

# The Q Function and Baseband Data Communication

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# 1 Eye Diagram for a Digital Communication Channel

## 1.1 Eye diagram

A so-called *eye diagram* is the pattern that originates from overlapping a digital signal over the length of one or more transmitted bits. For several types of digital modulation the plot would show a series of round shapes (eyes) delimited by two rails, hence the name. The rails are formed by the low-frequency components derived by the bit sequences 00 and 11, whereas the round shapes are formed by the rising and falling edges of the high-frequency bit sequences 01 and 10.

This tool is commonly used to investigate several performance measures of a transmission channel, including noise level, distortion, inter-symbol interference, and synchronization errors.

An eye diagram can be generated with an oscilloscope by enabling infinite persistence, and setting the trigger to react on a separate clock signal.

## 1.2 c5ce2.m: explanation

Here follows a thoroughly commented version of the provided `c5ce2.m` MATLAB script. The code below generates and plots the eye diagrams of four band-limited signals composed of random sequences of bits.

Listing 1: `../scripts/1/c5ce2.m`

```
% clean figure and load signal package (only for Octave)
clf
pkg load signal

% simulation parameters:
% - nr of symbols (must be divisible by 4)
% - nr of samples per symbol
% - filter cutoff values (normalized bandwidth)
nsym = 1000;
nsamp = 50;
bw = [0.4 0.6 1 2];

% for each filter cutoff value..
for k = 1:length(bw)
    % generate filter coefficients using one of the cutoff values
    lambda = bw(k);
    [b,a] = butter(3,2*lambda/nsamp);

    % allocate space for total bit sequence
```

```

l = nsym*nsamp;
y = zeros(1,l-nsamp+1);

% initialize random output vector with +1 and -1
x = 2*round(rand(1,nsym))-1;

% for each overlap..
for i = 1:nsym
    % place one symbol into vector y, leaving nsamp samples of
    % spacing
    kk = (i-1)*nsamp+1;
    y(kk) = x(i);
end
% zero-order hold
datavector = conv(y,ones(1,nsamp));

% apply filter to the total sequence
filtout = filter(b, a, datavector);

% splice sequence into sub-sequences of 4 symbols
datamatrix = reshape(filtout, 4*nsamp, nsym/4);

% discard the first 5 sub-sequences
datamatrix1 = datamatrix(:, 6:(nsym/4));

% plot eye diagram
subplot(length(bw),1,k)
plot(datamatrix1, 'k')

% format: print axis labels, legend, and set plot range
ylabel('Amplitude')
axis([0 4*nsamp -1.4 1.4])
legend(['Bn□=□', num2str(lambda)])
if k == 4
    xlabel('t/Tsamp')
end
end

```

### 1.3 Channel model

In order to analyze the performances of a given transmission system, a model of the adopted transmission channel has to be devised.

For the purpose of this simulation, the most interesting characteristic of the channel is its bandwidth, and therefore its suitability for transmitting information at a specific rate. The limitation in bandwidth is obtained by feeding the signal into a third-order Butterworth low-pass filter.

The channel is therefore characterized by its *normalized bandwidth*, which represents the bandwidth as a percentage of the data bit rate; e.g if the bit rate is 8000Hz and the normalized bandwidth is 0.5, the channel will have a cutoff frequency of 4000Hz.

The MATLAB tools for generating digital filters (`butter`, `ellip`, `cheby1...`) accepts the normalized frequency as input argument, which is the cutoff frequency in terms of the *Nyquist frequency* (half of the sampling frequency). The normalized bandwidth is proportional to the normalized frequency by a factor equivalent to the number of samples per bit.

## 1.4 c5ce2.m: different bandwidths

The following section will present a modified version of the previously introduced script, which can plot the eye diagrams for the normalized bandwidth 0.15, 0.3, 0.7, 1.2, and 4.

In order to do so, the input vector of `bw` has been changed to reflect the given values, together with the subplot format, and x-axis label clause.

Listing 2: `../scripts/1/c5ce2_augm.m`

```
% clean figure and load signal package (only for Octave)
clf
pkg load signal

% simulation parameters:
% - nr of symbols (must be divisible by 4)
% - nr of samples per symbol
% - filter cutoff values (normalized bandwidth)
nsym = (100)*4;
nsamp = 50;
bw = [0.15 0.3 0.7 1.2 4];

% for each filter cutoff value..
for k = 1:length(bw)
    % generate filter coefficients using one of the cutoff values
    lambda = bw(k);
    [b,a] = butter(3,2*lambda/nsamp);

    % allocate space for total bit sequence
    l = nsym*nsamp;
    y = zeros(1,l-nsamp+1);

    % initialize random output vector with +1 and -1
    x = 2*round(rand(1,nsym))-1;

    % for each overlap..
```

```

for i = 1:nsym
    % place one symbol into vector y, leaving nsamp samples of
    % spacing
    kk = (i-1)*nsamp+1;
    y(kk) = x(i);
end
% zero-order hold
datavector = conv(y, ones(1, nsamp));

% apply filter to the total sequence
filtout = filter(b, a, datavector);

% splice sequence into sub-sequences of 4 symbols
datamatrix = reshape(filtout, 4*nsamp, nsym/4);

% discard the first 5 sub-sequences
datamatrix1 = datamatrix(:, 6:(nsym/4));

% plot eye diagram
subplot(length(bw), 1, k)
plot(datamatrix1, 'k')

% format: print axis labels, legend, and set plot range
ylabel('Amplitude')
axis([0 4*nsamp -1.4 1.4])
legend(['Bn□=□', num2str(lambda)])
if k == length(bw)
    xlabel('t/Tsamp')
end
end

```

In order to properly visualize the resulting plots, the original bandwidth values have been removed. In order to view all the eight eye diagrams, line 11 can be substituted with the following:

Listing 3:

```
bw = [0.15 0.3 0.7 1.2 4];
```

## 1.5 c5ce2.m: plots

This section will elaborate on the structure and implications of the eye diagrams generated using the scripts above.

The plots are generated by the original and augmented versions of the script respectively, and show the eye diagram for an antipodal baseband transmission at various channel bandwidths.

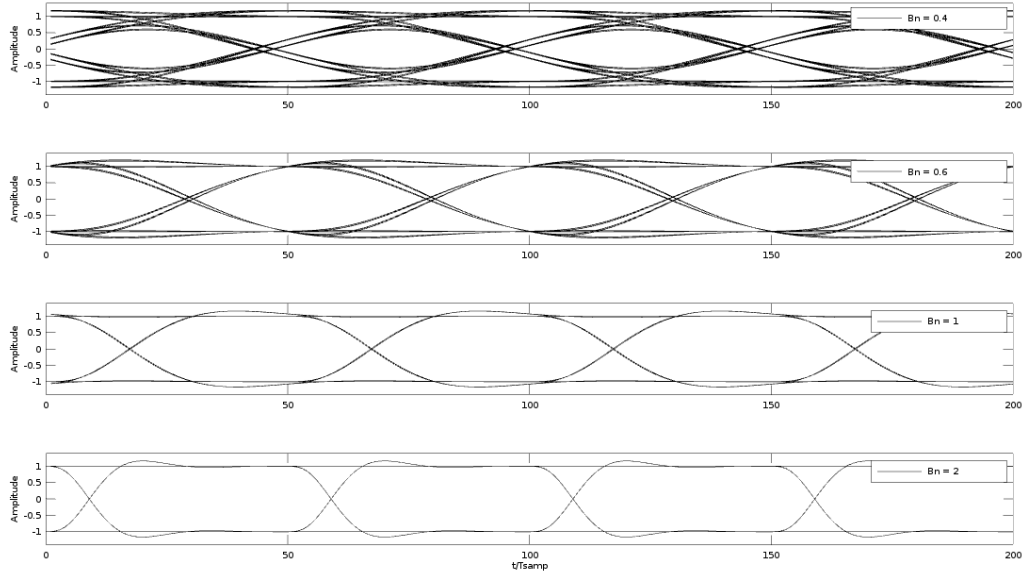


Figure 1: Eye diagrams of signals with filter coefficients 0.4, 0.6, 1, and 2

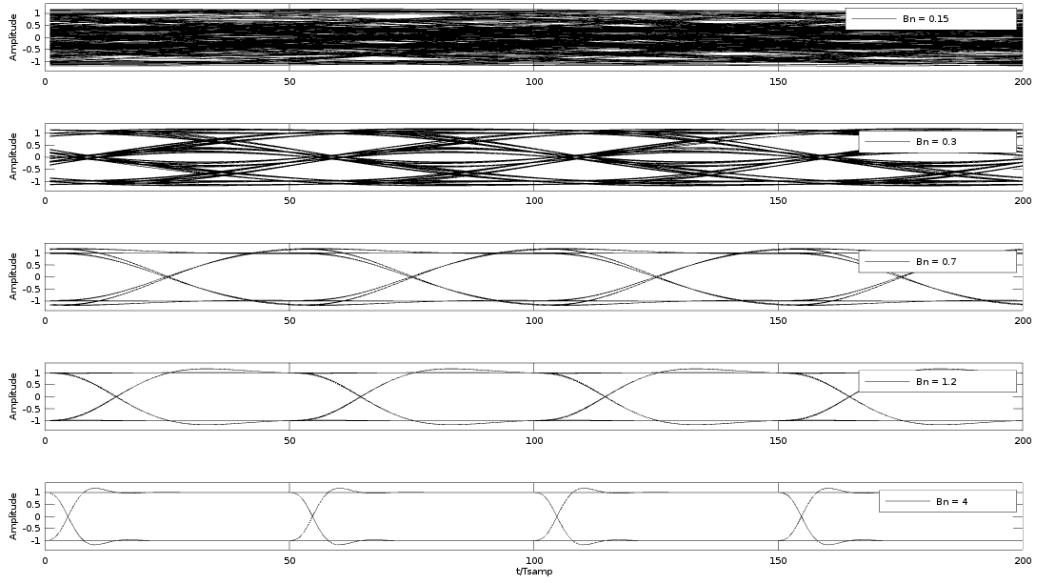


Figure 2: Eye diagrams of signals with filter coefficients 0.15, 0.3, 0.7, 1.2, and 4

When observing an eye diagram, the two main measures of the system performance are the amplitude jitter  $A_j$  and the timing jitter  $T_j$ . The former

measures the difference in amplitude between the low-frequency and high-frequency components of the signals, sampled where the eye is at its maximum opening. Low channel bandwidth causes the eye to shrink and amplitude jitter to increase, which in turns increases the bit error probability. The latter parameter represents the time interval at which the zero-crossing may occur. It is caused by inter-symbol interference, and a high value could result in synchronization issues.

The eye diagrams from both scripts clearly show how a low bandwidth results in overall more jittery and less intelligible signals. For the normalized bandwidth value of 0.15, the inter-symbol interference is so high that the eye figures become completely indistinguishable, which would likely result in an extremely high bit error probability (close to 0.5).

For channel bandwidth values higher than the bit rate, the resulting eye figures are particularly wide with negative amplitude jitter and no timing jitter, suggesting that there is no inter-symbol interference. Specifically, higher bandwidth values yield steeper slew rates. However, it is worth noting how increasing the available bandwidth beyond the bit rate may cause more noise to enter the system, negatively affecting the overall signal-to-noise figure and bit error probability.

It is also possible to notice the time offset of the eye shapes across the different plots, caused by the delay introduced by the channel frequency response.



## 2 The Q function

### 2.1 Normal probability density function 2.1

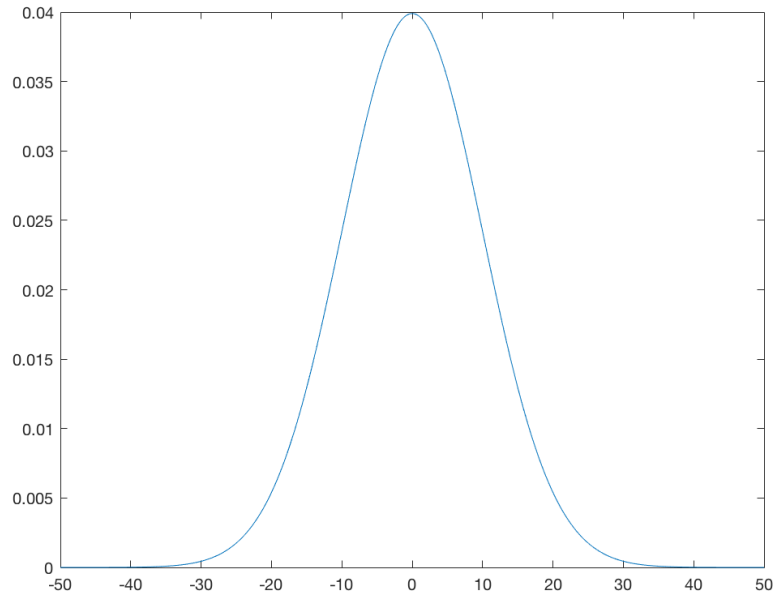


Figure 3: Normal Gaussian pdf graph with defined intervals

Normal gaussian distribution is one of the most important concept in Communication system and in statistics. It is used as a powerful tool when investigating random signals in communication systems. Such as investigating behaviour and application of noise signals.

The other reason is that because it is characterised by the limited variables. Such as mean  $\mu$  and  $\sigma$ . It is easier compute and understand when we apply on communication systems.

All the variables that has been used in graphingpdf.m has been explained below;

**mu** . This is the mean value ( $\mu$ ) for the normal probability density function.

**sigma** This the sensible standard deviation number. ( $\sigma$ ).

**MAX** 50; Maximum x value that x vector will get

**MIN** -50; Minimum x value that x vector will get

Also general formula for gaussian pdf is ;  $y=f(x|\mu, \sigma) = (\frac{1}{\sigma\sqrt{2\pi}})e^{\frac{-(x-\mu)^2}{2(\sigma)^2}}$   
As we can see and understand from the variables above and the formula that all variables has been specified and we only need  $\sigma$ ,  $\mu$  and range.

## 2.2 Explanation of Q(u) function in relation to the normal probability density function 2.2

As we have explained and investigated probability density density function above . We can easily link  $Q(u)$  function by exploiting the properties of cumulative distribution function and Probability density function.

- Q function is the 1- minus the cumulative distribution function of the standardised normal variable.
- Gaussian pdf with unit variance and zero mean is  $R=(\frac{1}{\sqrt{2\pi}})e^{\frac{-(x)^2}{2}}$
- And corresponding cumulative distribution function become;  $P = \int_{-\infty}^x Z$
- In last step Gaussian  $Q$  function defined as  $Q(x) = 1 - P(x) = \int_x^{-\infty} Z$

## 2.3 Preparing a script file for plotting the $Q(u)$ function for argument values of relevance to the detection problems for digital communication receivers and inserting them in Appendix 2.3 and 2.4

Script file that has been created for the assignment 2.3 and 2.4 has been added to the Appendix. Important concept has been explained and relevant digital communication input values specified.

## 2.4 2.5

### 2.4.1 Complementary error function

## 2.5 Plots

Plotting for the questions 2.3 and 2.4

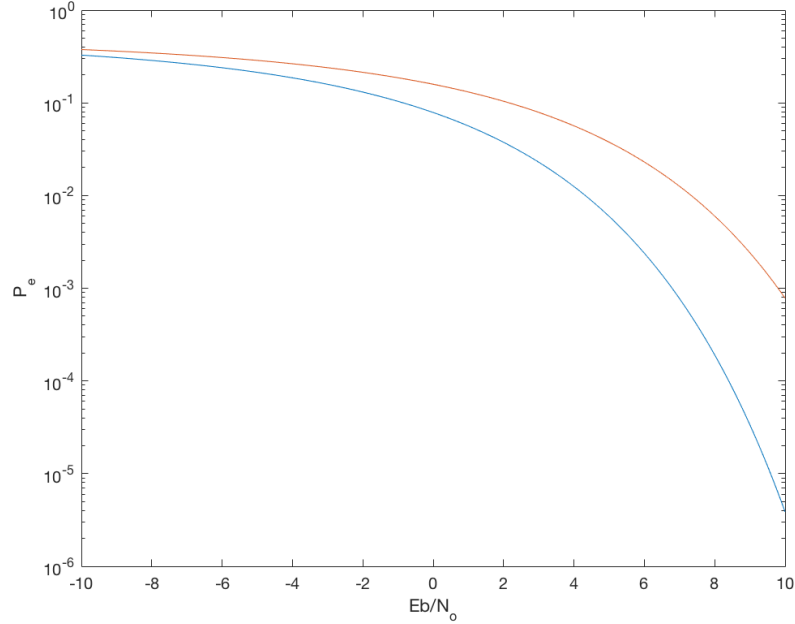


Figure 4: Plot for the  $Q(u)$  function relevant to the argument values for the detection problems for digital communication receivers

### 3 Source Code

Here we have the properly prepared MATLAB codes for the second part of the second project. It has been used for observations, calculations and comparing with specified commands that given in this project.

The code belows computes and graphs normal( Gaussian) probability density function (pdf) in an appropriate intervals

Listing 4: ../scripts/2/graphingpdf.m

```
%graphing PDF function with random variables
mu=0;
sigma=10;
MAX = 50;
MIN = -50;
STEP = (MAX - MIN) / 1000;
PDF = normpdf(MIN:STEP:MAX, mu, sigma);
plot (MIN:STEP:MAX, PDF)
```

For the question 2.3 and 2.4 we have created the following code below. This code will plot the  $Q(u)$  function for argument values relevant to the

digital communication receivers. As we know from previous chapters that there are two common argument values :

- $R_12 = 0$  (Orthogonal Signals)
- $R_12 = -1$  (Antipodal Signals)

We also used  $z_db$  for the ratio for  $E_b/N_o$ . We should also notice that  $z_db$  is dimensionless ratio.

After that we have investigated the graph for further investigation to see whether we have achieved a satisfactory results for the assignment 2.3 and 2.4. Mathematical calculations are matching up with MATLAB simulation results.

Listing 5: ../scripts/2/qfunction.m

```
z_db = -10:.1:10;
r12 = [-1 0];
z = db2pow(z_db);
p_e(1,:) = qfunc(sqrt((1 - r12(1))*z));
p_e(2,:) = qfunc(sqrt((1 - r12(2))*z));
semilogy(z_db, p_e(1,:))
hold on
semilogy(z_db, p_e(2,:))
xlabel('Eb/N_o')
ylabel('P_e')
```

## 4 The Matched Filter Base Band Receiver

### 4.1 Additive white gaussian noise model

### 4.2 `c8ce1a.m`: explanation

### 4.3 Extend the code in `c8ce1a.m` such that the number of input correlations can be up to 8. Presently 4 is the maximum of input arguments 3.4

For the part 3.4 `question35.m` MATLAB code has been created. It can be seen that code is able take 8 correlation input. It is accessible from the appendix part of this report.

### 4.4 Apply the following correlation coefficients in the `c9ce1a` script $[-1 -0.75 -0.500.50.750.8.995]$ and insert the resulting plot in the project report. Explain the plot and verify that it is correct 3.5

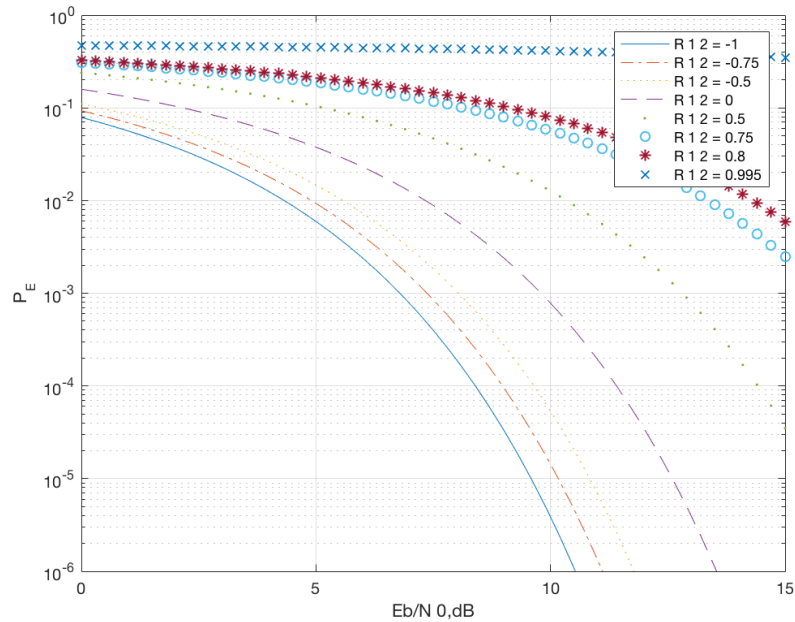


Figure 5:  $P_E$  over  $E_b/N_o$  graph for investigating how it will change with introduced correlation coefficients

4.5 Explain how a matched-filter receiver can be implemented by a correlator. The explanation should include the most important formulas leading to the matched filter implementation using a cross correlation Part 3.6

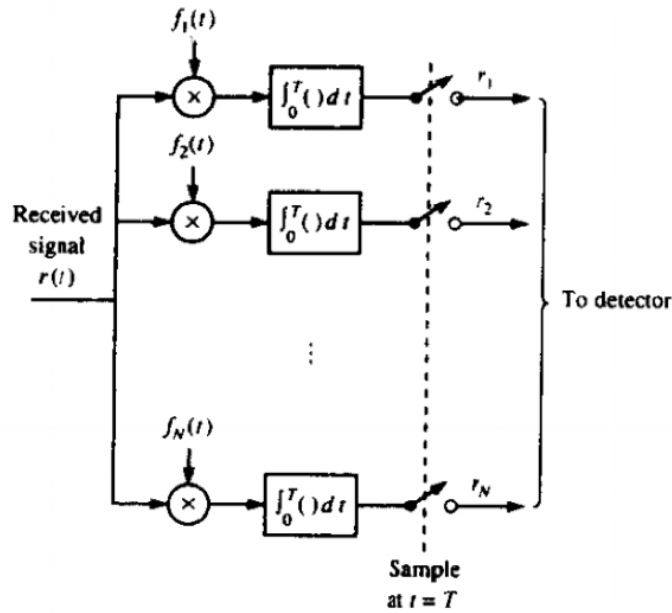


Figure 6: Correlator Matched Filter

As we can from the Figure 3 that we can implement the matched filter by implementing the correlation detectors. The cross-correlator does the cross-correlation between the noisy signal and noiseless signal.

If we look at the matched filter, it does the convolution between the received signal and the time-reversed copy of the reference signal. We can mathematically prove that both matched filter and the cross-correlator will give us same output signal values. To prove this we will derive mathematical derivation.

Suppose we have received  $r(t)$  and is passed through parallel bank of  $N$  crosscorrelators? which basically compute the prediction of  $r(t)$  onto the  $N$ -basis functions as  $f_n(t)$ . This can be seen in Figure 3.

For cross-correlator signal is now represented by  $s_m(t)$  with components  $s_{mk}(t)$ .  $s_{mk}(t)$  values depend on how many  $M_{signal}$  is transmitted. Now other

components of cross-correlator will be explained below for the mathematical derivation for the output signal.

- $\int_0^T r(t)f_k(t) = \int_0^T [s_m(t) + n(t)]f_k(t)dt$
- $r_k(t) = s_m(t) + n_k(t)$
- $\int_0^T s_m(t)f_k(t)$  for the values of  $k = 1, 2, 3, 4, 5, \dots$
- $\int_0^T n(t)f_k(t)$  for the values of  $k = 1, 2, 3, 4, 5, \dots$

Now our signal has been represented by the  $s_m(t)$  and the  $n(k)$  (random variables as noise)

Now the signal can be represent  $r(t)$  as

- $r(t) = \sum_{k=1}^N s_m(t)f_k(t) + n(t)' + \sum_{k=1}^N n_k(t)f_k(t)$
- $\sum_{k=1}^N r_k(t)f_k(t)$

As we can see that  $n(t)'$  is irrelevant to which signal is going to transmitted. The decision will be made upon entirely on correlator output and the basis functions.

Now we can compare with Matched filter output.

For matched filter, we use  $N$  bank linear filters. Impulse responses of the  $N$  filters are;

- $h_k(t) = f_k(T - t)$  for intervals between ,  $0 < t < T$

The output filters become:

- $y_k(t) = \int_0^T r(\tau)f_k(T - \tau)d(\tau)$
- $\int_0^T r(\tau)f_k(T - t + \tau)d(\tau)$  for the  $k = 1, 2, 3, 4, 5, \dots N$

If we sample outputs of the linears at  $t = T$

- $y_k(T) = \int_0^T r(\tau)f_k(\tau)d(\tau)$  which will eventually become  $r_k$

Hence our theoretical approach has been proved by mathematical derivation of matched filter and cross-correlator. Both have same output even though calculations have been done in a different way for both.

## 4.6 Explain what the consequences are with respect to a matched-filter receiver if there is noise on the timing synchronisation in the receiver Part 3.7

Even if we manage to recover the timing, it does not guarantee that the correct operation of data-aided frequency estimation algorithms. The reason is that for is the presence of noise on the timing synchronisation in the receiver. Due to fact that for a frequency offset in order of  $1/T$  the signal will be severely distorted when it passed through the matched filter. Severely distorted matched filter will not able to maximise the signal to noise ratio in the presence of additive noise.

### 4.6.1 Example for the Part 3.7

For example, if our noise signal is delta, sampling the matched filter's output at some time  $T + \delta$  ( where represents a receiver timing offset due to introduction of noise) will significantly reduce the effective SNR seen by subsequent receiver blocks. This example that linked to theory above proves that it is important to keeping receiver timing offset close to zero as possible and thus delivers motivation for the inclusion of a timing recovery loop in the receiver.

## 4.7 Plots

## 4.8 Source Code

For the question 3.5 following code has been created to take relevant correlation coefficients

Listing 6: ../scripts/2/question35.m

```
% file: c9ce1
% Bit error probability for binary signaling;
% vector of correlation coefficients allowed
%
clf
R12 = input('Enter_vector_of_desired_R_1_2_values;_<=_8_values_')
);
A = char('-', '-.', ':', '--', '.', 'o', '*', 'x');
LR = length(R12);
z_dB = 0:.3:15;
z = db2pow(z_dB);
for k = 1:LR
    P_E=qfn(sqrt(z*(1-R12(k)))); % Probability of error for
    vector of z-values
```



```

                                % Plot probability of error
                                versus Eb/N0 in dB
semilogy(z_dB,P_E,A(k,:)),axis([0 15 10^(-6) 1]),xlabel('Eb/
N_0,dB'),ylabel('P_E'),...
if k==1
hold on; grid % Hold plot for plots for other values of R12
end
end
end
if LR == 1 % Plot legends for R12 values
legend(['R_1_2_=_',num2str(R12(1))])
elseif LR == 2
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
])
elseif LR == 3
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
],['R_1_2_=_',num2str(R12(3))])
elseif LR == 4
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
],['R_1_2_=_',num2str(R12(3))],['R_1_2_=_',num2str(R12(4))])
elseif LR == 5
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
],['R_1_2_=_',num2str(R12(3))],['R_1_2_=_',num2str(R12(4))],['R_
1_2_=_',num2str(R12(5))])
elseif LR == 6
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
],['R_1_2_=_',num2str(R12(3))],['R_1_2_=_',num2str(R12(4))],['R_
1_2_=_',num2str(R12(5))],['R_1_2_=_',num2str(R12(6))])
elseif LR == 7
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
],['R_1_2_=_',num2str(R12(3))],['R_1_2_=_',num2str(R12(4))],['R_
1_2_=_',num2str(R12(5))],['R_1_2_=_',num2str(R12(6))],['R_
1_2_=_',num2str(R12(7))])
elseif LR == 8
legend(['R_1_2_=_',num2str(R12(1))],['R_1_2_=_',num2str(R12(2))
],['R_1_2_=_',num2str(R12(3))],['R_1_2_=_',num2str(R12(4))],['R_
1_2_=_',num2str(R12(5))],['R_1_2_=_',num2str(R12(6))],['R_
1_2_=_',num2str(R12(7))],['R_1_2_=_',num2str(R12(8))])
end
end

```