< Summary of lumped equivalent circuit >

Complex impedance Z_f

$$Z_f = \frac{1}{Z_{dc}} + \frac{1}{Z_b}, \qquad Z_f = \frac{Z_b Z_{dc}}{Z_b + Z_{dc}} \quad \cdots (1)$$

where

$$Z_b = R_b + \frac{1}{i\omega C} \quad : \quad C \text{ is capacitance}$$

Let $m = \dfrac{R_{dc}}{R_{dc} + R_b}$ and $\tau = (R_b + R_{dc})\, C$

Then $$Z_f = R_{dc} - R_{dc} \cdot m \left(1 - \frac{1}{i\omega\tau} \right)$$

Note...

This is the same form of Cole-Cole model with $c = 1$

✓ Frequency effect (FE) $= \dfrac{Z_{f\,min} - Z_{f\,max}}{Z_{f\,max}}$

Assume $Z_{f\,min} = R_{dc}$ & $Z_{f\,max} = Z_f$

$$\underline{FE} = \frac{R_{dc}}{R_b}$$

$R_b \downarrow \rightarrow FE \uparrow$

More conductive FE increase

✓ Time domain effect (Step-off)

$$V_t \doteq V_s(t) = V_p \left[\frac{R_{dc}}{R_{dc}+R_b} \right] e^{-t/\tau}$$

where $V_p = IR_{dc}$ & $V_t = \hat{\mathcal{L}}^{-1}[IZ_f]$

$\mathcal{L}^{-1}$ ⤳ Laplace transform operator

Define chargeability M' as V_t at $t=0$

$$M' = \frac{V_s}{V_p} = \frac{R_{dc}}{R_{dc}+R_b} = m$$

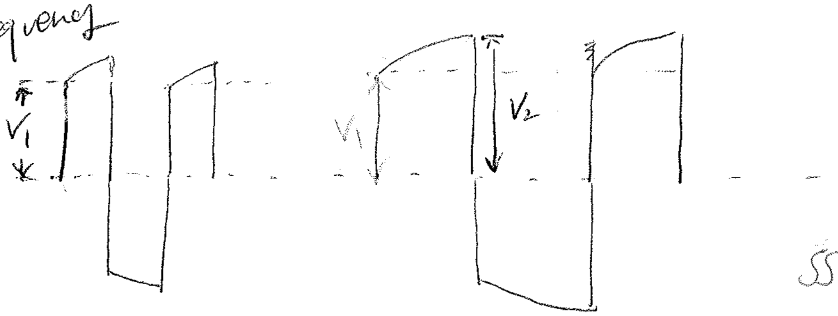
$\sigma_0 < \sigma_\infty$

$\rho_0 > \rho_\infty$

Thus,

$$FE = \frac{R_{dc}}{R_b} = \frac{m}{1-m} \approx \frac{M}{1+M} \;??$$

Frequency



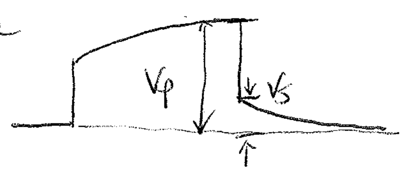
$f=0,\ f=\infty$

$\Delta V = V_2 - V_1$

$$FE = \frac{V_2 - V_1}{V_2} \;??$$

SS

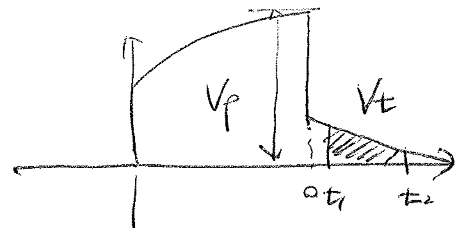
Time



$$M = \frac{V_s}{V_p} \leftrightarrow \frac{\Delta V}{V_2}$$

Define chargeability as

$$M^2 = \frac{1}{V_p} \int_{t_1}^{t_2} V_t \, dt$$



$$M^2 = m\int_0^{\infty} e^{-\frac{1}{\tau}t} \, dt = m\tau = R_{dc} C$$

$\therefore$ Thus $m = M^1 \neq \boxed{M^2 \cong m\tau}$

M^2 is measure of C, capacitance