

Bits, Bytes, and Integers

Instructors:

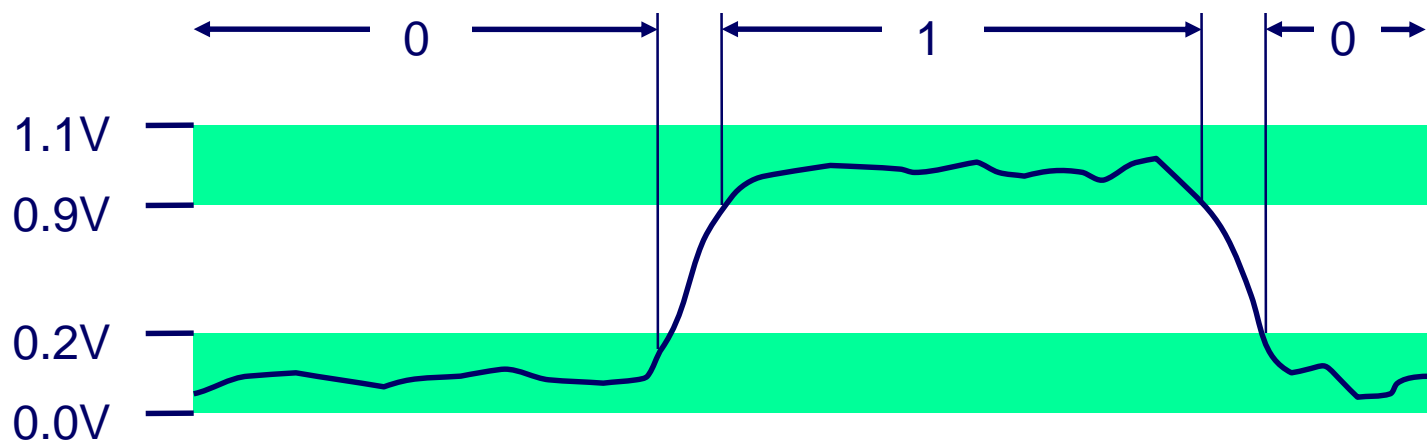
Sungyong Ahn

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- **Representations in memory, pointers, strings**

Everything is bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
 - Computers determine what to do (instructions)
 - ... and represent and manipulate numbers, sets, strings, etc...
- **Why bits? Electronic Implementation**
 - Easy to store with bistable elements
 - Reliably transmitted on noisy and inaccurate wires



For example, can count in binary

■ Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Encoding Byte Values

■ Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
- Hexadecimal 00_{16} to FF_{16}
 - Base 16 number representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Write $FA1D37B_{16}$ in C as
 - `0xFA1D37B`
 - `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
<code>char</code>	1	1	1
<code>short</code>	2	2	2
<code>int</code>	4	4	4
<code>long</code>	4	8	8
<code>float</code>	4	4	4
<code>double</code>	8	8	8
<code>long double</code>	–	–	10/16
<code>pointer</code>	4	8	8

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Boolean Algebra

■ Developed by George Boole in 19th Century

- Algebraic representation of logic
 - Encode “True” as 1 and “False” as 0

And

- $A \& B = 1$ when both $A=1$ and $B=1$

$\&$	0	1
0	0	0
1	0	1

Or

- $A | B = 1$ when either $A=1$ or $B=1$

$ $	0	1
0	0	1
1	1	1

Not

- $\sim A = 1$ when $A=0$

\sim	
0	1
1	0

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

\wedge	0	1
0	0	1
1	1	0

General Boolean Algebras

■ Operate on Bit Vectors

- Operations applied bitwise

01101001	01101001	01101001	
& 01010101	01010101	^ 01010101	~ 01010101
<u> </u>	<u> </u>	<u> </u>	<u> </u>
01000001	01111101	00111100	10101010

■ All of the Properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

■ Representation

- Width w bit vector represents subsets of $\{0, \dots, w-1\}$
- $a_j = 1$ if $j \in A$

▪ 01101001 $\{0, 3, 5, 6\}$

▪ 76543210

▪ 01010101 $\{0, 2, 4, 6\}$

▪ 76543210

■ Operations

- | | | |
|----------------------------|----------|------------------------|
| ▪ & Intersection | 01000001 | $\{0, 6\}$ |
| ▪ Union | 01111101 | $\{0, 2, 3, 4, 5, 6\}$ |
| ▪ ^ Symmetric difference | 00111100 | $\{2, 3, 4, 5\}$ |
| ▪ ~ Complement | 10101010 | $\{1, 3, 5, 7\}$ |

Bit-Level Operations in C

■ Operations $\&$, $|$, \sim , \wedge Available in C

- Apply to any “integral” data type
 - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

■ Examples (Char data type)

- $\sim 0x41 \rightarrow 0xBE$
 - $\sim 0100\ 0001_2 \rightarrow 1011\ 1110_2$
- $\sim 0x00 \rightarrow 0xFF$
 - $\sim 0000\ 0000_2 \rightarrow 1111\ 1111_2$
- $0x69 \& 0x55 \rightarrow 0x41$
 - $0110\ 1001_2 \& 0101\ 0101_2 \rightarrow 0100\ 0001_2$
- $0x69 | 0x55 \rightarrow 0x7D$
 - $0110\ 1001_2 | 0101\ 0101_2 \rightarrow 0111\ 1101_2$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Contrast: Logic Operations in C

■ Contrast to Logical Operators

- `&&`, `||`, `!`
 - View 0 as “False”
 - Anything nonzero as “True”
 - Always return 0 or 1
 - Early termination

■ Examples (char data type)

- `!0x41 → 0x00`
- `!0x00 → 0x01`
- `!!0x41 → 0x01`

- `0x69 && 0x55 → 0x01`
- `0x69 || 0x55 → 0x01`
- `p && *p` (avoids null pointer access)

Contrast: Logic Operations in C

■ Contrast to Logical Operators

- `&&`, `||`, `!`
 - View 0 as “False”
 - Anything nonzero
 - Always returns 0 or 1
 - **Early exit**

■ Example

- `!0x41` → 0
- `!0x00` → 1
- `!!0x41` → 1

- `0x69 && 0x55` → `0x01`
- `0x69 || 0x55` → `0x01`
- `p && *p` (avoids null pointer access)

**Watch out for `&&` vs. `&` (and `||` vs. `|`)...
one of the more common oopsies in
C programming**

Shift Operations

■ Left Shift: $x \ll y$

- Shift bit-vector x left y positions
 - Throw away extra bits on left
 - Fill with 0's on right

■ Right Shift: $x \gg y$

- Shift bit-vector x right y positions
 - Throw away extra bits on right
- Logical shift
 - Fill with 0's on left
- Arithmetic shift
 - Replicate most significant bit on left

■ Undefined Behavior

- Shift amount < 0 or \geq word size

Argument x	01100010
$\ll 3$	00010000
Log. $\gg 2$	00011000
Arith. $\gg 2$	00011000

Argument x	10100010
$\ll 3$	00010000
Log. $\gg 2$	00101000
Arith. $\gg 2$	11101000

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음수의 표현

■ Signed-magnitude

- $109 = 01101101_2 \rightarrow 11101101 = -109$

■ One's complement

- $109 = 01101101_2 \rightarrow 10010010 = -109$

```
01101101
+ 10010010
-----
11111111
```

■ Two's complement

- $109 = 01101101_2 \rightarrow 10010011 = -109$

```
01101101
+ 10010011
-----
100000000
```


Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x = 15213;
short int y = -15213;
```

Sign
Bit



■ C short 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

■ Sign Bit

- For 2's complement, most significant bit indicates sign
 - 0 for nonnegative
 - 1 for negative

Two-complement: Simple Example

	-16	8	4	2	1	
10 =	0	1	0	1	0	$8+2 = 10$

	-16	8	4	2	1	
-10 =	1	0	1	1	0	$-16+4+2 = -10$

Two-complement Encoding Example (Cont.)

x = 15213: 00111011 01101101
y = -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum	15213		-15213	

Numeric Ranges

■ Unsigned Values

- $UMin = 0$
000...0
- $UMax = 2^w - 1$
111...1

■ Two's Complement Values

- $TMin = -2^{w-1}$
100...0
- $TMax = 2^{w-1} - 1$
011...1

■ Other Values

- Minus 1
111...1

Values for $W = 16$

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

■ Observations

- $|TMin| = TMax + 1$
 - Asymmetric range
- $UMax = 2 * TMax + 1$

■ C Programming

- `#include <limits.h>`
- Declares constants, e.g.,
 - `ULONG_MAX`
 - `LONG_MAX`
 - `LONG_MIN`
- Values platform specific

Unsigned & Signed Numeric Values

X	$B2U(X)$	$B2T(X)$
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

■ Equivalence

- Same encodings for nonnegative values

■ Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

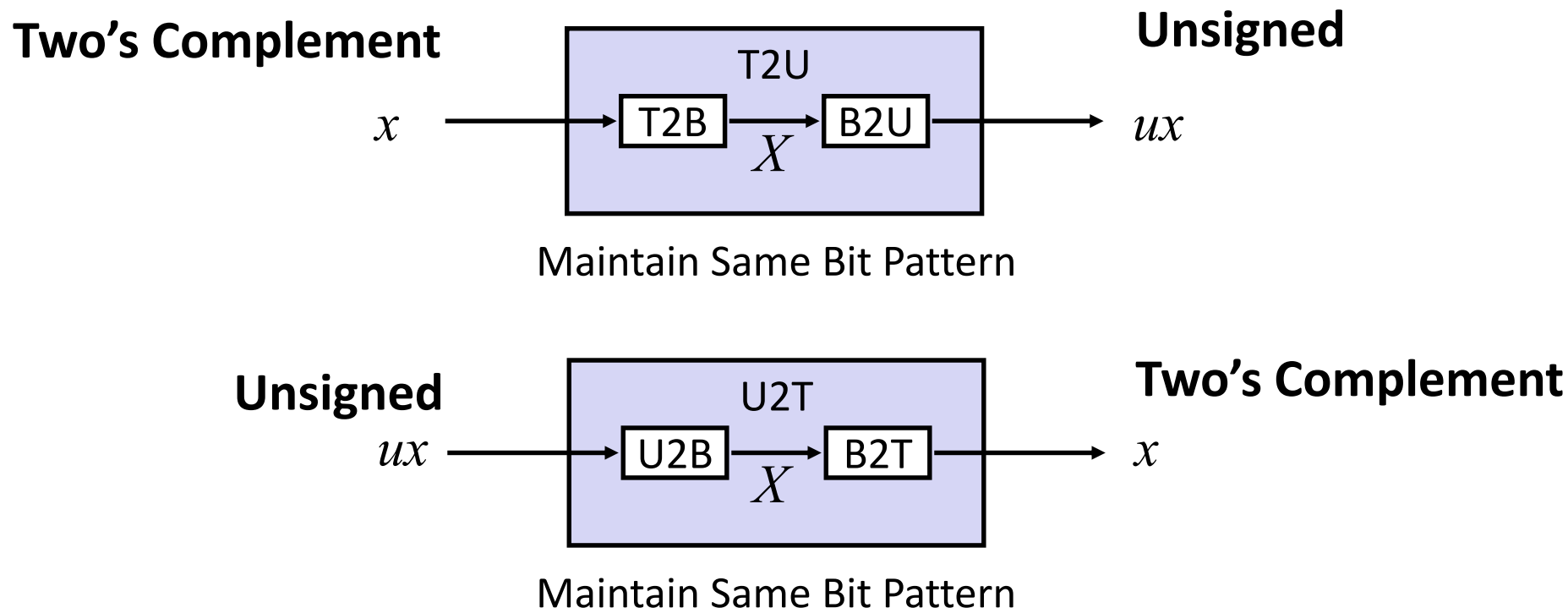
■ \Rightarrow Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$
 - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$
 - Bit pattern for two's comp integer

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Mapping Between Signed & Unsigned



- Mappings between unsigned and two's complement numbers:
Keep bit representations and reinterpret

Mapping Signed \leftrightarrow Unsigned

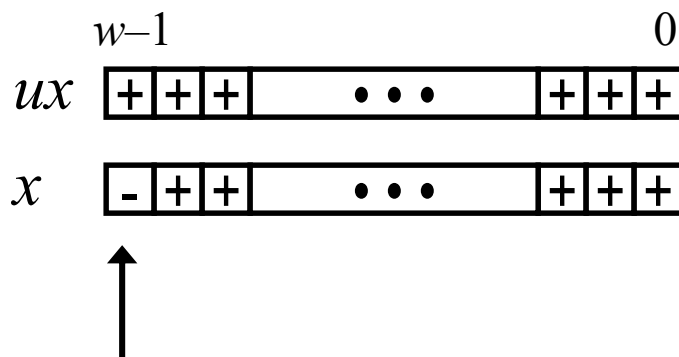
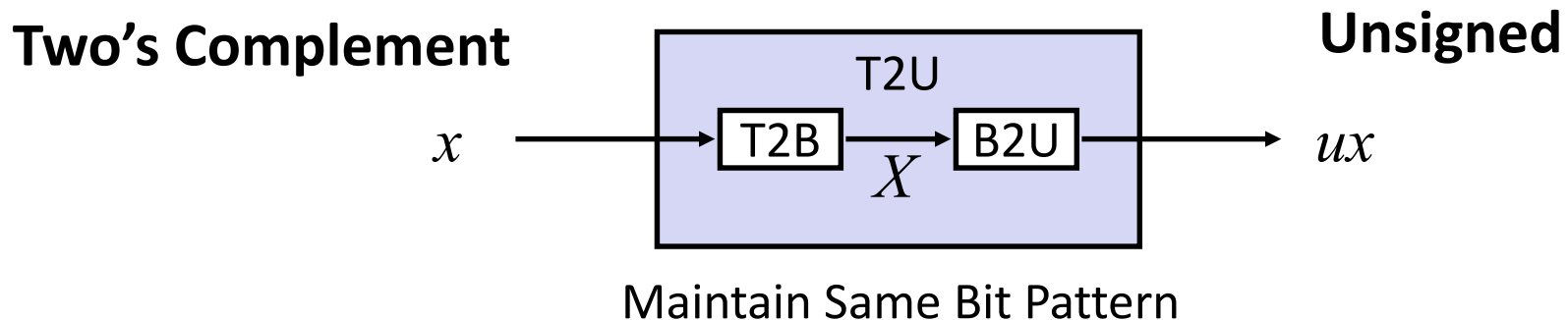
Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

\rightarrow **T2U** \rightarrow
 \leftarrow **U2T** \leftarrow

Mapping Signed \leftrightarrow Unsigned

Bits	Signed		Unsigned
0000	0	\longleftrightarrow =	0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8	\longleftrightarrow +/- 16	8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Relation between Signed & Unsigned

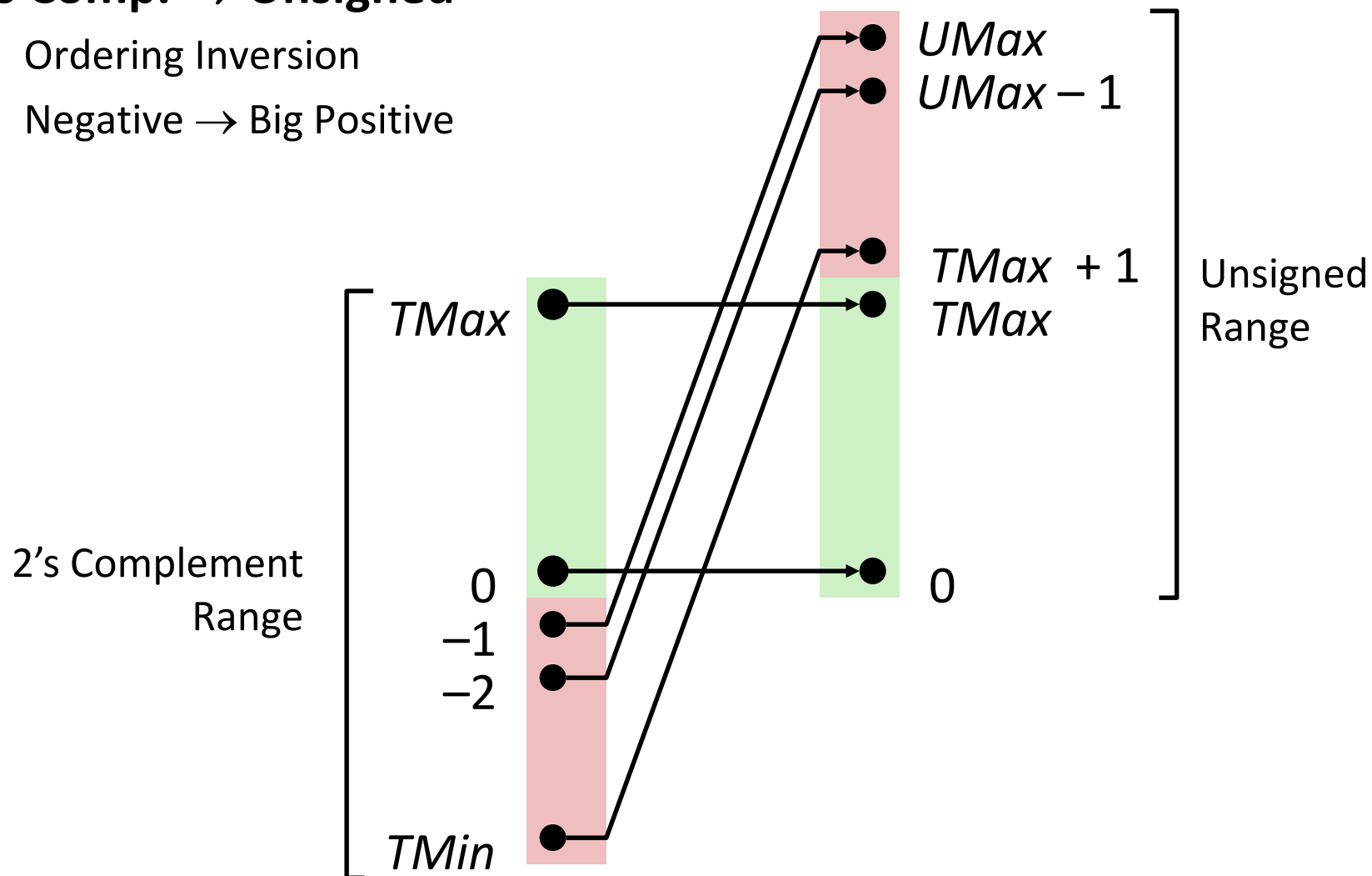


Large negative weight
becomes
Large positive weight

Conversion Visualized

■ 2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



Signed vs. Unsigned in C

■ Constants

- By default are considered to be signed integers
- Unsigned if have “U” as suffix

`0U, 4294967259U`

■ Casting

- Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

- Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

Casting Surprises

■ Expression Evaluation

- If there is a mix of unsigned and signed in single expression,
signed values implicitly cast to unsigned
- Including comparison operations $<$, $>$, $==$, $<=$, $>=$
- Examples for $W = 32$: **TMIN = -2,147,483,648**, **TMAX = 2,147,483,647**

■ Constant ₁	Constant ₂	Relation	Evaluation
0	0U		
-1	0		
-1	0U		
2147483647	-2147483647-1		
2147483647U	-2147483647-1		
-1	-2		
(unsigned)-1	-2		
2147483647	2147483648U		
2147483647	(int) 2147483648U		

Summary

Casting Signed \leftrightarrow Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w
- Expression containing signed and unsigned int
 - `int` is cast to `unsigned`!!

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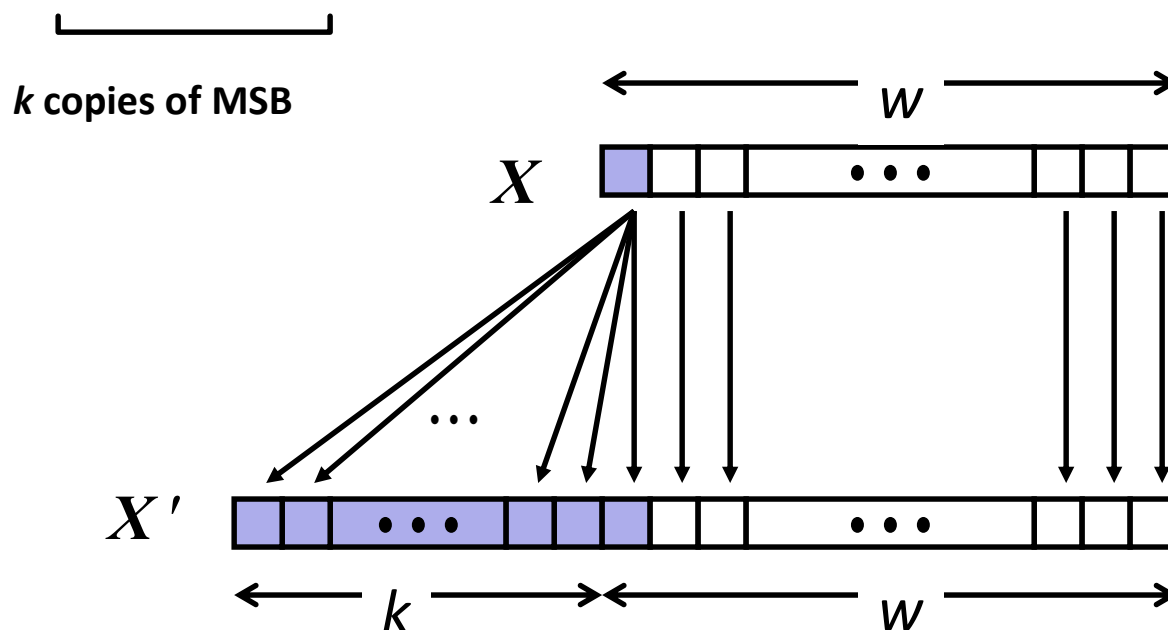
Sign Extension

■ Task:

- Given w -bit signed integer x
- Convert it to $w+k$ -bit integer with same value

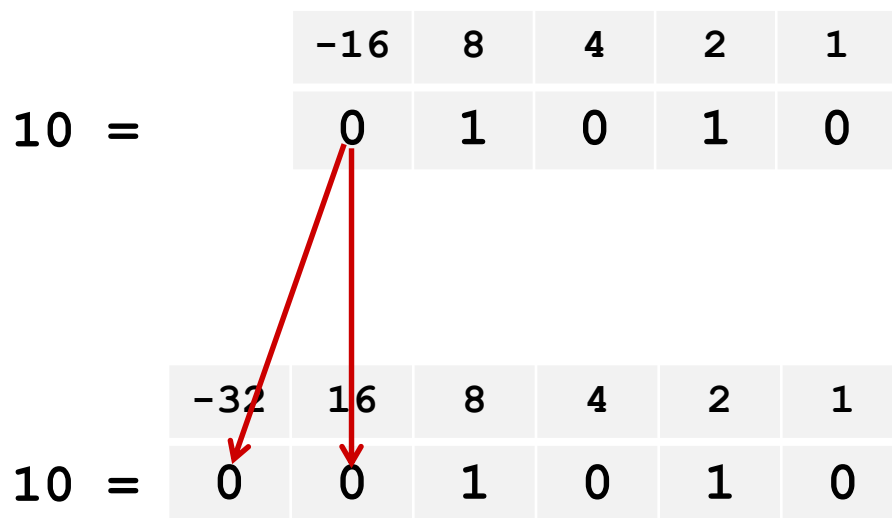
■ Rule:

- Make k copies of sign bit:
- $X' = \underbrace{x_{w-1}, \dots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \dots, x_0$

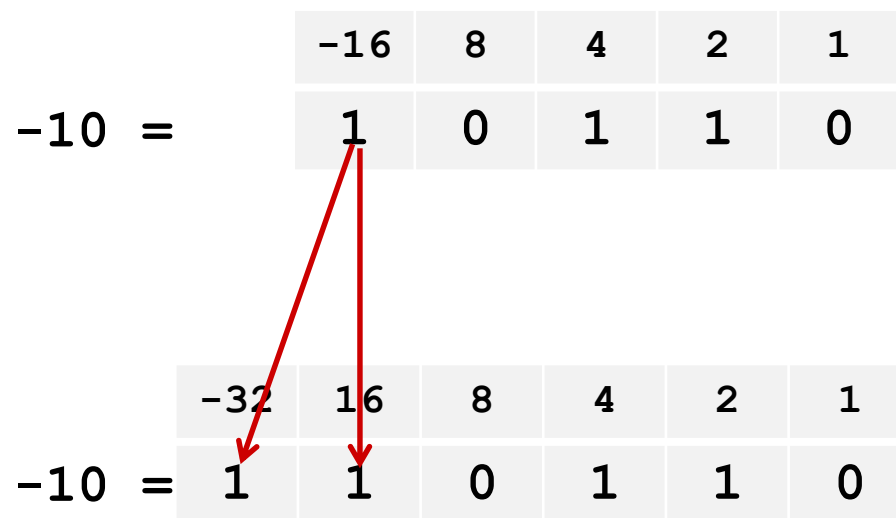


Sign Extension: Simple Example

Positive number



Negative number



Sign Extension Example

```
short int x = 15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

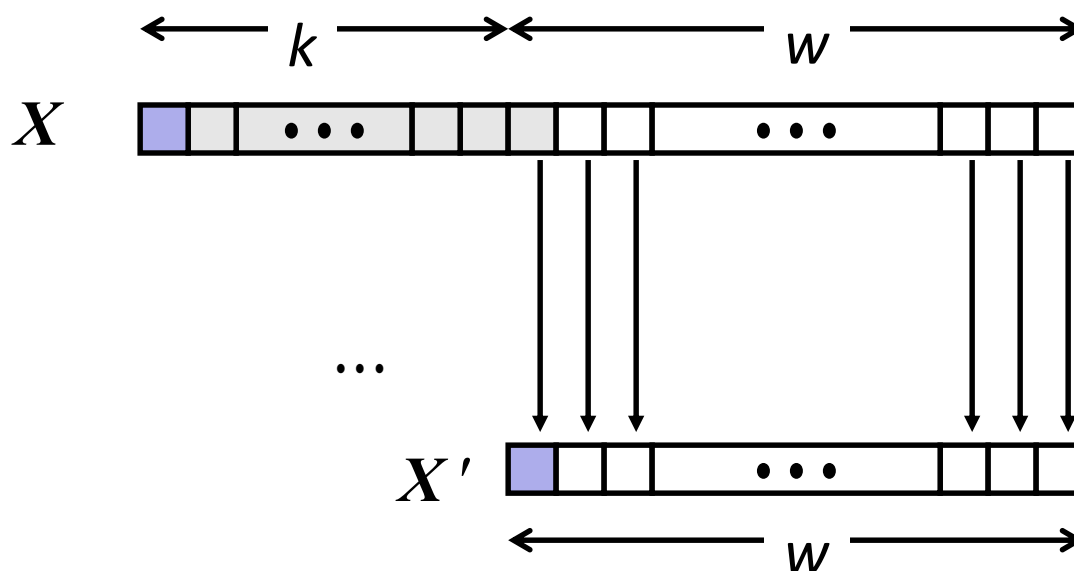
Truncation

■ Task:

- Given $k+w$ -bit signed or unsigned integer X
- Convert it to w -bit integer X' with same value for “small enough” X

■ Rule:

- Drop top k bits:
- $X' = x_{w-1}, x_{w-2}, \dots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

$$2 \bmod 16 = 2$$

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$-6 \bmod 16 = 26 \text{U} \bmod 16 = 10 \text{U} = -6$$

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$10 \bmod 16 = 10 \text{U} \bmod 16 = 10 \text{U} = -6$$

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

$$-10 \bmod 16 = 22 \text{U} \bmod 16 = 6 \text{U} = 6$$

Summary:

Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small numbers yields expected behavior

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Unsigned Addition

Operands: w bits

u

$+ v$

True Sum: $w+1$ bits

$u + v$

Discard Carry: w bits

$\text{UAdd}_w(u, v)$

■ Standard Addition Function

- Ignores carry output

■ Implements Modular Arithmetic

$$s = \text{UAdd}_w(u, v) = u + v \bmod 2^w$$

unsigned char

```

      1110 1001
    + 1101 0101
    -----
  1 1011 1110
    -----
    1011 1110
  
```

```

      E9
    + D5
    -----
    1BE
    -----
     BE
  
```

```

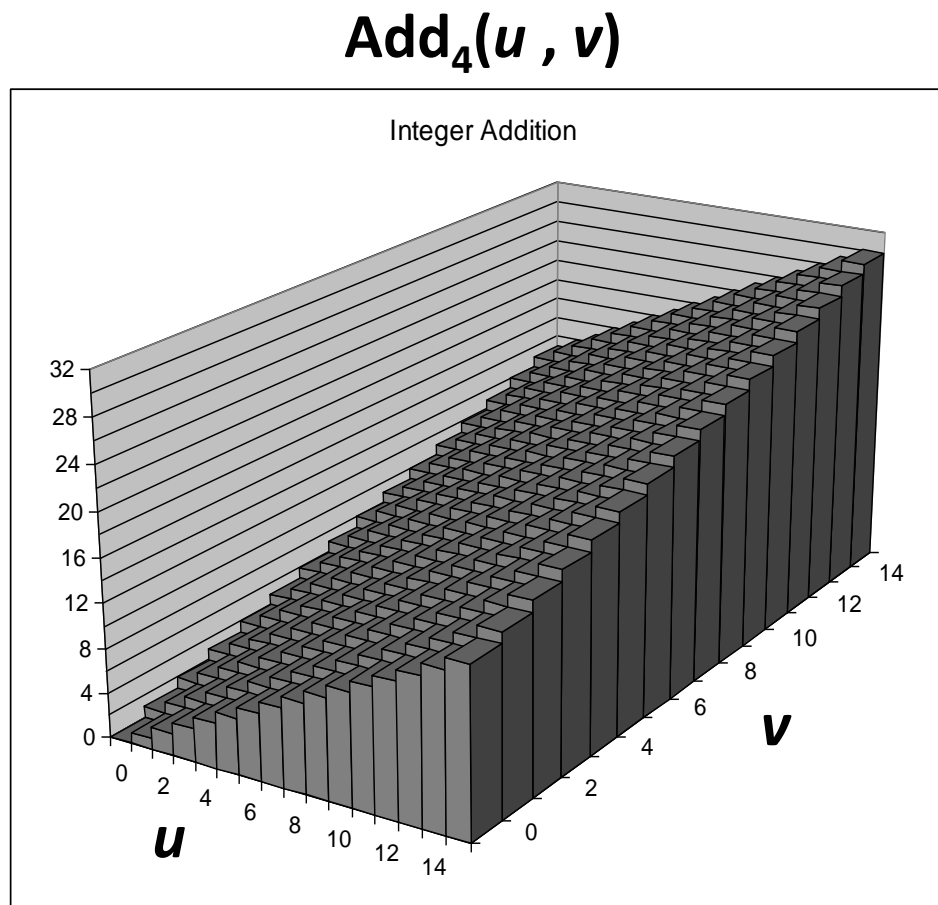
      233
    + 213
    -----
     446
    -----
     190
  
```

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualizing (Mathematical) Integer Addition

■ Integer Addition

- 4-bit integers u, v
- Compute true sum $\text{Add}_4(u, v)$
- Values increase linearly with u and v
- Forms planar surface

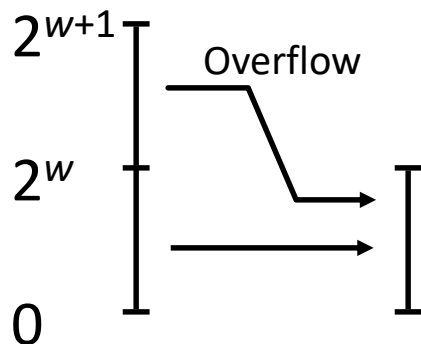


Visualizing Unsigned Addition

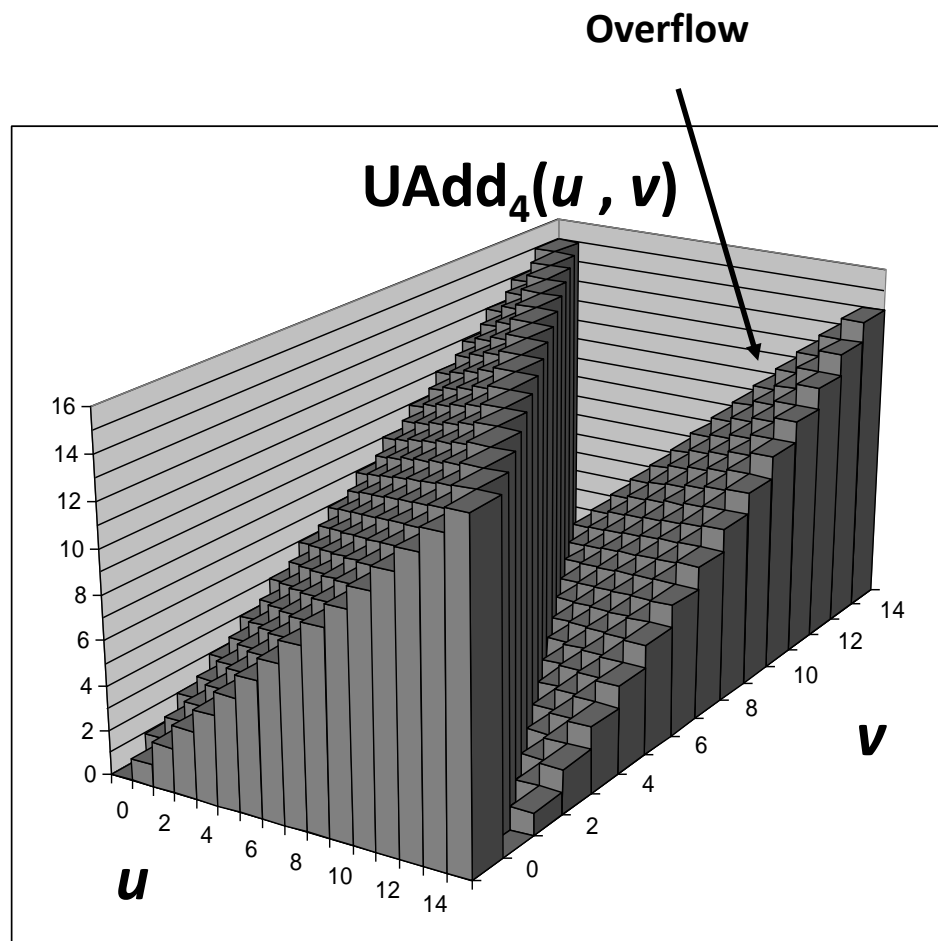
■ Wraps Around

- If true sum $\geq 2^w$
- At most once

True Sum



Modular Sum



Two's Complement Addition

Operands: w bits

u

$+$ v

True Sum: $w+1$ bits

$u + v$

Discard Carry: w bits

$\text{TAdd}_w(u, v)$

■ TAdd and UAdd have Identical Bit-Level Behavior

- Signed vs. unsigned addition in C:

```
int s, t, u, v;
```

```
s = (int) ((unsigned) u + (unsigned) v);
```

```
t = u + v
```

- Will give $s == t$

```

      1110 1001
    + 1101 0101
    -----
  1 1011 1110
    -----
    1011 1110
  
```

```

      E9
    + D5
    -----
    1BE
    -----
    BE
  
```

```

      -23
    + -43
    -----
    -66
    -----
    -66
  
```

TAdd Overflow

■ Functionality

- True sum requires $w+1$ bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer

0 111...1

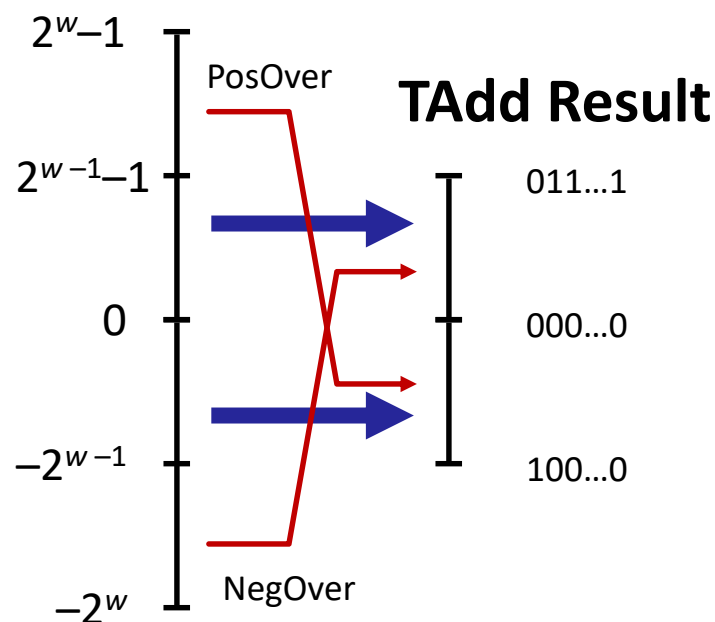
0 100...0

0 000...0

1 011...1

1 000...0

True Sum



Visualizing 2's Complement Addition

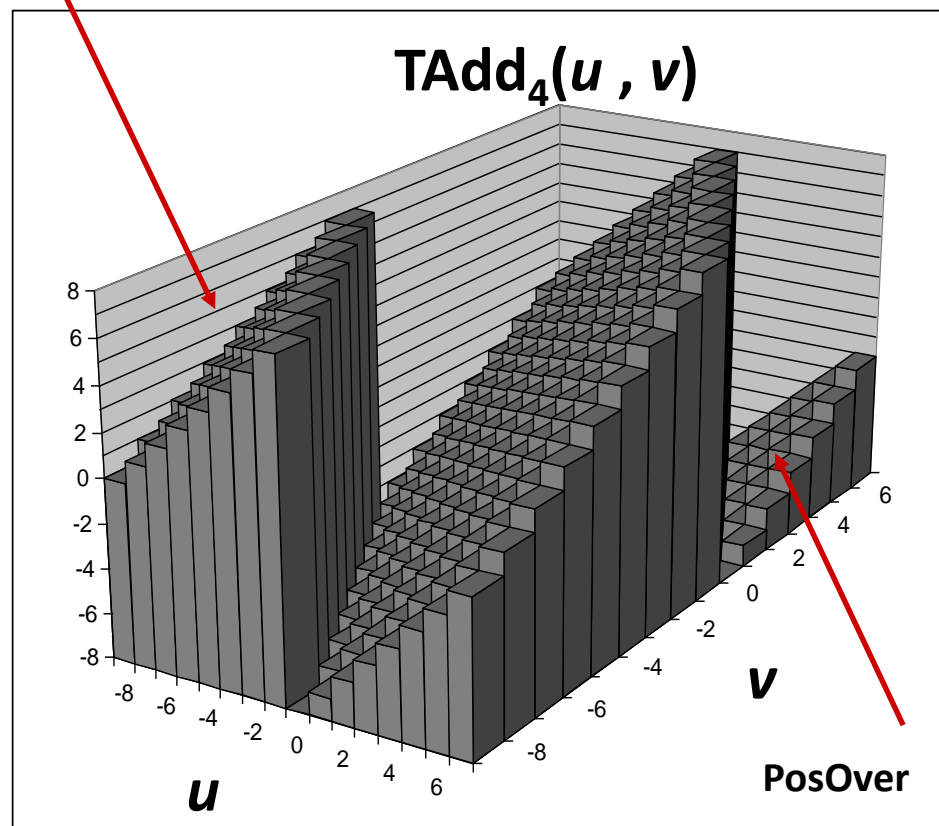
■ Values

- 4-bit two's comp.
- Range from -8 to +7

■ Wraps Around

- If $\text{sum} \geq 2^{w-1}$
 - Becomes negative
 - At most once
- If $\text{sum} < -2^{w-1}$
 - Becomes positive
 - At most once

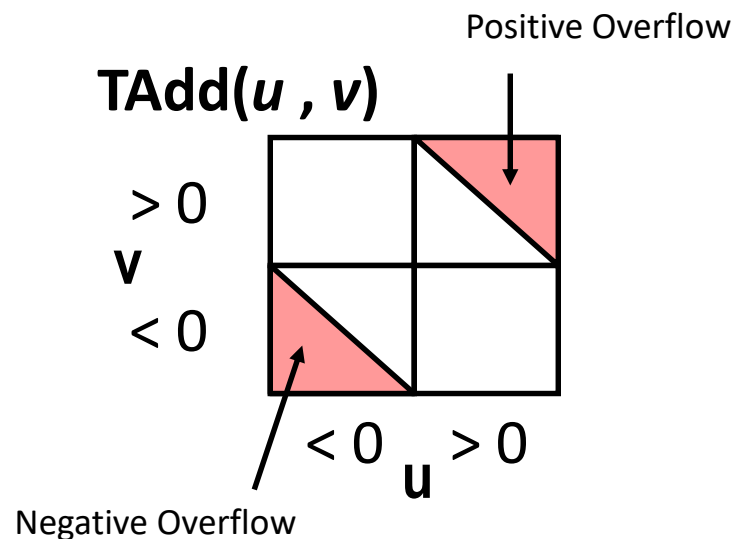
NegOver



Characterizing TAdd

■ Functionality

- True sum requires $w+1$ bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer

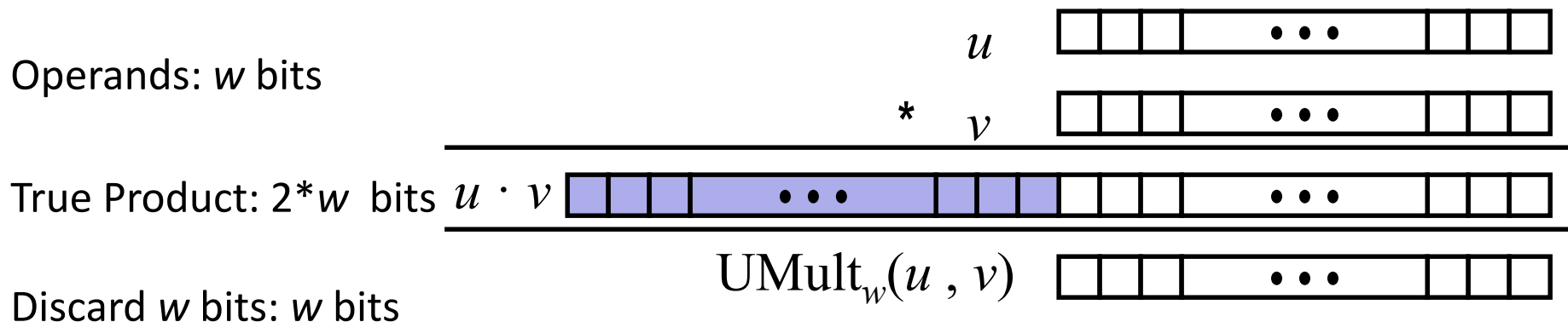


$$TAdd_w(u, v) = \begin{cases} u + v + 2^w & u + v < TMin_w \text{ (NegOver)} \\ u + v & TMin_w \leq u + v \leq TMax_w \\ u + v - 2^w & TMax_w < u + v \text{ (PosOver)} \end{cases}$$

Multiplication

- **Goal: Computing Product of w -bit numbers x, y**
 - Either signed or unsigned
- **But, exact results can be bigger than w bits**
 - Unsigned: up to $2w$ bits
 - Result range: $0 \leq x * y \leq (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
 - Two's complement min (negative): Up to $2w-1$ bits
 - Result range: $x * y \geq (-2^{w-1}) * (2^{w-1} - 1) = -2^{2w-2} + 2^{w-1}$
 - Two's complement max (positive): Up to $2w$ bits, but only for $(TMin_w)^2$
 - Result range: $x * y \leq (-2^{w-1})^2 = 2^{2w-2}$
- **So, maintaining exact results...**
 - would need to keep expanding word size with each product computed
 - is done in software, if needed
 - e.g., by “arbitrary precision” arithmetic packages

Unsigned Multiplication in C



■ Standard Multiplication Function

- Ignores high order w bits

■ Implements Modular Arithmetic

$$\text{UMult}_w(u, v) = u \cdot v \bmod 2^w$$

$$\begin{array}{r}
 1110 \ 1001 \\
 * 1101 \ 0101 \\
 \hline
 1100 \ 0001 \ 1101 \ 1101 \\
 1101 \ 1101 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 E9 \\
 * D5 \\
 \hline
 C1DD \\
 DD \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 233 \\
 * 213 \\
 \hline
 49629 \\
 221 \\
 \hline
 \end{array}$$

Signed Multiplication in C

Operands: w bits

u

*

v

True Product: $2w$ bits $u \cdot v$

Discard w bits: w bits

$\text{TMult}_w(u, v)$

■ Standard Multiplication Function

- Ignores high order w bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

	1110	1001
*	1101	0101
<hr/>		
	0000	0011 1101 1101
<hr/>		
		1101 1101

	E9	-23
*	D5	* -43
<hr/>		
	03DD	989
<hr/>		
	DD	-35

Power-of-2 Multiply with Shift

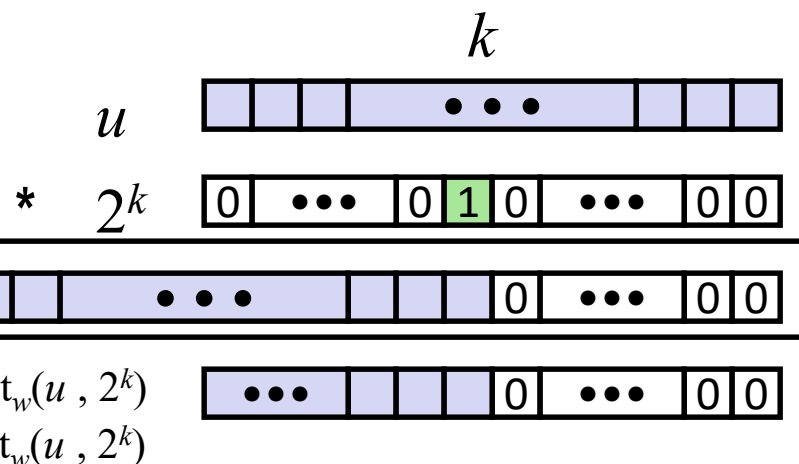
■ Operation

- $u \ll k$ gives $u * 2^k$
- Both signed and unsigned

Operands: w bits

True Product: $w+k$ bits

Discard k bits: w bits



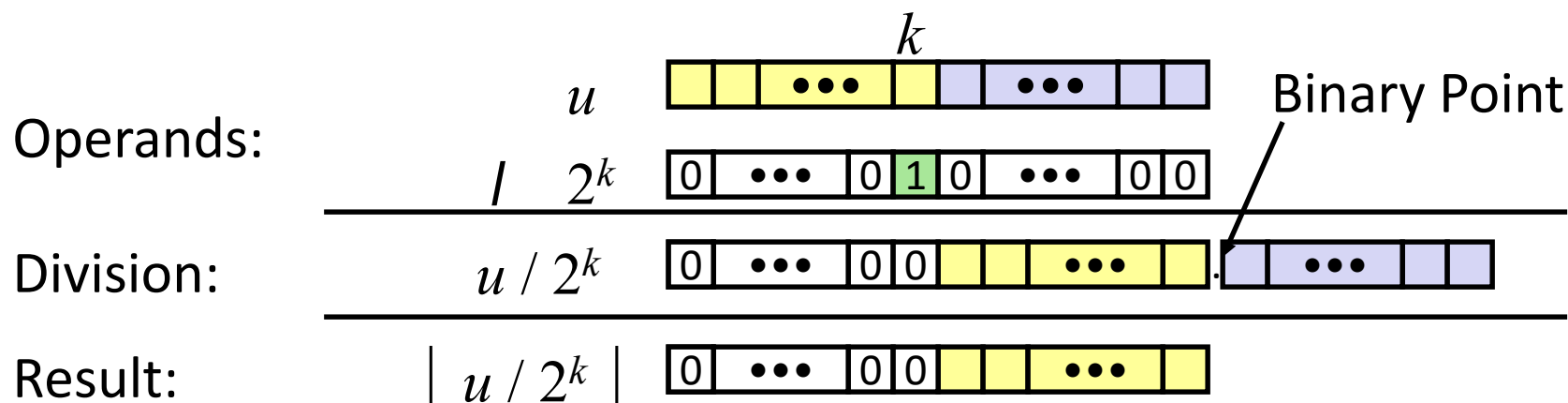
■ Examples

- $u \ll 3 == u * 8$
- $(u \ll 5) - (u \ll 3) == u * 24$
- Most machines shift and add faster than multiply
 - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

■ Quotient of Unsigned by Power of 2

- $u \gg k$ gives $\lfloor u / 2^k \rfloor$
- Uses logical shift

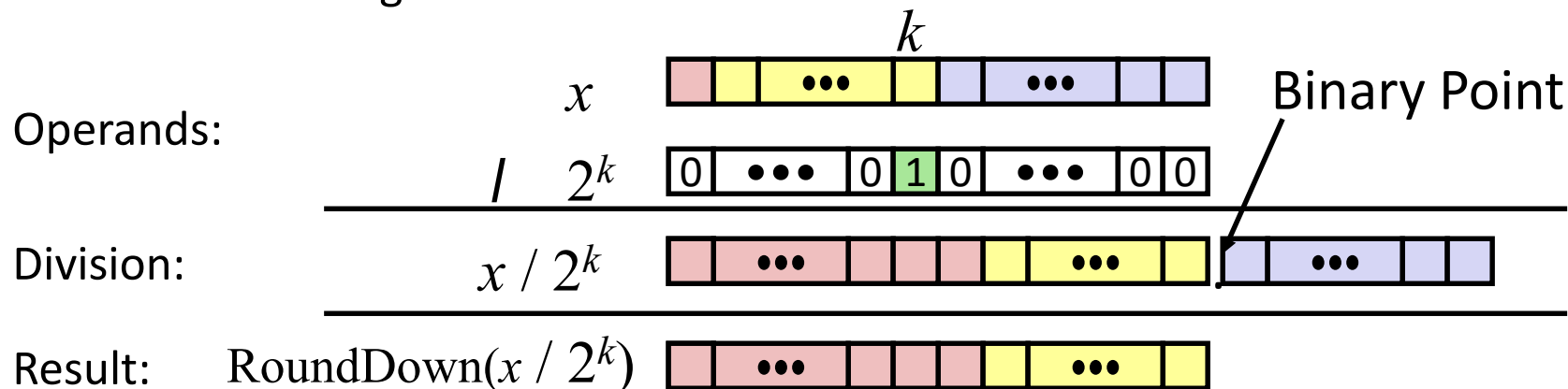


	Division	Computed	Hex	Binary
x	15213	15213	3B 6D	00111011 01101101
x >> 1	7606.5	7606	1D B6	00011101 10110110
x >> 4	950.8125	950	03 B6	00000011 10110110
x >> 8	59.4257813	59	00 3B	00000000 00111011

Signed Power-of-2 Divide with Shift

■ Quotient of Signed by Power of 2

- $x \gg k$ gives $\lfloor x / 2^k \rfloor$
- Uses arithmetic shift
- Rounds wrong direction when $u < 0$



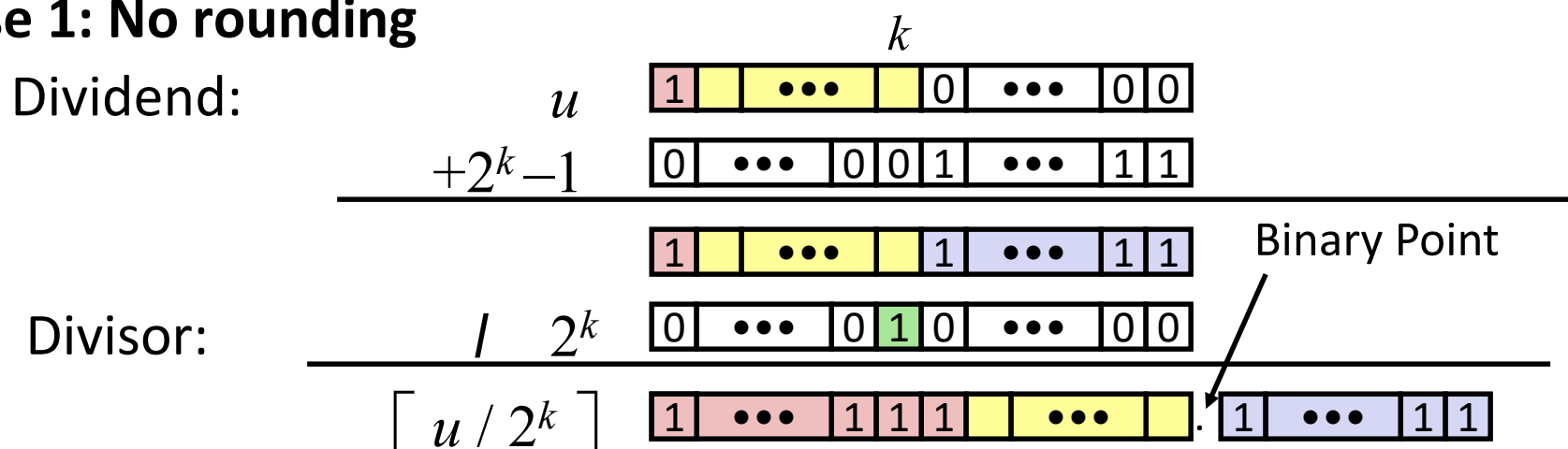
	Division	Computed	Hex	Binary
y	-15213	-15213	C4 93	11000100 10010011
y >> 1	-7606.5	-7607	E2 49	11100010 01001001
y >> 4	-950.8125	-951	FC 49	11111100 01001001
y >> 8	-59.4257813	-60	FF C4	11111111 11000100

Correct Power-of-2 Divide

■ Quotient of Negative Number by Power of 2

- Want $\lceil x / 2^k \rceil$ (Round Toward 0)
- Compute as $\lfloor (x+2^k-1) / 2^k \rfloor$
 - In C: $(x + (1 \ll k) - 1) \gg k$
 - Biases dividend toward 0

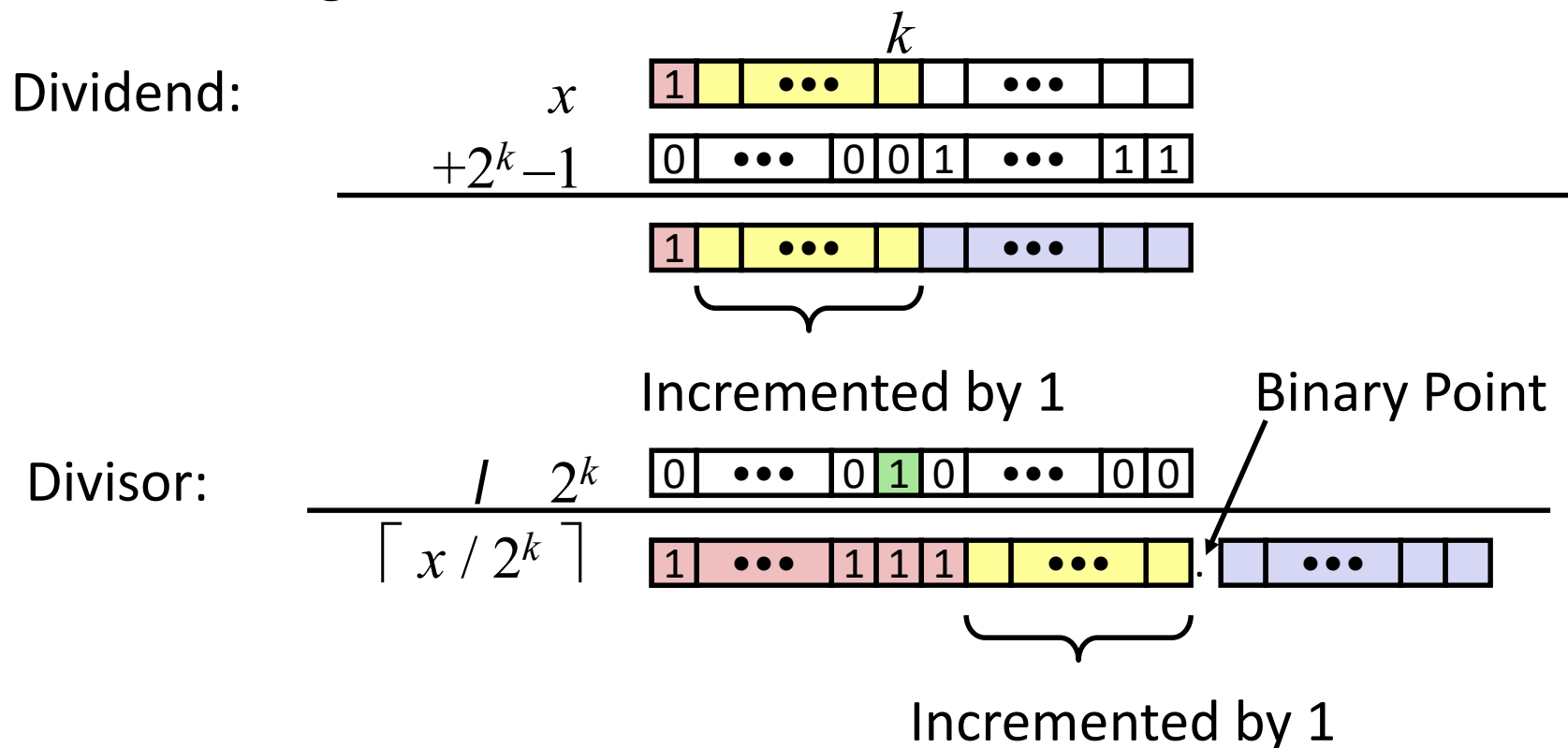
Case 1: No rounding



Biassing has no effect

Correct Power-of-2 Divide (Cont.)

Case 2: Rounding



Biasing adds 1 to final result

Negation: Complement & Increment

■ Negate through complement and increase

$$\sim x + 1 == -x$$

■ Example

- Observation: $\sim x + x == 1111\dots111 == -1$

$$\begin{array}{r}
 x \quad \boxed{10011101} \\
 + \quad \sim x \quad \boxed{01100010} \\
 \hline
 -1 \quad \boxed{11111111}
 \end{array}$$

x = 15213

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011
y	-15213	C4 93	11000100 10010011

Complement & Increment Examples

x = 0

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

x = TMin

	Decimal	Hex	Binary
x	-32768	80 00	10000000 00000000
~x	32767	7F FF	01111111 11111111
~x+1	-32768	80 00	10000000 00000000

Canonical counter example

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- **Integers**
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - **Summary**
- Representations in memory, pointers, strings

Arithmetic: Basic Rules

■ Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod 2^w
 - Mathematical addition + possible subtraction of 2^w
- Signed: modified addition mod 2^w (result in proper range)
 - Mathematical addition + possible addition or subtraction of 2^w

■ Multiplication:

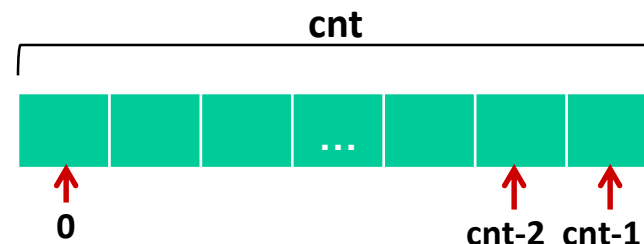
- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod 2^w
- Signed: modified multiplication mod 2^w (result in proper range)

Why Should I Use Unsigned?

■ *Don't* use without understanding implications

- Easy to make mistakes

```
unsigned i;  
for (i = cnt-2; i >= 0; i--)  
    a[i] += a[i+1];
```



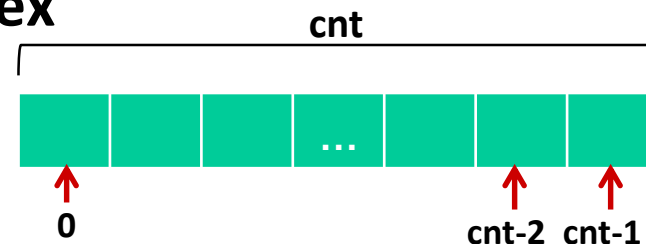
- Can be very subtle

```
#define DELTA sizeof(int)  
int i;  
for (i = CNT; i-DELTA >= 0; i-= DELTA)  
    . . .
```

Counting Down with Unsigned

■ Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
    a[i] += a[i+1];
```



■ See Robert Seacord, *Secure Coding in C and C++*

- C Standard guarantees that unsigned addition will behave like modular arithmetic
 - $0 - 1 \rightarrow UMax$

■ Even better

```
size_t i;
for (i = cnt-2; i < cnt; i--)
    a[i] += a[i+1];
```

- Data type `size_t` defined as unsigned value with length = word size
- Code will work even if `cnt = UMax`
- What if `cnt` is signed and `< 0`?

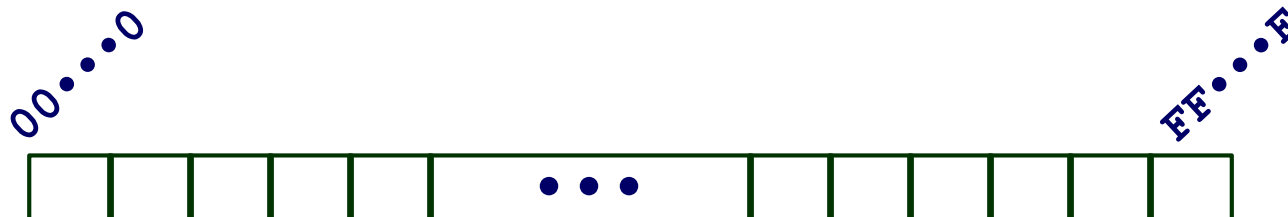
Why Should I Use Unsigned? (cont.)

- **Do Use When Performing Modular Arithmetic**
 - Multiprecision arithmetic
- **Do Use When Using Bits to Represent Sets**
 - Logical right shift, no sign extension
- **Do Use In System Programming**
 - Bit masks, device commands,...

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- **Representations in memory, pointers, strings**

Byte-Oriented Memory Organization



■ Programs refer to data by address

- Conceptually, envision it as a very large array of bytes
 - In reality, it's not, but can think of it that way
- An address is like an index into that array
 - and, a pointer variable stores an address

■ Note: system provides private address spaces to each “process”

- Think of a process as a program being executed
- So, a program can clobber its own data, but not that of others

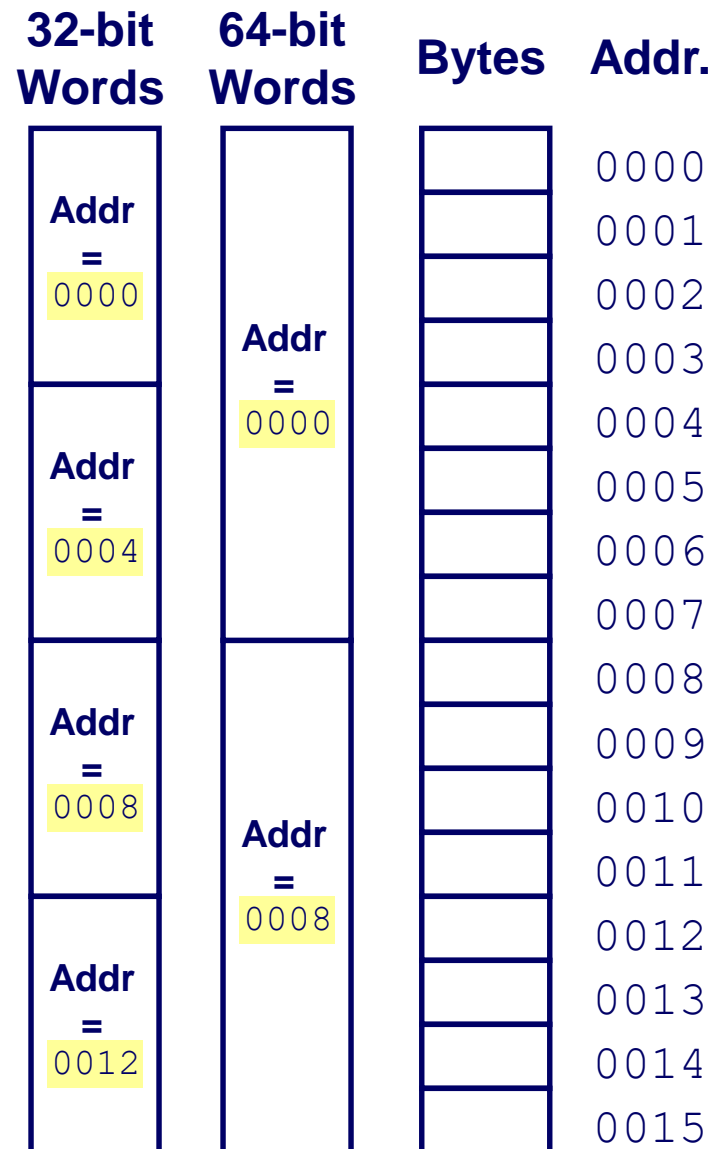
Machine Words

- **Any given computer has a “Word Size”**
 - Nominal size of integer-valued data
 - and of addresses
 - Until recently, most machines used 32 bits (4 bytes) as word size
 - Limits addresses to 4GB (2^{32} bytes)
 - Increasingly, machines have 64-bit word size
 - Potentially, could have 18 EB (exabytes) of addressable memory
 - That's 18.4×10^{18}
 - Machines still support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes

Word-Oriented Memory Organization

■ Addresses Specify Byte Locations

- Address of first byte in word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
<code>char</code>	1	1	1
<code>short</code>	2	2	2
<code>int</code>	4	4	4
<code>long</code>	4	8	8
<code>float</code>	4	4	4
<code>double</code>	8	8	8
<code>pointer</code>	4	8	8

Byte Ordering

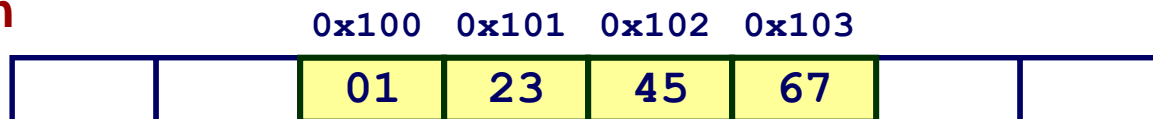
- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
 - Big Endian: Sun, PPC Mac, Internet
 - Least significant byte has highest address
 - Little Endian: x86, ARM processors running Android, iOS, and Linux
 - Least significant byte has lowest address

Byte Ordering Example

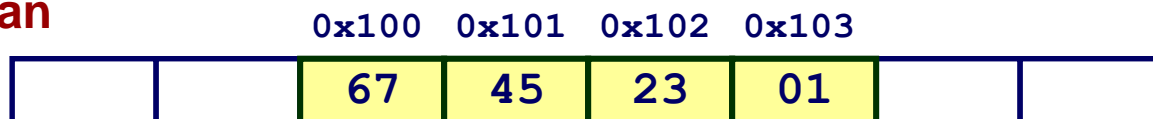
■ Example

- Variable x has 4-byte value of 0x01234567
- Address given by &x is 0x100

Big Endian



Little Endian



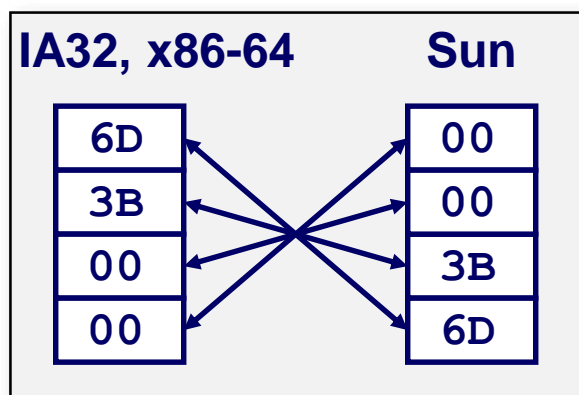
Representing Integers

Decimal: 15213

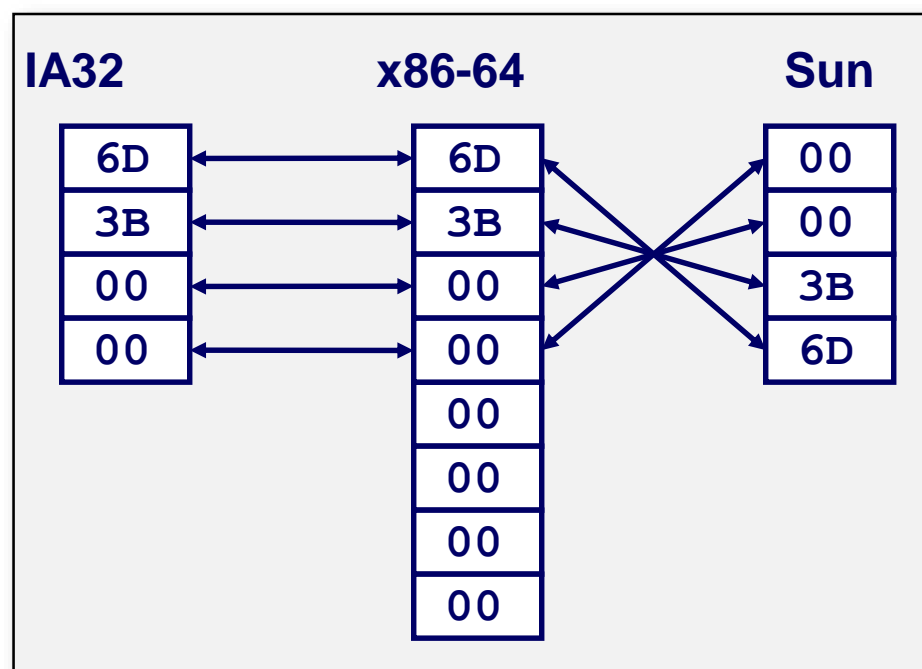
Binary: 0011 1011 0110 1101

Hex: 3 B 6 D

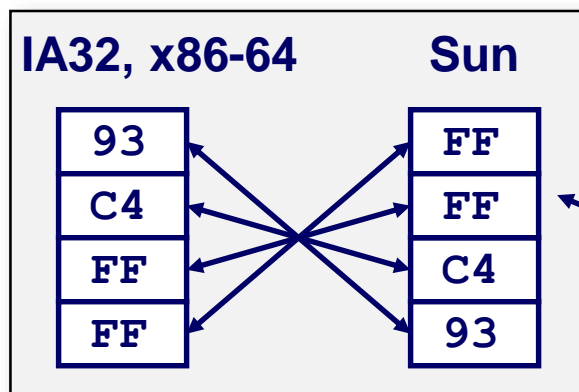
`int A = 15213;`



`long int C = 15213;`



`int B = -15213;`



Two's complement representation

Examining Data Representations

■ Code to Print Byte Representation of Data

- Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
    size_t i;
    for (i = 0; i < len; i++)
        printf("%p\t0x%.2x\n", start+i, start[i]);
    printf("\n");
}
```

Printf directives:

%p: Print pointer

%x: Print Hexadecimal

show_bytes Execution Example

```
int a = 15213;  
printf("int a = 15213;\n");  
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;  
0x7fffb7f71dbc    6d  
0x7fffb7f71dbd    3b  
0x7fffb7f71dbe    00  
0x7fffb7f71dbf    00
```

Representing Pointers

```
int B = -15213;  
int *P = &B;
```

Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

Different compilers & machines assign different locations to objects

Even get different results each time run program

Representing Strings

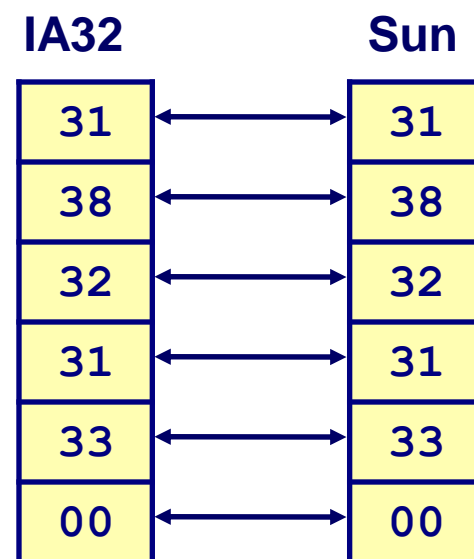
```
char S[6] = "18213";
```

■ Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Character "0" has code 0x30
 - Digit i has code $0x30+i$
- String should be null-terminated
 - Final character = 0

■ Compatibility

- Byte ordering not an issue



Reading Byte-Reversed Listings

■ Disassembly

- Text representation of binary machine code
- Generated by program that reads the machine code

■ Example Fragment

Address	Instruction Code	Assembly Rendition
8048365:	5b	pop %ebx
8048366:	81 c3 ab 12 00 00	add \$0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl \$0x0,0x28(%ebx)

■ Deciphering Numbers

- Value: 0x12ab
- Pad to 32 bits: 0x000012ab
- Split into bytes: 00 00 12 ab
- Reverse: ab 12 00 00

Integer C Puzzles

Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

- `x < 0` \rightarrow `((x*2) < 0)`
- `ux >= 0`
- `x & 7 == 7` \rightarrow `(x<<30) < 0`
- `ux > -1`
- `x > y` \rightarrow `-x < -y`
- `x * x >= 0`
- `x > 0 && y > 0` \rightarrow `x + y > 0`
- `x >= 0` \rightarrow `-x <= 0`
- `x <= 0` \rightarrow `-x >= 0`
- `(x|-x)>>31 == -1`
- `ux >> 3 == ux/8`
- `x >> 3 == x/8`
- `x & (x-1) != 0`