Checking The Action

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Explicitly checking that the energy ?! of a Manton-like solution goes to infinity for a quotient theory

NB Some references: 1// Manton: Topology in the Weinberg Salam theory | M
Manton & Klinkhamer: A Saddle-Point Solution in the Weinberg-Salam Theory | MK

ANSATZ

$$U_\infty = \frac{1}{r}\begin{pmatrix} z & x+iy \\ -x+iy & z \end{pmatrix}$$

$$\frac{ig}{2}\sigma\cdot W_\mu = -a(r)\, U_\infty (\partial_\mu U_\infty^\dagger) = A_\mu$$ [defined this way for consistency w/ M]

LAGRANGIAN

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu h \partial^\mu h}_{1} + \underbrace{\frac{v^2}{4}F(h)^2\, \mathrm{Tr}[D_\mu U D^\mu U^\dagger]}_{2} - \underbrace{\frac{1}{4}\mathrm{Tr}[W_{\mu\nu}W^{\nu\mu}]}_{3}$$

where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - ig$

Do the following:

- Set a static solution for h [$\lambda \to \infty$ limit]
- Set $U = U_\infty$ [eg M]
- Construct a covariant derivative: $D_\mu = \partial_\mu - \frac{ig}{2}\sigma\cdot W_\mu$

$$\sigma\cdot W_\mu = -\frac{2i}{g} f(r)\, \partial_\mu U_\infty U_\infty^\dagger \quad [MK]$$

$$= \partial_\mu + iA_\mu \qquad \frac{ig}{2}\sigma\cdot W_\mu = f(r)\,\partial_\mu U_\infty U_\infty^\dagger = -f(r)\, U_\infty \partial_\mu U_\infty^\dagger$$

Since Sphalerons are static solutions;

$$E = \int d^3x\, \mathcal{L}$$

1. Assuming static solution for h, contribution to energy is:

$$E_h = -\int d^3x \frac{1}{2}(\partial_\mu h)^2 = 0$$

2. Calculating some pieces which contribute to the stress-energy tensor:

- $D_\mu U_\infty = \partial_\mu U_\infty + i a(r)\, U_\infty (\partial_\mu U_\infty^\dagger) U_\infty$
  $= \partial_\mu U_\infty - i a(r)\, \partial_\mu U_\infty U_\infty^\dagger U_\infty$
  $= (1 - i a(r))\, \partial_\mu U_\infty$
- $D_\mu U_\infty D^\mu U_\infty^\dagger = \partial_\mu U_\infty \partial^\mu U_\infty^\dagger (1 + a(r)^2)$
- $\mathrm{Tr}[D_\mu U_\infty D^\mu U_\infty^\dagger] = \frac{(1+a(r)^2)}{r^2}$ [See mathematica notebook]

$$E = \int d^3x \left(\frac{v^2 F(h)^2}{4}\frac{(1+a(r)^2)}{r^2}\right) = \frac{v^2}{4}\cdot 4\pi^2 \cdot \int dr\, (1+a(r)^2) \longrightarrow \text{CURIOUS SIGN !!}$$

3. Taking term as calculated

3 Taking term as calculated