# Satellite Link Budgets and $P_E$

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# December 8, 2016

# Contents

1	Link budget model	<b>2</b>
	1.1 Free-space loss	2
	1.2 Effective radiated power	3
	1.3 Link budget margin	
2	$SNR$ and $P_E$	3
	2.1 Calculation of signal-to-noise ratio	3
	2.2 Calculation of bit error probability	
	2.3 Impact of parameters $d, \lambda, B$	
	2.4 Bit error probability at different transmission powers	
3	Alternative modulation methods	6
	3.1 Method 1:	6
	3.2 Method 12	
4	Source code	6

## 1 Link budget model

A link budget model is a way of estimating the power  $P_R$  of a received signal by taking into account the possible sources of gain and loss over the trasmission. Since the aforementioned gains and losses are typically expressed in the form of ratios, the calculation of a link budget becomes trivial if employing decibels

In the case of a satellite link, the following factors are taken into account in the budget:

$$P_{R,dB} = 20 \log_{10} \left( \frac{4\pi d}{\lambda} \right) + ERP_{dB} + G_{R,dB} - L_{0,dB}; \tag{1}$$

 $\left(\frac{4\pi d}{\lambda}\right)^2$  Free-space loss

ERP Effective radiated power

 $G_R$  Gain of receiver antenna

 $L_0$  Other losses (e.g. atmospheric absorption)

The following subsections will attempt to explain some of these factors.

#### 1.1 Free-space loss

The free-space loss is the signal strength loss occurring in a line-of-sight path through free space (usually air), without accounting for reflection or diffraction. The equation for the free-space loss is given by:

$$FSPL = \left(\frac{4\pi d}{\lambda}\right)^2 \tag{2}$$

where d represents the distance between transmitter and receiver, and  $\lambda$  is wavelength of the signal. In decibels, it can also be expressed as:

$$FSPL_{dB} = 20\log_{10}(d) + 20\log_{10}(f) - 147.55 \tag{3}$$

The equation results in a loss term, meaning that its value will always be confined between 0 and 1, or always negative when expressed in decibels.

It is clearly noticeable how the equation resembles that of the sphere surface area; with higher values of distance or signal frequency, the loss increases.

#### 1.2 Effective radiated power

The effective radiated power is the equivalent power transmitted equally in all directions, from a theoretical spherically radiating source, also known as isotropic radiator. It is obtained by:

$$ERP_{dB} = 10\log_{10}(P_T) + G_{T,dB} \tag{4}$$

where  $P_T$  is the output power of the transmitter and  $G_T$  is the gain of its antenna in respect to the isotropic radiator model.

#### 1.3 Link budget margin

The link margin is the additional amount of attenuation that a transceiving system can tolerate while still functioning. It is recommended to include a margin of a few decibels in the link budget, in order to account for unpredicted losses.

In the case of satellite communication, the margin might comprise of rain fade or atmospheric absorption. Other common sources of signal degradation which are included in the link margin are components tolerances and wear.

## 2 SNR and $P_E$

## 2.1 Calculation of signal-to-noise ratio

The SNR of the satellite transmission can be calculated as:

$$SNR_{dB} = P_{R,dB} - P_{int,dB} \tag{5}$$

where  $P_R$  is the previously introduced power of the received signal, and  $P_{int}$  is the power of the receiver output noise. The latter is provided by:

$$P_{int} = kT_R B \tag{6}$$

where  $T_R$  is the noise temperature of the receiver, B is the bandwidth of the transmission, and k is the Boltzmann constant.

### 2.2 Calculation of bit error probability

The bit error probability of a biphase-shift keying (BPSK) transmission is calculated as:

$$P_E = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{7}$$

It is possible to extrapolate the required energy-per-bit-to-noise spectral density ratio  $E_b/N_0$  from the SNR figure. Multiplying numerator and denominator by the bit duration  $T_b$  gives:

$$SNR = \frac{P_R T_b}{k T_R T_b} \to \frac{E_b}{N_0 B T_b} \tag{8}$$

In case of a binary BPSK (biphase-shift keying) transmission, the bandwidth B is  $2/T_b$ , so the factor  $BT_b$  is 2.

#### 2.3 Impact of parameters d, $\lambda$ , B

This section will investigate the different roles of the receiver and transmitter distance, the wavelength of the signal, and its bandwidth, in the calculation of the signal-to-noise ratio and the bit error probability. The following figures will show the surfaces generated by the combination of two of the aforementioned parameters:

- $\bullet$  d (distance) is given values between 1km and 10000km
- $\lambda$  (wavelength) is given values for frequencies between 300MHz and 300GHz
- B (bandwidth) is given values between 10kHz and 100kHz

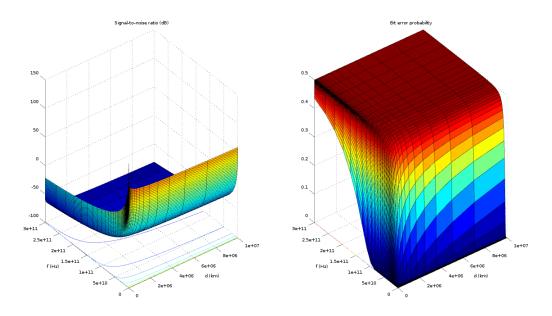


Figure 1: Histogram of randn function

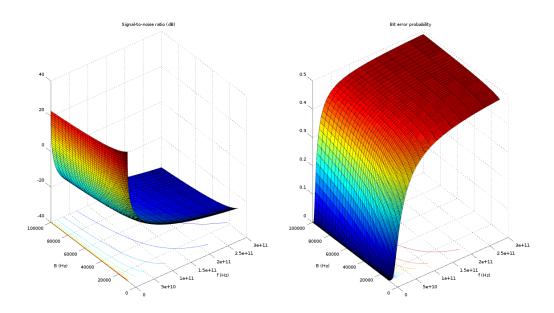


Figure 2: Histogram of randn function

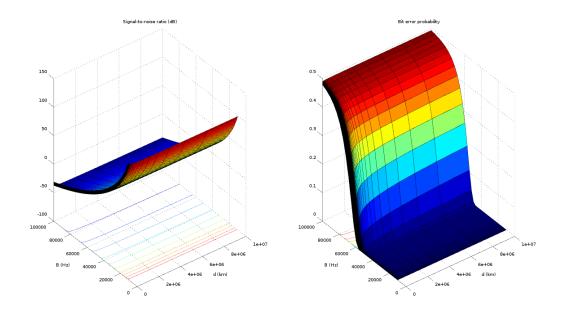


Figure 3: Histogram of randn function

- 2.4 Bit error probability at different transmission powers
- 3 Alternative modulation methods
- 3.1 Method 1:
- 3.2 Method 12

#### 4 Source code

Here follows a thoroughly commented version of the MATLAB script used for the above calculations and observations. The code below computes the signal-to-noise ratio and bit error probability for the free-space transmission characterized by its input parameters.

% script setup
clear all
pkg load communications

%% constants % standard temperature

```
t_0 = 290;
% boltzmann constant
k = 1.38064852e - 23;
% B * T_{-}b
bt_bdb = 3;
%% input parameters
d = 41e6;
lambda = 0.15;
bw = 50e3;
p_{-}t = 100;
g_t_d = 30;
a_r = 0;
g_r_d = 0;
l_0 - db = 3;
t_{-r} = 700;
\% mag2db: 20 * log(x)
\% pow2db: 10 * log(x)
% calculate free-space loss
fsl_db = mag2db(lambda / (4 * pi * d));
% calculate received signal power (A.70)
p_rdb = fsl_db + pow2db(p_t) + g_tdb + g_rdb - l_0db;
% calculate noise level power
p_{int_{db}} = pow2db(k * t_{r} * bw); \% (A.72)
\% p_{-}int_{-}db = pow2db(k * t_{-}0) + pow2db(t_{-}r / t_{-}0) + pow2db(bw);
% (A.73)
% calculate SNR at receiver output
snr_0 db = p_r db - p_int_db
\% calculate energy-per-bit-to-noise spectral density ratio
eb_n0_db = snr_0_db + bt_b_db;
% calculate bit error probability
p_e = qfunc(\mathbf{sqrt}(2 * db2pow(eb_n0_db)))
```