Lab

Part 0: Soldering Your Mic Board

The first thing you will do today is solder the mic board that you will be using for the project. This circuit serves as the audio front-end to your car, so **it is imperative that it is well-made.** A poorly-made mic board will cause much grief when it comes time to record and classify the voice commands you will use to control your car, which translates into a car that is extremely difficult to control.
Please watch the short video in the lab notebook to see a demonstration of proper soldering technique for your mic board. Here's a quick outline of the process:

1. Wet the sponge in your soldering station so you can clean your soldering iron. This is necessary for the iron to heat evenly.

2. Allow your iron to become fully hot. If you touch a bit of solder to the tip, it should immediately melt.

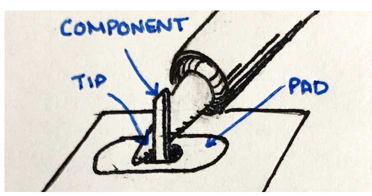
3. Clean your soldering iron before getting started by wiping it on the sponge. If it is particularly dirty, dip the tip in the flux or tip tinner to remove the old solder, tin the tip by applying a small amount of solder to it, and repeat until clean.

4. Tin the tip with a **tiny** amount of solder to improve heat transfer and reduce wear to the tip. This solder should **not** be enough to solder a component to the board: that's not the purpose of tinning the iron.

5. Start with the **lowest-profile** components on your board: the resistors. Place the proper resistors through the holes so that they are **completely flush** with the board, and bend the wires a bit to hold them in place. If you are using a soldering stand, use one of the clips to hold the resistor flush to the board; otherwise, lay the board on the desk so that the desk is holding the resistor flush to the board (this is why it's important to go from lowest- to highest-profile. It's less important if you are using a soldering stand, but it still does make it a bit easier to place components.

6. Make sure the board and component are securely held in place, and then pick up your iron in one hand and a moderate length of solder in the other.

7. Hold the soldering iron so that the **side** of the tip rests at the juncture between the metal via (be careful not to let the iron touch the plastic!) and the component, like so:



8. Heat them both evenly for just a few seconds — **don't hold the iron there for longer than 5 seconds**, because you can damage the board and the component if you allow them to get too hot. Less heating time is better: you want to heat the board and component for the minimum amount of time that still allows you to achieve a good joint.

9. Then, feed the solder in at the juncture of the board, the component, and the iron: it should flow quickly around the hole, adhering to both the hole and the component. If it only adheres to one of the two, the other isn't hot enough. Use just enough to make a sturdy joint: the hole around the component should be fully filled in, and a bit of solder should rise up the component's leg so that the joint looks like a Hershey kiss.

10. Move on to the next joint! Don't forget to clean and re-tin the tip every time you finish soldering a component.

Please Note: The EECS department uses lead-free solder. Traditional solder is typically 60% tin and 40% lead, while lead-free solder is typically over 99% tin (the remaining percentage is usually copper). If you have soldered with leaded solder in the past, please be aware that lead-free soldering has some important differences:

1. With traditional solder, "hot" (i.e., conductive) joints are shiny and "cold" (nonconductive) joints are mottled or have a less-than-mirror shine. While a hot traditional joint should look like the shiny side of a piece of aluminum foil, a hot lead-free joint looks like the non-shiny side.
2. Traditional solder melts at around 365-370° Fahrenheit, while lead-free solder melts at around 440°, so, clearly, you will have to set the soldering iron to a significantly higher temperature to work with lead-free solder. Your iron working temperature will be a few hundred degrees hotter than the solder's melting temperature, so set your iron about 100 degrees higher than your usual temperature to work with lead-free solder. You will also have to hold the iron to the joint for a little bit longer: if you're working at 600-700° F with leaded solder, a joint takes only a second or two to complete, while at 700-800° F with unleaded solder, a joint will take about twice as long. (This is still very quick, so be careful not to overheat your joints. Overheating your joints can burn your board, lift the solder pads from the board, or damage sensitive components.)
3. **Important:** The soldering iron tip is especially sensitive to oxidation when working with lead-free solder. Make sure to clean and re-tin your tip after soldering ***every*** component.
4. In order to avoid cross-contamination (in compliance with the Restriction of Hazardous Substances, or RoHS, policy), you should not use the same tip to work with leaded solder that you use to work with lead-free solder.

Part 1: Intro to RC Circuits

There are two approaches to conceptualizing and analyzing AC circuits, or, for our purposes, any circuit in which voltage and current change over time:

1. Voltage and/or current vs. time (time domain)
2. Amplitude vs. frequency (frequency domain)

You will cover the frequency-domain approach starting next week in lecture, so we will focus on time-domain analysis and building intuition for the frequency-domain view today in lab.

Capacitors: Physical Intuition

Charging

Let's look closely at the parallel plate capacitor. Consider this situation: you have two parallel metal plates with no difference of charge. You place them very close together, and then hook up a battery between them. The battery first pushes a single electron onto one of the plates. The electron's electric field repels other electrons, and because the plates are very close together, it repels electrons on the opposite plate, *inducing* a positive charge on the other plate. The battery has a more powerful electric field than the electrons around our new electron, so it stays put on the first plate, which now has a charge Q of -1e, while the second plate has a charge of +1e. The battery continues pushing electrons toward our first plate until the combined electric field of all the electrons on the first plate is as strong as the battery's, thereby preventing the battery from adding any more electrons to the plate. The voltage between the capacitor's plates is now equal to the battery's voltage, and the capacitor has reached steady state. If we replace the battery with a resistor or wire, the capacitor will discharge so that the plates return to their original state of having no difference in charge. The process described in this paragraph is probably very familiar to you already, but wait — there's more.

What happens when we charge and discharge a capacitor very quickly?

Now, imagine you only allow the capacitor to charge for a very short period of time before you discharge it. The capacitor does not fully charge, so the voltage between the plates does not reach the battery's voltage before the cap is allowed to discharge. Now, imagine you continuously decrease the amount of time the capacitor is allowed to charge before you discharge it, until the charge difference between the plates over the full charge-discharge cycle remains very close to zero. This implies that the voltage difference between the two plates is also virtually zero.

Which (extremely) basic component has zero voltage difference across it?

The ideal wire.

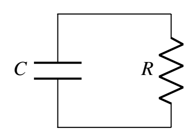
Therefore, **if we have an infinitely fast charge-discharge cycle, the capacitor behaves like an ideal wire.** The operative phrase in that sentence is ***behaves like***: *no current actually flows* ***through*** *the capacitor.*

How could we accomplish this without physically flipping a component in the circuit?
"Outsource" the flipping of polarity to the input signal: flip the *signal's* polarity instead of a component's.

Basic Time-Domain Analysis: Differential Equations
Below are the differential equations that model electric behavior in capacitors and inductors:

$$i_C(t) = C\frac{dv_C(t)}{dt} \qquad\qquad v_L(t) = L\frac{di_L(t)}{dt}$$

Capacitors and inductors are often referred to as ***duals*** *of each other. Can you guess why?*
Today, we will focus on RC circuits. Let's begin with analyzing the following circuit:



Let's say we want to find voltage through the capacitor as a function of time. Assume the capacitor is charged (we are examining the discharging behavior). Using both Ohm's law and the capacitor differential equation above, we can set up the following equations:

$$C\frac{dv_C(t)}{dt} = i_C(t) = -\frac{v_C(t)}{R}$$

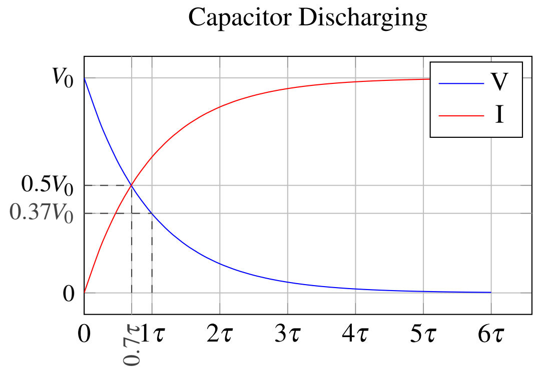
Now, we can drop the $i_C(t)$ term :

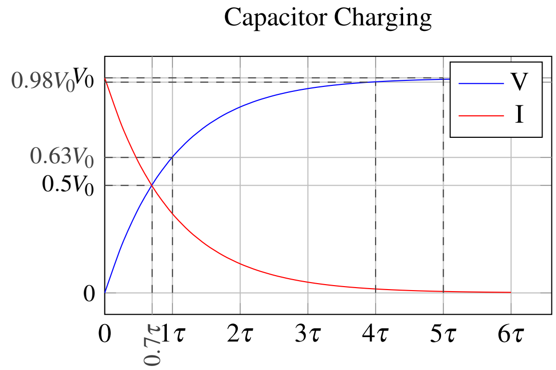
$$C\frac{dv_C(t)}{dt} = -\frac{v_C(t)}{R}$$

Let's step back from the math for a moment and think about this problem intuitively.
As negative charge collects on one plate of the capacitor, it becomes more and more difficult to force more electrons onto that plate. In fact, electric force is linearly related to charge: if each electron has its own electric field, and this electron is attempting to join with a group of electrons, the group's collective field will increase by precisely the magnitude of the new electron's field. Therefore, as more electrons collect on the plate, it becomes linearly more difficult to add more electrons to the plate: the change in voltage between the plates is linearly proportional to the change in charge, and change in charge with respect to time is defined as current. This is shown directly in the differential equation that relates voltage and current through a capacitor. Our goal, though, is to find voltage as a function of time. So, we now consider the second equation above. To solve the differential equation, we need to find a function that relates voltage linearly (i.e., by a constant factor) to its own derivative. So, let's think: which function do you know of that, when you take its derivative, is the same function multiplied by a constant factor? (For those of you who have already taken Math 54 and are familiar with the concept of eigenvalues and eigenfunctions, think: what is the eigenfunction for the differentiation operator?)
Solving the ODE, we find:

Discharging: $V = V_0 e^{-\frac{t}{RC}}$ Charging: $V = V_0(1 - e^{-\frac{t}{RC}})$

which follows from our intuition, as the derivative of the exponential is the exponential itself (and the exponential is an eigenfunction of the differentiation operator). A review of linear constant-coefficient differential equations will be included in appendix A of this note, and Appendix B will contain a brief sketch of the concept of the eigenfunction. We plot the charging and discharging functions below. Note the relationship between current and voltage.





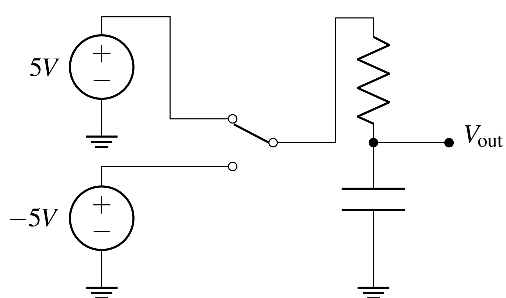
You can see the **"5RC rule-of-thumb"** illustrated here: a capacitor charges to (or discharges from) the voltage applied to it (to zero) within 0.7% of its final value within 5 time constants.

The RC Time Constant and the Cutoff Frequency
The RC time constant $\tau = RC$ is the time required (in seconds) to charge the capacitor through the resistor from 0 to approximately 63.2% of the applied voltage: $V_\tau = (1 - \frac{1}{e})V_{\text{applied}}$. The notion of time constant is not confined to circuits — in fact, it is the main characteristic unit of a first-order linear time-invariant (LTI) system.[1] An LTI system's response to a step input (i.e., its step response) is highly useful in characterizing the system's behavior. The cutoff frequency f_c is the frequency at which energy flowing through a system begins to attenuate or reflect instead of passing through. The circuit's *half-power point*, or the point where the output power drops to half of its peak value (measured in decibels, this would be the peak power **-3 dB**), is designated as the cutoff frequency. This occurs when the output voltage drops to $\frac{1}{\sqrt{2}}$ of the peak value.
The cutoff frequency is related to the time constant τ as follows:

$$\omega_c = \frac{1}{RC} = \frac{1}{\tau} \quad \text{(radians/sec)} \qquad\qquad f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau} \quad \text{(hertz)}$$

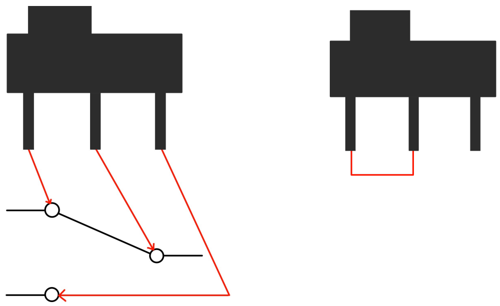
So, as you can see, the RC time constant and the cutoff frequency (in radians/sec) are simple inverses of each other.

Manually Switching Your Circuit
Your switch has three nodes:



The schematic nodes map to your switch pins as follows:

[1] Don't panic if you don't know what "first-order linear time-invariant system" means yet — you will learn about it later this semester. When you look over this note at the end of the semester ☺, hopefully that will invoke a new layer of understanding.



The two pins over which the bump is centered are connected. The middle pin is always connected to some input, so in this case it should be our output. So, flipping the switch changes the input voltage from $5V$ to $-5V$ or vice-versa.

Now you are ready to complete the lab! Go to the Jupyter notebook and complete parts 2 and 3.

Appendix A: Linear Constant-Coefficient Differential Equations

In this appendix, we will review homogenous linear constant-coefficient differential equations of the first order within a linear-algebraic framework. For an extended discussion (and an extension of the solution to nth-order differential equations, please see section 2.7 of Friedberg's *Linear Algebra*.

First, let's start with some definitions.

<u>Def.</u> A **differential equation** in some function $y(t)$ is an equation involving y, t, and derivatives of y.

<u>Def.</u> A **linear constant-coefficient differential equation** is an equation of the form

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y^{(1)} + a_0 y = f$$

where $a_0, a_1, \ldots, a_{n-1}, a_n$ are constants and f is some function. When f is 0, we say the LCCDE is homogeneous. The number in the superscript parentheses in the equation above represents the order of the derivative (i.e., $y^{(1)}$ is the first derivative of y, $y^{(n)}$ is the n^{th} derivative, etc.)

<u>Def.</u> An operator A is a **linear operator** if it satisfies the conditions of additivity ((1)) and homogeneity ((2)), i.e.,

$$A(f+g) = A(f) + A(g) \tag{1}$$

$$A(cf) = cA(f) \tag{2}$$

for functions f and g and scalar c.

<u>Def.</u> The **differential operator** on the polynomial $p(t)$ over C is denoted $p(D)$, and its order is given by the order of the polynomial $p(t)$. We use the derivative operation to define a mapping

$$D : C^\infty \to C^\infty$$

$$D(x) = x' \quad \text{for} \quad x \in C^\infty$$

Particularly important is the definition of the differential operator. The differential operator is a particularly useful construction because it lets us redefine differential equations in a linear-algebraic context, which tends to be more intuitive for many EE/CS students.

First, we will show that D is a linear operator. Intuitively, from calculus, we know this is true because the derivative of the sum of two differentiable functions is the sum of their derivatives, and the derivative of the multiplication of a differentiable function and a constant is equal to said constant times the derivative of the function. We will now show this more formally for the sake of clarity. Consider any polynomial function over C of the form

$$p(t) = a_n t^n + a_{n-1} t^{n-1} + \ldots + a_1 t + a_0$$

Defining D as follows:

$$p(D) = a_n D^n + a_{n-1} D^{n-1} + \ldots + a_1 D + a_0 I$$

where I is the identity operator, D is a linear operator on C because the derivative of $p(t)$ (where $p(t)$ is a sum of the scaled differentiable functions t^n, t^{n-1}, ... , t), denoted $p(D)$, is equal to the sum of the scaled derivatives of the functions t^n, t^{n-1}, ... , t by definition, and thereby satisfies both additivity and homogeneity.
Using the differential operator, we can rewrite the homogeneous LCCDE

$$y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y^{(1)} + a_0 y = 0$$

as

$$(D^n + a_{n-1} D^{n-1} + \ldots + a_1 D + a_0 I)(y) = 0$$

(*A reminder: pay particular attention to the notation, as an exponent in parentheses indicates a derivative, while an exponent not in parentheses is a standard exponent.*)
We can define the *auxiliary polynomial* that will help us solve the LCCDE. The homogeneous LCCDE is re-written as:

$$p(D)y = 0$$

It can be shown that if $c_1, ..., c_n$ are distinct roots of $p(D)$, $\{e^{c_1 x}, e^{c_2 x}, ..., e^{c_n x}\}$ are the basis functions of the solutions to the original LCCDE. This means that any answer to LCCDE will be of the form $\alpha_1 e^{c_1 x} + \alpha_2 e^{c_2 x} + ... + \alpha_n e^{c_n x}$. It can also be shown that if $(t - c_k)^n$ is the *auxiliary polynomial* of the LCCDE, $\{e^{c_k x}, xe^{c_k x}, ..., x^{n-1} e^{c_k x}\}$ are the basis functions of the solution. Therefore, to solve an LCCDE, one can just compute the roots of the *auxiliary polynomial* and form the solution as the linear combination of basis functions.

Example:

Suppose we want to find all $y = f(x)$ that solve the following LCCDE:

$$y^{(3)} - 4y^{(2)} + 5y^{(1)} - 2y = 0$$

The *auxiliary polynomial is*

$$t^3 - 4t^2 + 5t - 2 = (t-1)^2 (t-2)$$

Therefore, $\{e^x, xe^x, e^{2x}\}$ is a basis for the solution space. Thus any solution has the form

$$f(x) = \alpha_1 e^x + \alpha_2 x e^x + \alpha_3 e^{2x}$$

v1.0 Written by Mia Mirkovic (2019). Revised by Kourosh Hakhamaneshi (Spring 2020).