Assignment 8

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Problem 2

Our goal is to identify the optimal supply S that minimizes your Expected Cost $g(S)$, given by the following:

$$g_1(S) = E[\max(x-S,0)] = \int_{-\infty}^{\infty} \max(x-S,0)\cdot f(x)\cdot dx = \int_{S}^{\infty} (x-S)\cdot f(x)\cdot dx$$
$$g_2(S) = E[\max(S-x,0)] = \int_{-\infty}^{\infty} \max(S-x,0)\cdot f(x)\cdot dx = \int_{-\infty}^{S} (S-x)\cdot f(x)\cdot dx$$
$$g(S) = p\cdot g_1(S) + h\cdot g_2(S)$$

$$g(S) = p\cdot g_1(S) + h\cdot g_2(S) = p\cdot \int_{S}^{\infty} (x-S)\cdot f(x)\cdot dx + h\cdot \int_{-\infty}^{S} (S-x)\cdot f(x)\cdot dx$$

Using integration by parts ($\int udv = uv - \int vdu$):

$$g(S) = p\cdot (x-S)F(x)\Bigg|_{x=S}^{x=\infty} - p\cdot \int_{S}^{\infty} F(x)dx + h\cdot (S-x)F(x)\Bigg|_{x=-\infty}^{x=S} + h\cdot \int_{-\infty}^{S} F(x)dx$$

We take derivative of $g(S)$ w.r.t S and equate to 0 (F.O.C):

$$-p + p\cdot F(S^*) + h\cdot F(S^*) = 0$$

$$\Rightarrow (p+h)F(S^*) = p \Rightarrow F(S^*) = \frac{p}{p+h}$$

$$\Rightarrow S^* = F^{-1}(\frac{p}{p+h})$$