Homework 4

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I.

1. It is not correct because the conclusion of the fact the house from Minneapolis is colored in pink must be only that x=y. So, the corrected version is:

    ∃x House(x) ∧ In(x,Minneapolis) ∧ ∀y [(House(y) ∧ In(y,Minneapolis) ∧ Color(x,Pink)) ⇒ (x=y)]

2. The construction is not correct, because it says that for any apartment, there is a house which is bigger than it. Or, the required construction says that one house (i.e. a specific house) is bigger than all apartments. The corrected version is: **∃y∀x House(y) ∧ Apartment(x) ⇒ Bigger(y,x)**

3. The current construction says that all apartments are cheaper than some houses. Therefore, the corrected version is **∀x,y Apartment(x) ∧ House(y) ⇒ Cheaper(x,y)**

4. The current statement says there is a specific house which is more expensive than some farms. Or, equivalently, that some farms are cheaper that a particular house. The corrected version is **∃x Farm(x) ∧ ∃y House(y) ∧ Cheaper(x,y)**

5. It is not correct because says that, if bathrooms exist, then there is one bathroom which can be found in all houses. To correct it, we have to swap the ∧ and ⇒. The corrected version is:

    ∀x house(x) ⇒ (∃y bathroom(y) ∧ in(x,y))

II.

"Only pink objects are in the box" is logically equivalent with "If objects are NOT pink, then they are NOT in the box" which is converted into $\neg$Pink(x) $\rightarrow \neg$ InBox(x).

Or, the contrapositive of this is InBox(x) $\rightarrow$ Pink(x)
The construction can be also regarded as Pink(x) if InBox(x) or InBox(x) only if Pink(x)
From the provided logical expressions, only **1.** represents a correct translation.

2. actually says that all objects are pink and reside into the box.

3. actually says there is an object which is both pink and resides into the box.

4. actually says that only objects from the box are pink.

5. actually says there is a pink object which is also into the box.

III.

The conversion to CNF format follows the following steps:

- Elimination of implications
- Move inwards the negation
- Standardization of variables
- Skolemization
- Universal quantifiers get dropped
- disjunctions gets distributed over conjunctions

1. $[B \lor (A \land C)] \Rightarrow (B \lor \neg A)$
   $\equiv \neg[B \lor (A \land C)] \lor (B \lor \neg A) \equiv [\neg B \land \neg(A \land C)] \lor (B \lor \neg A)$
   $\equiv [\neg B \land (\neg A \lor \neg C)] \lor (B \lor \neg A)$
   $\equiv (B \lor \neg A \lor \neg A \lor \neg C) \land (B \lor \neg A \lor \neg B)$
   $\equiv (B \lor \neg A \lor \neg C) \land (\neg A \lor T)$

$\equiv (B \lor \neg A \lor \neg C) \land T$

$\equiv B \lor \neg A \lor \neg C$

(along the path, we have used idempotent law, complement law twice when simplifying $B \lor \neg B = T$ and $\neg A \lor T = T$)

2. $[Q \land (S \lor R)] \Rightarrow W$

   $\equiv \neg[Q \land (S \lor R)] \lor W$

   $\equiv \neg Q \lor \neg(S \lor R) \lor W$

   $\equiv (\neg Q \lor W) \lor (\neg S \land \neg R)$

   $\equiv (\neg Q \lor W \lor \neg S) \land (\neg Q \lor W \lor \neg R)$

3. $\forall p \, [[Pet(p) \land \exists c \, [Owner(c,p) \lor Feeds(c,p)]] \Rightarrow Happy(p)]$

   $\equiv \forall p[\neg Pet(p) \lor \forall c \, [\neg Owner(c,p) \land \neg Feeds(c,p)] \lor Happy(p)]$

   $\equiv [(\neg Pet(p) \lor \neg Owner(c,p)) \land (\neg Pet(p) \lor \neg Feeds(c,p))] \lor Happy(p)$

   $\equiv (Happy(p) \lor \neg Pet(p) \lor \neg Owner(c,p)) \land (Happy(p) \lor \neg Pet(p) \lor \neg Owner(c,p))$

4. $\forall x \, \exists y \, \forall z \, [P(x,y,z) \Rightarrow \exists u \, Q(x,u)]$

   $\equiv \forall x \, \exists y \, \forall z[\neg P(x,y,z) \lor \exists u \, Q(x,u)]$

   $\equiv \forall x \, \exists y \, \forall z \, [\neg P(x,y,z) \lor Q(x, R(x))]$

   $\equiv \forall x \, \forall z \, [\neg P(x, S(x), z) \lor Q(x,R(x))]$

   $\equiv \neg P(x, S(x), z) \lor Q(x,R(x))$

   (we have used skolemization for u and y)

IV.

To make the structure more clear, use letters "a, b, and c" to replace letters like u, v, w, y, z, etc. The statement still holds after the replacement, and it's easier for proof.

Prove by resolution with refutation "¬ H(C)":

1. ¬F(b,c) $\lor$ ¬H(C)   premise
2. ¬G(a) $\lor$ ¬F(a,b) $\lor$ F(b,c)   premise
3. F(A,B)   premise
4. G(A)   premise

5. H(C)   rev. Goal
6. ¬F(b,c)   1, 5
7. ¬G(a) V ¬F(a,b)   2, 6
8. ¬G(a)   1, 3
9. ⊥   4, 8

V.

First, convert all statement into cnf format:

1. Bill hates all cats which eat birds
    ≡ ∀x ∀y Bird(y) ∧ Cat(x) ∧ Eat(x,y) → Hate(Bill,x)
    ≡ ¬Bird(y) V ¬Cat(x) V ¬Eat(x,y) V Hate(Bill,x)

2. Felix is a cat
    ≡ Cat(Felix)

3. Felix ate a bird
    ≡ ∃y (Bird(y) ∧ Eat(Felix,y))
    ≡ ¬ (¬Bird(y) V ¬Eat(Felix,y))

4. Bill hates Felix
    ≡ Hate(Bill, Felix)

Prove by resolution with refutation "Hate(Bill, Felix)":

1. ¬Bird(y) V ¬Cat(x) V ¬Eat(x,y) V Hate(Bill,x)   premise
2. Cat(Felix)   premise
3. ¬(¬Bird(y) V ¬Eat(Felix,y))   premise
4. ¬Hate(Bill, Felix)   rev. Goal
5. ¬Bird(y) V ¬Cat(Felix) V ¬Eat(Felix,y) V Hate(Bill,Felix)   insulation on 1
6. ¬Bird(y) V ¬Cat(Felix) V ¬Eat(Felix,y)   4, 5
7. (¬Bird(y) V ¬Eat(Felix,y)) V ¬Cat(Felix)   rearrange 6
8. ¬Cat(Felix)   3, 7
9. ⊥   2, 8