Artificial Intelligence

Machine Learning
Naive Bayes

posterior likelihood Prior

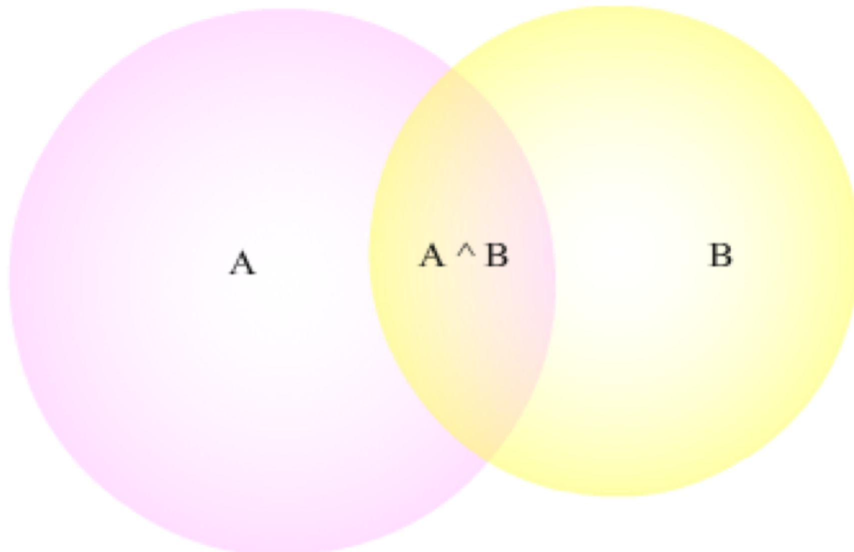
$$P(A/B) = \frac{P(B/A) * P(A)}{P(B)}$$

evidence

Outline

1. Generative models
2. Naive Bayes Classifier.
3. Setting
4. Example
5. Estimating probabilities

Conditional Probability



$$p(A|B) = \frac{p(A \wedge B)}{p(B)}$$

$$p(A \wedge B) = p(A|B) * p(B)$$

Bayes Rule

Writing $p(A \wedge B)$ in two different ways:

$$p(A \wedge B) = p(B|A) * p(A)$$

$$p(A \wedge B) = p(A|B) * p(B)$$

Bayes Rule

Writing $p(A \wedge B)$ in two different ways:

$$p(A \wedge B) = p(B|A) * p(A)$$

$$p(A \wedge B) = p(A|B) * p(B)$$

$$\color{blue}{p(A|B)} = \frac{p(B|A) * \color{red}{p(A)}}{p(B)}$$

Bayes Rule

Writing $p(A \wedge B)$ in two different ways:

$$p(A \wedge B) = p(B|A) * p(A)$$

$$p(A \wedge B) = p(A|B) * p(B)$$

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B)}$$

$p(A|B)$ is called posterior (posterior distribution on A given B.)
$p(A)$ is called prior.
$p(B)$ is called evidence.
$p(B|A)$ is called likelihood.

Bayes Rule

	A	not A	Sum
B	P(A and B)	P(not A and B)	P(B)
Not B	P(A and not B)	P(not A and not B)	P(not B)
	P(A)	P(not A)	1

- This table divides the sample space into 4 mutually exclusive events.
- The probability in the margins are called marginals and are calculated by summing across the rows and and the columns.

Bayes Rule

	A	not A	Sum
B	P(A and B)	P(not A and B)	P(B)
Not B	P(A and not B)	P(not A and not B)	P(not B)
	P(A)	P(not A)	1

- This table divides the sample space into 4 mutually exclusive events.
- The probability in the margins are called marginals and are calculated by summing across the rows and and the columns.

Another form:

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B|A) * p(A) + p(B|\neg A) * p(\neg A)}$$

Example of Using Bayes Rule

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B|A) * p(A) + p(B|\neg A) * p(\neg A)}$$

A: patient has cancer.

B: patient has a positive lab test.

Example of Using Bayes Rule

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B|A) * p(A) + p(B|\neg A) * p(\neg A)}$$

A: patient has cancer.

B: patient has a positive lab test.

$p(A) = 0.008$ $p(\neg A) = 0.992$

Example of Using Bayes Rule

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B|A) * p(A) + p(B|\neg A) * p(\neg A)}$$

A: patient has cancer.

B: patient has a positive lab test.

$p(A) = 0.008$ $\quad$ $p(\neg A) = 0.992$

$p(B|A) = 0.98$ $\quad$ $p(\neg B|A) = 0.02$

Example of Using Bayes Rule

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B|A) * p(A) + p(B|\neg A) * p(\neg A)}$$

A: patient has cancer.

B: patient has a positive lab test.

$p(A) = 0.008$ $\quad$ $p(\neg A) = 0.992$

$p(B|A) = 0.98$ $\quad$ $p(\neg B|A) = 0.02$

$p(B|\neg A) = 0.03$ $\quad$ $p(\neg B|\neg A) = 0.97$

Example of Using Bayes Rule

$$p(A|B) = \frac{p(B|A) * p(A)}{p(B|A) * p(A) + p(B|\neg A) * p(\neg A)}$$

A: patient has cancer.

B: patient has a positive lab test.

$p(A) = 0.008$

$p(B|A) = 0.98$

$p(B|\neg A) = 0.03$

$p(\neg A) = 0.992$

$p(\neg B|A) = 0.02$

$p(\neg B|\neg A) = 0.97$

$$p(A|B) = \frac{0.98 * 0.008}{0.98 * 0.008 + 0.03 * 0.992} = 0.21$$

Why probabilities?

Why are we bringing here Bayes rule?

Why probabilities?

Why are we bringing here Bayes rule?

Recall Classification framework:

Given: Training data: $(x_1, y_1), \ldots, (x_n, y_n) / x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{Y}$.
Task: Learn a classification function: $f : \mathbb{R}^d \longrightarrow \mathbb{Y}$

Why probabilities?

Why are we bringing here Bayes rule?

Recall Classification framework:

Given: Training data: $(x_1, y_1), \ldots, (x_n, y_n) / x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{Y}$.
Task: Learn a classification function: $f : \mathbb{R}^d \longrightarrow \mathbb{Y}$

Learn a mapping from x to y.
We would like to find this mapping $f(x) = y$ through $p(y|x)$!

Discriminative Algorithms

- **Discriminative Algorithms**:
  - Idea: model $p(y|x)$, conditional distribution of y given x.
  - In Discriminative Algorithms: find a decision boundary that separates positive from negative example.
  - To predict a new example, check on which side of the decision boundary it falls.
  - Model $p(y|x)$ directly.

Generative Algorithms

- **Generative Algorithms** adopt a different approach:
  - Idea: Build a model for what positive examples look like. Build a different model for what negative example look like.
  - To predict a new example, match it with each of the models and see which match is best.
  - Model $p(x|y)$ and $p(y)$!
  - Use Bayes rule to obtain $p(y|x) = \frac{p(x|y)p(y)}{p(x)}$.

Generative Algorithms

- **Generative Algorithms** adopt a different approach:
  - Idea: Build a model for what positive examples look like. Build a different model for what negative example look like.
  - To predict a new example, match it with each of the models and see which match is best.
  - Model $p(x|y)$ and $p(y)$!
  - Use Bayes rule to obtain $p(y|x) = \frac{p(x|y)p(y)}{p(x)}$.
  - To make a prediction:

$$\text{argmax}_y p(y|x) = \text{argmax}_y \frac{p(x|y)p(y)}{p(x)}$$

$$\text{argmax}_y p(y|x) \approx \text{argmax}_y p(x|y)p(y)$$

Naive Bayes Classifier

- Probabilistic model.
- Highly practical method.
- Application domains to natural language text documents.
- Naive because of the strong independence assumption it makes (not realistic).
- Simple model.
- Strong method can be comparable to decision trees and neural networks in some cases.

Setting

- A training data (x_i, y_i), x_i is a feature vector and y_i is a discrete label.
- d features, and n examples.
- Example: consider document classification, each example is a documents, each feature represents the presence or absence of a particular word in the document.
- We have a training set.
- A new example with feature values $x_{new} = (a_1, a_2, \cdots, a_d)$.
- We want to predict the label y_{new} of the new example.

Setting

$$y_{new} = \mathsf{argmax}_{y \in \mathbb{Y}} \quad p(y|a_1, a_2, \cdots, a_d)$$

Setting

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y|a_1, a_2, \cdots, a_d)$$

Use Bayes rule to obtain:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; \frac{p(a_1, a_2, \cdots, a_d|y) * p(y)}{p(a_1, a_2, \cdots, a_d)}$$

Setting

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y|a_1, a_2, \cdots, a_d)$$

Use Bayes rule to obtain:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; \frac{p(a_1, a_2, \cdots, a_d|y) * p(y)}{p(a_1, a_2, \cdots, a_d)}$$

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(a_1, a_2, \cdots, a_d|y) * p(y)$$

Setting

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y | a_1, a_2, \cdots, a_d)$$

Use Bayes rule to obtain:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; \frac{p(a_1, a_2, \cdots, a_d | y) * p(y)}{p(a_1, a_2, \cdots, a_d)}$$

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; {\color{red} p(a_1, a_2, \cdots, a_d | y)} * {\color{blue} p(y)}$$

Can we estimate these two terms from the training data?

Setting

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y|a_1, a_2, \cdots, a_d)$$

Use Bayes rule to obtain:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; \frac{p(a_1, a_2, \cdots, a_d|y) * p(y)}{p(a_1, a_2, \cdots, a_d)}$$

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(a_1, a_2, \cdots, a_d|y) * p(y)$$

Can we estimate these two terms from the training data?

1. $p(y)$ can be easy to estimate: count the frequency with which each label y occurs in the training data.

Setting

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y|a_1, a_2, \cdots, a_d)$$

Use Bayes rule to obtain:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; \frac{p(a_1, a_2, \cdots, a_d|y) * p(y)}{p(a_1, a_2, \cdots, a_d)}$$

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(a_1, a_2, \cdots, a_d|y) * p(y)$$

Can we estimate these two terms from the training data?

1. $p(y)$ can be easy to estimate: count the frequency with which each label y.
2. $p(a_1, a_2, \cdots, a_d|y)$ is not easy to estimate unless we have a very very large sample. (We need to see every example many times to get reliable estimates)

Naive Bayes Classifier

Makes a simplifying assumption that the feature values are conditionally independent given the label.

Given the label of the example, the probability of observing the conjunction $a_1, a_2, \cdots, a_d$ is the product of the probabilities for the individual features:

$$p(a_1, a_2, \cdots, a_d | y) = \prod_j p(a_j | y)$$

Naive Bayes Classifier

Makes a simplifying assumption that the feature values are conditionally independent given the label.
Given the label of the example, the probability of observing the conjunction $a_1, a_2, \cdots, a_d$ is the product of the probabilities for the individual features:

$$p(a_1, a_2, \cdots, a_d | y) = \prod_j p(a_j | y)$$

Naive Bayes Classifier:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y) \prod_j p(a_j | y)$$

Naive Bayes Classifier

Makes a simplifying assumption that the feature values are conditionally independent given the label.

Given the label of the example, the probability of observing the conjunction $a_1, a_2, \cdots, a_d$ is the product of the probabilities for the individual features:

$$p(a_1, a_2, \cdots, a_d | y) = \prod_j p(a_j | y)$$

Naive Bayes Classifier:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y) \prod_j p(a_j | y)$$

Can we estimate these two terms from the training data?

Naive Bayes Classifier

Makes a simplifying assumption that the feature values are conditionally independent given the label.

Given the label of the example, the probability of observing the conjunction $a_1, a_2, \cdots, a_d$ is the product of the probabilities for the individual features:

$$p(a_1, a_2, \cdots, a_d | y) = \prod_j p(a_j | y)$$

Naive Bayes Classifier:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} \;\; p(y) \prod_j p(a_j | y)$$

Can we estimate these two terms from the training data?

Yes!

Algorithm

Learning: Based on the frequency counts in the dataset:

1. Estimate all $p(y)$, $\forall y \in \mathbb{Y}$.
2. Estimate all $p(a_j|y)$ $\forall y \in \mathbb{Y}$, $\forall a_i$.

Classification: For a new example, use:

$$y_{new} = \text{argmax}_{y \in \mathbb{Y}} p(y) \prod_j p(a_j|y)$$

Note: No model per se or hyperplane, just count the frequencies of various data combinations within the training examples.

Example

Highest Degree	Work Experience	Favorite Language	Needs Work Visa	Hire
Bachelors	Mobile Dev	Objective-C	TRUE	yes
Masters	Web Dev	Java	FALSE	yes
Masters	Mobile Dev	Java	TRUE	yes
PhD	Mobile Dev	Objective-C	TRUE	yes
PhD	Web Dev	Objective-C	TRUE	no
Bachelors	UX Design	Objective-C	TRUE	no
Bachelors	Mobile Dev	Java	FALSE	yes
PhD	Web Dev	Objective-C	FALSE	no
Bachelors	UX Design	Java	FALSE	yes
Masters	UX Design	Objective-C	TRUE	no
Masters	UX Design	Java	FALSE	yes
PhD	Mobile Dev	Java	FALSE	no
Masters	Mobile Dev	Java	TRUE	yes
Bachelors	Web Dev	Objective-C	FALSE	no

Highest Degree	Work Experience	Favorite Language	Needs Work Visa	Hire
Masters	UX Design	Java	TRUE	?

Can we predict the class of the new example?

Example

$$y_{new} = \mathsf{argmax}_{y \in \{yes,no\}} p(y) * p(Masters|y) * p(UX\ \ Design|y) * p(Java|y) * p(TRUE|y)$$

$$p(yes) = 8/14 = 0.572$$

$$p(no) = 6/14 = 0.428$$

Example

$$y_{new} = \text{argmax}_{y \in \{yes, no\}} p(y) * p(Masters|y) * p(UX\ \ Design|y) * p(Java|y) * p(TRUE|y)$$

$$p(yes) = 8/14 = 0.572$$

$$p(no) = 6/14 = 0.428$$

Conditional probabilities:

$$p(masters|yes) = 4/8 \quad p(masters|no) = 1/6$$

$$p(UX\ \ Design|yes) = 2/8 \quad p(UX\ \ Design|no) = 2/6$$

$$p(Java|yes) = 6/8 \quad p(Java|no) = 1/6$$

$$p(TRUE|yes) = 4/8 \quad p(TRUE|no) = 3/6$$

Example

$$y_{new} = \text{argmax}_{y \in \{yes, no\}} p(y) * p(Masters|y) * p(UX\ \ Design|y) * p(Java|y) * p(TRUE|y)$$

$$p(yes) = 8/14 = 0.572$$

$$p(no) = 6/14 = 0.428$$

Conditional probabilities:

$$\begin{array}{rl} p(masters|yes) = 4/8 & p(masters|no) = 1/6 \\ p(UX\ \ Design|yes) = 2/8 & p(UX\ \ Design|no) = 2/6 \\ p(Java|yes) = 6/8 & p(Java|no) = 1/6 \\ p(TRUE|yes) = 4/8 & p(TRUE|no) = 3/6 \end{array}$$

$$p(yes) * p(Masters|yes) * p(UX\ \ Design|yes) * p(Java|yes) * p(TRUE|yes) = 0.026$$

$$p(no) * p(Masters|no) * p(UX\ \ Design|no) * p(Java|no) * p(TRUE|no) = 0.002$$

Example

$$y_{new} = \text{argmax}_{y \in \{yes,no\}} p(y) * p(Masters|y) * p(UX\ \ Design|y) * p(Java|y) * p(TRUE|y)$$

$$p(yes) = 8/14 = 0.572$$

$$p(no) = 6/14 = 0.428$$

Conditional probabilities:

$$p(masters|yes) = 4/8 \quad p(masters|no) = 1/6$$

$$p(UX\ \ Design|yes) = 2/8 \quad p(UX\ \ Design|no) = 2/6$$

$$p(Java|yes) = 6/8 \quad p(Java|no) = 1/6$$

$$p(TRUE|yes) = 4/8 \quad p(TRUE|no) = 3/6$$

$$p(yes) * p(Masters|yes) * p(UX\ \ Design|yes) * p(Java|yes) * p(TRUE|yes) = 0.026$$

$$p(no) * p(Masters|no) * p(UX\ \ Design|no) * p(Java|no) * p(TRUE|no) = 0.002$$

$$y_{new} = yes$$

Estimating probabilities

m-estimate of the probability:

$$p(a_j|y) = \frac{n_c + m * p}{n_y + m}$$

where:

n_y: total # examples for which the class is y.

n_c: total # examples for which the class is y and feature $x_j = a_j$.

m: called *equivalent sample size*

Estimating probabilities

m-estimate of the probability:

$$p(a_j|y) = \frac{n_c + m * p}{n_y + m}$$

where:

n_y: total # examples for which the class is y.

n_c: total # examples for which the class is y and feature $x_j = a_j$.

m: called *equivalent sample size*

Intuition: Augment the sample size by m virtual examples, distributed according to prior p (prior estimate of each value). If prior is unknown, assume uniform prior: if a feature has k values, we can set $p = \frac{1}{k}$.

Example: $P(java|no) = \frac{1}{6}$ Its m-estimate for $m = 10$ in which favorite language is uniformly distributed with $\frac{1}{2}$ So 'java' occurs uniformly in 5 out of the 10 additional samples. $P(java|no) = \frac{1+(1/2)\cdot 10}{6+10}$

Text Classification

Learning to classify text. Why?

- Learn which news articles are of interest
- Learn to classify web pages by topic
- Classify Spam from non Spam emails
- Naive Bayes is among most effective algorithms
- What attributes shall we use to represent text documents?

Text Classification

- Given a document (corpus), define an attribute for each word position in the document.
- The value of the attribute is the English word in that position.
- To reduce the number of probabilities that needs to be estimated, besides NB independence assumption, we assume that: The probability of a given word w_k occurrence is independent of the word position within the text. That is:

  $$p(x_1 = w_k|c_j), p(x_2 = w_k|c_j), \cdots$$

  estimated by:

  $$p(w_k|c_j)$$

Text Classification

- m-estimate of the probabilities:

$$p(w_k|c_j) = \frac{n_k + 1}{n_j + |Vocabulary|}$$

  where:
  n_j: total #word positions in all training examples of class c_j.
  n_k: number of times the word w_k is found in among these n_j word positions.

- The following function learns the probabilities $P(w_k/c_j)$ describing the probability that a randomly drawn word from a document with class c_j is the English word w_k. It also learn the class priors $P(c_j)$.

Text Classification

Learn_Naive_Bayes_texte(Examples, C)

Input: Examples is a set of document with classes. C is the set of classes.

1. Collect all words, punctuations and tokens occuring in the Examples. Let the pertinent vocabulary be V.

2. Calculate $P(c_j)$ and $P(w_k/c_j)$.

   - For each class c_j in C
     - $docs_j \leftarrow$ the subset of documents from Examples for which the label=c_j
     - $P(c_j) \leftarrow \frac{|docs_j|}{|Examples|}$
     - $text_j \leftarrow$ a single document concatenation of all documents in doc_j
     - $n_j \leftarrow$ total number of distinct word positions in $text_j$
     - for each word w_k in V
       * $n_k \leftarrow$ number of times word w_k appears in $text_j$
       * $P(w_k/c_j) \leftarrow \frac{n_k+1}{n_j+|V|}$

Output: all $P(c_j)$ and $P(w_k/c_j)$.

Text Classification

Classify_Naive_Bayes_text(Doc)

Return the estimated label for the document Doc. a_i denotes the word found in the ith position within Doc.

- positions $\leftarrow$ all word positions in Doc that contain token found in V.
- Return c_{Doc} where:

$$c_{Doc} =_{c_j \in C} P(c_j) \prod_{i \in positions} P(a_i / c_j)$$

Example

Classification of Radio and TV sentences.

TV:

1. TV programs are not interesting – TV is annoying.
2. Kids like TV.
3. We receive TV by radio waves.

Radio:

1. It is interesting to listen to the radio.
2. On the waves, kids programs are rare.
3. The kids listen to the radio; it is rare!

Vocabulary: V={TV, program, interesting, kids, radio, wave, listen, rare}

Example

$p(C_{Tv}) = 3/6 = 0.5 \qquad p(C_{Radio}) = 3/6 = 0.5$

$n_{TV} = 9 \qquad n_{Radio} = 11$

$w \in \mathcal{V}$	Class "TV"			Class "Radio"		
	n_{TV}	n_w	$p(w\|C_{TV})$	n_{Radio}	n_w	$p(w\|C_{radio})$
TV	9	4	(4+1)/(9+8)	11	0	1/(11+8)
program	9	1	(1+1)/(9+8)	11	1	2/(11+8)
Interesting	9	1	(1+1)/(9+8)	11	1	2/(11+8)
kids	9	1	(1+1)/(9+8)	11	2	3/(11+8)
radio	9	1	(1+1)/(9+8)	11	2	3/(11+8)
wave	9	1	(1+1)/(9+8)	11	1	2/(11+8)
listen	9	0	(0+1)/(9+8)	11	2	3/(11+8)
rare	9	0	(0+1)/(9+8)	11	2	3/(11+8)

Example

$p(C_{Tv}) = 3/6 = 0.5 \qquad p(C_{Radio}) = 3/6 = 0.5$

$n_{TV} = 9 \qquad n_{Radio} = 11$

$w \in \mathcal{V}$	Class "TV"			Class "Radio"		
	n_{TV}	n_w	$p(w\|C_{TV})$	n_{Radio}	n_w	$p(w\|C_{radio})$
TV	9	4	(4+1)/(9+8)	11	0	1/(11+8)
program	9	1	(1+1)/(9+8)	11	1	2/(11+8)
Interesting	9	1	(1+1)/(9+8)	11	1	2/(11+8)
kids	9	1	(1+1)/(9+8)	11	2	3/(11+8)
radio	9	1	(1+1)/(9+8)	11	2	3/(11+8)
wave	9	1	(1+1)/(9+8)	11	1	2/(11+8)
listen	9	0	(0+1)/(9+8)	11	2	3/(11+8)
rare	9	0	(0+1)/(9+8)	11	2	3/(11+8)

x_{new} = "I saw the radio of my lungs on TV"

The only words in the vocabulary are Radio and TV.

$p(C_{Tv}) \times p(TV|C_{Tv}) \times p(radio|C_{Tv}) = 0.5 \times 0.3 \times 0.12 = 0.018$

$p(C_{Radio}) \times p(TV|C_{Radio}) \times p(radio|C_{Radio}) = 0.5 \times 0.05 \times 0.15 = 0.003$

$y_{new} = C_{Tv}$

Linearity of NB?

Is Naive Bayes a linear separator?

Linearity of NB?

Is Naive Bayes a linear separator?

Yes!

Linearity of NB?

Suppose we have a binary classification case with binary features.

Source: Avi Pfeer. Naive Bayes and Autoclass.

$$P(C=T) = \theta_C$$

$$\begin{aligned}
P(C=F) &= 1-\theta_C \\
P(X_i=T \mid C=T) &= \theta_i^T \\
P(X_i=F \mid C=T) &= 1-\theta_i^T \\
P(X_i=T \mid C=F) &= \theta_i^F \\
P(X_i=F \mid C=F) &= 1-\theta_i^F
\end{aligned}$$

Linearity of NB?

Now we can use a trick. If we encode T as 1 and F as 0, so x_i is either 0 or 1, we can write the following:

$$P(X_i = x_i \mid C = c) = (\theta_i^c)^{x_i}(1 - \theta_i^c)^{1-x_i}$$

Now, a naive Bayes model classifies x as true if

$$P(C = T)\prod_i P(X_i = x_i \mid C = T) \geq P(C = F)\prod_i P(X_i = x_i \mid C = F)$$

I.e.

$$\theta_C \prod_i (\theta_i^T)^{x_i}(1 - \theta_i^T)^{(1-x_i)} \geq (1 - \theta_C)\prod_i (\theta_i^F)^{x_i}(1 - \theta_i^F)^{(1-x_i)}$$

Linearity of NB?

Now take logs:

$$\begin{array}{ll} \ln\theta_C & +\sum_i(x_i\ln\theta_i^T + (1-x_i)\ln(1-\theta_i^T)) \quad \geq \\ \ln(1-\theta_C) & +\sum_i(x_i\ln\theta_i^F + (1-x_i)\ln(1-\theta_i^F)) \end{array}$$

Rearranging terms,

$$\begin{array}{l} \ln\theta_C + \sum_i \ln(1-\theta_i^T) + \sum_i(\ln\theta_i^T - \ln(1-\theta_i^T))x_i \geq \\ \ln(1-\theta_C) + \sum_i \ln(1-\theta_i^F) + \sum_i(\ln\theta_i^F - \ln(1-\theta_i^F))x_i \end{array}$$

Linearity of NB?

We get

$$w_0 + \sum_{i=1}^{n} w_i x_i \quad \geq \quad 0$$

where

$$w_0 = \ln \theta_C + \sum_i \ln(1 - \theta_i^T) - [\ln(1 - \theta_C) + \sum_i \ln(1 - \theta_i^F)]$$

and

$$w_i = \ln \theta_i^T - \ln(1 - \theta_i^T) - [\ln \theta_i^F - \ln(1 - \theta_i^F)]$$

Naive Bayes for text

1. Naive Bayes is a linear classifier.

2. Incredibly simple and easy to implement.

3. Works wonderful for text.

Credit

- Machine Learning. Tom Mitchell 1997.