CW5: Collision and Internal Energy

I. Practice for function call

(1) First, you need to prove the following formulae for elastic collision between two hard spheres of the same mass, for their velocities just before and right after the elastic collision happens.

$$\vec{v}_1' = \vec{v}_1 + \frac{(\vec{v}_2 - \vec{v}_1)\cdot(\vec{x}_1 - \vec{x}_2)}{|\vec{x}_1 - \vec{x}_2|^2}(\vec{x}_1 - \vec{x}_2)$$

$$\vec{v}_2' = \vec{v}_2 + \frac{(\vec{v}_1 - \vec{v}_2)\cdot(\vec{x}_2 - \vec{x}_1)}{|\vec{x}_2 - \vec{x}_1|^2}(\vec{x}_2 - \vec{x}_1)$$

You can then use the formulae to simulate the collision between two objects.

```
from visual import *

size = [0.15, 0.10]
mass = [0.1, 0.1]
colors = [color.yellow, color.green]
position = [vector(1, 0.2, 0), vector(0, 0, 0)]
velocity = [vector(-0.5, 0, 0), vector(0, 0, 0)]

def af_col_v(v1, v2, x1, x2):                                    # function after collision velocity
   v1_prime =  (Fill in the formula)
   v2_prime =  (Fill in the formula)
   return (v1_prime, v2_prime)

scene = display(width = 800, height = 800, x = 600, y = 100, background = (0.3, 0.3, 0))
ball_reference = sphere (pos = (0,0,0), radius = 0.02, color=color.red)          #to mark a still point
balls =[]
for i in [0, 1]:
   balls.append(sphere(pos = position[i], radius = size[i],  color=colors[i]))
   balls[i].v = velocity[i]

dt = 0.001
while True:
   rate(1000)

   for ball in balls:
      ball.pos += ball.v *dt
   if (abs(balls[0].pos - balls[1].pos) <= size[0]+size[1] and
      dot(balls[0].pos-balls[1].pos, balls[0].v-balls[1].v) <= 0) :
      (balls[0].v, balls[1].v) =  af_col_v (balls[0].v, balls[1].v, balls[0].pos, balls[1].pos)
```

Notice in the last line of the program,

`(balls[0].v, balls[1].v) =  af_col_v (balls[0].v, balls[1].v, balls[0].pos, balls[1].pos)`

This is the function call named as “called by position”, that is, each argument is mapped by position in order, such as balls[0].v is mapped to v1 in al_col_v(….), balls[1].v to v2, balls[0].pos to x1, and balls[1].pos to x2 so on.

You can call the function by “called by keyword”, which is done in the following:

`(balls[0].v, balls[1].v) =  af_col_v (v1=balls[0].v, v2=balls[1].v, x1=balls[0].pos, x2=balls[1].pos)`

Here, each argument is called explicitly by the keyword. This way, the order is irrelevant and it is ok to write

`(balls[0].v, balls[1].v) =  af_col_v (v1=balls[0].v, x1=balls[0].pos, v2=balls[1].v, x2=balls[1].pos)`

(2) Before and after the collision, find and print the kinetic energies of both balls.

(3) Find out the elastic collision formulae between two hard spheres of different mass. Implement this in Python and simulate the collision in the above program for mass=[0.2, 0.1]. Print the kinetic energies of both balls before and after the collision.

II. Homework Submission: **(Suggestion: do it step by step, make sure you finish 1 correctly then go to do 2)**

1. Now we will simulate a ball (balls[2]) collides elastically with a spring-ball system, which is composed of two balls (balls[0] and balls[1]) connected by a spring. Here are the parameters. This is a combination of the above practice with a previous homework (homework 3).

```
r = 0.3                          #spring original length
k = 25.0                         #spring constant
size = [0.05, 0.04, 0.03]        #ball radius
mass = [0.3, 0.4, 0.3]           #ball mass
colors = [color.yellow, color.green, color.blue]                  #ball color
position = [vector(0, 0, 0), vector(0, -r, 0), vector(0.25,-0.45, 0)]      #ball initial position
velocity = [vector(0, 0, 0), vector(0, 0, 0), vector(-0.2, 0.42, 0)] #ball initial velocity
```

2. Before the collision, print the kinetic energy of balls[2] and its momentum.

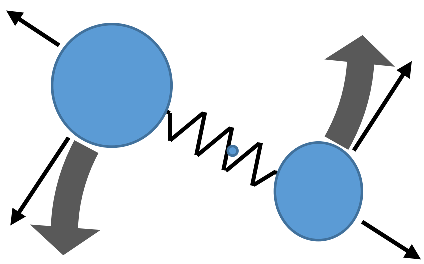
After the collision, for every period of the oscillation of the ball-spring system, print the following for the spring-ball system: (Here ‘internal’ means that if we see, from far away, the spring-ball system, we won’t be able to see the detailed structure of the system, thus called ‘internal’.)

(1) Internal Vibrational Potential Energy averaged over a period = the averaged energy stored in the spring

(2) Internal Vibrational Kinetic Energy averaged over a period = the averaged kinetic energy associated to the two balls’ motions relative to the center of mass and parallel to the spring axis.

(3) Internal Rotational Kinetic Energy averaged over a period = the averaged kinetic energy associated to the two ball’s motions relative to the center of mass and perpendicular to the spring axis.

(4) Center of Mass Kinetic Energy = $\frac{1}{2} M_{spring-ball\ system} \cdot v^2_{center\ of\ mass}$



(5) Kinetic Energy of ball[2]

(6) Total Energy = Summation of (1) to (5)

(7) Linear Momentum of the spring-ball system

(8) Linear Momentum of ball [2]

(9) Total Linear Momentum = (7) + (8)

Of course, you will see the conservation of total energy and total linear momentum before and after the collision. What other interesting facts do you see?

You may need several Vpython’s vector methods (http://vpython.org/contents/docs/vector.html)