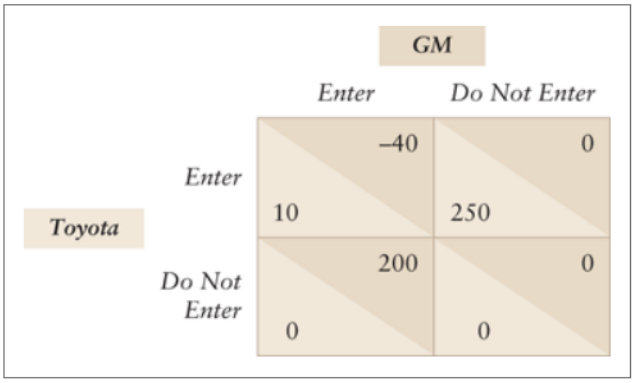
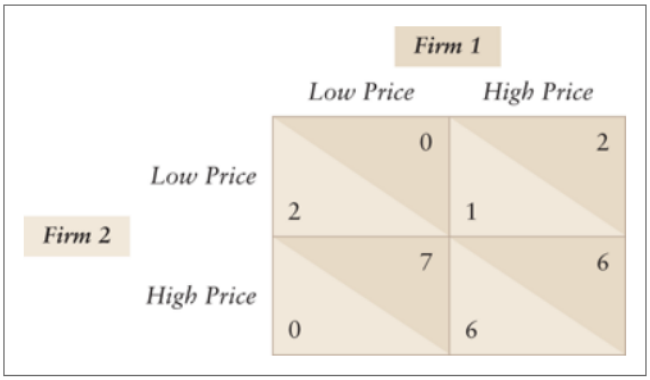
6 Game Theory

***1.4** Suppose that Toyota and GM are considering entering a new market for electric automobiles and that their profits (in millions of dollars) from entering or staying out of the market are



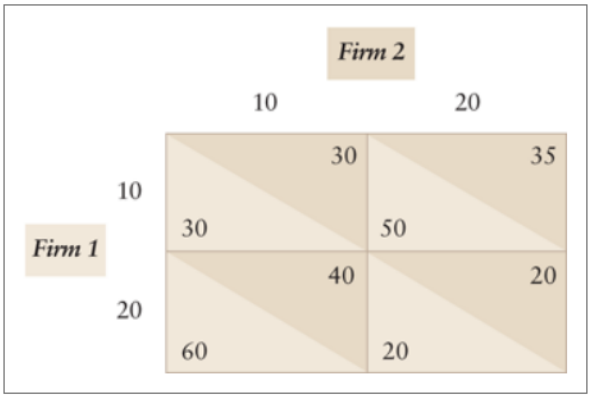
If the firms make their decisions simultaneously, do either or both firms enter? How would your answer change if the U.S. government committed to paying GM a lump-sum subsidy of $50 million on the condition that it would produce this new type of car?

***1.12** Two firms face the following payoff matrix:



Given these profits, Firm 2 wants to match Firm 1's price, but Firm 1 does not want to match Firm 2's price. Does either firm have a dominant strategy? Does this game have a unique, pure-strategy Nash equilibrium? Identify all pure- and mixed-strategy Nash equilibria. (*Hint*: See Solved Problems 14.1 and 14.2.) **A**

*3.1 Two firms are planning to sell 10 or 20 units of their goods and face the following profit matrix:



a. What is the Nash equilibrium if both firms make their decisions simultaneously?

b. Draw the game tree if Firm 1 can decide first. What is the outcome? Why?

c. Draw the game tree if Firm 2 can decide first. What is the outcome? Why?

*3.13 Before entry, the incumbent earns a monopoly profit of $\pi_m = \$10$ (million). If entry occurs, the incumbent and entrant each earn the duopoly profit, $\pi_d = \$3$. Suppose that the incumbent can induce the government to require all firms to install pollution-control devices that cost each firm $4. Show the game tree. Should the incumbent urge the government to require pollution-control devices? Why or why not?

Solutions:

1.4 We start by checking for dominant strategies. Given the payoff matrix, Toyota always does at least as well by entering the market. If GM enters, Toyota earns 10 by entering and 0 by staying out of the market. If GM does not enter, Toyota earns 250 if it enters and 0 otherwise. Thus entering is Toyota's dominant strategy. GM does not have a dominant strategy. It wants to enter if Toyota does not enter (earning 200 rather than 0), and it wants to stay out if Toyota enters (earning 0 rather than –40). Because GM knows that Toyota will enter (entering is Toyota's dominant strategy), GM stays out of the market. Toyota's entering and GM's not entering is a Nash equilibrium. Given the other firm's strategy, neither firm wants to change its strategy. Next, we examine how the subsidy affects the payoff matrix and dominant strategies. The subsidy does not affect Toyota's payoff, so Toyota still has a dominant strategy: It enters the market. With the subsidy, GM's payoff if it enters increases by 50: GM earns 10 if both enter and 250 if it enters and Toyota does not. With the subsidy, entering is a dominant strategy for GM. Thus, both firms' entering is a Nash equilibrium.

1.12 The firms do not have dominant strategies and the game does not have a pure-strategy Nash equilibrium.

A mixed strategy is one in which each player chooses among possible actions according to probabilities it assigns. Equate expected payoffs to determine the mixed-strategy equilibrium. In particular, if Firm 2 picks the low price with probability θ_2, Firm 1's expected payoff (*EP*) from picking the low price is

$$EP(\text{Low}) = 7 - 7\theta_2$$

and Firm 1's expected payoff from picking the high price is

$$EP(\text{High}) = 2\theta_2 + 6(1 - \theta_2)$$

$$EP(\text{High}) = 6 - 4\theta_2.$$

Firm 1's expected payoffs are equal when

$$7 - 7\theta_2 = 6 - 4\theta_2$$

$$1 = 3\theta_2$$

$$\theta_2 = 0.333.$$

Similarly, if Firm 1 picks the low price with probability θ_1, then Firm 2's expected payoff from picking the low price is

$$EP(\text{Low}) = 2\theta_1 + (1 - \theta_1)$$

$$EP(\text{Low}) = 1 + \theta_1,$$

and Firm 2's expected payoff from picking the high price is

$$\text{EP(High)} = 6 - 6\theta_1.$$

Firm 2's expected payoffs are equal when

$$1 + \theta_1 = 6 - 6\theta_1$$

$$7\theta_1 = 5$$

$$\theta_1 = 0.714.$$

Thus, the mixed-strategy Nash equilibrium is for Firm 1 to pick the low price with probability 0.714 and for Firm 2 to pick the low price with probability 0.333.

3.1

(a) There are two Nash equilibria (the off diagonals). If either firm produces 20 while the other produces 10, neither player has an incentive to change strategies given the strategy of the other player.

(b) If Firm 1 moves first and picks Q = 10, then Firm 2 will pick Q = 20, and Firm 1's profit will be \$50. If Firm 1 moves first and picks Q = 20, then Firm 2 will pick Q = 10, and Firm 1's profit will be \$60. Profit of \$60 is greater than \$50, so Firm 1 will pick Q = 20, and Firm 2 will pick Q = 10.

(c) If Firm 2 moves first and picks Q = 10, then Firm 1 will pick Q = 20, and Firm 2's profit will be \$40. If Firm 2 moves first and picks Q = 20, then Firm 1 will pick Q = 10, and Firm 2's profit will be \$35. Profit of \$40 is greater than \$35, so Firm 2 will pick Q = 10, and Firm 1 will pick Q = 20.

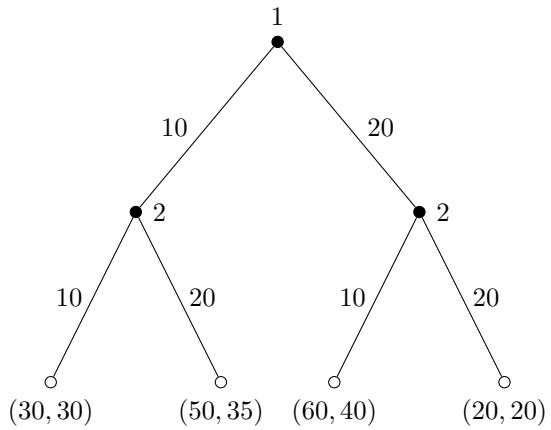


Figure 1: (b)

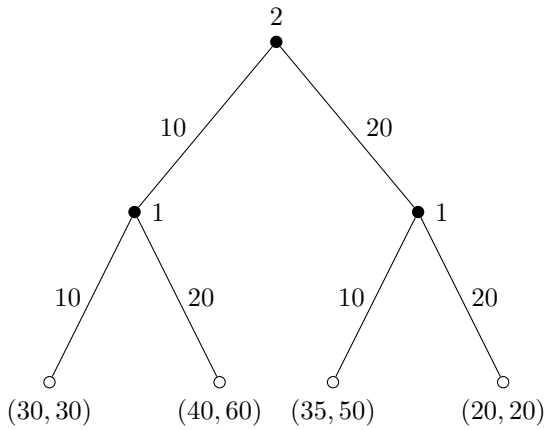
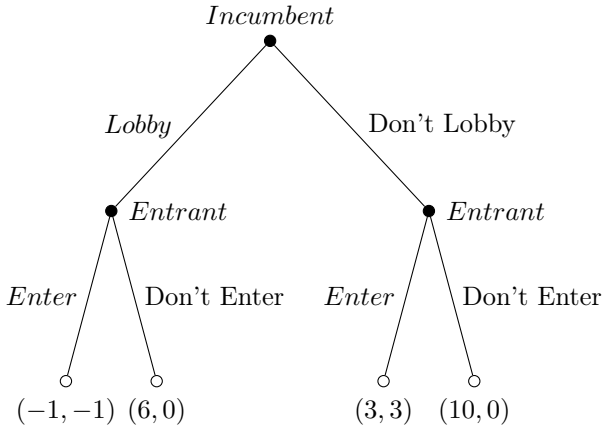


Figure 2: (c)

3.13 The extensive form of the game is



Notice that the strategy set of Incumbent is (Lobby, Don't lobby), the strategy set of Entrant is (Enter if Incumbent lobbies, Do not enter if Incumbent lobbies, Enter if Incumbent doesn't lobby, Do not enter if Incumbent doesn't lobby): that is {Enter,Enter},{Enter,Do Not Enter},{Do not Enter, Enter} and {Do not Enter, Do not Enter}. If Incumbent lobbies, then the Entrant would prefer not to Enter because -1 $<$ 0 and thus Incumbent's payoff is 6. If Incumbent does not lobby, then the Entrant would prefer to Enter because 3 $>$ 0 and thus Incumbent's payoff is 3. Payoff of \$6 is greater than \$3, so Incumbent should lobby the government to require all firms to install pollution-control devices and Entrant will not enter as a result. This is the outcome of the subgame perfect Nash equilibrium(SPNE):({Lobby}; {Do not enter if lobbying and Enter if no lobbying}). **You should be aware that** this SPNE is not the only Nash equilibrium. There is one other NE: ({Don't Lobby}; {Enter if lobbying and Enter if no lobbying}). It is an equilibrium only because if there is lobbying, the Entrant is choosing to Enter which is not realistic (since -1$<$0). This is not a realistic equilibrium. It is based on empty threat.