COMP 2804 – Assignment 3

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2.

$$S = \{[1,1],[1,2],[2,1],[2,2]\} \text{ each pair has equal probability if using fair coins}$$

$$\begin{aligned} Pr(A) &= Pr([1,1]) \\ &= 1 \div 4 \\ &= 0.25 \end{aligned} \qquad \begin{aligned} Pr(B) &= Pr([1,2],[2,1]) \\ &= 2 \div 4 \\ &= 0.5 \end{aligned} \qquad \begin{aligned} Pr(C) &= Pr([2,2]) \\ &= 1 \div 4 \\ &= 0.25 \end{aligned}$$

The sum of $Pr(A) = Pr(B) = Pr(C)$ must equal 1 since these events cover the entire sample space. Therefore, the probability of each must be one third.

$$Pr(A) = p \cdot q \qquad Pr(B) = p(1-q) + q(1-p) \qquad Pr(C) = (1-p)(1-q)$$

$$\begin{aligned} Pr(A) &= Pr(C) \\ p \cdot q &= (1-p)(1-q) \\ 0 &= 1 - q - p \\ p &= 1 - q \end{aligned}$$

$$p = 1 - q \tag{1}$$
$$p \cdot q = 1 \div 3 \tag{2}$$

Now I will solve equation 2 using the p value calculated above.

$$\begin{aligned} p \cdot q &= 1 \div 3 \\ (1-q) \cdot q &= 1 \div 3 \\ q^2 - q + 1 \div 3 &= 0 \end{aligned}$$

Trying to solve this using the quadratic formula, results in imaginary numbers. So the probabilities of events A, B, and C cannot be equal.

3. •

$$\begin{aligned} Pr(E|E \cup N) &= \frac{Pr(E \cap (E \cup N))}{Pr(E \cup N)} \\ &= \frac{p}{p + q - p \cdot q} \end{aligned} \qquad \begin{aligned} &= \frac{Pr(E)}{Pr(E \cup N)} \\ &= \frac{p}{p(1-q) + q} \end{aligned}$$

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$$\begin{aligned} Pr(E \cap N|E \cup N) &= \frac{Pr((E \cap N) \cap (E \cup N))}{Pr(E \cup N)} \\ &= \frac{Pr(E \cap N)}{Pr(E \cup N)} \\ &= \frac{p \cdot q}{p + q - p \cdot q} \end{aligned}$$

4.

$$Pr(A_1) = \frac{N(A_1)}{N(All)} = \frac{(n-1)!}{n!} \qquad Pr(A_2) = \frac{N(A_2)}{N(All)} = \frac{(n-1)!}{n!} \qquad Pr(A_1 \cap A_2) = \frac{\binom{n-2}{n-2}}{\binom{n}{n}} = \frac{(n-2)!}{n!}$$

$$\begin{aligned} Pr(A) &= Pr(A_1 \cup A_2) \\ &= P(A_1) + Pr(A_2) - Pr(A_1 \cap A_2) \\ &= \frac{(n-1)!}{n!} + \frac{(n-1)!}{n!} - \frac{(n-2)!}{n!} \\ &= \frac{2[(n-1)!] - (n-2)!}{n!} \end{aligned}$$

$$Pr(B_1) = \frac{N(B_1)}{N(All)} = \frac{(n-1)!}{n!} \qquad Pr(B_2) = \frac{N(B_2)}{N(All)} = \frac{(n-1)!}{n!} \qquad Pr(B_1 \cap B_2) = \frac{\binom{n-2}{n-2}}{\binom{n}{n}} = \frac{(n-1)!}{n!}$$

$$\begin{aligned} Pr(B) &= Pr(B_1 \cup B_2) \\ &= P(B_1) + Pr(B_2) - Pr(B_1 \cap A_2) \\ &= \frac{(n-1)!}{n!} + \frac{(n-1)!}{n!} - \frac{(n-1)!}{n!} \\ &= \frac{(n-1)!}{n!} \end{aligned}$$

5. Independent means it is not reliant on the outcome of the other, that means Pr(A) = 0 or Pr(A) = 1

6.

$$S = \{RRR, RRB, RBR, RBB, BRR, BRB, BBR, BBB\}$$

$$\begin{aligned} Pr(A) &= \{RRR, RRB, RBR, RBB\} \qquad \text{if } P_1 \text{ guesses Red, similar for Blue} \\ &= 4 \div 8 \\ &= 0.5 \end{aligned}$$

I will explain what happens in each possible event using the algorithm, and state whether the game was a success. R means the player said their hat was red, B means blue and P means they passed their turn.

$$\begin{aligned} E_1 &= RRR & E_2 &= RRB & E_3 &= RBR & E_4 &= RBB \\ BBB &= Loss & PPB &= Win & PBP &= Win & RPP &= Win \end{aligned}$$

$$\begin{aligned} E_5 &= BRR & E_6 &= BRB & E_7 &= BBR & E_8 &= BBB \\ BPP &= Win & PRP &= Win & PPR &= Win & RRR &= Loss \end{aligned}$$

$Pr(B) = 6 \div 8 = 0.75$

7.

$$Pr(A_0) = 1 \div 2^{n+1}$$

This is the inverse of the total number of subsets because only 1 out of all subsets is the empty set, the one where you have decided not to include each person 2^m ways of doing this.

All A_i events have the same probabilities because every student has an equal amount of chances to be selected in a subset. Then if they're selected for the subset X they have another uniformly random chance of winning the six-pack of cider.
Prove that:

$$Pr(A_1) = \frac{1 - \frac{1}{2^{n+1}}}{n+1}$$

This equation can be separated into numerator and denominator and what the equation comes from becomes clear. $\frac{1}{n+1}$ is the chance of being selected to win. The $1 - \frac{1}{2^{n+1}}$ is the person's chance of being in the subset. These have to be multiplied to get the total chance of winning.

$$A_1 = B_0 \cup B_1 \cup \ldots \cup B_n$$

8.

$$S = \{1, 2, 3, 4, 5, 6\} \qquad A = \{2, 4, 6\} \qquad B = \{1, 3, 5\} \qquad C = \{1, 2, 3, 4\}$$

To check if a two events are independent of each other, the probability of their intersection must equal the product of their individual probabilities.

$$P(A \cap B) = 0 \qquad P(A) \cdot P(B) = 0.5 \cdot 0.5 = 1/4$$

Events A and B are not independent.

$$P(A \cap C) = 1/3 \qquad P(A) \cdot P(C) = 0.5 \cdot 4/6 = 1/3$$

Events A and C are independent.

$$P(B \cap C) = 1/3 \qquad P(B) \cdot P(C) = 0.5 \cdot 4/6 = 1/3$$

Events B and C are independent.

9. Let $k = 2logn$

$$\begin{aligned} 2 \div n &\geq \frac{n - [2logn] + 1}{2^{[2logn]-1}} \\ 2logn &\geq logn \\ 2 &\geq 1 \end{aligned}$$