Chapters 9.3-9

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Modulation Schemes not Requiring Coherent References

In this section, now we consider two modulation schemes that you do not need to require the acquisition of a local reference signal in phase coherence with the received carrier.

Differential Phase-Shift Keying (DPSK)

- The implementation of a such a scheme presupposes two things;
 - 1 The unknown phase perturbation on the signal varies slowly that the phase is constant from one signalling interval to next.
 - The phase during a given signalling interval bears a known relationship to the phase during the preceding signalling interval bears a known relationship to the phase during the preceding signalling interval.

Table 9.3 Differential Encoding Example	e
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Message sequence:		1	0	0	1	1	1	0	0	0
Encoded sequence:	1	1	0	1	1	1	1	0	1	0
Reference digit:	1									
Transmitted phase:	0	0	π	0	0	0	0	π	0	π

Differential Encoding Message Sequence

- An arbitrary reference binary digit is being selected as an initial digit of the sequence
- For each digit , the present digit used as a reference
- 0 in the message sequence is encoded as a transition from state of reference digit to the opposite state in the encoded message sequence
- 1 encoded as no change of state

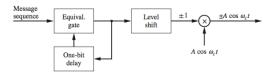
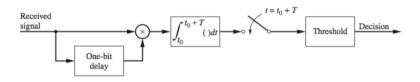


Figure 9.16 Block diagram of a DPSK modulator.

Figure for Differential Encoding Message Sequence

Table 9.4 Truth Table for the Equivalence Operation

Input 1 (Message)	Input 2 (Reference)	Output
0	0	1
0	1	0
1	0	0
1	1	1



Differential Encoding Message Sequence

- After the reference bit and plus the first encoded bit, signal input become $S_1 = A\cos(\omega_c)t$ and $R_1 = A*\cos(w_c)*t$
- Than the output correlator is; $v_1 = \int_0^T A^2 \cos^2(\omega_c t) dt$ which eventually become $\frac{1}{2}A^2T$
- The optimum detector for binary will become $I = x_{\nu}x_{\nu} 1 + y_{\nu}y_{\nu} 1$
- Without a loss of of generality, we can choose $\theta = 0$; we found outputs at t = 0 to be:
 - $x_0 = \frac{AT}{2} + n_1$ and $y_0 = n_3$ and where $n_1 = \int_{-T}^0 n(t) \cos^2(\omega_c t) dt$
 - $n_3 = \int_{-T}^0 n(t) \sin^2(\omega_c t) dt$. Similarly, at the time t = T, the outputs are; $x_1 = \frac{AT}{2} + n_2$ and $y_1 = n_4$
 - $n_2 = \int_0^T n(t) \cos^2(\omega_c t) dt$
 - $n_4 = \int_0^T n(t) \sin^2(\omega_c t) dt$



Important Figure for Differential Encoding

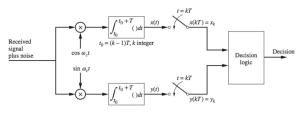


Figure 9.18
Optimum receiver for binary differential phase-shift keying.

If $\ell > 0$, the receiver chooses the signal sequence

$$s_1(t) = \begin{cases} A\cos(\omega_c t + \theta), & -T \le t < 0\\ A\cos(\omega_c t + \theta), & 0 \le t < T \end{cases}$$
(9.95)

as having been sent. If $\ell < 0$, the receiver chooses the signal sequence

$$s_2(t) = \begin{cases} A\cos(\omega_c t + \theta), & -T \le t < 0\\ -A\cos(\omega_c t + \theta), & 0 \le t < T \end{cases}$$
(9.96)

- It follows as n_1 , n_2 , n_3 and n_4 are uncorrelated and zero-mean Gaussian random variables with variances $\frac{N_0 T}{4}$ and they are independent.
- Expression for Probability error $P_E = Pr[(\frac{AT}{2} + n_1)(\frac{AT}{2} + n_2) + n_3 n_4 < 0]$
- We can define new gaussian random variables such as:

$$\omega_{1} = \frac{n_{1}}{2} + \frac{n_{2}}{2}$$

$$\omega_{2} = \frac{n_{1}}{2} - \frac{n_{2}}{2}$$

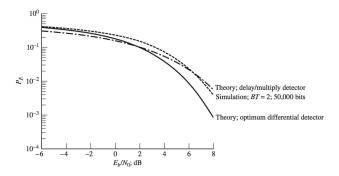
$$\omega_{3} = \frac{n_{3}}{2} + \frac{n_{4}}{2}$$

$$\omega_{4} = \frac{n_{3}}{2} - \frac{n_{4}}{2}$$

- Probability can be written in terms of Gaussian variables: $P_F = Pr[(\frac{AT}{2} + \omega_1)^2 + (\omega_3)^2 < (\omega_2^2 + \omega_4^2)]$
- Gaussian variables will also let us define the Ricean random variables. Ricean random variable will become: $R_1 = \sqrt{(\frac{AT}{2} + \omega_1)^2 + \omega_3^2}$
- Also Rayleigh random variable will become $R_2 = \sqrt{\frac{\omega_2^2}{\omega_4^2}}$
- If we also define the bit energy E_b as $A^2 \frac{A^2 T}{2}$ will give; $P_E = \frac{1}{2} e^{(\frac{-E_b}{N_0})}$ for the optimum DPSK receiver.
- \blacksquare At the large values $\frac{-E_b}{N_0}$ values of ; $P_E=Q[\sqrt{\frac{-E_b}{N_0}}]=Q[\sqrt{z}]$



■ Following result obtained by using the asymptotic approximation; $P_E = \frac{e^{(-E_b/N_0)}}{2\sqrt{\pi \frac{E_b}{N_c}}}$



Comparison of Digital Modulation Systems



Multipath Interference



Equalization



Equalization by Zero Forcing



Equalization by Minimum Mean-Squared Error



Tap Weight Ajustment (LMS Algorithm)

