Exam presentation
Assignment 2.2 and 2.3

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Eye diagram

- Plot composed by overlaying segments of different bit sequences
- Can be generated with an oscilloscope
- Shows effects of *inter-symbol interference*
- Provides a qualitative measure of the system performance
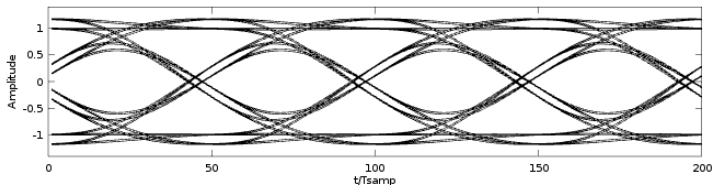
Eye diagram characteristics

Figure: *Eye diagram of baseband antipodal signal*

A Difference between high and low levels

A_j Difference between A and the eye opening

T_j Deviations from ideal timing

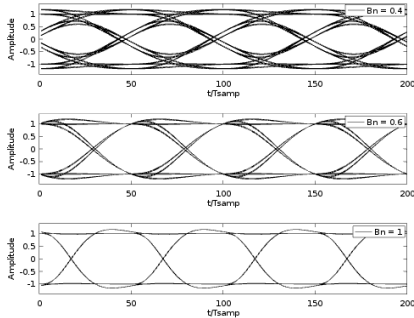T_b Bit time period

Eye diagram at different bandwidths

Figure: *Eye diagram for normalized bandwidths 0.3, 0.7, 1.2*

- Low BW: high amplitude and timing jitter
- High BW: no ISI, chances of higher noise

Q function 2.2

In this part of the assignment we will investigate the Normal (Gaussian) probability density function, $Q(u)$ function and it's relationship with complementary error function. It will also show how these theories will be related to the current communication systems by given assignment questions. Things we will look at are:

- Normal(Gaussian) Probability Density Function
- $Q(u)$ function
- $Q(u)$ function and it's relationship with complementary error function.

Probability Density Function

Normal distribution/Gaussian distribution is a really important and in fact most commonly used distribution in statistics. It is important because:

- Almost all variables are distributed approximately normally. They are approximately close
- Second reason is that statistical tests are derived from normal distribution and also work well if the distribution is approximately normal.
- Another reason is that it is only just characterised by two variables;
    - It's mean μ and and standard deviation σ

For the communication systems; Noise is an error or undesired random disturbance of a useful information in communication channel. The noise is a summation of unwanted or disturbing energy from natural and sometimes man-made sources.

Gaussian Noise

If we look at simple basic model for the net effect at the receiver of noise in the communication system is to assumed additive, Gaussian noise. In this model we have two signal components one is deterministic signal and the second component is the noise term, and is a quantity drawn from a Gaussian probability distribution with mean 0 and some variance and it is independent of the transmitted signal.

Q2.1 Plotting gaussian pdf and explain important variables

Now we can put our theory in a practice in this given question, we have created the MATLAB file named graphingpdf.m which can be accessible from the report file. In this assignment important variables are

mu . This is the mean value (μ) for the normal probability density function.

sigma This the sensible standard deviation number. (σ).

MAX 50; Maximum x value that x vector will get

MIN -50; Minimum x value that x vector will get
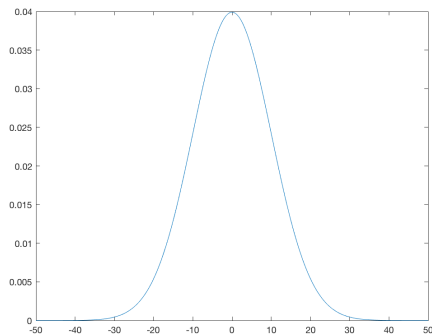
Graph for Gaussian PDF

Figure: *Normal Gaussian pdf graph with defined intervals*

Explanation of $Q(u)$ function

$Q(u)$ function is a important variable for the Gaussian probability density function and cumulative distribution function. If we talk about $Q(u)$ function, there are two important points before we are going to derive the $Q(u)$ function.

The $Q(u)$ function represents the area under the tail of a standard normal random variable and widely tabulated. Some interesting properties of the $Q - function$ are

- $Q(0) = 1/2$
- $Q(-\infty) = 1$

Linking $Q(u)$ function to Gaussian probability density function and cumulative distribution function

Now we can link $Q(u)$ function to Gaussian probability density function and cumulative distribution function

- The Gaussian probability density function of unit variance and zero mean is $Z(x) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-x^2}{4t})$
- And corresponding cumulative distribution function is $P(x) = \int_{-\infty}^{x} Z(t)dt$
- The Gaussian Q function is defined as:
    - $Q(x) = 1 - P(x) = \int_{x}^{-\infty} Z(t)dt$

Assignment related question for the $Q(u)$ function

To prove our theory behind, we have constructed the $Q(u)$ function plot that will able to take defined argument values of relevance to the detection problems for digital communication receivers. MATLAB filed called `qfunction.m` has been created for this assigment. For the next step Two important argument has been chosen for this assignment, those are:

- $R_1 2 = 0 (Orthogonal Signals)$
- $R_1 2 = -1 (Antipodal Signals)$
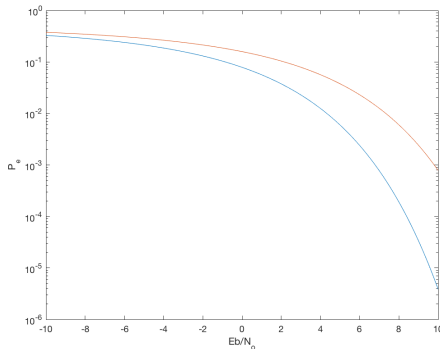
Graph for $Q(u)$ function assignment

Figure: *Plotting $Q(u)$ function relevant to detection problems in digital communication receivers*

Relationship between inverse $Q(u)$ function and Complementary Error Function

We know that $Q(u)$ function is :

- $Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} \exp(\frac{-y^2}{2}) dy$

And we have also learnt that complementary error function(erfc) is:

- $erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty \exp(-x^2) dx$

From the limits of the integrals in previously defined $Q(z)$ function and $erfc(z)$ function that we can conclude that Q function is directly related to erfc. Mathmetically by combining Q function and $erfc$ we get the following Q function that directly related to erfc:

- $Q(z) = \frac{1}{2} erfc(\frac{z}{\sqrt{(2)}})$

MATLAB code for user-defined Q function

We also note that MATLAB does not have built in function.So we have created a MATLAB file that uses erfc function to do the calculations and simulation process related to the $Q(u)$ function. It is accessible from the Source code section under the name of qfn.m.

Assignment 2.3

In assignment three, we will look into optimising our filter by using the Q function and error functions that we have learnt from the part 2 of this Assignment. The important things we will specifically look in these chapter are:

- Additive White Gaussian Noise for Matched Filter
- Matched Filter
- Correlator Filter
- Noise and Shape related problems in Matched Filter

Additive White Gaussian Noise for Matched Filter

For a working principle, The signal is multiplied by a locally stored reference copy ,and integrated over time. To understand the principles of a matched filter receiver for binary data in white Gaussian noise, thus using the so called AWGN (Additive White Gaussian Noise) model. We have firstly introduced the figure below.
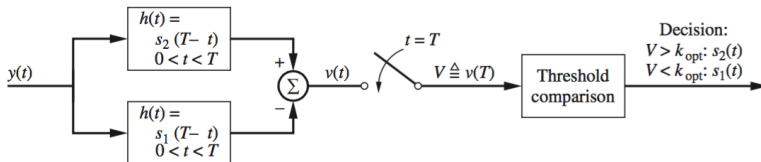

Figure: *AWGN for Matched Filter*

Principles of AWGN for Matched Filter

The matched filter correlates the incoming signal with a locally stored reference copy of the transmit waveform. The matched filter maximizes the signal-to-noise ratio for a known signal. It can be shown to be the optimal detector if

- The channel produces Additive White Gaussian Noise (AWGN)
- The channel is linear and time-invariant (LTI)
- An exact time reference is available, the signal amplitude as a function of time is precisely known.

In our report, we have explicitly explained math behind how to apply these principles to achieve maxisimed signal to noise ratio.

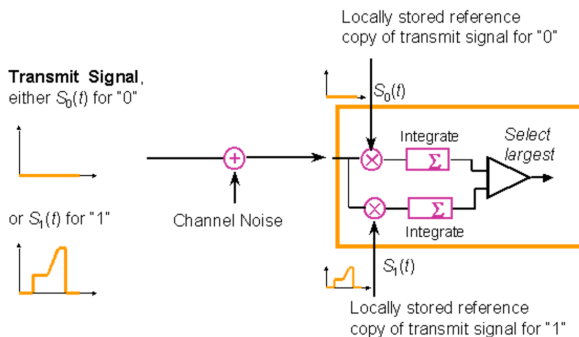# Another example on how system works for AWGN Matched Filter

Figure: *Working Principle of AWGN Matched Filter*

Assignment Questions that related to the Matched Filter

There are several questions that has been done in regards to the Matched Filter. The first question was understanding how `c8ce1` works as code.

Firstly, code was not working properly so we have changed it and there were not enough explanation for the process and what it does.

Basically code was calculating delay estimate, power in both reference vector and measurement vector, finding max for the cross-correlation matrix by using sine waves. It has both reference signal and measurement signal. We also note that measurement signal has slightly higher frequency component. For more detailed explanation and processed results can be done by running code with different given reference signal and measurement signal. It is accessible from the report.

Creating a User Defined Q function

To run our code for assignments 3.4 and 3.5 we have created a user defined qfn function. The code has been created as a `qfn.m` which can be accessible from the report.

Question 3.4 and 3.5

For the questions 3.4 and 3.5 we have modified and changed the
`ce9e1.m` MATLAB script. Code was broken and it was only taken
4 inputs. We have modified in a way that it can take up 8 inputs
also we have graphed with given 8 correlation coefficients. Those
were; $[-1, -0.75, -0.5, 0, 0.5, 0.75, 0.8, 0.995]$. To prove that we
did correct calculation and simulation graph in the next slight can
be seen.

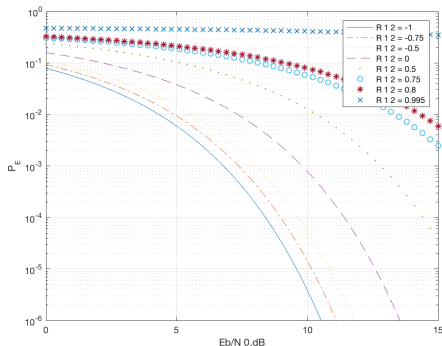# Graph with 8 Correlation Coefficient Inputs

Figure: P_E over N_o graph for 8 correlation coefficient Inputs

Construction of Matched filter by a Correlator

To understand this concept, we have provided a Correlator Match Filter below. The cross-correlator does the cross-correlation between the noisy signal and noiseless signal.
The difference between Correlator and matched filter, matched filter does the convolution between the received signal and the time-reversed copy of the the reference signal but important point is both of them will give us same output.
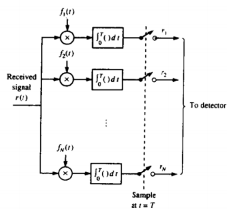

Figure: Correlator matched Filter

Cross-Correlation Functions

- $\int_0^T r(t)f_k(t) = \int_0^T [s_m(t) + n(t)]f_k(t)dt$
- $r_k(t) = s_m(t) + n_k(t)$
- $\int_0^T s_m(t)f_k(t)$ for the values of $k = 1, 2, 3, 4, 5, ...$
- $\int_0^T n(t)f_k(t)$ for the values of $k = 1, 2, 3, 4, 5.....$

Now our signal has been represented by the $s_m(t)$ and the $n(k)$ (random variables as noise)

Now the signal can be represent $r(t)$ as

- $r(t) = \sum\limits_{k=1}^{N} = s_m(t)f_k(t) + n(t)' + \sum\limits_{k=1}^{N} = n_k(t)f_k(t)$
- $\sum\limits_{k=1}^{N} = r_k(t)f_k(t)$

As we can see that $n(t)'$ is irrelevant to which signal is going to transmitted. The decision will be made upon entirely on correlator output and the basis functions.

Deriving Matched Filter Functions

Now we can compare with Matched filter output. For matched filter, we use N bank linear filters. Impulse responses of the N filters are;

- $h_k(t) = f_k(T - t)$ for invertals between , $0 < t < T$

The output filters become:

- $y_k(t) = \int_0^T r(\tau) f_k(T - \tau) d(\tau)$
- $\int_0^T r(\tau) f_k(T - t + \tau) d(\tau)$ for the $k = 1, 2, 3, 4, 5, ...N$

If we sample outputs of the linears at $t = T$

- $y_k(T) = \int_0^T r(\tau) f_k(\tau) d(\tau)$ which will eventually become r_k

Hence our theoritical approach has been proved by mathematical derivation of matched filter and cross-correlator. Both have same output even though calculations have been done in a different way for both.

Noise on the timing Synchronization in The Receiver

Even if we manage to recover the timing, it does not guarantee
that the correct operation of data-aided frequency estimation
algorithms. The reason is that for is the presence of noise on the
timing synchronisation in the receiver. Due to fact that for a
frequency offset in order of $1/T$ the signal will be severely distorted
when it passed through the matched filter. Severely distorted
matched filter will not able to maximise the signal to noise ratio in
the presence of additive noise.

BER does not depend signal waveshape

A filter is said to be matched to signal if it's impulse responses is the time-reversed replica of that signal. This is equivalent saying that the frequency response of the the signal is the same as the amplitude response of the signal which is matched. Output is the same as the auto-correlation function of the input signal.

So if we combine this theory with the output S/N ratio, we can see that output of the matched filter does not depend on the waveshape, but following parameter:

- E/N_o

And we know that BER is linked to S/N ratio and Q function and formula is;

- $BER = Q(\sqrt{(\frac{d^2}{2N_o})})$

Which proves the idea of that BER does not depend on signal waveshape.

Link budget model

- A way of estimating the power of a received signal
- Takes into account all the gains and losses of transmitter, channel, and receiver

$$P_R = \left(\frac{\lambda}{4\pi d}\right)^2 \frac{P_T G_T G_R}{L_0}; \qquad (1)$$

- In decibels:

$$P_{R,dB} = 20 \log_{10}\left(\frac{4\pi d}{\lambda}\right) + ERP_{dB} + G_{R,dB} - L_{0,dB}; \qquad (2)$$

$(\frac{4\pi d}{\lambda})^2$ Free-space loss

$ERP = P_T G_T$ Effective radiated power

G_R Gain of receiver antenna

L_0 Other losses, link budget margin

SNR calculation

- Signal-to-noise ratio in decibels:

$$SNR_{dB} = P_{R,dB} - P_{int,dB} \qquad (3)$$

P_R Calculated using link budget model

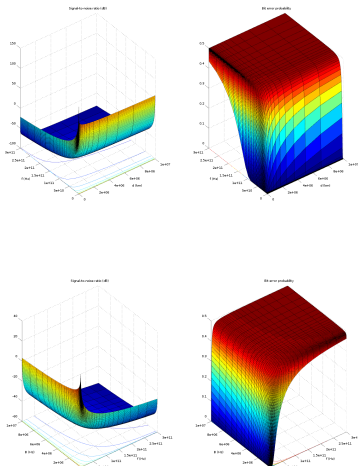P_{int} Noise power, proportional to the receiver noise temperature and the transmission bandwidth

P_E calculation

- Bit error probability for BSFK transmissions:

$$P_E = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \qquad (4)$$

1. ratio E_b/N_0 derived from $SNR = \rightarrow \frac{E_b}{N_0 B T_b}$
2. For binary BPSK, $B = 2/T_b$
3. Factor BT_b is 2, or 3 dB

Impact of d, λ, B

- SNR and P_E are negatively affected by:
  - Higher distance d
  - Lower wavelength λ
  - Wider bandwidth B (lower influence)

Impact of P_T

- P_E for P_T at 50 W, 5 W, and 500 mW:

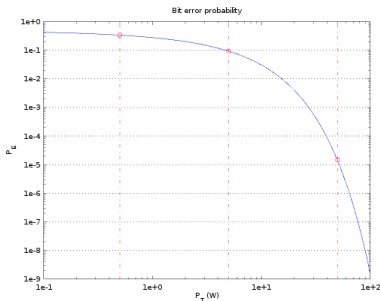$$1.4062\text{e-}05 \quad 9.2684\text{e-}02 \quad 3.3768\text{e-}01 \tag{5}$$

Figure: P_E *over values of* P_T

Alternative modulation: ASK
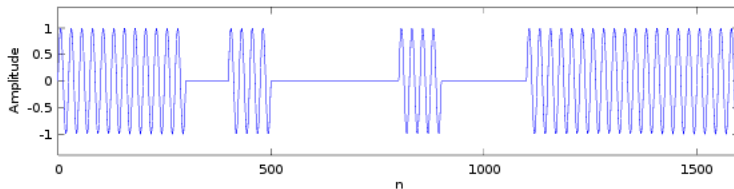
- Amplitude-shift keying

Figure: *Bit stream modulated using ASK*

- 0-bit represented as 0
- 1-bit represented as $A\cos(2\pi f_c t)$

Alternative modulation: ASK (P_E calculation)

- Correlation coefficients:
    - $\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_{-\infty}^{\infty} s_1(t) s_2(t) dt = 0$
    - $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$
- SNR to E_b/N_0: conversion factor $BT_b = 2$
- Bit error probability:

$$P_E = Q\left(\sqrt{(1 - R_{12})\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad (6)$$

Alternative modulation: FSK

- Frequency-shift keying

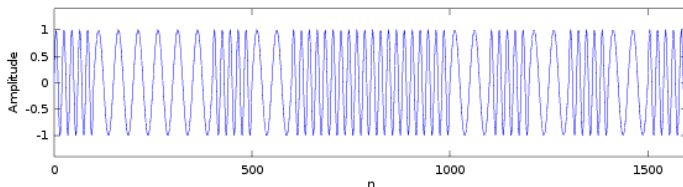

Figure: *Bit stream modulated using FSK*

- 0-bit represented as $A\cos(\omega_c t)$
- 1-bit represented as $A\cos((\omega_c + \Delta\omega)t)$
- Assumptions: $\omega_c = \frac{2\pi n}{T}$ and $\Delta\omega = \frac{2\pi m}{T}$

Alternative modulation: FSK (P_E calculation)

- Correlation coefficient $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$
- SNR to E_b/N_0: conversion factor $BT_b = 2.5$
- Bit error probability:

$$P_E = Q\left(\sqrt{(1 - R_{12}) \frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{7}$$