Introduction

This lab is designed to be done in two weeks. In this lab, you will design three filters to pass different sections of the audible frequency spectrum to separate LEDs so they will appear to flash in time to the music — in other words, you'll have made your very own color organ!

To do this, you will select your desired cutoff frequencies and calculate the appropriate resistor and capacitor values to build filters with said cutoff frequencies.

The audible range is actually a somewhat small spectrum of frequencies, as demonstrated below:

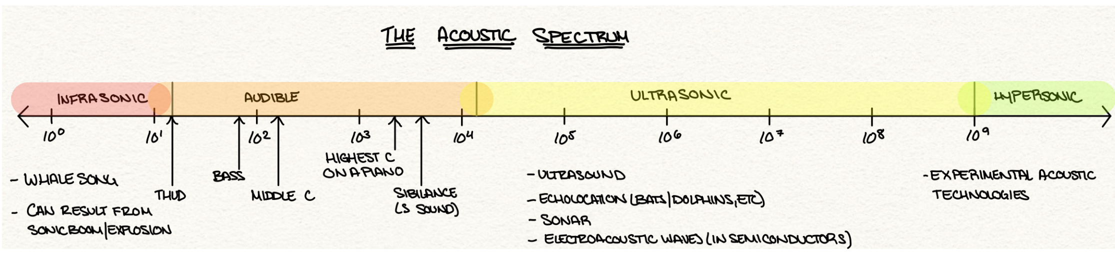


Figure 1: Sketch of the acoustic spectrum.

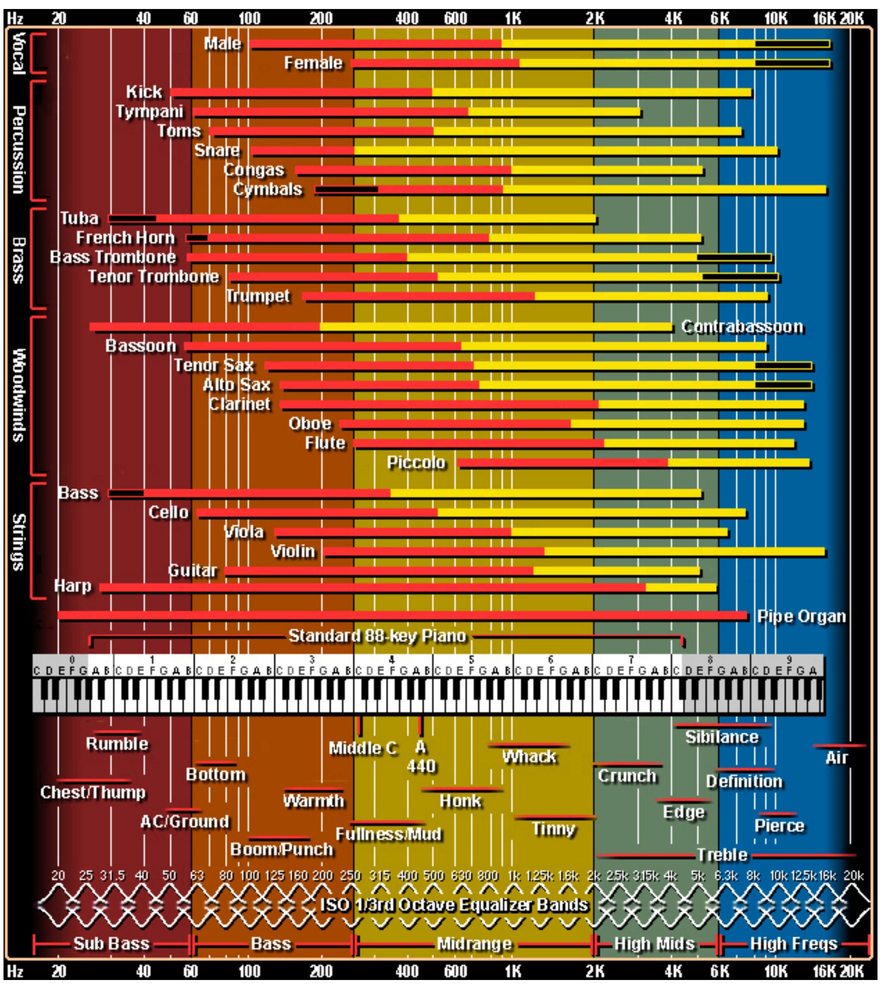


Figure 2: Expansion of the audible range of the acoustic spectrum.

Sanity check question: *What challenge does the relatively small size of the audible spectrum create? Remember, the word "cutoff" in the phrase "cutoff frequency" is somewhat of a misnomer; the cutoff frequency indicates the point at which the signal power is attenuated by half, not the point at which it is fully eliminated.* ***Hint****: Think about separation in frequency domain.*

Note: Acoustic waves are *not* electromagnetic waves: sound waves are mechanical and therefore need a medium through which to propagate, whereas EM waves do not need a medium[1]: they can propagate through the vacuum of space.

You will be targeting the bass, midrange, and treble (aka "high mids"-"high freqs") sections depicted in Figure 2 above, which we define as follows:

Bass	0-500 Hz
Midrange	1000-5,000 Hz
Treble	6,000-20,000 Hz

Ultimately, these frequency ranges are **guidelines**: the goal of this lab is to independently light up your LEDs (with little to no overlap — two LEDs should not light up at the same pure frequency, and "dead zones" should be minimal/imperceptible). Also, you may notice that these ranges do not align exactly with those displayed in figure 2. After completing Part 1 of the lab, you will see why that is.

Lab 4: Color Organ I

Part 1: Frequency Response of the Speaker-Microphone System

The system you are building today is not limited to just the color organ circuit: you must also consider the ability of your speaker to reliably reproduce the desired frequency at a volume large enough to excite the microphone, and the ability of your microphone to respond to the desired frequency. You must also consider that in some ranges, the signal will be highly attenuated, if picked up at all, and compensate for that when you design your color organ (*How can I add gain to some frequencies and not others? Which frequencies should I choose? How can I create sharp cutoffs to minimize both overlap and dead zones?*). To gain the necessary information to design a working color organ, we first identify the speaker-microphone system's frequency response. We will do this empirically: you will sweep over a range of audio frequencies and record the amplitude of the received wave at that frequency.

Now you are ready to complete the first part of the lab! Go to the ipython notebook and complete Part 1.

Part 2: A *Bass*-ic Color Organ

Now we are ready to begin building the complete color organ circuit! The finished product will look something like this:

[1] This bothered early scientists, so they came up with the concept of the aether (subsequently decommissioned in 1897).

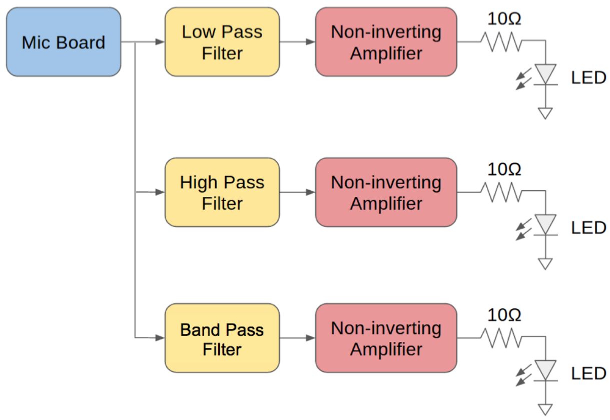


Figure 3: High-level overview of completed color organ.

Not only is this a larger circuit than those you have built in previous labs, but you will also be extending it at the end of Color Organ Part II (next part), so **be sure to plan ahead when constructing your circuit. Keep your circuit clean.**

The first filter you will build will isolate the bass frequencies, so it is a low-pass filter. You will use only one capacitor, so it is first-order.

> **2.1.** As a layout exercise, sketch on a piece of paper how you plan to allocate the space on your breadboard.

Part 3: A *Treble*-some Color Organ

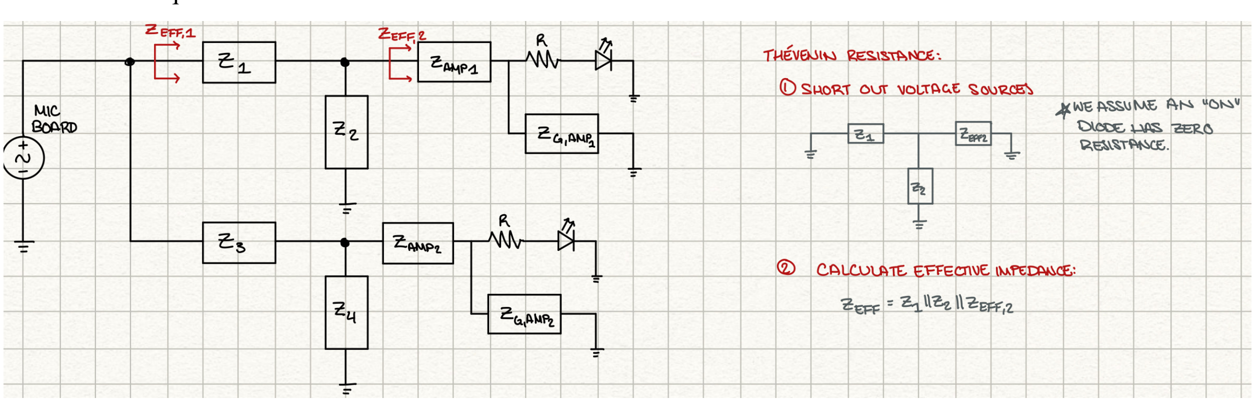
Now, you will build a first-order high-pass filter to isolate the treble frequencies.

> ***Sanity check question:*** *Do we need to put buffers in front of each filter now that we have two connected in parallel? I.e., does placing the filters in parallel affect their respective cutoff frequencies? Think: is the microphone signal you're trying to process a current signal or a voltage signal?*

To approach this question, we'll consider the topic of impedance "looking in" to a particular node or point in a circuit.

"Looking in" ? Electrons can't see!

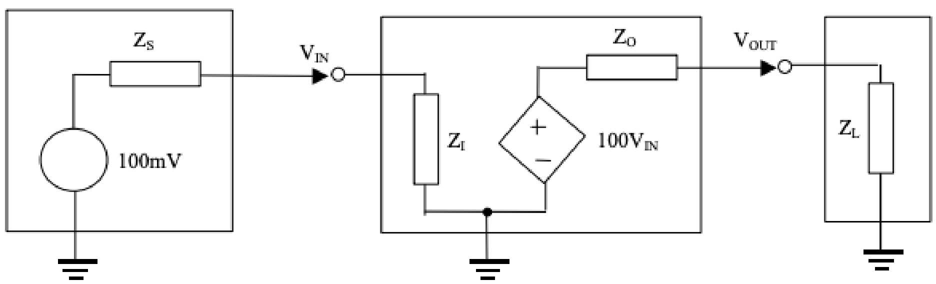
True, they can't. But, the concept of impedance "looking in" to a specific node is a very useful one (and one you'll see very often in circuits classes like EE 105 and EE 140, especially in transistor circuits). The impedance "looking in" to a node or part of a circuit is simply the Thévenin equivalent impedance of that part of the circuit. Here's another quick refresher on Thévenin:



Loading

Recall the impedance characteristics of the ideal op-amp: its input impedance is infinite and its output impedance is 0. This allows it to act like an ideal voltmeter at the input and supply infinite current at its output. But what happens if we don't have those characteristics?

Let's assume we have a noninverting amplifier with a gain of 100, with input impedance Z_I and output impedance Z_o, as in the schematic below. The middle box represents the amplifier.



We now see that V_{in} depends on the source output impedance Z_S and the amplifier input impedance Z_I, and V_{out} depends on the amplifier output impedance Z_O and the load impedance Z_L. Recalling the voltage divider equation,

$$V_{out} = (100V_{IN})\frac{Z_L}{Z_L + Z_O}$$

But $100V_{IN}$ is our desired V_O! To keep that approximately correct and avoid "loading" the output and reducing the voltage noticeably from what we expect, we need Z_L to be considerably larger than Z_O to keep $\frac{Z_L}{Z_L+Z_O}$ as close to 1 as possible.

This is why you set the "output load" on the signal generator to "High-Z": by doing so, you are telling it to expect a high-impedance load. The function generators in the lab (rip) have a 50-ohm output impedance, while the oscilloscope probes are high-impedance, so when the function generator is set to "High-Z," you can probe it with the oscilloscope and see the output voltage you expect (the one you explicitly set). If you set the function generator to "50 Ohm", it expects a 50 Ohm load. Since this is equal to its output impedance, V_{out} would be halved, so the function generator compensates for this by doubling its output voltage in 50 Ohm mode.

Buffers

You can think of a buffer as providing an impedance transformation between two *cascaded* circuits. When you observe an undesired loading effect between two circuits, placing a buffer between them changes the load impedance of the first circuit to a very high value and the source impedance of the second to a very low value in accordance with (approximately) ideal op-amp characteristics. This allows you to build very modular circuits easily, without having to do lots of ugly algebra.

Something to ponder: *When/where would you use buffers in your color organ? What improvements could you make that would require you to add additional buffers?*

Appendix A: Impedance and Reactance

Capacitors and inductors are **reactive** components: their behavior depends on frequency. But, both components are *linear*. You are likely familiar with the concept of linearity, i.e., that a linear system (or linear transformation) must exhibit both *superposition* (the result of adding two inputs and then feeding the result through the system is the same as the result of feeding the two inputs into the system individually and then adding the outputs) and *homogeneity* (the result of multiplying the input by a constant and feeding it through the system is the same as that of feeding the input through the system and then multiplying by a constant). Linearity is a property with many important consequences, the most important of which is likely that *when a sinusoid of frequency f is fed into a linear circuit, the output will be a sinusoid at the same frequency (though amplitude and phase may change).* This property makes the frequency response a useful characteristic: the frequency response defines how the output voltage depends on the input voltage at a particular frequency, and this relationship can be fully expressed by some constant numerical value at that point.

Impedance

Impedance (**Z**) can be thought of as "generalized resistance": it includes both *reactance* (X) and *resistance* (R). In reactive elements, voltage and current are always 90° out of phase, whereas in resistive elements, voltage and current are always in phase. Impedance is mathematically defined as follows:

$$Z = R + jX$$

The magnitude of Z gives the ratio of amplitudes of voltage to current, and the polar angle of Z gives the phase angle between current and voltage.

By observing both the complex impedances and the differential equations governing inductive and capacitive behavior, we see that these are indeed linear components. We will intuitively derive the reactance of each by imagining each in a simple circuit in series with a sinusoidal voltage source ($V(t) = V_0 \sin(\omega t)$), as follows:

Let's start with the capacitor. The current through the capacitor is given by

$$I(t) = C\frac{dV}{dt} = C\omega V_0 \cos(\omega t)$$

The current has amplitude $\omega C V_0$, and the current *leads* the voltage by 90°, since the voltage is given by a sine wave and the current by a cosine, and a cosine is just a sine shifted back by 90°; therefore, we can say the cosine is 90° ahead of the sine.

Disregarding phase (considering amplitude only), the current is

$$I = \omega C V = \frac{V}{X_C} \implies \frac{|V|}{|I|} = \frac{1}{\omega C} = X_C$$

The analysis of an inductor follows the same pattern: imagine an inductor in the place of the capacitor in our earlier imaginary circuit, with a sinusoidal voltage source such that the current is $I(t) = I_0 \sin(\omega t)$. Using the differential equation, we find the voltage across the inductor to be

$$V(t) = L\frac{dI(t)}{dt} = L\omega I_0 \cos(\omega t)$$

Now, the voltage is a cosine while the current is a sine, so we can say the voltage leads the current by 90° in the inductor.

We have now derived the impedances and reactances for both the capacitor and the inductor:

$$Z_C = \frac{1}{j\omega C} \qquad X_C = \frac{1}{\omega C} \qquad Z_L = j\omega L \qquad X_L = \omega L$$

Why complex numbers?

Since we need to specify both magnitude and phase shift of our voltages and currents at any point in the circuit, a single number is not an adequate representation. While we could explicitly write them as $V(t) = A\sin(\omega t + \Phi)$, it is more convenient to take advantage of the geometry of complex numbers to *represent* our voltages and currents: the complex number representation keeps the magnitude and phase neatly separated so that we can just add or subtract them directly instead of having to add or subtract sinusoidal functions of time.

We will use voltage as an example to review the complex representation:

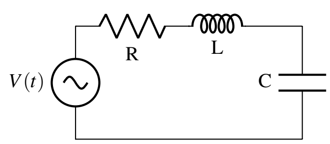
$$V_0\cos(\omega t+\phi) \quad \Longleftrightarrow \quad V_0 e^{j\phi} \ \text{(polar)} \quad = \quad a+jb, \quad a=V_0\cos(\phi), \quad b=V_0\sin(\phi) \ \text{(rectangular)}$$

$$\textbf{Euler's formula:} \quad e^{j\phi}=\cos(\phi)+j\sin(\phi)$$

To find the actual voltage and current, multiply the complex representation by $e^{j\omega t}$ and take the real part, for example, $V_0\cos(\omega t+\phi)=\Re\{e^{j\omega t}(V_0 e^{j\phi})\}$.

Deriving Transfer Functions

We can use the impedance formulas that were mentioned earlier to derive the transfer function of filters. Let's go through an example. Consider the following circuit:



We can compute the voltage across the capacitor similar to voltage dividers, and using impedance instead of resistance values:

$$\begin{aligned} V_C(j\omega) &= \frac{Z_C(j\omega)}{Z_C(j\omega)+Z_L(j\omega)+Z_R(j\omega)} V_s(j\omega) \\ &= \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+j\omega L+R} V_s(j\omega) \\ &= \underbrace{\frac{1}{1+RC(j\omega)+LC(j\omega)^2}}_{H(j\omega)} V_s(j\omega) \end{aligned}$$

$H(j\omega)$ is called the transfer function of the system. To fully specify the transfer function, input and output should also be specified. In the example above, input is Vs and output is the voltage across the capacitor. This transfer function determines what happens to a single tone input at a given frequency; in particular, how much will it be attenuated (or amplified) and how much will it delay.

Appendix A: WTPh is Phase?

While magnitude is a rather intuitive concept, many people struggle to understand phase: why we care about it, how to calculate it, and how to represent it.

Calculating phase

Close Encounters of the Third Kind: The all-pass filter

We finally come to the third kind of first-order filter: the all-pass filter. This filter (ideally) does not influence magnitude as a function of frequency, but it does influence phase[2].

So why do we even define a cutoff frequency for all-pass filters?

Recall that a phase shift of 180° corresponds to an inversion of the signal: all-pass filters are frequency selective

[2] Common uses of all-pass filters are in equalizing delays or any system that requires phase shaping. They are commonly applied in electronic music production.

References

Horowitz, P. and Hill, W. (2015). *The Art of Electronics*. 3rd ed. Cambridge: Cambridge University Press, ch 1.
Sedra, A. and Smith, K. (2015). *Microelectronic Circuits.* 7th ed. New York: Oxford University Press, ch 17.

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