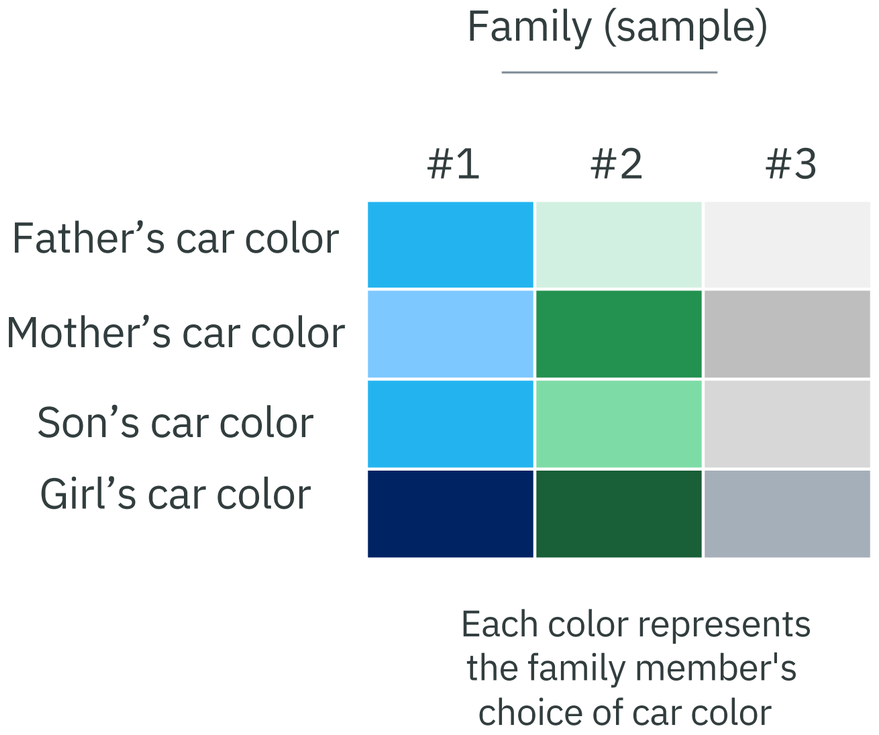
Portfolio Allocation

A Hierarchical Clustering (HC) approach

1. A Theoretical Overview
2. How to use HC for portfolio allocation

How hierarchical clustering works ?

Case Study

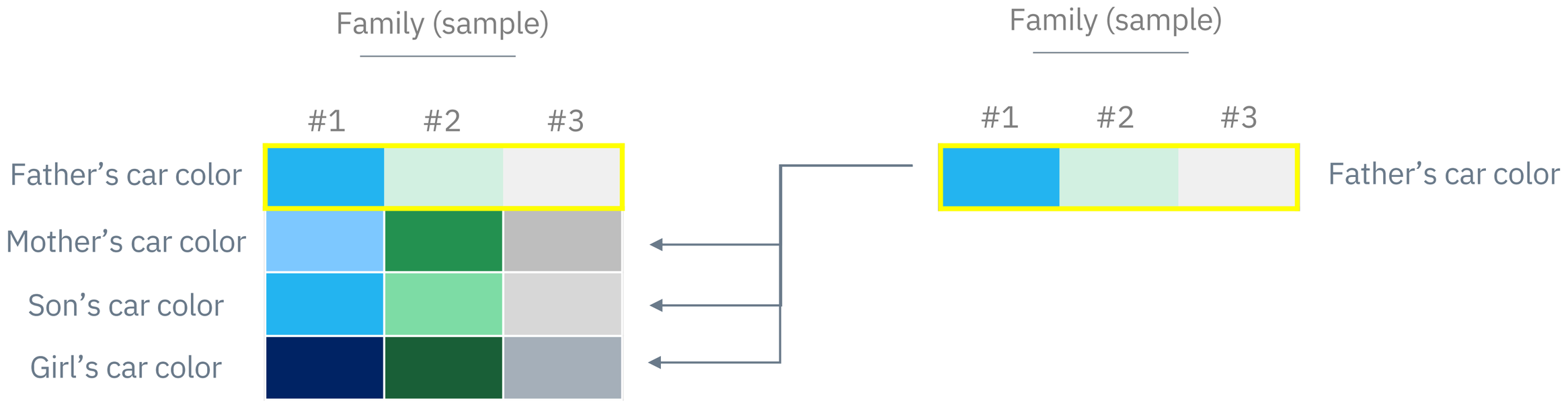


Idea is to reorder the rows to creates clusters of the family members car color

How hierarchical clustering works ?

Case Study

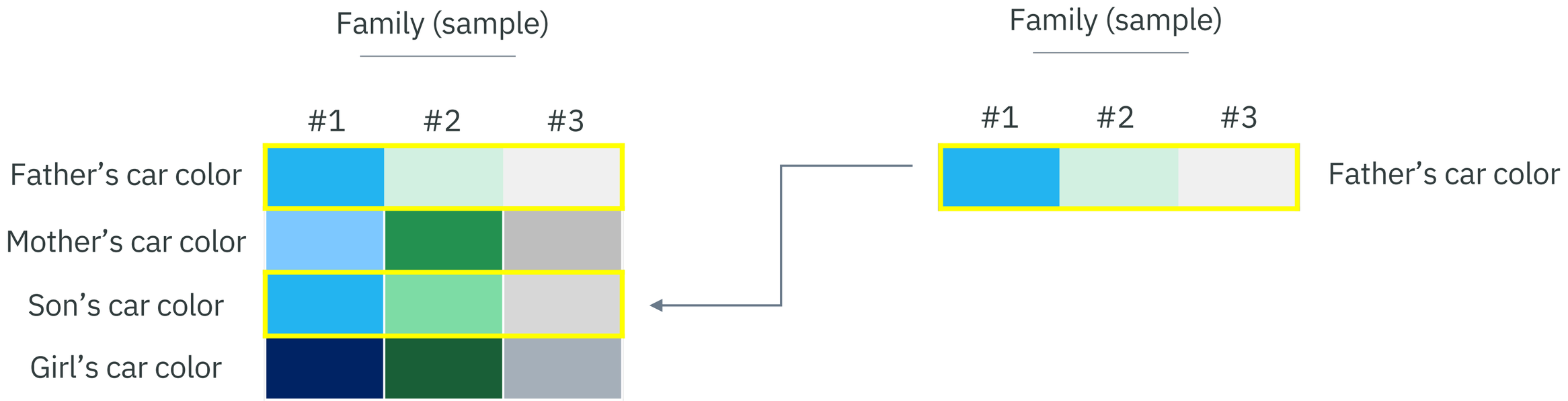
Which family member made the color choice most similar to the fathers?



How hierarchical clustering works ?

Case Study

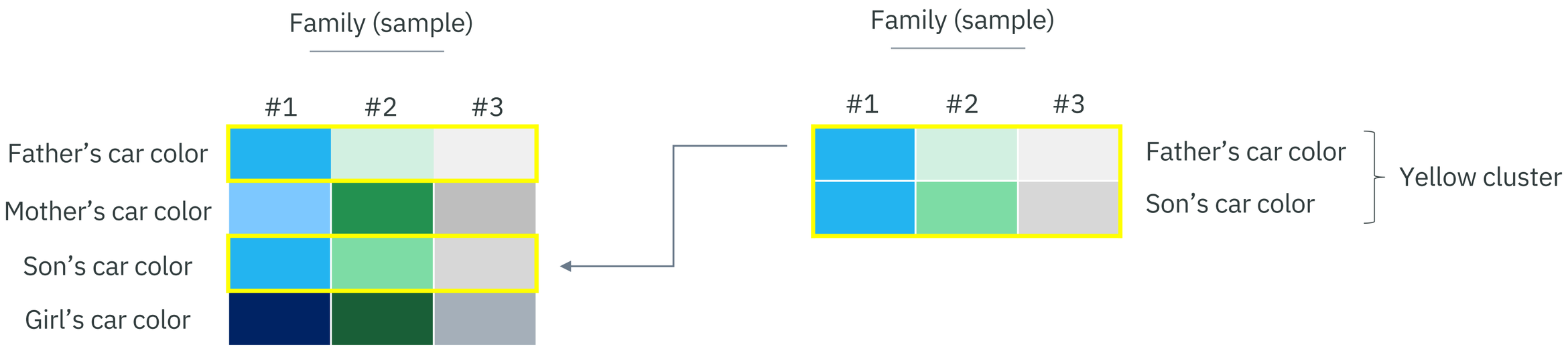
Sons colors choice is the most similar



How hierarchical clustering works ?

Case Study

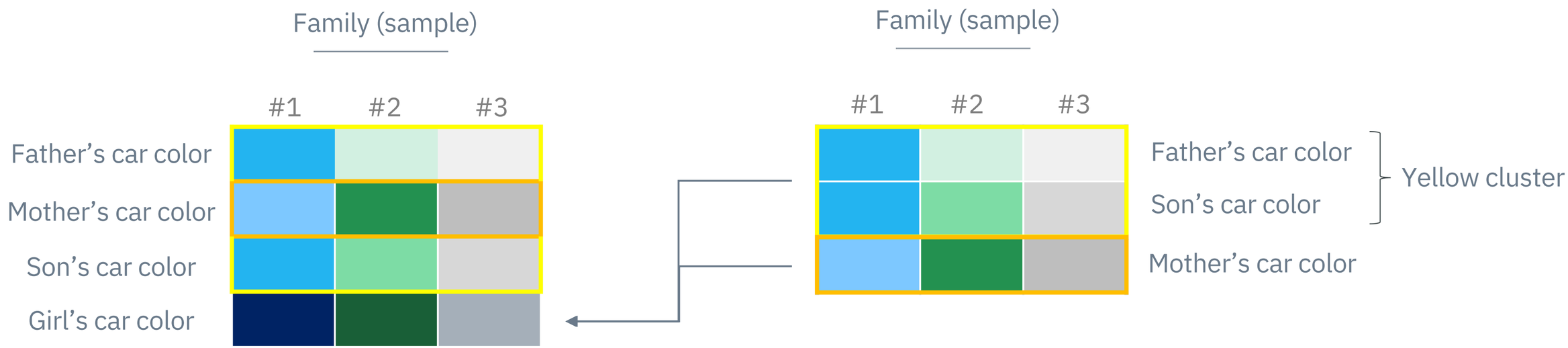
Fathers and sons chose the most similar car colors, so they created a cluster



How hierarchical clustering works ?

Case Study

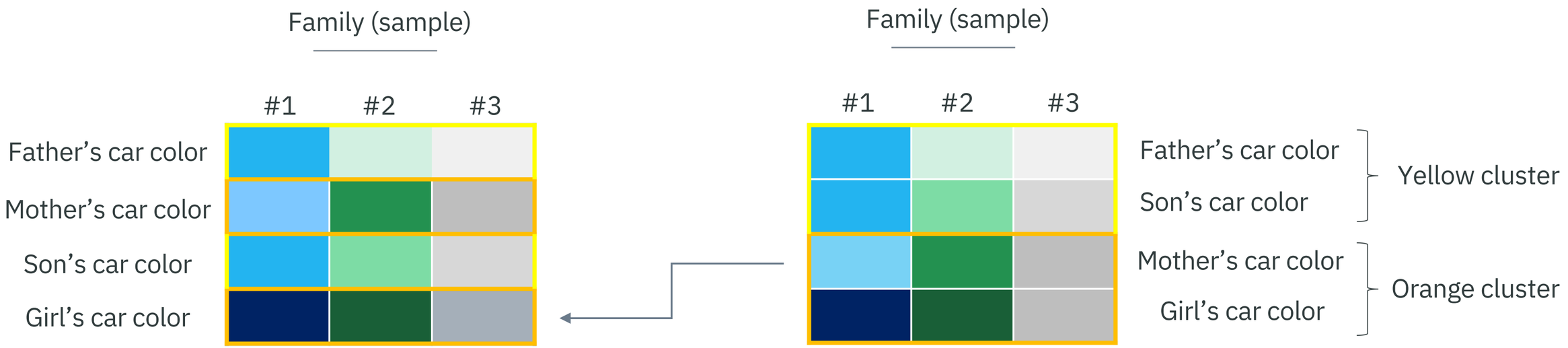
Which family member made the color choice most similar to the mothers ?



How hierarchical clustering works ?

Case Study

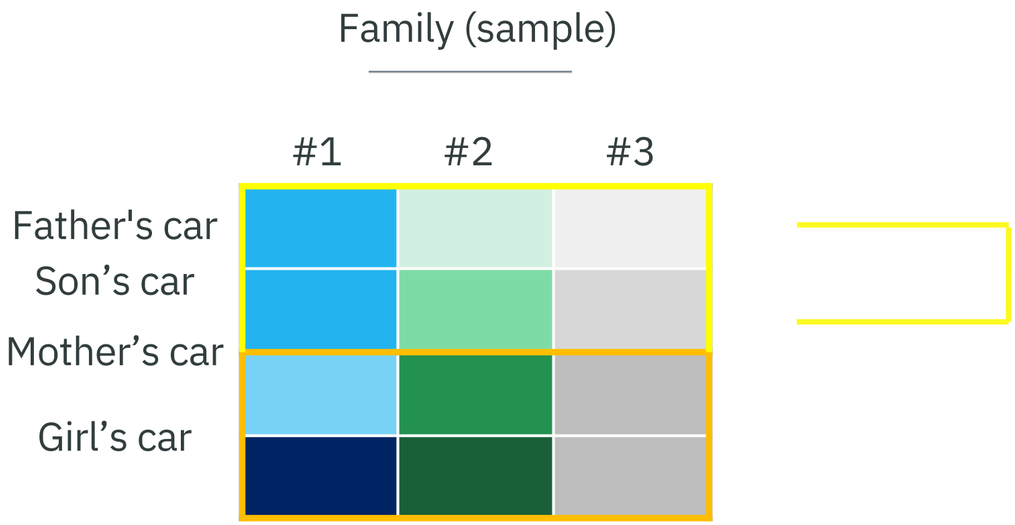
Mothers and girls chose the most similar car colors, so they created a cluster



How hierarchical clustering works ?

Case Study

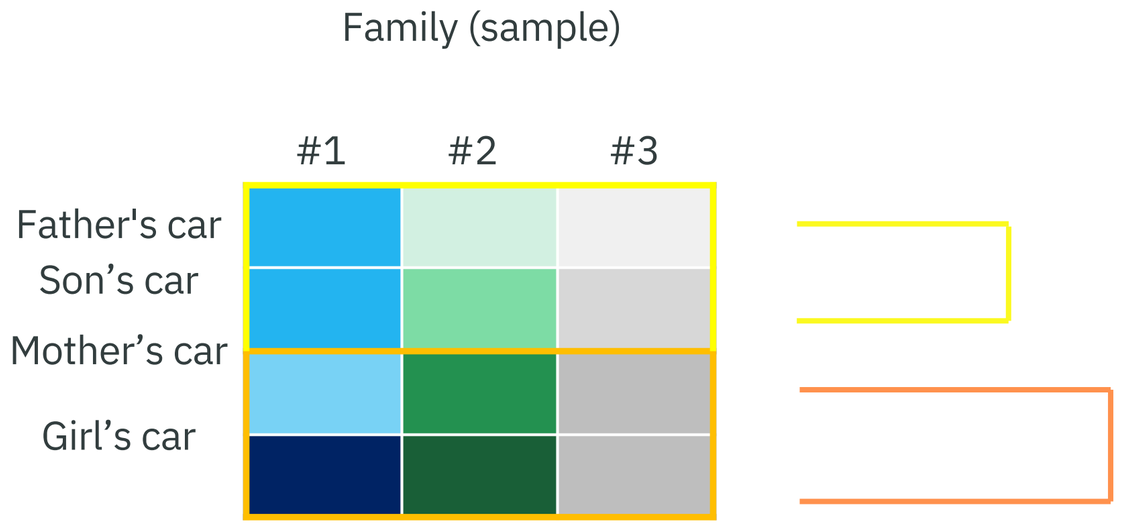
Let’s recap: Fathers and sons chose the most similar car colors,
so they are cluster together



How hierarchical clustering works ?

Case Study

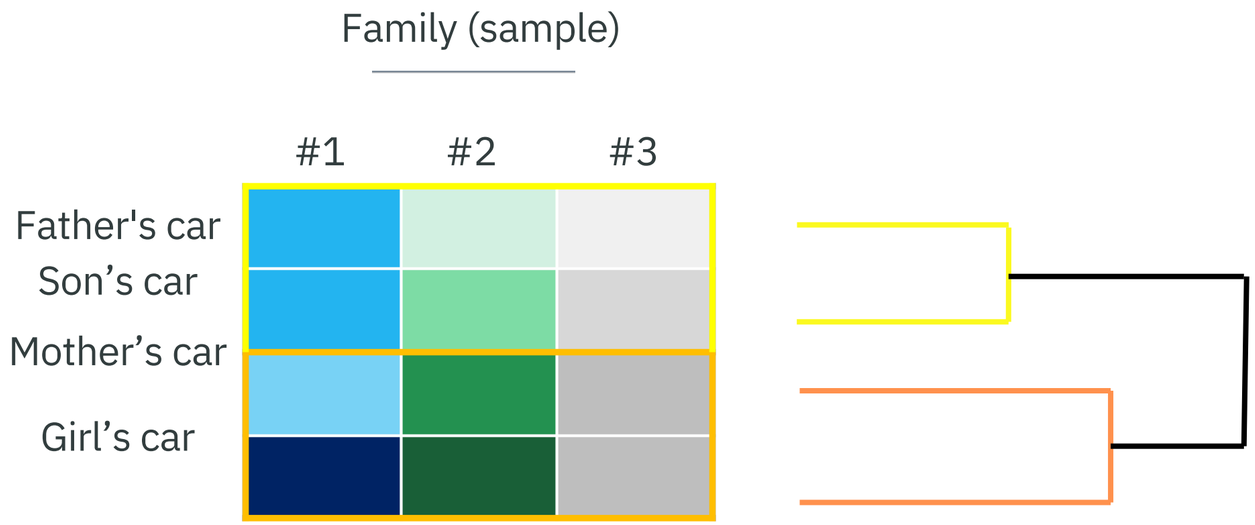
Let’s recap: mothers and sons chose the most similar car colors, so they are cluster together



How hierarchical clustering works ?

Case Study

Hierarchy has been created !



HC allocation process: A Case Study

Several steps

i. Compute correlation matrix on the returns

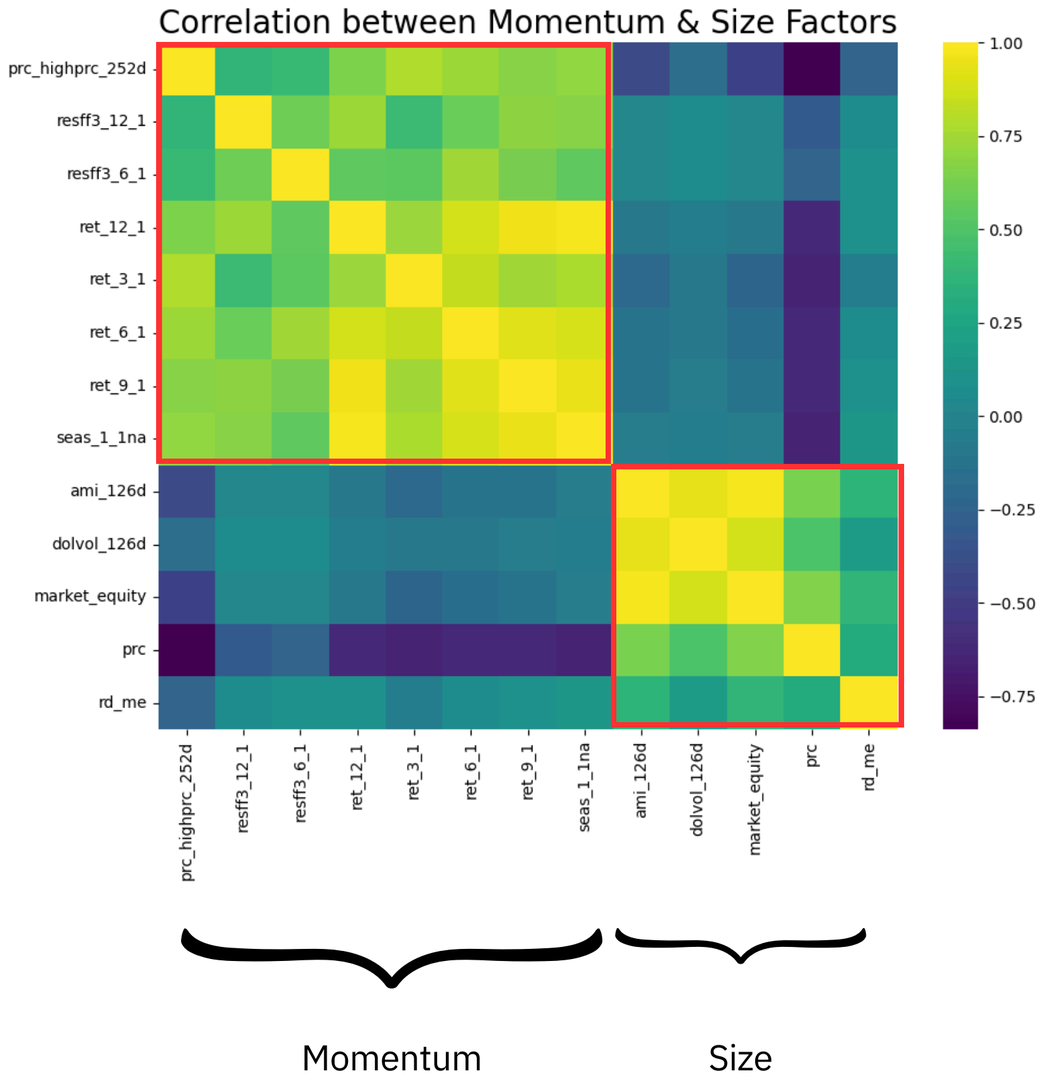
ii. Compute hierarchy between assets

iii. Propagate weights through the hierarchy

HC allocation process: A Case Study

Several steps

i. Compute correlation matrix on the returns

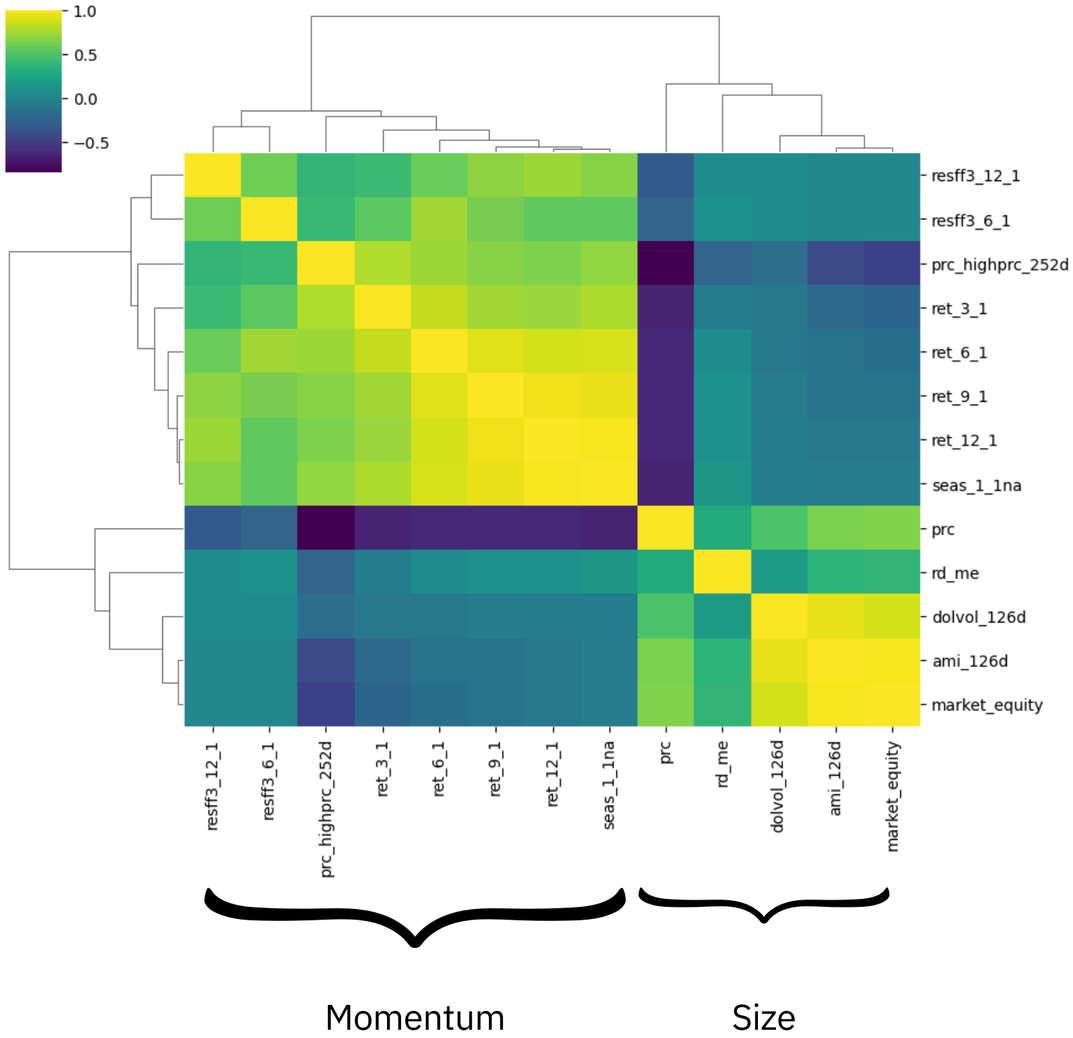


HC allocation process: A Case Study

Several steps

i. Compute correlation matrix on the returns

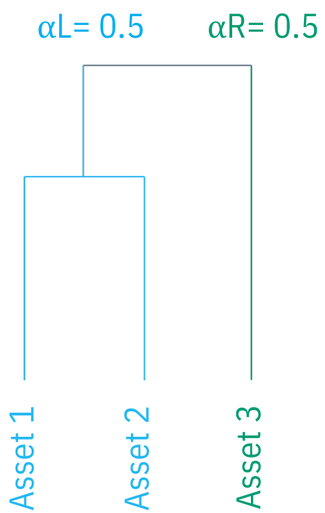
ii. Compute hierarchy between assets



HC allocation process: A Case Study

Step 3:Propagate weights through the hierarchy (theory)

Equal Risk Allocation



α_L : weights propagate through the left legs

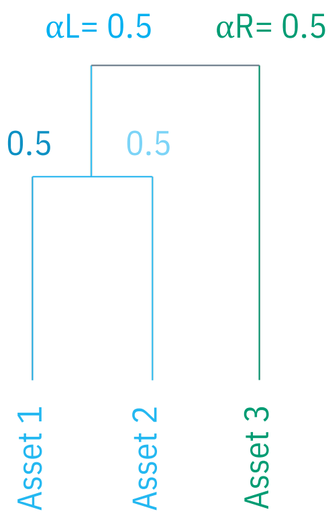
α_R : weights propagate through the right legs

$$\alpha L = \alpha R = 0.5$$

HC allocation process: A Case Study

Step 3:Propagate weights through the hierarchy (theory)

Equal Risk Allocation



Weight(*asset 1*) = 0.5 x 0.5= 0.25

Weight(*asset 2*) = 0.5 x 0.5= 0.25

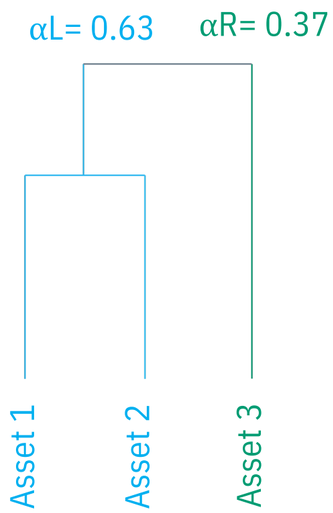
Weight(*asset 3*) = 0.5

Sum is 1

HC allocation process: A Case Study

Step 3:Propagate weights through the hierarchy (theory)

Inverse Variance Allocation



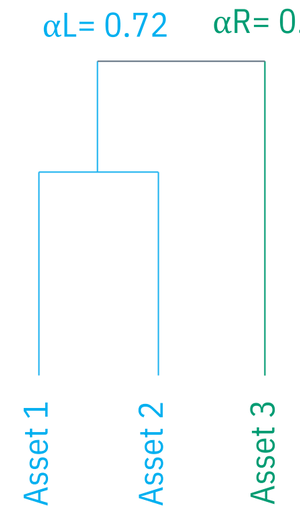
Blue cluster is less volatile than the green one, so he receives more weight

$$\alpha_L = \frac{\frac{1}{Var(asset\ 1,asset\ 2)}}{\frac{1}{Var(asset\ 1,asset\ 2)}+\frac{1}{Var(asset\ 3)}}$$

$$\alpha_R = \frac{\frac{1}{Var(asset\ 3)}}{\frac{1}{Var(asset\ 1,asset\ 2)}+\frac{1}{Var(asset\ 3)}}$$

Inverse Variance & Performance Allocation

Blue cluster is less volatile than the green one and has better perfromance so he receives more weight



$$\alpha L = (1-\mu)\times\frac{\frac{1}{Var(asset\ 1,asset\ 2)}}{\frac{1}{Var(asset\ 1,asset\ 2)}+\frac{1}{Var(asset\ 3)}} + \mu\times\frac{Rets(asset\ 1,asset\ 2)}{Rets(asset\ 1,asset\ 2)+Rets(asset\ 3)}$$

$$\alpha_R = (1-\mu)\times\frac{\frac{1}{Var(asset\ 1,asset\ 2)}}{\frac{1}{Var(asset\ 1,asset\ 2)}+\frac{1}{Var(asset\ 3)}} + \mu\times\frac{Rets(asset\ 3)}{Rets(asset\ 1,asset\ 2)+Rets(asset\ 3)}$$

Inverse variance factor

Performance factor