Opinion Dynamics via Infinity Pre-Bundles

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SUNY New Paltz

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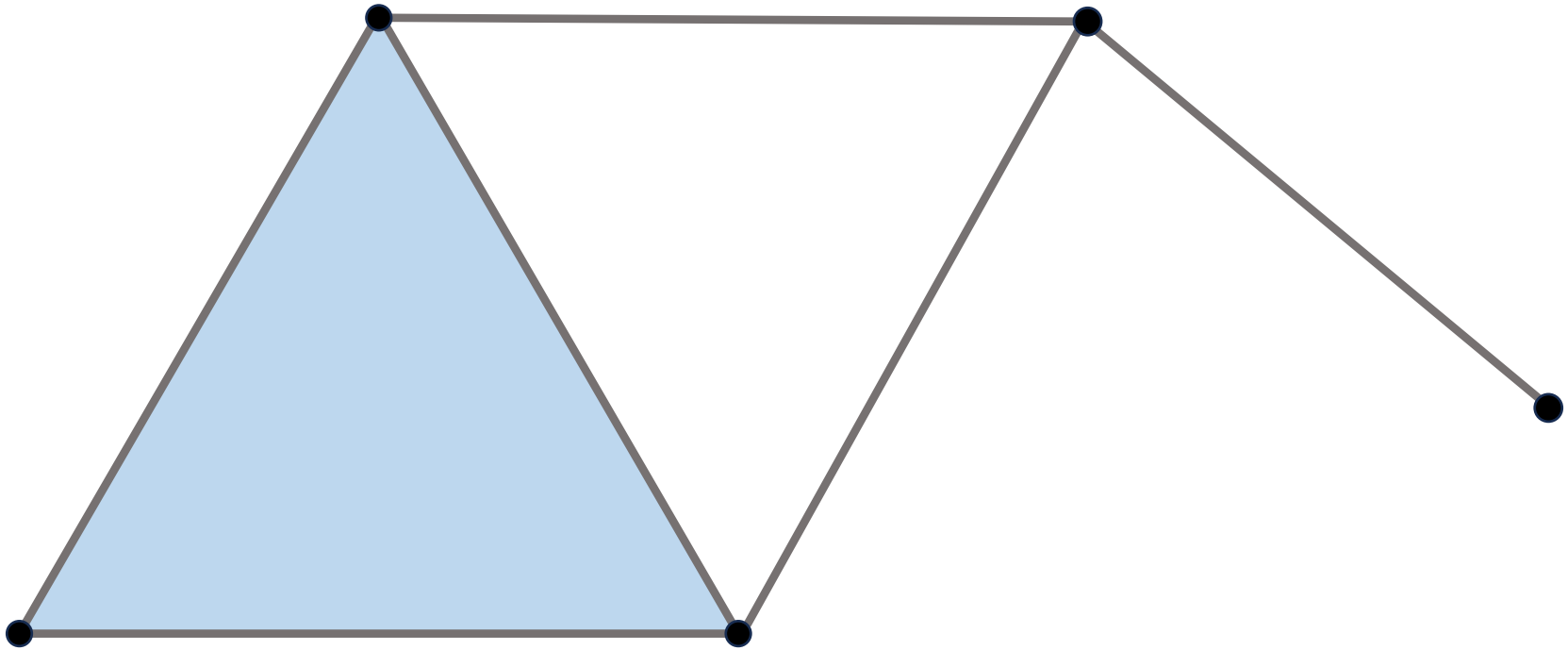
Preliminaries



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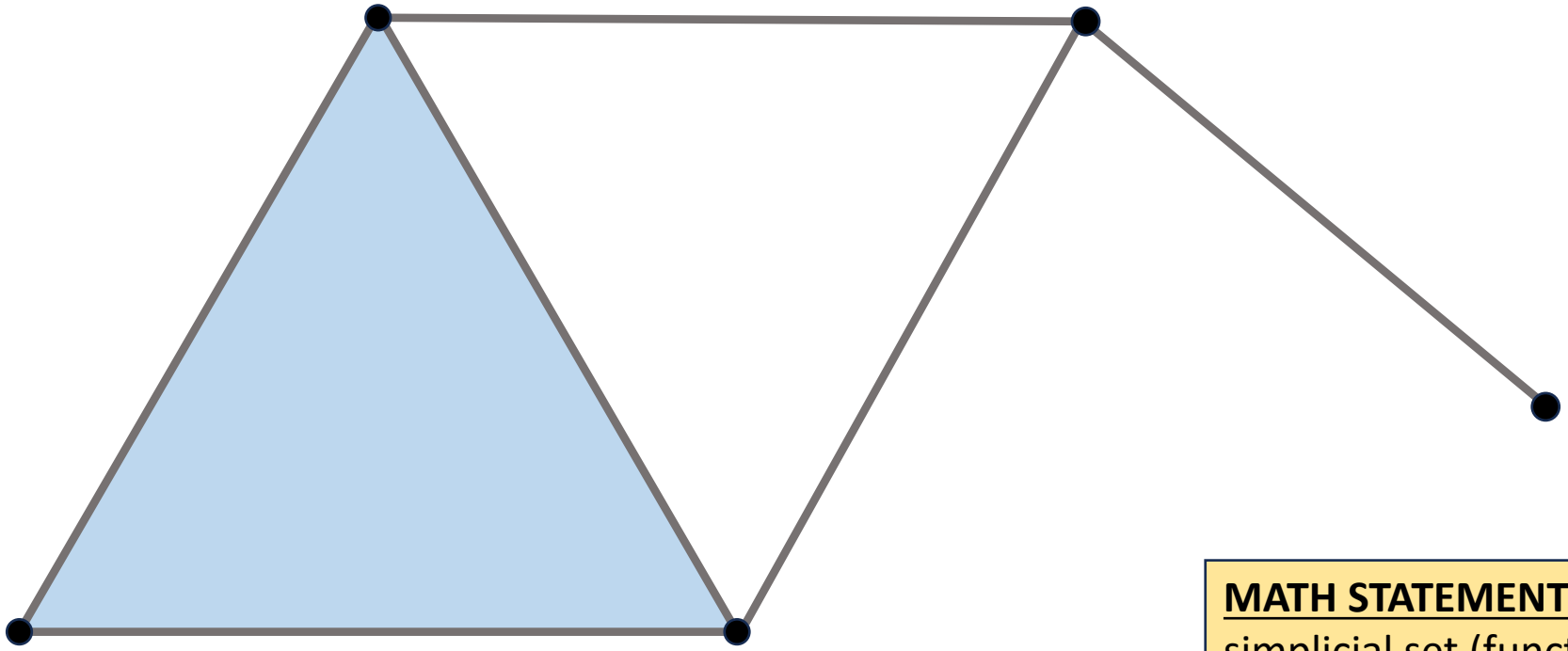
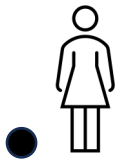


MATH STATEMENT:
simplicial set (functor)

$$X_{\cdot}: \Delta^{op} \rightarrow Set$$

Preliminaries



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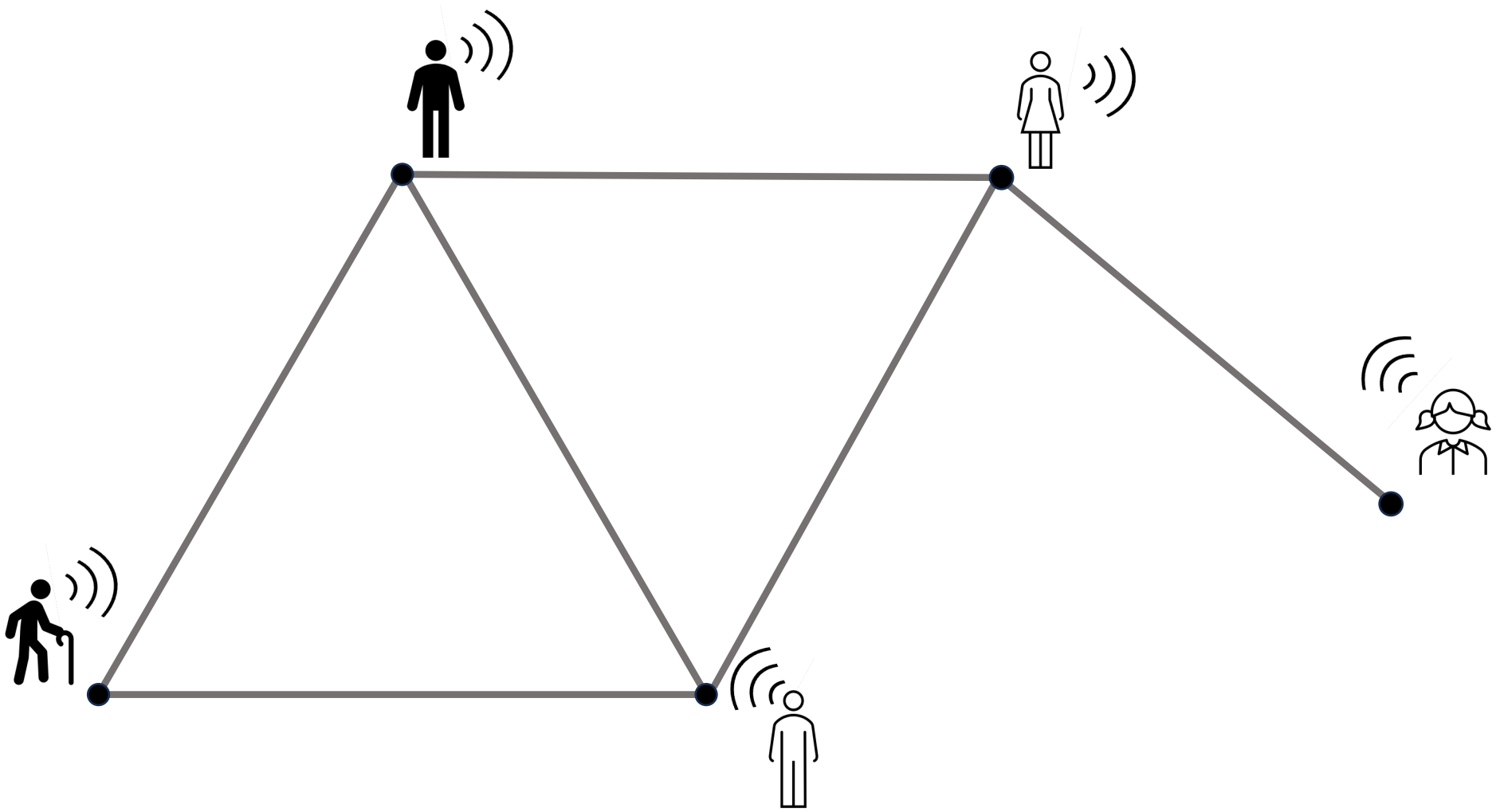

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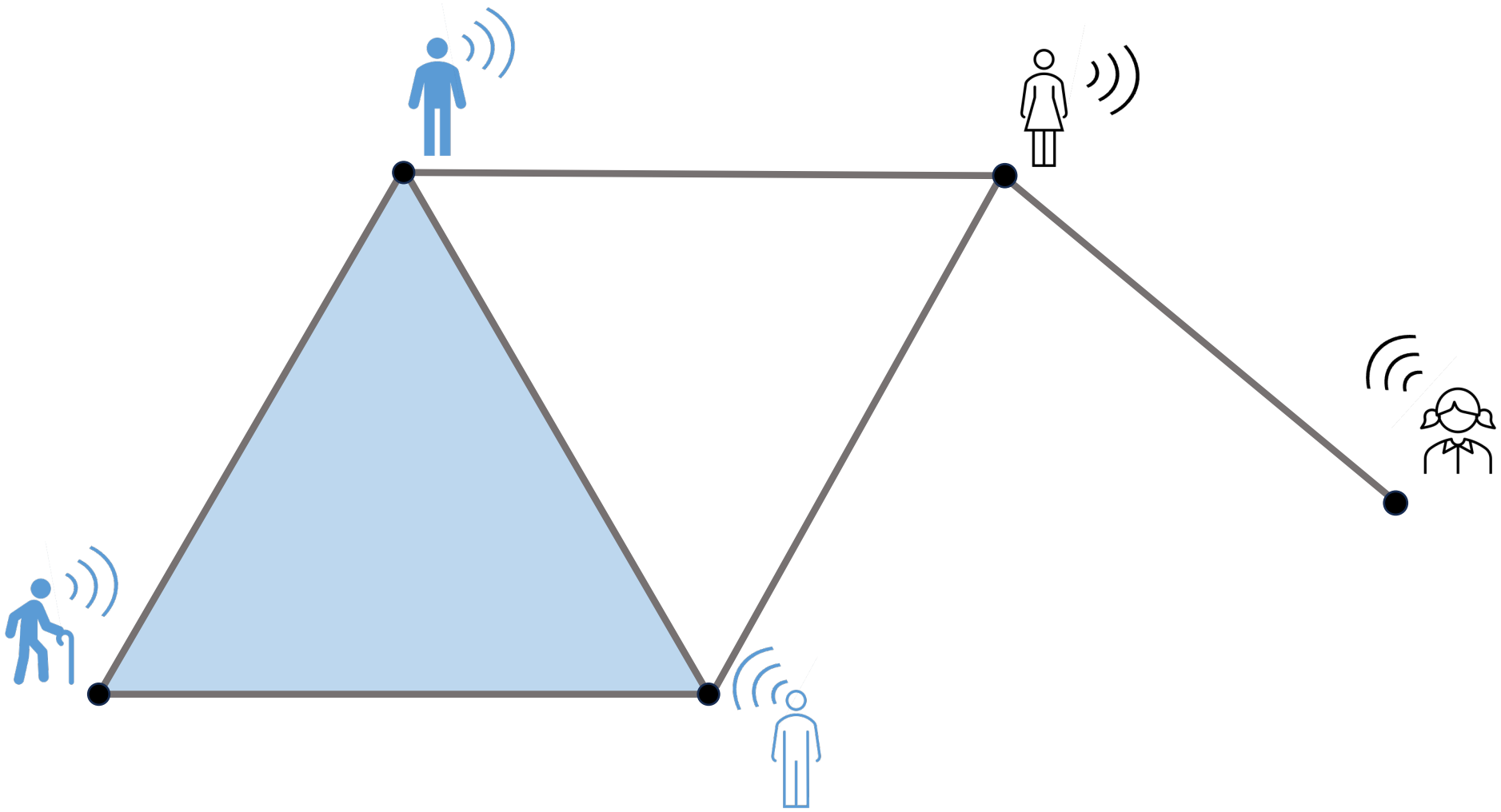

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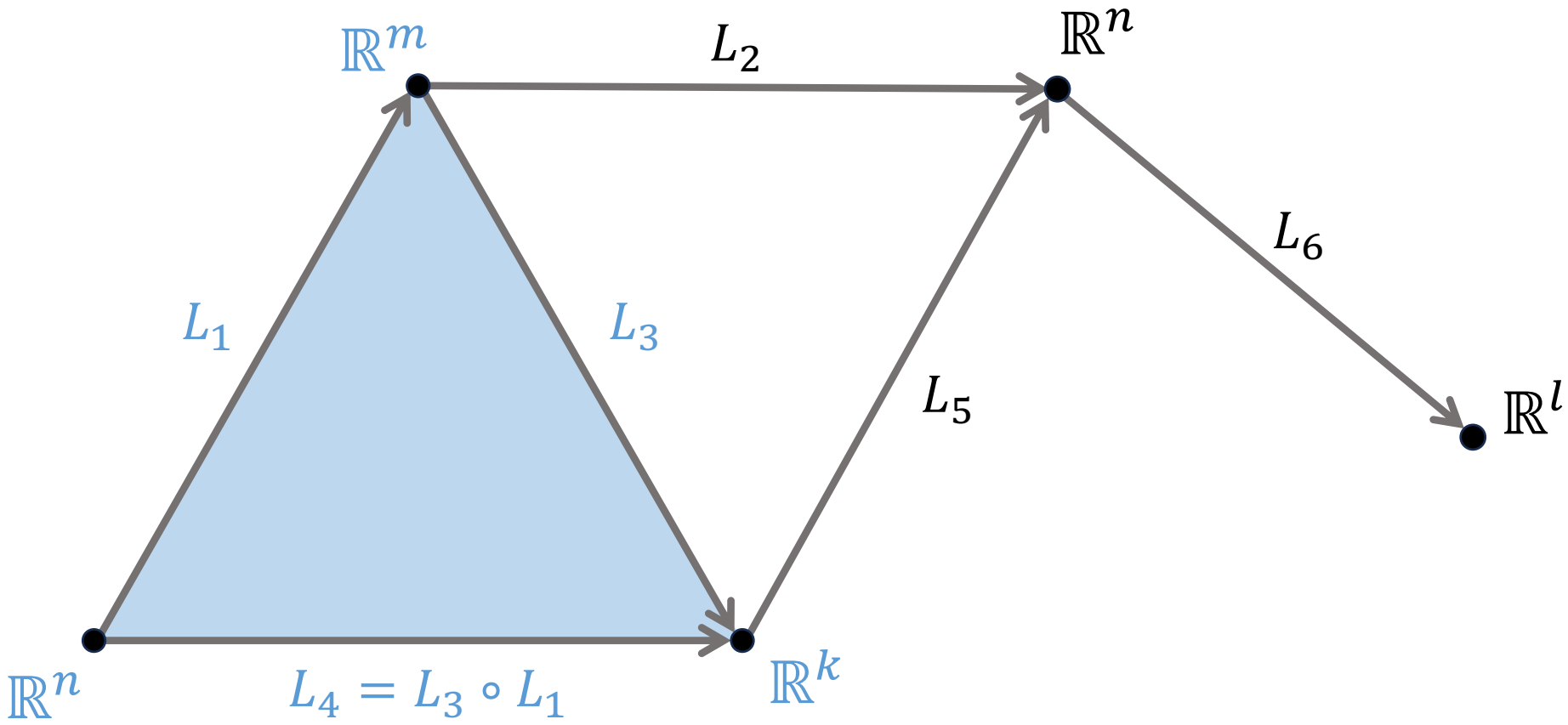
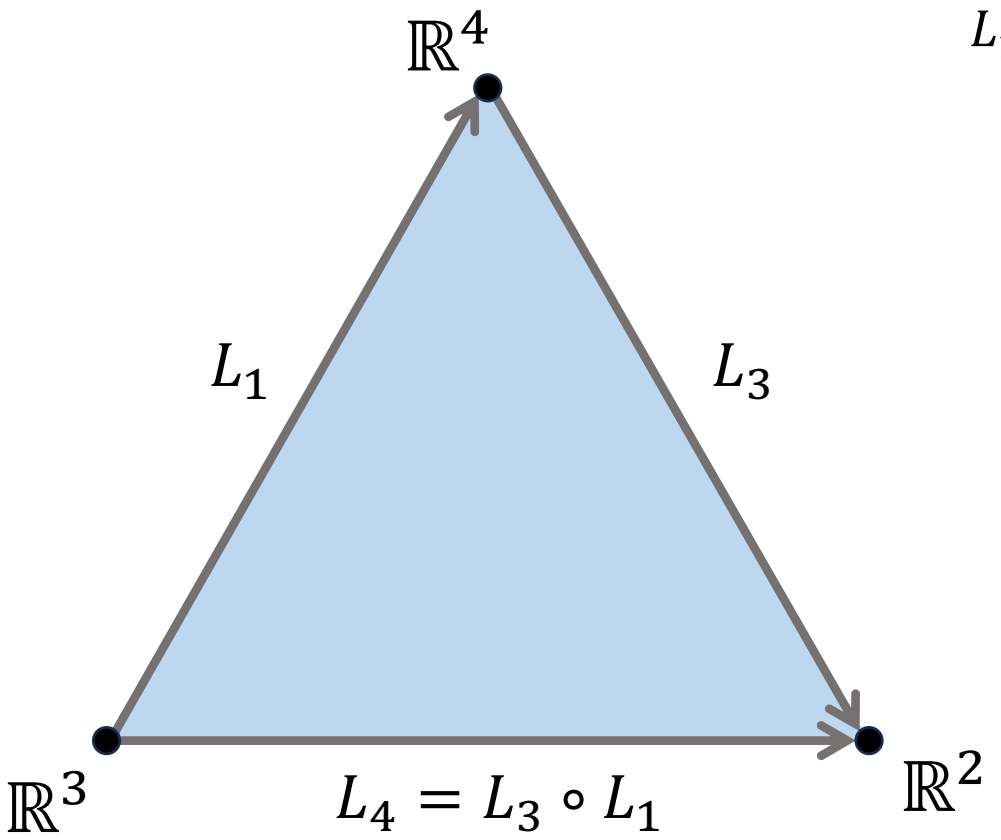


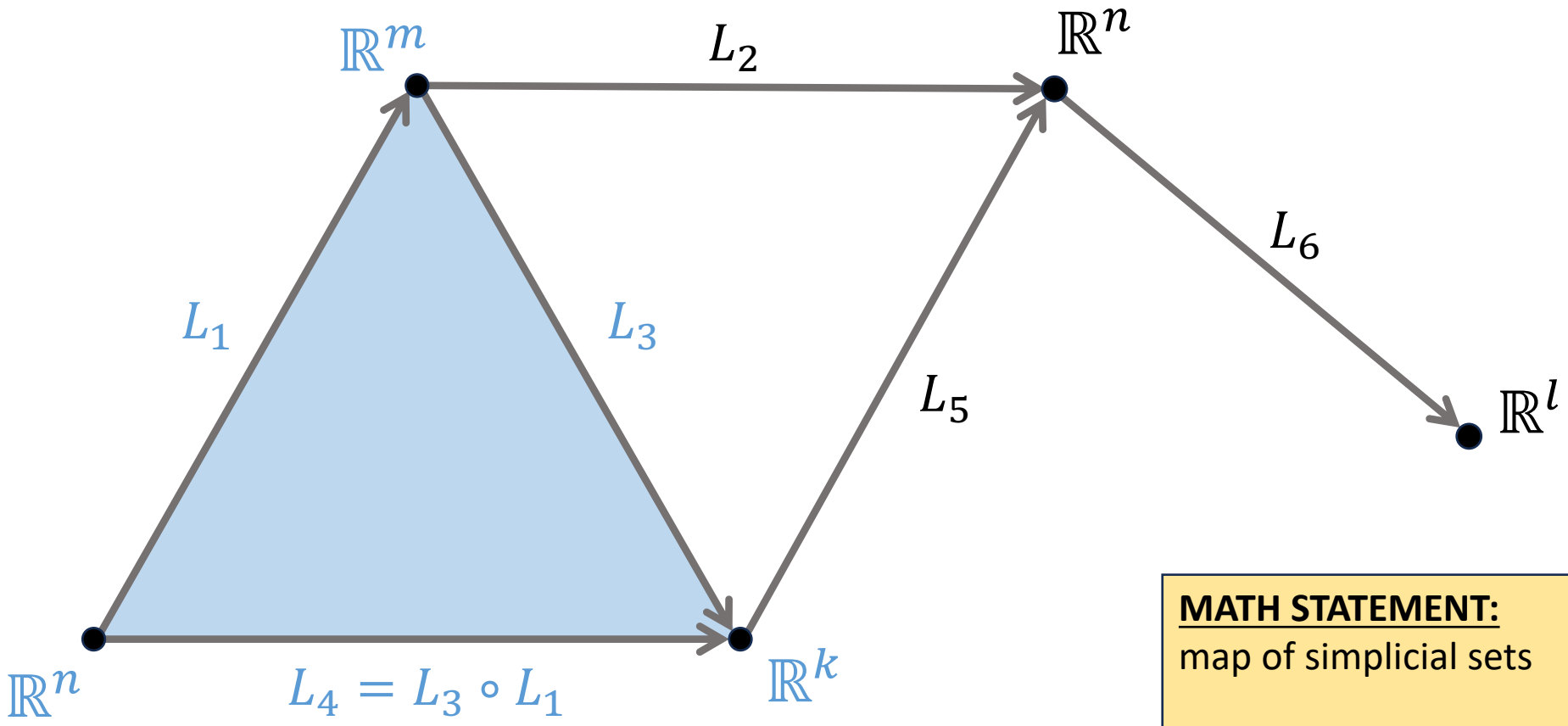
An Example



$$L_1 = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & 2 & -2 \\ 1 & 2 & 1 & 1 \end{pmatrix} \qquad L_3 = \begin{pmatrix} -3 & -2 \\ 1 & 0 \\ 0 & 2 \\ 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & 2 & -2 \\ 1 & 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} -3 & -2 \\ 1 & 0 \\ 0 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ -10 & 0 \\ 0 & -1 \end{pmatrix} = L_4$$

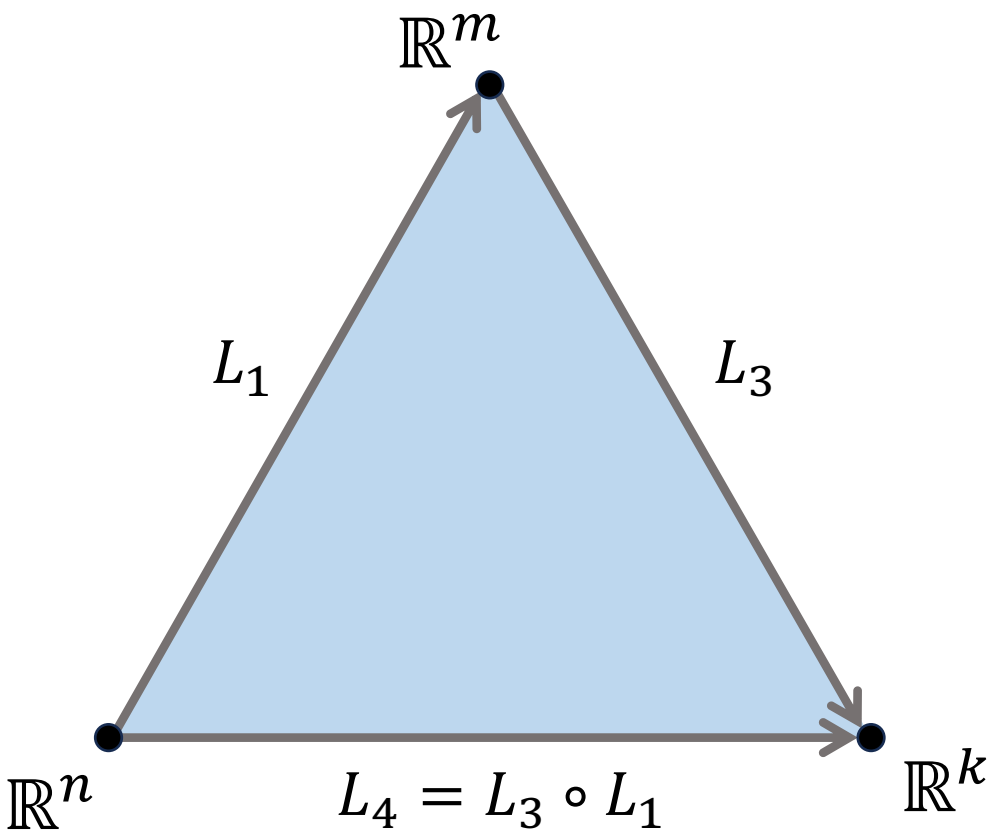
Preliminaries



MATH STATEMENT:
map of simplicial sets

$$\mathcal{F}: X_\cdot \to NVect$$

Preliminaries



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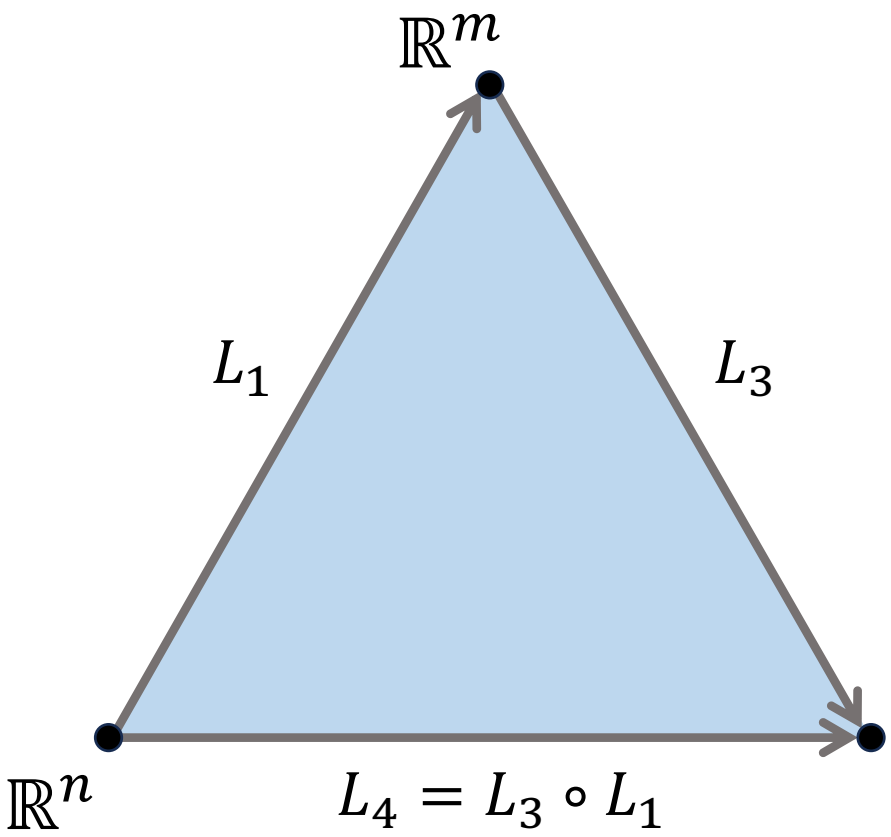
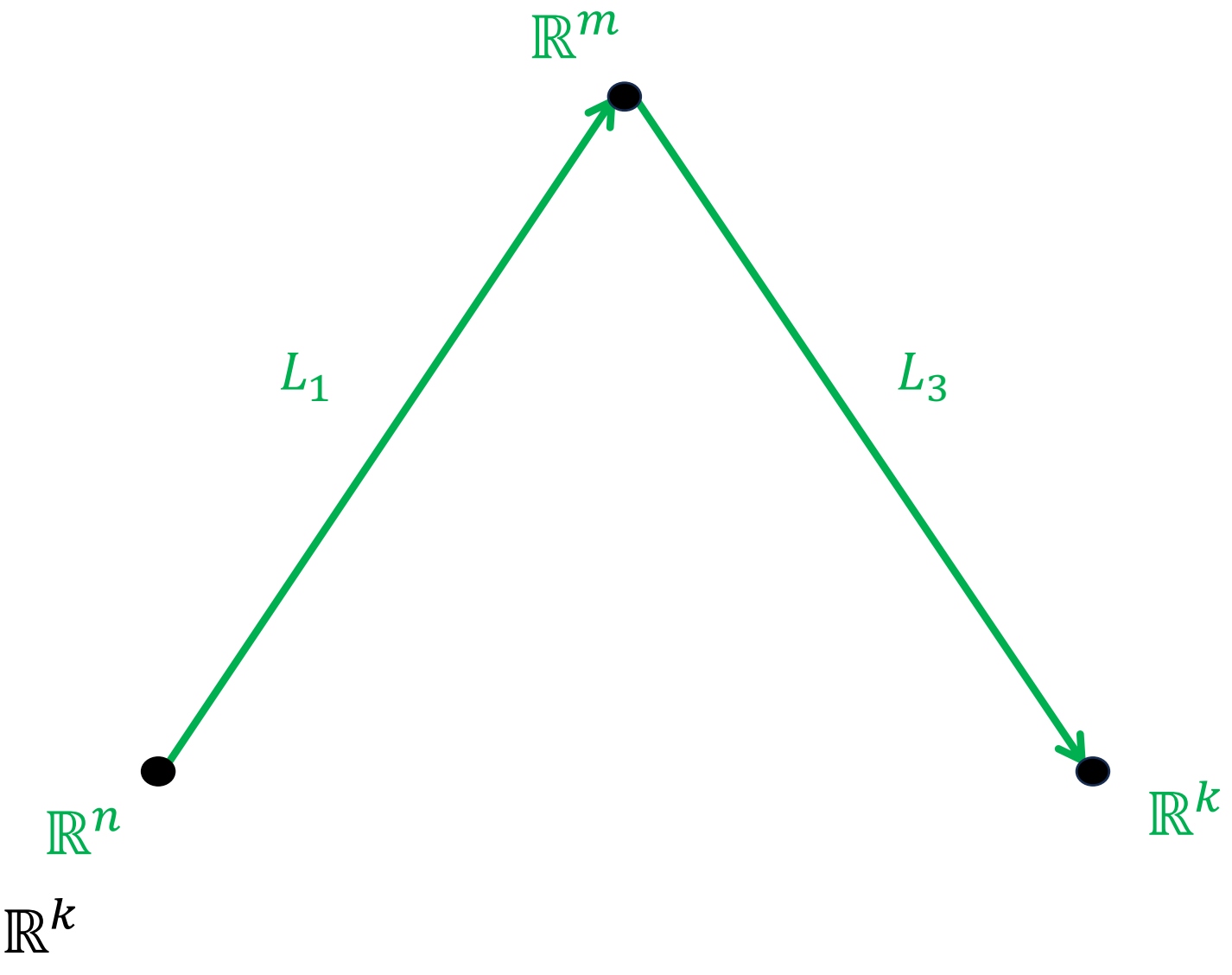
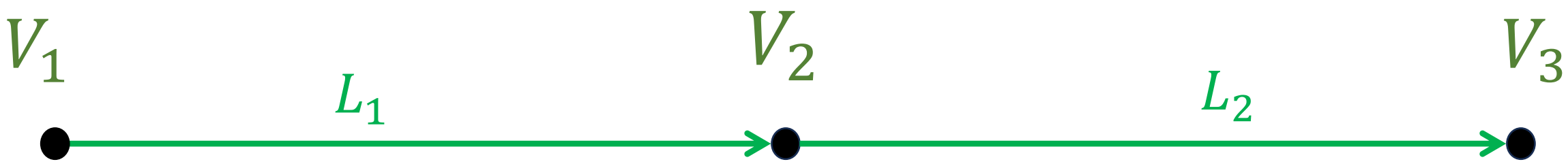




Preliminaries

chain complex



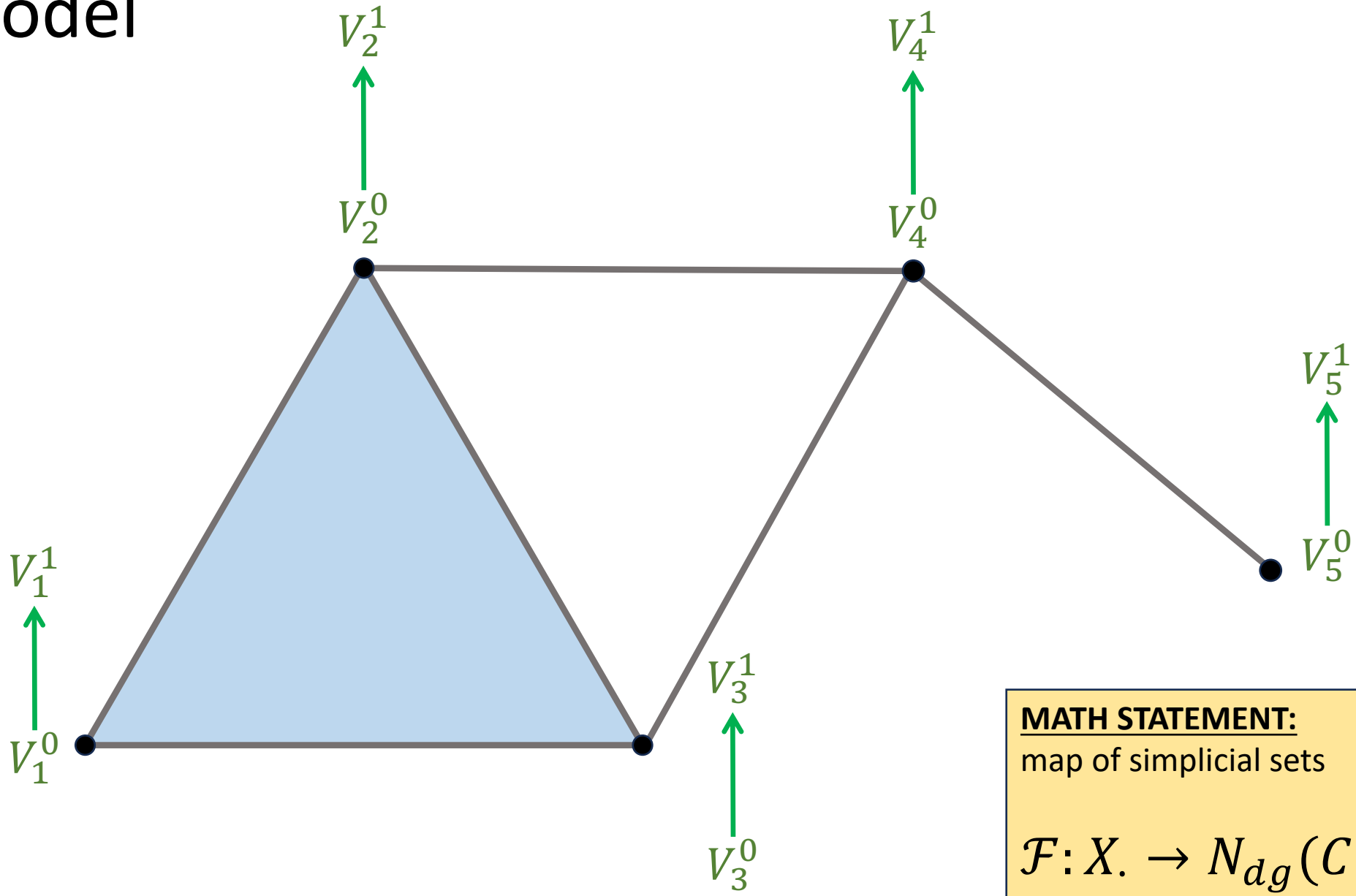
Purpose of Model

Major Theorems:

1. $x(t)$, heat equation using Laplacian converges to coherent sections in kernel of Laplacian, $\lim_{t\to\infty} x(t) \in \ker(\mathcal{L})$

2. $x(t)$ with stubborn agents also evolves to a predictable subspace

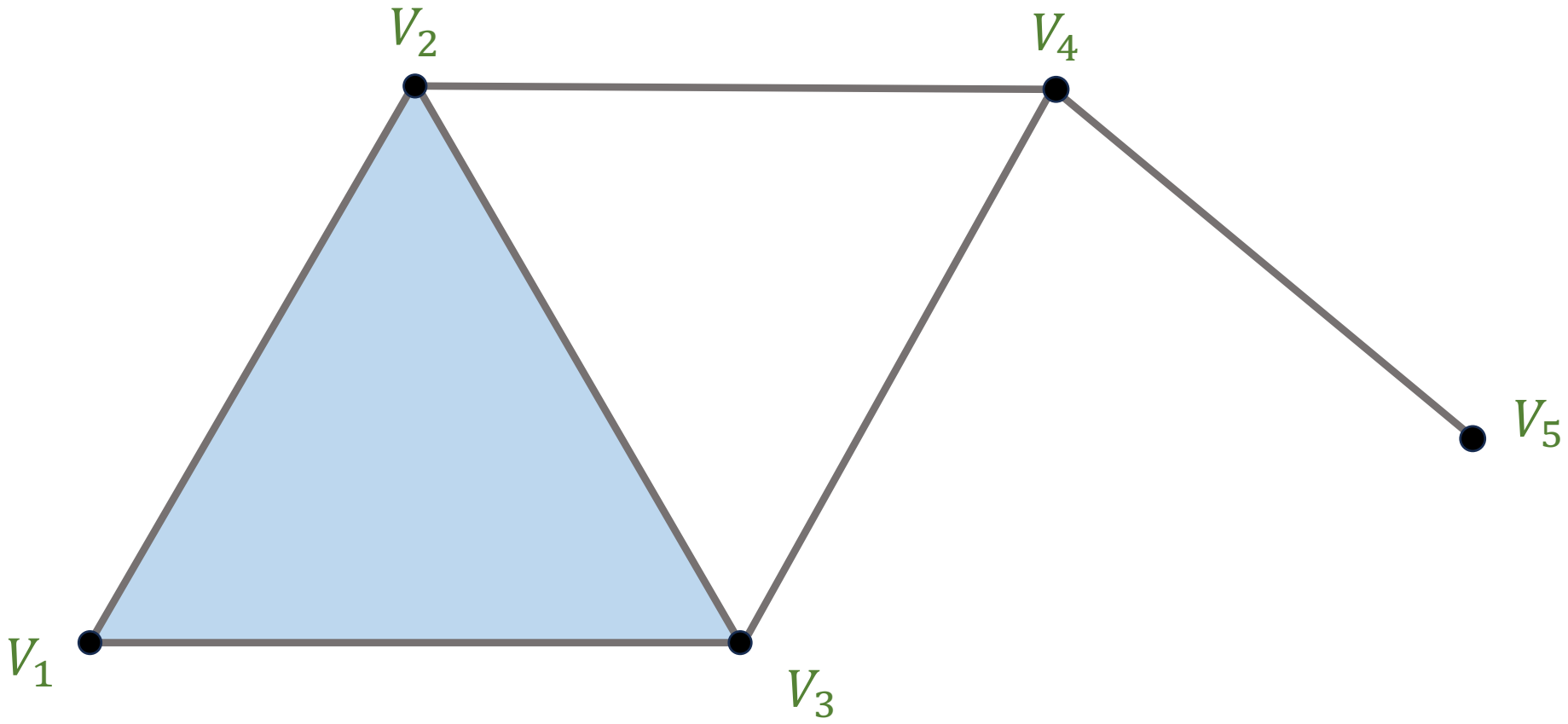
Our Model



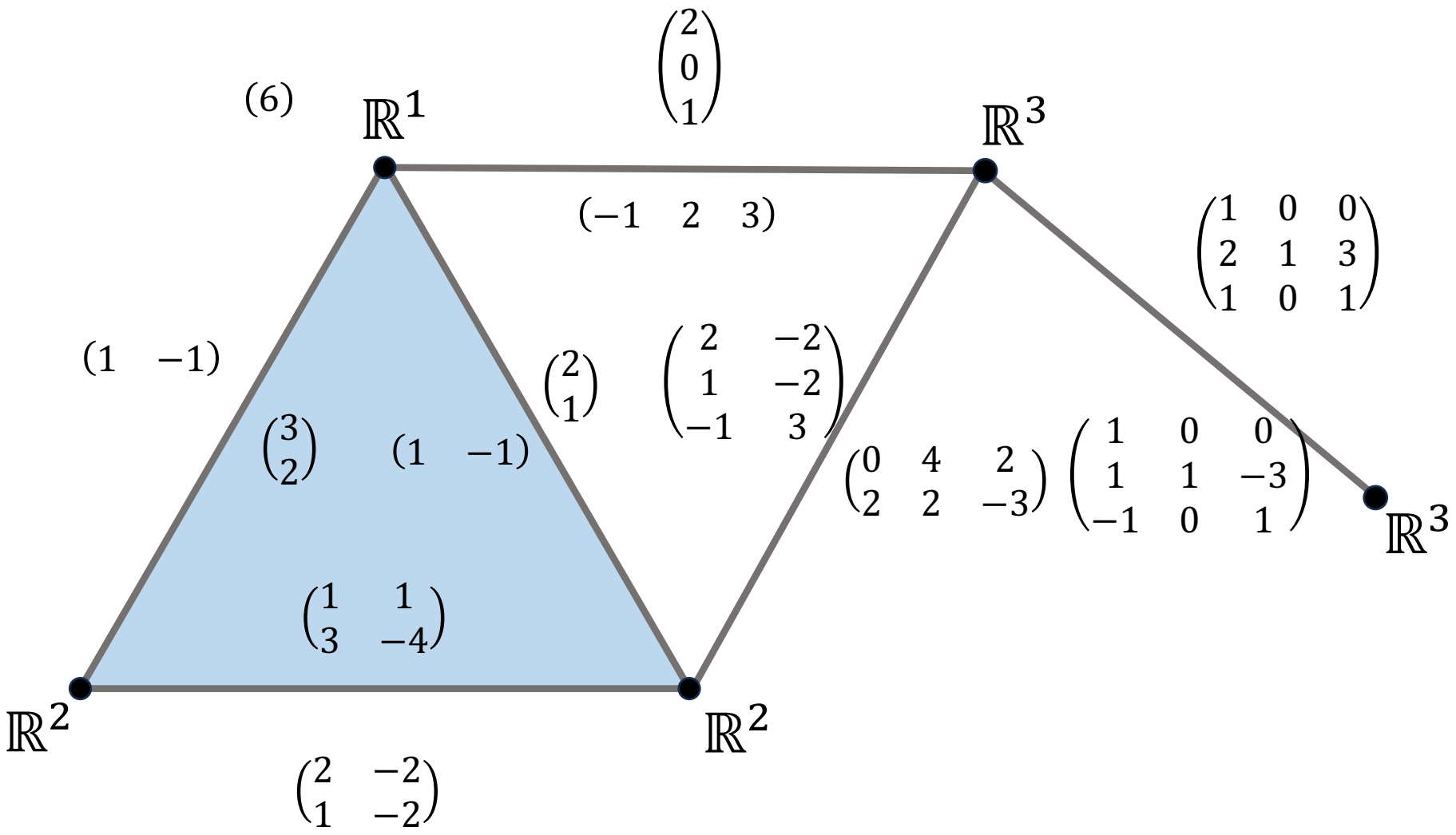
MATH STATEMENT:
map of simplicial sets

$$\mathcal{F}: X_\cdot \to N_{dg}(Ch^\cdot)$$

Our Model: Set Up



Example: coherent sections



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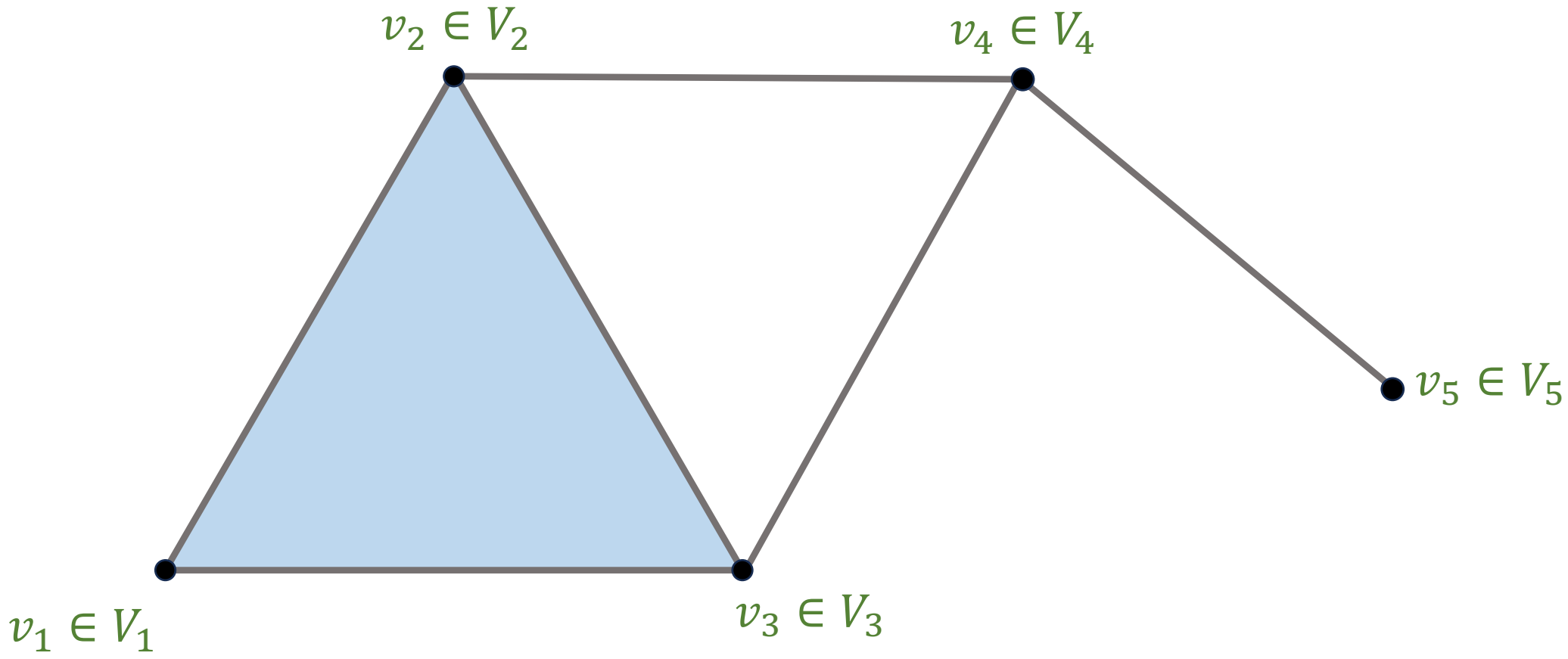

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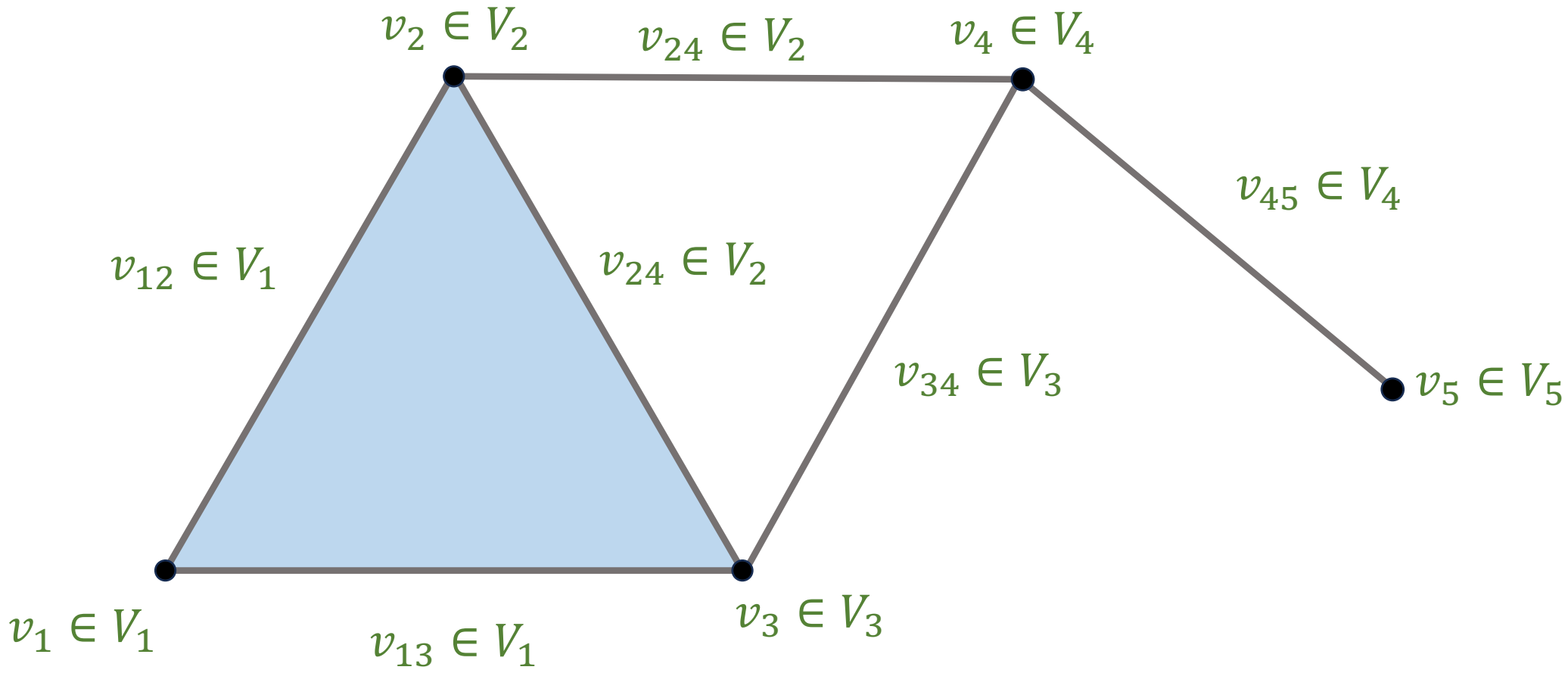

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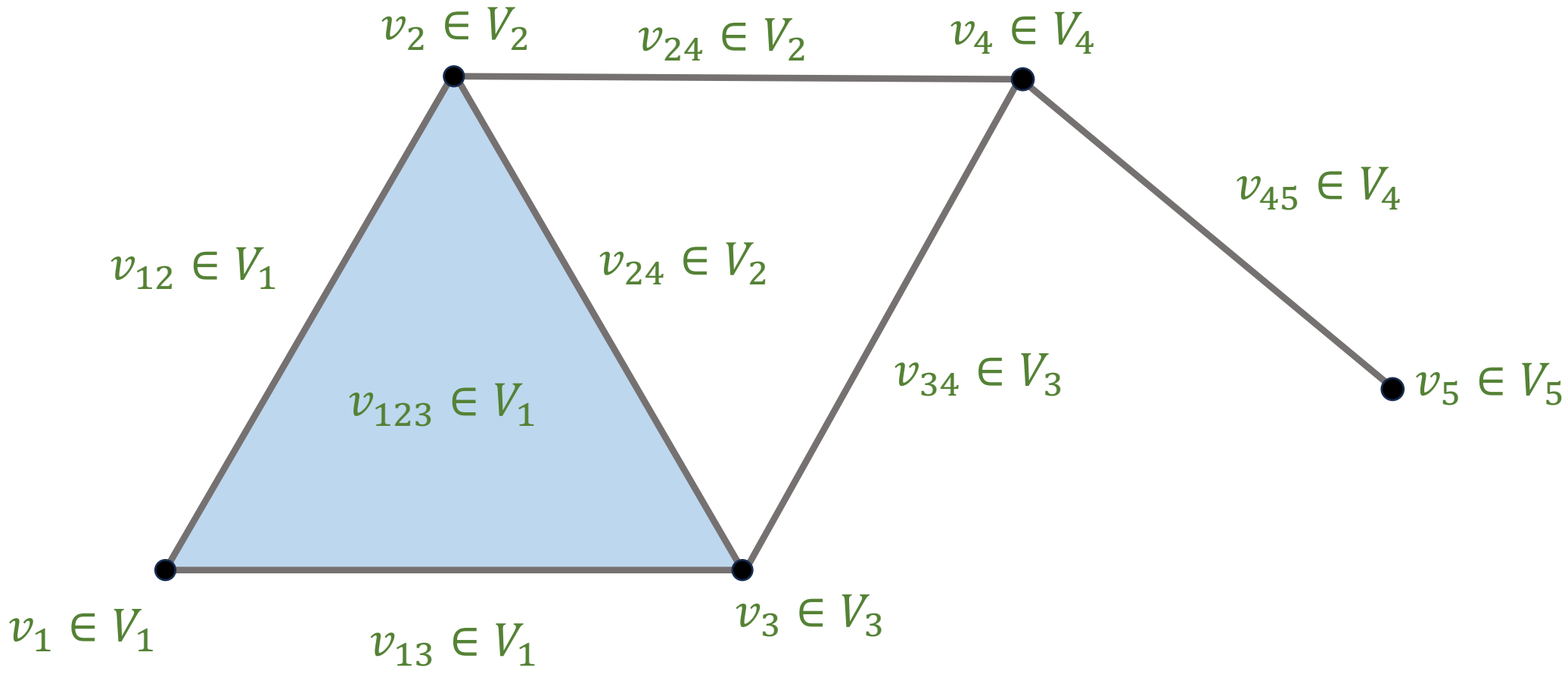

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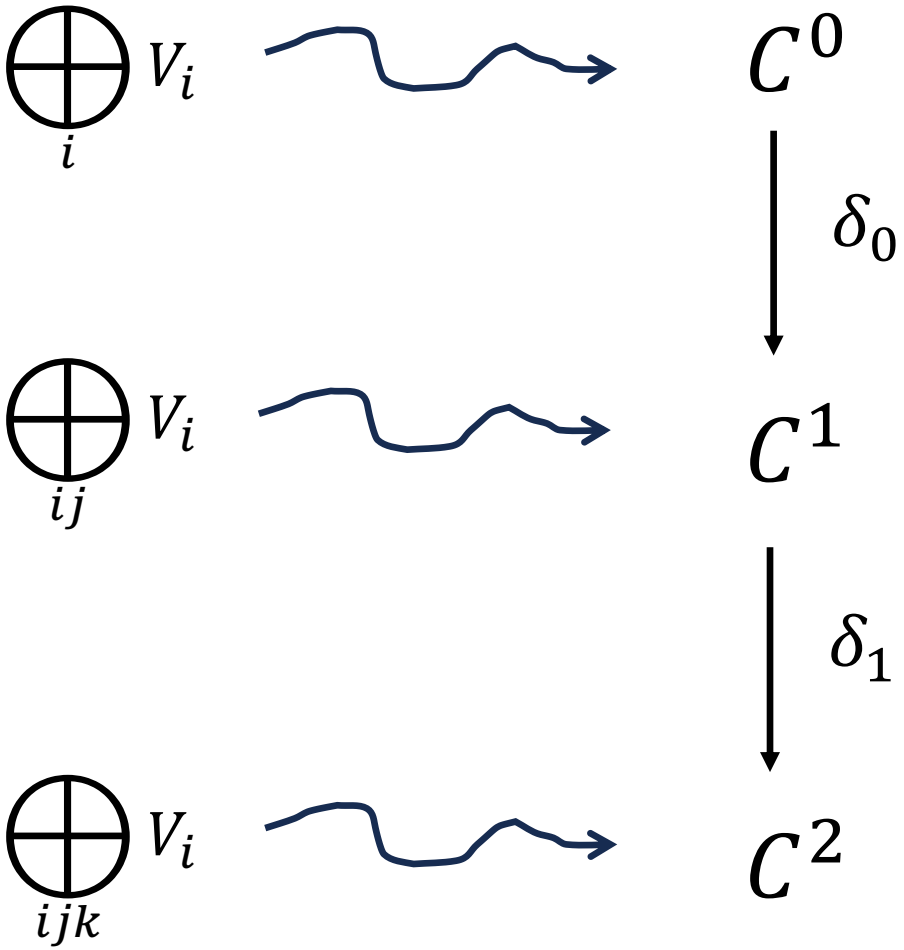

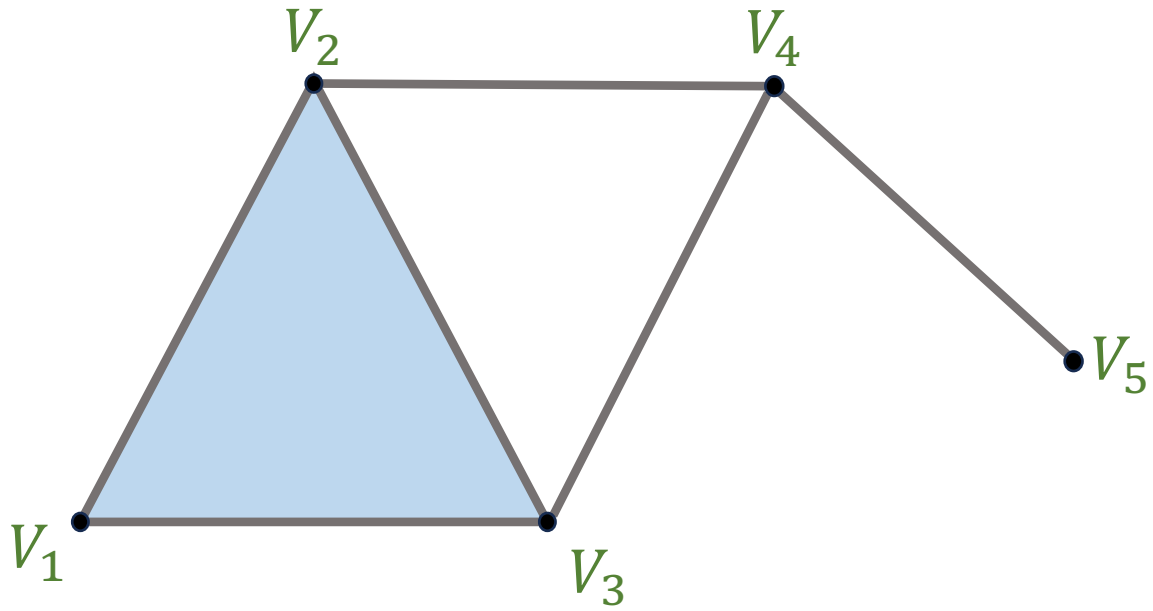

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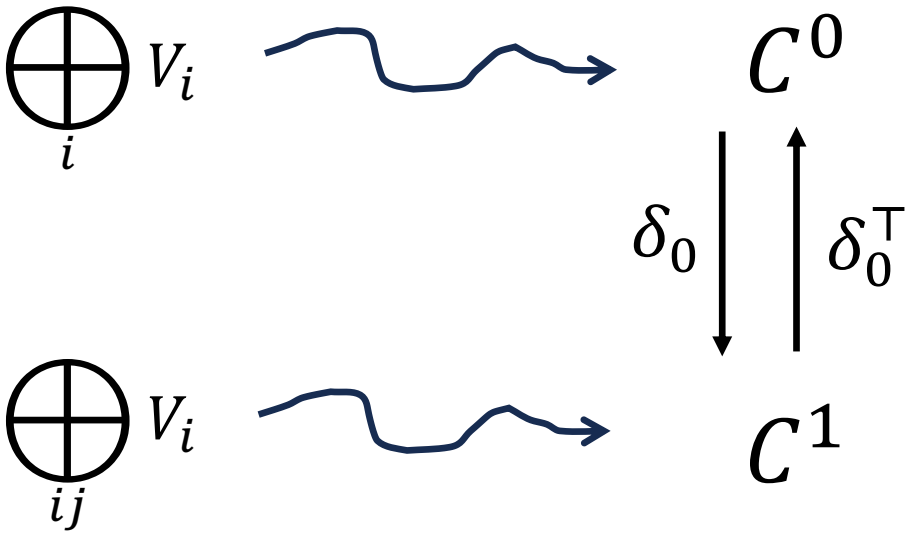
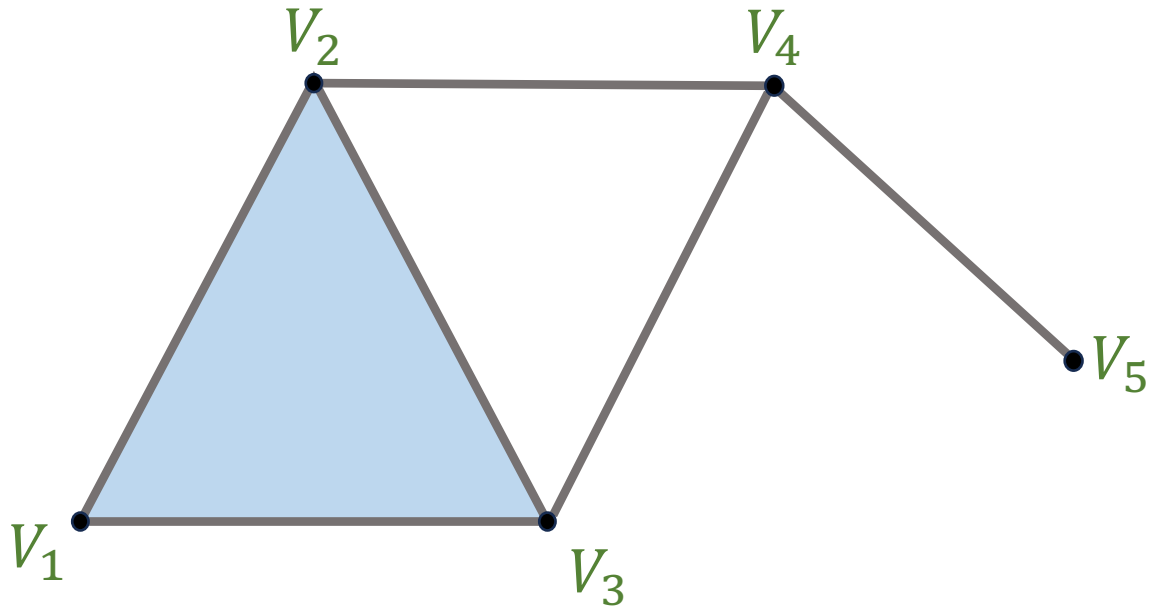




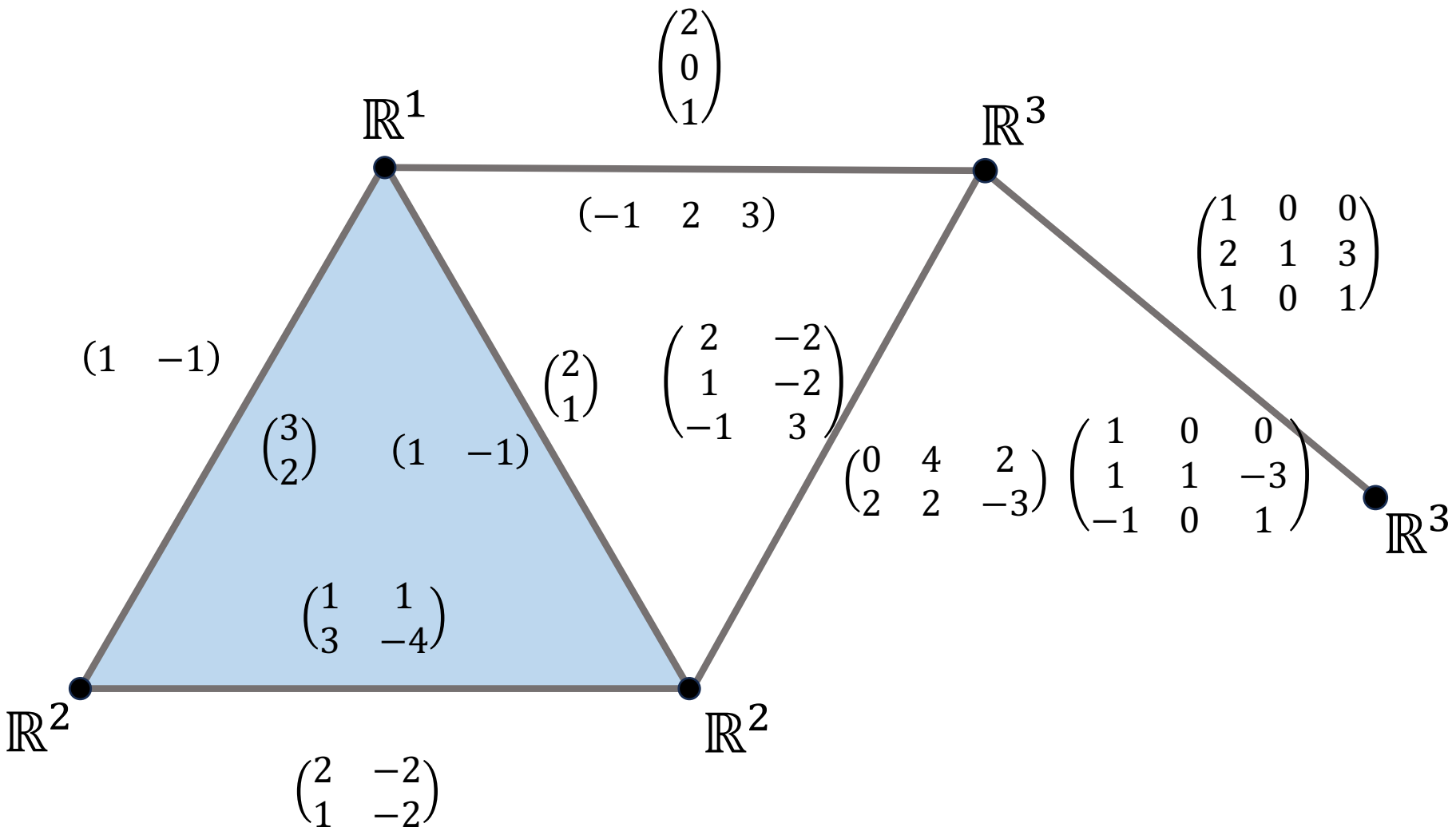
$$\delta_0 := v_i - L_{ij} v_j$$

MATH STATEMENT:

The Laplacian

$$\mathcal{L} = \delta^\top \circ \delta : C^0 \to C^0$$

Example: coherent sections

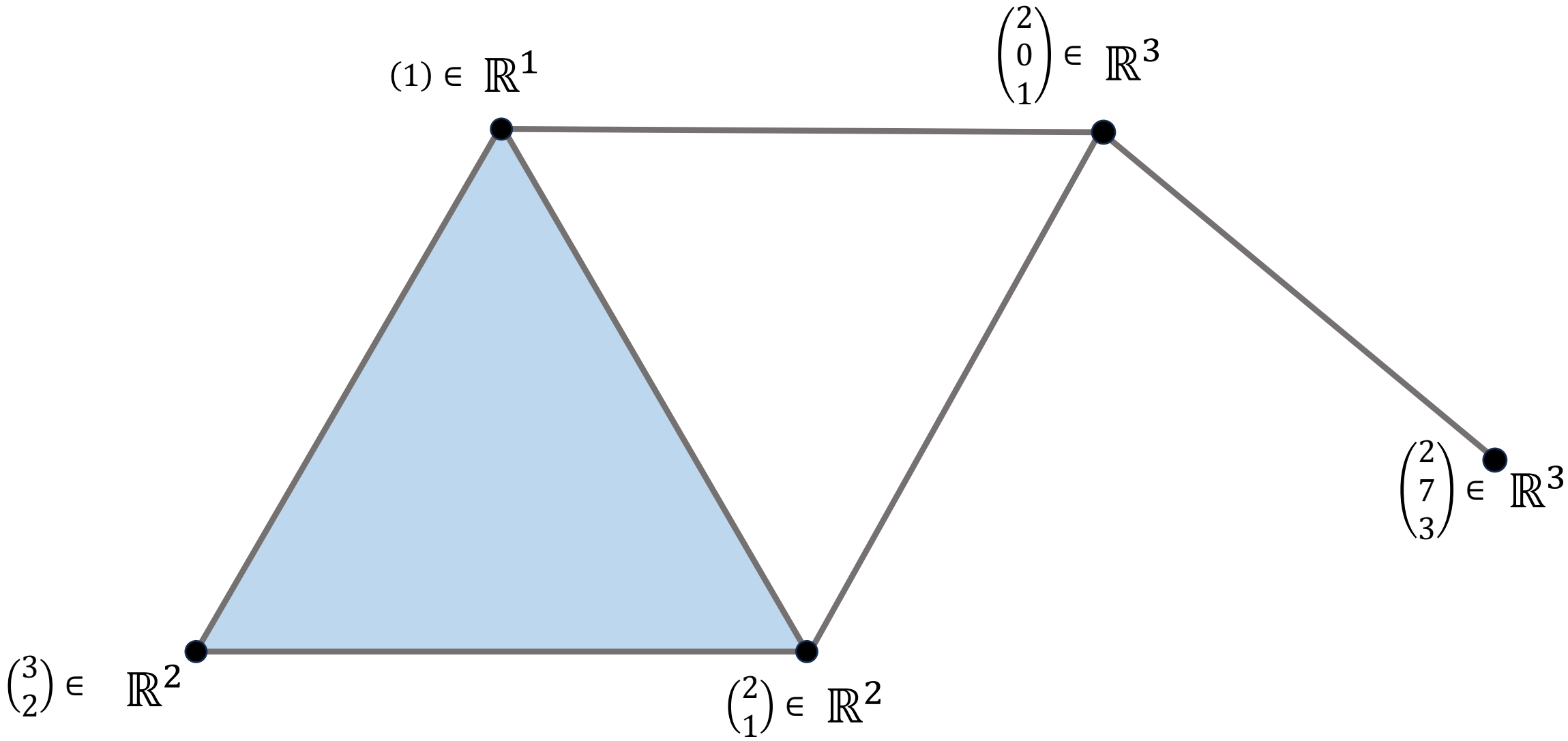


Example: coherent sections

$$\mathcal{L} = \begin{pmatrix} 8 & -6 & -1 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6 & 8 & 0 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 14 & -3 & -1 & -3 & -2 & -4 & 0 & 0 & 0 \\ -2 & 2 & -3 & 9 & 2 & -1 & -3 & -2 & 0 & 0 & 0 \\ -2 & -1 & -1 & 2 & 13 & -3 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -3 & -1 & -3 & 12 & 6 & 11 & -2 & -2 & -1 \\ 0 & 0 & -2 & -3 & 0 & 6 & 12 & 9 & -1 & -2 & 3 \\ 0 & 0 & -4 & -2 & -2 & 11 & 9 & 23 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 & 0 & -2 & -1 & 1 & 4 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & -2 & -2 & -3 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 & -1 & 3 & -2 & -4 & -3 & 11 \end{pmatrix}$$

Kernel= $\begin{pmatrix} 3 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ 0 \\ 1 \\ 2 \\ 7 \\ 3 \end{pmatrix}$

Example: coherent sections



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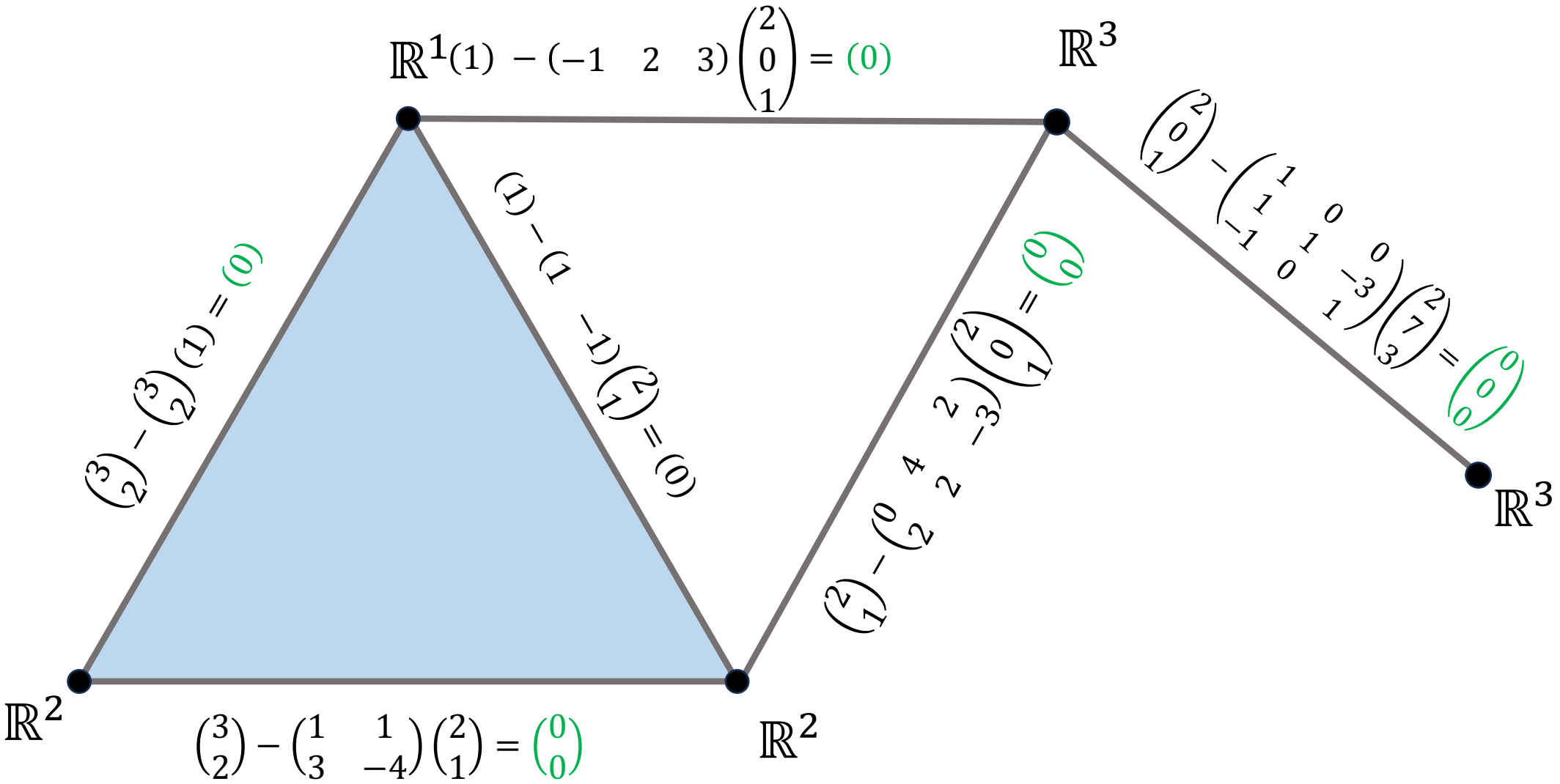
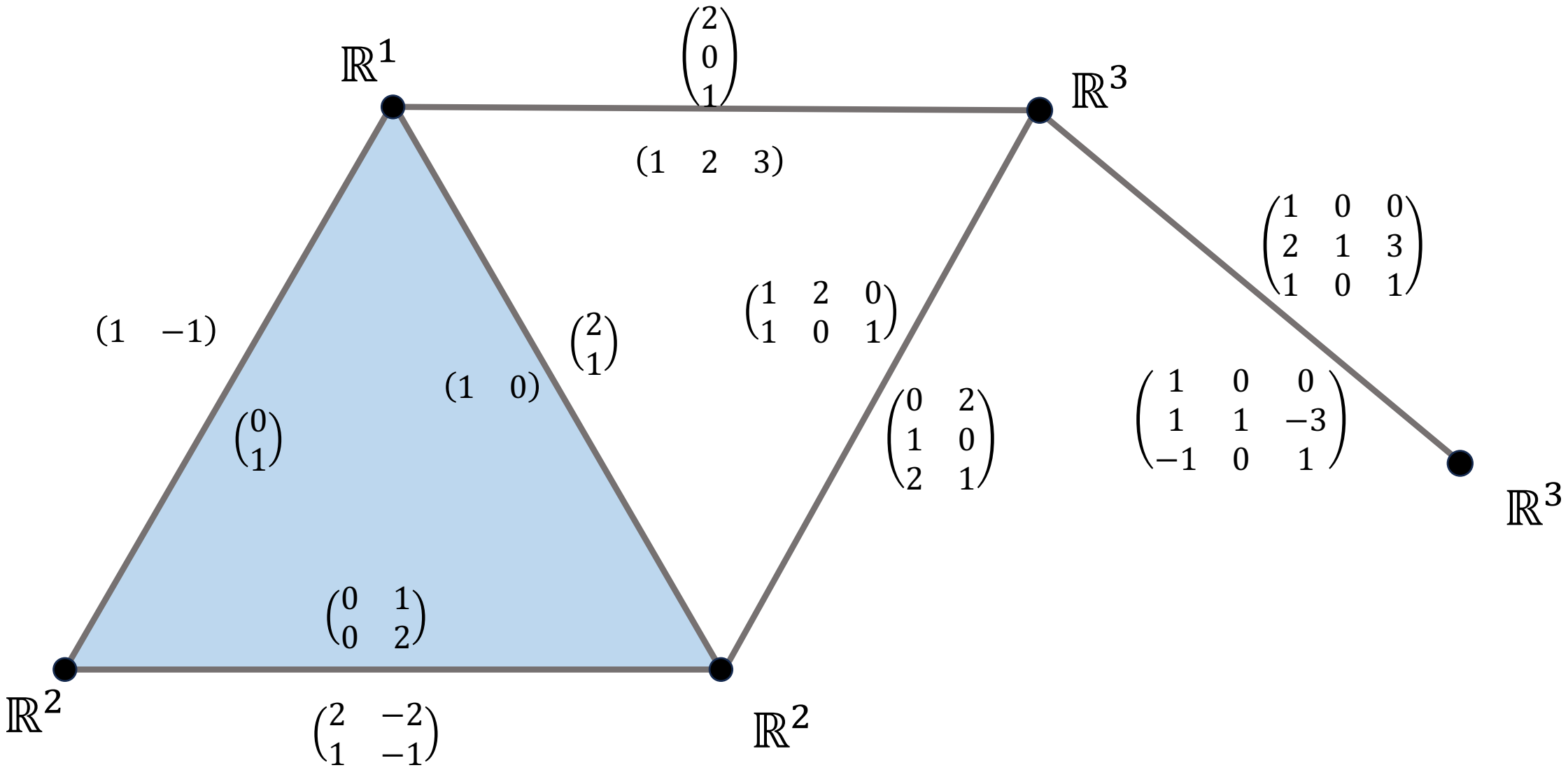


Example: noncoherent sections



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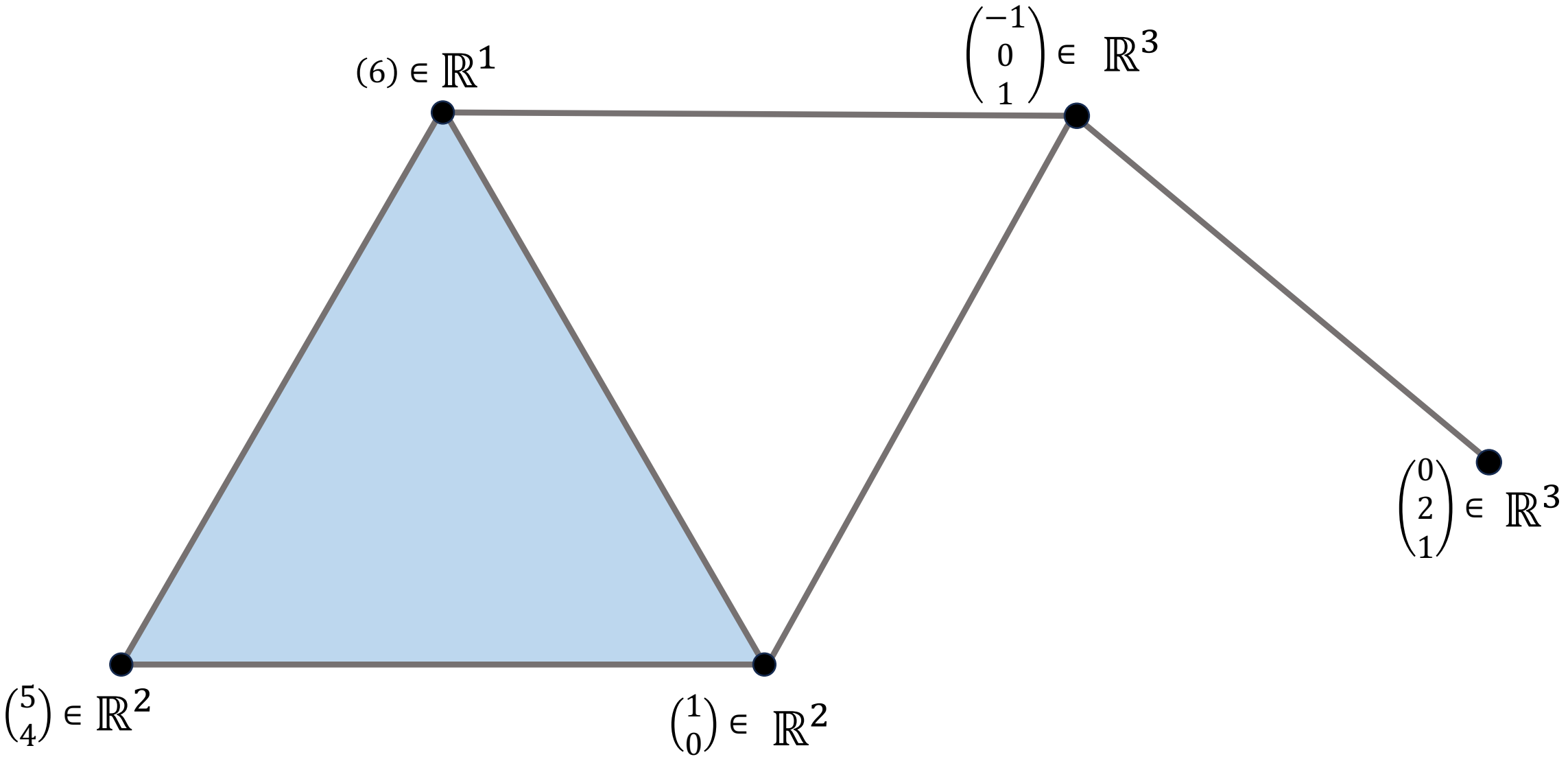

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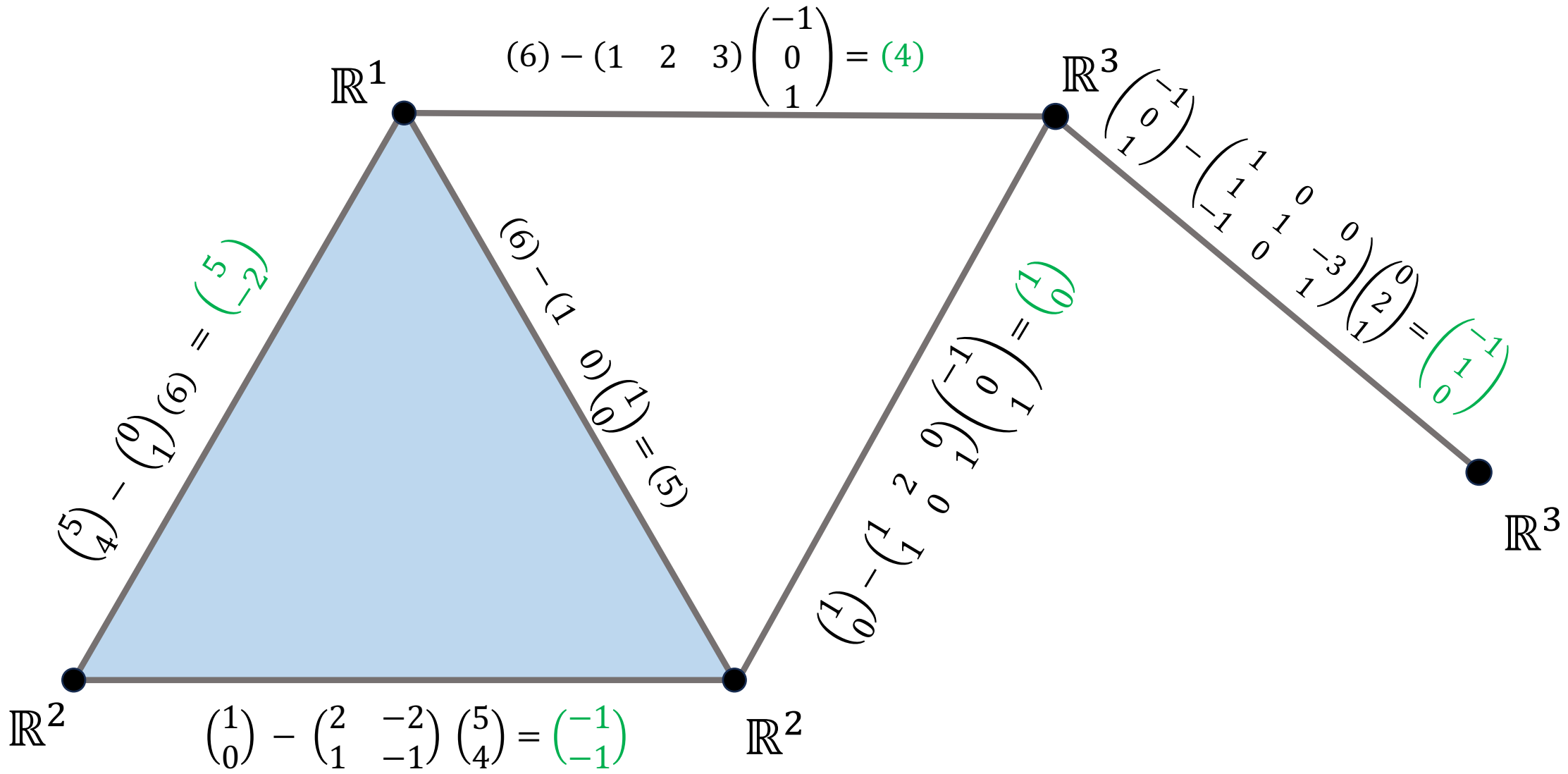

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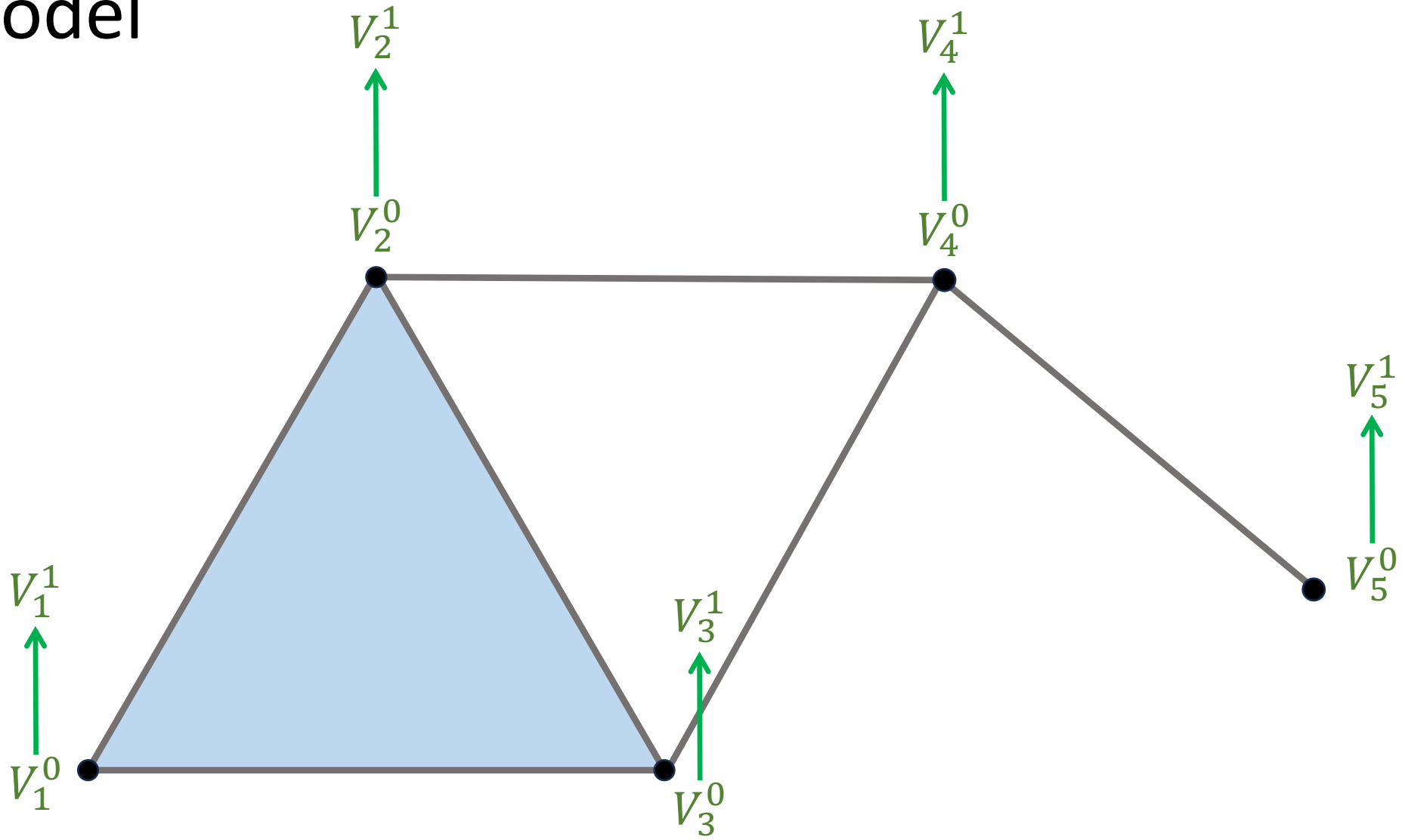

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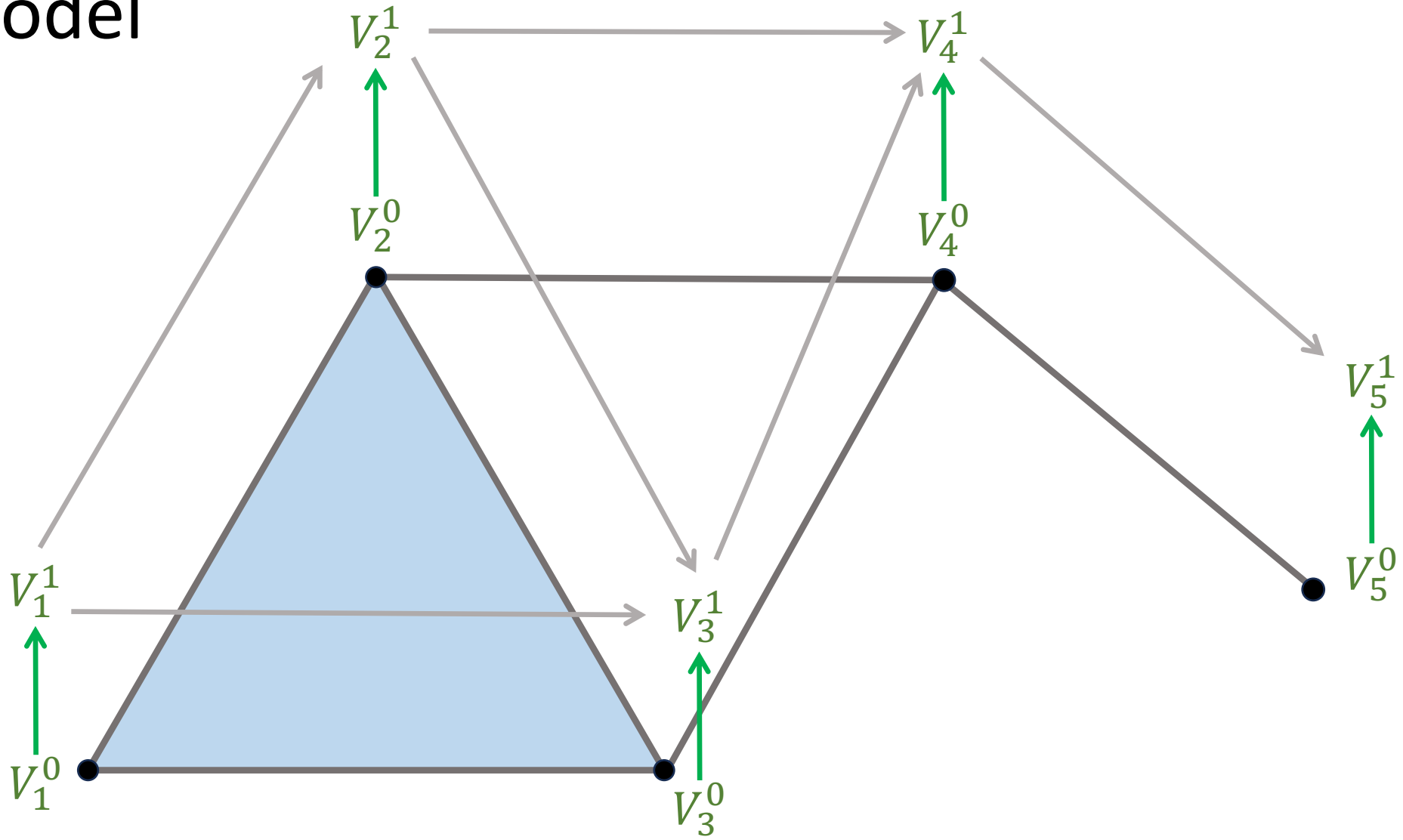

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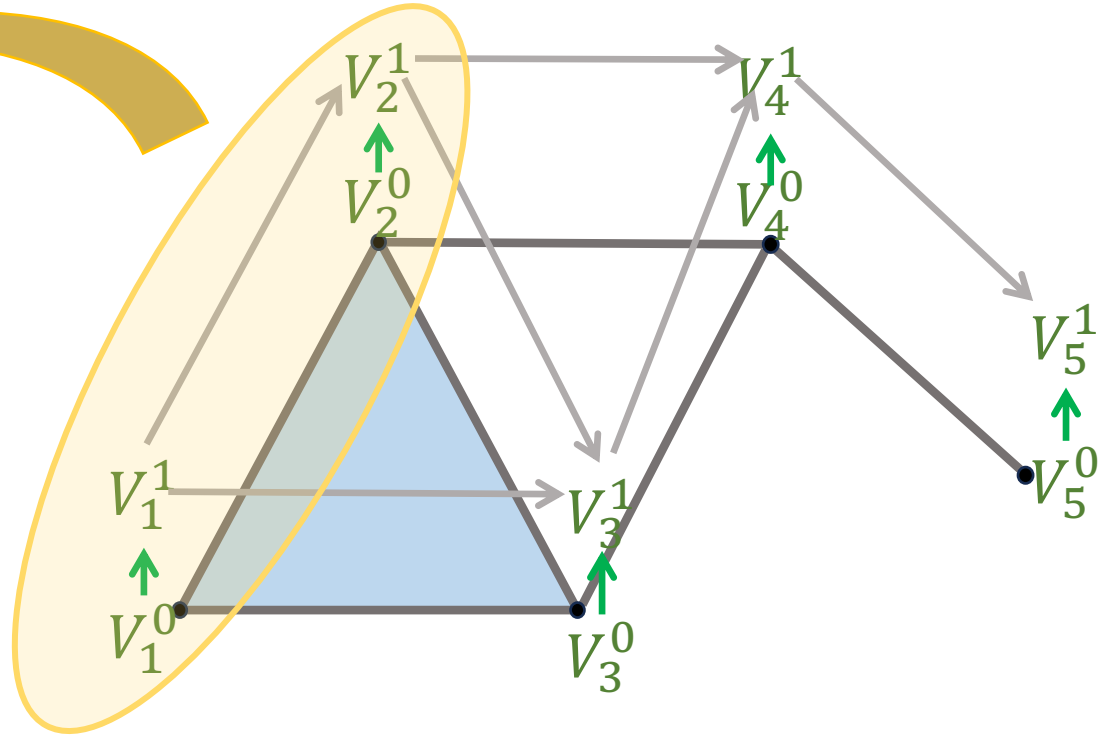
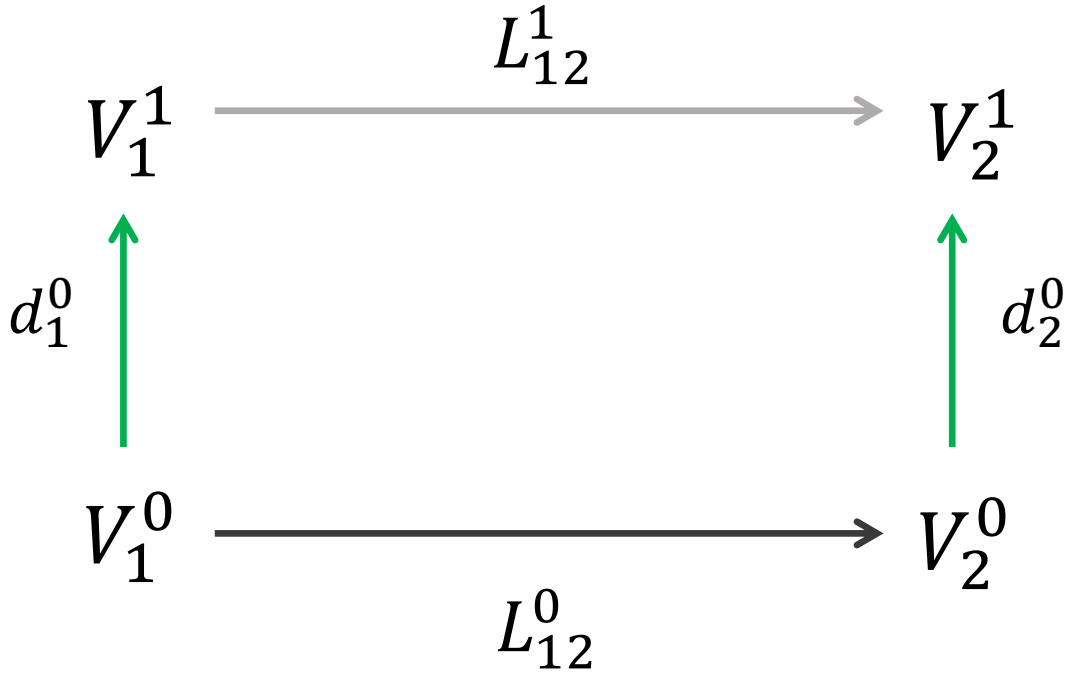
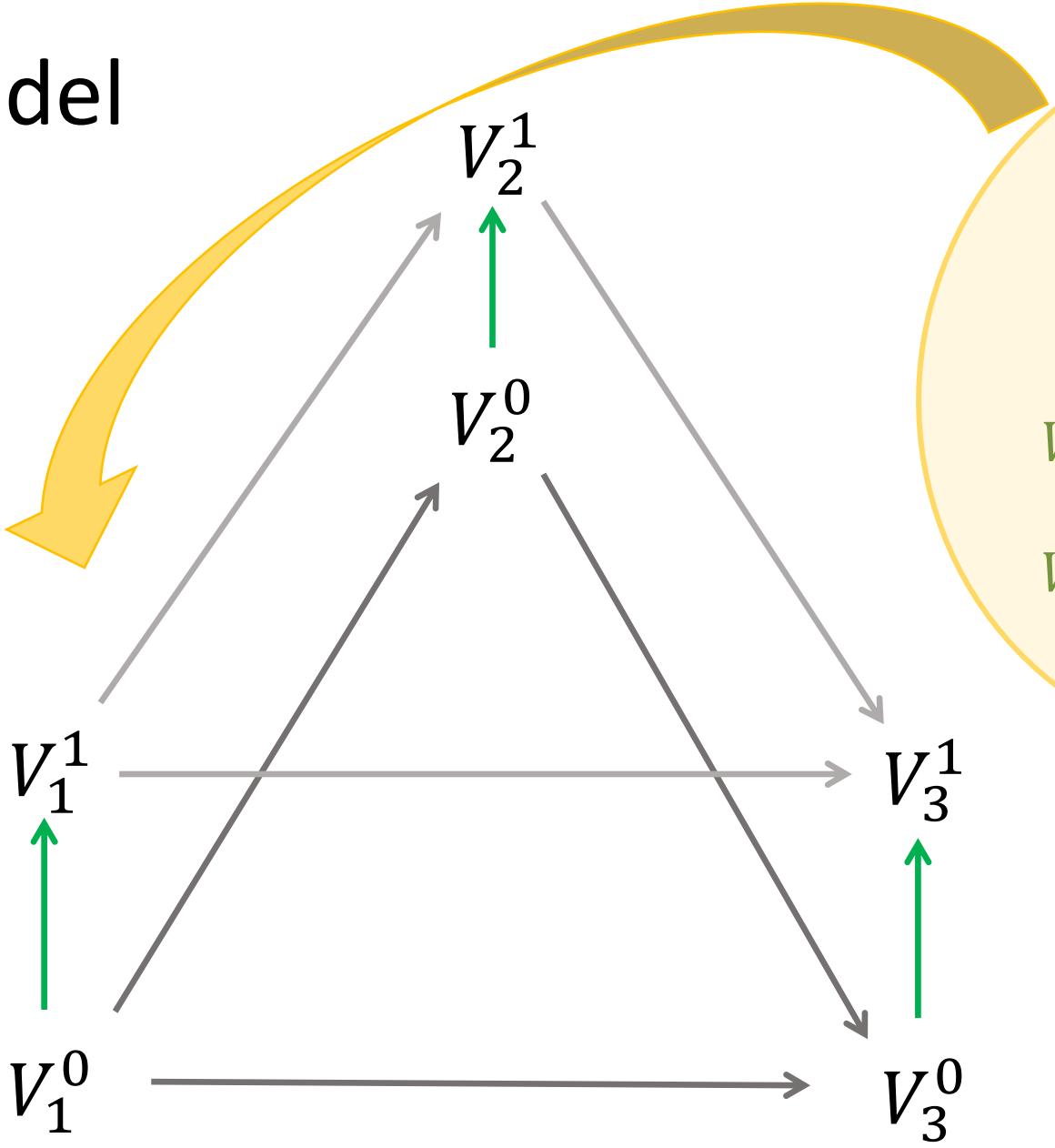
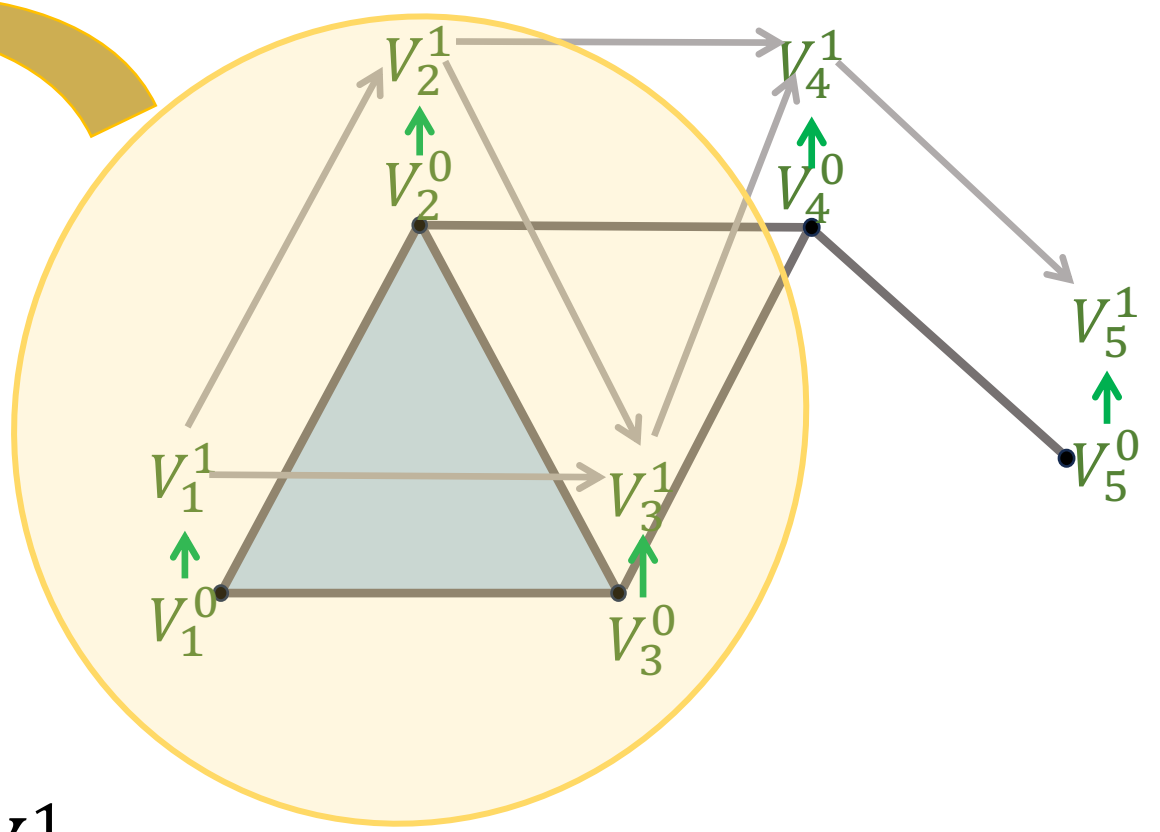

V_2^1
V_4^1
V_5^1
V_2^0
V_4^0
V_5^0
V_1^1
V_3^1
V_1^0
V_3^0


L_{12}^1
V_1^1
V_2^1
d_1^0
d_2^0
V_1^0
V_2^0
L_{12}^0

Our Model





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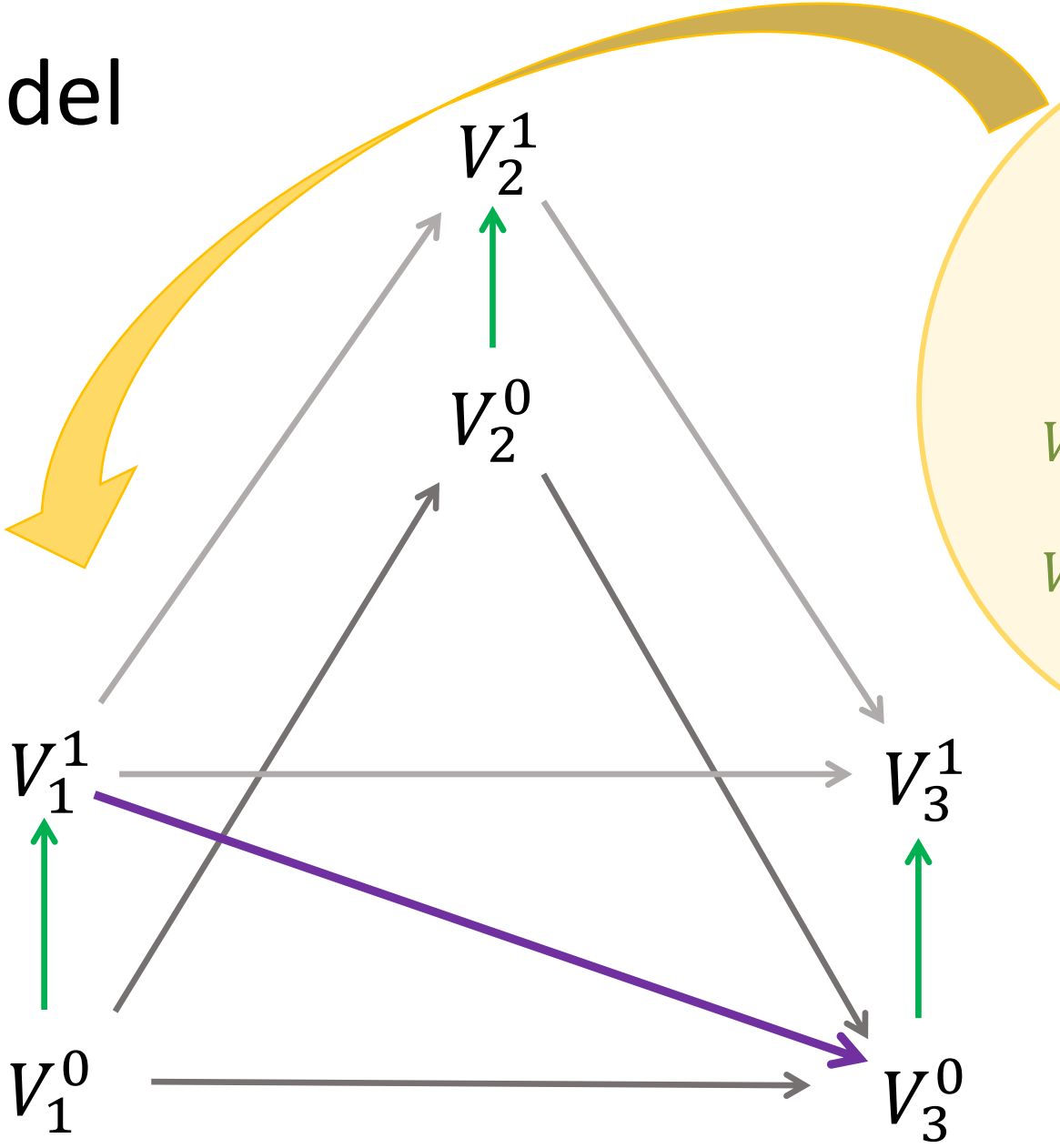

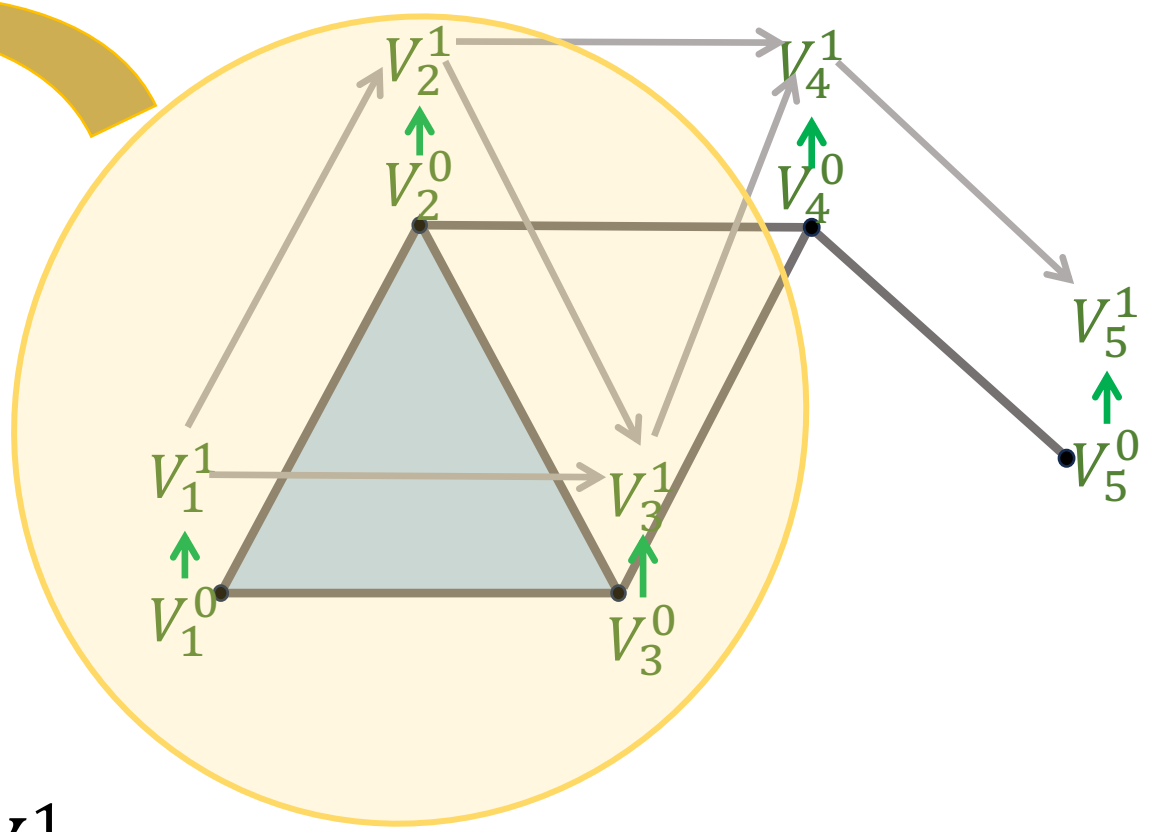

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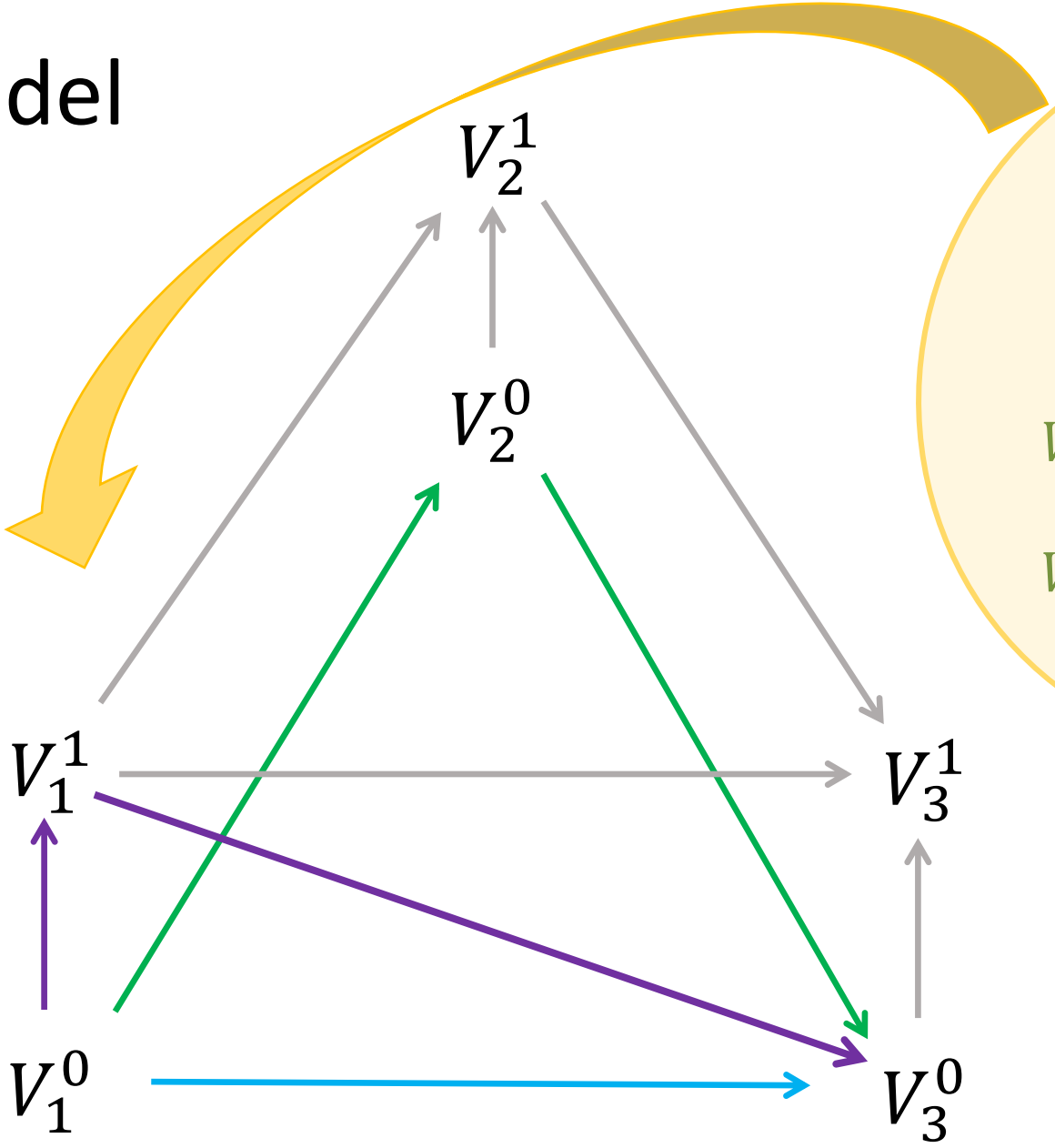

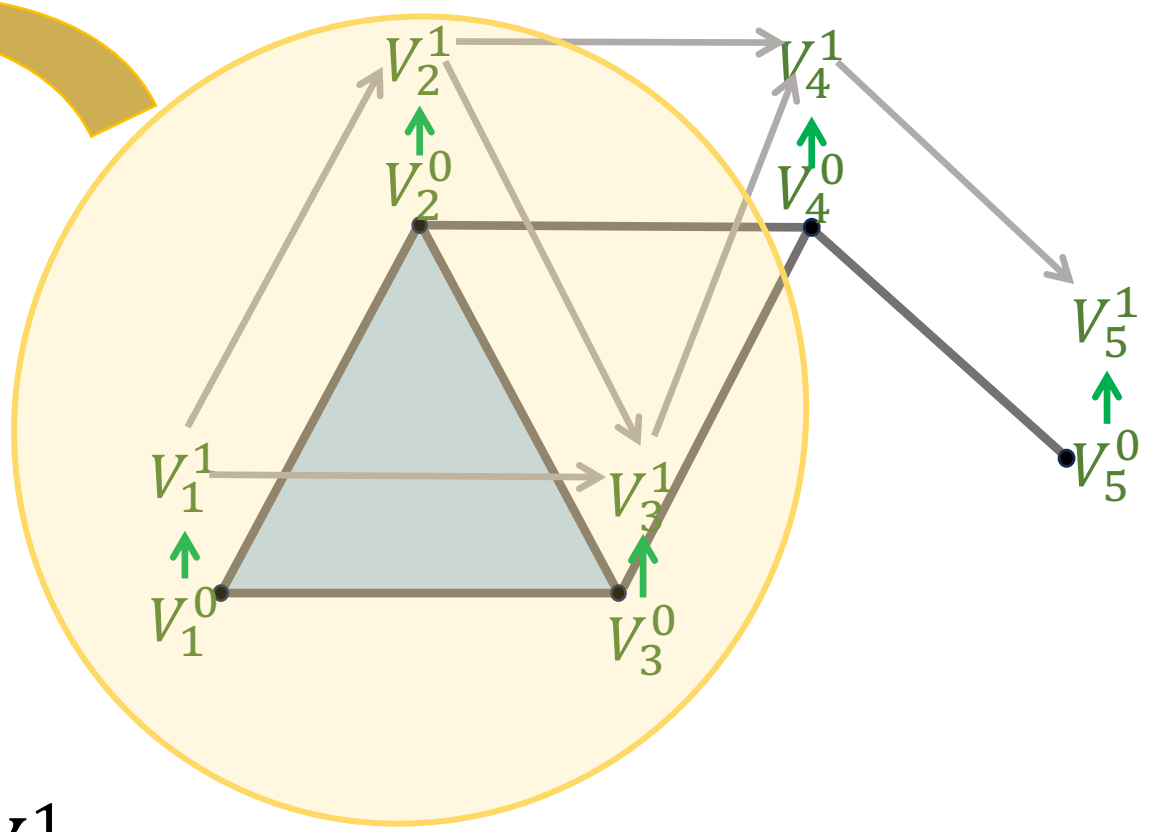

Recall: previous theorems

1. $x(t)$, heat equation using Laplacian converges to coherent sections in kernel of Laplacian, $\lim_{t\to\infty} x(t) \in \ker(\mathcal{L})$

2. $x(t)$ with stubborn agents also evolves to a predictable subspace

Future Work:

Interpretation of chain complex over vertices

References

- *Category theory in Context*

  -Emily Riehl

- *Discrete Vector Bundles with Connection and the Bianchi Identity*

  -Daniel Berwick-Evans, Anil N. Hirani, Mark D. Schubel

- *Opinion Dynamics on Discourse Sheaves*

  -Jakob Hanson, Robert Ghrist

- *Toward a Spectral Theory of Cellular Sheaves*

  -Jakob Hanson, Robert Ghrist

Thank you !