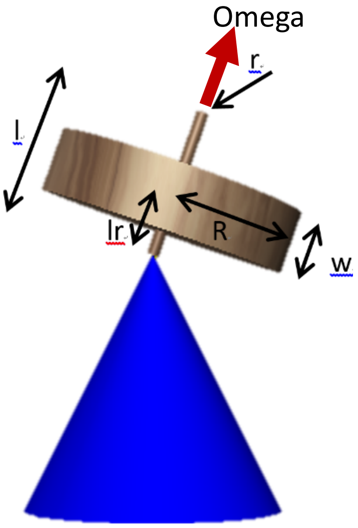
Computer Work for Unit 7 Precession



This homework has two problems.

I. Spinning top Precession:

```
from visual import *

g = 9.8                         # m/s^2
M, R, w = 0.5, 0.10, 0.05       # mass, radius, thickness of the central part
I = 0.5 * M * R ** 2            # rotational inertia of the spinning top
l, r = 0.12, 0.005              # length, radius of the MASSLESS shaft
theta = 70 * pi / 180.0         # initial upright angle relative to the horizon
Omega = 10*2*pi * vector(cos(theta), sin(theta), 0)         # initial angular velocity
lr = 0.045                      # the distance from the bottom to the center of mass of the spinning top

scene = display(width = 1000, height = 1000, range = 0.6, background = (0.5, 0.5, 0))

spintop = frame()
shaft = cylinder(frame=spintop, pos=(0,0,0), axis =(l,0,0), radius=r, length=l,  material = materials.wood)
disk = cylinder(frame=spintop, pos=(lr-w/2,0,0), axis = (w, 0, 0), radius=R, material = materials.wood)

spintop.pos = (0,0,0)

base = cone(pos=(0,-0.2,0), axis=(0,0.2,0), color = color.blue, radius=0.1)
dt = 0.0002
while True:
   rate(5000)

   spintop.axis = Omega
   delta_angle = mag(Omega) * dt
   spintop.rotate(angle=delta_angle, axis=spintop.axis)
```

(codes in bold case) **spintop** is drawn by combining the **disk** and the **shaft** in a **frame** called **spintop**. The original position of any **frame** is at (0,0,0) and its **axis** oriented in (1,0,0). Therefore, the **shaft** and the **disk** are drawn with their positions and orientations relative to the original setting of the **frame**. Then, when the **frame**'s position or axis are changed, anything belonging to the **frame**, will have the same change. This then shows the **spintop** as in the above figure. To make **spintop** rotate, we use the method **rotate(angle..., axis...)** in the while loop.

(1) Simulate the precession of **spintop**.

First calculate the angular momentum $\vec{L}$ of the spintop. Then in the while loop, calculate the torque $\vec{\tau}$ exerted on the spin top by the gravitational force, and then find the angular momentum change due to the torque $\Delta\vec{L} = \vec{\tau}dt$. Finally, $\vec{L}\ += \ \Delta\vec{L}$. With the new angular momentum $\vec{L}$, you can obtain the new angular velocity $\vec{\Omega}$ (Omega) of the spintop.

(2) Find the period of the precession in your program when the spintop makes a complete circle of the precession. Compare it to the theoretical value. (see https://goo.gl/OtWMNd, or http://hyperphysics.phy-astr.gsu.edu/hbase/top.html)

II. Earth Precession:

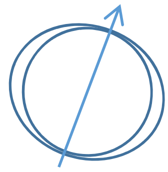
The Earth is significantly more dense near its center. The average density of the Earth is 5.5 g/cm3, whereas the average density of the rock near its surface is only 3.8 g/cm3. Find the thickness of each layer on the internet and

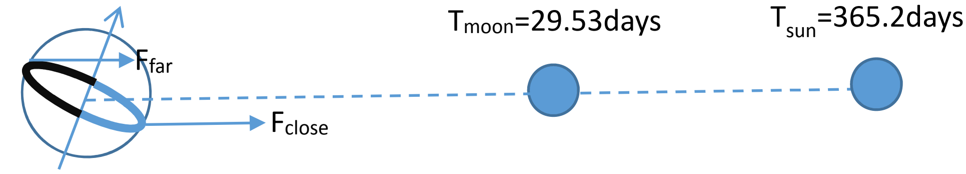
(1) Calculate the rotational inertia of the Earth.

(2) With the rotation of the Earth, calculate the angular momentum of the Earth, the direction of which is inclined $23.5°$ from the perpendicular direction of the sun-earth-moon plane.

The polar radius of the Earth is 6357 km, and the equatorial radius is 6378 km. For simplicity, we will model the Earth as a sphere of radius 6357 km plus a band of mass running around its equator. We can model the extra mass, which is the mass of the volume between the ellipsoid and the perfect sphere, as if it was on the belt around the equator.

(3) Estimate the mass of the "belt." **(*see the bottom comment)**





(4) As shown in the figure, every part of the belt has different distance from the moon and the sun, depending on the position of the moon and the sun. As a result, the gravitational pull from the sun and from the moon will be larger for the parts (lighter tone) close to the moon or the sun and the less for the parts (darker tone) away from the moon or the sun. This generates a torque that affects the Earth's angular momentum $\vec{L}$. When the moon or the sun orbits to different positions, the magnitude of the gravitational pull for different parts and the total torque change. However, the directions of all the torques are the same. Therefore, summing all the effects, you know, as in Problem 1, gives the precession of the Earth.

To analyze the problem more easily, we take the center of the Earth as the origin, and let the moon and sun "orbit" around the earth, with their own distances and periods, respectively. Assume now the inclination is toward x-axis. Simulate the precession of the rotation of the Earth. What is the period of the precession?

To speed up the simulation, you need to separate the simulation into two parts.
(5) Calculate for each year, on the average, the change of the angular momentum due to the "orbiting" moon $\Delta\vec{L}_{by\ moon} = \sum_{365days} \Delta\vec{\tau}_{moon}\Delta t$, and due to the "orbiting" sun $\Delta\vec{L}_{by\ sun} = \sum_{365days} \Delta\vec{\tau}_{sun}\Delta t$? The combined effect per year is $\Delta\vec{L} = \Delta\vec{L}_{by\ moon} + \Delta\vec{L}_{by\ sun}$. Very important: You will find $\Delta\vec{L}$ is a vector always perpendicular to $\vec{L}$.
(6) Then you use $\vec{L}_{next\ year} = \vec{L}_{this\ year} + \vec{\tau}\Delta t$ to obtain the angular momentum for the next year, and then the year after, and later years. Continue doing so, you will eventually have the direction of $\vec{L}$ to come back to the same direction as the one that you start doing the simulation.
*Of course, this way of estimating the mass distribution around the earth is inaccurate, yet it gives reasonable result estimating the precession period of the Earth. If you can find a better way to estimate the mass distribution and then to calculate the more accurate torques exerted by the Sun and the moon, your result will definitely more accurate.

Homework submission:
Program (1): Complete the spintop simulation, print the period of the precession from the simulation and from the theory.
Program (2): Complete the calculation in a series of steps to obtain the precession period of the earth. In the program, print the rotational inertia of the earth, the angular momentum (value) of the earth, the angular momentum change due to the moon for a year $\Delta\vec{L}_{by\ moon}$, the angular momentum change due to the sun for a year $\Delta\vec{L}_{by\ sun}$, and the precession period of the earth.
Zip them in a file and submit it together.