Principles of Digital Data Transmission in Noise

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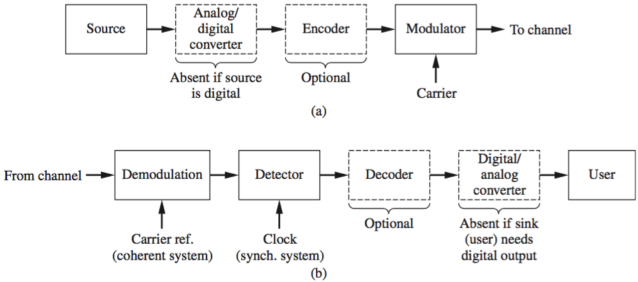
Principles of Digital Data Transmission in Noise

- In this chapter, we are concerned with the transmission of information from sources that produce discrete-valued symbols.
- Throughout this chapter, we will make the assumption that source symbols occur with equal probability. Many discrete-time sources naturally produces symbols with equal probability.

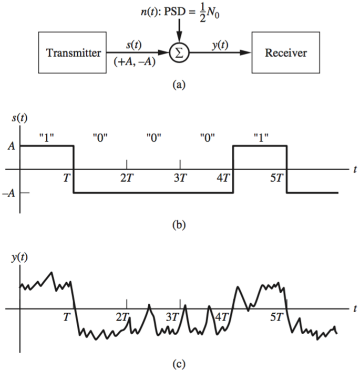
Block Diagram of Digital Data Tranmission System



- Whether a source is purely digital or analog that converted to digital , it may be advantegous to add or remove redundant digits to the digital signal. This process referred as forward error-correction coding
- We can see from the figlure that modulator input take on one of only two possible values. This system will referred as "binary". If it takes M ¿ 2 possible values, system will be referred as M-ary.
- Also system will be referred as "coherent" if a local reference is available for demodulation that in phase with the transmitter carrier. Otherwise it will be called "noncoherent".
- Also if the system has a periodic signal and synching with transmitted sequence of digital signals than system will be synchronous if not, system will be called asynchronous.

Baseband Data Tranmission In White Gaussian Noise

- During data transmission, receiver is to decide whether the transmitted signal was A or -A during each bit period.
- Practical way of determining this is to pass the signal-pulse noise through a lowpass predectition filter. If the sample greater than zero than A was transmitted if not , -A was transmitted.



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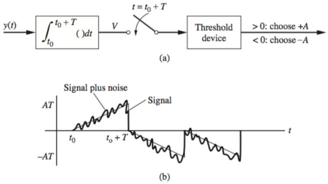
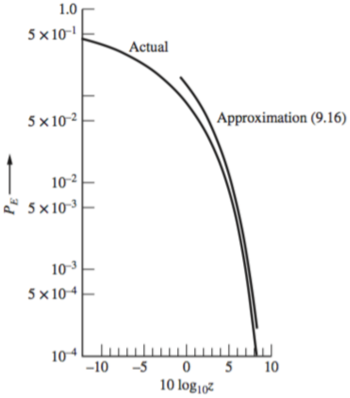


How well does this receiver will perform ?

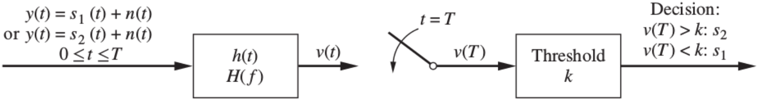
- As we discussed before ,useful criterion of performance is probability of error.
- Probability of error through approximation is: $P_E = Q * \sqrt{2 * A^2 * T/N_o}$
- Our important parameters are; $A^2 * T/N_0 = z$
- E_b is called the energy per bit that carries one bit of information.
- We also now that rectangular pulse of duration T seconds ahas amplitude spectrum $ATsincTf$ and that $B_p = 1/T$ is a rough measure of it's bandwidth. Thus our calculation will become: $E_b/N_o = A^2/N_o * (1/T) = A^2/(N_0 * B_p)$. This can be interpreted as the ratio of signal power to noise power in the signal bandwidth.

Plot of of P_E versus z



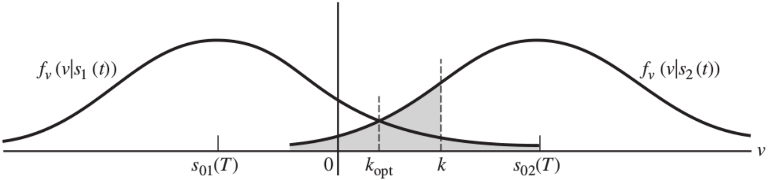
Binary data with arbitrary signal shapes - bit detection and noise

- Bit values represented by arbitrary signals: $s_1(t)$ and $s_2(t)$
- Received signal $y(t)$ can be either $s_1(t) + n(t)$ or $s_2(t) + n(t)$
- Signal goes into filter $H(f)$, output is $v(t) = n_0(t) + s_{01}(t) or s_{02}(t)$
- If $v(T) > k$, $s_2(t)$ was sent, otherwise $s_1(t)$ was sent
- Error occurs if:
  - $s_1(t)$ was sent but $v(T) = n_0(T) + s_{01}(T) > k$
  - $s_2(t)$ was sent but $v(T) = n_0(T) + s_{02}(T) < k$



Binary data with arbitrary signal shapes - error probability

- $N = n_0(T)$ is a stationary gaussian random variable with:
  - $\mu = 0$
  - $\sigma^2 = \int_{-\infty}^{\infty} \frac{1}{2} N_0 |H(f)|^2 df$
- Sampler output is also a stationary gaussian random variable:
  - $\mu = s_{01}(t)$ or $\mu = s_{02}(t)$
  - σ^2 same as N
- Probability of error given $s_1(t)$: $\int_k^{\infty} \frac{e^{-(v - s_{01}(T))^2/2\sigma_0^2}}{\sqrt{2\pi\sigma_0^2}} dv$
- Average probability of error: $P_E = \frac{1}{2} P(E|s_1(t)) + \frac{1}{2} P(E|s_2(t))$



Binary data with arbitrary signal shapes - error probability 2

- Probability of error can be reduced by choosing optimal value of k
- If symbols are equally likely, the optimal value of k lies at the interception of the probability distribution functions, i.e. $k_{opt} = \frac{s_{01}(T)+s_{02}(T)}{2}$
- The average probability of error becomes: $P_E = Q(\frac{s_{01}(T)+s_{02}(T)}{2\sigma_0})$ where $Q(x) = 1 - \Phi(x)$
- P_E decreases when the distances between $s_{01}(T)$ and $s_{02}(T)$ increases