Quantum Computing

Mia west

Session Plan

1. Why quantum computing could be really useful.
2. Qubits, entanglement & the circuit model of quantum computing.
3. IBM quantum computer.
4. A little (by necessity) computing exercise ☺.

1. What is Quantum Computing & Why is it Useful?

“Quantum Computation & Quantum Information”

M. Nielsen, I. Chuang

This Is Really Good!!

Quantum Physics

1. Quantum states can be represented by wavefunctions.

e.g. $|\psi\rangle$.

Quantum Physics

2. For a Hamiltonian H with eigenvalues i & eigenvectors $|i\rangle$, any superposition of these eigenvectors is a valid quantum state.

$$|\psi\rangle = \sum_i c_i |i\rangle$$

Quantum Physics

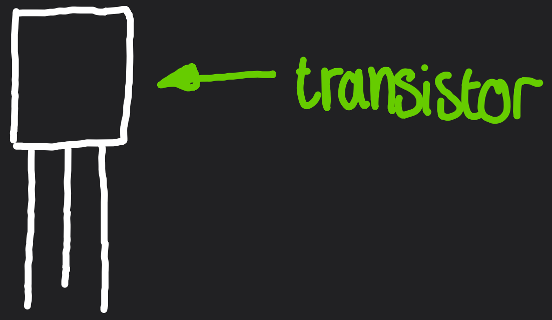
3. When you take a measurement of energy relating to the state $|\psi\rangle$:

   a) The result will be an eigenvalue of H; i with probability $|C_i|^2$

   b) The state will collapse into the corresponding eigenstate $|i\rangle$

What Does This Mean For Coding?

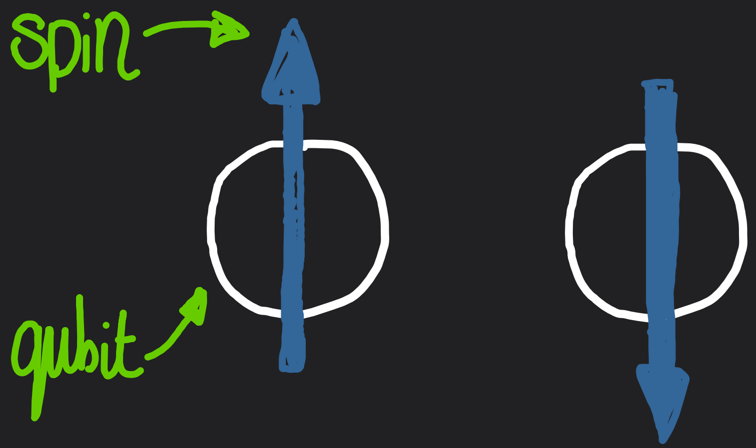
Bit (macroscopic)



1 or 0

Qubit (microscopic)

spin-up spin-down



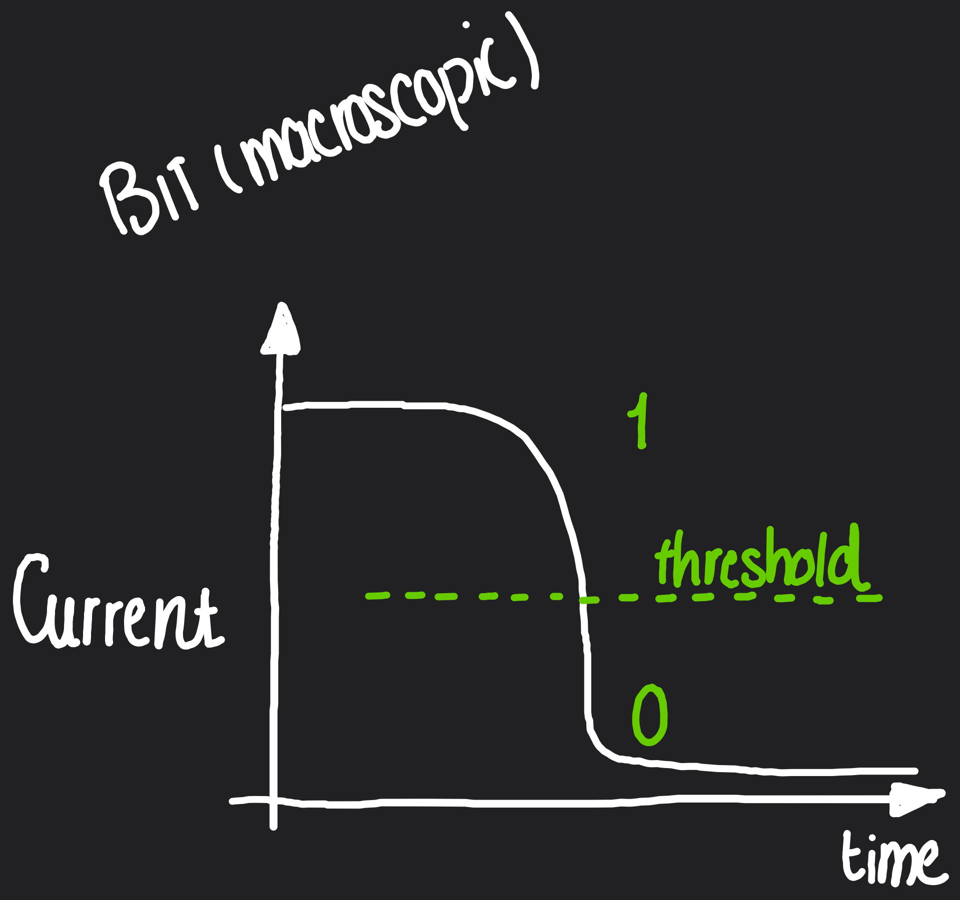
$|1\rangle$

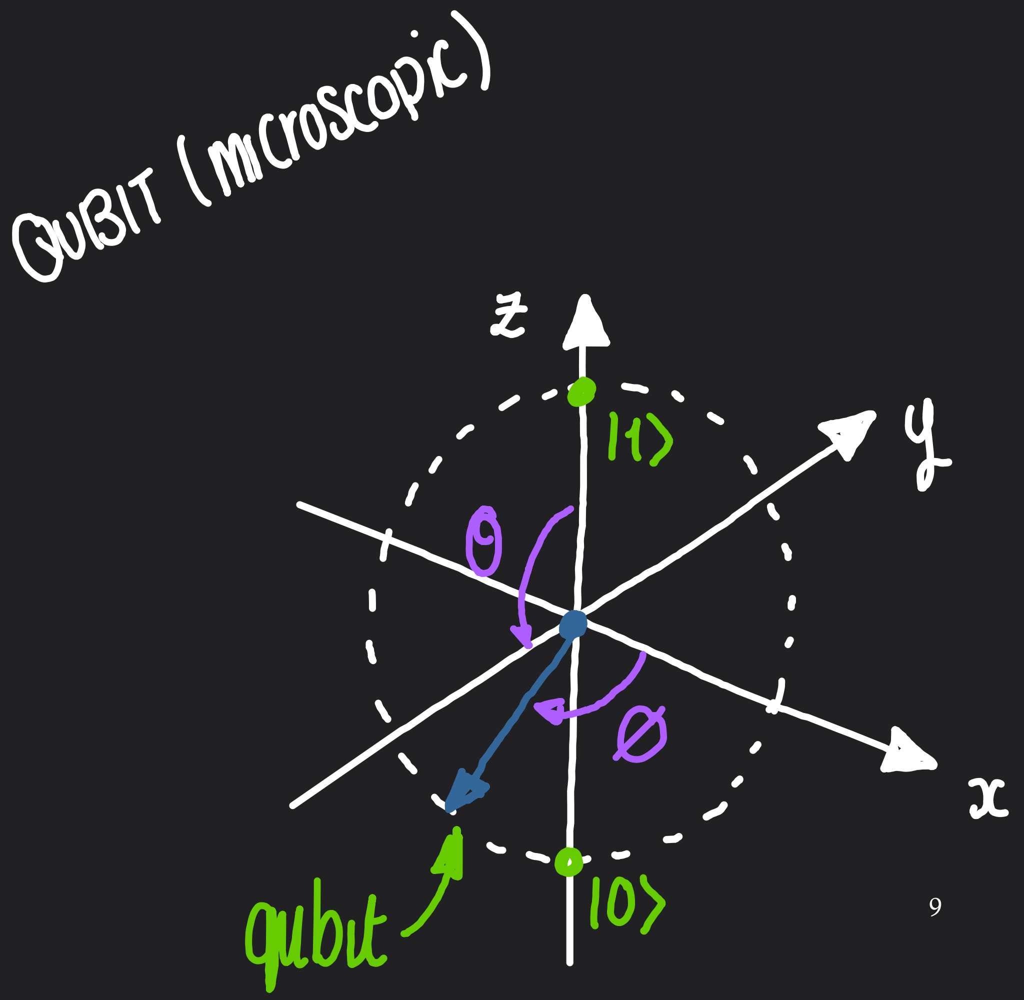
$|0\rangle$

... or a superposition

eg $(|0\rangle + |1\rangle)/\sqrt{2}$

What Does This Mean For Coding?





What Could This Be?

Superconducting quantum computer.

- Cooper pairs confined to an electrostatic box

  (# Cooper pairs becomes a good quantum number)

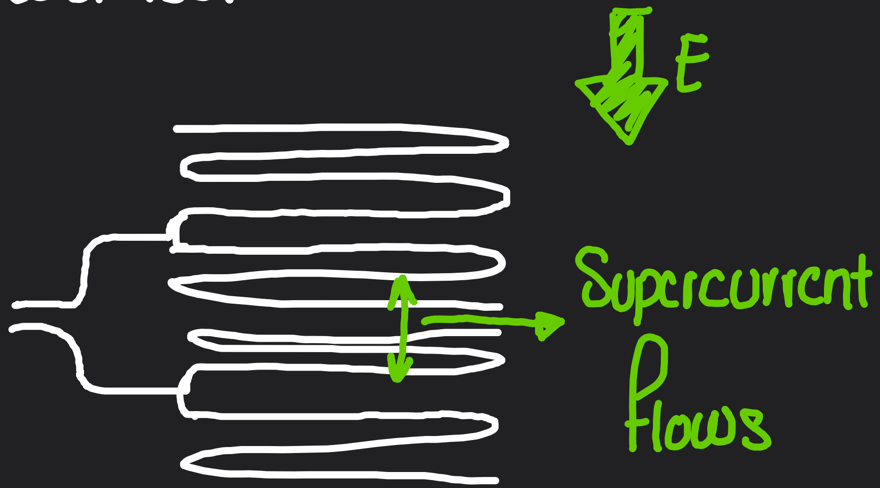
- Single qubit gates by modulating box potential

  Josephson junctions to couple qubits ⟶

- Measurement from measuring electric charge

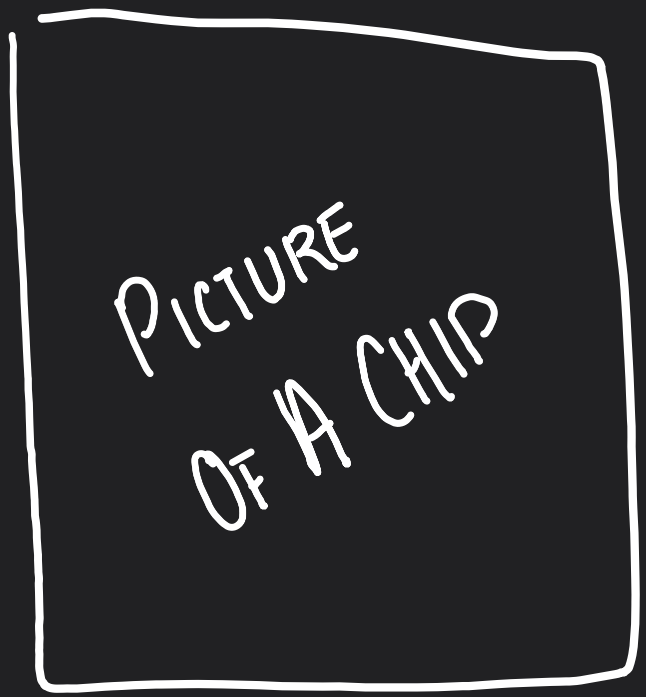
apply E-field to couple qubits

JOSEPHSON JUNCTION



Two superconductors are placed in proximity, with a barrier between them.

Many Bits



Why is Quantum Cool?

3 BITS...

Choose a '1' or '0' for each bit.

e.g. 110

3 QUBITS...

Choose an 'amplitude' for each possible combination of bits.

e.g. $1/\sqrt{2}$ in $|011\rangle$

$1/\sqrt{2}$ in $|101\rangle$

0 in all others.

3 qubits store 2^3 complex numbers.

Why is Quantum Useful?

SPEED

e.g. Shor's prime factorisation algorithm has exponential speedup compared to the best classical algorithm.

Why is Quantum Useful?

MODELS

e.g. Modelling molecules which may have useful properties for medicine.

2. The Circuit Model of Quantum Computing

Representing a Qubit

As a ket:

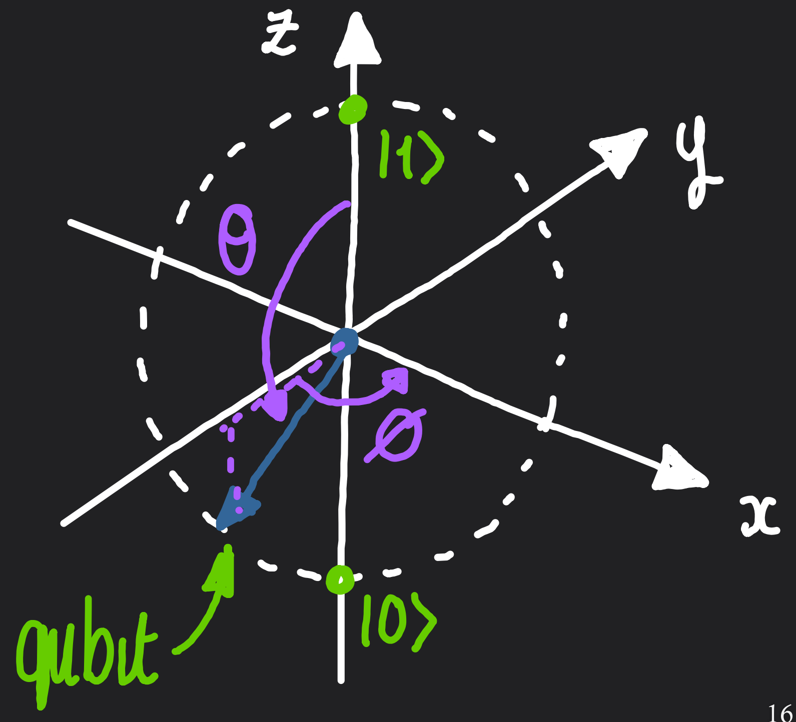
$$|q\rangle = a|0\rangle + b|1\rangle$$

where $|a|^2 + |b|^2 = 1$. Or...

$$|q\rangle = \cos\theta/2\,|0\rangle + e^{i\phi}\sin\theta/2\,|1\rangle$$

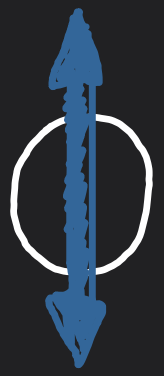
As a vector: $|q\rangle = \begin{array}{c} |0\rangle \\ |1\rangle \end{array}\begin{pmatrix} a \\ b \end{pmatrix}$

As a bloch sphere:



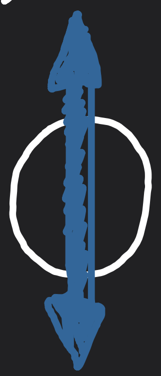
Many Qubits

QUBIT 0



$$\frac{|0\rangle_0+|1\rangle_0}{\sqrt{2}}$$

QUBIT 1



$$\frac{|0\rangle_1+|1\rangle_1}{\sqrt{2}}$$

Qubit 0

$$\frac{|00\rangle+|01\rangle+|10\rangle+|11\rangle}{2}$$

Qubit 1

State Vector

The state vector tracks the amplitudes for each system configuration.

- Usually normalised to 1
- Have 2^n complex amplitudes for n bits.
- Classical state vector has all entries '0' except one '1'.

e.g. for 3 qubits:

$$\begin{array}{c} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{array} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix}$$

Quantum Operations

Now that we have our basis, would like to apply some operations to it.

- Single qubit gates: H*, Pauli X, Y & Z, S*, T*.
- Multiple qubit gates: CNOT*, C-Z, Toffoli*.
- Measurement gate: M.

[See sheet!]

* = Universal set of quantum gates.

Single Qubit Gates

May be $|1\rangle, |0\rangle$ or a superposition

NOTATION: $|q\rangle$ Quantum state of a single qubit q

——— Circuit (time goes left to right)

$-\boxed{G}-$ Quantum gate 'G'

Pauli X Gate (Not Gate) $-\boxed{X}-$ OR $-\otimes-$

Begin with a single qubit in the $\left\{ \begin{array}{l} |0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \\ |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \end{array} \right\}$ state & apply an X:

CIRCUIT

$|0\rangle - \boxed{X} - |1\rangle$

$|1\rangle - \boxed{X} - |0\rangle$

STATE VECTOR

X Gate.

$$\begin{pmatrix}1\\0\end{pmatrix} \xrightarrow{X} \overbrace{\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}^{\text{X Gate.}}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \xrightarrow{X} \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

Hadamard Gate $-\boxed{H}-$

Use the Hadamard gate to create a superposition

$$|0\rangle \;-\boxed{H}-\; \frac{(|0\rangle+|1\rangle)}{\sqrt{2}}$$

$$|1\rangle \;-\boxed{H}-\; \frac{(|0\rangle-|1\rangle)}{\sqrt{2}}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \xrightarrow{H} \overbrace{\tfrac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}}^{\text{Hadamard gate.}}\begin{pmatrix}1\\0\end{pmatrix} = \tfrac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \xrightarrow{H} \tfrac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \tfrac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Measurement

1. Choose a measurement basis

   ↳ Usually choose $|0\rangle, |1\rangle$ basis

2. Measurement collapses qubit into $|0\rangle$ or $|1\rangle$ state, with probability:

$$P_{|0\rangle} = |\langle 0|q\rangle|^2 \qquad P_{|1\rangle} = |\langle 1|q\rangle|^2$$

e.g. if $|q\rangle = a|0\rangle + b|1\rangle$: $P_{|0\rangle} = |a|^2$ & $P_{|1\rangle} = |b|^2$

3. Superposition collapses into the measured state.

Measurement

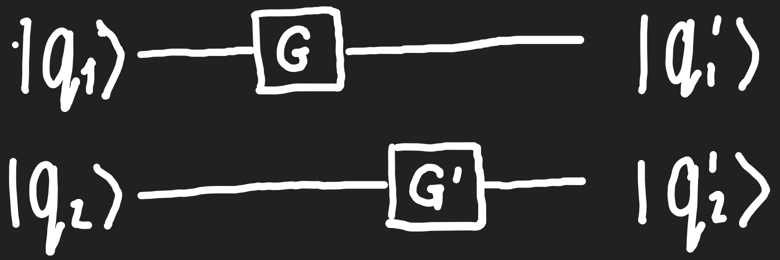
Example:

① $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

② $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

③ 0 with probability 50%
1 with probability 50%

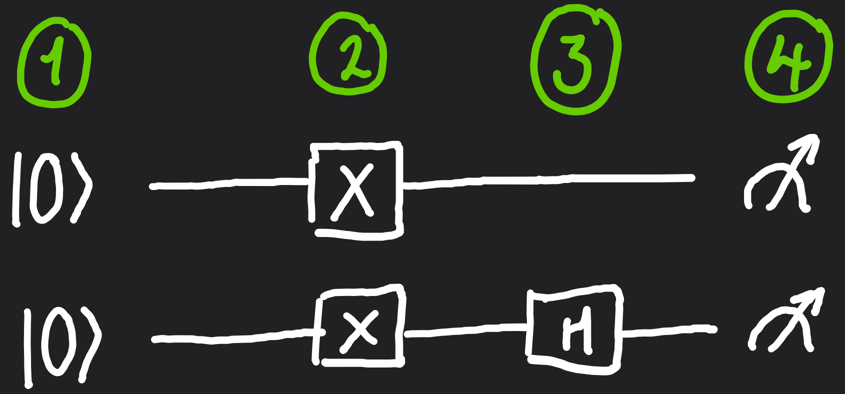
Multiple Qubits



Tensor product

$$|q_1'\rangle \otimes |q_2'\rangle$$
$$= (G \otimes \mathbb{1})(\mathbb{1} \otimes G')\,|q_1\rangle \otimes |q_2\rangle$$

Example Circuit



1

$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3

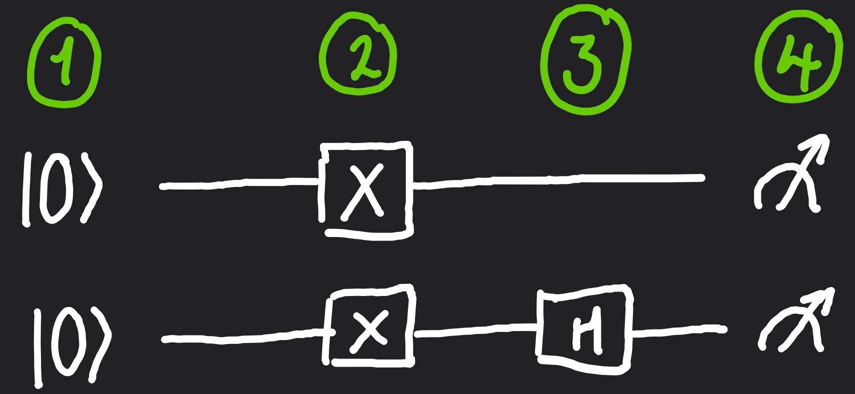
$$\frac{1}{\sqrt{2}} \begin{pmatrix} ? \end{pmatrix}$$

4

Qubit 0 :

Qubit 1 :

Example Circuit



1

$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

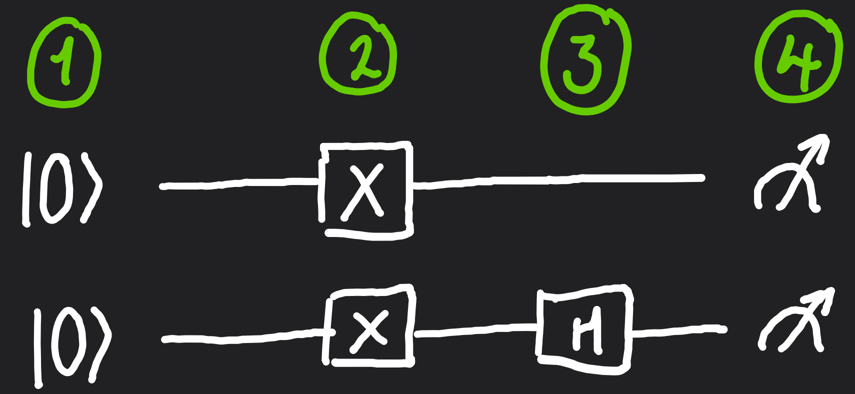
$$= \frac{1}{\sqrt{2}} (|10\rangle - |11\rangle)$$

4

Qubit 0 :

Qubit 1 :

Example Circuit



CAN YOU EXPLAIN THIS BETTER?!

1

$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

2

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} (|10\rangle - |11\rangle)$$

4

Qubit 0 : 1 100%

Qubit 1 : 0 50%

1 50%

Entanglement

When the quantum state of each particle of the group cannot be described **independently** of the others.

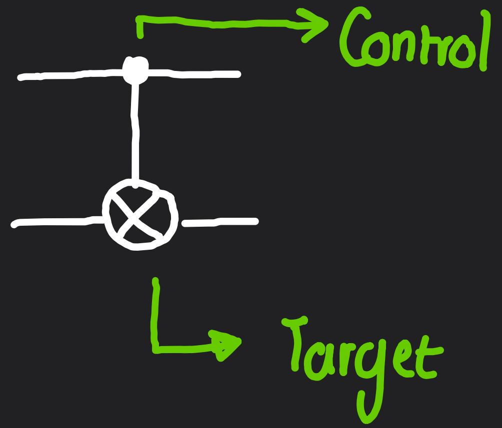
Entanglement

When the quantum state of each particle of the group cannot be described **independently** of the others.

eg $\frac{1}{\sqrt{2}}(|10\rangle - |11\rangle) = \frac{1}{\sqrt{2}} |1\rangle_0 \otimes (|0\rangle - |1\rangle)_1$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq \frac{1}{\sqrt{2}} |q\rangle_0 \otimes |q'\rangle_1$

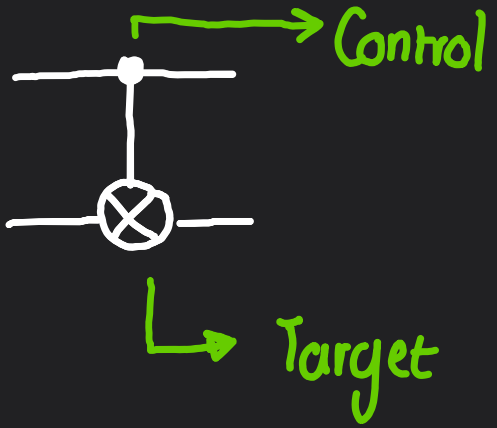
CNOT Gate



Generate these corellations using a 'Controlled Not' gate:

$$\text{If the 'control' qubit is } \begin{cases} |0\rangle_c & \text{do nothing} \\ |1\rangle_c & \text{apply Not gate to target} \end{cases}$$

CNOT Gate



Generate these corellations using a 'Controlled Not' gate:

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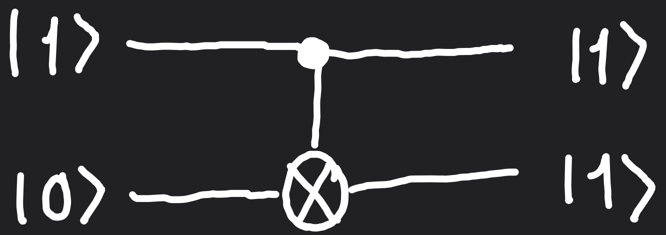
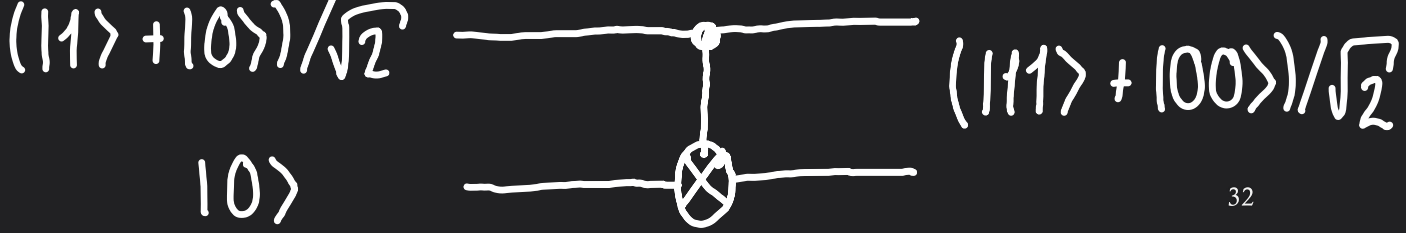
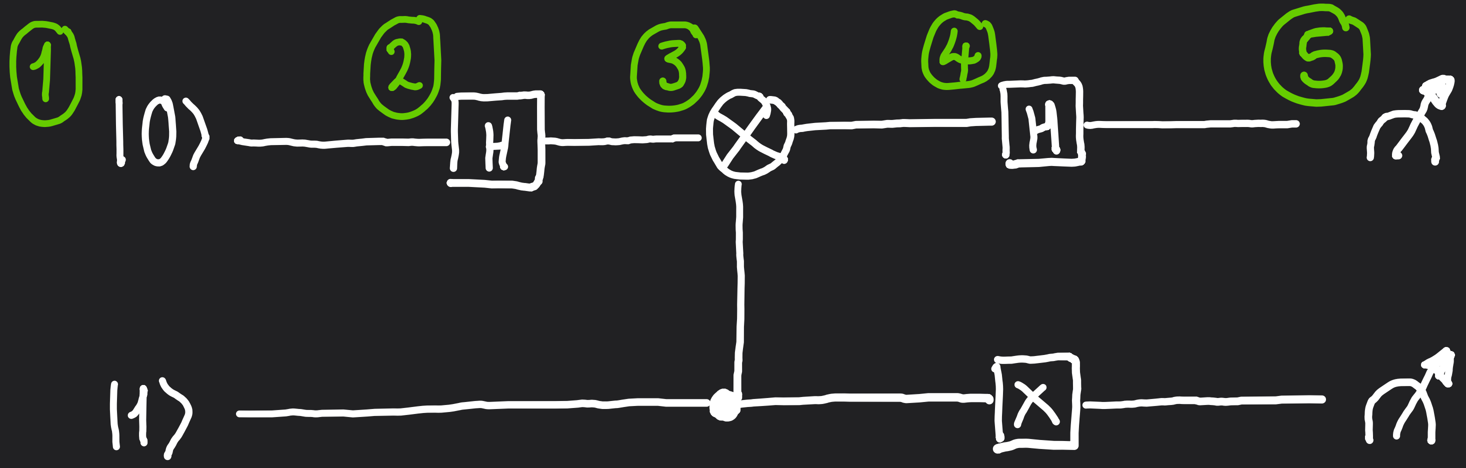




Simple Example



1

$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

2

$$\begin{pmatrix} ? \end{pmatrix}$$

3

$$\begin{pmatrix} \; \end{pmatrix}$$

4

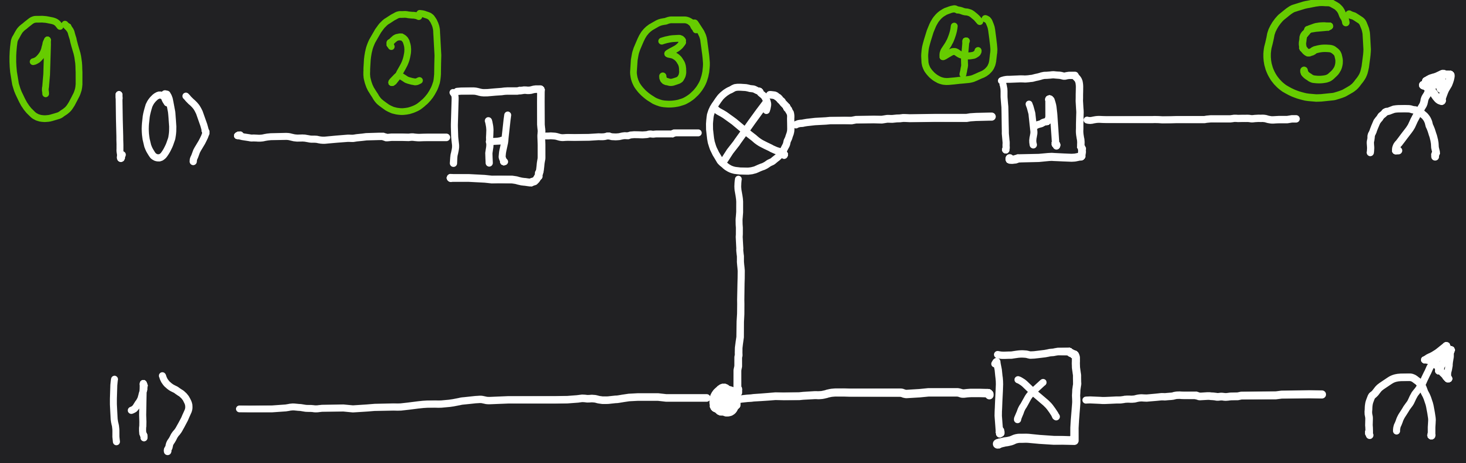
$$\begin{pmatrix} \; \end{pmatrix}$$

5

Qubit 0:

Qubit 1:

Simple Example



1

$$\begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$|01\rangle$

2

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$\frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$

3

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$\frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$

4

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$|00\rangle$

5

Qubit 0: 1 100%

Qubit 1: 1 100%

3. IBM Quantum Computer

IBM Quantum Computers

ibm_washington Exploratory

Details

127
Qubits

64
QV

'quantum volume'

850
CLOPS

'circuit layer ops. per sec'

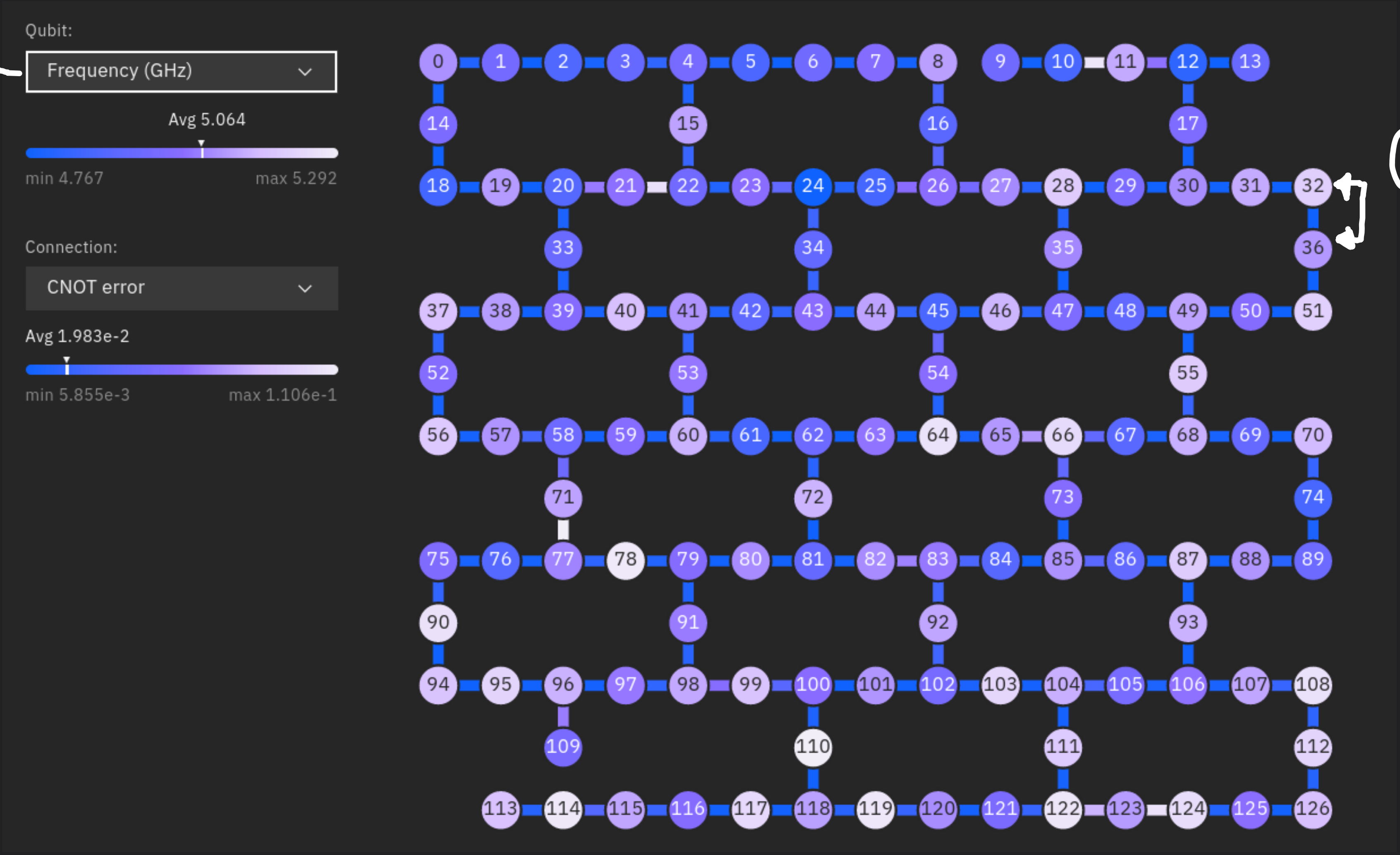
Status:	Online
Total pending jobs:	1437 jobs
Processor type ⓘ:	Eagle r1
Version:	1.1.0
Basis gates:	CX, ID, RZ, SX, X

It can build other gates from these

Avg. CNOT Error:	1.983e-2
Avg. Readout Error:	3.170e-2
Avg. T1:	100.83 us
Avg. T2:	96.74 us
Supports Qiskit Runtime:	No

IBM Quantum Computers

1/'lifetime'



Can use a multi-qubit gate between these!

Talk on $\underbrace{\text{adiabatic quantum computing}}_{\text{Really cool!!}}$ for particle physics

FRIDAY @ 2pm

by Khadeejah Bepari.

4. Exercise

Now go to:

github.com/miarobin/CodingClubQC

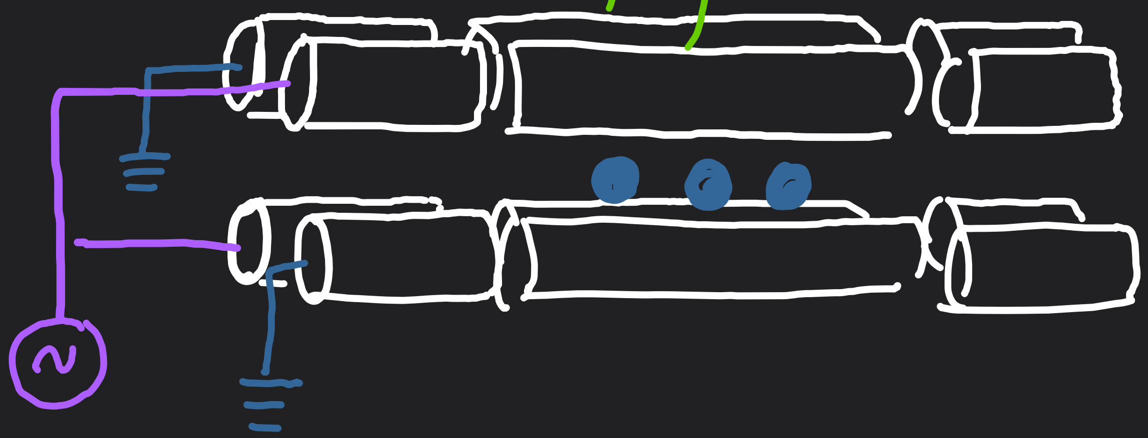
& read README.md

What Could This Be?

Trapped ion quantum computer.

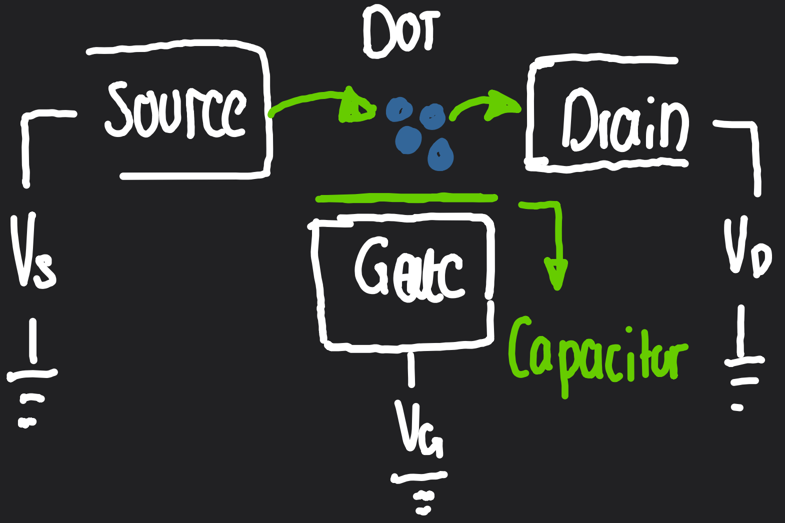
- Trapped ions oscillate (SHO) and act as a *phonon* ($\hbar\omega_z$ quanta)
- Single qubit gates by applying external magnetic field.

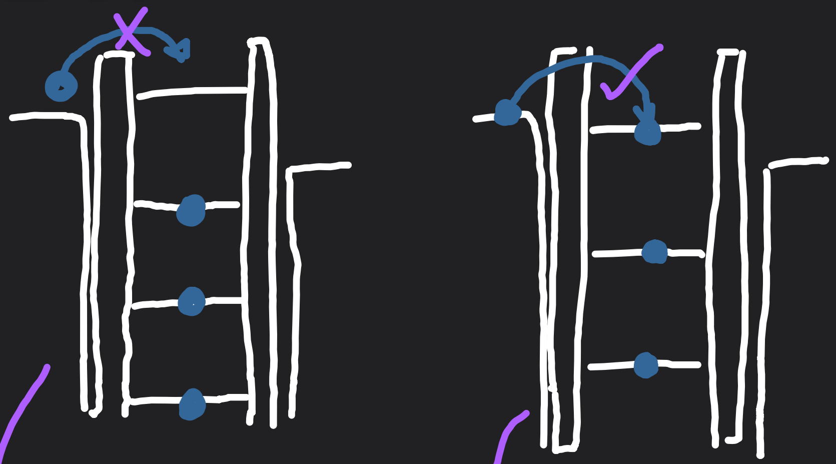
Static potential confines in 2D.



What Could This Be?

Quantum dots / Artifical Atoms.





N electrons in dot

N & N-1 electrons in dot

Energy levels look like an atoms.

Control dot occupation by controlling capacitance