First-order Methods for Large-scale Optimal Sensor Placement

Y. S. Sun[a], H. W. Huang[a], M. Yamashita[b]
[a]Department of Bridge Engineering, Tongji University, Shanghai, China
[b]Department of Mathematical and Computing Sciences, Tokyo Institute of Technology, Tokyo, Japan

***Abstract*—This paper discuss the problem of selecting a set of k optimal sensor locations from a large candidate set of n locations for the purpose of identification and correlation of large structures. Given that solving this problem by evaluating the performance of each of the choices is not practical when n and/or k are large, a method employing first-order methods is presented to provide an approximate solution, as well as a tighter lower bound of all possible choices for large-scale cases. The optimization is based on the newly developed "doubly nonnegative relaxation" (DNN) and is solved by iteratively solving its augmented Lagrangian. The final sensor configuration maximizes the determinant of the Fisher information matrix corresponding to the target modal partitions, which leads to better estimates and correlations. Simulations are carried out to verify its effectiveness.**

I. Introduction And Background

SENSOR selection problems arise in many fields, within civil structures, robotics, chemical plant control, etc. Under economic constraints, a limited number of sensors have to be located such that the best possible set of observations is achieved. Though the NP-hardness of this problem has not been established, and there're global optimization techniques which can give exact solutions, they often take too long time for only moderate values of n and k.

According to [1], the sensor selection is achieved by maximizing the determinant of the Fisher Information Matrix.

$$\begin{aligned}\text{(P1)}\quad \min\quad & -\log\det \textstyle\sum_{i=1}^{n} y_i\, \phi_i \phi_i^T \\ \text{s.t.}\quad & y_i \in \{0,1\} \\ & \textstyle\sum_{i=1}^{n} y_i = k,\ \mathrm{i} = 1,\dots,\mathrm{n}\end{aligned}$$

An iterative algorithm called the Effective Independence (EI) method proposed in [1] is used to extract most independent information about the mode shapes of structures from the vibration data. The EI method provides only a sub-optimal solution without any guarantees or bounds on the performance that is achievable. The convex relaxation presented in [2] can be used to solve large dimensional problems efficiently.

$$\begin{aligned}\text{(P2)}\quad \min\quad & -\log\det \textstyle\sum_{i=1}^{n} y_i\, \phi_i \phi_i^T \\ \text{s.t.}\quad & 0 \le y_i \le 1 \\ & \textstyle\sum_{i=1}^{n} y_i = k,\ \mathrm{i} = 1,\dots,\mathrm{n}\end{aligned}$$

Combined with heuristics, a solution and a lower bound of the objective function can be obtained. But the relaxation from binary constraints to box constraints in this method is too loose, which lowers the optimality of the solution.

In this paper, a tighter relaxation of the constraints is applied, so that a tighter lower bound of this particular objective function is expected to be obtained together with optimal solutions for practical applications. The newly proposed doubly nonnegative (DNN) relaxation [3] of the problem is proved to be capable of tightening the relaxation.

$$\begin{aligned}\text{(P3)}\quad \min\ & -\log\det\left(\textstyle\sum_{i=2}^{n+1} X_{1,i}\, \phi_i \phi_i^T\right) \\ \text{s.t.}\ & H_i \cdot X = b_i,\ i = 0,1 \\ & \mathrm{X} \in S_+^{2n+1} \cap N^{2n+1}\end{aligned}$$

The popular primal-dual interior-point method can be used to solve such problems, but it demands very high computational effort for large-scale cases, typically more than 30 or so variables would render a problem unsolvable. As a result, first-order methods are needed to solve problems with much more variables, where its augmented Lagrangian is derived to handle the linear constraints [5]. The spectral projected gradient (SPG) method proposed in [4] can be applied to solve the augmented Lagrangian with the variable being constrained to the DNN cone. This is called an inner loop. After a solution is obtained, the Lagrangian multipliers are updated and another inner loop is started, which is referred to as the outer loop. In addition, the projection of a matrix onto the DNN cone is realized by an accelerated proximal gradient method [3].

II. Results

The proposed method shows very good convergence on different data sets. For one example, the method is applied to the 64 degree of freedom mode shape matrix subtracted from the FE model of Kezhushan Bridge. A Lower value of the objective function is obtained compared to LP relaxation, proving to be a better selection of sensor locations. Besides, its variables are much closer to their ideal values, 0 or 1, much better than those fractions given by the basic convex relaxation in [2]. In summary, the method is very promising for sensor selection as well as problems with similar structures.

References

[1] Kammer D C. Sensor placement for on-orbit modal identification and correlation of large space structures[J]. Journal of Guidance, Control, and Dynamics, 1991, 14(2): 251-259.

[2] Joshi S, Boyd S. Sensor selection via convex optimization[J]. Signal Processing, IEEE Transactions on, 2009, 57(2): 451-462.

[3] Kim S, Kojima M, Toh K C. A Lagrangian-DNN relaxation: a fast method for computing tight lower bounds for a class of quadratic optimization problems[J]. Preprint, http://www. optimization-online. org/DB_HTML/2013/10/4073. html, 2013.

[4] Birgin E G, Martínez J M, Raydan M. Nonmonotone spectral projected gradient methods on convex sets[J]. SIAM Journal on Optimization, 2000, 10(4): 1196-1211.

[5] Nocedal J, Wright S J. Numerical Optimization[M]. Springer New York, 2006.