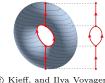
TDA Workshop Exercise Day 1

Exercise 1 Kepler Mapper - Getting Started

Revisit the example from the lecture and play around with different filter functions, noise-levels and covers.

Exercise 2 The Reeb Graph

Let X be a topological space and $f: X \to \mathbb{R}$ a continuous function. The Reeb Graph $R_f(X)$ is the space obtained by identifying $x, y \in f^{-1}(c)$, whenever they lie in the same connected component of the level set $f^{-1}(c)$.



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- (a) Determine the Reeb Graph of the standard embedding of a torus $T = S^1 \times S^1$ in \mathbb{R}^3 for different filter functions f. (E.g. rotate the torus before projecting to the z-axis, or come up with your own filter).
- (b) Use tadasets.torus() to construct the Mapper graph of a noisy torus for several predefined and/or custom filters.
- (c) Add a puncture to the torus by removing (enough) points in a certain area. How does this affect the Reeb and Mapper graphs?

Exercise 3 Kepler Mapper - Digits Datasets

In this exercise we will investigate a dataset of digits and make ourselves familiar with some advanced Kepler Mapper functionalities.

```
for copyable code see https://kepler-mapper.scikit-tda.org/generated/gallery/plot_digits.html
```

The dataset we will use is part of sklearn.datasets and can be accessed via:

```
from sklearn import datasets
import matplotlib.pyplot as plt

#Load the digits dataset
digits = datasets.load_digits()

#Display the first digit
plt.figure(1, figsize=(3, 3))
plt.imshow(digits.images[0], cmap=plt.cm.gray_r, interpolation='nearest')
plt.show()
```

Make yourself familiar with the data: Look at the data structure, plot the first few digits.

TDA Workshop Exercise Day 1

We will now create custom tooltips. This will help interpreting the html-output, since datapoints will not be listed by their index, but by the actual image instead.

```
import io
import sys
import base64
import numpy as np
try:
    from scipy.misc import imsave, toimage
except ImportError as e:
    print("imsave_requires_you_to_install_pillow._Run_'pip_install_pillow'_and_then_try_again.")
# Create images for a custom tooltip array
tooltip_s = []
for image_data in digits.data:
    output = io.BytesIO()
    img = toimage(image_data.reshape((8, 8))) # Data was a flat row of 64 "pixels".
    img.save(output, format="PNG")
    contents = output.getvalue()
    img\_encoded = base64.b64encode(contents)
    img_tag =
            """ < img~src = "data: image/png; base 64, \{\}" > """. \mathbf{format} (img\_encoded. decode('utf-8')) 
    tooltip_s.append(img_tag)
    output.close()
tooltip_s = np.array(tooltip_s) # need to make sure to feed it as a NumPy array, not a list
```

Now it's time to apply Kepler Mapper to the dataset. We use a t-SNE filter function with 2 components (reduces data to 2 dimensions) that is provided by sklearn.

Finally we will create two visualizations that will be written to html. Note that below we also increase the graph_gravity for a tighter graph-look.

TDA Workshop Exercise Day 1

We can also have a look at the data with the help of matplotlib.pyplot:

```
# Matplotlib examples
import matplotlib.pyplot as plt
km.draw_matplotlib(graph, layout="spring")
plt.show()
```

Exercise 4 Kepler Mapper - The Breast Cancer Dataset
The dataset used by Nicolau et al (2011)¹ is accessible at
https://www.kaggle.com/uciml/breast-cancer-wisconsin-data
Have a look at the data and experiment with it.

A view instructions, in particular two possible filter functions, can be found in the Kepler Mapper Gallery at

https://kepler-mapper.scikit-tda.org/generated/gallery/plot_breast_cancer.html

https://www.pnas.org/content/108/17/7265