1 Elasticity and Monopoly

Practice Question 1. A statistician estimates the demand for pizzas (Q_1) to be given by:

$$Q_1 = 20 + 0.1m - 2p_1 + 0.5p_2$$

Where m is income, p_1 is the price of pizzas and p_2 is the price of a bucket of fried chicken.

(a) Suppose $m = 200$ and $p_2 = 10$. Find the price elasticity of demand when $p_1 = 10$ and explain this in words. At this price, is the demand for pizza elastic or inelastic?

(b) Suppose $m = 200$ and $p_1 = 10$. Find the cross-price elasticity of demand when $p_2 = 10$, and explain this in words. Is fried-chicken a substitute for pizza?

(c) Suppose $p_1 = 10$ and $p_2 = 10$. Find the income elasticity of demand when $m = 200$, and explain this in words. At this income, is pizza a necessity or a luxury good?

(d) Now fix $m = 200$ and $p_2 = 10$. Suppose Domino's Pizza dominates the whole pizza market, find the MR function. What is the relationship between MR and price elasticity of demand? Verify it when $p_1 = 10$.

Solutions:

(a) Note that

$$Q_1 = 20 + 0.1(200) - 2P_1 + 0.5(10)$$
$$\Rightarrow Q_1 = 20 + 20 - 2P_1 + 5$$
$$\Rightarrow Q_1 = 45 - 2P_1$$
$$\varepsilon = \frac{\Delta Q_1}{\Delta P_1} \cdot \frac{P_1}{Q_1} = (-2) \cdot \frac{10}{45 - 2(10)} = -0.8$$

Holding all others constant, if the price of pizzas goes up by 1%, the quantity of pizzas demanded falls by 0.8%.$\Rightarrow$ Inelastic, since $|\varepsilon| < 1$

(b) Now

$$Q_1 = 20 + 0.1(200) - 2(10) + 0.5P_2$$
$$\Rightarrow Q_1 = 20 + 0.5P_2$$
$$\Rightarrow \varepsilon_{12} = \frac{\Delta Q_1}{\Delta P_2} \cdot \frac{P_2}{Q_1} = 0.5 \cdot \frac{10}{20 + 0.5(10)} = 0.2$$

Holding all others constant, if a bucket of fried chicken goes up by 1% in price, the quantity of pizzas demanded rises by 0.2%. $\Rightarrow$ Substitute, since $P_2 \uparrow, X_1 \uparrow, \varepsilon_{12} > 0$.

(c) Now

$$Q_1 = 20 + 0.1m - 2(10) + 0.5(10)$$
$$\Rightarrow Q_1 = 5 + 0.1m$$
$$\Rightarrow \varepsilon_m = \frac{\Delta Q_1}{\Delta m} \cdot \frac{m}{Q_1} = 0.1 \cdot \frac{200}{5 + 0.1(200)} = 0.8$$

Holding all others constant, if income rises by 1%, the quantity demanded of pizzas rises by 0.8%. $\Rightarrow$ Necessity, since $\varepsilon_m > 0$(normal),but $\varepsilon_m < 1$.

(d) Since $Q_1 = 45 - 2P_1 \Rightarrow P_1 = 22.5 - 0.5Q_1$

$$\Rightarrow MR = 22.5 - Q_1$$

when $P_1 = 10, Q_1 = 45 - 2(10) = 25, \varepsilon = -0.8$(see part a)

$$MR = 22.5 - 25 = -2.5$$

Notice that marginal revenue is negative where the demand curve is inelastic, $-1 < \varepsilon \leq 0$. We know $MR = P(1 + \frac{1}{\epsilon})$, and this relationship is confirmed by checking

$$-2.5 = 10(1 - \frac{1}{0.8}) = 10 \times (-0.25)$$

Practice Question 2. Consider a monopoly selling a product with the following inverse demand

$$p = 270 - 3Q$$

(a) The monopoly is producing $Q = 50$. Is the following statement **True or False**:"It is not possible to establish whether or not the monopoly is maximizing profits since we do not know the monopoly's cost function".

(b) Determine the price charged by the monopoly if the marginal cost of production is

$$MC(Q) = 3Q$$

(c) Determine the socially optimal outcome.

(d) Determine the deadweight loss of the monopoly.

(e) Determine the impact of a deadweight loss of a per unit tax $t = \$18$ on the monopoly's production.

Solutions:

(a) The statement is **False**! At $Q = 50$, $p = 270 - 3(50) = 120$, then the price elasticity of demand is

$$\Rightarrow \varepsilon = \frac{\Delta Q}{\Delta p} \cdot \frac{P}{Q} = -\frac{1}{3} \cdot \frac{120}{50} = -0.8$$

So the **demand is inelastic** at $Q = 50$ and therefore the monopolist can increase her profit just by reducing output. The monopoly is thus not maximizing profits.

Another possible explanation could be that a monopolist **will never** produce where the demand curve is inelastic $(-1 < \varepsilon \leq 0)$ as the marginal revenue is negative:

$$MR(Q) = 270 - 6Q$$

$$\Rightarrow MR(50) = 270 - 6 \times 50 = -30 < 0$$

(b) The monopoly will set

$$MR = MC$$

$$\Rightarrow 270 - 6Q = 3Q$$

$$\Rightarrow Q^m = 30, \quad p^m = 270 - 3(30) = 180$$

(c) The socially optimal outcome will be

$$p = MC$$

$$\Rightarrow 270 - 3Q = 3Q$$

$$\Rightarrow Q^* = 45, \quad p^* = 3(45) = 135$$

(d) At the monopoly output $Q^m = 30$, $MC(Q^m) = 3(30) = 90$, so the deadweight loss of the monopoly is

$$DWL = \frac{(p^m - MC(Q^m))(Q^* - Q^m)}{2} = \frac{(180 - 90)(45 - 30)}{2} = 675$$

(e) The monopoly produces such that

$$270 - 6Q = 3Q + 18$$

$$\Rightarrow 9Q = 252$$

$$\Rightarrow Q^t = 28, \quad p^t = 270 - 3(28) = 186$$

At $Q^t = 28$, $MC(Q^t) = 3(28) = 84$. Therefore the deadweight loss under the tax is

$$DWL^t = \frac{(p^t - MC(Q^t))(Q^* - Q^t)}{2} = \frac{(186 - 84)(45 - 28)}{2} = 867$$

The tax resulted in an increase of the DWL by

$$\Delta DWL = 867 - 675 = 192$$