

Exam presentation

Assignment 2.1 and 3

Riccardo Miccini¹

¹Technical University of Denmark
Digital Communication

December 16, 2016

Overview

- Assignment 2
 - Eye diagrams
 - Q function
 - Matched filter
- Assignment 3
 - Link budget model
 - SNR and P_E
 - Alternative modulations

Eye diagram

- Characteristics
- Impact of bandwidth

Eye diagram

- Plot composed by overlaying segments of different bit sequences
- Can be generated with an oscilloscope
- Shows effects of *inter-symbol interference*
- Provides a qualitative measure of the system performance

Eye diagram characteristics

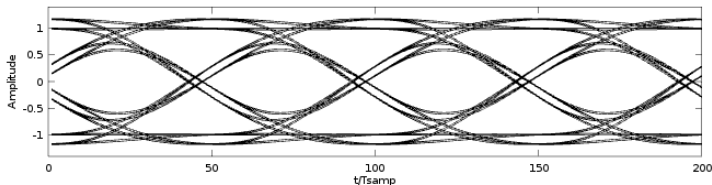


Figure: Eye diagram of baseband antipodal signal

- A Difference between high and low levels
- A_j Difference between A and the eye opening
- T_j Deviations from ideal timing
- T_b Bit time period

Eye diagram at different bandwidths

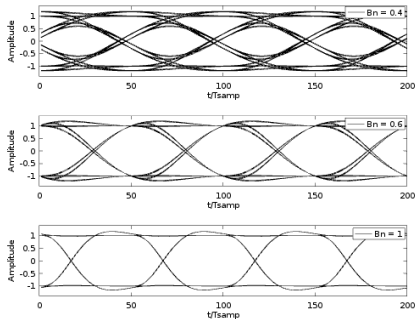


Figure: Eye diagram for normalized bandwidths 0.3, 0.7, 1.2

- Low BW: high amplitude and timing jitter
- High BW: no ISI, chances of higher noise

Q function

- Normal *pdf*
- Q function in relation to normal *pdf*
- Q function in relation to complementary error function
- Inverse Q function

Normal probability density function

- Continuous function representing likelihood of its argument
- Useful to analyze phenomena of unknown distribution due to *central limit theorem*
- General form:

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

μ Average value

σ Standard deviation

Q function

- Represents the tail probability of $\varphi(x)$ (standard normal distribution)
- Definition of $Q(x)$ through $\varphi(x)$:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du \quad (2)$$

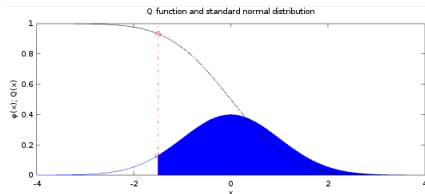


Figure: Relation between Q function and standard normal distribution

Q function and complementary error function

- Error function $\text{erf}(x)$: probability of normally-distributed random variable X ($\mu = 0$, $\sigma^2 = \frac{1}{2}$) to be in the range $[-x, x]$
- Complementary error function $\text{erfc}(x) = 1 - \text{erf}(x)$
- Definition of $Q(x)$ through $\text{erfc}(x)$:

$$Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad (3)$$

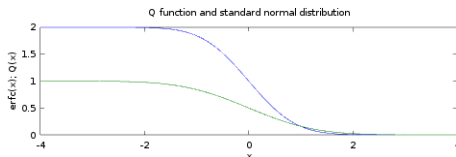


Figure: Q function (green) and complementary error function (blue)

Inverse Q function

- $Q^{-1}(x)$ is the value u for which $Q(u) = x$

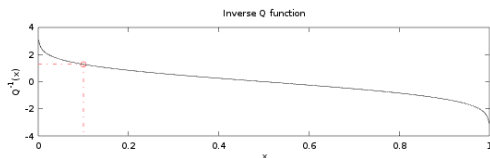


Figure: Inverse Q function

- If the argument x represents a bit error probability, $Q^{-1}(x)$ is proportional to the SNR

Matched filter



Link budget for satellite communication

- Link budget model
- SNR calculation
- P_E calculation
- Impact of d , λ , B , and P_T
- Alternative modulation techniques: ASK and FSK

Link budget model

- A way of estimating the power of a received signal
- Takes into account all the gains and losses of transmitter, channel, and receiver

$$P_R = \left(\frac{\lambda}{4\pi d} \right)^2 \frac{P_T G_T G_R}{L_0}; \quad (4)$$

- In decibels:

$$P_{R,dB} = 20 \log_{10} \left(\frac{4\pi d}{\lambda} \right) + ERP_{dB} + G_{R,dB} - L_{0,dB}; \quad (5)$$

Link budget model variables

$(\frac{4\pi d}{\lambda})^2$ Free-space loss

$ERP = P_T G_T$ Effective radiated power

G_R Gain of receiver antenna

L_0 Other losses, link budget margin

SNR calculation

- Signal-to-noise ratio in decibels:

$$SNR_{dB} = P_{R,dB} - P_{int,dB} \quad (6)$$

P_R Calculated using link budget model

P_{int} Noise power, proportional to the receiver noise temperature and the transmission bandwidth

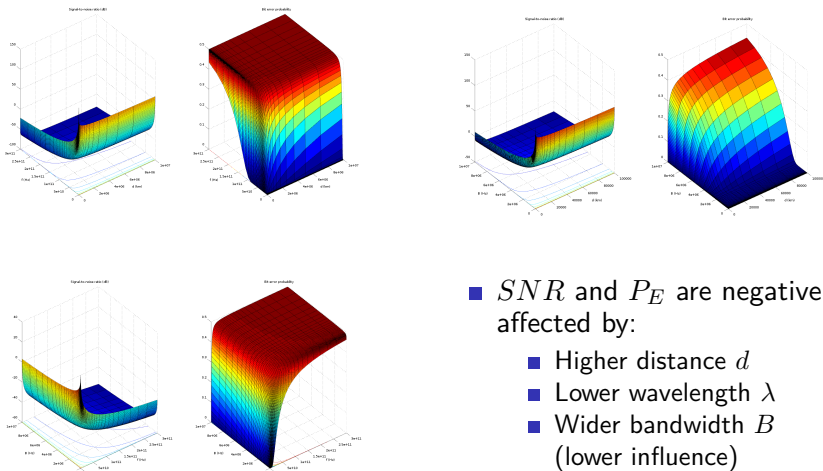
P_E calculation

- Bit error probability for BSFK transmissions:

$$P_E = Q \left(\sqrt{\frac{2E_b}{N_0}} \right) \quad (7)$$

- 1 ratio E_b/N_0 derived from $SNR \Rightarrow \frac{E_b}{N_0 BT_b}$
- 2 For binary BPSK, $B = 2/T_b$
- 3 Factor BT_b is 2, or 3 dB

Impact of d , λ , B



■ SNR and P_E are negatively affected by:

- Higher distance d
- Lower wavelength λ
- Wider bandwidth B (lower influence)

Impact of P_T

- P_E for P_T at 50 W, 5 W, and 500 mW:

$$1.4062\text{e-}05 \quad 9.2684\text{e-}02 \quad 3.3768\text{e-}01 \quad (8)$$

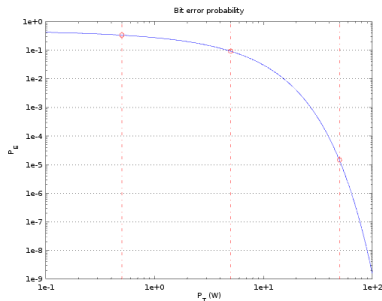


Figure: P_E over values of P_T

Alternative modulation: ASK

■ Amplitude-shift keying

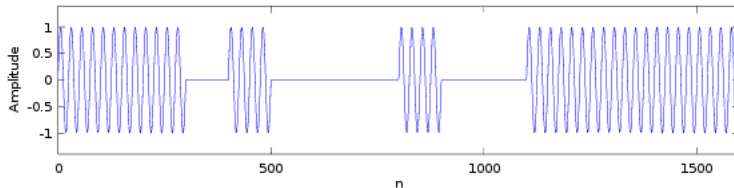


Figure: *Bit stream modulated using ASK*

- 0-bit represented as 0
- 1-bit represented as $A \cos(2\pi f_c t)$

Alternative modulation: ASK (P_E calculation)

- Correlation coefficients:

- $\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_{-\infty}^{\infty} s_1(t) s_2(t) dt = 0$

- $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$

- SNR to E_b/N_0 : conversion factor $BT_b = 2$

- Bit error probability:

$$P_E = Q \left(\sqrt{(1 - R_{12}) \frac{E_b}{N_0}} \right) = Q \left(\sqrt{\frac{E_b}{N_0}} \right) \quad (9)$$

Alternative modulation: FSK (P_E calculation)

- Correlation coefficient $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$
- SNR to E_b/N_0 : conversion factor $BT_b = 2.5$
- Bit error probability:

$$P_E = Q \left(\sqrt{(1 - R_{12}) \frac{E_b}{N_0}} \right) = Q \left(\sqrt{\frac{E_b}{N_0}} \right) \quad (10)$$