Moral Hazard_Single Task

Moral hazard: hidden action

2.1.1 Standard model: symmetric information

P: principal; A: agent; x_i: production of agent i; $a \in A$: effort of agent A; $c(a)$ is convex; $c' > 0$, $c'' \leq 0$;

all utilities are VNM formula; $\underline{U}$ is A's preserved utility

P's object function: $V(x, w) = v(x - w)$ $v' > 0, v'' \leq 0$ concave

A's object function: $U(w, a) = u(w) - c(a)$ $u' > 0, u'' \leq 0$ concave

P's mathematic program:

$$\max_{\{a, w(x_i)\}} \sum_{i=1}^{n} p_i(a) v(x_i - w(x_i)) \tag{1}$$

$$s.t. \sum_{i=1}^{n} p_i(a) u(w(x_i)) - c(a) \geq \underline{U} \tag{2}$$

The lagrange function:

$$\operatorname*{Max}_{a, w(x_i)\}} \sum_{i=1}^{n} p_i(a) v\left(x_i - w\left(x_i\right)\right) + \lambda \left[\sum_{i=1}^{n} p_i(a) u\left(w\left(x_i\right)\right) - c(a) - \underline{U}\right] \tag{3}$$

According to concave program's rule

$$\lambda = \frac{v'\left(x_i - w^{FB}\left(x_i\right)\right)}{u'\left(w^{FB}\left(x_i\right)\right)}, i \in \{1, 2, \ldots, n\} \tag{4}$$

r_P and r_A is P's and A's Arrow-Pratt meassure of absolute risk aversion respectively, The bigger r is, the more risk averse principal/agent is

$$\frac{dw^{FB}}{dx_i} = \frac{r_P}{r_P + r_A} \tag{5}$$

1. $r_P = 0, r_A >> 0$, $\frac{dw}{dx} = 0$, fixed wage
2. $r_A = 0$, $\frac{dw}{dx} = 1$, A get all marginal production
3. The bigger r_A is ,the bigger proportion of fixed wage is.

Proposition :

1. If P is risk neutral and A is risk averse, P should offer A constant wage provide full insurance;
2. If A is risk neutral and P is risk averse, P should sell his firm to A at a certain price;
3. If both P and A are risk neutral, P should sell his firm to A;
4. If both P and A are risk averse, they share risk according to a certain proportion.

Proposition 1-3 are special case of proposition 4.

2.1.2 Standard model: asymmetric information

$a \in \left\{a^{H}, a^{L}\right\}, c\left(a^{H}\right)>c\left(a^{L}\right)$

$$ for all the $k=1,2, \ldots, n-1$

$$\begin{aligned} \sum_{i=1}^{k} p_{i}^{H} &< \sum_{i=1}^{k} p_{i}^{L} \\ \sum_{i=1}^{n} p_{i}^{H} &= \sum_{i=1}^{n} p_{i}^{L}=1 \end{aligned} \tag{6}$$

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$$\begin{aligned} \sum_{i=1}^{n} p_{i}^{H} u\left(w\left(x_{i}\right)\right)-c\left(a^{H}\right) \geq \sum_{i=1}^{n} p_{i}^{L} u\left(w\left(x_{i}\right)\right)-c\left(a^{L}\right) \\ \Rightarrow \sum_{i=1}^{n}\left(p_{i}^{H}-p_{i}^{L}\right) u\left(w\left(x_{i}\right)\right) \geq c\left(a^{H}\right)-c\left(a^{L}\right) \end{aligned} \tag{7}$$

So, the P' 's problem is

$$\begin{aligned} & \operatorname{Max}_{w\left(x_{i}\right)} \sum_{i=1}^{n} p_{i}^{H} v\left(x_{i}-w\left(x_{i}\right)\right) \\ & \text{s.t. (IR)} \sum_{i=1}^{n} p_{i}^{H}(a) u\left(w\left(x_{i}\right)\right)-c\left(a^{H}\right) \geq \underline{U} \\ & \text{(IC)} \sum_{i=1}^{n}\left(p_{i}^{H}-p_{i}^{L}\right) u\left(w\left(x_{i}\right)\right) \geq c\left(a^{H}\right)-c\left(a^{L}\right) \end{aligned} \tag{8}$$

According to concave program's rule

$$\frac{v^{\prime}\left(x_{i}-w\left(x_{i}\right)\right)}{u^{\prime}\left(w\left(x_{i}\right)\right)}=\lambda+\mu\left(1-\frac{p_{i}^{L}}{p_{i}^{H}}\right) \tag{9}$$

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$$\beta=\frac{1}{1+r b \sigma^{2}} \tag{10}$$

β：提成比例； r：风险规避程度； b努力的边际成本（b越大表明能力越低）； σ^2产出的方差