Exam presentation

Assignments 2 and 3

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Overview

- Assignment 2
  - Eye diagrams
  - Q function
  - Matched filter
- Assignment 3
  - Link budget model
  - SNR and P_E
  - Alternative modulations

2.1 - Eye diagram

- Characteristics
- Impact of bandwidth

Eye diagram

- Plot composed by overlaying segments of different bit sequences
- Can be generated with an oscilloscope
- Shows effects of *inter-symbol interference*
- Provides a qualitative measure of the system performance

Eye diagram characteristics

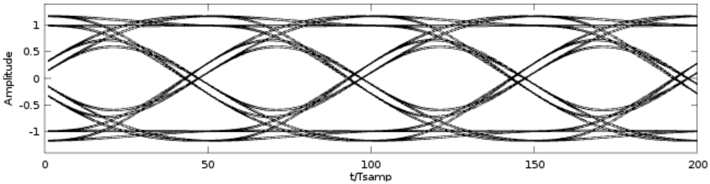


Figure: *Eye diagram of baseband antipodal signal*

- A Difference between high and low levels
- A_j Difference between A and the eye opening
- T_b Bit time period
- T_j Deviations from ideal timing

Eye diagram at different bandwidths

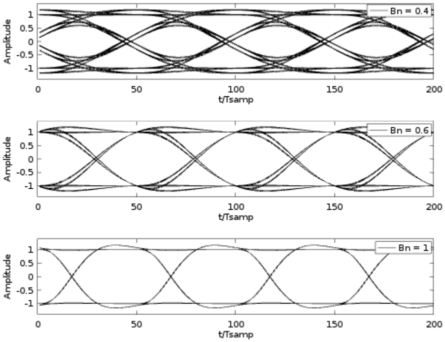


Figure: *Eye diagram for normalized bandwidths 0.3, 0.7, 1.2*

- Low BW: high amplitude and timing jitter
- High BW: no ISI, chances of higher noise

2.2 - Q function

- Normal *pdf*
- Q function in relation to normal *pdf*
- Q function in relation to complementary error function
- Inverse Q function

Normal probability density function

- Continuous function representing likelihood of its argument
- Useful to analyze unknown phenomena due to *central limit theorem*
- General form:

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{1}$$

μ Average value

σ Standard deviation

Q function

- Represents the tail probability of $\varphi(x)$ (standard normal distribution)
- Definition of $Q(x)$ through $\varphi(x)$:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du \tag{2}$$

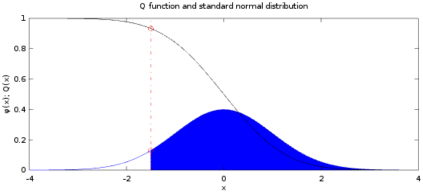


Figure: *Relation between Q function and standard normal distribution*

Q function and complementary error function

- Error function $\mathrm{erf}(x)$: probability of normally-distributed random variable X ($\mu = 0$, $\sigma^2 = \frac{1}{2}$) to be in the range $[-x, x]$
- Complementary error function $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$
- Definition of $Q(x)$ through $\mathrm{erfc}(x)$:

$$Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right) \tag{3}$$

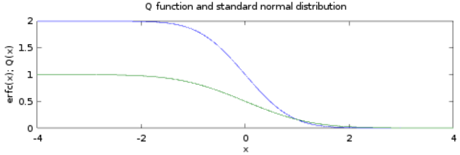


Figure: *Q function (green) and complementary error function (blue)*

Inverse Q function

- $Q^{-1}(x)$ is the value u for which $Q(u) = x$

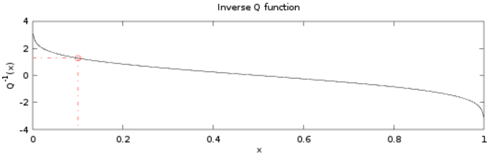


Figure: *Inverse Q function*

- If the argument x represents a bit error probability, $Q^{-1}(x)$ is proportional to the SNR

2.3 - Matched filter receiver

- *AWGN* model
- Matched filter
- Implementations
- Bit error probability P_E
- P_E at different correlation coefficients

AWGN model

- Additive white Gaussian noise
  - Additive: received signal is sum of transmitted signal and noise
  - White: flat power spectrum, $r_{xx}(k) \neq 0$ only for $k = 0$
  - Gaussian: normally-distributed samples

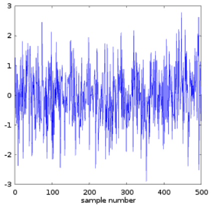


Figure: *Example waveform*

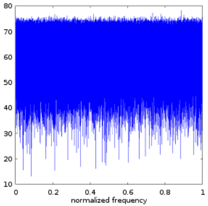


Figure: *Frequency response*

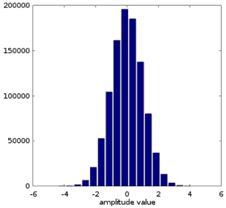


Figure: *Samples histogram*

Matched filter

- System for detecting incoming symbols
- Improves performances in presence of AWGN
- Requires reference signals $s_1(t)$ and $s_2(t)$

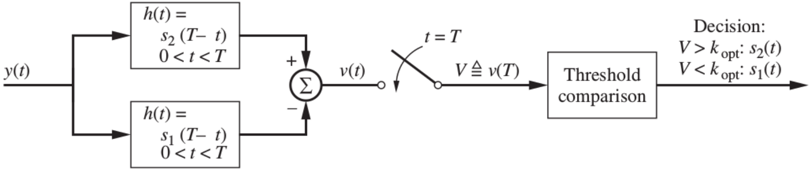


Figure: *Matched filter block diagram*

Implementation through convolution

- Received signal $y(t)$ contains AWGN
- $y(t)$ is convoluted with inverse reference signal:
  - $h(t) = s_1(T - t)$
  - $h(t) = s_2(T - t)$
- Resulting signals are summed into $v(t)$ and sampled at $t = T$
- $v(T)$ is compared against threshold k_{opt}:
  - If $v(T) > k_{opt}$, the incoming waveform was $s_2(t)$
  - If $v(T) < k_{opt}$, the incoming waveform was $s_1(t)$

Implementation through cross-correlation

- Cross-correlation: signals are multiplied and integrated
- Convolution with $h(t)$ is equivalent to cross-correlation:

$$v(t) = h(t) * y(t) = \int_0^T s(T-\tau)y(t-\tau)d\tau \tag{4}$$

- Cross-correlation: easier to implement analogically
- Convolution: easier to implement with DSPs

Bit error probability

- Bit error probability for matched filter receiver:

$$P_E = Q\left(\frac{\zeta}{2}\right) \tag{5}$$

- Maximum value of ζ:

$$\zeta_{max} = \sqrt{\frac{2}{N_0}\int_{-\infty}^{\infty} |S_2(f) - S_1(f)|^2 df} \tag{6}$$

- *Parseval's theorem*: the integral of the square of a function is equal to the integral of the square of its transform
- The integral of the squares of $s_1(t)$ and $s_2(t)$ are the energies of the two signals:

$$\zeta_{max}^2 = \frac{2}{N_0}\left(E_1 + E_2 - 2\sqrt{E_1 E_2}\rho_{12}\right) \tag{7}$$

Bit error probability (2)

- Correlation coefficient ρ_{12} expresses the signals similarity:

$$\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_{-\infty}^{\infty} s_1(t) s_2(t) dt \tag{8}$$

- Setting $E_b = \frac{1}{2}(E_1 + E_2)$ and $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b}\rho_{12}$
- Substituting ζ_{max} into P_E equation:

$$P_E = Q\left(\sqrt{\frac{E_1 + E_2 - 2\sqrt{E_1 E_2}\rho_{12}}{2N_0}}\right) = Q\left(\sqrt{(1 - R_{12})\frac{E_b}{N_0}}\right) \tag{9}$$

Bit error probability at different correlation coefficients

- Coefficient R_{12} goes from -1 to 1
  - -1: Uncorrelated symbol waveforms (antipodal), lowest P_E
  - 0: Orthogonal signals
  - 1: Highest correlation possible, i.e. identical symbol waveforms

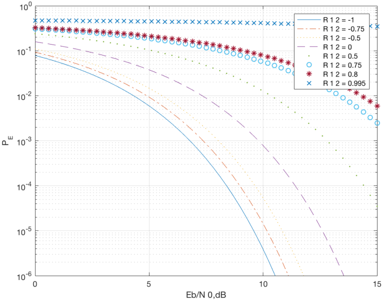


Figure: P_E *at different correlation coefficients*

3 - Link budget for satellite communication

- Link budget model
- SNR calculation
- P_E calculation
- Impact of of d, λ, B, and P_T
- Alternative techniques: ASK and FSK

Link budget model

- A way of estimating the power of a received signal
- Takes into account all the gains and losses of transmitter, channel, and receiver

$$P_R = \left(\frac{\lambda}{4\pi d}\right)^2 \frac{P_T G_T G_R}{L_0}; \tag{10}$$

- In decibels:

$$P_{R,dB} = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) + ERP_{dB} + G_{R,dB} - L_{0,dB}; \tag{11}$$

Link budget model variables

$\left(\frac{4\pi d}{\lambda}\right)^2$ Free-space loss

$ERP = P_T G_T$ Effective radiated power

G_R Gain of receiver antenna

L_0 Other losses, link budget margin

SNR calculation

- Signal-to-noise ratio in decibels:

$$SNR_{dB} = P_{R,dB} - P_{int,dB} \quad (12)$$

P_R Calculated using link budget model

P_{int} Noise power, proportional to the receiver noise temperature and the transmission bandwidth

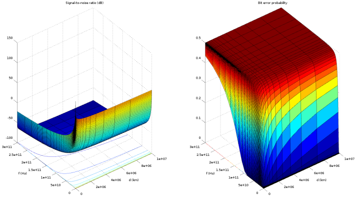
P_E calculation

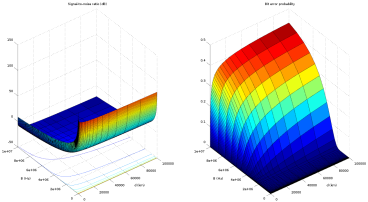
- Bit error probability for BSFK transmissions:

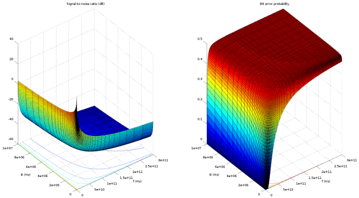
$$P_E = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{13}$$

1. ratio E_b/N_0 derived from $SNR = \longrightarrow \frac{E_b}{N_0 B T_b}$
2. For binary BPSK, $B = 2/T_b$
3. Factor BT_b is 2, or 3 dB

Impact of d, λ, B







- SNR and P_E are negatively affected by:
  - Higher distance d
  - Lower wavelength λ
  - Wider bandwidth B (lower influence)

Impact of P_T

- P_E for P_T at 50 W, 5 W, and 500 mW:

$$\texttt{1.4062e-05 9.2684e-02 3.3768e-01} \tag{14}$$

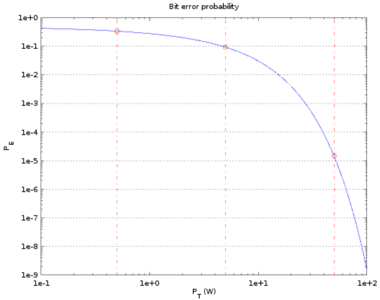


Figure: P_E *over values of* P_T

Alternative modulation: ASK

- Amplitude-shift keying

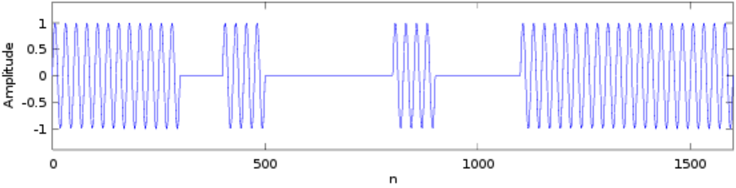


Figure: *Bit stream modulated using ASK*

- 0-bit represented as 0
- 1-bit represented as $A\cos(2\pi f_c t)$

Alternative modulation: ASK (P_E calculation)

- Correlation coefficients:
  - $\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_{-\infty}^{\infty} s_1(t) s_2(t) dt = 0$
  - $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$
- SNR to E_b/N_0: conversion factor $BT_b = 2$
- Bit error probability:

$$P_E = Q\left(\sqrt{(1 - R_{12})\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{15}$$

Alternative modulation: FSK

- Frequency-shift keying

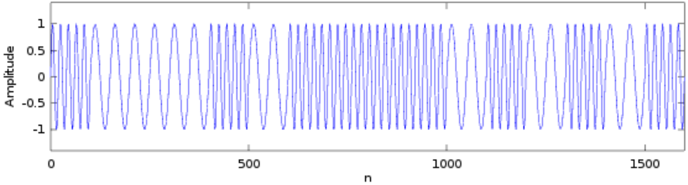


Figure: *Bit stream modulated using FSK*

- 0-bit represented as $A\cos(\omega_c t)$
- 1-bit represented as $A\cos((\omega_c + \Delta\omega)t)$
- Assumptions: $\omega_c = \frac{2\pi n}{T}$ and $\Delta\omega = \frac{2\pi m}{T}$

Alternative modulation: FSK (P_E calculation)

- Correlation coefficient $R_{12} = \frac{\sqrt{E_1 E_2}}{E_b} \rho_{12} = 0$
- SNR to E_b/N_0: conversion factor $BT_b = 2.5$
- Bit error probability:

$$P_E = Q\left(\sqrt{(1 - R_{12})\,\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{16}$$