Suppose that the outcome of game i is multinomial with single trial; that is $\mathbf{y}_i \sim MN(1; \mathbf{p}_i)$, where $\mathbf{y}_i = (y_{i1}, y_{i2}, y_{i3})$ and $\mathbf{p}_i = (p_i^H, p_i^D, p_i^A)$. Suppose that the probabilities are given by

$$\begin{aligned}
\operatorname{logit}(p_i^H) &= \log(o_i^H) + \delta^H = lo_i^H + \delta^H \\
\operatorname{logit}(p_i^D) &= \log(o_I^D) + \delta^D = lo_i^D + \delta^D \,,
\end{aligned}$$

where o_i^H are the odds of a home win based on the opta probabilities. Then the likelihood function based on n games is

$$L(\boldsymbol{\delta}) = \prod_{i=1}^{n} \left(p_i^H\right)^{y_{i1}} \left(p_i^D\right)^{y_{i2}} \left(p_i^A\right)^{y_{i3}}$$

and hence the log-likelihood is

$$\log L(\boldsymbol{\delta}) = \sum_{i=1}^{n} \left[y_{i1} \log p_i^H + y_{i2} \log p_i^D + y_{i3} \log p_i^A \right] \,. \tag{1}$$

Note that the model implies that the probabilities are given by

$$\begin{aligned}
p_i^H &= \frac{\exp(lo_i^H + \delta^H)}{\exp(lo_i^H + \delta^H) + \exp(lo_i^D + \delta^D) + 1} \\
p_i^D &= \frac{\exp(lo_i^D + \delta^D)}{\exp(lo_i^H + \delta^H) + \exp(lo_i^D + \delta^D) + 1} \\
p_i^A &= \frac{1}{\exp(lo_i^H + \delta^H) + \exp(lo_i^D + \delta^D) + 1} \,.
\end{aligned}$$

Hence (1) can be written as

$$\log L(\boldsymbol{\delta}) = \sum_{i=1}^{n} \left\{ y_{i1}(lo_i^H + \delta^H) + y_{i2}(lo_i^D + \delta^D) - \log \left[\exp(lo_i^H + \delta^H) + \exp(lo_i^D + \delta^D) + 1 \right] \right\}$$

This formula can be used to determine the gradient vector and Hessian of the log-likelihood (with respect to $\boldsymbol{\delta}$). Specifically,

$$\begin{aligned}
\frac{\partial}{\partial \delta^H} \log L &= \sum_{i=1}^{n} (y_{i1} - p_I^H) \\
\frac{\partial}{\partial \delta^D} \log L &= \sum_{i=1}^{n} (y_{i2} - p_I^D)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2}{\partial \delta^{H2}} &= -\sum_{i=1}^{n} p_i^H (1 - p_i^H) \\
\frac{\partial^2}{\partial \delta^{D2}} &= -\sum_{i=1}^{n} p_i^D (1 - p_i^D) \\
\frac{\partial^2}{\partial \delta^H \partial \delta^D} &= -\sum_{i=1}^{n} p_i^H p_i^D \,.
\end{aligned}$$