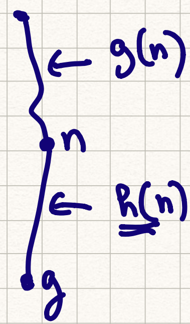
2/14/2019

## Few more things About heuristics

- Greedy search
- $A^*$ algorithm
- $h$ : heuristics.

$$f(n) = g(n) + h(n)$$




$h$ is admissible iff $\forall n \quad h(n) \le h^*(n)$

estimated cost from n to goal

True cost from n to the goal.

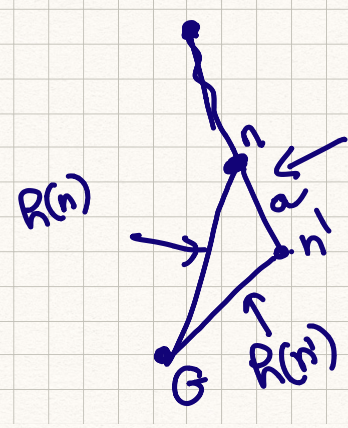
thm1: If $h$ is admissible $\longrightarrow$ $A^*$ using $h$ is optimal for tree search.

## How about graph Search?

We need a stronger condition for the optimality of $A^*$.

## Def (Consistency or monotonicity)

$h(n)$ is consistent $\forall n \forall n'$ Successors of $n$ IFF

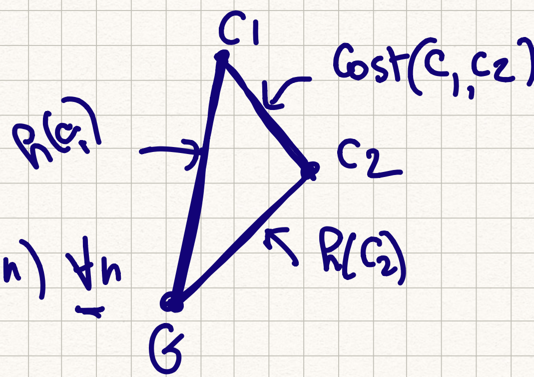



$$h(n) \le \underbrace{cost(n, n', a)}_{\Downarrow} + h(n')$$

$$\boxed{h(n) \leq g(n') - g(n) + h(n')}$$

Triangular inequality!

$h_{SLD}$




✓ admissible $h(n) \leq h^*(n) \ \forall n$

✓ consistent $h(C_1) \leq cost(C_1, C_2) + h(C_2)$.

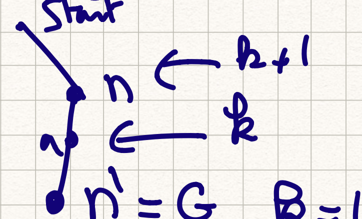
<u>lemma</u> Every consistent heuristic is admissible.

$\forall n, n' \ \underline{h(n) \leq c(n, n', a) + h(n')} \implies \underline{h(n) \leq h^*(n)}$

$n'$ successor of $n$.

Proof by induction on $k$: #nodes on the optimal path to the goal.

<u>$k = 1$.</u>




$h(G) = 0$

$$h(n) \leq c(n, G, a) + \underbrace{h(G)}_{0}$$

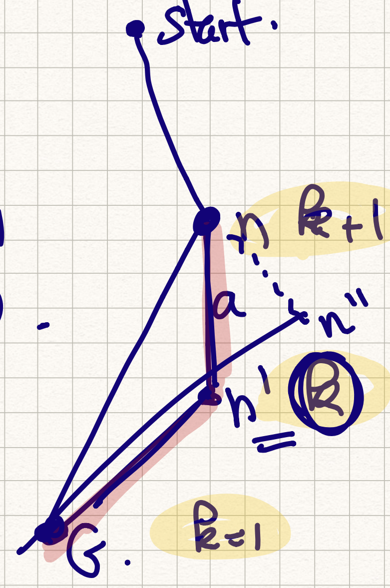
$\leq \quad h^*(n) \implies$ $k=1$ $h$ is admissible. for $k=1$

<u>inductive hypothesis</u>

Suppose $n'$ is at $k$ steps from the goal

$h(n') \leq h^*(n')$

$h$ is consistent and admissible for $n'$ that is far $k$ steps from the goal.




<u>Prove</u> $\quad h(n) \leq h^*(n)$

$n$ is at $k+1$ steps from the goal.

$$h(n) \leq c(n, n', a) + h(n')$$

$$\leq \underbrace{c(n, n', a) + h^*(n')}$$

because $h(n') \leq h^*(n)$ by ind. hyp.

$$h(n) \leq h^*(n)$$

therefore $h$ is admissible for $k+1$.

by induction, $h$ is admissible $\forall n$.

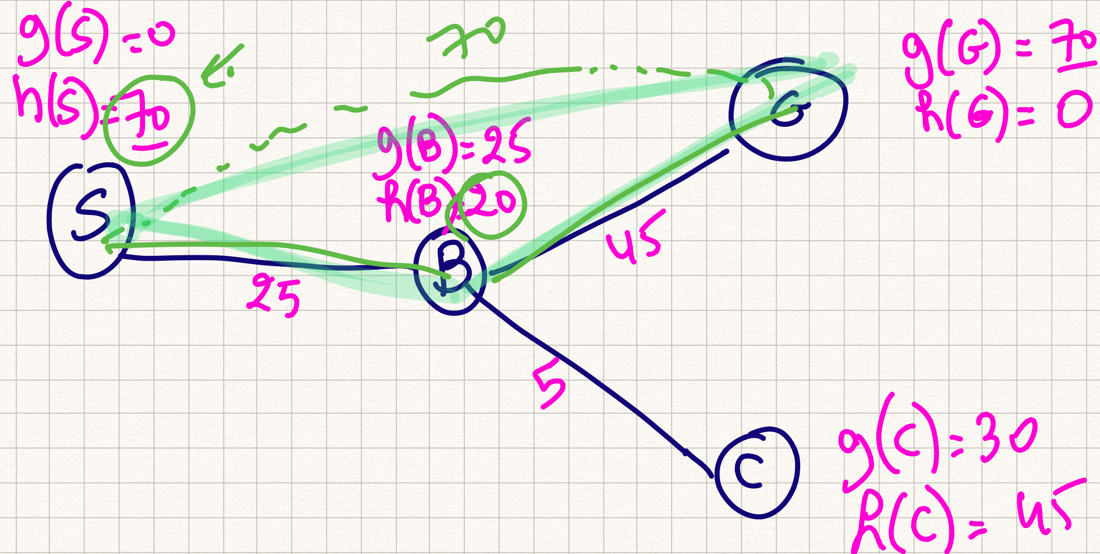
<u>Question</u> Converse true ?

$\forall n \; h(n) \leq h^*(n) \implies h(n) \leq cost(n, n', a) + h(n')$

$\forall n'$ succ of $n$

To disproof, Find a counter example, where $h$ is admissible but not consistent.

Counter example:




$h(S) \le h^*(S)$ $\quad 70 \le 70$

$h(B) \le h^*(B)$

$h(C) \le h^*(C)$

$$\underset{70}{h(S)} \not\le \underset{25}{c(S,B)} + \underset{20}{h(B)}$$

<u>thm</u> $\boxed{\text{A* using a consistent heuristic is optimal for graph search.}}$

for the Proof of the thm, and more about Heuristics:

- Russel / Norvig 97.
- J. Pearl Heuristics 1985 (book)
- Dechter & Pearl 1985

"Generalized BF Search strategies and the optimality of A*"

Further questions:

1) - Find another admissible but inconsistent Heuristic.

2) - Can you turn an inconsistent Heuristic into a consistent one?

The end.