Condorcet Jury theorem : example

1. Consider the simple example of 3 voters. $\boxed{n=3}$
   - Let x, y, z be the three voters.
   - Let $p(x)$ be the probability of x to be correct.
   - Let $p(y)$ " " " " y " " ".
   - Let $p(z)$ " " " " z " " ".
   - Let $p(c)$ be the probability of the crowd to be correct.

2. Suppose $\boxed{p(x) = p(y) = p(z) = 3/4}$.

3. Since the majority voting is used, we will need $\boxed{m=2}$ voters to be <u>at least</u> $m = \frac{n+1}{2} = 2$ correct.

For the majority to be correct, we need (in general)

m voters to be right <u>OR</u>

$m+1$ " " " " OR

⋮

n voters to be right.

In this case, we need 2 voters correct or 3 voters correct.

(4) $$P(C) = \sum_{k=m}^{n} \text{Probability of the votes correct} \quad \ldots (\text{I})$$

In this example: $\underbrace{\text{probability of 2 are correct}}_{\text{Case 1}}$ +
$\underbrace{\text{probability of 3 are correct}}_{\text{Case 2}}$.

(5) **Case 1** 2 voters are correct.

$\binom{3}{2}$ cases
- xy are correct but z is incorrect ...(1)
- xz " " " y " " . (2)
- yz " " " x " " . (3)

$$P(xy \text{ correct and } z \text{ incorrect}) = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}.$$

Same for (2) and (3). $\quad p \times p \times (1-p)$

therefore P(2 voters exactly are correct) =

$$\binom{3}{2} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$$

in general

$$\binom{n}{k} p^k (1-p)^{n-k}$$

(6) **Case 2** 3 voters are correct.

$$\binom{3}{3} \times p \times p \times p = \binom{3}{3} p^3$$

So in general we can re-write (I) as :

$$\sum_{k=m}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

$$p(c) = \binom{3}{2} \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} + \binom{3}{3} \left(\frac{3}{4}\right)^3$$

$$= 3\left(\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4}\right) + \left(\frac{3}{4}\right)^3 = 0.84$$

Using three voters with a probability of $\frac{3}{4}$ each will lead to a probability of the crowd of 0.84 which is higher than the individual probabilities of $\frac{3}{4}$.