CSC310: Algorithms and Data Structures

Fall 2020

Final Exam

1. Given an array A of **real** numbers and a number S, you are asked to find a subarray of **smallest length** whose sum is at least S. Give a most efficient algorithm for this task in each of the following cases:

   (i) A is sorted in ascending order.

   (ii) A is not sorted.

2. Shikri wants to get his money out of the bank! He realized his debit card can be used to buy stuff but not to withdraw money. So he decided to open a home appliances store and use his card to buy as many items as his bank account allows, aiming at a maximum possible total profit. For his shopping, Shikri made a list of all the available items, along with the price of each item and the profit he can make (per item).

   For a budget of W, Shikri wants your help by providing an algorithm to compute the maximum profit he can make.

   Give the pseudocode of a most-efficient algorithm for this problem and analyze your algorithm (assuming that prices are positive integers).

3. Christmas is approaching fast and Santa did not yet work on his gift distribution plan! For this, he seeks your help. Santa has one bag that can hold a maximum of Mkg in gifts' weight. Each gift g_i has a weight w_i, and obviously Santa wants to take the maximum number of gifts he can hold.

   Given the weight and value of n gifts and the total weight, M, the bag can hold, Santa needs you to design an algorithm that tells him which gifts he should put in his bag such that the total number of gifts is maximized given that the total weight of the selected gifts should not exceed M. What is the worst-case running time of your algorithm? Justify.

4. The economic crisis affected Farid's tourism company, so he decided to go to China and work there since he speaks Chinese very well and he knows almost all the cities and the various touristic places/attractions. He decided to organize tours between pairs of cities, so he identified a number of cities that tourists like to visit and made a deal with the bus lines. The cost of accompanying a tourist varies between pairs of neighboring cities and in some cases where the trip is to a city that is not so popular, he can make money by having someone take the bus with him! In other words, the cost would be negative between some pairs of cities. After collecting the cost of taking a bus between each pair of adjacent cities, Faird wants to advertise for all his tours. For this he needs to compute the cheapest trip between every pair of cities (not necessarily adjacent pairs). Given a list of adjacent cities along with the cost of a bus between each such pair, you are asked to write a most-efficient algorithm to help Farid with this task. How does your algorithm determine whether Farid can make a huge wealth out of this business (of some unlimited value, without taking money from tourists)? Justify.

5. In the 2-Packing problem you are given a graph G and a positive integer k as input, and the objective is to find a 2-Packing of size at least k. A 2-Packing is simply a set of vertices any two of which are at distance > 2 from each other: if u and v belong to a 2-packing then $N[u] \cap N[v] = \phi$. In other words, if a vertex is in a 2-packing P then neither its neighbors nor their neighbors (neighbors of neighbors) can be in P. Give a most-efficient 2-Packing algorithm with a running time that is asymptotically better than $O(1.6^n)$. Analyze the running time of your algorithm. Can you do better? Prove your answer.