

2 Price Regulation

Practice Question 3. Suppose a firm has a cost curve equal to

$$C = 600 + 10Q$$

Then the marginal cost is $MC = 10$ and the average cost is $AC = 600/Q + 10$. Suppose the inverse demand curve is given by

$$p = 400 - 5Q$$

- (a) If regulators set the price equal to the marginal cost, what would be the firm's loss?
- (b) If regulators set the price equal to the average cost, what would be the price? What would be the deadweight loss associated with average cost price regulation?

Solutions:

- (a) If regulators set the price equal to the marginal cost, then

$$p = MC$$

$$\Rightarrow 400 - 5Q = 10$$

$$\Rightarrow Q^* = 78, \quad p^* = 10$$

Therefore,

$$\pi = p^*Q^* - C = 78 \times 10 - (600 + 10 \times 78) = -600$$

- (b) If regulators set the price equal to the average cost, then

$$p = AC$$

$$\Rightarrow 400 - 5Q = \frac{600}{Q} + 10$$

$$\Rightarrow Q^* = 76.4, \quad p^* = 18$$

(Here when you solve the quadratic equation above, you will get two solutions. You may wonder why we only choose $Q = 76.4$. The answer to this is very simple: as a regulator, you would of course choose a lower price and a corresponding larger output rather than a much higher price.)

Then the deadweight loss associated with this average cost price regulation is

$$DWL = \frac{(78 - 76.4)(18 - 10)}{2} = 9.6$$

Practice Question 4. The inverse demand curve a monopoly faces is $p = 30 - Q$. The firm's total cost function is $C(Q) = 0.5Q^2$ and thus marginal cost function is $MC(Q) = Q$.

- (a) Determine the monopoly quantity, price and profit, and calculate the CS, PS and social welfare under the monopoly.
- (b) Determine the socially optimal outcome and calculate the CS, PS and social welfare under the social optimum.

- (c) Calculate the deadweight loss due to the monopolist behaviour of this firm.
- (d) Assume that the government puts a price ceiling on the monopolist at $P = 18$. Determine and plot the demand and marginal revenue function of the monopoly under this regulation. How much output will the monopolist produce? What will be the profit of the monopolist? Determine the impact of this price ceiling on the deadweight loss.
- (e) Assume that the government put a price ceiling on the monopolist in order to maximize the total (i.e. consumer plus producer) surplus. What price ceiling should it choose? How much output will the monopolist produce at this price ceiling? What will the profit of the monopolist be? What is the DWL?

Solutions:

- (a) The marginal revenue function for the monopoly is

$$MR = 30 - 2Q$$

The monopoly will set

$$MR = MC$$

$$\Rightarrow 30 - 2Q = Q$$

$$\Rightarrow Q^m = 10, \quad p^m = 30 - 10 = 20$$

Then the monopoly profit will be

$$\pi^m = p^m Q^m - C(Q^m) = 20 \times 10 - 0.5(10)^2 = 150$$

At the monopoly output $Q^m = 10$ and monopoly price $p^m = 20$, the CS and PS are

$$CS^m = \frac{(30 - 20) \times 10}{2} = 50$$

$$PS^m = \frac{10 \times 10}{2} + (20 - 10) \times 10 = 150 = \pi^m$$

The social welfare under the monopoly is

$$W^m = CS^m + PS^m = 50 + 150 = 200$$

- (b) The socially optimal outcome will be

$$p = MC$$

$$\Rightarrow 30 - Q = Q$$

$$\Rightarrow Q^* = 15, \quad p^* = 15$$

The CS and PS under the social optimum are

$$CS^* = \frac{(30 - 15) \times 15}{2} = 112.5$$

$$PS^* = \frac{15 \times 15}{2} = 112.5$$

The social welfare under the social optimum is thus

$$W^* = CS^* + PS^* = 225$$

(c) so the deadweight loss of the monopoly is

$$DWL = W^* - W^m = 225 - 200 = 25$$

Alternatively, if you don't calculate the CS, PS and W under the monopoly and social optimum, you can directly obtain the deadweight loss of the monopoly

$$DWL = \frac{(p^m - MC(Q^m))(Q^* - Q^m)}{2} = \frac{(20 - 10)(15 - 10)}{2} = 25$$

(d) With a price ceiling at $p = 18$, the demand function becomes

$$p = \begin{cases} 18 & \text{if } 0 \leq Q \leq 12 \\ 30 - Q & \text{if } 12 \leq Q \leq 30 \end{cases}$$

(How do you get $Q = 12$? Set $p = 30 - Q = 18 \Rightarrow Q = 12$.)

The marginal revenue thus becomes

$$MR = \begin{cases} 18 & \text{if } 0 \leq Q \leq 12 \\ 30 - 2Q & \text{if } 12 < Q \leq 30 \end{cases}$$

Notice that there will be a jump at $Q = 12$. Since at $Q = 12$, $MR(12) = 30 - 2(12) = 6 < MC(12) = 12 < 18$, MC is passing through the jump of the MR and therefore the monopoly will produce at $Q^r = 12$ and charge a price at $p^r = 18$. (Intuitively, you can think that the monopoly can't charge its optimal price at $p^m = 20$ due to the price ceiling, and thus it will choose the maximum price it is allowed to charge: $P = 18$.)

The resulting monopoly profit is

$$\pi^r = p^r Q^r - C(Q^r) = 18 \times 12 - 0.5(12)^2 = 144$$

The deadweight loss under the price ceiling is

$$DWL = \frac{(p^r - MC(Q^r))(Q^* - Q^r)}{2} = \frac{(18 - 12)(15 - 12)}{2} = 9 < 25$$

The DWL has thus decreased.

(e) We know from part (b) that the socially optimal price is 15. If the monopoly is to sell at this price, then the total surplus will be maximized and there will be no deadweight loss. Thus the government has to set the price ceiling equal to

$$p = p^* = 15$$

The monopoly will produce at

$$Q = 30 - 15 = 15$$

Then its profit will be

$$\pi = 15 \times 15 - 0.5(15)^2 = 112.5$$

The DWL will be totally eliminated at this optimal price regulation

$$DWL = 0$$