Bisection Method in MATLAB

Numerical Computing – 7th Semester

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October 8, 2025

Motivation

- Many problems in engineering and science require solving nonlinear equations.
- Exact solutions are not always possible $\Rightarrow$ use numerical methods.
- Bisection Method is one of the simplest and most reliable root-finding methods.

Introduction

- Root-finding method in numerical analysis.
- Requires a continuous function $f(x)$.
- If $f(a)$ and $f(b)$ have opposite signs $\Rightarrow$ root lies in $[a, b]$.
- Based on the Intermediate Value Theorem.

Algorithm Steps

1. Start with interval $[a, b]$ such that $f(a) \cdot f(b) < 0$.
2. Compute midpoint: $c = (a + b)/2$.
3. Evaluate $f(c)$.
4. Update interval:
   - If $f(a) \cdot f(c) < 0$, root lies in $[a, c]$.
   - Else, root lies in $[c, b]$.
5. Repeat until $|X_n + 1 - X_n| < \varepsilon$ (tolerance).

Advantages

- Simple to understand and implement.
- Always converges if initial condition satisfied.
- Error reduces systematically.

MATLAB Code

```
f = @(x) x^3 - x - 2;    % Function
a = 1; b = 2;            % Interval
tol = 1e-5; maxIter = 100;

for i = 1:maxIter
c = (a+b)/2;
if f(a)*f(c) < 0
b = c;
else
a = c;
end
if abs(f(c)) < tol
break
end
end
disp(['Root approx = ', num2str(c)])
```

Example 1

- Function: $f(x) = x^3 - x - 2$
- Interval: $[1, 2]$
- Expected Root ≈ 1.5214

Example 2

- Function: $f(x) = \cos(x) - x$
- Interval: $[0, 1]$
- Expected Root ≈ 0.7391

MATLAB Code in interval not given

```
f = @(x) x^3 - 6*x + 4;
x0 = 0;
step = 1;
maxExpand = 50;
a = x0;
b = x0 + step;
found = false;
for k = 1:maxExpand
if f(a)*f(b) < 0
found = true;
break;
else
a = a - step;
b = b + step;
end
end
```

MATLAB Code Cont...

```
if ~found error('No sign change found. Try another initial
guess or function.');
end
tol = 1e-6;
maxIter = 100;
for i = 1:maxIter
c = (a+b)/2;
if f(a)*f(c) < 0
b = c;
else
a = c;
end
if abs(f(c)) < tol
break;
end
end
fprintf('Root approximation = %.6f\n', c);
```

Example 3

- Function: $f(x) = x^3 - 6x + 4$
- Root lie between: $[0, 1]$
- Expected Root ≈ 0.7192

Classwork

Task 1

Apply Bisection Method in MATLAB to solve:

$$f(x) = x^3 - 4x - 9 \quad \text{on interval } [2, 3]$$

Task 2

Use MATLAB to find the root of:

$$f(x) = \sin(x) - 0.5 \quad \text{on interval } [0, 2]$$

Homework

Assignment 1

Solve using Bisection Method:

$$f(x) = \ln(x) + x^2 - 3, \quad [1, 2]$$

Assignment 2

Solve using MATLAB:

$$f(x) = xe^{-x} - 0.1, \quad [0, 1]$$

Summary

- Bisection is reliable but slow.
- Works only if $f(a) \cdot f(b) < 0$.
- Converges linearly with guaranteed error bound.
- Very useful as a first step in numerical analysis.