Index

Miao Qiao
The University of Auckland

Outline

- Basic Concepts
- Ordered Indices
- B^+-Tree Index Files
- B-Tree Index Files
- Hashing
- Write-optimized indices
- Spatio-temporal Indexing

Basic Concepts

- Why use indexes?
  - To speed up access to desired data.
- How to indicate the desired data?
  - **Search Key** - attribute used to look up records in a file.
- The basic structure of an **index file**
  - records (called **index entries**) of the form

search-key	pointer

- Index Evaluation Metrics
  - Access types supported efficiently. E.g.,
    - Records with a specified value in the attribute
    - Records with an attribute value falling in a specified range of values.
  - Access time, Insertion time, Deletion time, Space overhead
- Types of indices:
  - **Ordered indices:** search keys are stored in sorted order
  - **Hash indices:** search keys are distributed uniformly across "buckets" using a "hash function".

Ordered Indices

- In an **ordered index,** index entries are stored sorted on the search key value.
- **Clustering index:** in a sequentially ordered file, the index whose search key specifies the sequential order of the file.
    - Also called **primary index**
    - The search key of a primary index is usually but not necessarily the primary key.
- **Secondary index**: an index whose search key specifies an order different from the sequential order of the file. Also called **nonclustering index.**
- **Index-sequential file:** sequential file ordered on a search key, with a clustering index on the search key.
- **Dense index** — Index record appears for every search-key value in the file. E.g. index on *ID* attribute of *instructor* relation
- **Sparse Index**: contains index records for only some search-key values.
    - Applicable when records are sequentially ordered on search-key
- To locate a record with search-key value *K* we:
    - Find index record with largest search-key value < *K*
    - Search file sequentially starting at the record to which the index record points

Ordered Indices (Dense and Sparse)

10101	
12121	
15151	
22222	
32343	
33456	
45565	
58583	
76543	
76766	
83821	
98345	

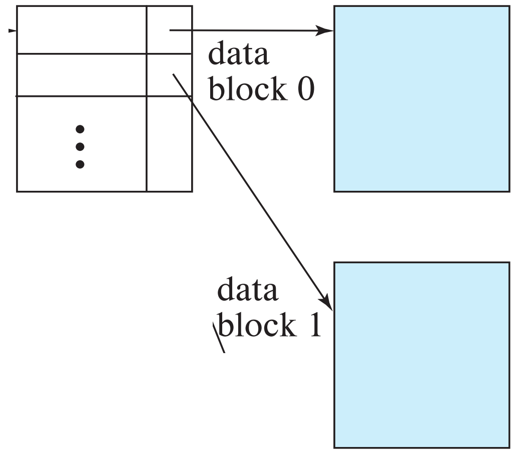
10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

10101	
32343	
76766	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

Sparse Index Files

- Compared to dense indices:
  - Less space and less maintenance overhead for insertions and deletions.
  - Generally slower than dense index for locating records.
- **Good tradeoff**:
  - for clustered index: sparse index with an index entry for every block in file, corresponding to least search-key value in the block.



  - For unclustered index: sparse index on top of dense index (multilevel index)

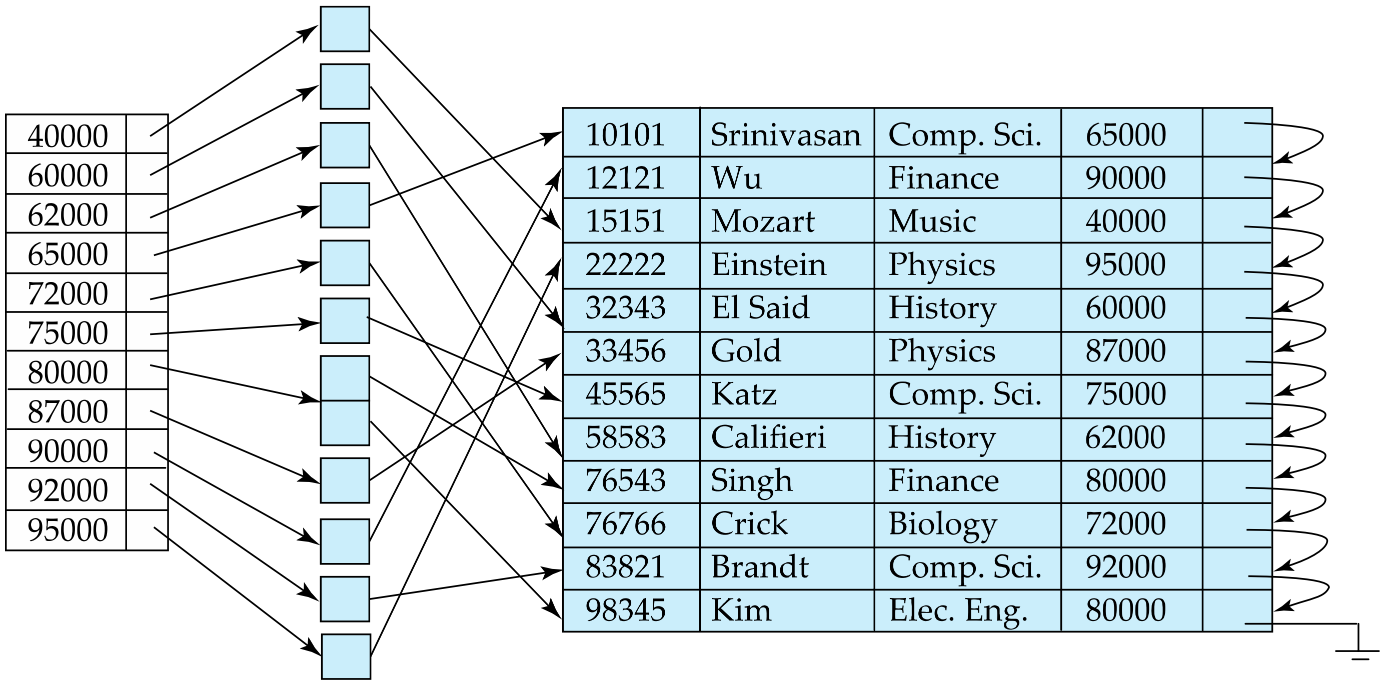
What type of index is this?

Biology	
Comp. Sci.	
Elec. Eng.	
Finance	
History	
Music	
Physics	

76766	Crick	Biology	72000	
10101	Srinivasan	Comp. Sci.	65000	
45565	Katz	Comp. Sci.	75000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	
12121	Wu	Finance	90000	
76543	Singh	Finance	80000	
32343	El Said	History	60000	
58583	Califieri	History	62000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
33465	Gold	Physics	87000	

Secondary Indices Example

- Secondary index on salary field of instructor

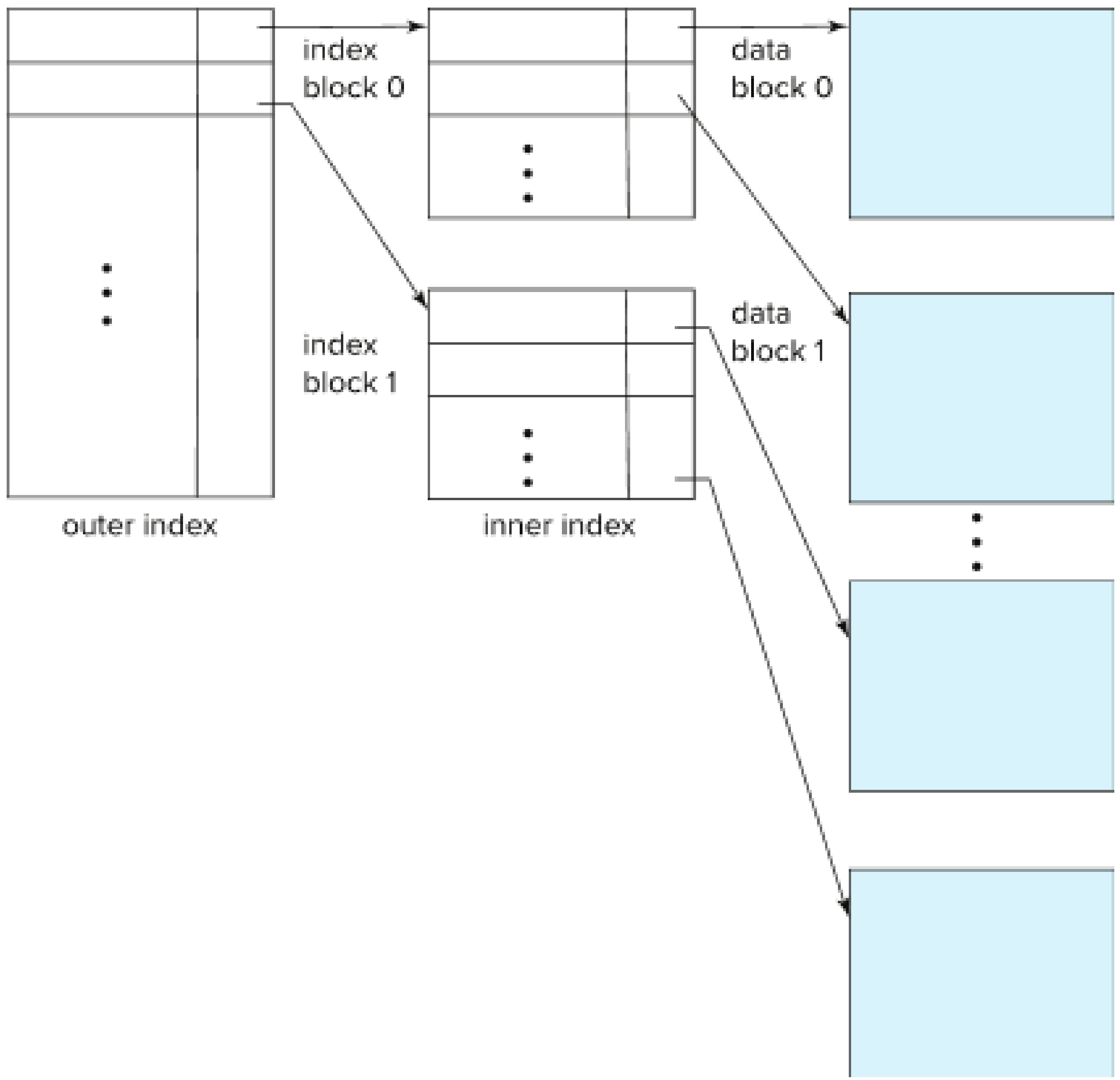


- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense

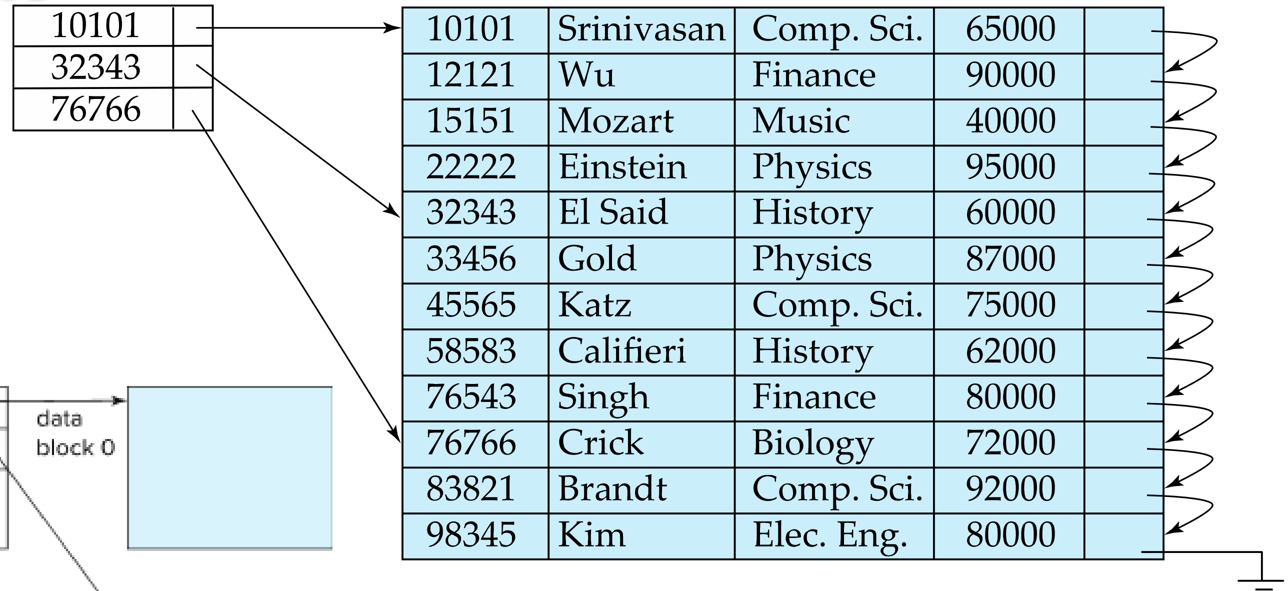
Discussions

- Clustering vs Nonclustering Indices
  - Sequential scan using clustering index is efficient, but a sequential scan using a secondary (nonclustering) index is expensive on magnetic disk
    - Each record access may fetch a new block from disk
    - Each block fetch on magnetic disk requires about 5 to 10 milliseconds
- Indices on Multiple Keys
  - Composite search key
    - E.g., index on *instructor* relation on attributes (*name, ID*)
    - Values are sorted lexicographically
      - E.g. (John, 12121) < (John, 13514) and
        (John, 13514) < (Peter, 11223)
    - Can query on just *name*, or on (*name, ID*)

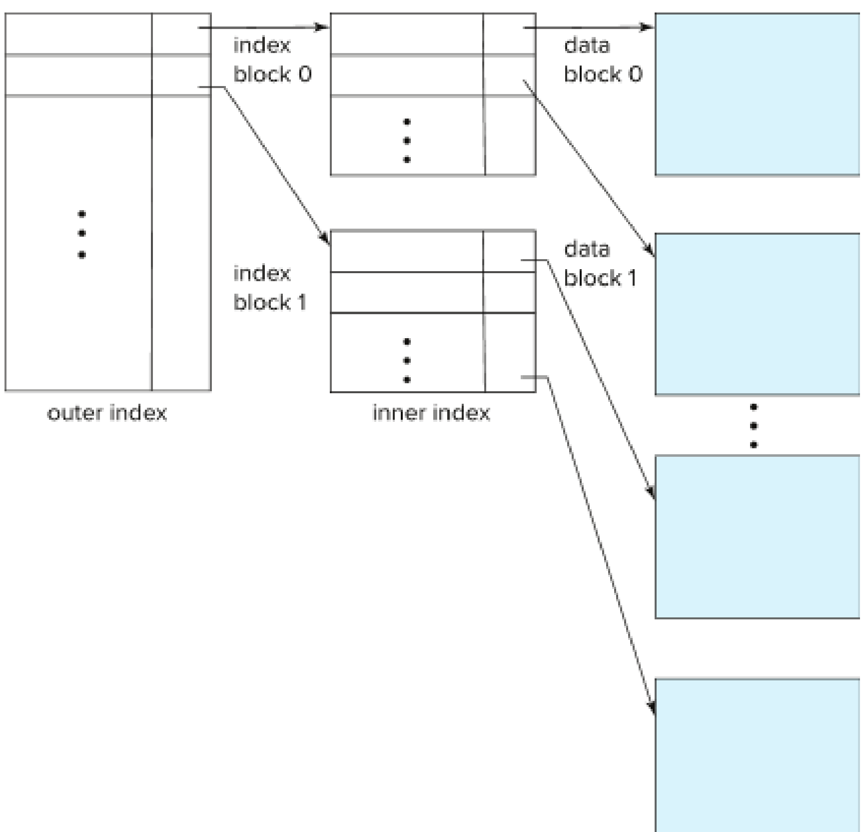
Multilevel Index



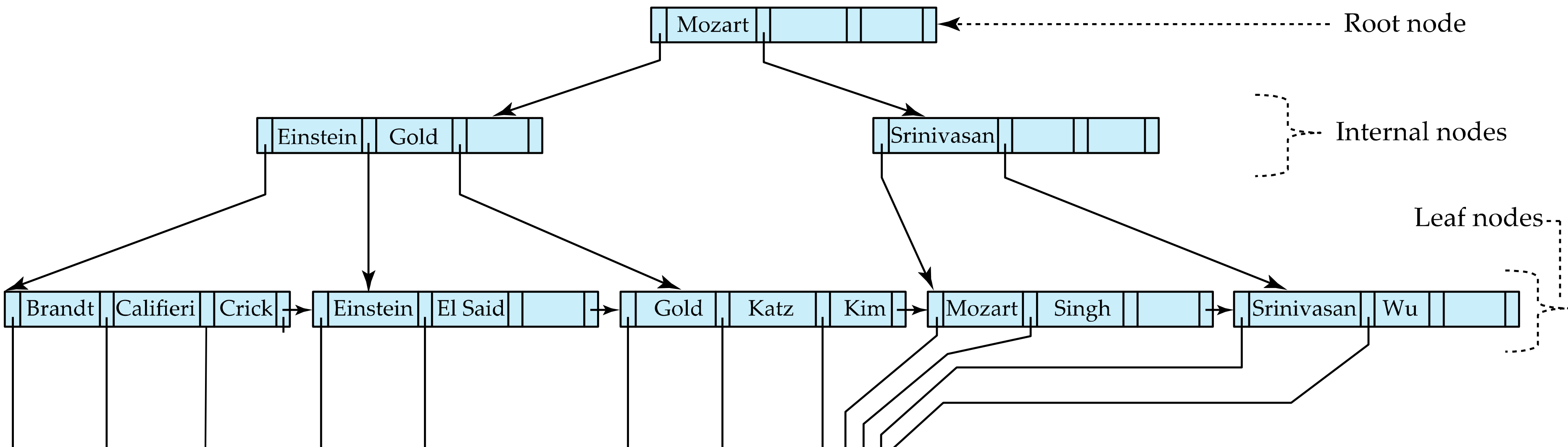
Index Update



10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000



B$^+$-Tree



10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	60000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

B+-Tree Index Files (Cont.)

A B+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and $n-1$ values
- Special cases:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and $(n-1)$ values.

B+-Tree Node Structure

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

$$K_1 < K_2 < K_3 < . . . < K_{n-1}$$

B⁺-Tree Node Structure

- Typical node

P_1	K_1	P_2	$\ldots$	P_{n-1}	K_{n-1}	P_n

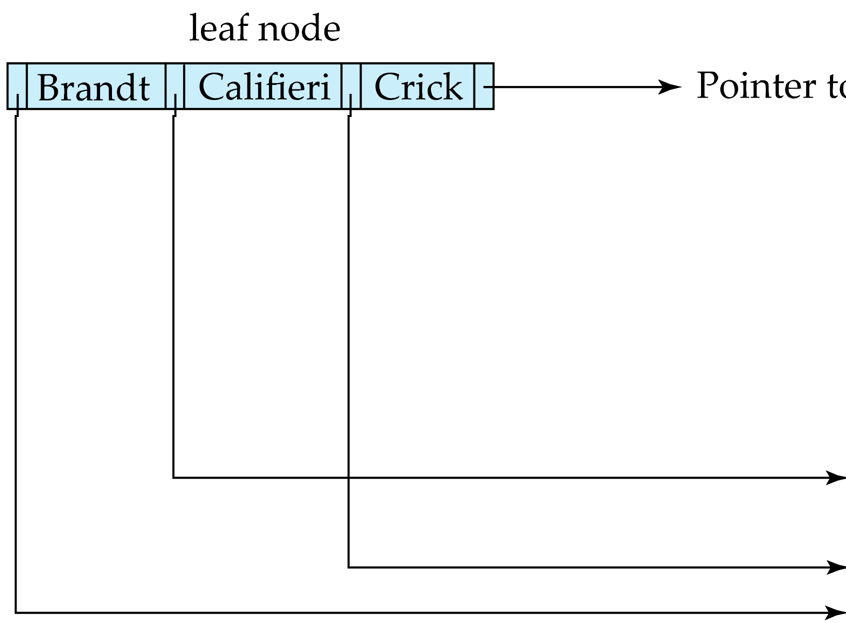
  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).

- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

(Initially assume no duplicate keys, address duplicates later)

Leaf Nodes in B^+-Trees (Secondary Index)

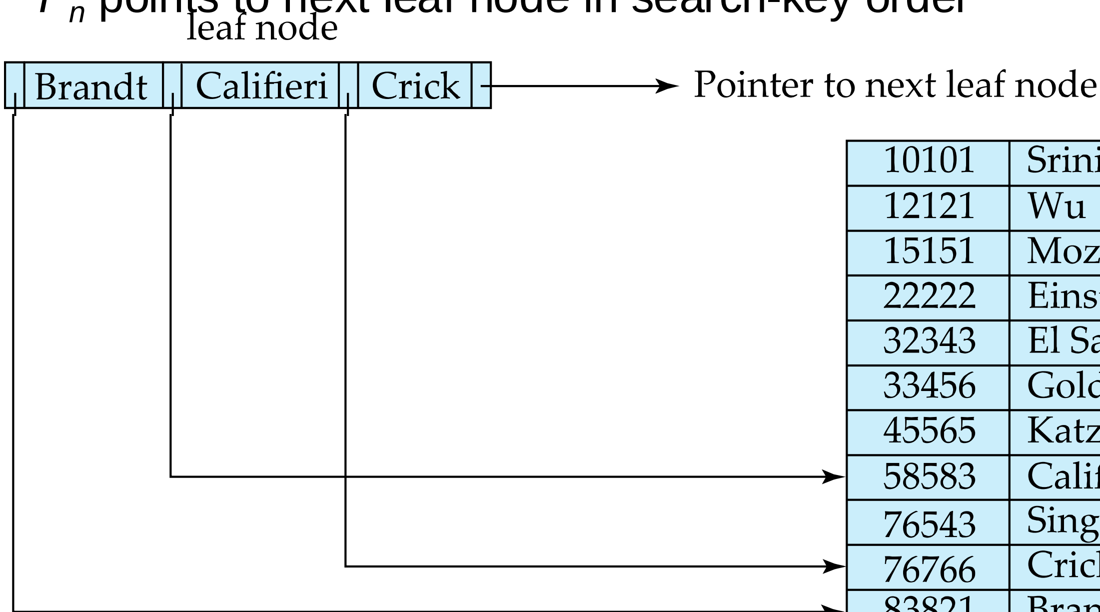


10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	60000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Leaf Nodes in B$^+$-Trees

Properties of a leaf node:

- For $i = 1, 2, \ldots, n-1$, pointer P_i points to a file record with search-key value K_i,
- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than or equal to L_j's search-key values
- P_n points to next leaf node in search-key order

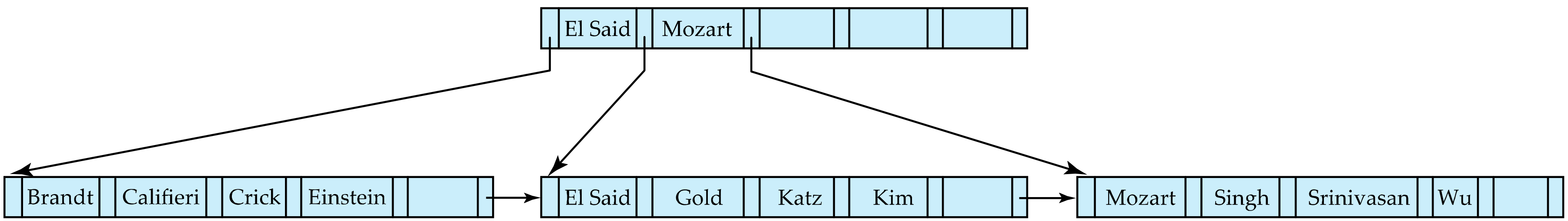


10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	60000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Non-Leaf Nodes in B⁺-Trees

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

- B⁺-tree for *instructor* file (n = 6)



- Leaf nodes must have between 3 and 5 values ($\lceil (n-1)/2 \rceil$ and n –1, with n = 6).
- Non-leaf nodes other than root must have between 3 and 6 children ($\lceil (n/2 \rceil$ and n with n =6).
- Root must have at least 2 children.

Non-Leaf Nodes in B$^+$-Trees

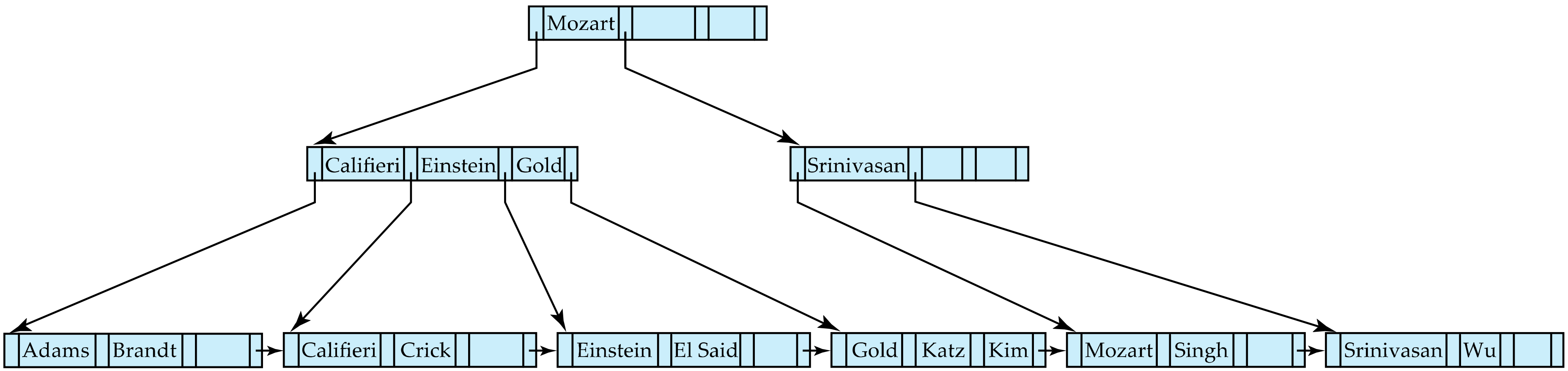
P_1	K_1	P_2	$\ldots$	P_{n-1}	K_{n-1}	P_n

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with *n* pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \leq i \leq n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}
  - General structure

Observations about B⁺-trees

- Since the inter-node connections are done by pointers, "logically" close blocks need not be "physically" close.
- The non-leaf levels of the B⁺-tree form a hierarchy of sparse indices.
- The B⁺-tree contains a relatively small number of levels
  - Level below root has at least $2*\lceil n/2 \rceil$ values
  - Next level has at least $2*\lceil n/2 \rceil * \lceil n/2 \rceil$ values
  - .. etc.
  - If there are K search-key values in the file, the tree height is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
  - thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

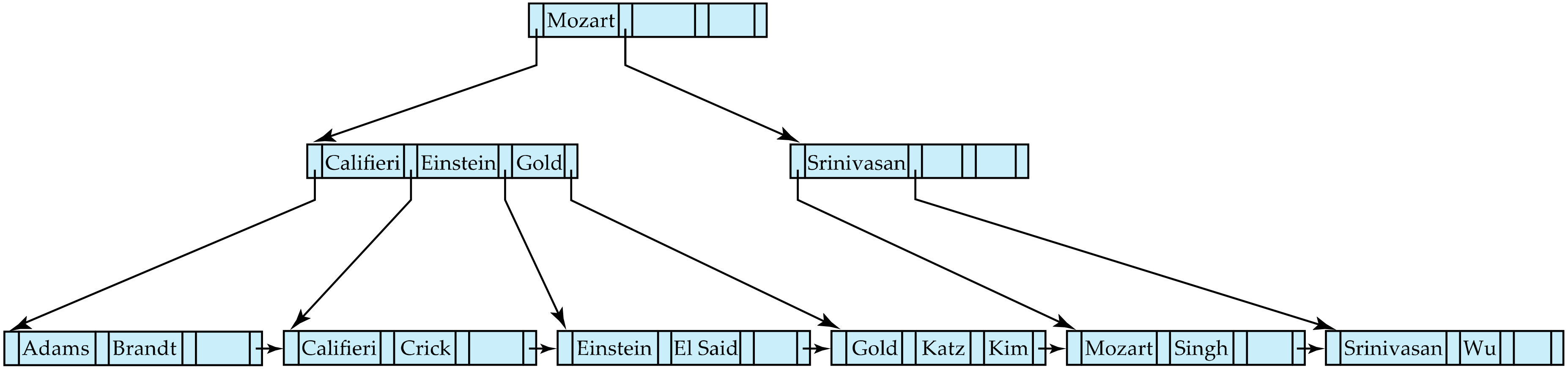
Queries on B$^+$-Trees



If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.

Queries on B$^+$-Trees (Cont.)

- **Range queries** find all records with search key values in a given range
  - See book for details of **function** *findRange*(*lb, ub*) which returns set of all such records
  - Real implementations usually provide an iterator interface to fetch matching records one at a time, using a *next*() function



Queries on B+-Trees (Cont.)

- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes
  - and n is typically around 100 (40 bytes per index entry).
- With 1 million search key values and $n = 100$
  - at most $log_{50}(1{,}000{,}000) = 4$ nodes are accessed in a lookup traversal from root to leaf.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - above difference is significant since every node access may need a disk I/O, costing around 20 milliseconds

Non-Unique Keys

- If a search key a_i is not unique, create instead an index on a composite key (a_i , A_p), which is unique
  - A_p could be a primary key, record ID, or any other attribute that guarantees uniqueness
- Search for $a_i = v$ can be implemented by a range search on composite key, with range (v, - ∞) to (v, + ∞)
- But more I/O operations are needed to fetch the actual records
  - If the index is clustering, all accesses are sequential
  - If the index is non-clustering, each record access may need an I/O operation

B⁺-Tree Insertion

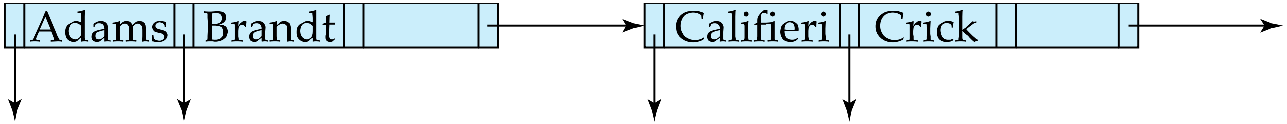
Assume record already added to the file. Let

- *pr* be pointer to the record, and let
- v be the search key value of the record

1. Find the leaf node in which the search-key value would appear
   1. If there is room in the leaf node, insert (v, *pr*) pair in the leaf node
   2. Otherwise, split the node (along with the new (*v, pr*) entry) as discussed in the next slide, and propagate updates to parent nodes.

B⁺-Tree Insertion (Cont.)

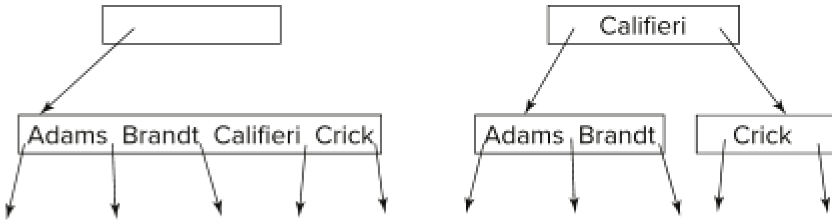
- Splitting a leaf node:
  - take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - let the new node be *p,* and let *k* be the least key value in *p.* Insert (*k*,*p*) in the parent of the node being split.
  - If the parent is full, split it and **propagate** the split further up.
- Splitting of nodes proceeds upwards till a node that is not full is found.
  - In the worst case the root node may be split increasing the height of the tree by 1.



Result of splitting node containing Brandt, Califieri and Crick on inserting Adams
Next step: insert entry with (Califieri, pointer-to-new-node) into parent

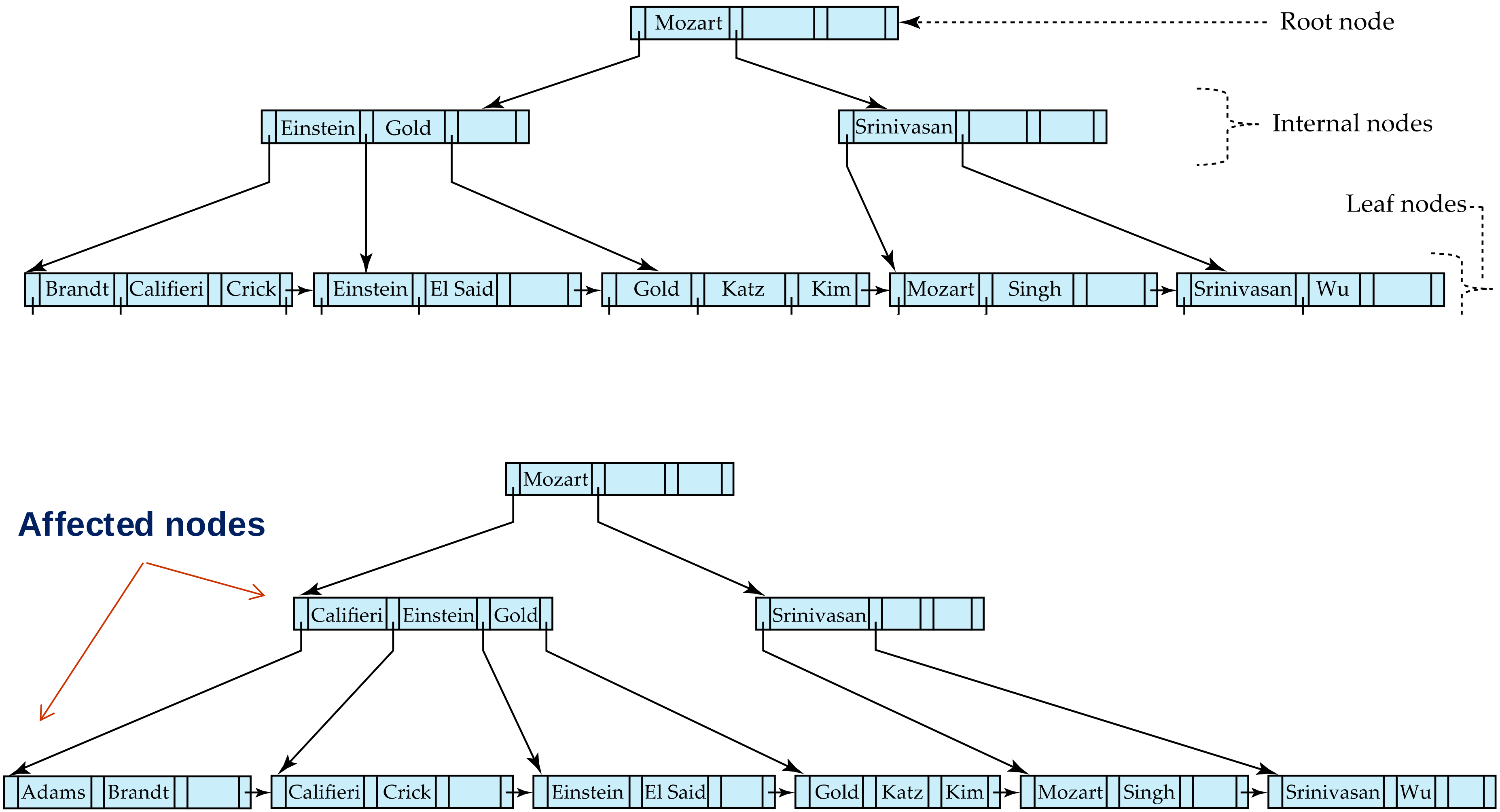
B⁺-Trees Insertion

- Splitting a non-leaf node: when inserting (k,p) into an already full internal node N
  - Copy N to an in-memory area M with space for n+1 pointers and n keys
  - Insert (k,p) into M
  - Copy $P_1, K_1, \ldots, K_{\lceil n/2 \rceil - 1}, P_{\lceil n/2 \rceil}$ from M back into node N
  - Copy $P_{\lceil n/2 \rceil + 1}, K_{\lceil n/2 \rceil + 1}, \ldots, K_n, P_{n+1}$ from M into newly allocated node N'
  - Insert $(K_{\lceil n/2 \rceil}, N')$ into parent N
- Example



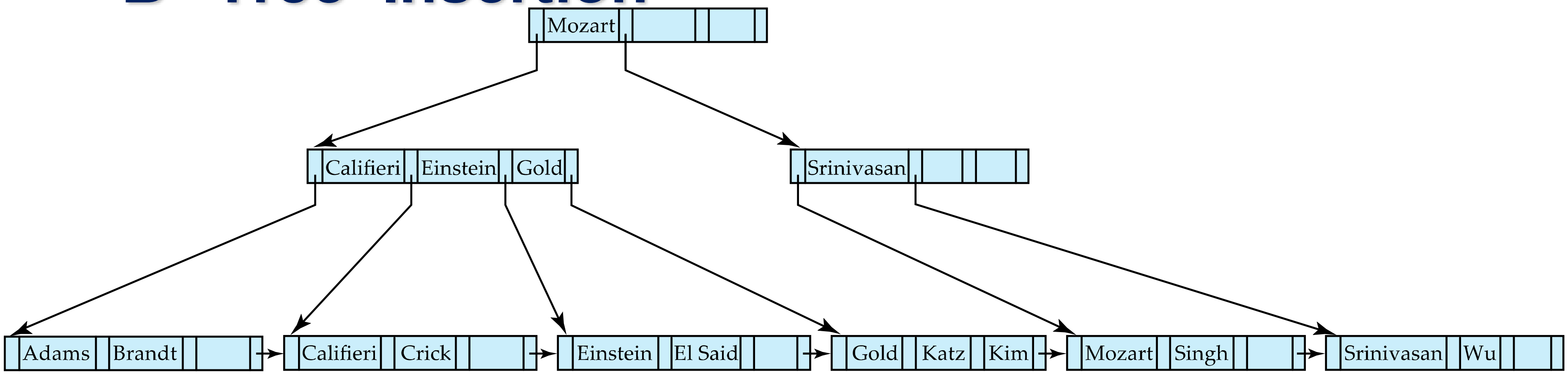
- **Read pseudocode in book!**

B+-Tree Insertion

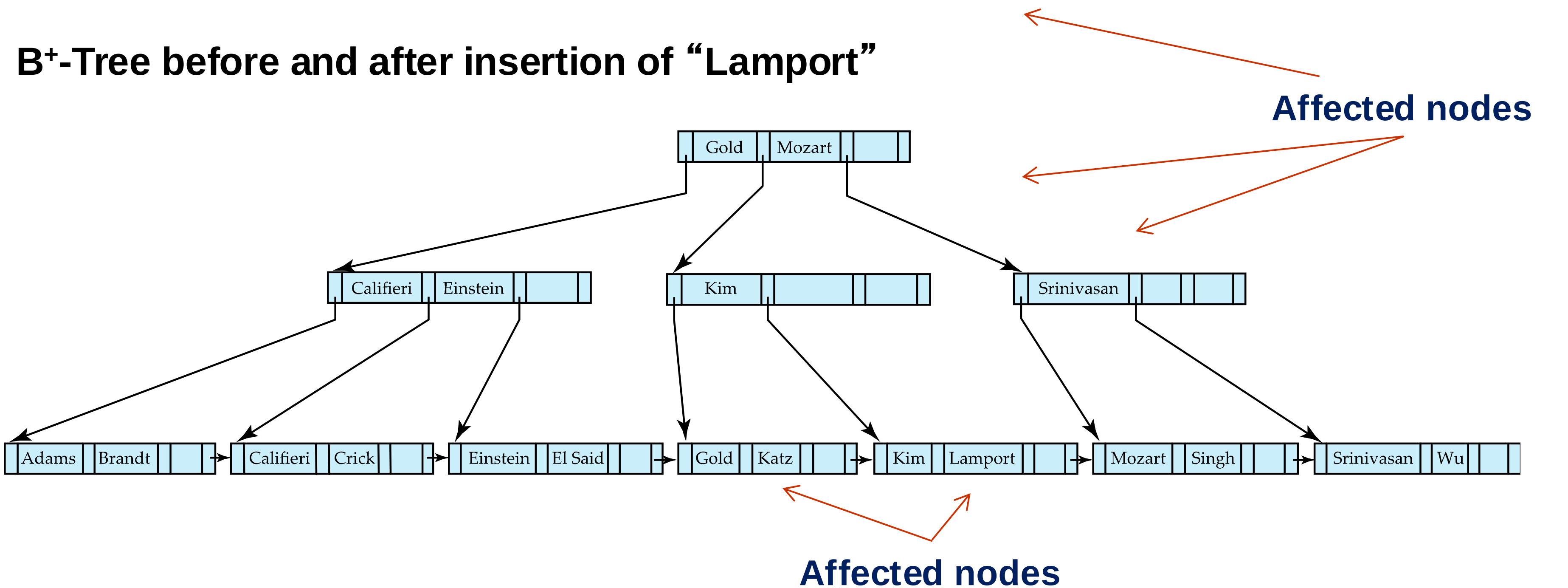


B+-Tree before and after insertion of “Adams”

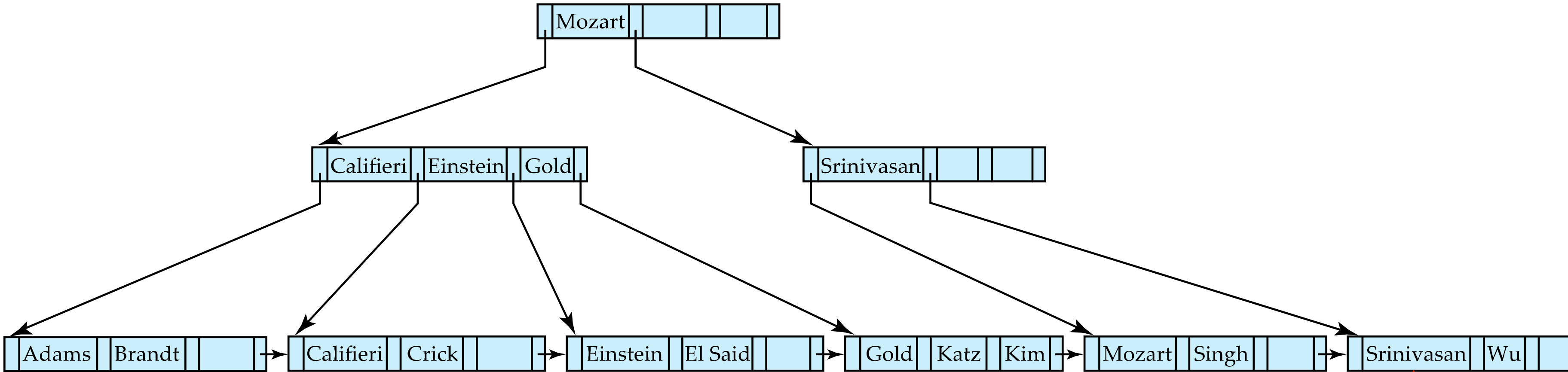
B+-Tree Insertion



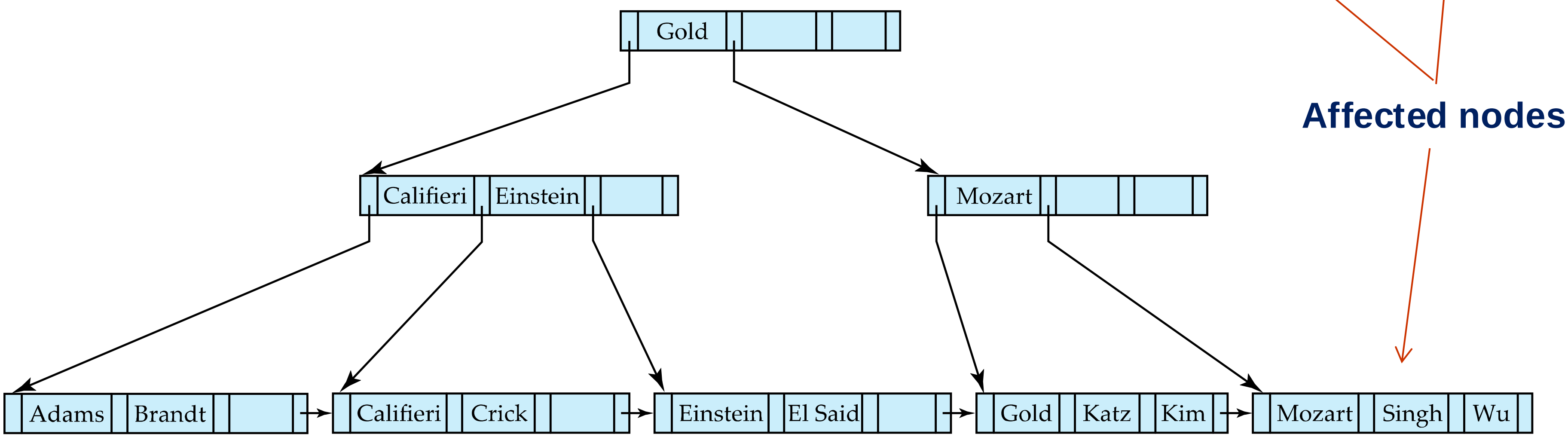
B+-Tree before and after insertion of "Lamport"



Examples of B+-Tree Deletion

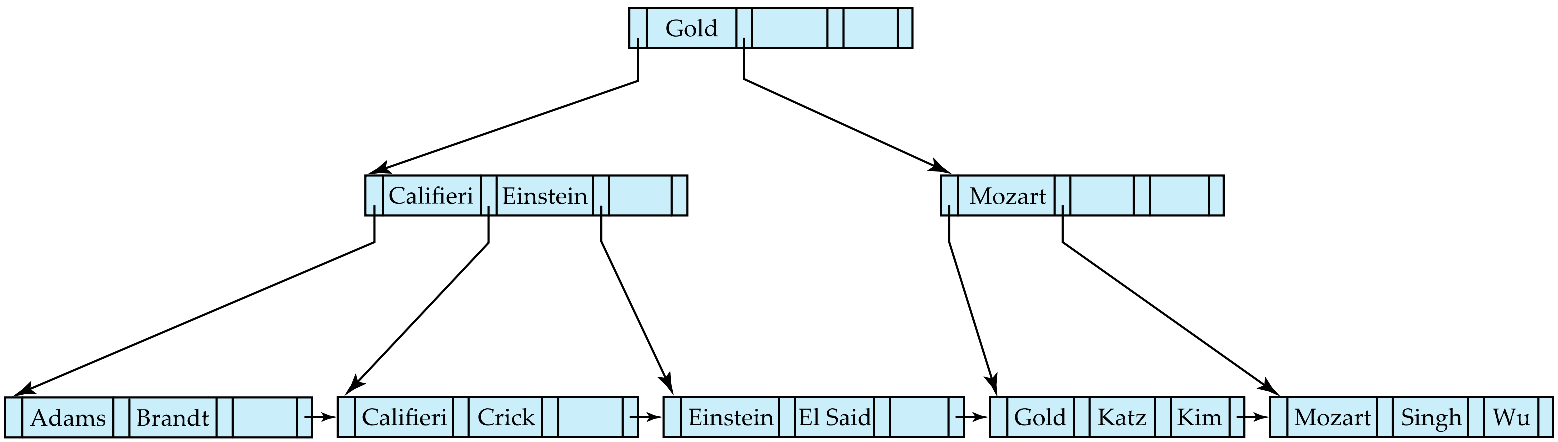


Before and after deleting "Srinivasan"

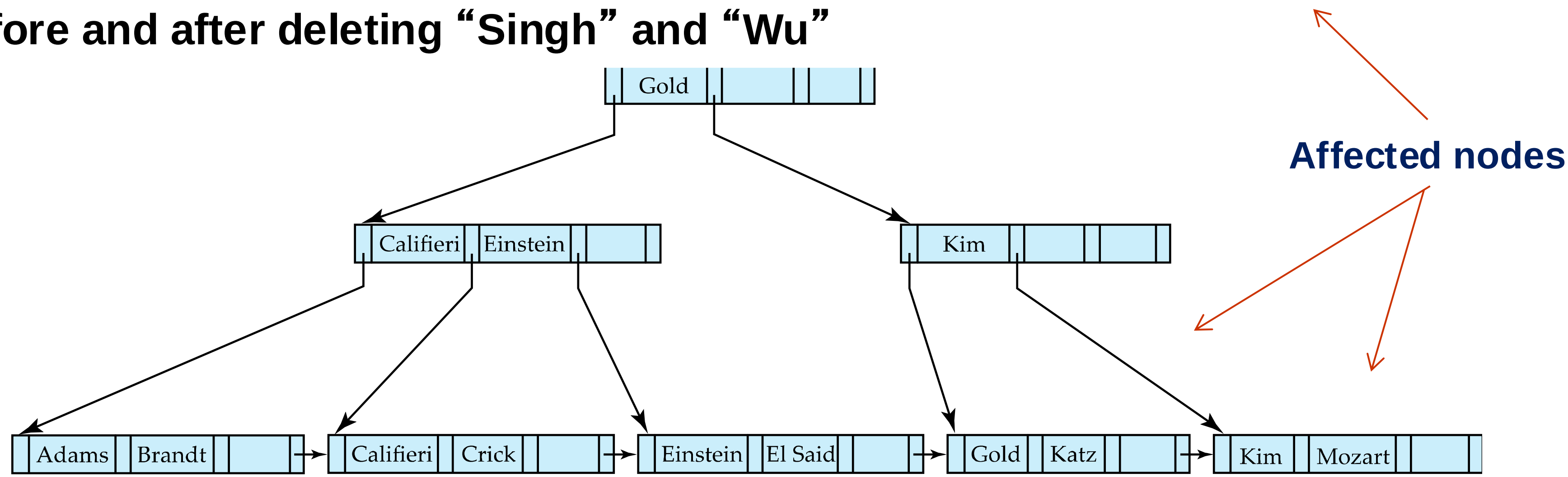


- Deleting "Srinivasan" causes **merging** of under-full leaves

Examples of B$^+$-Tree Deletion (Cont.)

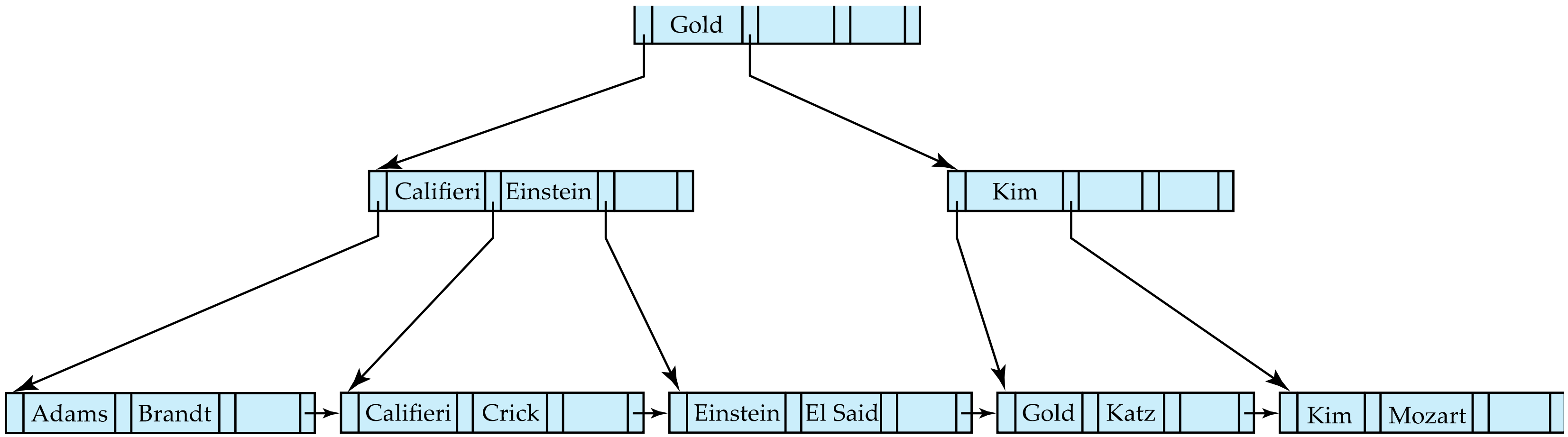


Before and after deleting “Singh” and “Wu”

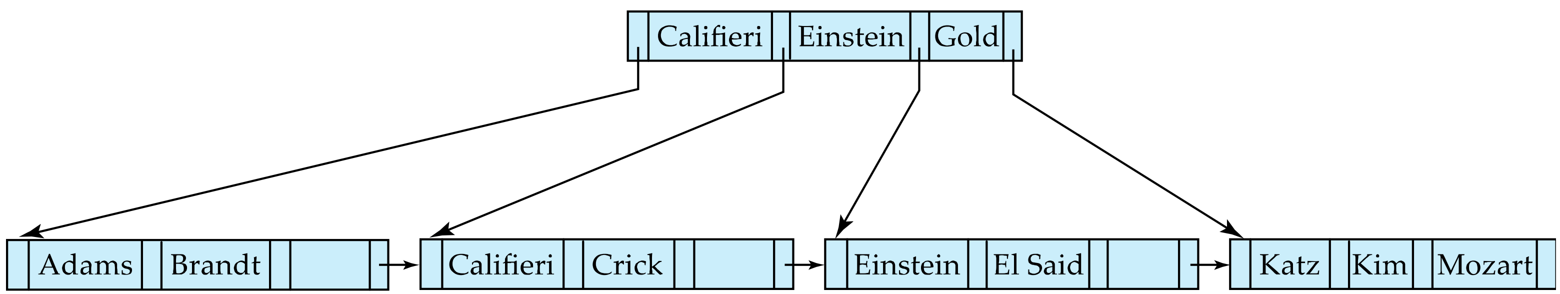


- Leaf containing Singh and Wu became underfull, and **borrowed a value** Kim from its left sibling
- Search-key value in the parent changes as a result

Example of B+-tree Deletion (Cont.)



Before and after deletion of “Gold”



- Node with Gold and Katz became underfull, and was merged with its sibling
- Parent node becomes underfull, and is merged with its sibling
  - Value separating two nodes (at the parent) is pulled down when merging
- Root node then has only one child, and is deleted

Updates on B+-Trees: Deletion

Assume record already deleted from file. Let *V* be the search key value of the record, and *Pr* be the pointer to the record.

- Remove (*Pr, V*) from the leaf node
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then ***merge siblings***:
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B$^+$-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

Complexity of Updates

- Cost (in terms of number of I/O operations) of insertion and deletion of a single entry proportional to height of the tree
  - With K entries and maximum fanout of n, worst case complexity of insert/delete of an entry is $O(\log_{\lceil n/2 \rceil}(K))$
- In practice, number of I/O operations is less:
  - Internal nodes tend to be in buffer
  - Splits/merges are rare, most insert/delete operations only affect a leaf node
- Average node occupancy depends on insertion order
  - 2/3rds with random, ½ with insertion in sorted order

Non-Unique Search Keys

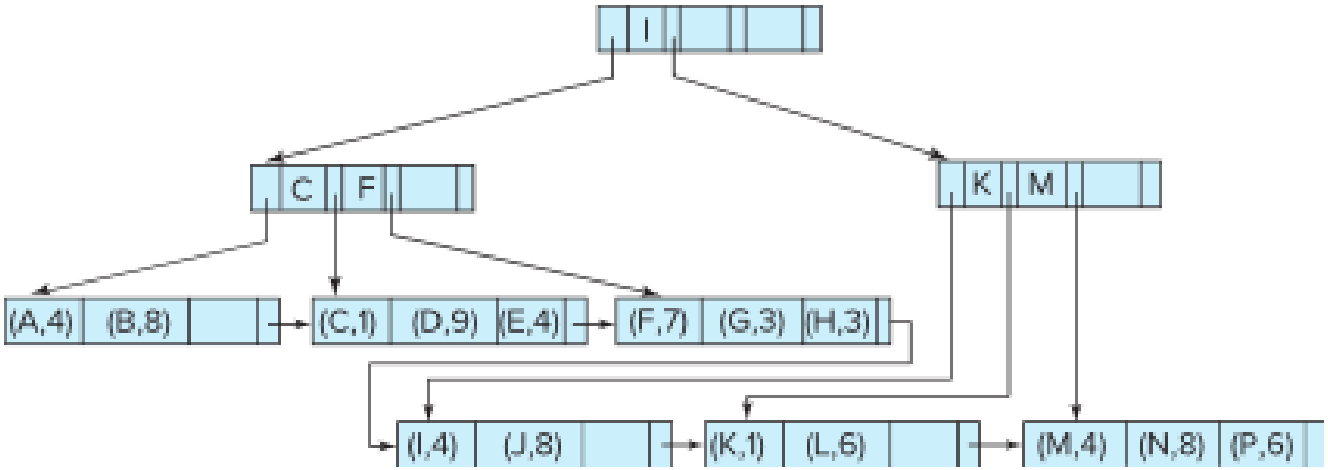
- Alternatives to scheme described earlier
  - Buckets on separate block (bad idea)
  - List of tuple pointers with each key
    - Extra code to handle long lists
    - Deletion of a tuple can be expensive if there are many duplicates on search key (why?)
      - Worst case complexity may be linear!
    - Low space overhead, no extra cost for queries
  - Make search key unique by adding a record-identifier
    - Extra storage overhead for keys
    - Simpler code for insertion/deletion
    - Widely used

B⁺-Tree File Organization

- B⁺-Tree File Organization:
  - Leaf nodes in a B⁺-tree file organization store records, instead of pointers
  - Helps keep data records clustered even when there are insertions/deletions/updates
- Leaf nodes are still required to be half full
  - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B⁺-tree index.

B⁺-Tree File Organization (Cont.)

- Example of B+-tree File Organization



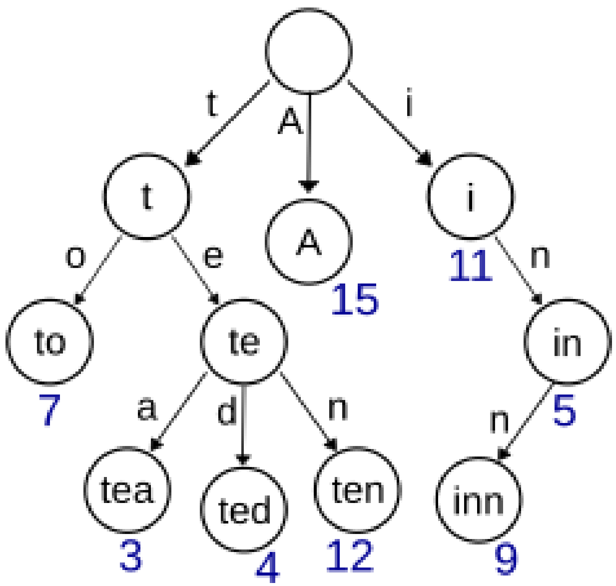
- Good space utilization important since records use more space than pointers.
- To improve space utilization, involve more sibling nodes in redistribution during splits and merges
  - Involving 2 siblings in redistribution (to avoid split / merge where possible) results in each node having at least $\lfloor 2n/3 \rfloor$ entries

Bulk Loading and Bottom-Up Build

- Inserting entries one-at-a-time into a B+-tree requires ≥ 1 IO per entry
  - assuming leaf level does not fit in memory
  - can be very inefficient for loading a large number of entries at a time (**bulk loading**)
- Efficient alternative 1:
  - sort entries first (using efficient external-memory sort algorithms)
  - insert in sorted order
    - a leaf needs to be written out only once
    - much improved IO performance, but most leaf nodes half full
- Efficient alternative 2: **Bottom-up B+-tree construction**
  - As before sort entries
  - And then create tree layer-by-layer, starting with leaf level
    - details as an exercise
  - Implemented as part of bulk-load utility by most database systems

Indexing Strings

- Variable length strings as keys
  - Variable fanout
  - Use space utilization as criterion for splitting, not number of pointers
- **Prefix compression**
  - Key values at internal nodes can be prefixes of full key
    - Keep enough characters to distinguish entries in the subtrees separated by the key value
      - E.g., “Silas” and “Silberschatz” can be separated by “Silb”
  - Keys in leaf node can be compressed by sharing common prefixes



FIN

Any questions?

Queries on B⁺-Trees

function *find*(*v*)

1. *C=root*
2. **while** (C is not a leaf node)
    1. Let *i* be least number s.t. $V \leq K_i$.
    2. **if** there is no such number *i then*
    3. Set *C = last non-null pointer in C*
    4. **else if** (v = C.K_i) Set C = P_{i+1}
    5. **else set** *C* = C.P_i
3. **if** for some *i*, $K_i = V$ **then** return C.P_i
4. **else** return null /* no record with search-key value *v* exists. */