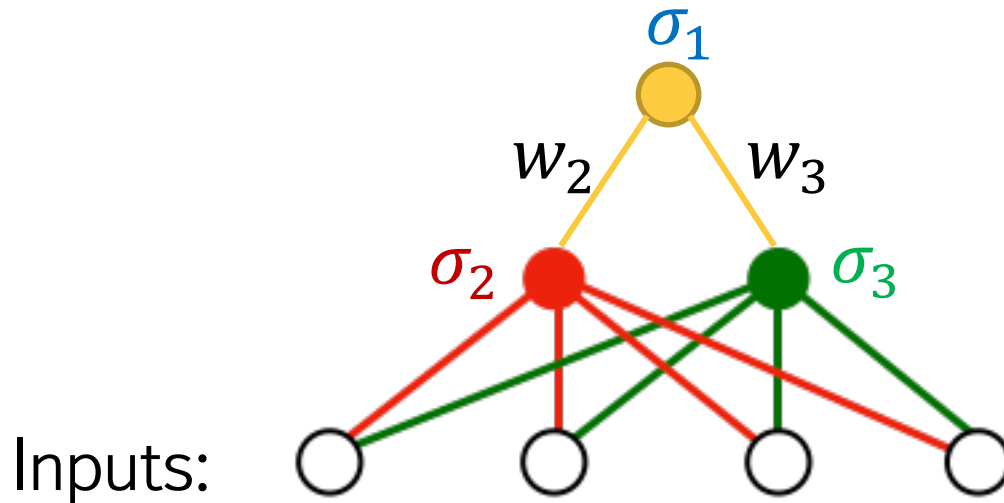
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σ_2 and σ_3 will always get the same updates!

- Maybe start with all zeros?

Weights initializations



$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \sigma_1} \sigma_1 (1 - \sigma_1) \sigma_2$$

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial \sigma_1} \sigma_1 (1 - \sigma_1) \sigma_3$$

σ_2 and σ_3 will always get the same updates!

- ~~Maybe start with all zeros?~~
- Need to break symmetry!
- Maybe start with small random numbers then?
- But how small? $0.03 \cdot \mathcal{N}(0,1)$?

Weights initializations

- Linear models work best when inputs are normalized.
- Neuron is a linear combination of inputs + activation.
- Neuron output will be used by consecutive layers.

Weights initializations

- Let's look at the neuron output **before activation**: $\sum_{i=1}^n x_i w_i$.
- If $E(x_i) = E(w_i) = 0$ and we generate weights independently from inputs, then $E(\sum_{i=1}^n x_i w_i) = 0$.
- But variance can grow with consecutive layers.
- Empirically this hurts convergence for deep networks!

Weights initializations

- Let's look at the variance of $\sum_{i=1}^n x_i w_i$:

Weights initializations

- Let's look at the variance of $\sum_{i=1}^n x_i w_i$: i.i.d. w_i and mostly uncorrelated x_i

$$\begin{aligned} \text{Var}(\sum_{i=1}^n x_i w_i) &= \\ &= \sum_{i=1}^n \text{Var}(x_i w_i) = \end{aligned}$$

Weights initializations

- Let's look at the variance of $\sum_{i=1}^n x_i w_i$: i.i.d. w_i and mostly uncorrelated x_i

$$\text{Var}(\sum_{i=1}^n x_i w_i) =$$

$$= \sum_{i=1}^n \text{Var}(x_i w_i) =$$

independent factors w_i and x_i

$$= \sum_{i=1}^n \left(\begin{array}{l} [E(x_i)]^2 \text{Var}(w_i) \\ + [E(w_i)]^2 \text{Var}(x_i) \\ + \text{Var}(x_i) \text{Var}(w_i) \end{array} \right) =$$

Weights initializations

- Let's look at the variance of $\sum_{i=1}^n x_i w_i$: i.i.d. w_i and mostly uncorrelated x_i

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w_i and x_i have 0 mean

$$= \sum_{i=1}^n \text{Var}(x_i) \text{Var}(w_i) = \text{Var}(x) [\mathbf{n} \mathbf{Var}(\mathbf{w})]$$

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We want this to be 1

Weights initializations

- Let's use the fact that $Var(aw) = a^2 Var(w)$.
- For $[n Var(aw)]$ to be 1 we need to multiply $\mathcal{N}(0,1)$ weights ($Var(w) = 1$) by $a = 1/\sqrt{n}$.
- Xavier initialization (Glorot et al.) multiplies weights by $\sqrt{2}/\sqrt{n_{in} + n_{out}}$.
- Initialization for ReLU neurons (He et al.) uses multiplication by $\sqrt{2}/\sqrt{n_{in}}$.

Batch normalization

- We know how to initialize our network to constrain variance.
- But what if it grows during backpropagation?
- Batch normalization controls mean and variance of outputs before activations.

Batch normalization

- Let's normalize h_i — neuron output before activation:

$$h_i = \gamma_i \frac{h_i - \mu_i}{\sqrt{\sigma_i^2}} + \beta_i \rightarrow 0 \text{ mean, unit variance}$$

Batch normalization

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- Where do μ_i and σ_i^2 come from? We can estimate them having **a current training batch!**

Batch normalization

- Let's normalize h_i — neuron output before activation:

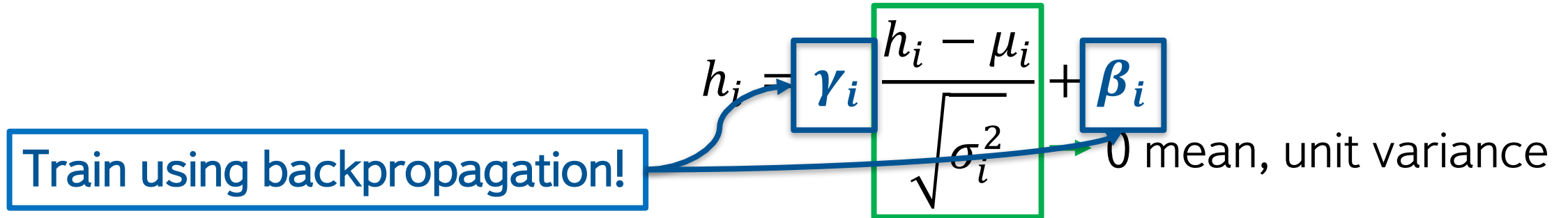
$$h_i = \gamma_i \left[\frac{h_i - \mu_i}{\sqrt{\sigma_i^2}} \right] + \beta_i \rightarrow 0 \text{ mean, unit variance}$$

- Where do μ_i and σ_i^2 come from? We can estimate them having **a current training batch!**
- During testing we will use an exponential moving average over batches:

$$0 < \alpha < 1 \quad \begin{aligned} \mu_i &= \alpha \cdot \mathbf{mean}_{\text{batch}} + (1 - \alpha) \cdot \mu_i \\ \sigma_i^2 &= \alpha \cdot \mathbf{variance}_{\text{batch}} + (1 - \alpha) \cdot \sigma_i^2 \end{aligned}$$

Batch normalization

- Let's normalize h_i — neuron output before activation:



- Where do μ_i and σ_i^2 come from? We can estimate them having a **current training batch**!
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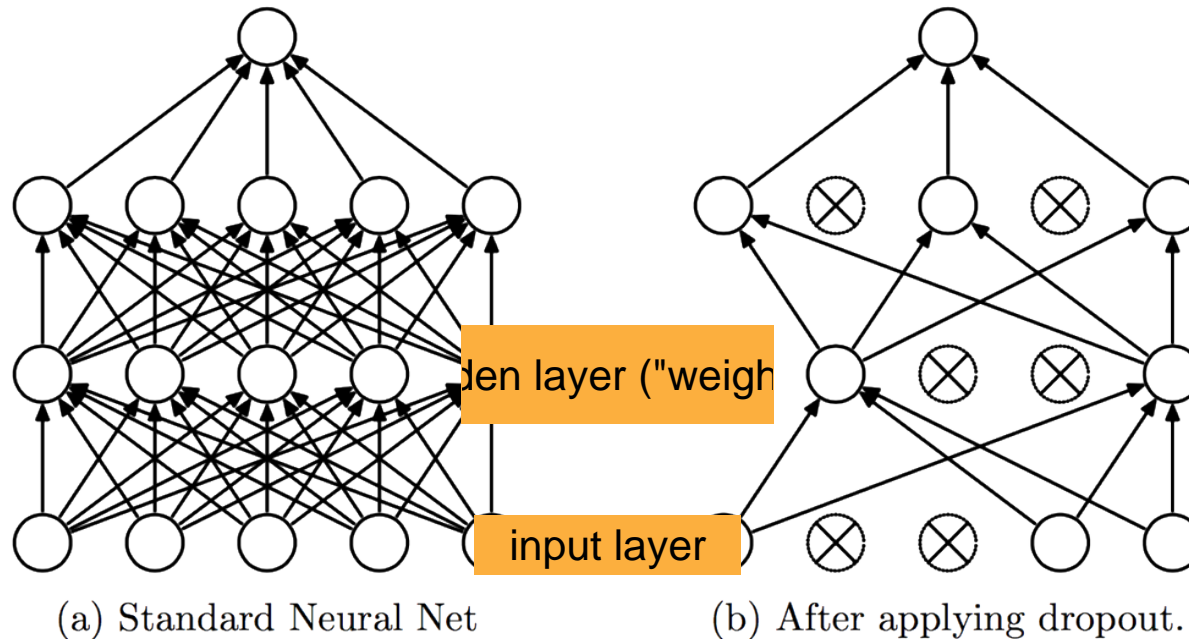
$$0 < \alpha < 1 \quad \mu_i = \alpha \cdot \mathbf{mean}_{\text{batch}} + (1 - \alpha) \cdot \mu_i$$
$$\sigma_i^2 = \alpha \cdot \mathbf{variance}_{\text{batch}} + (1 - \alpha) \cdot \sigma_i^2$$

Dropout

- A regularization technique to reduce overfitting.
- We keep neurons active (non-zero) with probability p .
- This way we sample the network during training and change only a subset of its parameters on every iteration.

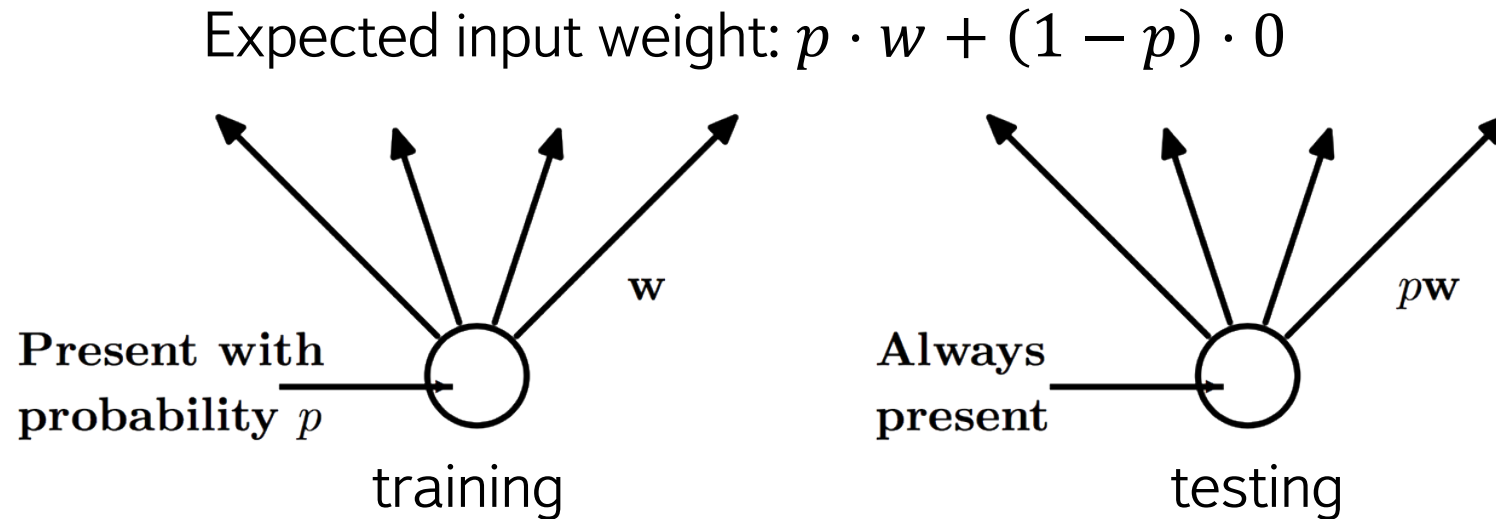
(hidden layer)--> single fea

value (each dot is different weight - eg. d



Dropout

- During testing all neurons are present but their outputs are multiplied by p to maintain the scale of inputs:



Nitish Srivastava, <http://www.cs.toronto.edu/~rsalakhu/papers/srivastava14a.pdf>

- The authors of dropout say it is similar to having an ensemble of exponentially large number of smaller networks.

Takeaways

- Use ReLU activation
- Use He et al. initialization
- Try to add batchnorm or dropout
- Try to augment your training data