On recovering IP information from EM data

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ABSTRACT

Extracting information about complex conductivity from electromagnetic surveys has been a continual goal in geophysics, especially for the mineral industry. The fundamental background of this technique and its application has been the focus of much research over the last few decades. Recently there has been a substantial resurgence of research into the technique that has been fuelled by the development of higher powered transmitters, development of new and better sensors, in particular sensitive magnetometers, and an enhanced ability to invert electromagnetic data in 3D. It is therefore timely to reassess past approaches to extract and invert IP data to determine efficient strategies to accomplish this goal. In this paper we review some basic strategies that have been implemented and re-evaluate them within the context of recent inversion capabilities and acquisition techniques.

Key words: electromagnetic, induced polarization, time domain, frequency domain, complex conductivity, Cole-Cole.

BACKGROUND

Maxwell's equation in the time domain, with displacement current neglected, can be written as

$$\nabla \times \vec{e} + \mu \vec{h}_t = 0 \tag{1}$$

$$\nabla \times \vec{h} - \vec{j} = \vec{s} \tag{2}$$

where $\vec{e}$ is the electric field, $\vec{h}$ is the magnetic field, μ is the magnetic permeability, $\vec{j}$ is the current density, and $\vec{s}$ is an electric source. A subscript t denotes the time derivative. In addition, we have a constitutive relationship $\vec{J} = \sigma(\omega)\vec{E}$, where $\vec{J}$ and $\vec{E}$ are the Fourier transforms of $\vec{j}$ and $\vec{e}$. For many materials the electrical conductivity is a function of frequency and the essential character of the dispersion curve is often described as a Cole-Cole relationship

$$\sigma(\omega) = \sigma_\infty \left[1 - \frac{\eta}{1+(1-\eta)(\iota\,\omega\tau)^c}\right] \tag{3}$$

In equation (3), $\sigma(\omega)$ is the conductivity at $\omega = \infty$; equivalently, it is the conductivity of the material that has no IP response. η is the magnitude of the chargeability, τ is a time constant, and c is a parameter that is related to the width of the frequency band over which the change in conductivity occurs. If the conductivity at zero frequency is denoted σ_0 by then $\sigma_0 = \sigma_\infty(1-\eta)$.

The Cole-Cole curve for representative parameters is plotted in Figure 1.

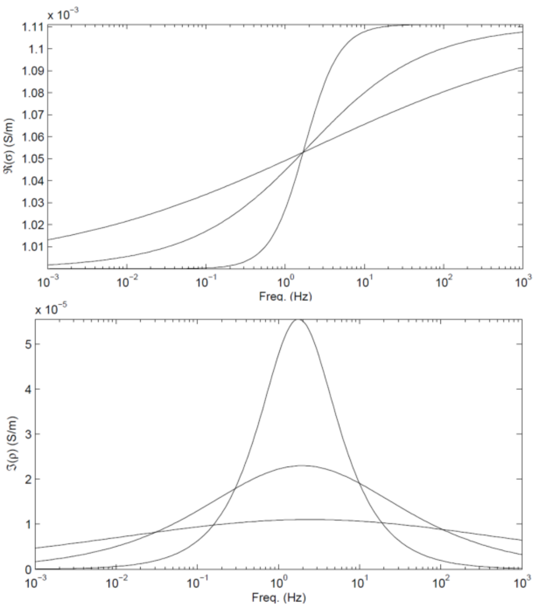

Figure 1. Real (top) and imaginary (bottom) parts of frequency dependent resistivities obeying Cole-Cole models with c = 1, c = 0.5 and c = 0.25. In all three cases, ρ_0 = 1000 Ωm, τ = 0.1, and η = 0.1 (σ_∞ = 1.11×10^{-3} *S/m*).

The effects of the frequency dependent conductivity carry over to the time domain. For instance, in a TEM experiment where the current is a square wave with a half duty cycle, the response of the system at early times (effectively high frequency) is controlled by σ_∞ while the response at late times (effectively low frequency) is related to σ_0.

The effects of chargeability can be measured in a variety of experiments. The sources can be grounded or inductive, data can be in the time or frequency domain, and either electric or magnetic sensors can be used. IP effects in the frequency domain are observed at low frequencies and in the time domain are manifest at late times. Each type of survey (Tx type and Rx type) deserves careful thought about how to extract the IP signal and invert it. For example, in a recent paper (Marchant et al., 2013), defined an IP datum for an inductive source and magnetic receivers that uses the scaled difference between the magnetic fields at two low frequencies. This ISIP datum is zero unless there is chargeability. An example data map is shown in Figure 2. These data can be inverted in 3D using a linearized sensitivity.

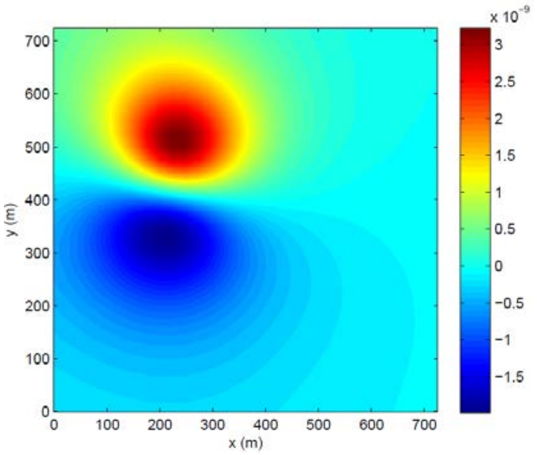

Figure 2. Example of ISIP data simulated above a chargeable block in a variable background. The chargeable target, centered at (200, 500), is clearly visible in the ISIP data.

Modeling IP effects

Solving Maxwell's equations with complex conductivity is straightforward in the frequency domain. Time-domain data can be simulated by solving Maxwell's equations for complex conductivity at many frequencies and performing a Fourier Transform. Alternatively we can solve Maxwell's equations directly in the time domain but we need to revisit Ohm's Law. The frequency and time representations of Ohm's Law incorporating a complex conductivity are

$$\vec{J}(\omega) = \sigma(\omega)\vec{E}(\omega) \quad (4)$$

$$\vec{j}(t) = \sigma(t)\otimes\vec{e}(t) \quad (5)$$

The multiplication of $\sigma(\omega)$and $\vec{E}(\omega)$ in the frequency domain becomes a convolution in the time domain, meaning that the currents flowing in the system at a given time depend upon the currents from all previous times. Directly performing this convolution has been the basis of some modeling techniques (Smith et al., 1988); however, this approach can quickly become memory limited when applied to large, three-dimension problems as the fields from previous times must be stored.

Another approach (Marchant et al., 2012) is to analytically transform the frequency dependent Ohm's law into a differential equation in the time domain. This allows for the time domain response to be modelled directly while considerably decreasing the required storage. In the simplest case, when $c = 1$ (the Debye model), Ohm's law in the time domain becomes

$$\sigma_0\vec{e} + \tau\sigma_0\vec{e}_t = \vec{j} + \tau(1-\eta)\vec{j}_t \quad (6)$$

A finite volume implementation of this technique was used to produce the synthetic data shown in Figure 3.

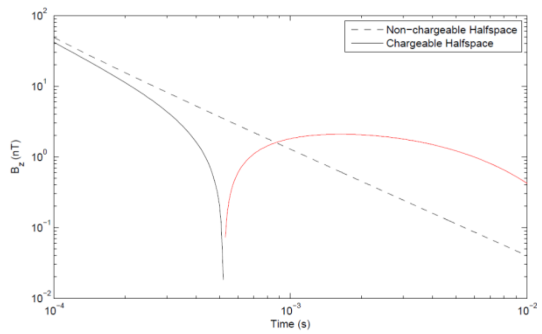

Figure 3. Vertical component of center-loop TEM data collected above a non-chargeable (dashed line) and chargeable (solid line) half-space. Negative data are shown in red. The chargeable model had Cole-Cole parameters of η = 0.1, τ = 1e-3, and c = 1. Both models had σ_∞= 0.1 S/m.

METHODOLOGY

With the ability to forward model in both frequency and time we consider how the complex conductivity affects the responses. For the frequency domain, equation (3), can be written

$$\sigma(\omega) = \sigma_\infty + \Delta\sigma(\omega) \quad (7)$$

$$\Delta\sigma(\omega) = -\sigma_\infty\tilde{\eta}(\omega) \quad (8)$$

$$\tilde{\eta}(\omega) = \frac{\eta}{1+(1-\eta)(\iota\omega\tau)^c} \quad (9)$$

$\tilde{\eta}(\omega)$ is a small, frequency dependent quantity that comprises composite effects of η, τ, and c.

For the time domain we can write a similar expression.

$$\Delta\sigma(t) = -\sigma_\infty\tilde{\eta}(t) \quad (10)$$

$\tilde{\eta}(t)$ is a small quantity that comprises the effects of η, τ, c and additionally aspects of the history of the electric fields at each cell. It too, is a pseudo-chargeabilty.

Since $\tilde{\eta}$ is a small quantity, $\Delta\sigma$ is small perturbation on σ_∞. Denoting the Maxwell operator (in frequency or time) by F and carrying out a first order Taylor expansion we have

$$F[\sigma_\infty + \Delta\sigma] = F[\sigma_\infty] + \frac{dF}{d\sigma}\Delta\sigma \qquad (11)$$

So the perturbation caused by the chargeability, or the IP datum at each individual time channel or frequency is given by

$$d^{\,IP} = -\frac{dF}{d\sigma}\sigma_\infty\tilde{\eta} \qquad (12)$$

$$d^{\,IP} = G\tilde{\eta} \qquad (13)$$

$$G = -\frac{dF}{d\,ln(\sigma)} \qquad (14)$$

Equation (12) shows that the IP data can be written as a linear functional of the sensitivities and a "pseudo-chargeability" ($\tilde{\eta}$). This formulation, which is standard for EIP surveys, is valid for all chargeability problems when the intrinsic chargeability η is small. We note that while equation (13) is valid, it may be also be desirable to further alter the definition of d^{IP}as was done for the ISIP data discussed previously.

We now concentrate upon time-domain problems since this is the most prevalent way in which data are collected. For simplicity, consider measured responses due to a step-off current. There are three time regimes of interest. The first is the steady state fields for the on-time. The relevant conductivity is σ_0 and information about this can only be obtained if a grounded source has been employed. The information content as a function of source and sensor types for steady state fields is listed in Table 1.

Table 1. Information for steady state fields.

Tx type	Rx type	Sensitive to:
Grounded	$\vec{e}$	σ_0 (DC resistivity)
	$\vec{h}$	σ_0 (MMR)
Inductive	$\vec{e}$	0 (no σ_0 information)
	$\vec{h}$	$\vec{h}$ in free space (no σ_0 information)

The second stage consists of early time measurements. The effects of induction are dominant and the relevant conductivity is σ_∞. Irrespective of whether we have a grounded or inductive source, or measuring E or H fields, we can invert these data to recover the electrical conductivity. For example see Oldenburg et al. (2013). This is a conductivity that we would like to obtain since this conductivity is used to compute the desired sensitivities as per equation (12). The procedure works so long as we restrict ourselves to sufficiently early times when any IP effects are non-existent or they are swamped by the strength of the induced signals. However, if IP effects are significant, then the resultant conductivity structure can be degraded. In some cases, for instance where there are zero crossings (negative transients) observed in coincident loop soundings, we know that the data cannot be explained through conductive effects. Figure 4 shows the line profile at different time channels for a field survey at Mt. Milligan. The late time channels associated with negative transients were omitted prior to carrying out a 3D inversion of the airborne EM data (Yang and Oldenburg, 2012).

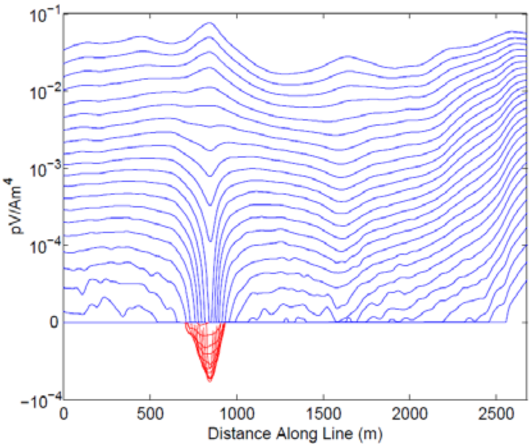

Figure 4. Example of a negative transient observed in a VTEM survey at the Mt. Milligan deposit in British Columbia. Data in red are negative.

The inversion, see Figure 5, was a success in that it delineated a resistive stock for the porphyry deposit that was consistent with geologic knowledge. Nevertheless, there is a resistive artifact that exists in the region where the negative transients were observed. That resistor is likely caused by IP effects that were not accounted for in the inversion.

The third stage of the data is late enough in time that inductive effects have decayed. These are the sought data that are to be inverted for chargeability. Unfortunately, if these responses are contaminated with inductive effects then erroneous chargeabilities will result. Subtracting inductive effects, generally referred to EM coupling removal, has spawned a wide variety of research over the last four decades (Dey and Morrison (1973); Fullagar et al. (2000); Routh and Oldenburg (2001)).

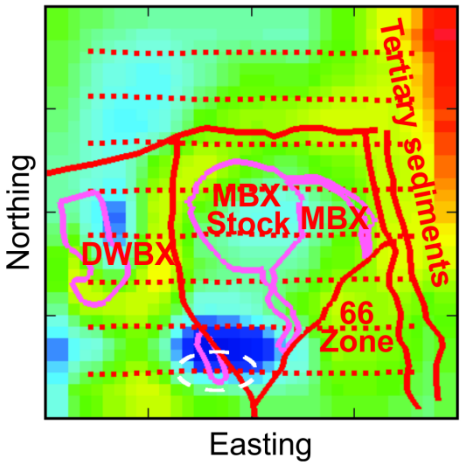

Figure 5. Depth slice of conductivity model produced by inverting VTEM data collected over the Mt. Milligan deposit in British Columbia, Canada. The location of observed negative transients is shown in the dashed-white circle.

DISCUSSIONS AND CONCLUSIONS

The overall problem of recovering information about the complex conductivity from EM surveys can be attacked from two viewpoints. Firstly, it is possible to set up the inverse problem to recover a full complex conductivity. Although this is attractive, it poses challenges because we are attempting to find a four-dimensional function, $\sigma(x, y, z;\ \omega)$. This increases the non-uniqueness of the problem. The regularization functional becomes more complicated as it now includes the frequency dependence for each cell. The real and imaginary parts of $\sigma(\omega)$ are Hilbert transform pairs, the real part is monotonic, and the imaginary part is smooth as a function of frequency. That a-priori information helps but there is still a great deal of latitude for determining a good regularization functional. Reduced models, like Cole-Cole representations, have potential and are popular. Instead of finding a function of frequency we have 4 parameters of $\sigma_\infty, \eta, \tau,$ and c for each cell. Although the parametric formulation seems to have simplicity, the resultant inverse problem is highly non-linear. Nevertheless, this has been used with some success.

In our work we concentrate on extracting information from the three stages of the time domain signals. Many aspects of this general problem have been addressed by others in previous years but our upgraded abilities in solving Maxwell's equations and 3D EM inverse problems opens the door to address some outstanding questions and to develop efficient strategies for extracting IP information. In particular one aspect common to all inversions of IP data is that a background conductivity is needed. The question is how to obtain this background information and how accurately must it be known? Also, when inverting for a real conductivity, contamination of the data by IP signals can lead to deleterious effects. Conversely, inverting for IP parameters can be greatly degraded if the data are contaminated with inductive responses. So there is both IP coupling and EM coupling that must be addressed. An additional issue is that any formulation used to recover a “pseudo-chargeability” yields numbers that are reflective of the relative chargeability effects in 3D space at a particular frequency or at a particular time. Extracting further information about intrinsic chargeability or additional Cole-Cole parameters requires further analysis.

In this paper we take a closer look at the above items and attempt to chart a practical inversion strategy to allow us to extract meaningful information from IP data.

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