Exam presentation

Assignment 2 and Assignment 3

Eren Can Gungor[1]

[1]Technical University of Denmark
Digital Communication

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The Eye Diagram for a Digital Communication Channel

- An eye diagram is used to see every possible symbol transition in one graph.
- One can analyze these transitions to see inter-symbol interference (ISI) effects.
- Generating an eye diagram requires plotting of overlapping signals of k-symbol duration where k is an integer more than one since we need to see the transitions between symbols.
- With real signals, it can be generated with an oscilloscope with time sweep frequency $1/kT_s$ where T_s is the symbol duration.
- With using an eye diagram, one can make comments on probability of error by looking at the eye opening.

- Channel model is assumed as a low pass filter.
- Butterworth filters with different cut-off frequencies are used as low pass filters to demonstrate the effects of ISI with an eye diagram.
- Normalized bandwidths are used as parameters of filters in the code. If normalized bandwidth is B and symbol rate is n_{sym} symbol per second then the bandwidth of the filter is given as Bn_{sym} Hz.
- We have sampling frequency $f_s = n_{sym}n_{samp}$ samples per second where n_{samp} is the sample number in a symbol. To obtain a filter with bandwidth Bn_{sym}, we should use normalized frequency $f_{normalized} = \frac{Bn_{sym}}{f_S/2} = 2B/n_{samp}$ (π rad/sec).

Eye diagram characteristics

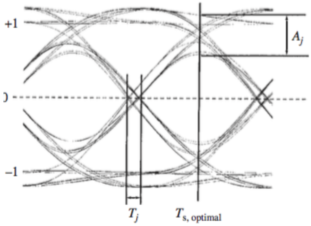

Figure: *Eye diagram of baseband antipodal signal*

- A Difference between high and low levels
- A_j Difference between A and the eye opening
- T_j Deviations from ideal timing
- T_b Bit time period

Eye diagram at different bandwidths

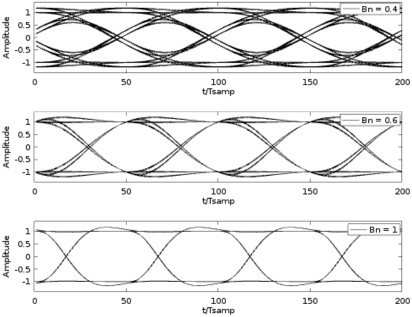

Figure: *Eye diagram for normalized bandwidths 0.3, 0.7, 1.2*

- ISI leads to T_j. increasing the signal bandwidth will result in increased slope that which will result in more open eye
- High BW: high amplitude and timing jitter.
- Low BW: no ISI, chances of higher noise

Eye Diagram Graph

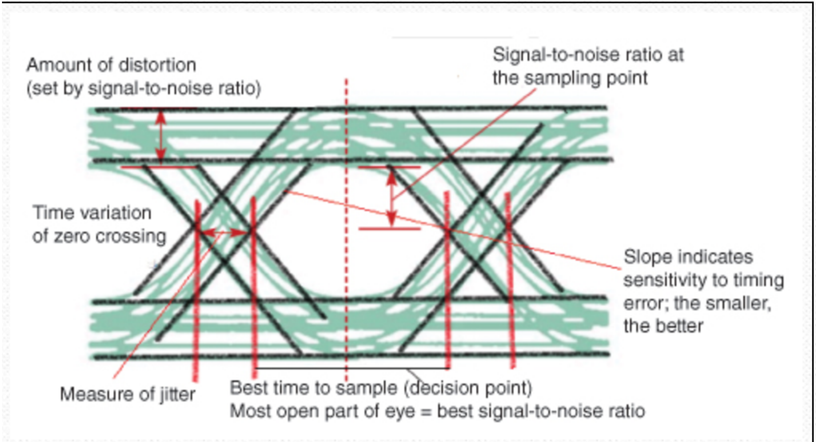

FIGURE 4. An eye diagram can help you interpret a signal and determine the best time for making a measurement

Q function 2.2

In this part of the assignment we will investigate the Normal (Gaussian) probability density function, $Q(u)$ function and it's relationship with complementary error function. It will also show how these theories will be related to the current communication systems by given assignment questions. Things we will look at are:

- Normal(Gaussian) Probability Density Function
- $Q(u)$ function
- $Q(u)$ function and it's relationship with complementary error function.

Probability Density Function

Normal distribution/Gaussian distribution is a really important and in fact most commonly used distribution in statistics. It is important because:

- Almost all variables are distributed approximately normally. They are approximately close
- Second reason is that statistical tests are derived from normal distribution and also work well if the distribution is approximately normal.
- Another reason is that it is only just characterised by two variables;
  - It's mean μ and and standard deviation σ

For the communication systems; Noise is an error or undesired random disturbance of a useful information in communication channel.The noise is a summation of unwanted or disturbing energy from natural and sometimes man-made sources.

Q2.1 Plotting gaussian pdf and explain important variables

Now we can put our theory in a practice in this given question, we have created the MATLAB file named`graphingpdf.m` which can be accessible from the report file. In this assignment important variables are

- **mu** . This is the mean value (μ) for the normal probability density function.
- **sigma** This the sensible standard deviation number. (σ).
- **MAX** 50; Maximum x value that x vector will get
- **MIN** -50; Minimum x value that x vector will get

Graph for Gaussian PDF

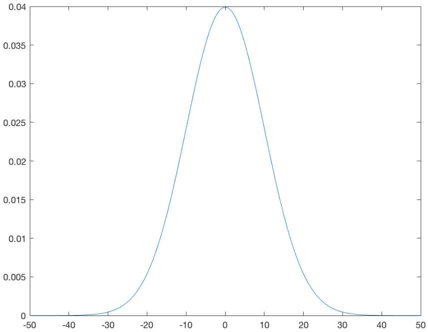

Figure: *Normal Gaussian pdf graph with defined intervals*

Explanation of $Q(u)$ function

Cumulative distribution function (CDF) shows the probability that random variable X is less than a value x,i.e., CDF $F(x) = Pr(X \leq x) = \int\limits_{-\infty}^{x} f(x)dx$ where $f(x)$ is normal probability density function. The probability that random variable X is larger than a value x defined as Q function. Therefore, $Q(u) = 1 - F(u)$. Normal distribution has a μ value 0 and σ value 1. By taking account these parameters, if we insert Gaussian PDF $f(x)$ into account Q function becomes $Q(u) = \int\limits_{u}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(\frac{-y^2}{2})dy$. For the general Gaussian PDF with μ and σ, a normalization should be done like $Q(\frac{u-\mu}{\sigma})$ as shown in book at page 295. In Fig. 3, shaded area shows the value of Q function.

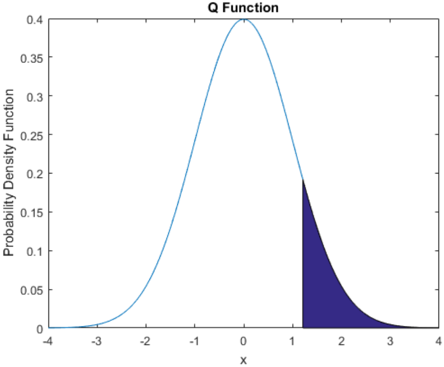

Figure: $Q(u)$ *graph*

Assignment related question for the $Q(u)$ function

To prove our theory behind, we have constructed the $Q(u)$ function plot that will able to take defined argument values of relevance to the detection problems for digital communication receivers. MATLAB filed called `qfunction.m` has been created for this assigment. For the next step Two important argument has been chosen for this assignment, those are:

- $R_{12} = 0(Orthogonal Signals)$
- $R_{12} = -1(Antipodal Signals)$

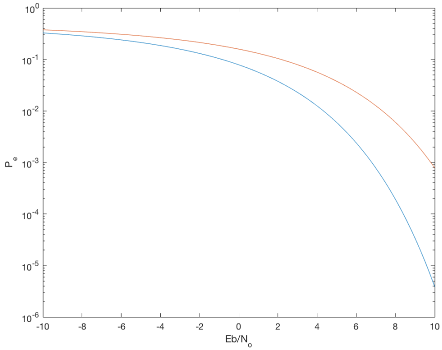

Figure: $Q(u)$ *graph*

Q inverse function and contemporary error function

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- $Q(u) = \int\limits_{u}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(\frac{-y^2}{2}) dy.$
- $erf(u) = \frac{2}{\sqrt{\pi}} \int\limits_{0}^{u} e^{-y^2} dy$
- $erfc(u) = 1 - erf(u) = \frac{2}{\sqrt{\pi}} \int\limits_{u}^{\infty} e^{-y^2} dy$
- $Q(u) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$
- $erfc(v) = 2Q(\sqrt{2}v)$

We also have inverse Q function that maps for any y value that gives $Q(x) = y$ to x ($Q^{-1}(y) = x$). These functions are defined in MATLAB and can be directly used with functions *qfunc(x), qfuncinv(x), erf(x) and erfc(x) commands.*

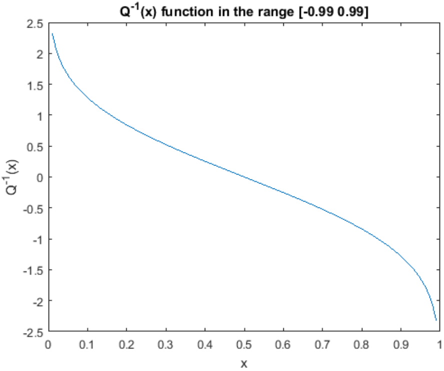


Figure: $Q(u)^(-1)$ *graph*

MATLAB code for user-defined Q function

We also note that MATLAB does not have built in function.So we have created a MATLAB file that uses erfc function to do the calculations and simulation process related to the $Q(u)$ function. It is accessible from the Source code section under the name of `qfn.m`.

Assignment 2 Part 3

In assignment two, part 3, we will look into optimising our filter by using the Q function and error functions that we have learnt from the part 2 of this Assignment. The important things we will specifically look in these chapter are:

- Additive White Gaussian Noise for Matched Filter
- Matched Filter
- Correlator Filter
- Noise and Shape related problems in Matched Filter

Additive White Gaussian Noise for Matched Filter

Matched filter receiver is an optimum receiver structure in terms of minimum prob- ability of error in communication systems. Derivation of the optimality of the strucure is given in the book. Block diagram of matched filter output for binary data in AWGN data is given in Fig.5 which is taken from the book page 409

Working Principle

For a working principle, The signal is multiplied by a locally stored reference copy ,and integrated over time. To understand the principles of a matched filter receiver for binary data in white Gaussian noise, thus using the so called AWGN (Additive White Gaussian Noise) model. We have firstly introduced the figure below.

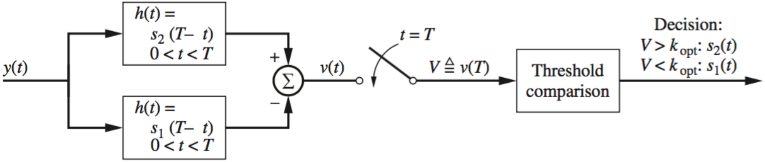


Figure: *AWGN for Matched Filter*

Principles of AWGN for Matched Filter

Received signal $y(t)$ should be passed through matched filters $s_1(T-t)$ and $s_2(T-t)$ and their difference must be sampled at $t = T$ and then a decision should be according to the optimal threshold value. $s_1(t)$ and $s_2(t)$ are original signals corresponds to binary data and $s_1(t) < s_2(t)$ is assumed. Optimal threshold value is $\frac{E_2 - E_1}{2}$ where E_2 and E_1 signal energies which are finite.

Another example on how system works for AWGN Matched Filter

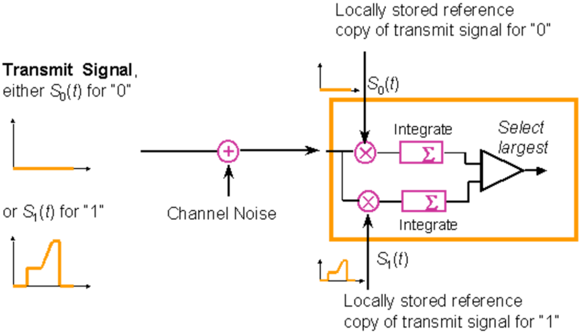


Figure: *Working Principle of AWGN Matched Filter*

Creating a User Defined Q function

To run our code for assignments 3.4 and 3.5 we have created a user defined `qfn.m` function. The code has been created as a `qfn.m` which can be accessible from the report.

Q 3.4 and 3.5 P_e Graph For Various Correlation Coefficients

We enter [-1 -0.75 -0.5 0 0.5 0.75 0.8 .995] matrix when we run script and it plots probability of error for different correlation values. For lower correlation values the probability of error decreases since the signals affect each other less. As the correlation values increase, the signals affect each other, e.g., if one is sent both matched filters high values of outputs and with noise the one that isn't sent may become higher easily. In Fig. 6, the results can be seen as expected.

Graph with 8 Correlation Coefficient Inputs

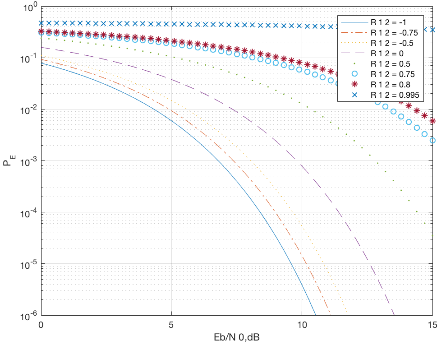


Figure: P_E *over* N_o *graph for 8 correlation coefficient Inputs*

Matched Filter and Correlator

In matched filter, $h(t) = s(T-t)$ and output of this filter can be written as convolution

$$h(t) * y(t) = \int_0^T s(T-\tau) y(t-\tau) d\tau$$

where $y(t)$ is received signal. Since system samples signals at t=T and with change of variables to $\alpha = T - \tau$

Derivation continues

we can rewrite above equation as

$$v(T) = \int_{0}^{T} s(\alpha)y(\alpha)d\alpha$$

where $v(T)$ is sampled version of convolution. The result is integration of the multiplication of received signal with original signal $s(t)$ and it is simply correlation of signals. Therefore, one can implement matched filter receiver with correlator as shown above.

Noise on the timing Synchronization in The Receiver

It is possible that the exact arrival time of received signal can be in error. That causes the sampling at wrong instances of matched filter output. That means receiver is no longer an optimal receiver because system samples output different point rather than it's maximum. That will cause a decrease of the amplitude of sampled signal and therefore, signal to noise ratio will also decrease. As a result probability of error becomes larger in timing synchronisation errors.

Bit Error Probability in Matched Filter

To find out bit error probability in a system with matched filter, we first need to define threshold. $s_1(T)$ and $s_2(T)$ are the inputs to the matched filter, $s_{01}(T)$ and $s_{02}(T)$ are the outputs of the matched filter. Then the equation becomes

- $s_{01}(T) = \int_{-\infty}^{\infty} h(\lambda)s_1(T-\lambda)d\lambda$
- $\int_{-\infty}^{\infty}(s_2(T-\lambda) - s_1(T-\lambda))s_1(T-\lambda)d\lambda$
- $\int_{-\infty}^{\infty} s_2(u)s_1(u)du - \int_{-\infty}^{\infty}(s_1(u))^2 du$
- $\sqrt{E_1E_2}\rho_{12} - E_1$

Continuing the derivation

and by the equations symmetric to the ones above we get
$s_{02}(T) = E_2 - \sqrt{E_1 E_2}\rho_{12}$
Then optimum threshold is the average of s_{01} and s_{02},
$k_{opt} = \frac{1}{2}(E_2 - E_1)$
We have probability of error equation
$P_e = P(Error|s_1(t))P(s_1(t)) + P(Error|s_2(t))P(s_2(t))$.
For the equiprobable symbol selection case we have
$P_e = P(Error|s_1(t))\frac{1}{2} + P(Error|s_2(t))\frac{1}{2}$
This can be formed as $P_e = Q(\frac{s_{02}(T) - s_{01}(T)}{2\sigma_0})$
if we make the selection of threshold as k_{opt}. We put values of s_{01} and s_{02} to the equation and it becomes $P_e = Q(\frac{E_1 + E_2 - 2\rho\sqrt{E_1 E_2}}{2\sigma_0}\Bigg)$

Link budget model

- A way of estimating the power of a received signal
- Takes into account all the gains and losses of transmitter, channel, and receiver

$$P_R = \left(\frac{\lambda}{4\pi d}\right)^2 \frac{P_T G_T G_R}{L_0}; \tag{1}$$

- In decibels:

$$P_{R,dB} = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) + ERP_{dB} + G_{R,dB} - L_{0,dB}; \tag{2}$$

Link budget model variables

$\left(\frac{4\pi d}{\lambda}\right)^2$ Free-space loss.Signal strength loss occurring in a line-of-sight path through free space (usually air), without accounting for reflection or diffraction.

$$FSPL = \left(\frac{4\pi d}{\lambda}\right)^2 \tag{3}$$

$ERP = P_T G_T$ Effective radiated power. Equivalent power transmitted equally in all directions, from a theoretical spherically radiating source.

$$ERP_{dB} = 10\log_{10}(P_T) + G_{T,dB} \tag{4}$$

G_R Gain of receiver antenna.

L_0 Other losses, link budget margin

SNR calculation

- Signal-to-noise ratio in decibels:

$$SNR_{dB} = P_{R,dB} - P_{int,dB} \tag{5}$$

P_R Calculated using link budget model

P_{int} Noise power, proportional to the receiver noise temperature and the transmission bandwidth

$$P_{int} = kT_RB \tag{6}$$

where T_R is the noise temperature of the receiver, B is the bandwidth of the transmission, and k is the Boltzmann constant.

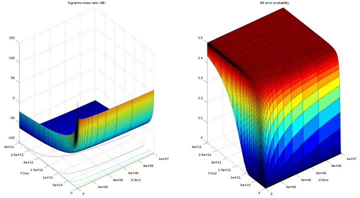
P_E calculation

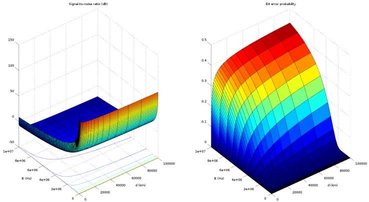
- Bit error probability for BSFK transmissions:

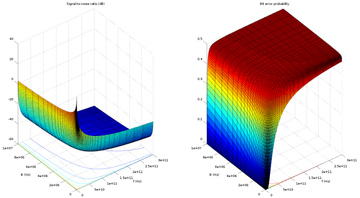
$$P_E = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{7}$$

1. ratio E_b/N_0 derived from $SNR = \longrightarrow \frac{E_b}{N_0 B T_b}$
2. For binary BPSK, $B = 2/T_b$
3. Factor BT_b is 2, or 3 dB

Impact of d, λ, B







- SNR and P_E are negatively affected by:
  - Higher distance d
  - Lower wavelength λ
  - Wider bandwidth B (lower influence)

Impact of P_T

- P_E for P_T at 50 W, 5 W, and 500 mW:

$$\texttt{1.4062e-05 9.2684e-02 3.3768e-01} \tag{8}$$

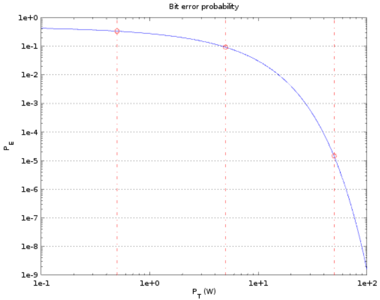


Figure: P_E *over values of* P_T

Alternative modulation: ASK

- Amplitude-shift keying

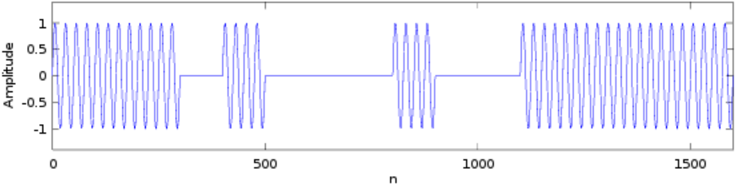


Figure: *Bit stream modulated using ASK*

- 0-bit represented as 0
- 1-bit represented as $A\cos(2\pi f_c t)$

Alternative modulation: ASK (P_E calculation)

- Correlation coefficients:
  - $\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_{-\infty}^{\infty} s_1(t) s_2(t) dt = 0$
  - $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$
- SNR to E_b/N_0: conversion factor $BT_b = 2$
- Bit error probability:

$$P_E = Q\left(\sqrt{\left(1 - R_{12}\right) \frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{9}$$

Whilst the formula is similar to the one previously introduced for the calculation of P_E in a BPSK transmission, it lacks a $\sqrt{2}$ term. Therefore, the signal-to-noise radio of a ASK transmission has to be $3dB$ higher in order to maintain the same performances.

Alternative modulation: FSK

- Frequency-shift keying

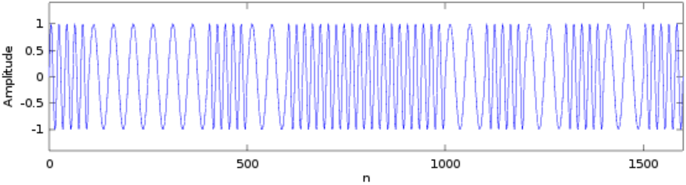


Figure: *Bit stream modulated using FSK*

- 0-bit represented as $A\cos(\omega_c t)$
- 1-bit represented as $A\cos((\omega_c + \Delta\omega)t)$
- Assumptions: $\omega_c = \frac{2\pi n}{T}$ and $\Delta\omega = \frac{2\pi m}{T}$

Alternative modulation: FSK (P_E calculation)

- Correlation coefficient $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b}\rho_{12} = 0$
- SNR to E_b/N_0: conversion factor $BT_b = 2.5$
- Bit error probability:

$$P_E = Q\left(\sqrt{(1 - R_{12})\,\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{10}$$

However, when deriving the $Eb/N0$ ratio from the signal-to-noise ratio, the required bandwidth is given by $2.5/T$, so the factor BTb becomes 2.5, or around $4dB$.