Homework 9

MA372 Introduction to Discrete Math

Due: Lesson 29 08 APR 2022

Instructions. Solve the following problems on material up through Lesson 27. Your solutions should be typeset using LaTeX for final submission and include a coversheet.

Note: Graphs may be included in your submission as images using the graphicx package along with the command \includegraphics{ }. If you wish to create your graphs using LaTeX, see the links under "Creating Graphs Using TikZ" in the LaTeX Resources folder on Blackboard.

1. For each of the following relations, determine if it is reflexive, symmetric, antisymmetric, and/or transitive. Explain your answers.

   a) On the integers $\mathbb{Z}$, define the relation $R = \{(a, b) |\ a - b \text{ is odd}\}$.

   b) On the set of all bit strings of length four, define the relation T where rTs if and only if the sum of the characters in r equals the sum of the characters in s.

2. On the set $\{0, 1, 2, 3\}$, define the relation $R = \{(0, 0), (0, 1), (1, 1), (1, 1), (1, 2), (2, 2), (2, 3)\}$. For this relation,

   i. represent the relation as a zero-one matrix,

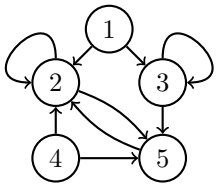
   ii. represent the relation as a directed graph, and

   iii. determine whether the relation is reflexive, symmetric, antisymmetric, and/or transitive.

3. Determine the relation on the set $A = \{1, 2, 3, 4, 5\}$ given by the zero-one matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

4. Determine the relation on the set $\{1, 2, 3, 4, 5\}$ given by the digraph below.



5. Show that the relation $\mathbb{Z}$ defined by $R = \{(a, b) |\ 2 \text{ divides } a - b\}$ is an equivalence relation and describe the distinct equivalence classes of the relation.

6. Let $A = \{0, 1, 2, 3, 4\}$.

   a) Do the sets $\{0, 1\}, \{2, 4\}, \{1, 3\}$ form a partition of A? Explain your answer.

   b) Determine the equivalence relation on A induced by the partition $\{0\}, \{1, 3, 4\}, \{2\}$.