Predictive Supervisory Control for Timed Discrete Event Systems under Communication Delays

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***Abstract*— In many cyber-physical systems, controllers and plants are located at different sites. Communications between a controller and a plant are carried out over a wired or wireless communication network. Communication delays in such systems must be addressed when designing controllers. In this paper, we model a cyber-physical system as a timed discrete event system. We investigate predictive supervisory control of timed discrete event systems with communication delays in both observation channel and control channel. We derive a necessary and sufficient condition for the existence of an admissible networked supervisor that ensures the language generated by the closed-loop system equals a given specification language. An example is given to illustrate the results.**

***Index Terms*— Discrete event systems, networked systems, cyber-physical systems, supervisory control, controllability, observability.**

I. Introduction

Cyber-physical systems are becoming increasingly popular in a wide range of applications, such as transportation networks, smart homes, smart grids, advanced automotive systems [1] [2] [3]. Since the controllers and plants in cyber-physical systems are usually located in different sites, the communications between controllers and plants are carried out via communication networks. As an inherent property of communication networks, communication delays cannot be ignored when designing controllers. Since cyber-physical systems often involve various events, and the systems are usually event driven. Hence, we model a cyber-physical system as a timed discrete event system and investigate supervisory control problem of timed discrete event systems under communication delays.

Supervisory control of discrete event systems has been investigated intensively since the publication of [4]. The goal of supervisory control is to design a supervisor which observes some observable events and control some controllable events of the plant so that some control objective, described by a specification language, is achieved. The control objective can be achieved if and only if the specification language is controllable [4] and observable [5]. Observability have been extended to co-observability [6] in decentralized control when several local supervisors are used.

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As networks become more and more widely used in cyber-physical systems and other systems, communication delays become an important issue in control as well as diagnosis of discrete event systems. For example, how to deal with communication delays in fault diagnosis in discrete event systems is investigated in [7] [8]. The impacts of communication delays among several local supervisors on decentralized supervisory control of discrete event systems are investigated in [9] [10] [11]. Communication delays between supervisors and plants are considered in [12] [13].

We systematically investigate supervisory control of networked discrete event systems with communication delays and losses in both observation channel and control channel. In [14], a general framework for networked control of discrete event systems is proposed. Network controllability and network observability are introduced to ensure the existence of a (non-predictive) supervisor. In [15], predictive supervisors are used to control networked discrete event systems. In general, a predictive supervisor can do better than a non-predictive supervisor. Necessary and sufficient condition is found in [15] for the existence of a predictive supervisor in terms of controllability and network observability. An algorithm to check network observability is developed in [16]. Since the delays are nondeterministic, control may also be nondeterministic. Deterministic supervisory control is investigated in [17]. It shows that in order for a deterministic supervisor to exist, a stronger condition, namely delay observability must be satisfied.

In all the works mentioned above, time is not explicitly modeled. To explicitly model time, timed discrete event systems are investigated in [18], where a special event, called tick, is introduced to represent the passage of a unit of time. The goal of supervisory control for timed discrete event systems is similar to that for (un-timed) discrete event systems, except that the time can be explicitly considered. Controllability, observability and co-observability are extended to timed discrete event systems in [18] [19] [20]. For better presentation, we call them T-controllability, T-observability and T-co-observability, respectively, in this paper.

To deal with communication delays in timed discrete event systems, non-predictive networked supervisory control of timed discrete event systems is investigated in [21], where necessary and sufficient condition for the existence of a networked supervisor is derived in terms of network T-controllability and network T-observability. The networked supervisor proposed in [21] is non-predictive. The non-predictive networked supervisor cannot predict the impact of communication delays in the control channel when determining control actions. It is proved in [15] that in (untimed) discrete event systems, predictive networked supervisors are better than non-predictive networked supervisors and no other networked supervisors are better than predictive networked supervisors.

In this paper, we investigate predictive network control of timed discrete event systems. Our approach is based on the delayed observer developed in [22] for timed discrete event systems. With the delayed observer, we can obtain the state estimate for a given (delayed) observation, that is, the set of all possible states the plant may be in. We propose a new method to obtain the predicted state estimate by considering delays on control channel. A state-estimate-based predictive supervisor is then introduced to control the timed discrete event system. To derive necessary and sufficient condition for the existence of an admissible networked supervisor, we generalize network T-observability by incorporating delays in control channel into the definition. We show that an admissible networked supervisor exists for a given specification language if and only if the language is T-controllable and network T-observable. Furthermore, we show that if an admissible networked supervisor exists, the proposed state-estimate-based predictive supervisor is such a supervisor.

The rest of the paper is organized as follows. Section II reviews timed discrete event systems. In Section III, we formally state the supervisory control problem of timed discrete event systems under communication delays. In Section IV, we propose a state-estimate-based predictive supervisor to solve the problem. A necessary and sufficient condition for the existence of an admissible networked supervisor is obtained in Section V. Finally we conclude the paper in section VI. II

II. Timed discrete event systems

A timed discrete event system is described by a timed automaton as

$$\widetilde{G} = (Q, \widetilde{\Sigma}, \rho, q_0),$$

where Q is the set of states, $\widetilde{\Sigma} = \Sigma \cup \{t\}$ is the set of events including $tick$, denoted by t, representing the elapse of a unit of time. We call Σ the set of activity events. Each activity event has a lower time bound and an upper time bound specifying when the event can occur. For the activity events whose upper bound is finite, we call them prospective events. The set of prospective events is denoted by Σ_{spe}. For the activity events whose upper bound is infinite, we call them remote events. The set of remote events is denoted by Σ_{rem}. With the above definitions, the set of events in timed discrete events systems can also be described as $\widetilde{\Sigma} = \Sigma_{spe} \cup \Sigma_{rem} \cup \{t\}$. $\rho : Q \times \widetilde{\Sigma} \to Q$ is the (partial) state transition function, and q_0 is the initial state. The state transition function is derived from the activity transition function (see [18] for details). In the usual way, we extend the transition function to $\rho : Q \times \widetilde{\Sigma}^* \to Q$. In addition, we use $\rho(q, s)!$ to mean that transition $\rho(q, s)$ is defined. The behavior of $\widetilde{G}$ is described by the language generated by $\widetilde{G}$ which is defined as

$$L(\widetilde{G}) = \{s \in \widetilde{\Sigma}^* : \rho(q_0, s)!\}.$$

For a string s, we use $Pr(s)$ to denote its prefix set, use $|s|$ to denote its length, and use $|t|(s)$ to denote the number of t in s. For example, for string $s = tt\sigma_1$, we have $Pr(s) = \{\varepsilon, t, tt, tt\sigma_1\}$, $|s| = 3$ and $|t|(s) = 2$.

The prefix closure of a language is the set of prefixes of strings in that language. A language is (prefix) closed if it equals its prefix closure. Hence, $L(\widetilde{G})$ is closed. We consider only closed language in this paper.

We assume that some events in Σ are observable while the other events are unobservable. The set of observable events are denoted by $\Sigma_o \subseteq \Sigma$ and the set of unobservable events are denoted by $\Sigma_{uo} = \Sigma - \Sigma_o$. Note that t is always observable. Hence, for the timed discrete event system, the observable event set is $\widetilde{\Sigma}_o = \Sigma_o \cup \{t\}$ and the unobservable event set is $\widetilde{\Sigma}_{uo} = \Sigma_{uo}$. The observation is the natural projection $P : \widetilde{\Sigma}^* \to \widetilde{\Sigma}_o^*$ defined as

$$P(\varepsilon) = \varepsilon \qquad P(s\sigma) = \begin{cases} P(s)\sigma & \sigma \in \widetilde{\Sigma}_o \\ P(s) & \sigma \notin \widetilde{\Sigma}_o \end{cases} .$$

For a language L, its projection is then defined as $P(L) = \{P(s) : s \in L\}$.

For languages L_1 and L_2, we define L_1/L_2 as

$$L_1/L_2 = \{s_1 : (\exists s_2 \in L_2) s_1 s_2 \in L_1\}.$$

As usual, we assume that not all the events are controllable. The set of controllable events is denoted by $\Sigma_c \subseteq \Sigma \subseteq \widetilde{\Sigma}$. For timed discrete event systems, we further assume that some events in $\widetilde{\Sigma}$ can be forced to occur in the sense that they can preempt t (but no other events). The set of forcible events is denoted by $\Sigma_f \subseteq \widetilde{\Sigma}$. The set of uncontrollable events is $\widetilde{\Sigma}_{uc} = \Sigma - \Sigma_c \subseteq \widetilde{\Sigma}$. Note that forcible events can be either controllable or uncontrollable.

In order to consider the supervisory control problem, we make the following two assumptions.

Assumption 1: Only a finite number of events can occur in one unit of time, that is, $\widetilde{G}$ is Σ-loop free:

$$(\forall q \in Q)(\forall s \in \Sigma^+) \rho(q, s) \neq q,$$

where $\Sigma^+ = \Sigma^* - \{\varepsilon\}$.

Assumption 2: The advance of time will never stop, that is, $\widetilde{G}$ is deadlock free:

$$(\forall q \in Q)(\exists \sigma \in \widetilde{\Sigma}) \rho(q, \sigma)!$$

Assumption 1 excludes the physically unrealistic possibility that activity events occur infinitely during one unit of time. Assumption 2 assumes that a timed discrete event system never "stops the time". At any state, either transitions with activity events are defined or at least the t transition is defined.

III. Problem statement

As discussed in [14], when we observe the occurrences of events via networks, communication delays are unavoidable. We assume that observation delays are nondeterministic and are upper-bounded by N_o[1]. We also assume that delays do not change the order of events occurred (first in, first out, or FIFO). Because delays are nondeterministic, even for the same string s occurred in $\widetilde{G}$, the observation becomes nondeterministic.

If the observation may be delayed for i units of time, we use $\Theta_i(s)$ to denote the set of observations for the special case where all events are observable. For a given string $s \in L(\widetilde{G})$, we divide it into substrings as

$$s = u_1 t u_2 t \cdots u_l t u_{l+1},$$

where $u_k \in \Sigma^*(k = 1, 2, \cdots, l+1)$ does not include t. That is, $|t|(u_k) = 0$. Note that u_k may be empty strings, that is, for some u_k, $u_k = \varepsilon$. For all substrings $u_k, k = 1, 2, \cdots, l$, any event in u_k may be delayed for 1 tick such that the event is observed after 1 tick. Let us rewrite $u_k t$ as

$$u_k t = \sigma_1 \sigma_2 \cdots \sigma_n t.$$

We define $\theta(u_k t)$ as

$$\theta(u_k t) = \{\sigma_1 \sigma_2 \cdots \sigma_n t, \sigma_1 \sigma_2 \cdots t \sigma_n, \cdots, t \sigma_1 \sigma_2 \cdots \sigma_n\}.$$

Particularly, if $u_k = \varepsilon$, we have $\theta(u_k t) = \{t\}$. Then we define $DELAY^1(s)$ as

$$DELAY^1(s) = \theta(u_1 t)\theta(u_2 t) \cdots \theta(u_l t) u_{l+1}.$$

We extend $DELAY^1(s)$ to the case of j ticks as

$$DELAY^j(s) = DELAY^1(DELAY^{j-1}(s)).$$

For all $s \in L(\widetilde{G})$, if the observation delays are upper-bounded by i, we define $\Theta_i(s)$ as follows.

$$\Theta_i(s) = DELAY^i(s)/\Sigma^*.$$

That is because due to observation delays, some activity events at the end of the string may not be observed yet.

For language $L(\widetilde{G})$, assume that observation delays are upper-bounded by N_o, the set of all possible observations under the delays are given by

$$\Theta_{N_o}(L(\widetilde{G})) = \cup_{s \in L(\widetilde{G})} \Theta_{N_o}(s).$$

Now we extend observation to the general case in which some events are unobservable by natural projection. We use $\Phi_{N_o}(s)$ to denote the set of possible observations. Clearly,

$$\Phi_{N_o}(s) = P(\Theta_{N_o}(s)).$$

We further extend the definition $\Phi_{N_o}(\cdot)$ from string s to language L as

$$\Phi_{N_o}(L) = \cup_{s \in L} \Phi_{N_o}(s).$$

[1] When we say that delays are upper-bounded by N, we mean that they are upper-bounded by N ticks.

The inverse projection is denoted as ${\Phi_{N_o}}^{-1}$. For an observation w, ${\Phi_{N_o}}^{-1}(w)$ is defined as

$${\Phi_{N_o}}^{-1}(w) = \{s \in L(\widetilde{G}) : w \in \Phi_{N_o}(s)\}.$$

A networked supervisor in timed discrete event system issues its control decisions based on its observations and is then defined as

$$S : \Phi_{N_o}(L(\widetilde{G})) \to 2^{\widetilde{\Sigma}}.$$

Given an observation $w \in \Phi_{N_o}(L(\widetilde{G}))$, $S(w)$ is the set of enabled events. Because uncontrollable events shall always be allowed to occur, we require that

$$(\forall w \in \Phi_{N_o}(L(\widetilde{G})))\widetilde{\Sigma}_{uc} \subseteq S(w). \tag{1}$$

As discussed above, t can be preempted (or disabled) by forcing a forcible event to occur. In the control structure of supervisory control, the supervisor specifies the set of enabled events. In order to be consistent with this structure, for any string $s \in L(\widetilde{G})$ and its observation $w \in \Phi_{N_o}(s)$, we require that if t in $\widetilde{G}$ is not enabled by the supervisor, that is $t \notin S(w)$, then the system must enforce some available forcible event to preempt t. So we require that

$$\begin{aligned}(\forall s \in L(\widetilde{G}))(\forall w \in \Phi_{N_o}(B_{N_c}(s)))t \in \widetilde{\Sigma}_{L(\widetilde{G})}(s) \\ \wedge t \notin S(w) \Rightarrow \widetilde{\Sigma}_{L(\widetilde{G})}(s) \cap \Sigma_f \cap S(w) \neq \emptyset,\end{aligned} \tag{2}$$

where $\widetilde{\Sigma}_{L(\widetilde{G})}(s) = \{\sigma \in \widetilde{\Sigma} : s\sigma \in L(\widetilde{G})\}$.

We say a supervisor is admissible if Equation (1) and Equation (2) are satisfied. We use S_a to denote an admissible networked supervisor.

If there are no communication delays in the control channel, given a string $s \in L(\widetilde{G})$, when we observe a string w, the control command $S(w)$ is executed immediately. However, when communication delays exist in the control channel which is assumed to be upper-bounded by N_c, the control command used currently can be any one of the control commands issued by S within the past N_c ticks.

For $s \in L(\widetilde{G})$, we denote the set of prefixes of s with delays less than N_c by $B_{N_c}(s)$, which is defined as

$$\begin{aligned}B_{N_c}(s) = \{s' \in L(\widetilde{G}) : (\exists s'' \in \widetilde{\Sigma}^*) s = s's'' \\ \wedge |t|(s'') \leq N_c\}.\end{aligned}$$

We extend B_{N_c} from string s to language L as

$$B_{N_c}(L) = \cup_{s \in L} B_{N_c}(s).$$

The inverse operation of B_{N_c} is denoted as ${B_{N_c}}^{-1}$, for a string $s' \in L(\widetilde{G})$, ${B_{N_c}}^{-1}(s')$ is defined as

$$\begin{aligned}{B_{N_c}}^{-1}(s') &= \{s \in L(\widetilde{G}) : s' \in B_{N_c}(s)\} \\ &= \{s \in L(\widetilde{G}) : (\exists s'' \in \widetilde{\Sigma}^*) s = s's'' \\ &\quad \wedge |t|(s'') \leq N_c\}.\end{aligned}$$

We extend ${B_{N_c}}^{-1}$ from string s to language L as

$${B_{N_c}}^{-1}(L) = \cup_{s' \in L} {B_{N_c}}^{-1}(s').$$

After the occurrence of s, the possible (delayed) control commands being used are $S(w)$ where $w \in \Phi_{N_o}(B_{N_c}(s))$. Under the control, an event σ is permitted to occur if it is enabled by one of the control commands, that is $(\exists w \in \Phi_{N_o}(B_{N_c}(s)))\sigma \in S(w)$.

We assume that the supervisor always uses the latest control command received and the initial control command is received without delays. Considering communication delays in both observation channel and control channel, the language generated by the supervised system is defined as follows.

Definition 1: The language $L(S/\widetilde{G})$ generated by the supervised system under observation delays upper-bounded by N_o as well as control delays upper-bounded by N_c is obtained recursively as follows.

1. The empty string belongs to $L(S/\widetilde{G})$. That is, $\varepsilon \in L(S/\widetilde{G})$.

2. If s belongs to $L(S/\widetilde{G})$, then for any $\sigma \in \widetilde{\Sigma}$, $s\sigma$ belongs to $L(S/\widetilde{G})$ if and only if $s\sigma$ is allowed in $L(\widetilde{G})$ and σ is enabled by the supervisor in one of the past N_c ticks:

$$(\forall s \in L(S/\widetilde{G}))(\forall \sigma \in \widetilde{\Sigma}) s\sigma \in L(S/\widetilde{G})$$
$$\Leftrightarrow s\sigma \in L(\widetilde{G}) \wedge (\exists w \in \Phi_{N_o}(B_{N_c}(s)))\sigma \in S(w).$$

We assume that the control objective is to achieve a given closed specification $K \subseteq L(\widetilde{G})$. Without loss of generality, we further assume that K is generated by a sub-automaton $H \sqsubseteq \widetilde{G}$, that is, $K = L(H)$ for some

$$H = (Q_H, \widetilde{\Sigma}, \rho_H, q_0),$$

where $Q_H \subseteq Q$ and $\rho_H = \rho|_{Q_H \times \widetilde{\Sigma}} \subseteq \rho$.

With these knowledges, the control problem is then formally stated as follows.

Control Problem of Timed Discrete Event Systems with Communication Delays: Consider a timed discrete event system $\widetilde{G}$ with communication delays in the observation channel upper-bounded by N_o, and communication delays in the control channel upper-bounded by N_c. For a non-empty closed specification language $K \subseteq L(\widetilde{G})$ modeled as a sub-automaton $H \sqsubseteq \widetilde{G}$, find an admissible networked supervisor S_a such that $L(S_a/\widetilde{G}) = K$.

IV. State-Estimate-Based Predictive Supervisor

For the control problem introduced in the previous section, we propose a state-estimate-based predictive supervisor as follows. We first introduce some notations.

When $s \in K$ occurs, the system is in state $\rho(q_0, s) = q$. At state q, we define

$$\gamma(q) = \{\sigma \in \Sigma : q \in Q_H \wedge \rho(q,\sigma) \in Q - Q_H\}.$$

$\gamma(q)$ is the set of events that moves the system from state q to some "illegal states" which should be disabled at state q. If no such events exist, $\gamma(q) = \emptyset$.

We extend the definition of γ from single state q to a subset of states. For $Q' \subseteq Q$, we define $\gamma(Q')$ as

$$\gamma(Q') = \cup_{q \in Q'} \gamma(q).$$

Given an observation $w \in \Phi_{N_o}(L(\widetilde{G}))$, let us determine what events need to be disabled after observing w. We first compute the set of states that the system may be in, called state estimate after observing w. We use $TE_H^{N_o}(w)$ to denote the state estimate within H after observing w. The definition of $TE_H^{N_o}(w)$ is given by

$$TE_H^{N_o}(w) = \{q \in Q : (\exists s \in L(H)) w \in \Phi_{N_o}(s)$$
$$\wedge \rho(q_0, s) = q\}.$$

To calculate state estimate $TE_H^{N_o}(w)$, we refer to our results in [22]. We first construct the delayed observer $H_{D,obs}^{N_o}$ based on H and N_o, which is described as

$$H_{D,obs}^{N_o} = (Y, \widetilde{\Sigma}_o, \zeta_H, y_0).$$

The state estimate $TE_H^{N_o}(w)$ is then obtained as

$$TE_H^{N_o}(w) = \zeta_H(y_0, w).$$

Consider the control delays, we should predict the states reachable within N_c ticks. Hence we define, for any state $q \in Q_H$, the set of states reachable within N_c ticks in H as

$$TR_H^{N_c}(q) = \{q' \in Q_H : (\exists s \in \widetilde{\Sigma}^*) |t|(s) \leq N_c$$
$$\wedge \rho_H(q,s) = q'\}.$$

After observing w, the state estimate considering the control delays is then given by

$$TR_H^{N_c}(TE_H^{N_o}(w)) = \cup_{q \in TE_H^{N_o}(w)} TR_H^{N_c}(q).$$

For state estimate $TR_H^{N_c}(TE_H^{N_o}(w))$, the set of events which should be disabled/preempted are:

$$\gamma(TR_H^{N_c}(TE_H^{N_o}(w))).$$

However, $t \in \gamma(TR_H^{N_c}(TE_H^{N_o}(w)))$ cannot be disabled/preempted unless Equation (2) always holds. That is, at any state $q \in TR_H^{N_c}(TE_H^{N_o}(w))$, t can be preempted if at least one forcible event is permitted to occur, or t cannot occur at state q as

$$(\Gamma_H(q) - \gamma(TR_H^{N_c}(TE_H^{N_o}(w)))) \cap \Sigma_f \neq \emptyset \vee t \notin \Gamma(q),$$

where $\Gamma_H(q) = \{\sigma : \rho_H(q,\sigma)!\}$ and $\Gamma(q) = \{\sigma : \rho(q,\sigma)!\}$ denoted the set of eligible events at state q in H and in $\widetilde{G}$, respectively.

Hence, for state estimate $TR_H^{N_c}(TE_H^{N_o}(w))$, t can be disabled/preempted if

$$\begin{aligned}&(\forall q \in TR_H^{N_c}(TE_H^{N_o}(w))) t \notin \Gamma(q) \vee \\ &(\Gamma_H(q) - \gamma(TR_H^{N_c}(TE_H^{N_o}(w)))) \cap \Sigma_f \neq \emptyset\end{aligned} \tag{3}$$

For state estimate $TR_H^{N_c}(TE_H^{N_o}(w))$, t cannot be disabled/preempted (that is, t should be enabled) if

$$\begin{aligned}&(\exists q \in TR_H^{N_c}(TE_H^{N_o}(w))) t \in \Gamma(q) \wedge \\ &(\Gamma_H(q) - \gamma(TR_H^{N_c}(TE_H^{N_o}(w)))) \cap \Sigma_f = \emptyset.\end{aligned} \tag{4}$$

Based on these discussions, let us propose the state-estimate-based predictive supervisor S_p as follows. For any observed string $w \in \Phi_{N_o}(L(\widetilde{G}))$, $S_p(w)$ is defined in Equation (5).

Propsition 1: The state-estimated-based predictive supervisor S_p defined in Equation (5) is an admissible supervisor.

$$S_p(w) = \begin{cases} (\widetilde{\Sigma} - \gamma(TR_H^{N_c}(TE_H^{N_o}(w)))) \cup \widetilde{\Sigma}_{uc} & \text{if } w \in \Phi_{N_o}(L(H)) \text{ and (3) holds} \\ (\widetilde{\Sigma} - \gamma(TR_H^{N_c}(TE_H^{N_o}(w)))) \cup \widetilde{\Sigma}_{uc} \cup \{t\} & \text{if } w \in \Phi_{N_o}(L(H)) \text{ and (4) holds} \\ \widetilde{\Sigma}_{uc} \cup \{t\} & \text{otherwise} \end{cases} . \quad (5)$$

V. EXISTENCE CONDITIONS

In order to capture the impacts of delays in both observation channel and control channel, we introduce network T-observability as follows.

Definition 2: Given a closed language $K \subseteq L(\widetilde{G})$ and observation delays upper-bounded by N_o and control delays upper-bounded by N_c, we say that K is network T-observable with respect to N_o, N_c, and $\widetilde{\Sigma}_o$ if

1) $(\forall \sigma \in \widetilde{\Sigma})(\forall s\sigma \in L(\widetilde{G}))s\sigma \in K \Rightarrow ((\exists w \in \Phi_{N_o}(B_{N_c}(s)))(\forall s' \in B_{N_c}^{-1}(\Phi_{N_o}{}^{-1}(w)))s' \in K \wedge s'\sigma \in L(\widetilde{G}) \Rightarrow s'\sigma \in K)$,

2) $(\forall st \in L(\widetilde{G}))s \in K \wedge st \notin K \Rightarrow (\forall w \in \Phi_{N_o}(B_{N_c}(s)))(\forall s' \in B_{N_c}^{-1}(\Phi_{N_o}{}^{-1}(w)))(s't \notin L(\widetilde{G}) \vee \widetilde{\Sigma}_K(s') \cap \Sigma_f \neq \emptyset)$.

Let us recall T-controllability defined in [18] as follows.

Definition 3: Given a closed language $K \subseteq L(\widetilde{G})$, we say K is T-controllable with respect to Σ_c and t if for all $s \in K$

1) $(\forall \sigma \in \widetilde{\Sigma})s\sigma \in L(\widetilde{G}) \wedge \sigma \in \widetilde{\Sigma}_{uc} \Rightarrow s\sigma \in K$,

2) $\widetilde{\Sigma}_K(s) \cap \Sigma_f = \emptyset \wedge t \in \widetilde{\Sigma}_{L(\widetilde{G})}(s) \Rightarrow t \in \widetilde{\Sigma}_K(s)$.

With T-controllability and network T-observability, we have the following theorem to solve the control problem of timed discrete event systems with communication delays.

Theorem 1: Consider a networked timed discrete event system $\widetilde{G}$ with communication delays. For a nonempty closed language $K \subseteq L(\widetilde{G})$ modeled as a sub-automaton $H \sqsubseteq \widetilde{G}$, there exists an admissible networked supervisor S_a such that $L(S_a/\widetilde{G}) = K$ if and only if K is T-controllable with respect to Σ_c and t, and K is network T-observable with respect to N_o, N_c, and $\widetilde{\Sigma}_o$. Furthermore, if the existence condition is satisfied, then the state-estimate-based predictive S_p is a supervisor such that $L(S_p/\widetilde{G}) = K$.

Let us use an example to illustrate these results.

Example 1: We consider a timed discrete event system $\widetilde{G}$ shown in Fig. 1. $\widetilde{\Sigma} = \{\alpha, \beta, \lambda, \mu, t\}$.

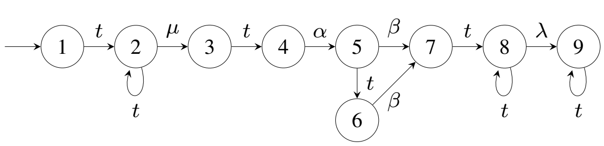

Fig. 1. A timed discrete event system $\widetilde{G}$

Assume that all events are observable, λ and μ are controllable, λ and β are forcible. That is $\widetilde{\Sigma}_o = \widetilde{\Sigma}$, $\Sigma_c = \{\lambda, \mu\}$, $\widetilde{\Sigma}_{uc} = \Sigma - \Sigma_c = \{\alpha, \beta\}$, and $\Sigma_f = \{\beta, \lambda\}$. We further assume that both observation delays and control delays are upper-bounded by 1.

The closed specification language K which is generated by a sub-automaton $H \sqsubseteq \widetilde{G}$ is shown in Fig. 2.

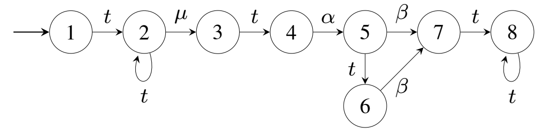

Fig. 2. The sub-automaton H

We first check T-controllability. The first part of T-controllability can be re-written as

$$(\forall s \in K)(\forall \sigma \in \widetilde{\Sigma})s\sigma \in (L(\widetilde{G}) - K) \Rightarrow \sigma \notin \widetilde{\Sigma}_{uc}.$$

The string s satisfying $s \in K \wedge s\sigma \in (L(\widetilde{G}) - K)$ are strings ending in state 8 and $\sigma = \lambda$. Since $\lambda \notin \widetilde{\Sigma}_{uc}$, the first part is satisfied. The second part of T-controllability can be re-written as

$$(\forall s \in K)t \in (\widetilde{\Sigma}_{L(\widetilde{G})}(s) - \widetilde{\Sigma}_K(s)) \Rightarrow \widetilde{\Sigma}_K(s) \cap \Sigma_f \neq \emptyset.$$

Since there is no $s \in K$ such that $t \in (\widetilde{\Sigma}_{L(\widetilde{G})}(s) - \widetilde{\Sigma}_K(s))$. The second part is satisfied.

To check if the first part of network T-observability holds, We use a string $s = t\mu t \in K$ as an example. For $\sigma = \alpha$, we have $s\sigma \in L(\widetilde{G})$, and $s\sigma \in K$. We calculate $B_{N_c}(s)$ and $\Phi_{N_o}(B_{N_c}(s))$ as

$$B_{N_c}(s) = \{t\mu t, t\mu, t\}.$$
$$\Phi_{N_o}(B_{N_c}(s)) = \{t\mu t, tt\mu, t\mu, t\}.$$

We take one observation $w = tut \in \Phi_{N_o}(B_{N_c}(s))$ to show that, for all $s' \in B_{N_c}{}^{-1}(\Phi_{N_o}{}^{-1}(w))$,

$$s' \in K \wedge s'\sigma \in L(\widetilde{G}) \Rightarrow s'\sigma \in K. \quad (6)$$

We have

$$B_{N_c}{}^{-1}(\Phi_{N_o}{}^{-1}(w)) = \{t\mu t, t\mu t\alpha, t\mu t\alpha\beta, t\mu t\alpha t, t\mu t\alpha t\beta, t\mu t\alpha\beta t, t\mu t\alpha\beta t\mu\},$$

in which $t\mu t \in K$, $t\mu t\alpha\beta \in K$ and $t\mu t\alpha\beta t \in K$.

For $s' = t\mu t$, we have $s'\sigma \in L(\widetilde{G})$ and $s'\sigma \in K$. For $s' = t\mu t\alpha\beta$ and $s' = t\mu t\alpha\beta t$, we have $s'\sigma \notin L(\widetilde{G})$. Hence, for all $s' \in B_{N_c}{}^{-1}(\Phi_{N_o}{}^{-1}(w))$ and $\sigma = \alpha$, Equation (6) holds. Similarly, we check that Equation (6) holds for all $s\sigma \in K$ and for all $\sigma \in \widetilde{\Sigma}$.

Since for all $s \in K$ and $st \in L(\widetilde{G})$, $st \in K$, the second part of network T-observability is also stisfied. Hence, K is network observable.

According to Theorem 1, the state-estimate-based predictive supervisor S_p is a solution. Let us construct it. We first construct the delayed observer of H, denoted by $H_{D,obs}^{N_o} = \{Y, \widetilde{\Sigma}_o, \zeta_H, y_0\}$, which is shown in Fig. 3.

TABLE I
$TR_H^{N_c}(y)$ AND $\gamma(TR_H^{N_c}(y))$ FOR DIFFERENT y.

y	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7
$TR_H^{N_c}(y)$	1,2,3	3,4,5,7	4,5,6,7	6,7,8	2,3,4,5,7	2,3,4,5,6,7,8	3,4,5,6,7,8	4,5,6,7,8
$\gamma(TR_H^{N_c}(y))$	$\emptyset$	$\emptyset$	$\emptyset$	$\{\lambda\}$	$\{\emptyset\}$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$

y	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}
$TR_H^{N_c}(y)$	5,6,7,8	5,6,7,8	6,7,8	8	6,7,8	7,8	8	7,8
$\gamma(TR_H^{N_c}(y))$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda\}$

For any observed string w, the state estimate $TE_H^{N_o}(w) = \zeta_H(y_0, w)$. Denote $\zeta_H(y_0, w) = y$. For all $y \in Y$, we calculate $TR_H^{N_c}(y)$ and $\gamma(TR_H^{N_c}(y))$ as in Table I.

$S_p(w)$ can then be determined from Fig. **??** and Table I. For example, if $w = tt\mu\alpha\beta$, we have $TE_H^{N_o}(w) = \zeta_H(y_0, w) = y_{13}$ and $TR_H^{N_c}(y_{13}) = \{7, 8\}$. we then have $\gamma(TR_H^{N_c}(y_{13})) = \{\lambda\}$. At state 7 or 8, t cannot be disabled/preempted. According to Equation (5),

$$S_p(w) = (\widetilde{\Sigma} - \gamma(TR_H^{N_c}(y_{13}))) \cup \widetilde{\Sigma}_{uc} \cup \{t\} = \{\alpha, \beta, \mu, t\}.$$

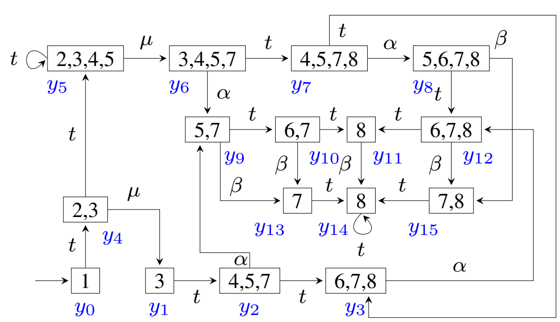

Fig. 3. The delayed observer $H_{D,obs}^{N_o}$

VI. Conclusion

In this paper, we investigate predictive supervisory control of timed discrete event systems under communication delays. Using timed discrete event systems allows us to consider communication delays in control and observation channels in terms of time explicitly. The main contributions are summarized as follows. (1) We develop a method to predict state estimates under control delays. (2) We derive a necessary and sufficient condition under which an admissible networked supervisor exists. (3) We propose an admissible state-estimate-based predictive supervisor which ensures that the timed discrete event system is safe.

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