4 Price Discrimination and Cournot Oligopoly

Practice Question 9 (Two-part Tariff)**.** Assume a monopoly faces two customers in a market. Customer 1 has an inverse demand of

$$p = 70 - q_1$$

and costumer 2 has an inverse demand of

$$p = 90 - q_2$$

The marginal cost of production is constant and equal to 25. The monopoly uses a lump-sum fee and a per unit charge as a pricing scheme.

(a) Consider the following statement: "If the monopoly is able to offer each customer a separate scheme (the scheme offered to customer 1 may differ from the scheme offered to customer 2), then the monopoly will charge a cheaper per-unit price to customer 1 since her demand is more elastic than the demand of customer 2." Is this statement true, false or uncertain. Explain.

(b) Suppose now that the monopoly cannot discriminate between the two customers, the same scheme must be offered to both customers. Write the objective of the monopoly as a function of the per unit price charged.

(c) Determine the optimal per unit price and lump-sum fee.

(d) Discuss the impact of the presence of customer 1 on customer 2's welfare.

Solutions:

(a) False. If the monopoly can use a consumer specific scheme, the per unit price would be the marginal cost of production which is constant and equal to 25. Thus it will be the same for both customers.

(b) Since now the monopoly cannot discriminate between the two customers, then when both customers buy, the objective function of the monopoly is

$$\pi = (90 - p)p + (70 - p)p - 25(90 - p + 70 - p) + 2L$$

where the lump-sum fee is

$$L = \frac{(70 - p)^2}{2}$$

(How do you get L? Since customer 1's CS is smaller than customer 2 at every price p, the monopoly will thus charge 1's CS if both customers are served.)
This objective function can be further simplified to

$$\Rightarrow \pi = (160 - 2p)p - 25(160 - 2p) + (70 - p)^2$$

$$\Rightarrow \pi = (p - 25)(160 - 2p) + (70 - p)^2$$

$$\Rightarrow \pi = -p^2 + 70p + 900$$

(c) The optimal unit price will be such that

$$\frac{d\pi}{dp} = -2p + 70 = 0$$

$$\Rightarrow p^* = 35$$

and thus the lump-sum fee is

$$L = \frac{(70-35)^2}{2} = 612.5$$

(d) Without customer 1, customer 2's welfare would be 0 instead of being positive when customer 1 is present. Why? Because he will be charged a per unit price of 25 and a lump-sum fee equal to

$$\frac{(90-25)^2}{2} = 2112.5$$

With the presence of customer 1, the lump-sum fee of 2112.5 would too expensive to get customer 1 to buy anything. In order to attract customer 1, the monopoly reduces the lump-sum and increases the per unit price, leaving some surplus to consumer 2. Therefore consumer 2 benefits from the presence of consumer 1.

Practice Question 10 (Bundling)**.** Suppose there are two types of customers (the number of each type of customers are equal). The maximum willingness to pay of each type, for each of two phone services (Talk, Data), is given by

	Consumer 1	Consumer 2
Talk	15	12
Data	20	25

Assume the marginal cost of the service provider is constant equal to 4 for offering the Talk service to each customer and 5 for offering the Data service to each customer.

(a) Without computing profits under different scenarios, discuss whether the firm will prefer to use bundling or not?

(b) Confirm your answer in part (a) by computing the profits with and without bundling.

Solutions:

(a) Yes, this is a case of negative correlation. Therefore bundling is more profitable.

(b) With bundling, the price of bundle should be 35, for a profit of

$$35 \times 2 - 2 \times (4+5) = 70 - 18 = 52$$

Without bundling, the price of Talk would be 12 and Data 20, for a profit of

$$(12-4) \times 2 + (20-5) \times 2 = 46$$

Practice Question 11 (Cournot Duopoly)**.** The jet aircraft industry is dominated by two major competitors: Airbus (A) and Boeing (B). Their costs functions are given by:

$$TC(y_A) = 20y_A$$
$$TC(y_B) = 20y_B$$

Assume there are no fixed costs. The inverse demand function for jets by major airlines is estimated to be

$$p(y) = 200 - y$$

(a) Find the best response functions for Boeing $BR_B(y_A)$ and Airbus $BR_A(y_B)$. Plot them in a graph.

(b) Find the market price of an aircraft, the level of individual and aggregate production in a Cournot-Nash equilibrium. Also find the level of profit of each individual firm.

(c) What is the deadweight loss associated with oligopolistic trading by the two firms?

Solutions:

(a) Boeing's profit function is

$$\pi_B(y_A, y_B) = (200 - y_A - y_B)y_B - 20y_B$$

The first-order condition is

$$\frac{\partial \pi_B}{\partial y_B} = 200 - y_A - 2y_B - 20 = 0$$
$$\Rightarrow 2y_B = 180 - y_A$$

Thus the best response function for Boeing is

$$y_B = BR_B(y_A) = \frac{180 - y_A}{2}$$

Do the same for Airbus, and then the best response function for Airbus is

$$y_A = BR_A(y_B) = \frac{180 - y_B}{2}$$

(b) The Cournot-Nash equilibrium level of output produced by each firm is obtained by solving the best response functions simultaneously. It will be the intersection of the two best response curves in the diagram. Then we have

$$y_A = y_B = 60$$

Total output is thus

$$y = y_A + y_B = 60 + 60 = 120$$

The associated market price is then

$$p = 200 - y = 200 - 120 = 80$$

The profit for each firm is

$$\pi_A = \pi_B = (80 - 20) \times 60 = 3600$$

(c) To find the DWL, we first need to find the Pareto efficient output, where $p = MC$. Then

$$\Rightarrow 200 - y = 20$$
$$\Rightarrow y^* = 180$$

The DWL is

$$DWL = \frac{1}{2}(80 - 20)(180 - 120) = 1800$$