

Problem 1 (75) Let S be a compact surface with no boundary. Suppose that p is the point on S where the maximum of x value occurs. What can we say about the sign of the Gauss curvature is at p ? (No proof needed)

Problem 2 (125) Let $(u, v) \mapsto R \subset S^2$ be a parameterization of a piece of the sphere containing the north pole. Argue that there it is impossible to have $E = 1$, $G = v^2$ and $F = 0$.

Problem 3 (100) Consider the surface arising as the graph of $z = \sqrt{1 - x^2}$ for $|x| < 1$. Try to find a parameterization of this surface so that $E = 1 = G$ and $F = 0$.

Problem 4 (100) Find a parameterization of any part of the plane $\{z = 0\}$ so that $F < 0$ at some points and $F > 0$ at other points.

Problem 5 (150) Consider the graph of the function $z = 2 + \sin(x)$ rotated around the x -axis. Give a point on S with negative Gauss curvature. Give a point on S with positive Gauss curvature. Give an example of a geodesic in S .

Problem 6 (150) Consider the surface $z = xy$, parameterized by (u, v, uv) . Find the Christoffel symbols. The curve (t, t, t^2) is parameterized in (u, v) coordinates by $\alpha(t) = (t, t)$. Using Christoffel symbols, compute the covariant derivative of $\alpha'(t)$ with respect to the surface S . Is $\alpha(t)$ a parameterized geodesic? Is the curve traced out by α a geodesic?

Problem 7 (100) Give an example of a line of curvature that is not a geodesic.

Problem 8 (125) Consider the surface defined by

$$z = \cosh x$$

rotated around the x axis, for some positive valued function f . Find the geodesic curvature k_g of the curve defined by $x = T$ as a function of T .