Lab 4: Color Organ II

In this part of the lab, we will explore cascade design, one of the most popular methods for designing active filters (filters that include active components such as op-amps).

If the output terminal of a filter circuit has a much lower (ideally zero) impedance than the input terminal of the filter circuit with which it is cascaded (whose impedance would ideally be infinite), *cascading does not change the transfer functions of the individual circuits* and the overall transfer function of the cascade is simply the product of the transfer functions of the individual circuits. This is why buffers are so useful in filter design.

Part 1: Caught in the Midrange

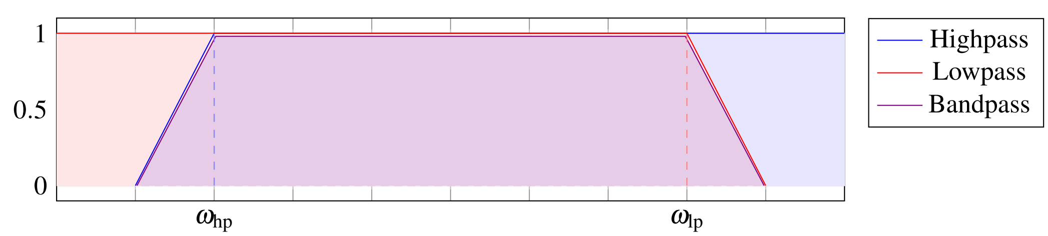
To build onto the color organ from Lab 3, you will build an active RC bandpass filter to isolate the midrange frequencies. You're familiar with first-order low- and high-pass filters already. But, can we make a first-order bandpass filter? As you probably expected, the answer is no[1]. We need to rely on our knowledge of loading and first-order filters to design an appropriate bandpass filter.

To do so, you will need to choose two cutoff frequencies ω_{hp} and ω_{lp}. Important questions to ask yourself while designing this filter include:

- Which of the two cutoff frequencies should be higher?
- What's a good amount of space to leave between your bandpass cutoffs and your bass and treble filter cutoffs?

The transfer functions of buffered cascaded filters multiply, so the frequency response of the bandpass is the multiplication of its component frequency responses:

Bandpass Frequency Response (Bode Approximation)



Ponder the following while you build your filters:

- What would a pulse train (i.e., square wave with its minimum at 0) look like if you passed it through a low-pass filter? What about a high-pass filter?
- Can you think of additional applications for these filters? Try thinking about more general signals, like stock prices or images.

You are now ready to start the lab! Go to the Jupyter notebook and complete Part 1.
The remainder of this note may help you for Parts 2 and 3.

Part 2: Filter Order

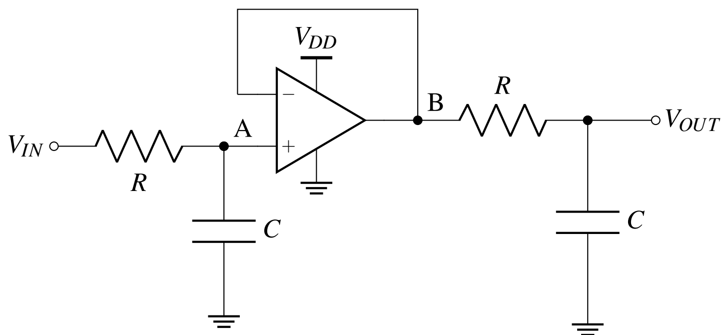
Both our low pass and high pass filters are what we call a "first order filter." This means that they have only one component which responds to frequency. For us, this is a capacitor.

A filter's order is equal to the order of its transfer function. Recall that the order of a polynomial is the highest power of the polynomial's variable. The order of a rational function (i.e., a ratio of polynomials) is defined as the maximum of the orders of its numerator and denominator. Since we only work with linear time-invariant (LTI) filters in this course, we can assert that a filter's order is equal to that of its transfer function.

[1] See Appendix B for a discussion of phase and the third kind of first-order filter: the all-pass.

You may have noticed that your filters still have some response to input past the cutoff frequency. Because both of our filters are first order filters, they attenuate the signal at the same rate (20dB per decade) If we want to attenuate at a faster rate, and therefore reduce the range past our cutoff frequency where we still see some output signal, we're going to need to add some more reactive components (we'll be using capacitors).

Below is a schematic for a second order low pass filter.



Let's derive the transfer function for this filter. To start, let's find the transfer function from V_{IN} to node A. This is just the basic, first-order low-pass filter transfer function, which we can find by plugging in the impedances of the resistor and capacitor into the voltage divider equation:

$$\frac{V_A}{V_{IN}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

This filter has a cutoff frequency of $\frac{1}{RC}$, which is where the pole occurs in the transfer function:

Generalizing the first-order filter

The general first-order (or **bilinear**, since it is linear in both the numerator and denominator) transfer function is as follows (recall, $s = j\omega$):

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

This filter has a pole at $s = \omega_0$ and a zero at $s = a_0/a_1$. *Think: What is the gain at $s = \infty$? What about at $s = 0$?* The gain at $s = 0$ is the DC gain, and the gain at $s = \infty$ is the high frequency gain. In this case, the high-frequency gain approaches a_1, and the DC gain is a_0/ω_0. The coefficients a_0 and a_1 determine what kind of filter we have. As an exercise, think of the relationships among the numerator coefficients that realize the different kinds of filters.

Generalizing the second-order filter

The reason that we might be interested in higher order filters is that we may need a better selectivity in the given frequency range. As we increase the order of the filter, the slope with which the magnitude decays to zero increases. For example in the acoustic domain the slope determines how well the undesired frequency band is rejected, and therefore determines the frequency selectivity.

Second-order filters can be either active or passive, but as we have discussed, making the filters active allows for greater modularity and ease of design. The general second-order (or **biquadratic**) transfer function is as follows:[2]

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

By applying the quadratic formula to the denominator, we can find the poles of this function. As with the first-order transfer function, the type of filter is determined by the numerator coefficients.

[2]There are several common ways of representing the coefficients in the general second-order transfer function, and a particularly common one is to write the coefficient of s in the denominator as 2ζ. This notation is especially frequently used with the general second-order **all-pole** transfer function, $T(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$.

What kinds of second-order filters can we make?

Second order filters include highpass, lowpass, bandpass, bandstop, notch, and allpass. As an exercise, think of the relationships among the numerator coefficients that realize the different kinds of filters.

The natural frequency, ω_0

The natural frequency is the frequency at which the system oscillates with no damping. Meaning that if there were no non-energy storing elements in the circuit (i.e. resistor) the circuit would have oscillated at that frequency. Mathematically this means that if $Q = 0$, at $\omega = \omega_0$ the denominator becomes zero. For example, the natural frequency of a series RLC circuit would be its frequency of oscillation with the resistance shorted (i.e., replaced with ideal wire). We will address natural frequency more in-depth later, so don't worry about it for now.

References

Horowitz, P. and Hill, W. (2015). *The Art of Electronics*. 3rd ed. Cambridge: Cambridge University Press, ch 1.

Sedra, A. and Smith, K. (2015). *Microelectronic Circuits.* 7th ed. New York: Oxford University Press, ch 17.

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