7 Externality and Public Goods

Practice Question 15 (Market Structure and Externality)**.** Suppose that the inverse demand curve for paper is

$$p = 200 - Q$$

the private marginal cost (unregulated competitive market supply) is

$$MC_p = 80 + Q$$

and the marginal harm from gunk is

$$MC_g = Q$$

(a) What is the unregulated competitive equilibrium?

(b) What is the social optimum? What specific tax (per unit of output or gunk) results in the social optimum?

(c) What is the unregulated monopoly equilibrium?

(d) How would you optimally regulate the monopoly? What is the resulting equilibrium?

Solutions:

(a) The unregulated competitive equilibrium is

$$MC_p = 80 + Q = p = 200 - Q$$

$$\Rightarrow Q = 60, \quad p = 200 - 60 = 140$$

(b) The social optimal is

$$MC_s = MC_p + MC_g = 80 + Q + Q = p = 200 - Q$$

$$\Rightarrow Q^* = 40, \quad p = 200 - 40 = 160$$

A specific tax of \$40 per unit results in this outcome:

$$t = MC_g(Q^*) = 40$$

(c) The unregulated monopoly output is the same as the socially optimal output:

$$MR = 200 - 2Q = MC_p = 80 + Q$$

$$\Rightarrow Q^m = 40 = Q^*, \quad p = 200 - 40 = 160$$

(d) The monopolist is already producing the socially optimal output level and thus does not require regulation.

Practice Question 16 (Private Provision of Public Goods)**.** Anna and Bess are assigned to write a joint paper within a 24-hour period about the Pareto optimal provision of public goods. Let t_A denote the number of hours that Anna contributes to the project and t_B the number of hours that Bess contributes. The numeric grade that Anna and Bess earn is a function,

$$23 \ln(t_A + t_B)$$

of the total number of hours that they contribute to the project. If Anna contributes t_A, then she has $(24 - t_A)$ hours in the day for leisure. Anna's utility function is

$$U_A = 23ln(t_A + t_B) + ln(24 - t_A)$$

and Bess's utility function is

$$U_B = 23ln(t_A + t_B) + ln(24 - t_B)$$

(a) If they choose the hours to contribute simultaneously and independently, what is the Nash equilibrium number of hours that each will provide?

(b) What is the number of hours each should contribute to the project that maximizes the sum of their utilities?

Solutions:

(a) In Nash equilibrium, each person maximizes her utility taking the number of hours the other works as given. Taking the partial derivative of U_A with respect to t_A and putting it equal to zero, we get

$$24t_A + t_B = 552$$

Taking the partial derivative of U_B with respect to t_B and putting it equal to zero, we get

$$24t_B + t_A = 552$$

Solving these two equations we get

$$t_A = t_B = 22.08$$

(b) To find the number of hours that maximizes the sum of utilities, we take the partial derivative of the sum, once with respect to t_A and once with respect to t_B, and put them equal to zero. We get the two equations,

$$47t_A + t_B = 1,104$$

$$t_A + 47t_B = 1,104$$

Solving these two equations we get

$$t_A = t_B = 23$$

Therefore Anna and Bess, while maximizing their utilities, would free ride.

Practice Question 17 (Public Goods)**.** Consider good x with two consumers. Consumer 1's MWTP is given by $MWTP_1 = 1 - Q_1$, while consumer 2's MWTP is $MWTP_2 = 2 - Q_2$.

1. Assume that x is not a public good, compute the social demand for this economy.

2. Assume that x is a public good.

   (a) Explain two characteristics of good x.

   (b) Compute the aggregate MWTP for this economy.

   (c) Suppose the marginal social cost is given by $MSC = 5Q$. Compute the social efficient output level Q^*.

(d) Is the market, without government intervention, going to produce Q^*, given that x is a public good? Explain why or why not?

Solutions:

1. x is a non-public good $\Rightarrow$ Horizontal summation at the same P.

$$P_1 = MWTP_1 = 1 - Q_1 \Rightarrow Q_1 = 1 - P$$
$$P_2 = MWTP_2 = 2 - Q_2 \Rightarrow Q_2 = 2 - P$$

When $0 \leq P \leq 1$,
$$Q_{agg} = Q_1 + Q_2 = 3 - 2P$$

When $1 \leq P \leq 2$,
$$Q_{agg} = Q_2 = 2 - P$$

So the social demand for this economy is

$$Q_{agg} = \begin{cases} 3 - 2P & \text{if } 0 \leq P \leq 1 \\ 2 - P & \text{if } 1 \leq P \leq 2 \end{cases}$$

2. (a) Non-rivalrous and non-excludable such that individuals cannot be effectively excluded from use and where use by one individual does not reduce availability to others.

(b) x is public good $\Rightarrow$ vertical summation at the same Q.
When $0 \leq Q \leq 1$,

$$MWTP_{agg} = MWTP_1 + MWTP_2 = 1 - Q + 2 - Q = 3 - 2Q$$

When $2 \geq Q \geq 1$,
$$MWTP_{agg} = MWTP_2 = 2 - Q$$

So, the aggregate MWTP for this economy is

$$MWTP_{agg} = \begin{cases} 3 - 2Q & \text{if } 0 \leq Q \leq 1 \\ 2 - Q & \text{if } 1 \leq Q \leq 2 \end{cases}$$

(c) $MSC = MWTP_{agg}$ (The MSC line will intersect the line (3-2Q)).

$$\Rightarrow 5Q = 3 - 2Q$$

$$\Rightarrow Q^* = \frac{3}{7}$$

(d) No.
Free-riding incentive of consumers means that individual consumers will pay less than their true MWTP.
Under-provision of x by the market because it is a public good.