The Q Function and Baseband Data Communication

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Contents

1 Eye Diagram for a Digital Communication Channel

1.1 Eye diagram

1.2 `c5ce2.m`: explanation

Here follows a thoroughly commented version of the provided `c5ce2.m` MATLAB script. The code below generates and plots the eye diagrams of four band-limited signals composed of random sequences of bits.

Listing 1: ../scripts/1/c5ce2.m

```
% clean figure and load signal package (only for Octave)
clf
pkg load signal

% simulation parameters:
%  - nr of symbols (must be divisible by 4)
%  - nr of samples per symbol
%  - filter cutoff values (normalized values)
nsym = 100;
nsamp = 50;
bw = [0.4 0.6 1 2];

% for each filter..
for k = 1:length(bw)
  % generate filter coefficients
  lambda = bw(k);
  [b,a] = butter(3,2*lambda/nsamp);

  l = nsym*nsamp;

  % Total sequence length
  y = zeros(1,l-nsamp+1);

  % Initalize random output vector with +1 and -1
  x = 2*round(rand(1,nsym))-1;

  % for each overlap..
  for i = 1:nsym
    % place symbols into vector y
    kk = (i-1)*nsamp+1;
    y(kk) = x(i);
  end
  % zero-order hold
  datavector = conv(y,ones(1,nsamp));
```

```
  % apply filter to complite sequence
  filtout = filter(b, a, datavector);

  % splice sequence into sub-sequences of 4 symbols
  datamatrix = reshape(filtout, 4*nsamp, nsym/4);

  % discart the first 6 sub-sequences
  datamatrix1 = datamatrix(:, 6:(nsym/4));

  % plot and format
  subplot(length(bw),1,k)
  plot(datamatrix1, 'k')
  ylabel('Amplitude')
  axis([0 200 -1.4 1.4])
  legend(['Bn␣=␣', num2str(lambda)])
  if k == 4
    xlabel('t/Tsamp')
  end
end
```

1.3 Channel model

1.4 c5ce2.m: different bandwidths

1.5 c5ce2.m: plots

This section will elaborate on the results and implications of the plots generated by the two scripts.

2 The Q function

2.1 Normal probability density function 2.1

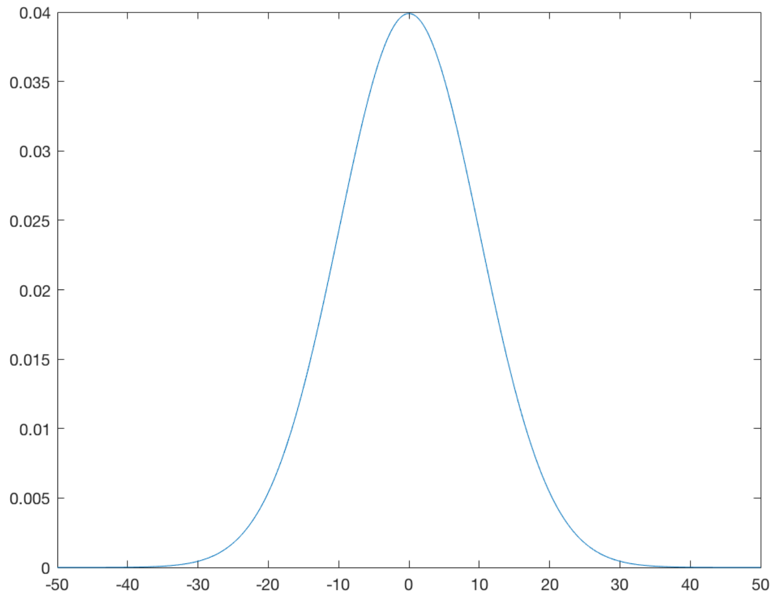

Figure 1: Normal Gaussian pdf graph with defined intervals

Normal gaussian distribution is one of the most important concept in Communication system and in statistics. It is used as a powerful tool when investigating random signals in communication systems. Such as investigating behaviour and application of noise signals.

The other reason is that because it is characterised by the limited variables. Such as mean μ and σ. It is easier compute and understand when we apply on communication systems.

All the variables that has been used in graphingpdf.m has been explained below;

mu . This is the mean value (μ) for the normal probability density function.

sigma This the sensible standard deviation number. (σ).

MAX 50; Maximum x value that x vector will get

MIN -50; Minimum x value that x vector will get

Also general formula for gaussian pdf is ; y=$f(x|\mu,\sigma) = (\frac{1}{\sigma\sqrt{2\pi}})e^{\frac{-(x-\mu)^2}{2(\mu)^2}}$ As we can see and understand from the variables above and the formula that all variables has been specified and we only need σ, μ and range.

2.2 Explanation of Q(u) function in relation to the normal probability density function 2.2

As we have explained and investigated probability density density function above . We can easily link $Q(u)$ function by exploiting the properties of cumulative distribution function and Probability density function.

- Q function is the 1- minus the cumulative distribution function of the standardised normal variable.
- Gaussian pdf with unit variance and zero mean is R=$(\frac{1}{\sqrt{2\pi}})e^{\frac{-(x)^2}{2}}$
- And corresponding cumulative distribution function become; $P = \int_{-\inf}^{x} Z$
- In last step Gaussian Q function defined as $Q(x) = 1 - P(x) = \int_{x}^{-\inf} Z$

2.3 Preparing a script file for plotting the $Q(u)$ function for argument values of relevance to the detection problems for digital communication receivers and inserting them in Appendix 2.3 and 2.4

Script file that has been created for the assignment 2.3 and 2.4 has been added to the Appendix. Important concept has been explained and relevant digital communication input values specified.

2.4 2.5

2.4.1 Complementary error function

2.5 Plots

Plotting for the questions 2.3 and 2.4

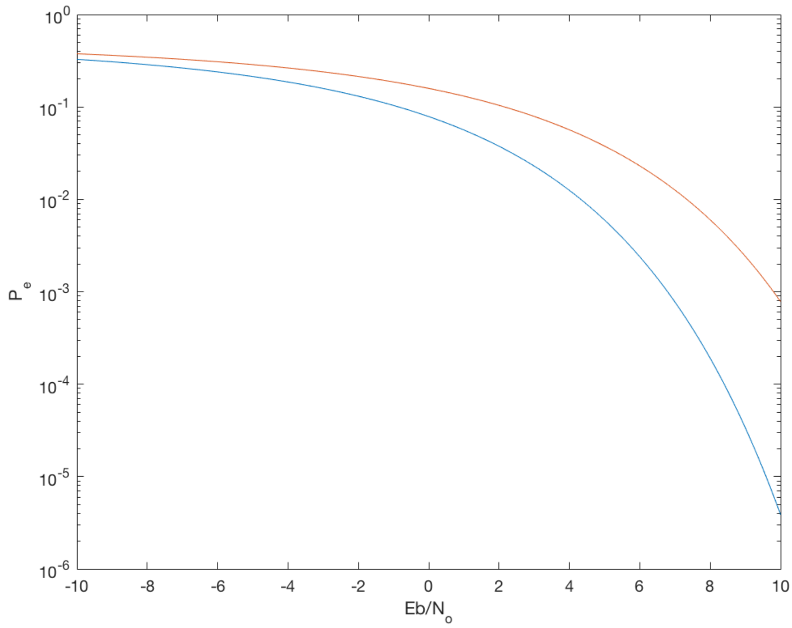

Figure 2: Plot for the $Q(u)$ function relevant to the argument values for the detection problems for digital communication receivers

3 Source Code

Here we have the properly prepared MATLAB codes for the second part of the second project. It has been used for observations, calculations and comparing with specified commands that given in this project.

The code belows computes and graphs normal(Gaussian) probability density function (pdf) in an appropriate intervals

Listing 2: ../scripts/2/graphingpdf.m

```
%graphing PDF function with random variables
mu=0;
sigma=10;
MAX = 50;
MIN = -50;
STEP = (MAX - MIN) / 1000;
PDF = normpdf(MIN:STEP:MAX, mu, sigma);
plot(MIN:STEP:MAX, PDF)
```

For the question 2.3 and 2.4 we have created the following code below. This code will plot the $Q(u)$ function for argument values relevant to the digital communication receivers. As we know from previous chapters that there are two common argument values :

- $R_12 = 0$ (Orthogonal Signals)
- $R_12 = -1$ (Antipodal Signals)

We also used z_db for the ratio for E_b/N_o. We should also notice that z_db is dimensionless ratio.

After that we have investigated the graph for further investigation to see whether we have achieved a satisfactory results for the assignment 2.3 and 2.4. Mathematical calculations are matching up with MATLAB simulation results.

Listing 3: ../scripts/2/qfunction.m

```
z_db = -10:.1:10;
r12 = [-1 0];
z = db2pow(z_db);
p_e(1,:) = qfunc(sqrt((1 - r12(1))*z));
p_e(2,:) = qfunc(sqrt((1 - r12(2))*z));
semilogy(z_db, p_e(1,:))
hold on
semilogy(z_db, p_e(2,:))
xlabel('Eb/N_o')
ylabel('P_e')
```

4 The Matched Filter Base Band Receiver

4.1 Additive white gaussian noise model

4.2 `c8ce1a.m`: explanation

4.3 Explain what the consequences are with respect to a matched-filter receiver if there is noise on the timing synchronisation in the receiver Part 3.7

Even if we manage to recover the timing, it does not guarantee that the correct operation of data-aided frequency estimation algorithms. The reason is that for is the presence of noise on the timing synchronisation in the receiver. Due to fact that for a frequency offset in order of $1/T$ the signal will be severely distorted when it passed through the matched filter. Severely distorted matched filter will not able to maximise the signal to noise ratio in the presence of additive noise.

4.3.1 Example for the Part 3.7

For example, if our noise signal is delta, sampling the matched filter?s output at some time $T+\delta$ (where represents a receiver timing offset due to introduction of noise) will significantly reduce the effective SNR seen by subsequent receiver blocks. This example that linked to theory above proves that it is important to keeping receiver timing offset close to zero as possible and thus delivers motivation for the inclusion of a timing recovery loop in the receiver.