On the Profitability of Cross-ownership in Cournot Nonrenewable Resource Oligopolies: Stock Size Matters

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Abstract

We examine the profitability of cross-ownership in a nonrenewable resource oligopolistic industry where firms compete as Cournot rivals. Assuming a subset of the oligopolists own a share in each other's profits, we show that a symmetric cross-ownership can be profitable for any number of participating firms, provided that the initial resource stock owned by each firm is small enough. This is in sharp contrast with the static case where for any levels of non-controlling minority shareholdings, a symmetric cross-ownership is never (always) profitable if the relative number of participating firms is below (above) some lower (upper) threshold. When the relative number of participating firms is in between the two thresholds, profitability of cross-ownership depends on the level of shareholdings. We also highlight that cross-ownership can be preferable to a horizontal merger in terms of Cournot competition. Not only is it more profitable to do so, more importantly, it constitutes a shrewd strategy to avoid the possible legal challenges. Finally, we show that cross-ownership may turn out to be relatively less detrimental to society in a nonrenewable resource industry than other industries where resource constraints are absent. Thus, a specific analysis is needed when dealing with industries where resource constraints play an important role.

Keywords: Cross-ownership, Profitability, Oligopoly, Shareholdings, Nonrenewable Resources, Resource Stock, Horizontal Merger, Competition Policy, Antitrust

JEL Codes: L13, L41, Q3

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1 Introduction

Nonrenewable resource industries have experienced intense and widely documented cross-ownership activities mainly through partial acquisitions and joint ventures.[1] As noted by Kumar (2012) and Benchekroun, Breton and Chaudhuri (2019), the volume of mergers and acquisitions has been historically and consistently much higher in the exhaustible sector than others. Many joint ventures exist in the nonrenewable resource sector, as firms often jointly own and/or develop a mine. For instance, in the global oil and gas industry, the top six multinational oil companies, i.e., ExxonMobile, British Petroleum (BP), Royal Dutch Shell, Chervon, Total and Eni, are more closely interconnected with each other than would be expected.[2] According to a report by Water Street Partners based on the source from Rystad Energy,[3] intriguingly large amounts of supermajor-to-supermajor joint-ventures exist in the production stage, let alone other stages such as exploration, refining, distributing and retailing. We seek to understand the incentives of rival firms to participate in cross-ownership and the levels of cross-shareholdings that will be profitable in nonrenewable resource industries. This will require investigating how ownership links between any rival firms may affect the use of a nonrenewable resource and whether increased cross-ownership will give rise to increased market power.

To better understand the role played by resource constraints, we start by examining the profitability of cross-ownership in a static game since it has not been addressed even in this benchmark framework.[4] Specifically, we consider a k-symmetric cross-ownership structure in an n-firm Cournot homogeneous-product model where a subset of $k \leq n$ firms engage in rival cross-shareholdings and each firm has an equal silent financial interest in the other firms, while the remaining $n - k$ firms stay independent. By examining the profitability of cross-ownership, we show that for any levels of non-controlling minority shareholdings, a k-symmetric cross-ownership is never profitable if the number of participating firms is below some lower threshold, but always profitable when the number of participating firms is above some upper threshold. When the number of participating firms is between these thresholds, the profitability of cross-ownership depends on the value of stakes that each

[1]When firms form a joint venture, it is usually majority-owned and operated by one firm and minority-held by the others. This translates into mutual shareholdings of one firm in another.

[2]Other notable examples include: BP holds a 19.75% stake in the Russian oil giant Rosneft; the Mexican state-owned petroleum company Pemex holds a 9.3% stake in the Spanish oil giant Repsol; China's state-owned Sinopec holds a 30% stake in Petrogal Brasil, and 40% in Repsol YPF Brasil, respectively.

[3]See https://www.waterstreetpartners.net/blog/the-web-of-partnerships-between-bp-chevron-eni-exxonmobil-shell-and-total.

[4]Viewing cross-ownership as "partial mergers", previous studies have focused mainly on the potential anticompetitive effects induced by cross-ownership, i.e., unilateral effects (Reynolds and Snapp, 1986; Bresnahan and Salop, 1986; Farrell and Shapiro, 1990; Flath, 1991, 1992; O'Brien and Salop, 2000; Dietzenbacher, Smid and Volkerink, 2000; Brito, Cabral and Vasconcelos, 2014; Brito, Ribeiro and Vasconcelos, 2014; Brito et al., 2018) and coordinated effects (Malueg, 1992; Gilo, Moshe and Spiegel, 2006; Brito, Ribeiro and Vasconcelos, 2018), and have thus proposed various modified measurement indexes—the Herfindahl-Hirschman Index and the Gross Upward Price Pressure Index—to account for it. However, none have addressed the issue of profitability of cross-shareholdings.

firm involved in cross-ownership holds in the other firms. Cross-ownership is then profitable only when the stakes are below a certain threshold. This result seems surprising as one would naturally think it should be always profitable for firms to participate in cross-ownership due to a less intensified competition. We thus define this result as a cross-ownership paradox, analogous to the merger paradox. In general, firms have no incentive to engage in cross-shareholdings if less than 50% of the firms in the industry participate. However, beyond that participation ratio, for example, with $n = 10$ and $k = 6$, cross-ownership is profitable provided that each of the 6 firms holds no more than 6.5% of the shares of any other firm; with $n = 9$ and $k = 6$, cross-ownership is profitable provided that each of the 6 firms holds no more than 12.5% of the shares of any other firm; and with $n = 8$ and $k = 6$, cross-ownership is profitable provided that each of the 6 firms holds less than 17.6% of the shares of any other firm. Moreover, cross-ownership is always profitable for any non-controlling minority shareholdings if more than 80% of the firms participate. Thus a k-symmetric cross-ownership is more likely to be profitable with lower levels of shareholdings for a lower participation ratio. The main intuition behind the result can be explained by cross-ownership theory and oligopoly theory. When a firm acquires a partial ownership stake in a rival, it has an incentive to compete less aggressively and thus unilaterally reduce its output. A larger shareholding by the firms that engage in the symmetric cross-ownership will induce them to further reduce output, triggering a more aggressive response by the outsiders in terms of strategic substitutes in Cournot competition. The increase in both the number and output of the outsiders more than offsets the benefit the cross-owners can receive from their reduction of output, thereby reducing the profitability of cross-ownership.

We show that the conclusions reached in the static benchmark above may not be extendable to the case of nonrenewable resource oligopolies. The output of each resource extracting firm, i.e., their cumulative extraction over time, is constrained by their limited initial resource stocks. As a result, current extraction and production affect the availability of reserves for future extraction and production (Hotelling, 1931). To capture the specificity of the nonrenewable resource sector, we use a dynamic game model in which firms compete *à* la Cournot while each firm faces a resource stock constraint. We use a continuous time framework with an endogenous time horizon. Following much of the existing literature on oligopoly models of nonrenewable resource markets (Salant, 1976; Lewis and Schmalensee, 1980; Loury, 1986; Benchekroun, Halsema and Withagen, 2009, 2010; Benchekroun, Breton and Chaudhuri, 2019), we adopt the open-loop strategies by which firms commit to a fixed time path of extraction. We acknowledge that open-loop Nash equilibrium (OLNE) is only time-consistent but not necessarily subgame perfect.[5] If firms have all the information about its own and competitors' stocks at any future dates, they would be able to adjust their production at each instant

[5] See Chapter 4 in Dockner et al. (2000) for more details.

of time, i.e., use closed-loop or Markov strategies. However, there are several reasons to justify the use of OLNE as noted in Benchekroun, van der Meijden and Withagen (2019). The first justification is the analytical tractability, as one has to resort to numerical methods to characterize a closed-loop equilibrium, but such methods suffer from the curse of dimensionality. The second is the prevalence of long-term contracts in nonrenewable resource markets so that actual extraction rates do not only depend on the actual resource stocks but also from the pre-committed supplies. Finally, requiring information on the vector of stocks at each moment can be quite unrealistic given the difficulty to gather that information. We then characterize an open-loop Nash-Cournot cross-ownership equilibrium (OL-NCOE) of the game and investigate the profitability of a k-symmetric cross-ownership in this context. We find that a k-symmetric cross-ownership can be profitable even when the participation ratio is below the lower threshold and is always profitable when above the lower threshold, provided that the initial resource stock owned by each firm is small enough. Moreover, the profitability increases in levels of cross-ownership when resource stock owned by each firm is small. This result sharply contrasts with the static model in which lower levels of cross-ownership seem more profitable. Unlike the static model in which outsiders respond to any increased shareholdings between cross-owners by aggressively increasing output and mitigating the cross-ownership participants' gain in market power, the limited resource stocks restrict the outsiders in their response. Consequently, when the stock is sufficiently small, a higher level of cross-ownership will generate a higher profitability.

In addition, a k-symmetric cross-ownership results in a slower extraction rate for the industry and induces the outsiders to exhaust their stocks earlier than the cross-ownership participants at any resource stock level. These findings indicate that the degree of concentration in supply will increase over time, and a group of cross-owners will eventually supply the resource before exhaustion. This result resembles the 'oil'igopoly theory (Loury, 1986; Polasky, 1992), which predicts that small firms will exhaust their stocks before large firms do, leading possibly to eventual monopolization of the market. The increased concentration over time induced by cross-ownership confers market power on those cross-owners. As such, the cross-ownership participants can raise prices more than in other industries without stock constraint, which provides an additional incentive to look at the exhaustible sector differently.

Our paper also contrasts cross-ownership with horizontal mergers. One of the seminal works in the literature is arguably the paper by Salant, Switzer and Reynolds (1983), who show that the seemingly profitable mergers between competing firms in the same industry can be unprofitable, which is known as the merger paradox. More specifically, when firms compete à la Cournot in an oligopolistic industry with linear demand and constant marginal cost of production, horizontal mergers are not profitable unless at least 80% of the industry participates in the merger. Since cross-ownership is often referred to as "partial mergers", one may wonder why firms do not engage in a full merger in the first place, as a merger totally eliminates the previous rivalry and can pool resources more efficiently. Foros, Kind and Shaffer (2011) answer this question by showing that in a spatial Salop 3-firm Bertrand model with differentiated products, the profitability of a partial cross-ownership that gives the acquirer corporate control over all pricing decisions could be much higher than that of a full merger because a partial ownership arrangement can greatly lessen competition when the firms' choices are strategic complements. Stühmeier (2016) extends their 3-firm setting with four or more firms, only to find that firms prefer a merger to a partial acquisition, because both neighbors to the entity respond differently to the acquisition. Thus he concludes that whether partial acquisition is preferable to a merger is sensitive to the intensity of competition in the market. However, these papers only consider Bertrand competition whereas there are numerous industries in which firms compete in a way that is more consistent with Cournot competition. Using models with price competition to investigate quantity competition would often end up with unreliable results and give misleading policy implications. Our paper thus provides a possible explanation as to why cross-ownership is preferable to a full merger in terms of Cournot competition. For example, as indicated earlier, when $k = 6$ and $n = 10$, cross-ownership is profitable provided that each of the 6 firms holds no more than 6.5% of the non-controlling minority shares of any other firm, while a horizontal merger of 6 firms is unprofitable.

This result also bears some practical considerations from a company's corporate strategy point of view. Not only is participating in cross-ownership more profitable than a horizontal merger, but—more importantly—it constitutes a "smart" way to avoid the possible legal challenges. While horizontal mergers are subject to substantial antitrust scrutiny and are often opposed by antitrust authorities, non-controlling minority shareholdings are either granted a de facto exemption from antitrust liability or have gone unchallenged by antitrust agencies (Gilo, 2000; Gilo, Moshe and Spiegel, 2006). Indeed, Nain and Wang (2018) document that fewer than 1% of the minority acquisitions are challenged by the Federal Trade Commission (FTC) or the Department of Justice (DOJ), and even fewer are blocked outright. Antitrust authorities of the European Union (EU) do not even have competence to investigate such cases.[6] As noted by Jovanovic and Wey (2014), in many merger cases, the acquiring firm often proposes a passive cross-ownership in the target firm before a full merger. This two-step covert takeover strategy has two central benefits: first, it evades merger scrutiny when antitrust authorities often give the green light to non-controlling minority shareholdings; second, it achieves the eventual

[6]It should be noted that "Articles 101 and/or 102 TFEU may apply to passive minority shareholdings in situations where there is evidence of an anticompetitive agreement or concerted practice among the investigated firms or the firms that are engaged in the acquisition of non-controlling stakes and/or one or more firms have a dominant position" (Fotis and Zevgolis, 2016). But European Commission also acknowledged its limited ability to use these Articles to intervene against minority shareholdings in the 2013 Consultation Paper and therefore does not cover all types of anti-competitive minority interests.

goal of a full acquisition on the basis of increasing consumer surplus approved by antitrust authorities. Therefore, firms may view cross-ownership as a more attractive corporate strategy, further explaining why firms want to engage in cross-shareholdings. Our analysis thus suggests that competition authorities should adapt their current lenient approach towards minority shareholdings to a stricter regulation.

In the absence of any possible efficiency gains, passive cross-shareholdings result in a welfare loss, and thus competition authorities should rule against them in accordance with a total surplus criterion.[7] However, when competition authorities need to make the tradeoffs between the possible efficiency gains and the welfare loss brought by cross-ownership, they should be cautious when ruling in the nonrenewable resource sector. As when the resource stock owned by each firm is small enough, cross-ownership results in a relatively smaller welfare loss than in a static Cournot oligopoly. This is because a group of cross-owners will monopolize the market after the outsiders deplete their resource stocks. As such, they can substantially raise the price, which slows down resource extraction and extends the date of exhaustion. As the resource becomes increasingly scarce, the extended periods of the use of the resource partially offset the negative effect of the higher price on social welfare.

The remainder of the paper is structured as follows. Section 2 presents first a static model used as a benchmark and then the dynamic model of a nonrenewable resource industry. Section 3 analyzes the profitability of cross-ownership. Section 4 provides a welfare analysis. Section 5 conducts a comparative static analysis. Finally, Section 6 concludes with the summary of our findings.

2 The Model and Preliminary Analysis

2.1 The Static Model

We consider an n-firm oligopolistic industry where firms compete $\grave{a}$ la Cournot. Demand is linear and given by $p = a - b\sum_{j=1}^{n} q_j = a - bQ$, where p is the market price and q_j is the output produced by firm j. Marginal costs are constant and identical across all firms, denoted by c with $a > c$. Assume that a subset of k firms ($2 \leq k \leq n$) engage in rival cross-shareholdings[8] and each firm has an equal silent financial interest in the other firms, while the remaining $n - k$ firms stay independent. Denote the set of firms as $J = \{1, 2, \cdots, n\}$, indexed by j, and use the subsets $I = \{1, 2, \cdots, k\}$, indexed

[7] We have assumed absence of efficiency gains throughout the paper to highlight the market power effect of cross-ownership.

[8] In an industry characterized by rival cross-shareholdings, the aggregate profits of a firm j include not only the stream of profits generated by the firm from its own operations, but also a share in its competitors' aggregate profits due to its direct and indirect ownership stakes in these firms (Flath, 1992; Gilo, Moshe and Spiegel, 2006). The aggregate profits can be interpreted as the accounting profits or the taxable profits of firm j. For example, say, if the corporate tax rate is 20%, then firm j must pay the government a tax amount of $0.2\Pi_j$.

by i and $O = \{k+1, \cdots, n\}$, indexed by o, referring, respectively, to the insiders and the outsiders to the cross-ownership. Then firm j's problem can be expressed as

$$\max_{q_j \geq 0} \Pi_j = \pi_j + v \sum_{i \neq j} \Pi_i = (p-c)q_j + v \sum_{i \neq j} \Pi_i$$

where $\pi_j = (p-c)q_j$ denotes firm j's operating profit and $v \geq 0$ represents firm j's fractional shareholdings in firm i for any $i \neq j$. Let $\boldsymbol{\Pi}$ and $\boldsymbol{q}$ denote the $n \times 1$ vectors of aggregate profits and outputs, respectively, and $\boldsymbol{D}$ denote the $n \times n$ cross-shareholding matrix, then the aggregate profit functions can be expressed in matrix form as

$$\boldsymbol{\Pi} = (p-c)\boldsymbol{q} + \boldsymbol{D}\boldsymbol{\Pi}.$$

Under the k-symmetric cross-ownership structure, $\boldsymbol{D} = \begin{bmatrix} \boldsymbol{A}_{kk} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{n-k} \end{bmatrix}$, where $\boldsymbol{A}_{kk}$ is a $k \times k$ matrix with element 0 in the diagonal and v off-diagonal. This set of n equations implicitly defines the aggregate profit for each firm. Then $\boldsymbol{I} - \boldsymbol{D} = \begin{bmatrix} \boldsymbol{B}_{kk} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{n-k} \end{bmatrix}$, where $\boldsymbol{B}_{kk}$ is a $k \times k$ matrix with element 1 in the diagonal and $-v$ off-diagonal, and $\boldsymbol{I}_{n-k}$ denote the $(n-k) \times (n-k)$ identity matrix. We make the following assumption:

Assumption 1. *Each firm seeks to maximize the value of its aggregate profits, but controls only its own production q_j, with rival shareholdings $v < \frac{1}{k-1}$, i.e., firms only have a silent financial interest or non-controlling minority stake in the rivals.*

Similar restriction can be found in Gilo, Moshe and Spiegel (2006) where the weight given to rivals' profits is bounded from above by $1/(n-1)$ when $k = n$. Assumption 1 guarantees that the aggregate stake of rivals in each cross-ownership participant, $(k-1)v$, is less than 1.[9] Under Assumption 1, matrix $(\boldsymbol{I} - \boldsymbol{D})$ is invertible,[10] which implies that it is possible to solve for the aggregate

[9] We don't allow v to be equal to $1/(k-1)$. The reason is that from the firm' corporate governance perspective, it makes little sense if each of the other $k-1$ firms holds a $1/(k-1)$ share of the k-th firm while the k-th firm can still make its own independent decision. It should also be noted that the k-firm merger outcome can be achieved with k-symmetric cross-ownership when $v = 1/(k-1)$, as if firms are maximizing the industry profits in that case.

[10] This follows from the properties of "Dominant Diagonal Matrices" (see, e.g.,Takayama (1985), Mathematical Economics, Cambridge University Press, page 381). According to Theorem 4.C.1 of that book, if an $n \times n$ matrix A has a dominant diagonal, then A^{-1} exists, where an $n \times n$ matrix A is said to have a dominant diagonal if there exists positive numbers $d_1, d_2, \cdots, d_n$ such that, for each j, we have

$$d_j |a_{jj}| > \sum_{i \neq j} d_i |a_{ij}|.$$

Clearly the matrix $\boldsymbol{I} - \boldsymbol{D} = A$ has a dominant diagonal because $a_{jj} = 1$ and $\sum_{i \neq j} a_{ij} < 1$.

profit functions:

$$\mathbf{\Pi} = (\boldsymbol{I} - \boldsymbol{D})^{-1}(p-c)\boldsymbol{q} = \begin{bmatrix} \boldsymbol{B}_{kk}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{n-k} \end{bmatrix} (a-c-bQ)\boldsymbol{q},$$ [11]

where $\boldsymbol{B}_{kk}^{-1}$ is given by the following matrix

$$\Omega \equiv \frac{1}{f(v)} \begin{bmatrix} 1-(k-2)v & v & \cdots & v \\ v & 1-(k-2)v & \cdots & v \\ \vdots & \vdots & \ddots & \vdots \\ v & v & \cdots & 1-(k-2)v \end{bmatrix}$$

with $f(v) = (1+v)\big(1-(k-1)v\big) > 0$. The aggregate profit function of firm $i \in I$ is

$$\Pi_i = \frac{a-c-bQ_{-i}-bq_i}{f(v)} \left[\big(1-(k-2)v\big)q_i + v \sum_{k \in I \setminus i} q_k \right],$$

while for firm $o \in O$, the aggregate profit function is

$$\Pi_o = (a - bQ_{-o} - bq_o - c)q_o$$

where $Q_{-j} = Q - q_j$. Firm j takes other firms' production Q_{-j} as given and chooses q_j to maximize its aggregate profit. The first order conditions are

$$\Big(1-(k-2)v\Big)\Big(a-c-bQ_{-i}-bq_i\Big) - b\left[\big(1-(k-2)v\big)q_i + v\sum_{k\in I\setminus i} q_k\right] = 0 \tag{1}$$

$$a - c - 2bq_o - bQ_{-o} = 0 \tag{2}$$

[11]Note that by the theory of partitioned matrices, if $\boldsymbol{B}_{kk}^{-1}$ exists, then

$$(\boldsymbol{I} - \boldsymbol{D})^{-1} = \begin{bmatrix} \boldsymbol{B}_{kk}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{n-k} \end{bmatrix}.$$

To see this is true, observe that

$$\begin{bmatrix} \boldsymbol{B}_{kk} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{n-k} \end{bmatrix} \begin{bmatrix} \boldsymbol{B}_{kk}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{n-k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{kk}\boldsymbol{B}_{kk}^{-1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{I}_{n-k} \end{bmatrix} = \boldsymbol{I}$$

Exploiting symmetry, the interior solution[12] yields the static Cournot equilibrium outputs:

$$q_i^v = \frac{(2-k)v+1}{(k+n+1-k^2)v+n+1}\frac{a-c}{b}, \quad q_o^v = \frac{1+v}{(k+n+1-k^2)v+n+1}\frac{a-c}{b}.$$

Thus, the equilibrium industry output is

$$Q_v = kq_i^v + (n-k)q_o^v = \frac{(-k^2+n+k)v+n}{(k+n+1-k^2)v+n+1}\frac{a-c}{b}.$$

Then, the equilibrium operating profit for a typical firm i is

$$\pi_i^v = (a-c-bQ_v)q_i^v = \frac{(1+v)\big(1-(k-2)v\big)}{\big((k+n+1-k^2)v+n+1\big)^2}\frac{(a-c)^2}{b}$$

and for a typical firm o is

$$\pi_o^v = (a-c-bQ_v)q_o^v = \frac{(1+v)^2}{\big((k+n+1-k^2)v+n+1\big)^2}\frac{(a-c)^2}{b}$$

2.2 The Case of a Nonrenewable resource industry: A Dynamic Model

The above model, however, cannot apply directly to the exhaustible resource sector, as the specificity of a nonrenewable resource (i.e., current extraction goes at the cost of future extraction) makes it inherently a dynamic problem. We consider an exhaustible resource industry involving n firms with the same initial stock endowments $S_{0j} = S$ and the same marginal cost of production c. Firms are oligopolists in the resource market where they compete $\grave{a}$ la Cournot. Let $q_j(t) \geq 0$ denote the extraction rate at time t for firm j. Demand for resource is stationary and linear with a choke price $a > c$, so that the inverse demand at time $t \geq 0$ for the extracted resource is given by $p(t) = a - bQ(t) = a - b\sum_{j=1}^{n} q_j(t)$. In an industry characterized by symmetric rival cross-shareholdings, the aggregate profits of firm j at time t is as follows:

$$\Pi_j(t) = \pi_j(t) + v\sum_{i\neq j}\Pi_i(t) = (p(t)-c)q_j(t) + v\sum_{i\neq j}\Pi_i(t)$$

Each firm j takes the supply paths of all other firms as given and maximizes the discounted sum of the aggregate profits, which consists of its operating profit and the share of profits obtained through

[12]Note that the denominator is positive because we have imposed the restriction that $v < 1/(k-1)$.

$$(k+n+1-k^2)v+n+1 = n+1+(n+1)v-k(k-1)v \geq 1+(n+1)v+n-k > 0$$

ownership interests in other firms, subject to its resource constraint:

$$\max_{q_j(t)\geq 0} \int_0^{\infty} e^{-rt}\left[(a-bQ(t)-c)q_j(t)+\sum_{i\neq j} v_{ji}\Pi_i(t)\right]dt$$

$$s.t. \quad \int_0^{\infty} q_j(t)dt \leq S_{0j}$$

We consider the k-symmetric cross-ownership structure as in the static model and make a similar assumption:

Assumption 2. *Each firm j seeks to maximize the discounted sum of the value of its aggregate profits, including returns on any shares held in rivals, but controls only its own production $q_j(t)$ with $v < \frac{1}{k-1}$ for all i, k, i.e., firms only have a silent financial interest or non-controlling minority stake in the rivals.*

Under Assumption 2, it is possible to solve for the aggregate profit equation at each time t, and thus the problem of all firms can be reformulated as

$$\max_{\boldsymbol{q}(t)\geq 0} \int_0^{\infty} e^{-rt}\left(\begin{bmatrix} \boldsymbol{B}_{kk}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{n-k} \end{bmatrix}\big(a-c-bQ(t)\big)\boldsymbol{q}(t)\right)dt$$

$$s.t. \quad \int_0^{\infty} \boldsymbol{q}(t)dt \leq \boldsymbol{S_0}(t)dt$$

where $\boldsymbol{S_0} = [S_{01}, S_{02}, \cdots, S_{0n}]'$. Let's write $Q(t) = q_j(t) + Q_{-j}(t)$. Then for a typical firm $i \in I$,

$$\max_{q_i(t)\geq 0} \int_0^{\infty} e^{-rt}\left[\frac{1}{1-(k-2)v-(k-1)v^2}\left((1-(k-2)v)q_i + v\sum_{k\in I\setminus i} q_k\right)\left(a-c-bQ_{-i}-bq_i\right)\right]dt$$

$$s.t. \quad \int_0^{\infty} q_i(t)dt \leq S_{0i}$$

while for a typical firm $o \in O$,

$$\max_{q_o(t)\geq 0} \int_0^{\infty} e^{-rt}\left[(a-bQ_{-o}-bq_o-c)q_o\right]dt$$

$$s.t. \quad \int_0^{\infty} q_o(t)dt \leq S_{0o}$$

We characterize an open-loop Nash-Cournot cross-ownership equilibrium (OL-NCOE) of this game. More precisely,

Definition 1 (Open-loop Nash-Cournot Cross-ownership Equilibrium (OL-NCOE))**.** *An n-tuple vec-*

tor of extraction paths $\boldsymbol{q} = (q_1, q_2, \cdots, q_k, q_{k+1}, \cdots, q_n)$ *with* $q(t) \geq 0$ *for all* $t \geq 0$ *is an open-loop Nash-Cournot cross-ownership equilibrium if*

(i) every extraction path is admissible and satisfies the corresponding resource constraint,

(ii) for all $i \in I$,

$$\int_0^\infty e^{-rt}\left[\frac{1}{1-(k-2)v-(k-1)v^2}\left(\big(1-(k-2)v\big)q_i + v\sum_{k\in I\setminus i} q_k\right)\left(a-c-bQ_{-i}-bq_i\right)\right]dt$$
$$\geq \int_0^\infty e^{-rt}\left[\frac{1}{1-(k-2)v-(k-1)v^2}\left(\big(1-(k-2)v\big)q_l + v\sum_{k\in I\setminus l} q_k\right)\left(a-c-bQ_{-i}-bq_l\right)\right]dt$$

for all q_l *satisfying the resource constraint, and*

(iii) for all $o \in O$,

$$\int_0^\infty e^{-rt}\Big[(a-bQ_{-o}-bq_o-c)q_o\Big]dt$$
$$\geq \int_0^\infty e^{-rt}\Big[(a-bQ_{-o}-bq_m-c)q_m\Big]dt$$

for all q_m *satisfying the resource constraint.*

We now proceed to characterize an OL-NCOE of the above-defined game. Let T_i and T_o denote the time at which firm $i \in I$ and firm $o \in O$ deplete their stocks, and denote by q_i and q_o the extraction paths of firm $i \in I$ and firm $o \in O$, respectively. Then,

Proposition 1. *Assume that the initial stocks of all firms are equal, i.e.,* $S_{0j} = S$, *then the* n*-tuple vector* $\boldsymbol{q}^{eq}$ *where* $q_j^{eq} = q_i$ *when* $j = 1, 2, \cdots, k$ *and* $q_j^{eq} = q_o$ *when* $j = k+1, \cdots, n$ *constitutes an*

OL-NCOE.

$$q_i(t) = \begin{cases} \dfrac{\Big(1-(k-2)v\Big)(a-c)\left[1+k+\Big(1-k(k-2)\Big)v-\Big((k+n+1-k^2)v+n+1\Big)e^{r(t-T_i)}+(n-k)(1+v)e^{r(t-T_o)}\right]}{\left[(k+n+1-k^2)v+n+1\right]\left[1+k+\Big(1-k(k-2)\Big)v\right]b} & \textit{for } 0 \le t \le T_o \\ \dfrac{(1-(k-2)v)(a-c)}{\left[1+k+\Big(1-k(k-2)\Big)v\right]b}\left[1-e^{r(t-T_i)}\right] & \textit{for } T_o \le t \le T_i \\ 0 & \textit{for } t \ge T_i \end{cases} \tag{3}$$

$$q_o(t) = \begin{cases} \dfrac{(a-c)(1+v)}{\left[(k+n+1-k^2)v+n+1\right]b}\left[1-e^{r(t-T_o)}\right] & \textit{for } 0 \le t \le T_o \\ 0 & \textit{for } t \ge T_o \end{cases} \tag{4}$$

where T_i and T_o are the unique solutions to

$$\int_0^{T_i} q_i(t)dt = S, \quad \int_0^{T_o} q_o(t)dt = S \tag{5}$$

Proof. See the Appendix. □

Proposition 1 shows that given an initial resource stock S, all firms will exhaust their stocks in finite time. Moreover, it can be shown that $T_i > T_o$ for all S and $0 < v < \frac{1}{k-1}$, i.e., the outsiders will deplete their stocks earlier than the insiders. This is in line with cross-ownership theory where, when a firm acquires a partial ownership stake in a rival, it has an incentive to compete less aggressively and thus unilaterally reduce its output, as one firm's gain may come at the loss of the other firms in which it has financial interests. This is also consistent with standard oligopoly theory where, for strategic substitutes, a reduction in cross-owners' outputs will result in an expansion of the outsider firms. As a result, each of the outsider firms tends to extract from its resource stock faster than each of the insider firms. Using the parameter values $a = 1, b = 1, c = 0$ and $r = 0.1$, Figure 1 plots the stock exhaustion dates (T_i, T_o) as a function of the initial resource stock S, of a typical insider firm $i \in I$ that engages in cross-ownership, and an outsider firm $o \in O$ that remains independent, respectively, for $k = 6, n = 9, v = 0.05$. Simulations using any combinations of k, n with $v < \frac{1}{k-1}$ and various values for the parameters a, b, c and r show that this result is qualitatively robust: a k-symmetric cross-ownership induces the outsiders to exhaust their stocks earlier than the cross-ownership participants for any resource stock level.

The equilibrium extraction path then consists of two phases: phase I from date 0 to T_o, and phase

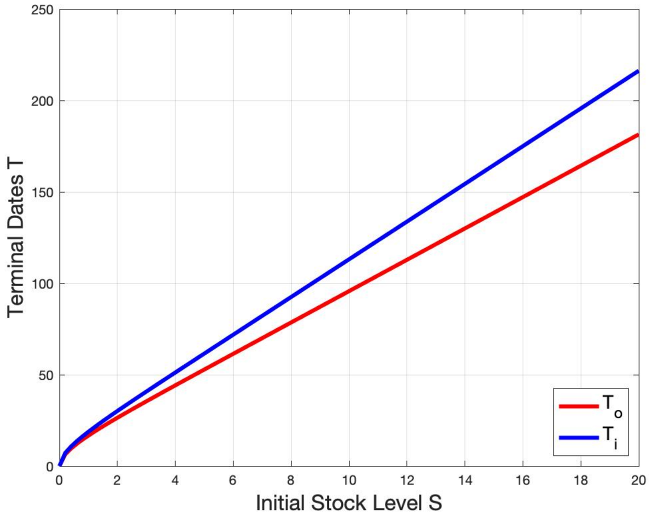
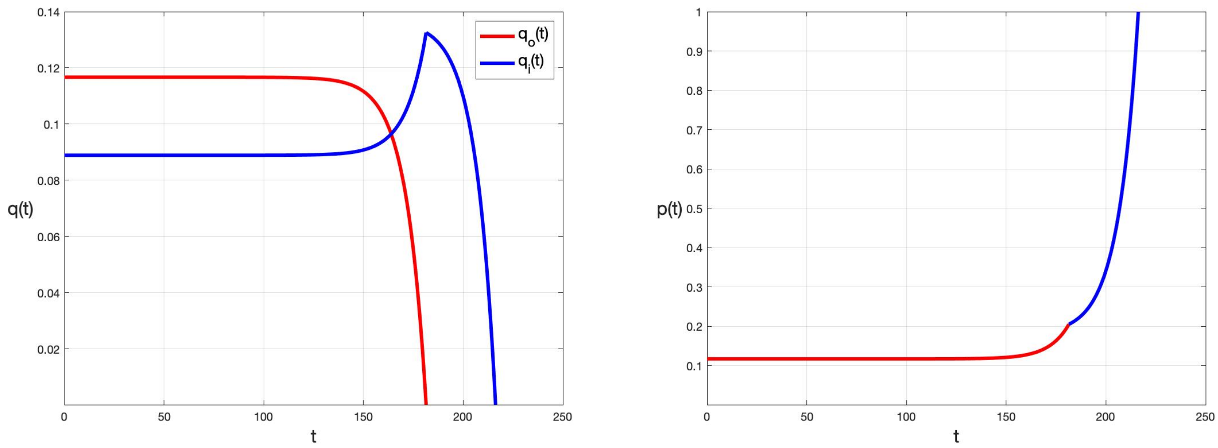


Figure 1: Terminal dates as a function of initial stock

II from T_o to T_i. During phase I, the extraction of all the n firms is positive until T_o, where the extraction and the stock of firms $o \in O$ vanish. During phase II, only firms $i \in I$ still own a positive stock, until T_i where the extraction and the stock of these remaining firms vanish. To illustrate these results, we use the same parameter values as in Figure 1 and plot in Figure 2 the equilibrium extraction paths of a typical insider firm $i \in I$ and an outsider firm $o \in O$ as well as the equilibrium price path for $n = 9, k = 6, v = 0.05$ and $S = 20$. As shown in Figure 2a, the outsiders start with a higher exploitation rates than the cross-owners, but as more resource gets depleted, the outsiders gradually decrease their production while the insiders steadily increase their output. When the outsiders exhaust their resource stocks, the resource is supplied only by the group of cross-owners. The degree of concentration in supply increases over time. These findings are in line with the 'oil'igopoly theory (Loury, 1986; Polasky, 1992), which predicts that small firms will exhaust their stocks before large firms do, leading to eventual monopolization of the market. The increased concentration over time induced by cross-ownership confers market power on those cross-owners. As a consequence, the cross-ownership participants can raise prices substantially higher than in other industries without stock constraint as shown in Figure 2b, thus providing an additional incentive to view the exhaustible sector differently. With the remaining stocks, the insiders gradually decrease their production until total depletion of the resource.

3 Profitability of Cross-ownership

In this section, we exploit the characterization of both the static Cournot equilibrium and the OL-NCOE in the above-defined game to investigate the profitability of the k-symmetric cross-ownership



(a) The OL-NCOE extraction path

(b) The OL-NCOE price path

Figure 2: The open-loop Nash-Cournot cross-ownership equilibrium (OL-NCOE)

in the industry. We define the profitability of cross-ownership in the static case as the difference between the equilibrium operating profits with and without cross-ownership, and in the dynamic case as the difference between the equilibrium discounted sum of operating profits with and without cross-ownership.[13] We first focus on the static case for a generic industry and formally define our findings that cross-ownership is profitable only when it involves a relatively large number of firms as the *cross-ownership paradox*, analogous to the *merger paradox* which refers to the seminal result in oligopoly theory that a horizontal merger is profitable only when it involves a relatively large number of firms (Salant, Switzer and Reynolds, 1983). We then compare cross-ownership with horizontal merger and provide some explanations as to why firms want to engage in cross-shareholdings instead of a full merger. Next, we move to focus on the exhaustible sector. Specifically, we numerically examine the profitability under different cross-ownership structures and show that a k-symmetric cross-ownership can be profitable even when the participation ratio $\frac{k}{n}$ is less than or equal to $\frac{k}{2k-1}$ and is always profitable when the participation ratio $\frac{k}{n}$ is greater than $\frac{k}{2k-1}$, provided that the resource stock owned by each firm is small enough for any levels of cross-ownership.

3.1 Profitability: the Static Case

The equilibrium operating profit for a typical firm i that participates in cross-ownership is given by

$$\pi_i^v(k,n,v) = (a - bQ_v - c)q_i^v = \frac{(1+v)\big(1-(k-2)v\big)}{\big((k+n+1-k^2)v+n+1\big)^2}\frac{(a-c)^2}{b},$$

[13]Here we use the operating profits (π_j) instead of the aggregate profits or accounting profits (Π_j) to compare with the case of a standard Cournot model. This is the usual distinction we make about the economic profits and accounting profits.

while that for a typical firm in the standard Cournot model without cross-ownership is

$$\pi_c = \pi_i^v(k,n,0) = \frac{1}{(n+1)^2}\frac{(a-c)^2}{b}$$

A k-symmetric cross-ownership is profitable if

$$G(k,n,v) = \pi_i^v(k,n,v) - \pi_i^v(k,n,0) > 0$$

We summarize in Proposition 2 the profitability of a $k-$symmetric cross-ownership in the static case:

Proposition 2. *For any $2 \leq k \leq n$ and $0 < v < \frac{1}{k-1}$, the profitability of a k-symmetric cross-ownership for Cournot competitors depends on the following scenarios:*

1. *If $\frac{k}{n} \leq \frac{k}{2k-1}$, then $G < 0$ for all $v \in (0, \frac{1}{k-1})$;*
2. *If $\frac{k}{2k-1} < \frac{k}{n} < \gamma(k) \equiv \frac{k}{k+\sqrt{k}-1}$, then $G > 0$ for $v < \bar{v}$ and $G < 0$ for $v \in (\bar{v}, \frac{1}{k-1})$, where $G(\bar{v}) = 0$ and $\bar{v} \equiv -\frac{(n+1)(2k-n-1)}{(n+1)(2k-n-1)-k^2(k-1)}$;*
3. *If $\frac{k}{n} > \gamma(k) \equiv \frac{k}{k+\sqrt{k}-1}$, then $G > 0$ for all $v \in (0, \frac{1}{k-1})$.*

Proof. See the Appendix. □

This result seems surprising as one would naturally think it should be always profitable for firms to participate in cross-ownership due to a less intensified competition. We thus define this result as a cross-ownership paradox, analogous to the merger paradox. A closer look at the lower threshold participation ratio $\left(\frac{k}{2k-1}\right)$ indicates that firms can never profit from cross-shareholdings if less than half of the firms in the industry participate. This 50-percent benchmark has also been addressed in Levin (1990)'s analysis of horizontal mergers under quite general conditions. However, our threshold includes ratios beyond only 50 percent, and crucially depends on both k and n. For example, when $k = 2$ and $n = 3$ (or $k/n = 66.7\%$), $k = 3$ and $n = 5$ (or $k/n = 60\%$), $k = 4$ and $n = 7$ (or $k/n = 57.1\%$), firms will also find any levels of cross-shareholdings unprofitable. A similar examination at the the upper threshold $\left(\frac{k}{k+\sqrt{k}-1}\right)$ demonstrates that the profitability of cross-ownership is always positive if the number of firms involved in cross-ownership is significant enough. In particular, we can show that this upper threshold is at least 80% ($\frac{k}{k+\sqrt{k}-1} = 80\%$ when $k = 4$, but for any other $k \geq 2$, $\frac{k}{k+\sqrt{k}-1} > 80\%$). The threshold of 80% coincides with the famous threshold determined in Salant, Switzer and Reynolds (1983) for the case of horizontal mergers, where they show that at a merger needs to involve at least 80% of the firms to be profitable. Finally, the second part of the cross-ownership paradox posits a large range of cross-shareholdings for which a k-symmetric cross-ownership can be profitable when the participation ratio is in between the lower threshold ($\frac{k}{2k-1}$) and upper threshold ($\frac{k}{k+\sqrt{k}-1}$).

We illustrate the findings of the above proposition with several numerical examples below where we fix the number of insiders at $k = 6$ and vary the number of firms in the industry from $n = 7, 8, 9, 10$ to 11, respectively. These examples will serve as benchmarks when analyzing the profitability of cross-ownership in the case of a nonrenewable resource industry. The lower and upper threshold participation ratios with $k = 6$ are respectively

$$\frac{k}{2k-1} = 0.5455 \quad \text{and} \quad \gamma(k) = \frac{k}{k+\sqrt{k}-1} = 0.8054.$$

Example 1. First, consider $n = 7$. Since

$$\frac{k}{n} = \frac{6}{7} = 0.8571 > \gamma(k) = 0.8054,$$

we have

$$G > 0, \forall\, v \in (0, \frac{1}{k-1}).$$

Example 1 shows that with $n = 7$ and $k = 6$, cross-ownership is always profitable for any admissible $v \in (0, \frac{1}{k-1})$.

Example 2. Consider $n = 8$, then

$$\frac{k}{2k-1} = 0.5455 < \frac{k}{n} = \frac{6}{8} = 0.75 < \gamma(k) = 0.8054$$

Thus $G > 0$ if and only if

$$v \leq \bar{v} \equiv -\frac{(n+1)(2k-n-1)}{(n+1)(2k-n-1) - k^2(k-1)} = 0.176$$

Example 2 shows that with $n = 8$ and $k = 6$, cross-ownership is profitable provided that each of the 6 firms holds no more than 17.6% of the shares of any other firm.

Example 3. Consider $n = 9$, then

$$\frac{k}{2k-1} = 0.5455 < \frac{k}{n} = \frac{6}{9} = 0.6667 < \gamma(k) = 0.8054$$

Thus $G > 0$ if and only if

$$v \leq \bar{v} \equiv -\frac{(n+1)(2k-n-1)}{(n+1)(2k-n-1)-k^2(k-1)} = 0.125$$

Example 3 shows that with $n = 9$ and $k = 6$, cross-ownership is profitable provided that each of the 6 firms holds no more than 12.5% of the shares of any other firm.

Example 4. Consider $n = 10$, then

$$\frac{k}{2k-1} = 0.5455 < \frac{k}{n} = \frac{6}{10} = 0.6 < \gamma(k) = 0.8054$$

Thus $G > 0$ if and only if

$$v \leq \bar{v} \equiv -\frac{(n+1)(2k-n-1)}{(n+1)(2k-n-1)-k^2(k-1)} = 0.065$$

Example 4 shows that with $n = 10$ and $k = 6$, cross-ownership is profitable provided that each of the 6 firms holds no more than 6.5% of the shares of any other firm.

Example 5. Finally, consider $n = 11$. Since

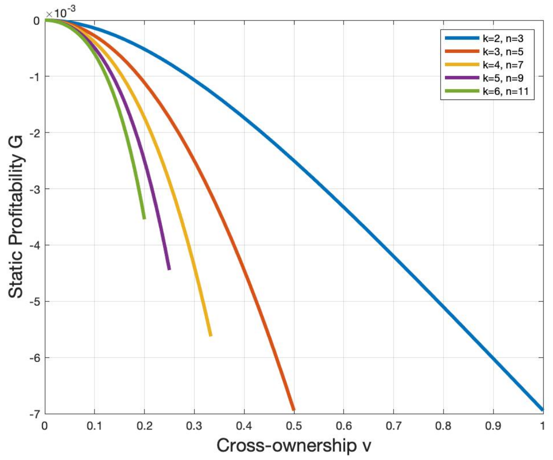
$$\frac{k}{n} = \frac{6}{11} = 0.5455 = \frac{k}{2k-1}$$

we have

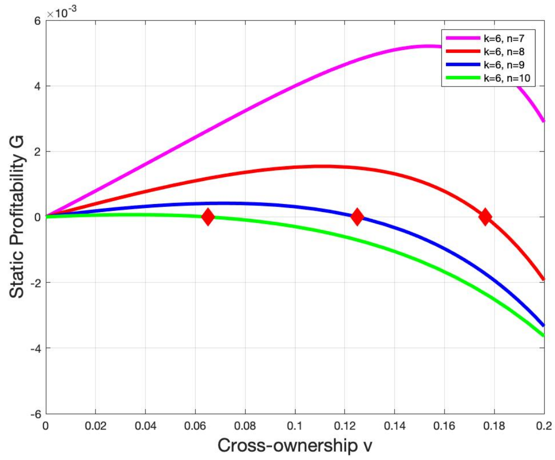
$$G < 0, \forall\, v \in (0, \frac{1}{k-1}).$$

Example 5 shows that with $n = 11$ and $k = 6$, cross-ownership is never profitable for any admissible $v \in (0, \frac{1}{k-1})$. To visualize these results, we also plot the static profitability G as a function of the level of cross-ownership v for different k and n in Figure 3. Specially, we have different combinations of k and n that satisfy $\frac{k}{n} \leq \frac{k}{2k-1}$ in Figure 3a and $\frac{k}{n} > \frac{k}{2k-1}$ in Figure 3b with $k = 6$ and $n = 7, 8, 9$ and 10, respectively. Clearly, these figures have validated our results.

The above numerical and graphic illustrations help clarify the intuition behind the "cross-ownership paradox". The profitability of cross-ownership depends on three competing forces. First, by partially internalizing previous rivalry, each of the cross-ownership participants reduces its quantity and thereby increases its profit. Second, given that firms' quantities are strategic substitutes, the outsider firms react by increasing their output, which reduces the profitability of cross-ownership. Third, a larger ownership stakes between cross-owners will lead to a greater output reduction, but this induces the outsiders to respond more aggressively, thereby reducing the profitability of cross-ownership. So



(a) $\frac{k}{n} \leq \frac{k}{2k-1}$



(b) $\frac{k}{n} > \frac{k}{2k-1}$

Figure 3: Static profitability as a function of cross-ownership

for a cross-ownership to be profitable, either the number of cross-ownership participants must be significant enough (i.e., the first effect dominates the latter two effects) or the number of cross-ownership participants is moderate but the shareholding is not too large (i.e., the first effect and third effect dominates the second effect).

An immediate result that follows Proposition 2 is the set of admissible levels of shareholdings on profitability, which we summarize as below:

Corollary 1. When $\frac{k}{n} > \frac{k}{2k-1}$, the set of admissible levels of shareholdings for which a k-symmetric cross-ownership is profitable decreases with the participation ratio k/n.

Proof. See the Appendix. $\square$

Indeed, throughout Example 1-4 and Figure 3b, we can observe that a k-symmetric cross-ownership is more likely to be profitable with lower levels of shareholdings for a lower participation ratio. The intuition behind this result is that a larger shareholding by the firms that engage in the symmetric cross-ownership will induce them to reduce output by more, but this triggers a more aggressive response by the outsiders in terms of strategic substitutes in Cournot competition. The increase in both the number and the output of the outsiders more than offsets the benefit the cross-owners can receive from their reduction of output, thereby reducing the profitability of cross-ownership. This result has shed light on the differences between cross-ownership and horizontal mergers, possibly explaining why firms may prefer to participate in cross-ownership than in a horizontal merger. For example, when $n = 10$ and $k = 6$, the profitability of a k-symmetric cross-ownership is positive provided that each of the 6 firms holds no more than 6.5% of the non-controlling minority shares of any other firm, while that of a horizontal merger of 6 firms is negative.

These findings also raise some practical considerations from a company's corporate strategy viewpoint. Not only is it more profitable to participate in cross-ownership than a horizontal merger, more importantly, it constitutes a "smart" way to avoid the possible legal challenges. In the US, partial cross-ownership arrangements are most often examined under Section 7 of the Clayton Act.[14] While Section 7 of the Clayton Act covers the acquisition of "any part" of the stock of another company, it also "shall not apply to persons purchasing such stock solely for investment" (Scott Morton and Hovenkamp, 2017). The ambiguity in the statutory language has left courts struggling to access the antitrust risk of those partial stock acquisitions, and thus provides very little guidance for antitrust practitioners to set forth any clear guidelines or parameters as to what the "safe" shareholdings are (O'Brien and Salop, 2000). As a result, antitrust authorities have adopted a lenient approach toward passive investments. As a matter of fact, Nain and Wang (2018) document that fewer than 1% of the minority acquisitions are challenged by FTC or DOJ, and even fewer are blocked. In the EU and most other jurisdictions, however, antitrust authorities have no competence to investigate such cases. As noted by Jovanovic and Wey (2014), in many merger cases, the acquiring firm often proposes to take a passive partial ownership stake in the target firm prior to a full merger. They show that antitrust authorities, which do not account for passive partial ownership acquisitions, create incentives among firms to engage in "sneaky takeovers", which proceed in two steps. First, the acquiring firm abstains from proposing a full acquisition, as this would harm consumers. Rather, it strategically acquires a passive partial ownership, which often goes unnoticed or unchallenged by the antitrust authorities. Second, the acquiring firm proposes a full takeover, which can then be viewed as consumer surplus increasing and accepted by the antitrust authorities. The consumer surplus increases because passive partial ownership reduces the necessary minimal synergy level that leaves consumer surplus unchanged by a merger, thus relaxing the synergy requirement for a merger to increase consumer surplus (Jovanovic and Wey, 2014). As a result, a larger set of such synergies would be supported by antitrust authorities. However, if the antitrust authorities evaluated the whole process, they would find that it is actually detrimental to consumers. Because this two-step strategy perfectly evades scrutiny, it can eventually achieve the goal of a full merger without any legal challenges, which further explains why firms may want to engage in cross-shareholdings. Viewing cross-ownership as a more attractive corporate strategy, firms disproportionately adopt it without any legal accountability, ultimately to the

[14] Acquisitions of voting securities can be also challenged under Section 1 of the Sherman Act, which prohibits contracts, combinations, or conspiracies in restraint of trade, but a plaintiff challenging an acquisition under Section 1 carries the burden of proving an actual anticompetitive effect through a restraint of trade, as well as concerted action (O'Brien and Salop, 2000). The Hart-Scott-Rodino (HSR) Act is also being used to evaluate certain transactions above a certain dollar threshold – including minority acquisitions – in the premerger notification program, but it specifically exempts from reporting requirements acquisitions solely for purposes of investment when the securities acquired or held do not exceed 10% of the outstanding voting securities of the issuer. See more at https://www.ftc.gov/enforcement/premerger-notification-program.

detriment of consumers. Competition authorities should thus reform their current lenient approach by subjecting minority shareholdings to a stricter scrutiny.

3.2 Profitability in the case of a Nonrenewable Resource Industry

We can now compute the value function of each firm $i \in I$ that engages in rival cross-shareholdings, which constitute a building block to analyze the profitability of cross-ownership in a nonrenewable resource industry. The equilibrium discounted sum of operating profits with a k-symmetric cross-ownership for a typical firm is given by:

$$\begin{aligned} V_S &= \int_0^{T_o} e^{-rt}\Big[(a - b\sum_{j=1}^{n} q_j - c)q_i\Big]dt + \int_{T_o}^{T_i} e^{-rt}\Big[(a - b\sum_{j=1}^{n} q_j - c)q_i\Big]dt \\ &= \int_0^{T_o} e^{-rt}\Big[(a - bkq_i - b(n-k)q_o - c)q_i\Big]dt + \int_{T_o}^{T_i} e^{-rt}\Big[(a - bkq_i - c)q_i\Big]dt \end{aligned}$$

where the equilibrium extraction paths for each phase are given by (3) and (4) and the exhaustion dates are solutions to (5). It will be useful to explicitly write down the equilibrium discounted sum of operating profits as a function of (k, n, v, S), but the expression is too cumbersome to report here. Instead we choose to numerically examine the profitability of the k-symmetric cross-ownership under two groups of participation ratios: $\frac{k}{n} \leq \frac{k}{2k-1}$ and $\frac{k}{n} > \frac{k}{2k-1}$. The equilibrium discounted sum of profits without cross-ownership for an individual firm is given by:

$$V_C = \int_0^{T_C} e^{-rt}\Big[(a - bnq_C - c)q_C\Big]dt$$

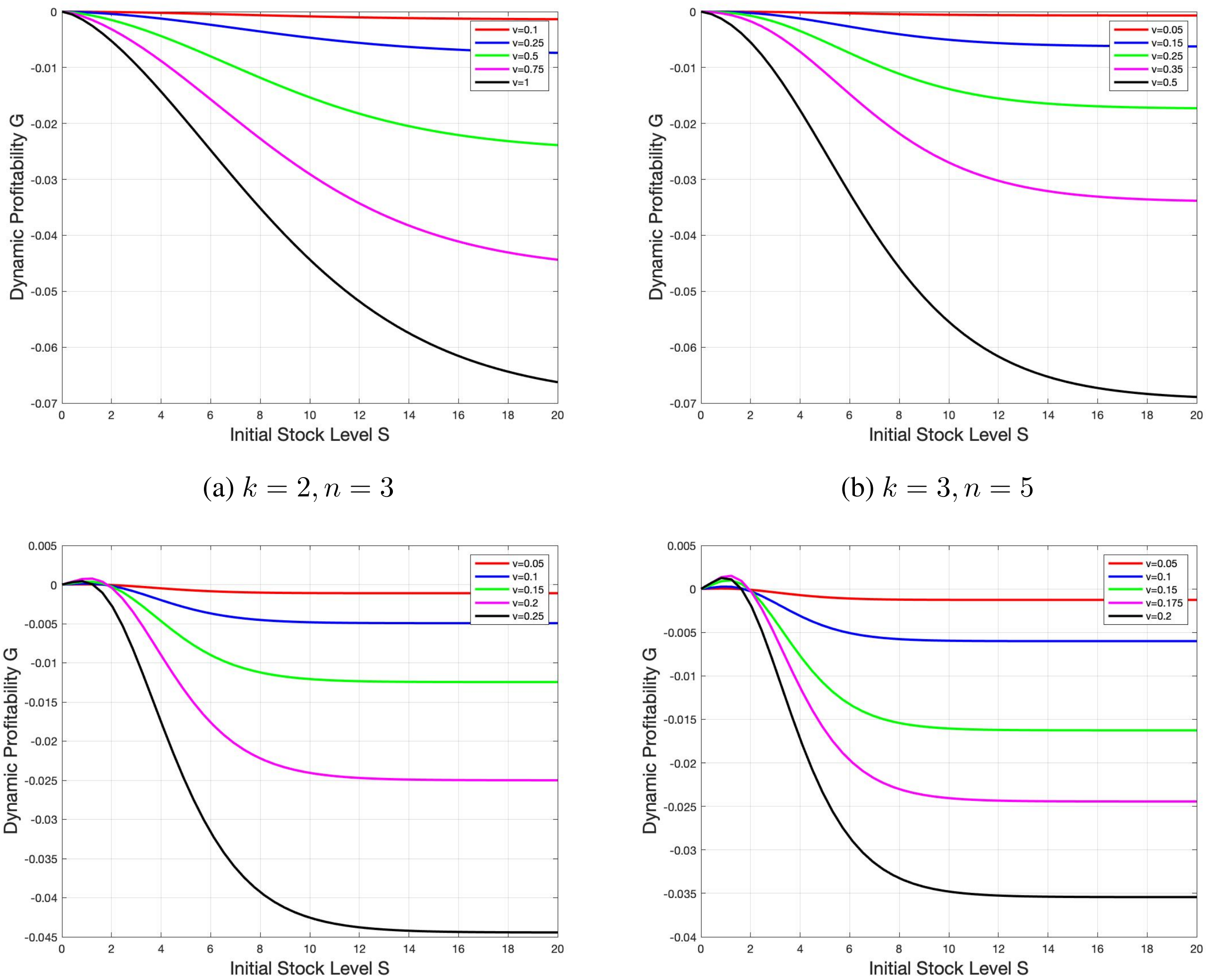
where

$$q_C(t) = \frac{a-c}{b(n+1)}\left[1 - e^{r(t-T_C)}\right], \quad \frac{a-c}{b(n+1)}\left(T_C - \frac{1}{r} + \frac{e^{-rT_C}}{r}\right) = S$$

Then a k-symmetric cross-ownership is profitable when

$$G(k, n, v, S) = V_S - V_C > 0$$

We use the same parameter values as in Figure 1 and illustrate in Figure 4 the gains resulting from a k-symmetric cross-ownership as a function of initial stock S for different levels of shareholdings when the participation ratio $\frac{k}{n} \leq \frac{k}{2k-1}$. While Figure 4a and 4b show that it is never profitable for firms to participate in cross-ownership for any levels of initial resource stock, Figure 4c and 4d indicate that the profitability of cross-ownership can be positive for any $v \in (0, \frac{1}{k-1})$ when the initial stock owned by each firm is small enough. Simulations using many other combinations of k and n



(a) $k = 2, n = 3$

(b) $k = 3, n = 5$

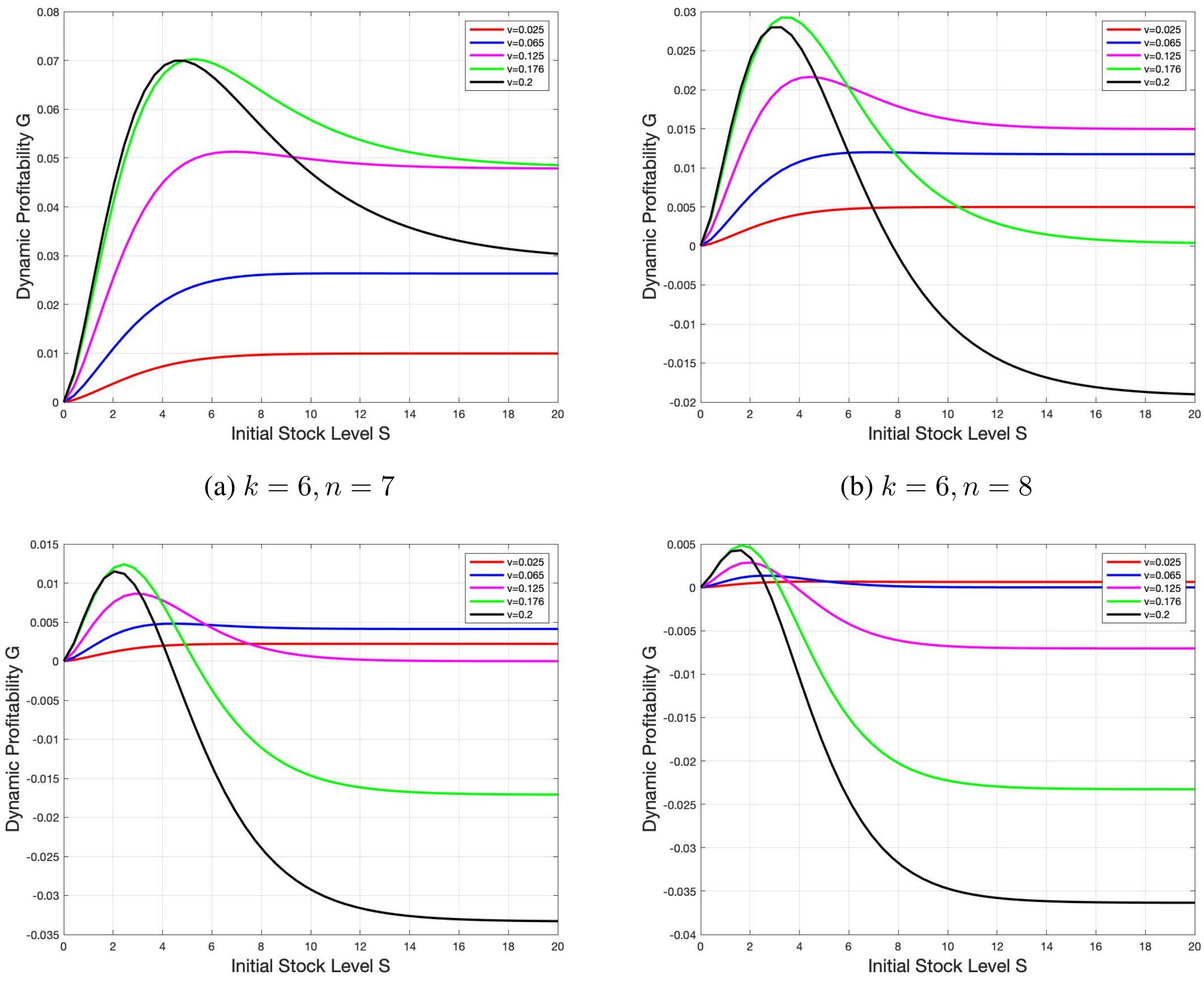
(c) $k = 5, n = 9$

(d) $k = 6, n = 11$

Figure 4: Profitability as a function of initial stock when $\frac{k}{n} \leq \frac{k}{2k-1}$

(i.e., for all $k = \frac{1}{2}n$ and $k \geq 6$; for all $k = \frac{1}{2}(n+1)$ and $n \geq 7$) satisfying $\frac{k}{n} \leq \frac{k}{2k-1}$ also show such findings. These similar findings mean that the previous static results do not necessarily carry over to our dynamic model.

We now move to check whether this result holds when the participation ratio $\frac{k}{n} > \frac{k}{2k-1}$. Specifically, Figure 5 illustrates the profitability resulting from a k-symmetric cross-ownership as a function of initial stock S when $k = 6$ and $n = 7, 8, 9$ and 10 respectively, using the same parameter values as in Figure 1. As a comparison, we refer back to Figure 3b, which illustrates the profitability as a function of cross-ownership v in the static case. With $n = 7$ and $k = 6$, cross-ownership is always profitable for any $v \in (0, \frac{1}{k-1})$ in the static model. The same holds true in the dynamic model for all resource stock levels. With $n = 8$ and $k = 6$, where the k-symmetric cross-ownership in the static model is not profitable if each of the 6 firms holds more than 17.6% of the shares of any other firm, it can be profitable in the dynamic model for any levels of cross-shareholdings $v \in (0, \frac{1}{k-1})$ as long as the stock of the firms is small. Moreover, compared to the static case where the k-symmetric



(a) $k = 6, n = 7$ (b) $k = 6, n = 8$

(c) $k = 6, n = 9$ (d) $k = 6, n = 10$

Figure 5: Profitability as a function of initial stock when $\frac{k}{n} > \frac{k}{2k-1}$

cross-ownership for which $k = 6$ and $n = 9$ is not profitable when $v > 12.5\%$, it can be profitable for any $v \in (0, \frac{1}{k-1})$ as long as the stock of the firms is small. In addition, whereas in the static case profitability with $k = 6$ and $n = 10$ is negative for $v > 6.5\%$, in the dynamic case it is always positive for any $v \in (0, \frac{1}{k-1})$ provided that the stock of the firms is small enough. We also observe that the profitability of a k-symmetric cross-ownership increases in v for all $v \in (0, \frac{1}{k-1})$ when the stock is small enough, but this increase in v does not hold if the initial stock is large. Simulations using a wide range of values of k and n satisfying $\frac{k}{n} > \frac{k}{2k-1}$ suggest that these findings are quite robust.

Clearly, some of the results from the cross-ownership paradox do not carry over to the case of nonrenewable resource industries. We therefore summarize these findings in Result 1, which is robust to different combinations of k and n and changes in parameter values.

Result 1. *The profitability of a k-symmetric cross-ownership can be positive even when the participation ratio $\frac{k}{n} \leq \frac{k}{2k-1}$ and is always positive when the participation ratio $\frac{k}{n} > \frac{k}{2k-1}$, provided that the initial resource stock owned by each firm is small enough. Moreover, the profitability of k-symmetric* *cross-ownership increases in $v \in (0, \frac{1}{k-1})$ for S positive and sufficiently small.*

Result 1 sharply contrast with the case of a standard Cournot model with cross-ownership but without resource stock constraints. In our earlier static settings, with linear demand and constant marginal cost, a k-symmetric cross-ownership can be profitable even if only 60% of the firms in the industry participate provided that the cross-shareholdings are small enough. However, in the presence of stock constraints, there exists a range of stock levels for which any levels of cross-ownership can be profitable —the higher the shareholdings, the higher the profitability. Unlike in the static Cournot model with cross-ownership, where outsiders respond to any increased shareholdings between cross-owners by aggressively increasing output and mitigating the cross-ownership participants' gain in market power, here the outsiders are restricted in their response due to their resource constraints. As a result, when the stock levels are sufficiently small, a larger level of cross-ownership will ensure a higher profitability. Within our context, the $n-k$ outsiders exhaust their stocks earlier than the cross-ownership participants, resulting in greater induced market power by cross-ownership than in the static model. A similar result can be found in Benchekroun, Breton and Chaudhuri (2019), who find that a merger is always profitable provided that the resource stock owned by each firm is small enough. The fact that the profitability of a k-symmetric cross-ownership is mostly positive when resource stock owned by each firm is small thus provides an explanation as to why there is so much cross-ownership in the exhaustible sector.

4 Welfare Analysis

Antitrust authorities may be concerned by profitable cross-ownership if it is detrimental to welfare. In this section, we first examine the welfare implications in the static case of the k-symmetric cross-ownership using the total surplus criterion, i.e., the sum of consumer surplus and producer surplus or industry profits, where industry profits are defined as the combined sum of the operating profits of the cross-ownership participants (belonging to the subset I of insiders), $k\pi_i^v$, and of the firms outside the cross-ownership (belonging to the subset O of outsiders), $(n-k)\pi_o^v$.[15] Subsequently, we compare the results obtained in the dynamic model (for a nonrenewable resource industry) with that in the static model. Finally, we provide some policy implications from our analysis.

The change in total surplus induced by the k-symmetric cross-ownership is given by:

$$\Delta TS = W_v - W_c = \frac{b}{2}Q_v^2 - \frac{b}{2}Q_c^2 + +k\pi_i^v + (n-k)\pi_o^v - n\pi_c$$

[15]We thank an anonymous reviewer for suggesting the conduct of welfare analysis using the total surplus criterion to be more in line with the existing literature. In a previous version, we focused on consumer surplus only.

where Q_v and Q_c are the equilibrium industry output with and without cross-ownership, respectively. After substitution, it yields

$$\Delta TS(k,n,v) = \frac{\left[v\Big(k(k-1)-2(n+1)\Big)-2(n+1)\right]kv(k-1)}{\Big((k+n+1-k^2)v+n+1\Big)^2(n+1)^2}\left[\frac{(a-c)^2}{2b}\right]$$

Proposition 3. *For any $2 \leq k \leq n$ and $v \in (0, \frac{1}{k-1})$, a k-symmetric cross-ownership is never welfare-improving when evaluated in accordance with a total surplus criterion.*

Proof. See the Appendix. □

This result is quite intuitive. When firms engage in rival cross-shareholdings, they will compete less aggressively with each other and thus unilaterally reduce their outputs, since any gains from the acquirers' own activities may be offset by a negative impact on the acquirers' share of the targets' profits. Although the outsiders expand their outputs as a response, the reduction in the outputs brought by cross-ownership more than offsets the increase. As a result, the industry output decreases and market price increases, thus increasing industry profits[16] but decreasing consumer surplus. However, the overall reduction from consumer surplus dominates the increase in industry profits, thereby resulting in a welfare loss.

We now turn to the welfare analysis in a nonrenewable industry. The consumer surplus generated by the exploitation of the nonrenewable resource under the k-symmetric cross-ownership structure is

$$\begin{aligned} CS_S &= \int_0^{T_i} e^{-rt}\left[\frac{b}{2}(\sum_{j=1}^{n} q_j)^2\right]dt \\ &= \int_0^{T_o} e^{-rt}\left[\frac{b}{2}(kq_i+(n-k)q_o)^2\right]dt + \int_{T_o}^{T_i} e^{-rt}\left[\frac{b}{2}(kq_i)^2\right]dt, \end{aligned}$$

while the industry profits are

$$\begin{aligned} PS_S &= \int_0^{T_i} e^{-rt}\left[(a-b\sum_{j=1}^{n} q_j - c)(kq_i+(n-k)q_o)\right]dt \\ &= \int_0^{T_o} e^{-rt}\Big[(a-bkq_i-b(n-k)q_o-c)(kq_i+(n-k)q_o)\Big]dt + \int_{T_o}^{T_i} e^{-rt}\Big[(a-bkq_i-c)kq_i\Big]dt, \end{aligned}$$

where the equilibrium extraction paths for each phase are given by (3) and (4) and the exhaustion

[16] Industry profits surge because of an increase in profits from both insiders and outsiders. The outsiders increase its profits as both market price and quantity increase. While the change in insiders' profits may seem ambiguous as market price increases but its output decreases, the insiders' profits actually increase otherwise they wouldn't have engaged in cross-shareholdings at the first place.

dates are solutions to (5). Thus, the welfare under the k-symmetric cross-ownership structure in a nonrenewable resource industry is given by

$$
\begin{aligned}
W_S &= CS_S + PS_S \\
&= \int_0^{T_o} e^{-rt}\left[\frac{b}{2}(kq_i + (n-k)q_o)^2 + (a - bkq_i - b(n-k)q_o - c)(kq_i + (n-k)q_o)\right]dt \\
&+ \int_{T_o}^{T_i} e^{-rt}\left[\frac{b}{2}(kq_i)^2 + (a - bkq_i - c)kq_i\right]dt.
\end{aligned}
$$

The total surplus generated by the exploitation of the nonrenewable resource under the standard Cournot model without cross-ownership is given by

$$
W_C = \int_0^{T_C} e^{-rt}\left[\frac{b}{2}(nq_C)^2 + n(a - bnq_C - c)q_C\right]dt,
$$

where

$$
q_C(t) = \frac{a-c}{b(n+1)}\left[1 - e^{r(t-T_C)}\right], \quad \frac{a-c}{b(n+1)}\left(T_C - \frac{1}{r} + \frac{e^{-rT_C}}{r}\right) = S.
$$

The competition authority determines the total surplus change induced by the k-symmetric cross-ownership in a nonrenewable resource industry:

$$
W(k, n, v, S) = W_S - W_C
$$

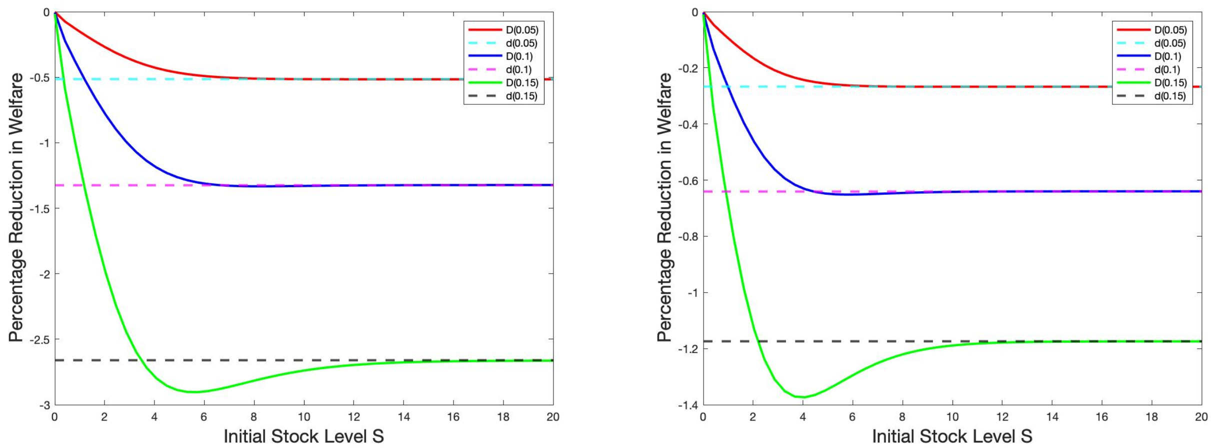
It will be useful to explicitly express W as a function of (k, n, v, S). Its expression is too cumbersome to report here. Instead, we choose to numerically examine the percentage welfare change of the k-symmetric cross-ownership in the dynamic case defined as

$$
D(v) = \frac{W_S - W_C}{W_C},
$$

and directly compare it with the static percentage welfare change defined as

$$
d(v) = \frac{W_v - W_c}{W_c}.
$$

When S is large enough, i.e., the resource is abundant, the dynamic percentage welfare change will asymptotically converge to the static result. Using the same parameter values as in Figure 1, we illustrate in Figure 6 the percentage welfare change resulting from a k-symmetric cross-ownership as a function of initial stock S for different levels of cross-ownership under participation ratios $\frac{k}{n} = \frac{6}{8}$ and $\frac{k}{n} = \frac{6}{10}$. The dashed and solid line denote the percentage welfare loss in the static and dynamic cases, respectively. Figure 6 indicates that a k-symmetric cross-ownership is never welfare-improving



(a) $k = 6$ and $n = 8$

(b) $k = 6$ and $n = 10$

Figure 6: Percentage welfare change as a function of initial stock

for all S based on a total surplus criterion. Simulations using a wide range of values of k and n with $v < \frac{1}{k-1}$ and of the parameters a, b, c and r show that this result is qualitatively robust. We also observe the following:

Result 2. *When the initial resource stock owned by each firm is small enough, the percentage welfare loss in the case of a nonrenewable resource oligopoly resulting from a k-symmetric cross-ownership is smaller than that in the static case.*

This result seems surprising, as one would expect the exact opposite: the welfare loss is larger in the dynamic case. This is because when resource stock owned by each firm is small enough, a k-symmetric cross-ownership induces the outsiders to exhaust their resource stocks before the cross-ownership participants. Consequently, a group of cross-owners will eventually monopolize the market, and thus the price can be raised higher than in a static model, resulting in more welfare loss. While result 2 may seem counterintuitive, the main intuition behind it is that although the cross-owners can raise the price higher, it also extends the duration of the resource that can be used. As the resource becomes increasingly scarce, its extended periods of use partially offset the negative effect of the higher price on the consumer surplus. Therefore, the loss in consumer surplus is relatively smaller in the dynamic case than the static case when S is small enough.[17] As a result, the smaller loss in consumer surplus due to increased scarcity and the increased profits due to higher price will lead to a smaller welfare loss in the case of a nonrenewable resource oligopoly than that in the static case.

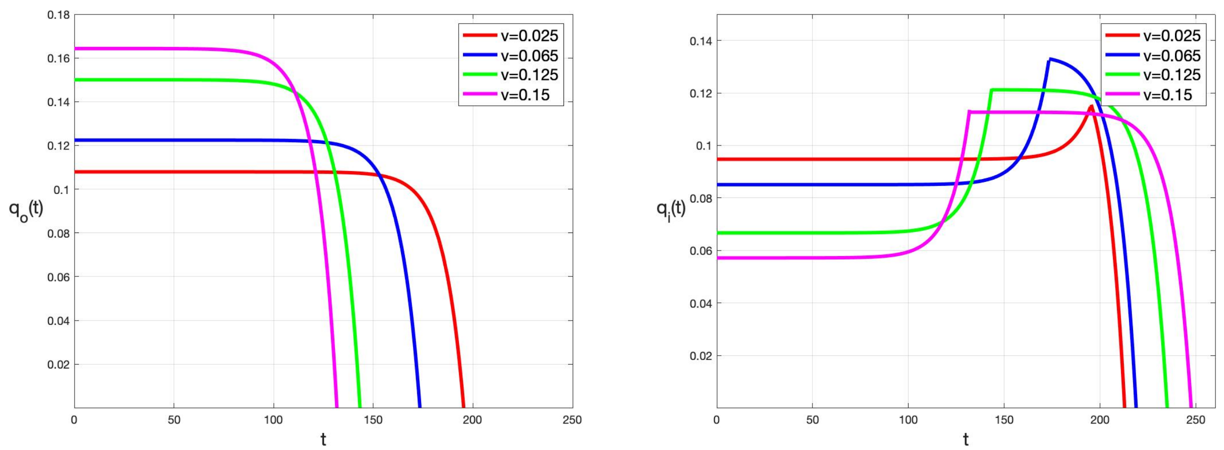
In the absence of any potential efficiency gains, our results thus suggest that passive minority

[17] In addition to the scarcity effect, the risk of future trade disruption may also favor a more conservationist extraction path at the cost of higher price, as emphasized in Hillman and Long (1983).

cross-shareholdings should be blocked by competition authorities according to a total surplus standard. However, cross-ownership is generally believed to bring efficiency gains. For example, partial cross-ownership "offers a means for providing and compensating capital to risky ventures, for solidifying buyer-seller relationships, for funding and exploiting joint R&D activities, and for appropriating the returns to technology transfer" (Reynolds and Snapp, 1986). From a financial perspective, partial cross-ownership can "help to reduce holdup costs, mitigate financing constraints, and facilitate greater innovation and relation-specific investment", thus improving in operating efficiency (Nain and Wang, 2018).[18] Thus, when competition authorities make the tradeoffs between the possible efficiency gains and the welfare loss brought by cross-ownership, they should be cautious when ruling in the nonrenewable resource sector. As when the resource stock owned by each firm is small enough, cross-ownership may turn out to be relatively less detrimental to society.

5 Comparative Static Analysis

In this section, we examine how a change in v, k and n impacts the exploitation rates (q_i and q_o), discounted consumer surplus (CS_S), discounted industry profits (PS_S) and ultimately discounted welfare (W_S) in a nonrenewable industry. [19] The results are illustrated by numerical simulations.

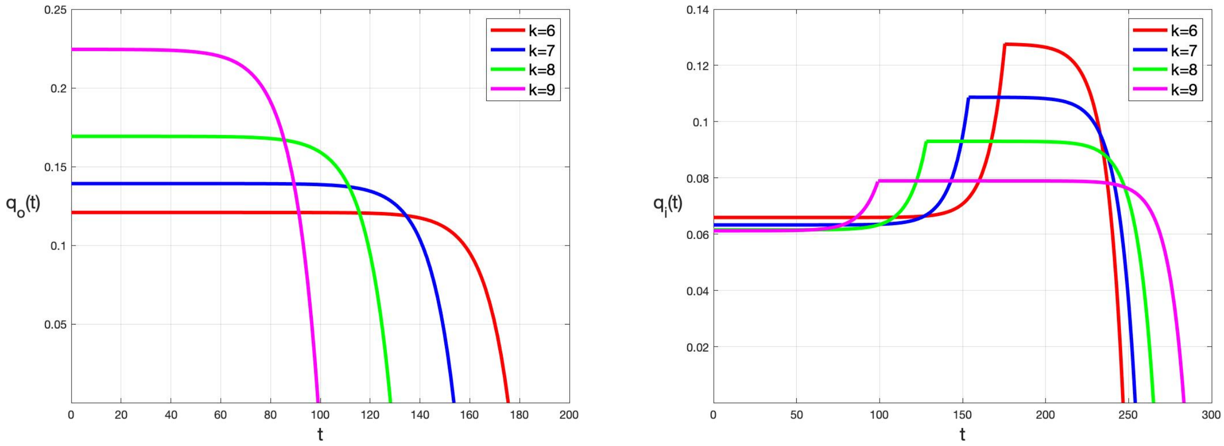


(a) Extraction path of firm $o \in O$ (b) Extraction path of firm $i \in I$

Figure 7: Comparative statics of exploitation rates with respect to v

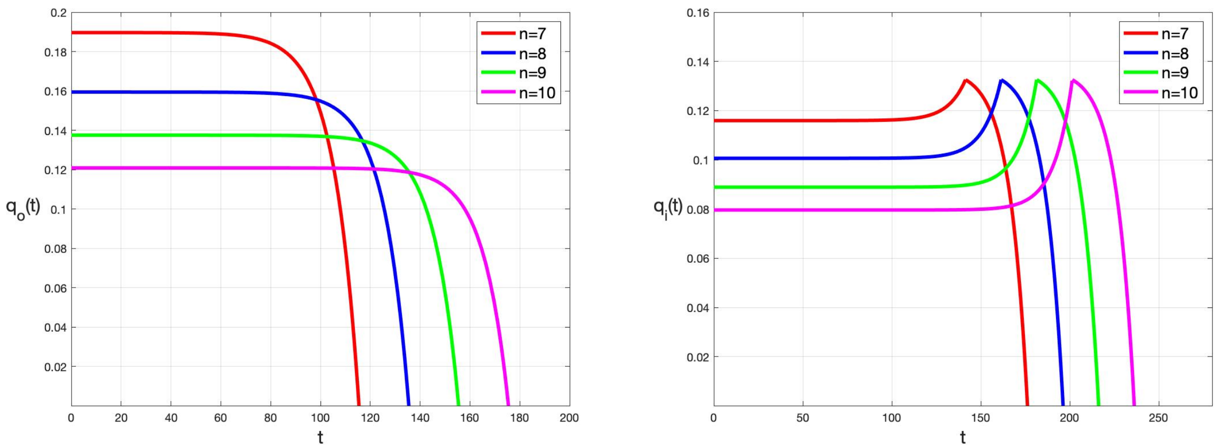
[18]It should be noted that these possible efficiency gains are also one of the reasons why firms want to participate in cross-ownership.

[19]We thank an anonymous reviewer for suggesting conducting this analysis.



(a) Extraction path of firm $o \in O$ (b) Extraction path of firm $i \in I$

Figure 8: Comparative statics of exploitation rates with respect to k



(a) Extraction path of firm $o \in O$ (b) Extraction path of firm $i \in I$

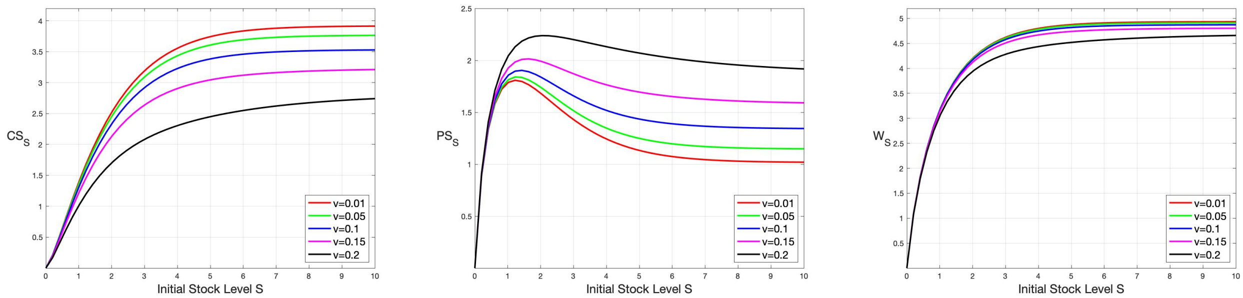
Figure 9: Comparative statics of exploitation rates with respect to n

We first conduct the comparative statics of exploitation rates (q_i and q_o) with respect to v, k and n, respectively. Using the parameter values $a = b = 1, c = 0, r = 0.1$ and $S = 20$, we plot the equilibrium extraction paths of a typical insider firm $i \in I$ and an outsider firm $o \in O$ for different levels of cross-ownership v when fixing $k = 6, n = 9$ in Figure 7, for different number of cross-ownership participants k when fixing $v = 0.1$ and $n = 10$ in Figure 8, and for different number of industry players n when fixing $k = 6$ and $v = 0.1$ in Figure 9. We observe the following result:

Result 3. *Ceteris paribus, an increase in v or k and a decrease in n will accelerate the speed at which the outsiders deplete their resource but delay the exhaustion of the cross-owners.*

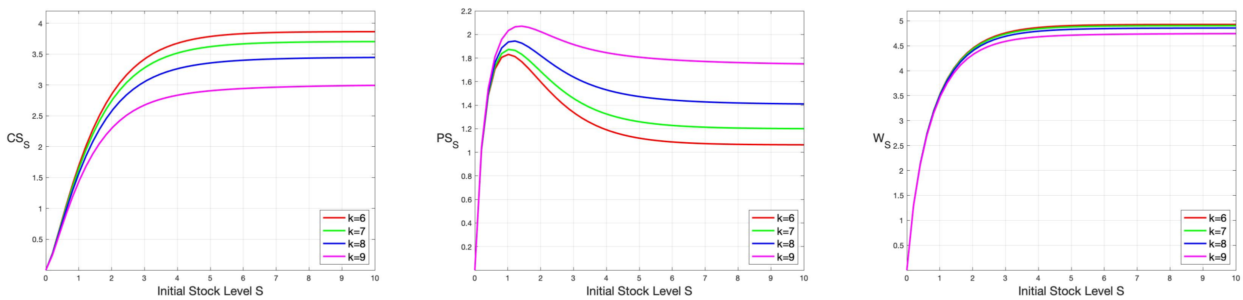
This is intuitive. An increased cross-ownership either in levels (v) or ratios (k/n) results in a weaker competition between insiders, but this induces the outsiders to compete more aggressively.

Each outsider starts with a higher exploitation rate and speeds up their resource exhaustion, while the insiders slow down their extraction due to their ownership stakes in rival firms and enjoy the cross-ownership conferred market power after the outsiders deplete their stocks, thereby delaying their resource exhaustion to a later date.



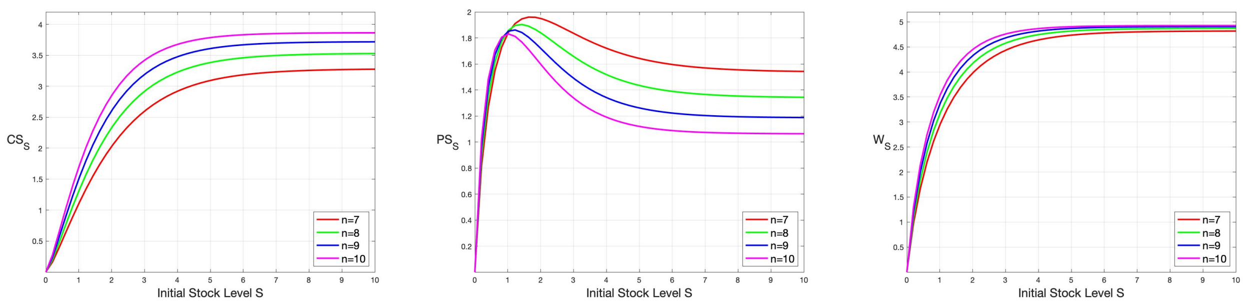
(a) CS as a function of initial stock (b) PS as a function of S (c) Welfare as a function of S

Figure 10: Comparative statics of CS_S, PS_S and W_S with respect to v



(a) CS as a function of S (b) PS as a function of S (c) Welfare as a function of S

Figure 11: Comparative statics of CS_S, PS_S and W_S with respect to k



(a) CS as a function of S (b) PS as a function of S (c) Welfare as a function of S

Figure 12: Comparative statics of CS_S, PS_S and W_S with respect to n

Using the parameter values $a = b = 1, c = 0$ and $r = 0.1$, we plot the discounted sum of consumer surplus, industry profits as well as welfare as a function of initial stock S for different v when fixing $k = 6$ and $n = 8$ in Figure 10, for different k when fixing $v = 0.1$ and $n = 10$ in Figure

11, and for different n when fixing $k = 6$ and $v = 0.1$ in Figure 12, respectively. From these figures, we observe the following result:

Result 4. *Ceteris paribus, an increase in v or k results in a decrease in consumer surplus, an increase in industry profits and ultimately a decrease in social welfare for all levels of resource stock S.*

Result 5. *Ceteris paribus, a decrease in n will lead to a reduction in both consumer surplus and social welfare for all levels of initial resource stock S but a rise in industry profits when S is large. However, for S positive and small enough, a decrease in n will result in less industry profits.*

Indeed, for any S, an increase in v or k will lead to a larger output reduction from the insiders. Even though the outsiders respond aggressively by expanding their outputs, the reduction from the cross-owners more than offsets the increase from the outsiders. As a result, the industry output decreases and market price increases. This brings down consumer surplus but increases industry profits. However, the overall reduction from consumer surplus always dominates the increase in industry profits, thereby resulting in a welfare loss.

The same intuition illustrated earlier (equivalently when k increases) applies for the effect of a decrease of n on consumer surplus, industry profits when S is large enough and social welfare. However, when the initial resource stock owned by each firm is small enough, as the number of players in the industry decreases, outsiders will be restricted in their response to the increased cross-ownership because they have a limited resource stock. The scarcity effect then overcomes the competition effect. Since the total industry resource stock is also reduced because of a reduction in n, total industry profits decrease.

6 Conclusion

We have shown that for a nonrenewable resource industry, the profitability of a k-symmetric cross-ownership can be positive for any participation ratios, provided that the initial resource stock owned by each firm is small enough. This outcome occurs because when the cross-owners reduce their output due to their ownership stakes in the rival firms, the outsiders are limited in their response in terms of increased output due to their finite resource stocks. Consequently, the cross-ownership participants may raise prices more than in other industries without stock constraint. These findings are in sharp contrast with those obtained in cases where resource constraints are absent. Indeed, we have shown in the static case that for any levels of non-controlling minority shareholdings, a k-symmetric cross-ownership is never profitable if the participation ratio is below some lower threshold, but always profitable when the participation ratio is above some upper threshold, while there exists a large range of shareholdings for which it can be profitable when the participation ratio is in between the lower and upper thresholds. We define the result that cross-ownership may be unprofitable as a cross-ownership paradox, analogous to the merger paradox. Our analysis shows that with symmetric firms, the cross-ownership paradox applies in nonrenewable industries only when the stock is large enough.

Our paper also highlights that cross-ownership can be preferable to a full merger in terms of Cournot competition. Not only is it more profitable to participate in the cross-ownership than a horizontal merger, more importantly, it constitutes a shrewd strategy to avoid the possible legal challenges. Thus competition authorities should adapt their current lenient approach towards minority shareholdings to a stricter scrutiny. Our analysis also shows that cross-ownership may turn out to be relatively less detrimental to consumers in a nonrenewable resource industry than other industries where resource constraints are absent. Thus, antitrust authorities should consider adapting its guidelines and conduct a specific examination when dealing with industries where inter-temporal constraints play an important role. These include, for example, industries where a common property renewable resource is exploited (Colombo and Labrecciosa, 2018), or where stock pollutants are generated (De Frutos and Martín-Herrán, 2019; Arguedas, Cabo and Martín-Herrán, 2020), or where the buildup of capital is a strategic decision (Feichtinger et al., 2005; Huisman and Kort, 2015; López and Vives, 2019), or where firms compete under price stickiness (Esfahani, 2019; Colombo and Labrecciosa, 2021).

Appendix

Proof of Proposition 1:

Proof. We characterize the OL-NCOE by using optimal control theory. The current value Hamiltonian associated with the problem of a typical firm $i \in I$ is given by

$$H_i(q_i, Q_{-i}, \lambda_i, t) = \frac{1}{1-(k-2)v-(k-1)v^2}\Bigg(\Big(1-(k-2)v\Big)q_i + v\sum_{k \in I \setminus i} q_k\Bigg)\Big(a-c-bQ_{-i}-bq_i\Big) - \lambda_i q_i,$$

while that for a typical firm $o \in O$ is

$$H_o(q_o, Q_{-o}, \lambda_o, t) = (a - bQ_{-o} - bq_o - c)q_o - \lambda_o q_o$$

Exploiting symmetry, the maximum principle yields the interior solution

$$\Big(1-(k-2)v\Big)(a-c) - \Big[1+k+\Big(1-k(k-2)\Big)v\Big]bq_i - \Big(1-(k-2)v\Big)(n-k)bq_o = \lambda_i\Big(1-(k-2)v-(k-1)v^2\Big) \tag{6}$$

$$a - c - (n-k+1)bq_o - bkq_i = \lambda_o \tag{7}$$

for $i = 1, 2, \cdots, k$ and $o = k+1, \cdots, n$, with

$$\frac{d\lambda_i}{dt} = r\lambda_i \tag{8}$$

$$\frac{d\lambda_o}{dt} = r\lambda_o \tag{9}$$

Solving for (q_i, q_o) from (6) and (7), then we get

$$q_i(t) = \frac{\Big(1-(k-2)v\Big)(a-c) - \Big(1-(k-2)v-(k-1)v^2\Big)(n-k+1)\lambda_i + \Big(1-(k-2)v\Big)(n-k)\lambda_o}{\Big((k+n+1-k^2)v+n+1\Big)b} \tag{10}$$

$$q_o(t) = \frac{(1+v)(a-c) + \Big(1-(k-2)v-(k-1)v^2\Big)k\lambda_i - \Big[1+k+\Big(1-k(k-2)\Big)v\Big]\lambda_o}{\Big((k+n+1-k^2)v+n+1\Big)b} \tag{11}$$

During the second phase where only firms $i \in I$ extract a positive quantity, the maximum principle yields

$$\Big(1-(k-2)v\Big)(a-c)-\Big[1+k+\Big(1-k(k-2)\Big)v\Big]bq_i=\lambda_i\Big(1-(k-2)v-(k-1)v^2\Big) \quad (12)$$

with

$$\frac{d\lambda_i}{dt}=r\lambda_i \tag{13}$$

Solving for q_i from (12), we obtain

$$q_i(t)=\frac{\Big(1-(k-2)v\Big)(a-c)-\Big(1-(k-2)v-(k-1)v^2\Big)\lambda_i}{\Big[1+k+\Big(1-k(k-2)\Big)v\Big]b} \tag{14}$$

The terminal dates T_i and T_o are endogenous and determined by

$$H_i(q_i(T_i),q_{-i}(T_i),\lambda_i(T_i),T_i)=0$$

for $i\in I$ and

$$H_o(q_o(T_o),q_{-o}(T_o),\lambda_o(T_o),T_o)=0$$

for $o\in O$. These terminal conditions along with the maximum principle imply that

$$q_i(T_i)=0, \quad q_o(T_o)=0 \tag{15}$$

From (8),(9) and (13) and continuity of the costate variable λ_i at T_o, we have

$$\lambda_i=\lambda_{i0}e^{rt} \quad \forall\, t\in[0,T_i] \tag{16}$$

$$\lambda_o=\lambda_{o0}e^{rt} \quad \forall\, t\in[0,T_o] \tag{17}$$

where λ_{i0} and λ_{o0} are determined using conditions (15) along with (14) and (11). From (14), we have

$$q_i(T_i)=\frac{\Big(1-(k-2)v\Big)(a-c)-\Big(1-(k-2)v-(k-1)v^2\Big)\lambda_{i0}e^{rT_i}}{\Big[1+k+\Big(1-k(k-2)\Big)v\Big]b}=0,$$

that is,

$$\lambda_{i0}=(a-c)\Big(1-(k-2)v\Big)\Big(1-(k-2)v-(k-1)v^2\Big)^{-1}e^{-rT_i}$$

and

$$\lambda_i = \lambda_{i0}e^{rt} = (a-c)\Big(1-(k-2)v\Big)\Big(1-(k-2)v-(k-1)v^2\Big)^{-1} e^{r(t-T_i)} \tag{18}$$

From (11), we have

$$q_o(T_o) = \frac{(1+v)(a-c)+\Big(1-(k-2)v-(k-1)v^2\Big)k\lambda_i(T_o)-\Big[1+k+\Big(1-k(k-2)\Big)v\Big]\lambda_o(T_o)}{\Big((k+n+1-k^2)v+n+1\Big)b} = 0,$$

that is,

$$\lambda_{o0} = \frac{a-c}{1+k+\Big(1-k(k-2)\Big)v}\Big[(1+v)e^{-rT_o}+k\Big(1-(k-2)v\Big)e^{-rT_i}\Big]$$

and

$$\lambda_o = \lambda_{o0}e^{rt} = \frac{a-c}{1+k+\Big(1-k(k-2)\Big)v}\Big[(1+v)e^{r(t-T_o)}+k\Big(1-(k-2)v\Big)e^{r(t-T_i)}\Big] \tag{19}$$

Substituting (18) and (19) into (10), (11) and (14) yields the Phase I ($0 \le t \le T_o$) and Phase II ($T_o \le t \le T_i$) equilibrium supply paths of all the firms as presented in (3) and (4). These equilibrium paths are determined as functions of the terminal times T_i and T_o, which are determined from the resource constraint conditions, i.e.,(5). It can be shown that such a non-linear system in (T_i, T_o) admits a unique solution, with $T_i \ge T_o$. A full proof is provided in the following.

The terminal dates T_i and T_o are determined from the resource constraint conditions. More specifically,

$$\int_0^{T_o} \frac{(1-(k-2)v)(a-c)\cdot A}{((k+n+1-k^2)v+n+1)[1+k+(1-k(k-2))v]b}dt + \int_{T_o}^{T_i} \frac{(1-(k-2)v)(a-c)\cdot B}{[1+k+(1-k(k-2))v]b}dt = S_{0i} \tag{20}$$

where

$$A = \Big[1+k+\Big(1-k(k-2)\Big)v-\Big((k+n+1-k^2)v+n+1\Big)e^{r(t-T_i)}+(n-k)(1+v)e^{r(t-T_o)}\Big]$$

$$B = \Big[1-e^{r(t-T_i)}\Big]$$

and

$$\int_0^{T_o} \frac{(a-c)(1+v)}{\Big((k+n+1-k^2)v+n+1\Big)b}\Big[1-e^{r(t-T_o)}\Big]dt = S_{0o} \tag{21}$$

From (20), we have

$$\frac{(1-(k-2)v)}{[1+k+(1-k(k-2))v]}\Big[\Big((k+n+1-k^2)v+n+1\Big)(e^{-rT_i}+rT_i-1)-(n-k)(1+v)(e^{-rT_o}+rT_o-1)\Big]$$

$$=\frac{\Big((k+n+1-k^2)v+n+1\Big)brS_{0i}}{(a-c)}$$

From (21), we have

$$(1+v)(e^{-rT_o}+rT_o-1)=\frac{\Big((k+n+1-k^2)v+n+1\Big)brS_{0o}}{(a-c)}$$

Same resource endowments $S_{0i}=S_{0o}=S$ yields

$$\frac{(1-(k-2)v)}{[1+k+(1-k(k-2))v]}\Big[\Big((k+n+1-k^2)v+n+1\Big)(e^{-rT_i}+rT_i-1)-(n-k)(1+v)(e^{-rT_o}+rT_o-1)\Big]$$

$$=(1+v)(e^{-rT_o}+rT_o-1)$$

or

$$(1-(k-2)v)\Big((k+n+1-k^2)v+n+1\Big)(e^{-rT_i}+rT_i-1)=(1+v)\Big((1-n(k-2))v+n+1\Big)(e^{-rT_o}+rT_o-1)$$

Note that

$$\begin{aligned}&(1-(k-2)v)\Big((k+n+1-k^2)v+n+1\Big)-(1+v)\Big((1-n(k-2))v+n+1\Big)\\&=-v(k-1)\big(k+v+2kv-k^2v+1\big)\\&=-v(k-1)\Big(k\big(1-(k-2)v\big)+v+1\Big)<0\quad\forall\, v\in(0,\frac{1}{k-1})\end{aligned}$$

Thus,

$$(1-(k-2)v)\Big((k+n+1-k^2)v+n+1\Big)<(1+v)\Big((1-n(k-2))v+n+1\Big).$$

Then we must have

$$f(T_i)=e^{-rT_i}+rT_i-1>f(T_o)=e^{-rT_o}+rT_o-1\quad \text{for } T_i>T_o$$

In other words, we need to show that $f(T) = e^{-rT} + rT - 1$ is an increasing function. Indeed,

$$f'(T) = -re^{-rT} + r = r(1 - e^{-rT}) > 0$$

Thus we have finished our proof.

$\square$

Proof of Proposition 2:

Proof. The profitability function G can be simplified as

$$G(k,n,v) = \frac{\Big[(n+1)(v+1)(2k-n-1) - k^2 v(k-1)\Big] v(k-1)}{\Big((k+n+1-k^2)v + n + 1\Big)^2 (n+1)^2} \left[\frac{(a-c)^2}{b}\right]$$

For any $v > 0$ and $2 \leq k \leq n$, the function G has the same sign as the function H, where

$$H\left(k,n,v\right) = \left(n+1\right)\left(v+1\right)\left(2k-n-1\right) - k^2 v\left(k-1\right)$$

This function is linear in v, indeed

$$H\left(k,n,v\right) = \left(n+1\right)\left(2k-n-1\right) + \Bigg(\left(n+1\right)\left(2k-n-1\right) - k^2\left(k-1\right)\Bigg)v,$$

which can be rewritten as

$$H\left(k,n,v\right) = H\left(k,n,0\right) + \Bigg(H\left(k,n,0\right) - k^2\left(k-1\right)\Bigg)v.$$

Clearly, when $H\left(k,n,0\right) \leq 0$, i.e. $2k - n - 1 \leq 0$ or $k \leq \dfrac{1+n}{2}$, we have $H\left(k,n,v\right) < 0$ and therefore $G < 0$. Thus, a necessary condition for cross-ownership to be profitable is that $k > \dfrac{1+n}{2}$.

We now show that $H\left(k,n,v\right)$ is a strictly decreasing function of v. Its slope is given by

$$\begin{aligned}
\frac{\partial H\left(k,n,v\right)}{\partial v} &= H\left(k,n,0\right) - k^2\left(k-1\right) \\
&= \left(n+1\right)\left(2k-n-1\right) - k^2\left(k-1\right) \\
&= \left(2k-2\right)n - n^2 + \big(2k - k^2\left(k-1\right) - 1\big) \\
&\equiv F(k,n)
\end{aligned}$$

Therefore, $F(k,n)$ is a quadratic inverted U-shaped function of n that we shall show is strictly decreasing in n for all $n \geq k$ and is negative for $n = k$, and thus negative for all $n \geq k$. Indeed

$$\frac{\partial F(k,n)}{\partial n} = 2k - 2 - 2n < 0 \quad \forall\, n \geq k$$

and at $n = k$,

$$F(k,k) = (k+1)(2k-k-1) - k^2(k-1) = -(k^2-k-1)(k-1) < 0$$

thus $F(k,n) < 0, \forall\, n \geq k$. So when $k > \frac{1+n}{2}$, for

$$H\left(k,n,v\right) = H\left(k,n,0\right) + \underbrace{\Big(H\left(k,n,0\right) - k^2\left(k-1\right)\Big)}_{=F(k,n)<0} v > 0,$$

we need

$$v < \bar{v} \equiv -\frac{H\left(k,n,0\right)}{H\left(k,n,0\right) - k^2\left(k-1\right)},$$

where $\bar{v}$ is the threshold shareholding such that $H\left(k,n,\bar{v}\right) = 0$ or $G(\bar{v}) = 0$. To sum up, $H\left(k,n,v\right)$ is a strictly decreasing linear function of v, and when $k > \frac{1+n}{2}$, we have $H\left(k,n,v\right) > 0$ or $G > 0$ if and only if $v < \bar{v} \equiv -\frac{H(k,n,0)}{(H(k,n,0)-k^2(k-1))}$.

We now determine when $\bar{v}$ is less than the upper bound of shareholdings $\frac{1}{k-1}$ by finding the sign of $H\left(k,n,\frac{1}{k-1}\right)$:

$$\begin{aligned}
H\left(k,n,\frac{1}{k-1}\right) &= (n+1)\left(2k-n-1\right) + \Big((n+1)\left(2k-n-1\right) - k^2\left(k-1\right)\Big)\frac{1}{k-1} \\
&= \frac{1}{k-1}\Big(k\left(n+1\right)\left(2k-n-1\right) - k^2\left(k-1\right)\Big) \\
&= \frac{1}{k-1}\Big(2k^2n + 2k^2 - kn^2 - 2kn - k - k^3 + k^2\Big)
\end{aligned}$$

or

$$(k-1)H\left(k,n,\frac{1}{k-1}\right) = -kn^2 + \left(2k^2 - 2k\right)n + \left(3k^2 - k^3 - k\right).$$

This $(k-1)H\left(k,n,\frac{1}{k-1}\right)$ function is a quadratic inverted U-shaped function of n and has two real roots n_1 and n_2 with

$$n_2 = k + \sqrt{k} - 1 > n_1 = k - \sqrt{k} - 1$$

and thus, it is strictly positive for $n \in (n_1, n_2)$ and negative for $n > n_2$. Since $n_1 < k < n_2$, by directly evaluating the sign of $(k-1)H\left(k,n,\frac{1}{k-1}\right)$ at $n = k$, we must have $(k-1)H\left(k,k,\frac{1}{k-1}\right) > 0$.

Therefore, for $n \in [k, n_2)$, we have $H\left(k, n, \frac{1}{k-1}\right) > 0 = H(k, n, \bar{v})$ and thus $\bar{v} > \frac{1}{k-1}$; for $n > n_2$, we have $H\left(k, n, \frac{1}{k-1}\right) < 0 = H(k, n, \bar{v})$ and thus $\bar{v} < \frac{1}{k-1}$, so there exists some $v \in (\bar{v}, \frac{1}{k-1})$ for which $G < 0$. To sum up:

1. For $k \leq \frac{1+n}{2}$, we have $G < 0$;
2. For $k > \frac{1+n}{2}$, there exists $\bar{v} \equiv -\frac{H(k,n,0)}{(H(k,n,0)-k^2(k-1))} > 0$ such that we have $G > 0$ if and only if $v < \bar{v}$, where

$$H(k, n, v) = (n+1)(2k - n - 1) + \big((n+1)(2k - n - 1) - k^2(k-1)\big)v.$$

Moreover, for $n \in [k, n_2)$, we have $\bar{v} > \frac{1}{k-1}$, therefore $G > 0$ for all admissible $v < \frac{1}{k-1}$. When $n > n_2$, then $G > 0$ for $v < \bar{v}$ and $G < 0$ for $v \in \left(\bar{v}, \frac{1}{k-1}\right)$, where $n_2 \equiv k + \sqrt{k} - 1$.

Note that the condition $k > \frac{1+n}{2}$ can be expressed as $\frac{k}{n} > \frac{k}{2k-1}$, and $n > n_2$ is equivalent to $\frac{k}{n} < \gamma(k) \equiv \frac{k}{k+\sqrt{k}-1}$, so we can draw the following conclusion: For any $2 \leq k \leq n$ and $0 < v < \frac{1}{k-1}$, the profitability of a k-symmetric cross-ownership for Cournot competitors depends on the following scenarios:

1. If $\frac{k}{n} \leq \frac{k}{2k-1}$, then $G < 0$ for all $v \in (0, \frac{1}{k-1})$;
2. If $\frac{k}{2k-1} < \frac{k}{n} < \gamma(k) \equiv \frac{k}{k+\sqrt{k}-1}$, then $G > 0$ for $v < \bar{v}$ and $G < 0$ for $v \in (\bar{v}, \frac{1}{k-1})$;
3. If $\frac{k}{n} > \gamma(k) \equiv \frac{k}{k+\sqrt{k}-1}$, then $G > 0$ for all $v \in (0, \frac{1}{k-1})$.

□

Proof of Corollary 1:

Proof. We focus on the case where $\frac{k}{n} > \frac{k}{2k-1}$ and show that $\bar{v} = -\frac{(n+1)(2k-n-1)}{(n+1)(2k-n-1)-k^2(k-1)}$ is strictly increasing in $y \equiv \frac{k}{n}$. We can rewrite $\bar{v}$ as

$$\bar{v} = -\frac{1}{\left(1 + \frac{k^2(k-1)}{(n+1)(n+1-2k)}\right)} = -\frac{1}{\left(1 + \frac{k^2(k-1)}{\left(\frac{k}{y}+1\right)\left(\frac{k}{y}+1-2k\right)}\right)}$$

Direct computation of $\frac{\partial \bar{v}}{\partial y}$ gives

$$\frac{\partial \bar{v}}{\partial y} = 2k^3 y (k-1) \frac{k(1-y) + y}{\left((k^3 - k^2 - 2k + 1) y^2 + 2k(1-k) y + k^2\right)^2} > 0.$$

□

Proof of Proposition 3:

Proof. The welfare change resulting from the k-symmetric cross-ownership is

$$\Delta TS(k,n,v) = \frac{\left[v\Big(k(k-1)-2(n+1)\Big)-2(n+1)\right]kv(k-1)}{\Big((k+n+1-k^2)v+n+1\Big)^2(n+1)^2}\left[\frac{(a-c)^2}{2b}\right]$$

For any $v > 0$ and $2 \leq k \leq n$, the function ΔTS has the same sign as the function Γ, where

$$\Gamma(k,n,v) = \Big(k(k-1)-2(n+1)\Big)v-2(n+1)$$

which is linear in v. Note that

$$\Gamma(k,n,0) = -2(n+1) < 0$$

and

$$\begin{aligned}\Gamma(k,n,\frac{1}{k-1}) &= \frac{1}{k-1}\Big(k(k-1)-2(n+1)\Big)-2(n+1)\\ &= k-2n-2-\frac{2(n+1)}{k-1} < 0\end{aligned}$$

Thus,

$$\Gamma(k,n,v) < 0 \Longleftrightarrow \Delta TS(k,n,v) < 0, \forall\, v \in (0,\frac{1}{k-1})$$

□

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