Influential Sustainability with Forgetting Mechanism in Social Networks

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Previous Work

Influence Maximization Problem

Find the k most influential nodes and trigger them at the same time

- Myopic - companies aim at maximizing profit in the long run

Influence Sustainability Problem

Select influential nodes and proper activation time slots

- Seed set is given
- Simplified influence diffusion model - nodes can only go from inactive state to active state

Goal & Assumptions

Goal

- Extend influence model - consider forgetting property
- Seed selection - choose seeds and their activation time simultaneously

Assumptions

- Cycle-based graph - cycles are prevalent
- Links are all unidirectional
  - Twitter - one way following
  - Mutual trust is difficult to form (financial incentive, virtual environment)
  - People are often influenced by someone superior to them

Forgetting Model

Deterministic

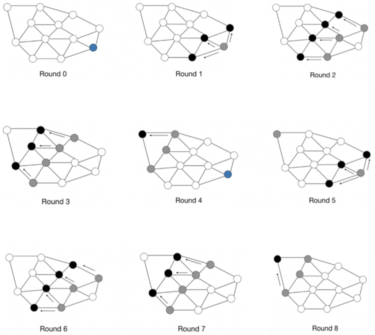
- $p(u, v) = 1$
- f_{avg}: number of rounds a node needs to forget something
- If v is activated at round t, v would still be in active state at round $t + i \ \forall i \in \{0, 1, ..., f_{avg} - 1\}$ and switch back to inactive state at round $t + f_{avg}$.

Stochastic

- $p(u, v) \in [0, 1]$
- p_f: probability a node would forget the product at the next round
- For each round t, if v is in active state at round t, v would go back to inactive state at round $t + 1$ with probability p_f.

Definition 1. Extended Independent Cascade Model (EIC)

- Based on Independent Cascade (IC) propagation model
- A node can be activated again once it becomes inactive
- Example - $f_{avg} = 2$, blue nodes: seeds, black nodes: newly activated nodes, gray nodes: active nodes, white nodes: inactive nodes



Definition 2. Stable Cycle (SC)

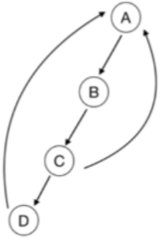
A cycle $[v_1, ..., v_{k-1}, v_k]$ appears as a stable cycle once in the propagation process if
$v_i \in NAN(t), v_{i+j} \in NAN(t+j)\ \forall j \in \{0, 1, ..., k-i\}$ and
$v_j \in NAN(t+k-i+j)\ \forall j \in \{1, ..., i\}$.

Definition 3. Permanently Stable Cycle (PSC)

A cycle $[v_1, ..., v_{k-1}, v_k]$ is a permanently stable cycle if for pair $(v_i, v_{i+1})\ \forall i \in \{1, 2, ..., k-1\}$ and (v_k, v_1), $v_{i+1}(v_1)$ would be activated at round $t+1$ if $v_i(v_k)$ is activated at round t in the propagation process.

Representations of Sustainability Example

- $f_{avg} = 1$, Seed = A



$C_1 : A - B - C$
$C_2 : A - B - C - D$

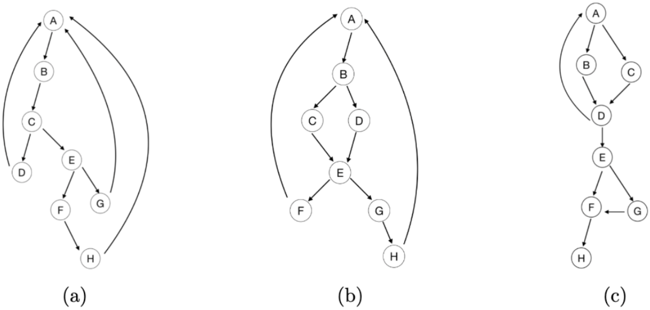
Round	NAN
0	A
1	B
2	C
3	AD
4	B
5	C
6	AD

Cycle-Based Graph

- Each node in the graph belongs to at least one cycle
- All links are unidirectional
- **Definiton 4**. Unique Node with Multiple In-Neighbors Cycle-Based Graph (UNMICB-G)
  - There is a unique node with multiple in-neighbors; the other nodes are with 1 in-neighbor
  - The unique node is an element of all cycles
- **Definition 5.** Multiple Nodes with Multiple In-Neighbors Cycle-Based Graph (MNMICB-G)
  - There are at least two nodes with multiple in-neighbors; the other nodes are with 1 in-neighbor
- MI-N: set of nodes with multiple in-neighbors
  UNMICB-G - $|\text{MI-N}| = 1$; MNMICB-G - $|\text{MI-N}| > 1$

Cycle-Based Graph Example

- (a) UNMICB-G; $|\text{MI-N}| = \{A\}$
- (b) MNMICB-G; $|\text{MI-N}| = \{A, E\}$
- (c) Not a cycle-based graph



Problem Formulation

- Find activating sequence $S = \{S_1, S_2, \ldots, S_t\}$ to solve the influential sustainability problem
- Node status:
  - Inactive
  - *NAN*
  - Active for stochastic scenario
  - $A(t,i),\ \ i = 1, 2, ..., f_{avg} - 1$ be the set of active nodes at round t which would switch into inactive state after i rounds for deterministic scenario
  - $V_s(t)$ stands for the status $\forall v \in V$ at round t with sequence of seed S.

Deterministic Scenario

- Activate all seeds at round 0 ($S = \{S_0\}, S_t = \emptyset \; \forall t > 0$)
- $NAN_s(t)$: the number of newly activated nodes at round t following the activating sequence S_0
- Study the problem for a period of time T ($T \gg 0$)
- If two seed sets both can let the propagation process last for T rounds, we then focus on how fast the propagation process enters into a steady state given a seed set.

Deterministic Scenario - Subproblem

Given a seed set S,

$$t_1^* = \min_{t_1, t_2} t_1 \qquad (P_1)$$

$$s.t.\ V_S(t_1) = V_S(t_2),\ t_1 < t_2,\ t_1,\ t_2 \in \{0, 1, ..., T\}$$

- The propagation process enters into a steady state sooner if t_1^* is smaller.
- If problem P_1 is infeasible given a seed set S, we force the objective function value to be T.

Deterministic Scenario - Main Problem

Define a function to consider the influence periods and stability simultaneously:

$$\varphi_T(S) = \sum_{t=0}^{T} 1_{\{x|x>0\}}(NAN_s(t)) + [1_{\{x|x>0\}}(NAN_s(T))] \cdot (T - t_1^*)$$

Definition 6. Given an interval of time T ($T \gg 0$) and a social network G, the target is to identify a seed set S_0 in order to maximize the sustainability and the stability of the influence spread. The objective function is as follows:

$$S_0^* = arg \max_{S_0} \varphi_T(S_0) \qquad (P_D)$$

Stochastic Scenario

- Propagation process may only last for several rounds because of the low propagation probability.
- Focus on the emergence of SC
- Identify a seed activating sequence S s.t. cycles appear as SC in the propagation process as many times as possible
- We define $SC_S(C)$ to be the times cycle C serves as an SC in the propagation process given the activating sequence S and let

$$\psi(S) = \sum_{C_i \in G} SC_S(C_i)$$

Stochastic Scenario - Main Problem

Definition 7. Given a budget of seed selection k and a social network G, the target is to identify an activating sequence $S = \{S_1, S_2, \ldots, S_t\}$ in order to maximize the times of the emergence of SC in the propagation process. The objective function is as follows:

$$S^* = arg\ \max_S \psi(S) \qquad (P_S)$$

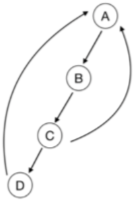
with $\sum_i |s_i| \leqslant k, s_i \in S^*$

Proposed Method - Deterministic

- f_{avg} must be less than all the length of cycles in the studied graph
- **Definition 8.** Sustainable Loop (SL)
  If $V_s(t) = V_s(t+h)$ for some $h > 0$, there is a sustainable loop $\{V_s(t), ..., V_s(t+1), ..., V_s(t+h-1)\}$ with h rounds with seed set S being activated round 0.
- If we detect an SL in the propagation process, node status would reiterate according to the pattern of SL as time goes by, i.e., $\{V_s(t), ..., V_s(t+h-1), V_s(t+h) = V_s(t), ..., V_s(t+h-1), V_s(t), ...\}$.

Sustainable Loop Example

- $f_{avg} = 2, S_0 = A, T = 7$
- $V_{\{A\}}(2) = V_{\{A\}}(5)$
- SL $= \{V_{\{A\}}(2), V_{\{A\}}(3), V_{\{A\}}(4)\}$ with 3 rounds



$C_1 : A - B - C$
$C_2 : A - B - C - D$

Round	NAN	$A(\cdot, 1)$	Inactive nodes
0	A	–	BCD
1	B	A	CD
2	C	B	AD
3	AD	C	B
4	B	AD	C
5	C	B	AD
6	AD	C	B
7	B	AD	C

Deterministic Scenario - UNMICB-G

- If we detect PSC or SL , we maintain the influence spread for a longer influence period.
- MI-N $= \{A\}$
- Focus on the rounds node A becomes NAN
- Goal: find a seed set s.t. one of the cycles in the UNMICB-G would be a PSC without interruption by the other cycles in the graph in the long run
- We divide all the UNMICB-G into two categories, the UNMICB-G composed of at least one cycle with a particular length form and others.

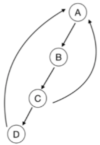
Deterministic Scenario - UNMICB-G

Proposition 1.

If there is a cycle C with length $n \cdot (f_{avg} + 1), n \in \mathbb{N}$ in the UNMICB-G , C would be a PSC by selecting nodes with $n \cdot (f_{avg} + 1), n \in \{0\} \cup \mathbb{N}$ hops from node A in UNMICB-G to be seeds at round 0.

Example

- $f_{avg} = 2$, $S_0 = \{A, D\}$, C_1 is a PSC
- SL $= \{V_{\{AD\}}(1), V_{\{AD\}}(2), V_{\{AD\}}(3)\}$



$C_1 : A - B - C$
$C_2 : A - B - C - D$

Round	NAN	$A(\cdot, 1)$	Inactive nodes
0	AD	–	BC
1	B	AD	C
2	C	B	AD
3	AD	C	B
4	B	AD	C
5	C	B	AD
6	AD	C	B
7	B	AD	C

- SL emerges and C_1 appears as a PSC faster in the propagation process.
- Solve problem P_D for UNMICB-G composed of at least one cycle with length form $n \cdot (f_{avg} + 1)$

Round	NAN	$A(\cdot, 1)$	Inactive nodes
0	A	–	BCD
1	B	A	CD
2	C	B	AD
3	AD	C	B
4	B	AD	C
5	C	B	AD
6	AD	C	B
7	B	AD	C

Round	NAN	$A(\cdot, 1)$	Inactive nodes
0	AD	–	BC
1	B	AD	C
2	C	B	AD
3	AD	C	B
4	B	AD	C
5	C	B	AD
6	AD	C	B
7	B	AD	C

Deterministic Scenario - UNMICB-G

- For the other cases in UNMICB-G, our solution would utilize SL and we focus on node A.
- Construct an SL based on a PSC in UNMICB-G
- Let $f'_{avg} = f_{avg} + 1$ and write the cycle length into the form $d_i = f'_{avg} \cdot n + m$ where $n, m \in \mathbb{N}$ and $0 \leqslant m < f'_{avg}$ (n is the multiple and m is the remainder)
- SL with $f'_{avg} \cdot n + m$ rounds would be constructed by leveraging a PSC with length $f'_{avg} \cdot n + m$ n times.

Deterministic Scenario - UNMICB-G

- Define an n dimensional activating interval vector $AIV^{n,m}$ to track the round interval of A becoming NAN
- $AIV^{n,m} = \underbrace{(f_{avg} + x_1, f_{avg} + x_2, ..., f_{avg} + x_{n-1}, f_{avg} + x_n)}_{n-dim}$

  where $\sum_{j=1}^{n} x_j = m, \ x_j \in \{0\} \cup \mathbb{N} \ \forall j$
- If the formulated SL is $\{V(t), V(t+1), ..., V(t+h-1)\}$ with $h = f'_{avg} \cdot n + m$, select $NAN(t) \subset V(t)$ as seeds at round 0.

Deterministic Scenario - UNMICB-G

Example

Construct a $3 \cdot 3 + 2$-round SL with $AIV_1^{3,2} = (2,3,3)$ and $AIV_2^{3,2} = (2,4,2)$ respectively.

SL Round	1	2	3
1	0	8	4
2	1	9	5
3	2	10	6
4	3	0	7
5	4	1	8
6	5	2	9
7	6	3	10
8	7	4	0
9	8	5	1
10	9	6	2
11	10	7	3

$AIV_1^{3,2} = (2,3,3)$

SL Round	1	2	3
1	0	8	3
2	1	9	4
3	2	10	5
4	3	0	6
5	4	1	7
6	5	2	8
7	6	3	9
8	7	4	10
9	8	5	0
10	9	6	1
11	10	7	2

$AIV_2^{3,2} = (2,4,2)$

Deterministic Scenario - UMNICB-G

- $AIV^{n,m} = (f_{avg} + x_1, f_{avg} + x_2, ..., f_{avg} + x_{n-1}, f_{avg} + x_n) = \underbrace{(f_{avg}, f_{avg}, ..., f_{avg}, f_{avg})}_{n-dim} + (x_1, x_2, ..., x_{n-1}, x_n)$
- $x^{n,m} := (x_1, x_2, ..., x_{n-1}, x_n);\ \sum_{j=1}^{n} x_j = m,\ \ x_j \in \{0\} \cup \mathbb{N}\ \forall j$

Method

- Choose a target cycle based on Least Common Multiple Method (LCM method)
- Determine $x^{n,m}$ based on Distribution Rule (DR)

Least Common Multiple Method (LCM Method)

- Let n_{lcm} be the least common multiple of $(n_1, n_2, ..., n_p)$
- Convert $f'_{avg} \cdot n_j + m_j$ to $f'_{avg} \cdot n_{lcm} + m'_j$, where $m'_j = \frac{n_{lcm}}{n_j} \cdot m_j$
- If $min\{m'_j\}_j = m'_r$, construct an SL based on C_r.
- Furthermore, if $gcd(n_r, m_r) \neq 1$, the SL constructed based on C_r is with $f'_{avg} \cdot \frac{n_r}{gcd(n_r,m_r)} + \frac{m_r}{gcd(n_r,m_r)}$ rounds.
- Utilize a cycle C_l with length $f'_{avg} \cdot n_l + m_l$ where $n_l = \frac{n_r}{gcd(n_r,m_r)}, m_l = \frac{m_r}{gcd(n_r,m_r)}$ to construct the SL

Example

- $f_{avg} = 7, f'_{avg} = 8$

$$
\begin{array}{c} f'_{avg} \cdot 3 + 2 \\ f'_{avg} \cdot 7 + 6 \\ f'_{avg} \cdot 9 + 4 \\ f'_{avg} \cdot 10 + 7 \\ f'_{avg} \cdot 12 + 7 \end{array}
\xRightarrow{lcm(3,7,9,10,12)=1260}
\begin{array}{c} f'_{avg} \cdot 1260 + 840 \\ f'_{avg} \cdot 1260 + 1080 \\ f'_{avg} \cdot 1260 + 560 \\ f'_{avg} \cdot 1260 + 882 \\ f'_{avg} \cdot 1260 + 735 \end{array}
$$

Distribution Rule (DR)

- $x^{n_l,m_l} := (x_1, x_2, ..., x_{n_l-1}, x_{n_l});\ \sum_{j=1}^{n_l} x_j = m_l,\ \ x_j \in \{0,1\}\ \forall j$
- Let $m_l = w \cdot n_l + r_l\ (r_l < n_l)$
  1. $w = 0, n_l - r_l > r_l$: $x^{n_l,m_l} = x^{n_l,r_l} = \left(\lfloor \frac{n_l}{r_l} \rfloor, 1\right) \odot \left(\lfloor \frac{n_l}{r_l} \rfloor + 1, 1\right) \odot \left(\lfloor \frac{n_l}{r_l} \cdot 2 \rfloor - \lfloor \frac{n_l}{r_l} \cdot 1 \rfloor, 1\right) \odot \left(\lfloor \frac{n_l}{r_l} \cdot 3 \rfloor - \lfloor \frac{n_l}{r_l} \cdot 2 \rfloor, 1\right) \odot ... \odot \left(\lfloor \frac{n_l}{r_l}(r_l - 1) \rfloor - \lfloor \frac{n_l}{r_l}(r_l - 2) \rfloor, 1\right)$
  2. $w = 0, n_l - r_l \leqslant r_l$: $x^{n_l,m_l} = x^{n_l,r_l} = \left(\lfloor \frac{1 \cdot n_l}{n_l - r_l} \rfloor, \lfloor \frac{1 \cdot n_l}{n_l - r_l} \rfloor - 1\right) \odot \left(\lfloor \frac{2 \cdot n_l}{n_l - r_l} \rfloor - \lfloor \frac{1 \cdot n_l}{n_l - r_l} \rfloor, \lfloor \frac{2 \cdot n_l}{n_l - r_l} \rfloor - \lfloor \frac{1 \cdot n_l}{n_l - r_l} \rfloor - 1\right) \odot ... \odot \left(\lfloor \frac{(n_l - r_l) n_l}{n_l - r_l} \rfloor - \lfloor \frac{(n_l - r_l - 1) n_l}{n_l - r_l} \rfloor, \lfloor \frac{(n_l - r_l) . n_l}{n_l - r_l} \rfloor - \lfloor \frac{(n_l - r_l - 1) n_l}{n_l - r_l} \rfloor - 1\right)$

3. $w \neq 0, n_l - r_l > r_l$: $x^{n_l,m_l} = x^{n_l,r_l} + \underbrace{(w, w, ..., w, w)}_{n_l - dim}$ where x^{n_l,r_l} is specified by 1.
4. $w \neq 0, n_l - r_l \leqslant r_l$: $x^{n_l,m_l} = x^{n_l,r_l} + \underbrace{(w, w, ..., w, w)}_{n_l - dim}$ where x^{n_l,r_l} is specified by 2.

Example

1. $n_l = 5, m_l = 2 \Rightarrow w = 0, n_l - r_l = 3 > 2 = r_l$
   $x^{n_l,m_l} = x^{n_l,r_l} = (1, 0, 1, 0, 0)$
2. $n_l = 5, m_l = 3 \Rightarrow w = 0, n_l - r_l = 2 \leqslant 3 = r_l$
   $x^{n_l,m_l} = x^{n_l,r_l} = (1, 0, 1, 1, 0)$
3. $n_l = 5, m_l = 7 \Rightarrow w = 1, n_l - r_l = 3 > 2 = r_l$
   $x^{n_l,m_l} = (1, 1, 1, 1, 1) + (1, 0, 1, 0, 0) = (2, 1, 2, 1, 1)$
4. $n_l = 5, m_l = 18 \Rightarrow w = 3, n_l - r_l = 2 \leqslant 3 = r_l$
   $x^{n_l,m_l} = (3, 3, 3, 3, 3) + (1, 0, 1, 1, 0) = (4, 3, 4, 4, 3)$

UNMICB-G Example

- $f_{avg} = 4, f'_{avg} = 5,\ C_l = f'_{avg} \cdot 9 + 4.$
- Possible shorter cycles vs. Possible longer cycles

Multiple	Remainder
8	4-7
7	4-7
6	3-7
5	3-7
4	2-7
3	2-7
2	1-7
1	1-7

Multiple	Remainder
9(0)	5-7(1-3)
10(1)	5-7(1-3)
11(2)	5-7(1-3)
12(3)	6-7(2-3)
13(4)	6-7(2-3)
14(5)	7(3)
15(6)	7(3)

- Check cycles with length $f'_{avg} \cdot 6 + 3,\ f'_{avg} \cdot 4 + 2,\ f'_{avg} \cdot 2 + 1$ would not break the SL constructed based on $f'_{avg} \cdot 9 + 4$; the other possible shorter cycles would not either since we suitably lower the remainder

- Suppose $x^{9,4} = (0, 1, 0, 0, 3, 0, 0, 0, 0)$

1	2(1)	3	4	5(3)	6
$f'_{avg} \cdot 9 + 4$	$f'_{avg} \cdot 8 + 4$	$f'_{avg} \cdot 7 + 3$	$f'_{avg} \cdot 6 + 3$	$f'_{avg} \cdot 5 + 3$	$f'_{avg} \cdot 4 + 0$
$f'_{avg} \cdot 5 + 4$	$f'_{avg} \cdot 4 + 4$	$f'_{avg} \cdot 3 + 3$	$f'_{avg} \cdot 2 + 3$	$f'_{avg} \cdot 1 + 3$	$f'_{avg} \cdot 9 + 4$

- DR $\Rightarrow x^{9,4} = (1, 0, 1, 0, 0, 1, 0, 1, 0)$

1(1)	2	3(1)	4	5	6(1)
$f'_{avg} \cdot 9 + 4$	$f'_{avg} \cdot 8 + 3$	$f'_{avg} \cdot 7 + 3$	$f'_{avg} \cdot 6 + 2$	$f'_{avg} \cdot 5 + 2$	$f'_{avg} \cdot 4 + 2$
$f'_{avg} \cdot 5 + 2$	$f'_{avg} \cdot 4 + 1$	$f'_{avg} \cdot 3 + 1$	$f'_{avg} \cdot 2 + 0$	$f'_{avg} \cdot 1 + 0$	$f'_{avg} \cdot 9 + 4$

- In SL table, at the rows with cycle length $f'_{avg} \cdot 9 + 4$ appearing, the remainder of multiple 6 would be either 3 or 2 ; the remainder of multiple 4 would be either 2 or 1, and the remainder of multiple 2 would be either 1 or 0.

UNMICB-G Seed Selection

- We previously select the NAN in SL Round 1 as seeds.
- We may also choose NAN in the other SL rounds to reach the same goal and further lower the number of seeds.
- Make sure node A does not emerge as NAN at the round which it is not supposed to be

UNMICB-G Seed Selection Example

- Cycle length $= 17$, $f_{avg} = 4$

SL Round	1	2	3
1	0	11	5
2	1	12	6
3	2	13	7
4	3	14	8
5	4	15	9
6	5	16	10
7	6	0	11
8	7	1	12
9	8	2	13
10	9	3	14
11	10	4	15
12	11	5	16
13	12	6	0
14	13	7	1
15	14	8	2
16	15	9	3
17	16	10	4

Cycle Length	Hop
$5 \cdot 1 + 1$	5
$5 \cdot 1 + 2$	6
$5 \cdot 1 + 3$	7
$5 \cdot 1 + 4$	8
$5 \cdot 2 + 2$	11
$5 \cdot 2 + 3$	12
$5 \cdot 3 + 3$	17(0)
$5 \cdot 3 + 4$	18(1)
$5 \cdot 4 + 3$	22(5)
$5 \cdot 4 + 4$	23(6)
$5 \cdot 5 + 4$	28(11)

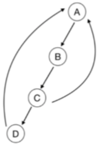
Algorithm 1 Select seeds for UNMICB-G

Input: d_i: cycle length $(i = 1, 2, ..., p)$; f_{avg}: forgetting round
Output: S: seed set (hops from node A)

for $i = 1$ to p **do**
  $n_i = \lfloor \frac{d_i}{f_{avg}+1} \rfloor$
  $m_i = d_i - (f_{avg} + 1) * n_i$
end for
compute $n_{lcm} = lcm(n_1, n_2, ..., n_p)$
for $i = 1$ to p **do**
  $m'_i = \frac{n_{lcm}}{n_i} * m_i$
end for
$r = \arg\min_i m'_i$
$n_l = \frac{n_r}{gcd(n_r, m_r)}$; $m_l = \frac{m_r}{gcd(n_r, m_r)}$
$s = 0$; $S = \varnothing$
set x^{n_l, m_l} an n_l dim vector following **(DR)**
$j = n_l$
while $s \leqslant \max_i (d_i - 1)$ **do**
  $S = S \cup \{s\}$
  $s+ = x_j^{n_l, m_l} + f_{avg} + 1$
  $j- = 1$
  if $j == 0$ **then**
    $j = n_l$
  end if
end while
return S

Deterministic Scenario - MNMICB-G

- Think about UNMICB-G first



$C_1 : A - B - C$
$C_2 : A - B - C - D$

Round	NAN	$A(\cdot, 1)$	Inactive nodes
0	AD	–	BC
1	B	AD	C
2	C	B	AD
3	AD	C	B
4	B	AD	C
5	C	B	AD
6	AD	C	B
7	B	AD	C

- Consider possible hop for nodes in MI-N
  - $A : \{0, 3, 4\}$; 3:A-B-C-A; 4: A-B-C-D-A

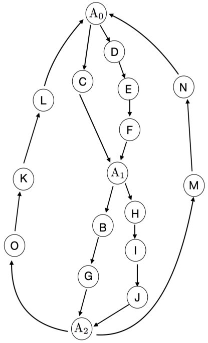
SL Round	Hop	Node
0	0 (3)	AD
1	1	B
2	2	C

  - Each node only take a role as a unique hop from A.

Deterministic Scenario - MNMICB-G

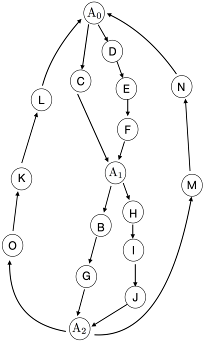
- The key is to find the unique hop for each node in MI-N.
- Consider a simpler case in the MNMICB-G. Each node in MI-N is component of all cycles.

Example



- WLOG, let A_0 be the starting point (hop 0)
- A_1 is with $\{2, 4\}$ hops from A_0.
- A_2 is with $\{3, 4\}$ hops from A_1.
- Let $f_{avg} = 6$, cycle:A_0-C-A_1-B-G-A_2-M-N would be a PSC.
  - A_1 is with 2 hops from A_0.
  - A_2 is with 3 hops from A_1.

MNMICB-G Example



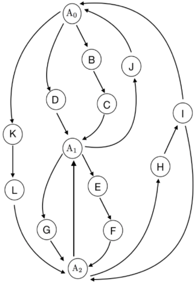
SL Round	Hop	NAN
1	0 8	A_0L
2	1	CD
3	2	A_1E
4	3	BFH
5	4	GI
6	5	A_2J
7	6	MO
8	7	KN

- A_1 can be activated by C and F, but F is invalid.
- A_2 can be activated by G and J, but J is invalid.

General MNMICB-G

- When choosing a cycle, some nodes in MI-N may not be covered.
- We need to recover that by adding some paths.

Example



- C_1 : A_0-D-A_1-J, A_2 is not covered.
  - p_1 : A_0-K-L-A_2
    $\{A_0 : 0, A_1 : 2, A_2 : 3\}$
  - p_2 : A_1-G-A_2
    $\{A_0 : 0, A_1 : 2, A_2 : 4\}$
  - p_3 : A_1-E-F-A_2
    $\{A_0 : 0, A_1 : 2, A_2 : 5\}$
- C_2 : A_2-H-I, A_0 & A_1 are not covered.
  - p_4 : $A_2 - A_1$ & p_5 : A_2-H-I-A_0
    $\{A_0 : 3, A_1 : 1, A_2 : 0\}$
  - p_6 : A_2-H-I-A_0-D-A_1
    $\{A_0 : 3, A_1 : 5, A_2 : 0\}$

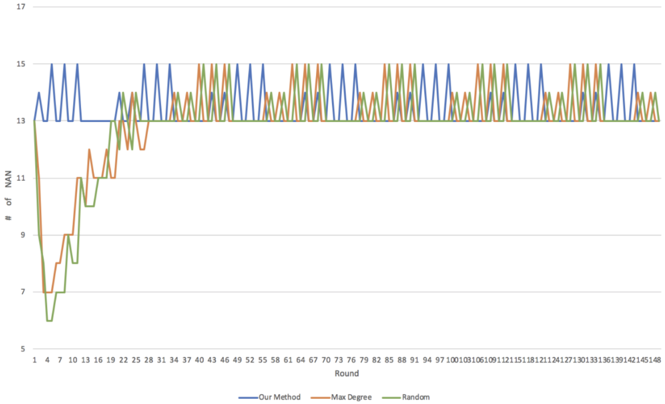
$\Rightarrow (C_1, \{p_1\}), (C_1, \{p_2\}), (C_1, \{p_3\}), (C_2, \{p_4, p_5\}), (C_2, \{p_6\})$

Stochastic Scenario

- Let rk_j be a measure for determining the priority of cycles we would utilize for solving our problem P_S
- Since propagation probability mostly dominates p_f in real-world, we define $rk_j = (\prod_{k=1}^{i-1} p(v_{j_k}, v_{j_{k+1}})) \cdot p(v_{j_i}, v_{j_1})$ for cycle $C_j = [v_{j_1}, v_{j_2}, ..., v_{j_i}]$.
- Denote cycle with highest rk_j as C_{1*}.
- Method: randomly selecting one node from C_{1*} and keep activating it until there is no budget
- $S = \{S_{t_0=0}, S_{t_1}, S_{t_2}, .., S_{t_{k-1}}\}$, t_j is the round when the propagation process $S_{t_{j-1}}$ triggers ends, i.e., for each node $v \in V \backslash \{S_{t_j}\}$, v is inactive at round t_j. And $S_{t_j} = v_{1*} \in C_{1*}\ \forall j \in \{0, 1, ...k-1\}$.

Experiments - Deterministic (UNMICB-G)

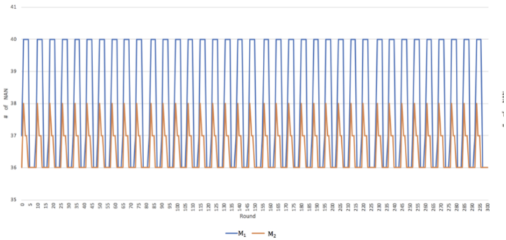
- UNMICB-G composed of 6 cycles with length 10,14,17,20,22,23
- $f_{avg} = 2$ and $T = 150$
- By LCM and DR, cycle with length $3 \cdot 7 + 1$ is selected and nodes with 0, 3, 6, 9, 12, 15, 18, 22 hops (13 nodes) are seeds.
- 3 methods: Our Method, Max Degree, Random

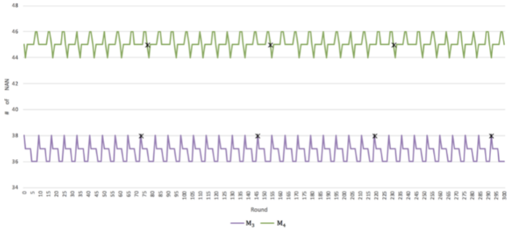


Same Multiple

- $f_{avg} = 7, \ T = 300$
- All cycles are with multiple 9.
- Denote $M = \{m_1, m_2, m_3, ..., m_j\}$ where $m_i \leqslant m_{i+1} \ \forall i \in \{1, 2, .., j-1\}$ to record the remainders
- Two cases
  1. $M_1 = \{1, 1, 3, 4, 4, 6, 6, 6, 6\}$ vs. $M_2 = \{1, 2, 3, 5\}$
     $\Rightarrow$ 73 vs.73 rounds
  2. $M_3 = \{1, 1, 1, 2, 2, 2, 2, 2, 6\}$ vs. $M_4 = \{5, 5, 5, 5, 6, 6, 6, 6\}$
     $\Rightarrow$ 73 vs.77 rounds

Same Multiple

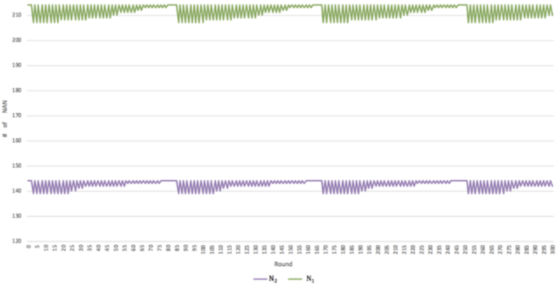


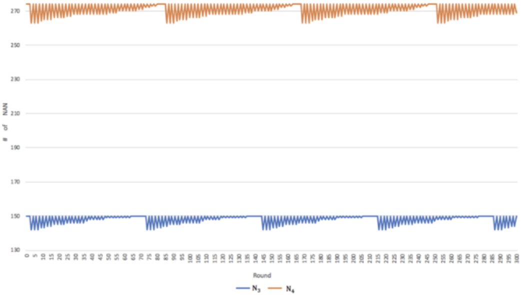


Same Remainder

- $f_{avg} = 1, \ T = 300$
- All cycles are with remainder 1.
- Denote $N = \{n_1, n_2, n_3, ..., n_j\}$ where $n_i \leqslant n_{i+1} \ \forall i \in \{1, 2, .., j-1\}$ to record the multiples.
- Two cases
  1. $N_1 = \{8, 16, 23, 25, 30, 32, 39, 41\}$ vs. $N_2 = \{11, 13, 15, 27, 37, 41\} \Rightarrow$ 83 vs.83 rounds
  2. $N_3 = \{7, 11, 15, 23, 35, 37, 47, 63, 71\}$ vs. $N_4 = \{7, 9, 15, 23, 27, 49, 55, 67, 71, 75, 79, 83\} \Rightarrow$ 143 vs.167 rounds

Same Remainder

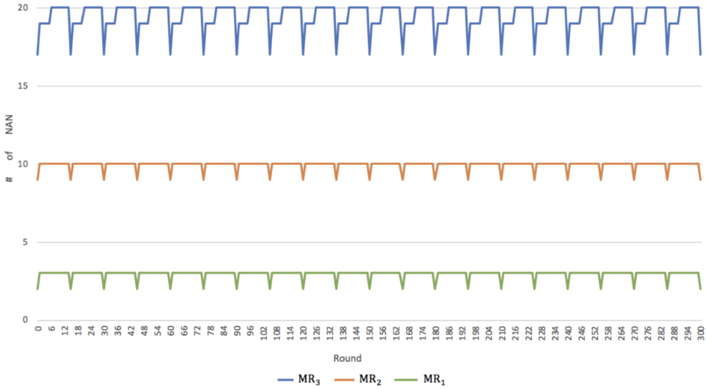




Greatest Common Divisor of Remainder and Multiple

- $f_{avg} = 12,\ T = 300$
- Three UNMICB-Gs
  - $MR_1 = \{(1,2),(2,4)\}$
  - $MR_2 = \{(3,6),(4,8),(4,8),(5,10),(5,10)\}$
  - $MR_3 = \{(1,2),(2,4),(2,4),(3,6),(4,8),(5,10),$ $(5,10),(5,10),(6,12),(6,12)\}$
- If we divide all the pairs by their greatest common divisor, all pairs would be $(1,2)$.
- SL of all three graphs are with 15 rounds.

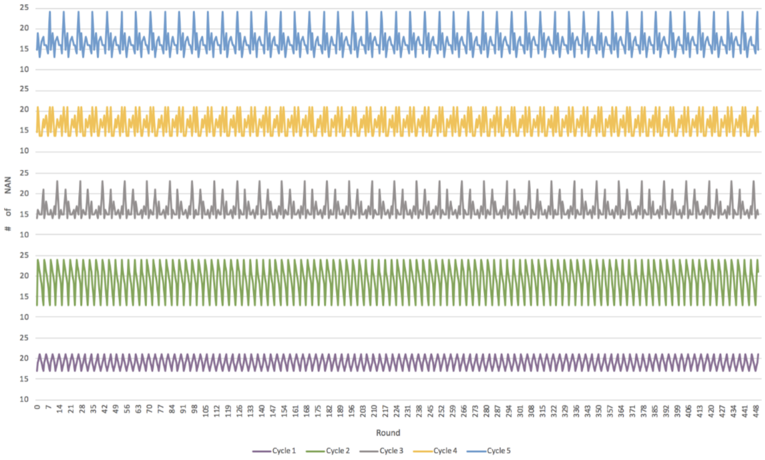
Greatest Common Divisor of Remainder and Multiple



Experiments - Deterministic (MNMICB-G)

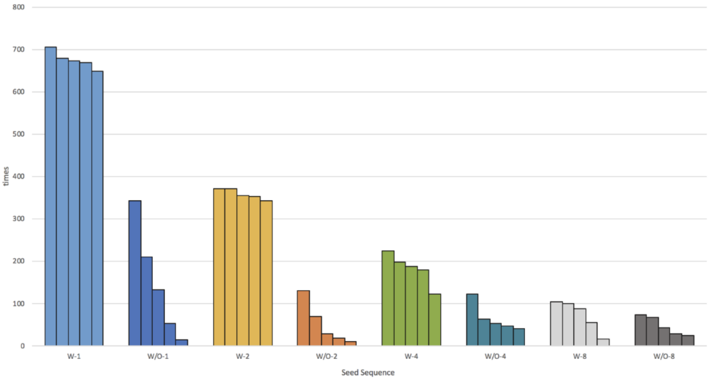
- 76 nodes, 104 edges, 1621 cycles
- $|\text{MI-N}| = 16$
- $f_{avg} = 3$ and $T = 450$
- By our method, we found that there are 98 cycles can be PSCs and successfully be based for formulating SL
- We solve the problem P_D with an optimal objective function value 451+(450-2)=899.

Experiments - Deterministic (MNMICB-G)



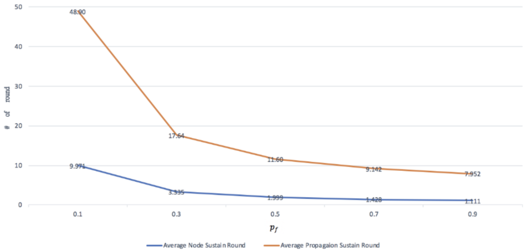
Experiments - Stochastic

- 76 nodes, 104 edges, 1621 cycles, $|\text{MI-N}| = 16$, $p_f = 0.5$, $k = 8$ and simulation times $= 10000$



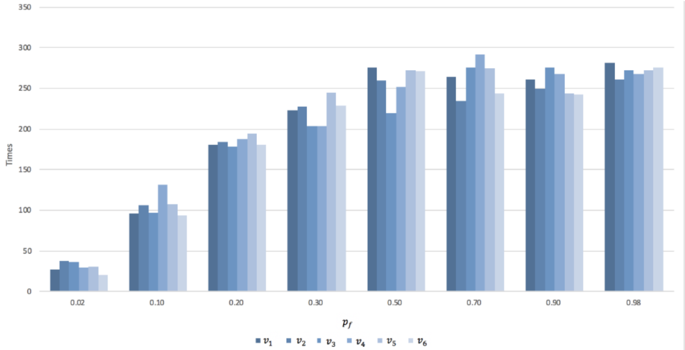
Forgetting Probability (p_f)

- If the forgetting probability is low, the propagation process ($\exists v \in V, v$ is active) would last for longer period and node would be active for more rounds.



Forgetting Probability (p_f)

- for SC to appear, node should switch into inactive state before a specific time which is more easily to be achieved if p_f is higher.
- Note that if p_f is high enough (0.50 in this case), the factor of p_f becomes less important when solving problem P_S.



Future Work

- Looser restrictions of graph structure
- f_{avg} can be different for $v \in V$