Homework 3

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I.

1. Cold $\Rightarrow$ Cold is valid.

Cold	Cold $\Rightarrow$ Cold
T	T
F	T

2. Cold $\Rightarrow$ Wet is neither.

Cold	Wet	Cold $\Rightarrow$ Wet
T	T	T
T	F	F
F	T	T
F	F	T

3. (Cold $\Rightarrow$ Wet) $\Rightarrow$ ($\neg$ Cold $\Rightarrow$ $\neg$ Wet) is neither.

Cold	Wet	Cold $\Rightarrow$ Wet	$\neg$ Cold $\Rightarrow$ $\neg$ Wet	(Cold $\Rightarrow$ Wet) $\Rightarrow$ ($\neg$ Cold $\Rightarrow$ $\neg$ Wet)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

4. Cold $\vee$ Wet $\vee$ $\neg$ Wet is valid.

Cold	Wet	¬ Wet	Cold ∨ Wet	Cold ∨ Wet ∨ ¬ Wet
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T

5. ((Cold ⇒ Wet) ⇒ (Wet ⇒ Cloudy)) is neither.

Cold	Wet	Cloudy	Cold ⇒ Wet	Wet ⇒ Cloudy	((Cold ⇒ Wet) ⇒ (Wet ⇒ Cloudy))
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

6. (Cold ⇒ Cloudy) ⇒ ((Cold ∧ Wet) ⇒ Cloudy) is valid.

Cold	Wet	Cloudy	Cold ⇒ Cloudy	Cold ∧ Wet	(Cold ∧ Wet) ⇒ Cloudy	(Cold ⇒ Cloudy) ⇒ ((Cold ∧ Wet) ⇒ Cloudy)
T	T	T	T	T	T	T
T	T	F	F	T	F	T

T	F	T	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

7. Warm ∨ Sunny ∨ (Warm ⇒ Sunny) is valid.

Warm	Sunny	Warm ∨ Sunny	Warm ⇒ Sunny	Warm ∨ Sunny ∨ (Warm⇒ Sunny)
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

8. (Warm ∧ Sunny) ∨ ¬ Sunny is neither.

Warm	Sunny	¬ Sunny	Warm ∧ Sunny	(Warm ∧ Sunny) ∨ ¬ Sunny
T	T	F	T	T
T	F	T	F	T
F	T	F	F	F
F	F	T	F	T

9. ((Rain ⇒ Wet) ∧ (Wet ⇒ Cold)) ⇒ (Rain ⇒ Cold) is valid.

Rain	Wet	Cold	Rain ⇒ Wet	Wet ⇒ Cold	(Rain ⇒ Wet) ∧ (Wet ⇒ Cold)	Rain ⇒ Cold	((Rain ⇒ Wet) ∧ (Wet ⇒ Cold)) ⇒ (Rain ⇒ Cold)

T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

10. ((Rain $\lor$ Wet) $\land$ ($\neg$ Wet $\lor$ Cold)) $\Rightarrow$ (Rain $\lor$ Cold) is valid.

Rain	Wet	Cold	Rain $\lor$ Wet	$\neg$ Wet $\lor$ Cold	(Rain $\lor$ Wet) $\land$ ($\neg$ Wet $\lor$ Cold)	Rain $\lor$ Cold	((Rain $\lor$ Wet) $\land$ ($\neg$ Wet $\lor$ Cold)) $\Rightarrow$ (Rain $\lor$ Cold)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

II.

1. No, it isn't. "DogSleeps $\land$ HouseWarm $\land$ NightQuiet" means "the dog sleeps, and the house is warm, and the night is quiet."

2. Yes, it is. "(DogSleeps ∧ HouseWarm) ⇒ NightQuiet" means exactly "If the dog sleeps and the house is warm, then the night is quiet."

3. No, it isn't. "(DogSleeps ∨ HouseWarm) ⇒ NightQuiet" means "If the dog sleeps or the house is warm, then the night is quiet."

4. No, it isn't. "NightQuiet ⇒ (DogSleeps ∧ NightQuiet)" means "If the night is quiet, then the dog sleeps and the house is warm."

5. No, it isn't. "¬ DogSleeps ∨ (¬ NightQuiet ∨ HouseWarm)" means "the dog doesn't sleep, or either the night is not quiet or the house is warm."

III.

First, we need to convert all these propositional clauses into CNF format:

1. (Cold ∧ Dry) ⇒ Pleasant
   ≡ ¬ (Cold ∧ Dry) ∨ Pleasant
   ≡ ¬ Cold ∨ ¬ Dry ∨ Pleasant
2. January ⇒ (Winter ∧ Wet)
   ≡ ¬ January ∨ (Winter ∧ Wet)
   ≡ (¬ January ∧ Winter) ∨ (¬ January ∧ Wet)
3. Winter ⇒ Dry
   ≡ ¬ Winter ∨ Dry
4. Winter ⇒ Cold
   ≡ ¬ Winter ∨ Cold
5. January
6. ¬ Pleasant (rep. Goal)

Prove by resolution with refutation "Pleasant":

1. ¬ Cold ∨ ¬ Dry ∨ Pleasant		premise
2. (¬ January ∧ Winter) ∨ (¬ January ∧ Wet)		premise
3. ¬ Winter ∨ Dry		premise
4. ¬ Winter ∨ Cold		premise
5. January		premise
6. ¬ Pleasant		rep. Goal
7. Winter ∧ Wet		2, 5
7a. Winter		7
7b. Wet		7
8. Cold		4, 7a

9. Dry	3, 7a
10. ¬ Dry ∨ Pleasant	1, 8
11. Pleasant	10, 9
12. ⊥	6, 11

- Combine #5 (January) with #2 (¬ January ∧ Winter) ∨ (¬ January ∧ Wet) to get #7 (Winter ∧ Wet)
- Separate #7 (Winter ∧ Wet) into #7a (Winter) and #7b (Wet)
- Combine #4 (¬ Winter ∨ Cold) and #7a (Winter) to get #8 (Cold)
- Combine #3 (¬ Winter ∨ Dry) and #7a (Winter) to get #9 (Dry)
- Combine #8 (Cold) and #1 (¬ Cold ∨ ¬ Dry ∨ Pleasant) to get #10 (¬ Dry ∨ Pleasant)
- Combine #9 (Dry) and #10 (¬ Dry ∨ Pleasant) to get #11 (Pleasant)
- Combine #11 (Pleasant) and rep. Goal (¬ Pleasant) to get #12 (⊥ Contradiction)

IV.

First, we need to convert all these propositional clauses into CNF format:
1. ¬ A ⇒ (B ∨ C)
   ≡ ¬¬ A ∨ (B ∨ C)
   ≡ A ∨ B ∨ C
2. A ⇒ B
   ≡ ¬ A ∨ B
3. ¬ (¬ B ⇒ D)
   ≡ ¬ (B ∨ D)
   ≡ ¬ B ∧ ¬ D
4. ¬ (C ∧ ¬ D) (rep. Goal)
   ≡ ¬ C ∨ D

Prove by resolution with refutation "C ∧ ¬ D":

1. A ∨ B ∨ C	premise
2. ¬ A ∨ B	premise
3. ¬ B ∧ ¬ D	premise

4. ¬ C ∨ D	premise
5. ¬ B	3
6. ¬ D	3
7. ¬ C	4, 6
8. A ∨ B	1, 7
9. A	8, 5
10. B	2, 9
11. ⊥	5, 10

- Separate #3 (¬ B ∧ ¬ D) into #5 (¬ B) and #6 (¬ D)
- Combine #1 (A ∨ B ∨ C) and #7 (¬ C) to get #8 (A ∨ B)
- Combine #8 (A ∨ B) and #5 (¬ B) to get #9 (A)
- Combine #2 (¬ A ∨ B) and #9 (A) to get #10 (B)
- Combine #5 (¬ B) and #10 (B) to get #11 (⊥ Contradiction)