

Fix a time period, and let R be the total blockreward over that time period.
Let q_1, q_2 and q_3 be the hashrates of each of the firms, and let

$$Q = q_1 + q_2 + q_3$$

the total hashrate.

The profit to each of the firms over the time period is given by (difficulty adjusted)

$$\begin{aligned}\pi_1 &= \frac{R}{q_1 + q_2 + q_3} q_1 - 3q_1. \\ \pi_2 &= \frac{R}{q_1 + q_2 + q_3} q_2 - 4q_2. \\ \pi_3 &= \frac{R}{q_1 + q_2 + q_3} q_3 - 5q_3\end{aligned}$$

(We are assuming the "unit of hash" is such that the time period times the hash rate yields that number of unit. This won't matter in the end.)

Now each firm will maximize their own profit.

Notice the following. We may fix q_2, q_3 and assume one or both is not zero, then consider the function $\pi_1(q_1, q_2, q_3)$ with q_1 as a variable. Now

$$\pi_1(0, q_2, q_3) = 0$$

and

$$\lim_{q_1 \rightarrow \infty} \pi_1(q_1, q_2, q_3) = -\infty$$

because the first term in π_1 is bound above by R , and the second term becomes increasingly negative. In fact the function π_1 is concave in the variable q_1 provided q_2 or q_3 positive.

Thus given q_2, q_3 , we can find a unique positive value q_1^* that maximizes $\pi_1(-, q_2, q_3)$.

This is determined by taking the derivative with respect to q_1 and setting it equal to zero: We get

$$\frac{R(Q - q_1)}{Q^2} - 3 = 0$$

Similarly, for any given q_1, q_3 we can argue there is a unique maximizer q_2^* for $\pi_2(q_1, -, q_3)$, etc.

Now we argue that there is a Nash equilibrium, that is a value where all three hold. We try to simultaneously solve the system of equations

$$\frac{R(Q - q_1)}{Q^2} - 3 = 0 \tag{1}$$

$$\frac{R(Q - q_2)}{Q^2} - 4 = 0 \tag{2}$$

$$\frac{R(Q - q_3)}{Q^2} - 5 = 0 \tag{3}$$

In order to do this, we sum the equations:

$$\frac{R(3Q - q_1 - q_2 - q_3)}{Q^2} = 12$$

or

$$\frac{2R}{Q} = 12$$

leading to

$$Q = \frac{R}{6}$$

Now knowing Q we can go back to equations (1)(2)(3) to get the respective quantities:

$$\frac{R\left(\frac{R}{6} - q_1\right)}{\left(\frac{R}{6}\right)^2} - 3 = 0$$

gives

$$q_1 = \frac{R}{12}$$

which is 50% of Q .

Similarly,

$$\begin{aligned} q_2 &= \frac{R}{18} \\ q_3 &= \frac{R}{36} \end{aligned}$$

correspondingly, 33.3% and 16.67%

As the reward functions are concave, this will be a unique Nash equilibrium.