1. Does there exist a differentiable function $g(x)$ such that $g(0) = -1$, $g(2) = 4$, and $g'(x) \leq 2$ for all x? Find an example or explain why it doesn't exist.

> **Solution:**
>
> If such a function existed, then by MVT we would have some c in the interval $(0, 2)$ such that
> $$g'(c) = \frac{g(2) - g(0)}{2 - 0} = \frac{4 - (-1)}{2} = \frac{5}{2}$$
> This is impossible since $g'(x) \leq 2$ for all x.

2. Determine whether $f(x)$ satisfies the hypothesis of the mean value theorem on the given interval. If so, find all numbers c in the interval so that $f(b) - f(a) = f'(c)(b - a)$.

   (a) $f(x) = \frac{1}{(x-1)^2}$, $[0, 2]$

   > **Solution:** No, f is not continuous on $[0, 2]$

   (b) $f(x) = x + \frac{4}{x}$, $[1, 4]$

   > **Solution:** $c = 2$

   (c) $f(x) = 4 + \sqrt{x - 1}$, $[1, 5]$

   > **Solution:** $c = 2$