Background

- Most categories of data and related representation can be organized as the graph structures, while the detailed information can be characterized as the vertices and edges.
- The connection among vertices are important for many data handling and processing in common, and enhancements are desired in most applications.

Graph Routes Detection

- The well-known **graph routes detection (or discovery)** aims to seek for hidden paths of graph that can represent the enhanced routes among data vertices.

$$L(A) = \left\{ B \in L \left| \begin{array}{l} (B)_{ij} \leq 0 \quad if \quad (A)_{ij} = 1 \\ (B)_{ij} = 0 \quad if \quad (A)_{ij} = 0 \end{array} \right. \quad for \ i \neq j \right\} \tag{1}$$

Graph Routes Detection

- The standard solutions rely on the optimization of approximation to data affinity of ground truth, and knowledge discovery is normally referred during iterative learning stepwise.
- The generalized learning of graph routes

$$\begin{aligned} J &= \arg\min_{B} tr\left(B\mathrm{K}\right) - \log\det\left(B\right) \\ s.t. \quad & B \in L\left(A\right) \end{aligned} \tag{2}$$

Related Work

The most representative methods can be listed as below.

- Discovery of Sparse Graph [Lake and Tenenbaum, 2010]
- Generalized Graph Processing [Pavez and Ortega, 2016]
- Quadratic Approximation of Graph [Hsieh, et. al., 2011] [Dong, et. al., 2016]

Graph Routes From Local and Global Entrances

- Following the traits of smooth graph signals, the optimization of smooth graph signals can be defined as

$$J_g = \arg\min_{L,Y} \|Y - X\|_{\mathrm{F}}^2 + \alpha tr\left(Y^T L Y\right) + \beta \|L\|_{\mathrm{F}}^2 \tag{3}$$

Graph Routes From Local and Global Entrances

- Derived from the dual learning of graph signals, another graph representation is brought to denote the detailed information, which is also known as the local graph [Cheng, et. al., 2009].

Graph Routes From Local and Global Entrances

- The additional constraints are devised to enhance the invariant characteristics of **local graph**.

$$J_l = \arg\min_{L_{bl}, Y} \|L_{bl} - L_{al}\|_{\mathrm{F}}^2 + \delta tr\left(Y^T L_{bl} Y\right) + \gamma \|L_{bl}\|_{\mathrm{F}}^2 \tag{4}$$

Graph Routes From Local and Global Entrances

- Thereafter, the whole objective function of the proposed method can be defined

$$
\begin{aligned}
J = \arg \min_{L, L_{bl}, Y} & \|Y - X\|_{\mathrm{F}}^2 + \|L_{bl} - L_{al}\|_{\mathrm{F}}^2 + \alpha tr\left(Y^T \right. \\
& \left. \delta tr\left(Y^T L_{bl} Y\right) + \beta \|L\|_{\mathrm{F}}^2 + \gamma \|L_{bl}\|_{\mathrm{F}}^2 \right. \\
s.t. \quad & (L)_{ij} = (L)_{ji} \leq 0, \qquad i \neq j \\
& L \cdot 1 = 0 \\
& (L_{bl})_{ij} = (L_{bl})_{ji} \leq 0, \quad i \neq j \\
& L_{bl} \cdot 1 = 0
\end{aligned}
\tag{5}
$$

Graph Routes From Local and Global Entrances

- To optimize such function, it is to learn the ideal solutions of L, L_{bl}, and Y respectively

$$
\begin{aligned}
&J(L) = \alpha Y^T Y L + \beta L^T L \\
&J(L_{bl}) = (1+\gamma) L_{bl}^T L_{bl} + \left(\delta Y^T Y - 2L_{al}^T\right) L_{bl} \\
&Y = X(\alpha L + \delta L_{bl} + I)^+
\end{aligned}
\tag{6}
$$

- To optimize the problem, it is to give the initial Y beforehand, and the principal components of original data X are referred in this work.

Graph Routes From Local and Global Entrances

Several issues:

- Firstly, the obtained L and L_{bl} are quite different from each other as each item is calculated separately.
- The obtained graphs actually depend on the graph signal filtering in theory, and the global and local graphs stand for the structure of filtered data with low and high frequency bands respectively.

GRLG Algorithm

Input: Signal $X \in \mathbb{R}^{d \times n}$, the dimensionality r, parameters $\alpha > 0$, $\beta > 0$, $\delta > 0$, $\gamma > 0$, the maximum iteration t.

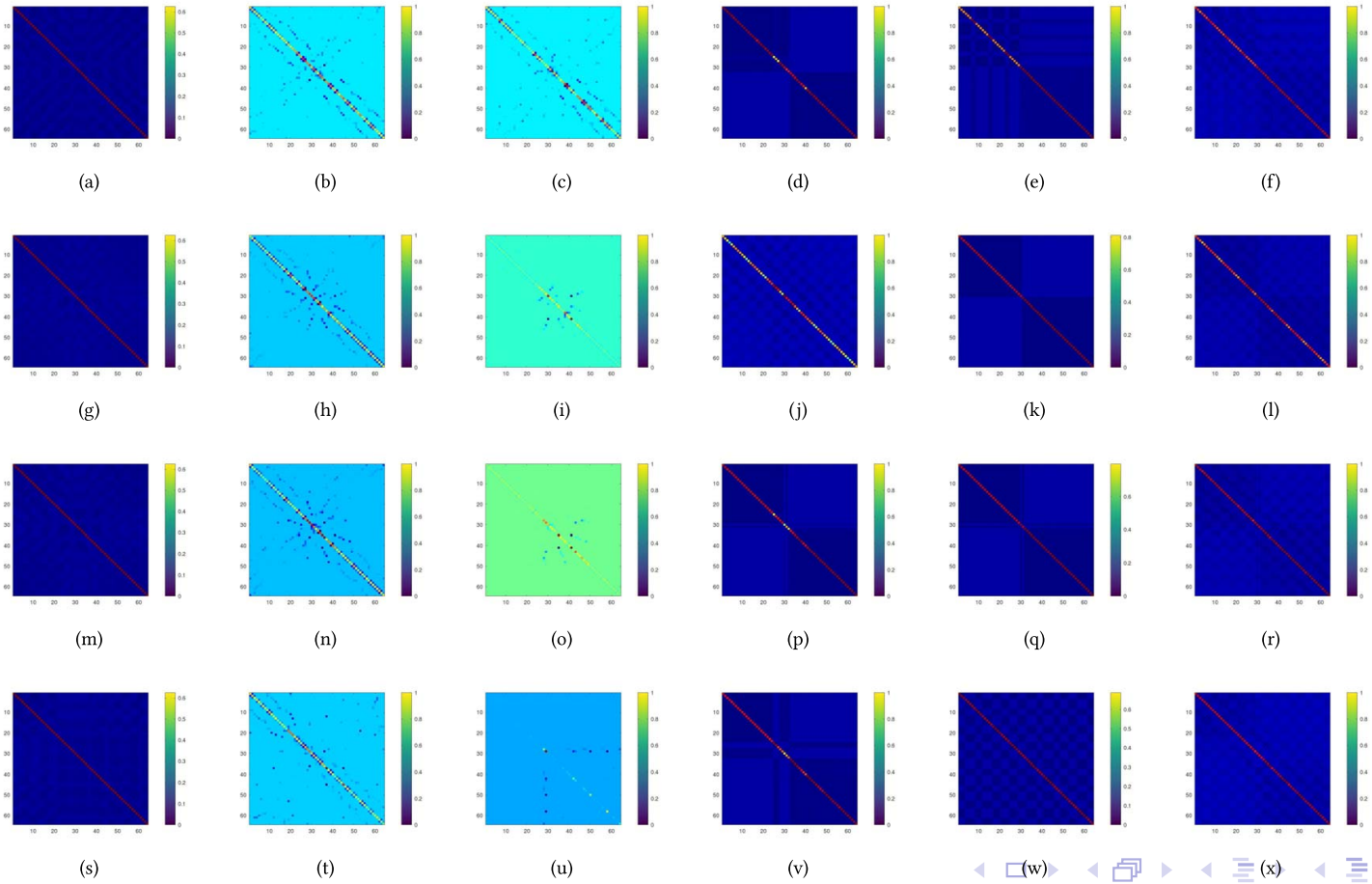
1. Calculate the local graph based on original data, and obtain the initial L_{al}.
2. Calculate the first r principal components of X, and obtain the initial Y.
3. While the maximum iteration t or stable results have never been reached,
   - Update the local graph, and the global graph.
   - Update the compact data.
4. Obtain the desired routes L_{bl} and L, as well as the refined data Y.

Experiments

- Several benchmark graph routes detection methods are involved
  - Generalized graph learning (GGL) [Egilmez et. al., 2017][Segarra et. al., 2017]
  - Combinational graph learning (CGL) [Egilmez et. al., 2017]
  - Graph learning of smooth signals (SIGREP) [Dong et. al., 2016]

Experiments

- The obtained graphs of different methods.



Experiments

Table: The obtained relative errors of different methods associated with involved representation and neighbors

Rep.	NN	GGL	CGL	SigRep	Local	Fusion
320	3	0.357	0.339	**0.003**	0.004	0.007
	5	0.354	0.338	**0.005**	0.006	0.008
	10	0.347	0.332	**0.01**	**0.01**	0.012
	Full	0.346	0.32	0.02	0.017	**0.012**
640	3	0.376	0.426	0.008	**0.003**	0.004
	5	0.374	0.424	0.011	**0.005**	0.007
	10	0.366	0.417	0.018	**0.009**	0.012
	Full	0.363	0.402	0.022	0.018	**0.012**
980	3	0.356	0.445	**0.004**	**0.004**	0.006
	5	0.354	0.442	0.007	**0.006**	0.008
	10	0.344	0.433	0.013	**0.01**	0.013
	Full	0.342	0.419	0.019	0.019	**0.014**
1280	3	0.358	0.315	0.005	**0.004**	0.006
	5	0.358	0.317	0.007	**0.005**	0.008
	10	0.348	0.311	0.012	**0.009**	0.012
	Full	0.347	0.261	0.018	0.019	**0.013**

Experiments

Table: The obtained modularity of different methods associated with involved representation and neighbors

Rep.	NN	GGL	CGL	SigRep	Local	Fusion
320	3	0.124	0.125	**1.794**	0.674	0.182
	5	0.115	0.12	**1.48**	0.349	0.181
	10	0.105	0.108	**0.954**	0.212	0.169
	Full	0.094	0.099	**0.694**	0.247	0.181
640	3	0.126	0.129	**0.291**	0.203	0.233
	5	0.108	0.109	**0.286**	0.239	0.216
	10	**0.978**	0.098	0.27	0.219	0.214
	Full	0.086	0.084	**0.22**	0.183	0.181
980	3	0.155	0.124	**1.253**	0.244	0.239
	5	0.125	0.105	**1.169**	0.187	0.205
	10	0.098	0.087	**0.665**	0.168	0.155
	Full	0.088	0.086	**0.916**	0.152	0.204
1280	3	0.157	0.182	**2.233**	0.258	0.22
	5	0.134	0.17	**1.262**	0.243	0.236
	10	0.115	0.18	**0.695**	0.234	0.232
	Full	0.097	0.144	**0.743**	0.191	0.305

Experiments

Table: The obtained F Score of different methods associated with involved representation and neighbors

Rep.	NN	GGL	CGL	SigRep	Local	Fusion
320	3	0.934	0.937	**0.939**	0.927	0.933
	5	0.909	0.906	0.906	0.89	**0.915**
	10	**0.857**	0.855	0.791	0.807	0.82
	Full	**0.904**	0.893	0.66	0.777	0.777
640	3	0.929	0.929	0.935	**0.938**	**0.938**
	5	0.899	0.897	0.898	**0.904**	0.902
	10	**0.844**	0.838	0.079	0.804	0.803
	Full	0.907	**0.909**	0.66	0.758	0.758
980	3	**0.93**	0.929	0.922	0.923	0.929
	5	**0.902**	0.899	0.871	0.884	0.887
	10	**0.853**	0.85	0.757	0.793	0.796
	Full	**0.9**	0.898	0.66	0.756	0.756
1280	3	0.925	0.927	0.909	**0.934**	0.928
	5	0.894	0.902	0.865	**0.904**	0.898
	10	0.847	**0.85**	0.775	0.817	0.812
	Full	**0.901**	0.887	0.66	0.756	0.756

Conclusion

- An improved approach to graph routes detection is proposed, which involves the local and global graphs into optimization.
- Several outstanding advantages:
  - The proposed method is able to learn the local and global routes simultaneously.
  - Exchange between different variants are available during iterative learning.
  - The fixed results can be obtained in each circle, and the theoretical background can be enhanced.

Thank You

for

Your Participant & The Best

@ ICBDT'23, Qingdao, China