3 Price Discrimination

Practice Question 5 (First-degree Price Discrimination)**.** If a monopoly faces an inverse demand curve of

$$p(Q) = 150 - Q,$$

has a constant marginal and average cost of $MC = 30$, and can perfectly price discriminate among the consumers.

(a) What is its profit? What are the consumer surplus, welfare, and deadweight loss? Is the allocation Pareto efficient?

(b) How would these results change if the firm were a single-price monopoly?

(c) Explain why many firms usually do not perfectly price discriminate in reality?

Solutions:

(a) If the monopolist can perfectly price discriminate, i.e., it knows exactly how much each consumer is willing to pay for each unit of its good, then the monopolist will charge each consumer his or her reservation price(the maximum amount a person would be willing to pay). In this case, the perfectly price-discriminating monopoly will capture all the consumer surplus, and its profit will be the total producer surplus.

$$p(Q) = 150 - Q = 30 \Rightarrow Q^* = 120$$

$$CS = 0, \qquad \pi = PS = \frac{1}{2} \times 120 \times 120 = 7200$$

The welfare will be

$$W = PS = 7200$$

The deadweight loss will be

$$DWL = 0$$

as the last unit is sold at the price, $p = 30$, that equals the marginal cost $MC = 30$. The outcome is Pareto efficient since there is no DWL.

(b) With a single-price monopoly which charges all its customers the same price because it cannot distinguish among them, it will set

$$MR = MC$$

$$\Rightarrow 150 - 2Q = 30$$

$$\Rightarrow Q^m = 60, \quad p^m = 150 - 60 = 90$$

$$\Rightarrow CS^m = \frac{(150-90) \times 60}{2} = 1800, \quad PS^m = (90 - 30) \times 60 = 3600 = \pi$$
$$W^m = 1800 + 3600 = 5400, \quad DWL = 7200 - 5400 = 1800$$

(c) Transaction costs are a major reason why these firms do not perfectly price discriminate: It is too difficult or costly to gather information about each customer's price sensitivity.

Practice Question 6 (Third-degree Price Discrimination)**.** A monopoly book publisher with a constant marginal cost and average cost of $MC = 9$ sells in only two countries and faces a linear inverse demand curve of $p_1 = 6 - 0.5Q_1$ in Country 1 and $p_2 = 15 - Q_2$ in Country 2.

(a) What price does the monopoly charge in each country, how much does it sell in each, and what profit does it earn in each with a ban against shipments between the countries?

(b) Suppose now there is free trade between Country 1 and Country 2, how will the results change?

(c) How would the analysis change if $MC = 1$?

Solutions:

(a) With a ban against shipments between the countries, the monopoly will be able to charge the monopoly price in each country. For profit-maximization, it will set $MR = MC$ in each country. In Country 1,

$$MR_1 = 6 - Q_1 = 9 \Rightarrow Q_1^* = 0, \quad p_1^* = 9(\text{ or any price higher than } 9), \quad \pi_1^* = 0$$

In Country 2,

$$MR_2 = 15 - 2Q_2 = 9 \Rightarrow Q_2^* = 3, \quad p_2^* = 15 - 3 = 12, \quad \pi_2^* = (12 - 9) \times 3 = 9$$

(b) Now imports are permitted so that price discrimination is impossible. If the monopoly cannot price discriminate, then it charges the same price, p, in both countries. We can determine the aggregate demand curve it faces by horizontally summing the demand curves in each country at a given price. Note that no books are sold in Country 1 at prices above 6, so the total demand curve equals Country 2's demand curve at prices above 6:

$$Q_{agg} = Q_2 = 15 - p \quad \text{for} \quad p \geq 6$$

In the range of prices where positive quantities are sold in each country($p \leq 6$), the total demand function is

$$Q_{agg} = Q_1 + Q_2 = 12 - 2p + 15 - p = 27 - 3p \quad \text{for} \quad p \leq 6$$

(You need to get the demand function for $Q_1 = Q_1(p)$ first:

$$p_1 = 6 - 0.5Q_1 \Rightarrow 0.5Q_1 = 6 - p_1 \Rightarrow Q_1 = 12 - 2p_1)$$

So the aggregate demand function is

$$Q_{agg} = \begin{cases} 15 - p & \text{for} \quad 6 \leq p \leq 15 \\ 27 - 3p & \text{for} \quad 0 \leq p \leq 6 \end{cases}$$

Then the inverse demand function is

$$p = \begin{cases} 15 - Q & \text{for} \quad 6 \leq p \leq 15 \quad \text{(Region 1)} \\ 9 - \frac{1}{3}Q & \text{for} \quad 0 \leq p \leq 6 \quad \text{(Region 2)} \end{cases}$$

So the marginal revenue function is

$$MR = \begin{cases} 15 - 2Q & \text{for} \quad 6 \leq p \leq 15 \\ 9 - \frac{2}{3}Q & \text{for} \quad 0 \leq p \leq 6 \end{cases}$$

At $MC = 9$, we set $MR = MC$, then

$$15 - 2Q = 9 \Rightarrow Q^m = 3, \quad p^m = 15 - 3 = 12 \in [6, 15], \quad \pi^m = (12 - 9) \times 3 = 9$$

$$9-\frac{2}{3}Q=9 \Rightarrow Q^m=0, \quad p^m=9>6(\text{ Not possible })$$

So the monopoly would choose to sell at

$$Q^m=3, \quad p^m=12$$

and make a profit of

$$\pi^m=9$$

(c) If $MC=1$, **with a ban** on the shipments,

$$MR_1=6-Q_1=1 \Rightarrow Q_1^*=5, \quad p_1^*=6-0.5(5)=3.5, \quad \pi_1^*=(3.5-1)\times 5=12.5$$

$$MR_2=15-2Q_2=1 \Rightarrow Q_2^*=7, \quad p_2^*=15-7=8, \quad \pi_2^*=(8-1)\times 7=49$$

Without a ban, we set $MR=MC$, then

$$15-2Q=1 \Rightarrow Q^m=7, \quad p^m=15-7=8 \in [6,15], \quad \pi^m=(8-1)\times 7=49$$

$$9-\frac{2}{3}Q=1 \quad \Rightarrow Q^m=12, \quad p^m=9-\frac{12}{3}=5 \in [0,6], \quad \pi^m=(5-1)\times 12=48<49$$

So the monopoly would choose to sell at

$$Q^m=7, \quad p^m=8$$

and make a profit of

$$\pi^m=49$$

Practice Question 7 (Non-linear Pricing)**.** Suppose a monopoly is able to use nonlinear pricing in a market where the inverse demand is

$$p=200-Q$$

The marginal and average cost of production of the monopoly is constant and equal to 50. The monopoly wants to set two prices depending on the quantity bought by a consumer.

(a) Write the monopoly's objective in terms of the quantity sold in block 1 (Q_1) and the total quantity sold (Q_2).

(b) Determine the price p_1 of the first block and p_2 of the second block, as well as the quantity sold under the first block and quantity sold in the second block.

(c) The monopoly is considering adding a very large number of blocks. As a representative of consumers, should you oppose this idea of adding more blocks?

Solutions:

(a) The monopoly's objective function is

$$\pi=p_1Q_1+p_2(Q_2-Q_1)-50Q_2$$

$$\Rightarrow \pi=(200-Q_1)Q_1+(200-Q_2)(Q_2-Q_1)-50Q_2$$

(b) The profit maximization conditions are: for a given Q_2, Q_1 is such that the profits' rate of change is

$$\frac{d\pi}{dQ_1} = 200 - 2Q_1 - (200 - Q_2) = 0 \Rightarrow Q_2 = 2Q_1$$

and for a given Q_1, Q_2 is such that the profits' rate of change is

$$\frac{d\pi}{dQ_2} = (200 - Q_2) - (Q_2 - Q_1) - 50 = 0 \Rightarrow Q_1 - 2Q_2 + 150 = 0$$

Solving these two FOCs yield

$$Q_1^* = 50, \quad Q_2^* = 100$$

So the quantity sold in block 1 and block 2 are

$$Q_1^* = 50, \quad Q_2^* - Q_1^* = 100 - 50 = 50$$

and the prices are

$$p_1^* = 200 - Q_1^* = 200 - 50 = 150, \quad p_2^* = 200 - Q_2^* = 200 - 100 = 100$$

(c) Yes, very large number of blocks yields an outcome close to perfect price discrimination, which means a consumers surplus close to zero.

Practice Question 8 (Non-linear Pricing)**.** Suppose a monopoly is able to use nonlinear pricing in a market where the inverse demand is

$$p = 200 - Q$$

The marginal and average cost of production of the monopoly is constant and equal to 50. The monopoly wants to set two prices depending on the quantity bought by a consumer.

(a) Write the monopoly's objective in terms of the price charged in block 1 (p_1) and price charged in block 2 (p_2).

(b) Determine the price p_1 of the first block and p_2 of the second block, as well as the quantity sold under the first block (Q_1) and quantity sold in the second block($Q_2 - Q_1$).

(c) The monopoly is considering adding a very large number of blocks. As a representative of consumers, should you oppose this idea of adding more blocks?

Solutions:

(a) The monopoly's objective function is

$$\pi = p_1 Q_1 + p_2(Q_2 - Q_1) - 50Q_2$$

$$\Rightarrow \pi = (200 - p_1)p_1 + [(200 - p_2) - (200 - p_1)]p_2 - 50(200 - p_2)$$

$$\Rightarrow \pi = (200 - p_1)p_1 + (p_1 - p_2)p_2 - 50(200 - p_2)$$

(b) The profit maximization conditions are: for a given p_2, p_1 is such that the profits' rate of change is

$$\frac{d\pi}{dp_1} = 200 - 2p_1 + p_2 = 0$$

and for a given p_1, p_2 is such that the profits' rate of change is

$$\frac{d\pi}{dp_2} = (p_1 - p_2) - p_2 + 50 = 0 \Rightarrow p_1 - 2p_2 + 50 = 0$$

Solving these two FOCs yield

$$p_1^* = 150, \quad p_2^* = 100$$

Then the quantities are

$$Q_1^* = 200 - 150 = 50, \quad Q_2^* = 200 - 100 = 100$$

So the quantity sold in block 1 and block 2 are

$$Q_1^* = 50, \quad Q_2^* - Q_1^* = 100 - 50 = 50$$

(c) Yes, very large number of blocks yields an outcome close to perfect price discrimination, which means a consumers surplus close to zero.