

HW7

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Question 1.

1. Let X_1, X_2, \dots, X_n be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k . Let Y denote the minimum of the X_i 's. Find the distribution of Y .

$$y = 1 \leq j \leq k, ((k-j+1)^n - (k-j)^n) / k^n$$

Question 2 2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part).

$P(\text{One failure in ten years}) = 1/10$

$P(\text{No failures in ten years}) = 9/10$

- a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years.)

```
p = 1/10
q = 9/10
n=8

year = 9
v = vector()
for (k in 1:year) {
  v[k] = p*(q^(k-1))
}

prob_ofeight = round(pgeom(n,p,lower.tail = F),3)

E_x = 1/p

sig = round(sqrt(1-p)/p,3)

prob_ofeight
## [1] 0.387

E_x
```

```
## [1] 10
sig
## [1] 9.487
```

- b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```
lam = 1/10

expo = round(exp(-n/10),3)
rprob=round(pexp(p,n,lower.tail = F),3)

gE_x = 1/lam

var = 1/lam^2

std = 1/lam
gE_x
## [1] 10
var
## [1] 100
std
## [1] 10
```

- c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

```
pbin<- round(pbinom(q,n,p),3)

bex <- round(n*p,3)

b_stdx <- sqrt(n*p*q)

bex
## [1] 0.8
b_stdx
## [1] 0.8485281
```

- d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```
plamb <- n*p
pe_x <- plamb
```

```
p_prob = round(ppois(q,plamb),3)
p_std = round(sqrt(plamb),3)
```

```
p_prob
```

```
## [1] 0.449
```

```
p_std
```

```
## [1] 0.894
```