Exam presentation

Assignment 2.2 and 2.3

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Q function 2.2

In this part of the assignment we will investigate the Normal (Gaussian) probability density function, $Q(u)$ function and it's relationship with complementary error function. It will also show how these theories will be related to the current communication systems by given assignment questions. Things we will look at are:

- Normal(Gaussian) Probability Density Function
- $Q(u)$ function
- $Q(u)$ function and it's relationship with complementary error function.

Probability Density Function

Normal distribution/Gaussian distribution is a really important and in fact most commonly used distribution in statistics. It is important because:

- Almost all variables are distributed approximately normally. They are approximately close
- Second reason is that statistical tests are derived from normal distribution and also work well if the distribution is approximately normal.
- Another reason is that it is only just characterised by two variables;
  - It's mean μ and and standard deviation σ

For the communication systems; Noise is an error or undesired random disturbance of a useful information in communication channel.The noise is a summation of unwanted or disturbing energy from natural and sometimes man-made sources.

Gaussian Noise

If we look at simple basic model for the net effect at the receiver of noise in the communication system is to assumed additive, Gaussian noise. In this model we have two signal components one is deterministic signal and the second component is the noise term, and is a quantity drawn from a Gaussian probability distribution with mean 0 and some variance and it is independent of the transmitted signal.

Q2.1 Plotting gaussian pdf and explain important variables

Now we can put our theory in a practice in this given question, we have created the MATLAB file named "`graphingpdf.m` which can be accessible from the report file. In this assignment important variables are

- **mu** . This is the mean value (μ) for the normal probability density function.
- **sigma** This the sensible standard deviation number. (σ).
- **MAX** 50; Maximum x value that x vector will get
- **MIN** -50; Minimum x value that x vector will get

Graph for Gaussian PDF

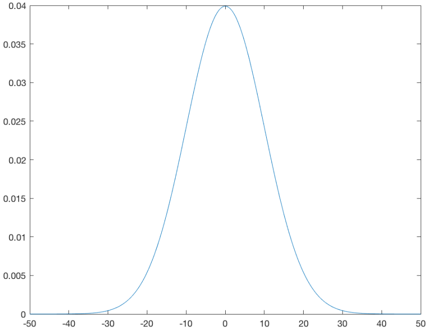

Figure: Normal Gaussian pdf graph with defined intervals

Explanation of $Q(u)$ function

$Q(u)$ function is a important variable for the Gaussian probability density function and cumulative distribution function. If we talk about $Q(u)$ function, there are two important points before we are going to derive the $Q(u)$ function.
The $Q(u)$ function represents the area under the tail of a standard normal random variable and widely tabulated. Some interesting properties of the $Q - function$ are

- $Q(0) = 1/2$
- $Q(-\infty) = 1$

Linking $Q(u)$ function to Gaussian probability density function and cumulative distribution function

Now we can link $Q(u)$ function to Gaussian probability density function and cumulative distribution function

- The Gaussian probability density function of unit variance and zero mean is $Z(x) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-x^2}{4t})$
- And corresponding cumulative distribution function is $P(x) = \int_{-\infty}^{x} Z(t)dt$
- The Gaussian Q function is defined as:
  - $Q(x) = 1 - P(x) = \int_{x}^{-\infty} Z(t)dt$

Assignment related question for the $Q(u)$ function

To prove our theory behind, we have constructed the $Q(u)$ function plot that will able to take defined argument values of relevance to the detection problems for digital communication receivers. MATLAB filed called `qfunction.m` has been created for this assigment. For the next step Two important argument has been chosen for this assignment, those are:

- $R_1 2 = 0 (Orthogonal Signals)$
- $R_1 2 = -1 (Antipodal Signals)$

Graph for $Q(u)$ function assignment

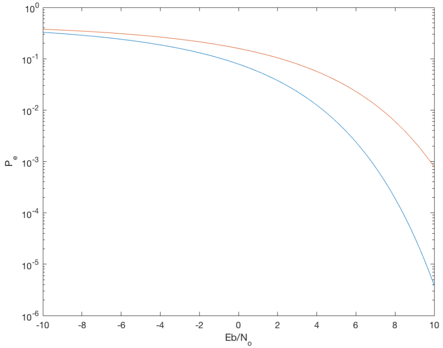

Figure: Plotting $Q(u)$ function relevant to detection problems in digital communication receivers

Relationship between inverse $Q(u)$ function and Complementary Error Function

We know that $Q(u)$ function is :

- $Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} \exp(\frac{-y^2}{2}) dy$

And we have also learnt that complementary error function(erfc) is:

- $erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty \exp(-x^2) dx$

From the limits of the integrals in previously defined $Q(z)$ function and $erfc(z)$ function that we can conclude that Q function is directly related to erfc. Mathmetically by combining Q function and $erfc$ we get the following Q function that directly related to erfc:

- $Q(z) = \frac{1}{2} erfc(\frac{z}{\sqrt{(2)}})$

MATLAB code for user-defined Q function

We also note that MATLAB does not have built in function.So we have created a MATLAB file that uses erfc function to do the calculations and simulation process related to the $Q(u)$ function. It is accesible from the Source code section under the name of `qfn.m`.

Assignment 2.3

In assignment three, we will look into optimising our filter by using the Q function and error functions that we have learnt from the part 2 of this Assignment. The important things we will specifically look in these chapter are:

- Additive White Gaussian Noise
- Matched Filter
- Correlator Filter
- Noise and Shape related problems in Matched Filter