FP2: Torso Pose Estimation on the HRP4 Humanoid Robot

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Introduction

Intro.

Kinematic Model

Let $\mathbf{x} = (\mathbf{p}_t^T, \mathbf{o}_t^T)^T$ be the pose of \mathcal{F}_t wrt \mathcal{F}_w . Let \mathcal{F}_s be the support foot frame with respect to the world frame and let \mathbf{o}_s be its orientation. Let $J(\mathbf{q}_s, \mathbf{o}_s)$ the Jacobian matrix of the kinematic map from the support frame \mathcal{F}_s to the torso frame \mathcal{F}_t .

Let's use the following kinematic model to describe the evolution of the state x through time:

$$\dot{x} = J(q_s, o_s)\dot{q}_s$$

with $\dot{\textbf{\textit{q}}}_{\scriptscriptstyle S}$ velocities of the support joints acting as control inputs.

The robot is equipped with a RGBD camera and an IMU, used to measure the pose of the torso frame:

$$y = h(x, q_n) = \begin{pmatrix} p_t \\ o_t \end{pmatrix}$$

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Extended Kalman Filter

It is now possible to define a discrete-time stochastic system:

$$x_{k+1} = x_k + TJ(q_{s,k}, o_s)\dot{q}_{s,k} + v_k$$

 $y_k = h(x_k, q_{n,k}) + w_k$

with k current timestep, T sampling interval, $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{V}_k)$ and $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{W}_k)$ zero-mean white Gaussian noises and $\mathbf{V}_k \in \mathbb{R}^{6 \times 6}$, $\mathbf{W}_k \in \mathbb{R}^{6 \times 6}$ their respective covariance matrices.

Extended Kalman Filter: Prediction

At each timestep k, a prediction $\hat{\mathbf{x}}_{k+1|k}$ is generated using the current estimate $\hat{\mathbf{x}}_k$:

$$\hat{x}_{k+1|k} = \hat{x}_k + J(q_{s,k}, o_s) \Delta q_{s,k}, \quad \Delta q_{s,k} = q_{s,k+1} - q_{s,k}$$

with $\Delta q_{s,k} \approx T \dot{q}_{s,k}$ obtained using encoder readings. The covariance prediction matrix is updated accordingly:

$$P_{k+1|k} = P_k + V_k$$

The predicted output associated to $\hat{x}_{k+1|k}$ is computed as well:

$$\hat{y}_{k+1|k} = h(\hat{x}_{k+1|k}, q_{n,k+1})$$

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Extended Kalman Filter: Correction

To correct the predicted state we need to compute the innovation:

$$\nu_{k+1} = y_{k+1} - \hat{y}_{k+1|k}$$

It is possible to determine the corrected state estimate by computing:

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{G}_{k+1} \mathbf{\nu}_{k+1}$$

with G_{k+1} Kalman gain matrix, defined as

$$G_{k+1} = P_{k+1|k} H_{k+1}^{\mathsf{T}} (H_{k+1} P_{k+1|k} H_{k+1}^{\mathsf{T}} + W_{k+1})^{-1}$$

The corrected covariance matrix is updated as well:

$$P_{k+1} = P_{k+1|k} - G_{k+1}H_{k+1}P_{k+1|k}$$

Note that, in our implementation, we defined V and W as:

$$V = diag\{5, 5, 5, 100, 100, 100\} \cdot 10^{-6}$$

$$W = diag\{5, 5, 5, 5 \cdot 10^{-2}, 5 \cdot 10^{-2}, 5 \cdot 10^{-2}\} \cdot 10^{-2}$$

Accelerometer Integration

Considering constant acceleration $\ddot{p}_{t,k}$ in an interval $[t_k, t_{k+1})$:

$$\begin{split} \dot{p}_{t,k+1} &= \dot{p}_{t,k} + \ddot{p}_{t,k}T \\ p_{t,k+1} &= p_{t,k} + \dot{p}_{t,k}T + \frac{1}{2}\ddot{p}_{t,k}T^2 \end{split}$$

At each timestep k, the accelerometer returns the linear acceleration ${}^ta_{t,k}$ expressed in \mathcal{F}_t . It must be transformed to \mathcal{F}_w :

$$\ddot{\boldsymbol{p}}_{t,k} = {}^{w}\boldsymbol{R}_{t}(\hat{\boldsymbol{x}}_{k})^{t}\boldsymbol{a}_{t,k}$$

with:

$${}^{\mathsf{W}}\mathsf{R}_{\mathsf{t}}(\hat{\mathsf{x}}_{\mathsf{k}}) = \mathsf{R}_{\mathsf{z}}(\gamma)\mathsf{R}_{\mathsf{y}}(\beta)\mathsf{R}_{\mathsf{x}}(\alpha)$$

where α , β and γ are, respectively, the roll, the pitch and the yaw angles of \hat{x}_k .

Gyroscope Integration

Considering a constant angular velocity in an interval $[t_k, t_{k+1})$:

$$o_{t,k+1} = o_{t,k} + T\dot{o}_{t,k}$$

At each timestep k, the gyroscope returns the angular velocity in ${}^t\omega_{t,k}$ expressed in \mathcal{F}_t . It must be transformed to \mathcal{F}_w :

$$\dot{\boldsymbol{o}}_{t,k} = \boldsymbol{T}(\hat{\boldsymbol{x}}_k)^t \boldsymbol{\omega}_{t,k}$$

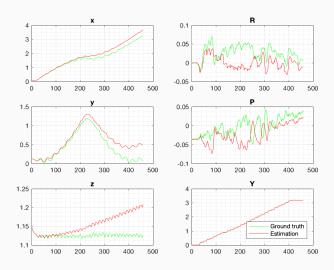
with:

$$T(\hat{x}_k) = \begin{pmatrix} \cos(\gamma)/\cos(\beta) & \sin(\gamma)/\cos(\beta) & 0\\ -\sin(\gamma) & \cos(\gamma) & 0\\ \cos(\gamma)\tan(\beta) & \sin(\gamma)\tan(\beta) & 1 \end{pmatrix}$$

where β and γ are, respectively, the pitch and the yaw angles of $\hat{\mathbf{x}}_k$.

EKF with IMU Integration

Figure 1: Experiments (5.1).



Filtering Linear Velocities

Add to the state an approximated estimation of the velocity in such a way to fix drift accumulation.

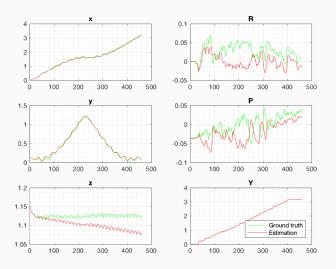
Velocity is obtained using position of the robot in 2 successive frames

$$\dot{\boldsymbol{p}}_{t,k} \approx \dot{\hat{\boldsymbol{p}}}_{t,k} \approx \dot{\hat{\boldsymbol{p}}}_{t,k-1} = \frac{\hat{\boldsymbol{p}}_{t,k} - \hat{\boldsymbol{p}}_{t,k-1}}{T}$$

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Filtering Linear Velocities

Figure 2: Improves the estimate of $(x, y)^T$ coordinates decreasing the error on z.



Remove integration and double integration from the calculus of the position.

Required stuff: RGBD camera posed on the head of the robot, identifiable landmarks with known positions.

TODO: QUI FORSE CI STA UNO SCREENSHOOT PER MOSTRARE LA CONFIGUARZINE... E FOTO TIPO TRILATERATION

Obtain the distance of the landmarks from the camera:

$$cam x = \frac{2p_x - w}{w}^{cam} z \cdot tan\left(\frac{\phi}{2}\right)$$
$$cam y = \frac{2p_y - h}{h}^{cam} z \cdot tan\left(\frac{\phi}{2}\right)$$
$$cam z = (\lambda_f - \lambda) \cdot p_z + \lambda$$

$$r_{i} = \left\| \begin{pmatrix} cam_{X_{i}} \\ cam_{Y_{i}} \\ cam_{Z_{i}} \end{pmatrix} \right\|^{2} + \overline{r} \quad (i = 1, 2, \dots, n)$$

FOTO TIPO CAMERA MODEL

Knowing position of each landmarks in the world frame $(x_i, y_i, z_i)^T$ and its distance from the camera r_i , find the position of the camera is equal to resolve an equation for landmark

$$(p_{h,x}-x_i)^2+(p_{h,y}-y_i)^2+(p_{h,z}-z_i)^2=r_i^2$$
 $(i=1,2,\ldots,n)$

which can be rewritten as linear system Ax = b:

$$A = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ \vdots & \vdots & \vdots \\ x_n - x_1 & y_n - y_1 & z_n - z_1 \end{pmatrix}, \quad x = p_n - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \quad b = \begin{pmatrix} b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{pmatrix}$$

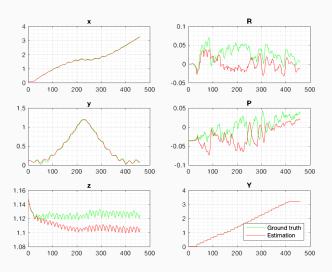
$$b_{k1} = \frac{1}{2} \left[r_1^2 + r_k^2 + (x_k - x_1)^2 + (y_k - y_1)^2 + (z_k - z_1)^2 \right] \quad (k = 2, 3, \dots, n)$$

Hence, the position of the torso can be determined by:

$$p_h = x + \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad p_t = p_h - {}^{w}R_t(\hat{x}_k)({}^{t}p_h - {}^{t}p_t)$$

We need at least 5 landmarks for the computation: 4 for obtain a square matrix, and at least 1 more for execute the pseudoinverse that bring results also in case of noisy measurements.

Figure 3: Improved estimation for x, y, z



MPC Loop Closure

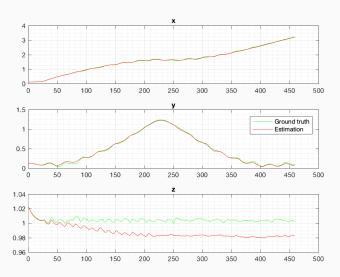
MPC Loop CLosure

The MPC need the position of the CoM with respect to the support foot to close the loop.

We want to estimate the same position but with the support foot reference frame with roll e pitch (wrt world frame) equal to zero.

MPC Loop CLosure

Figure 4: Todo.



MPC Loop CLosure

Given the estimate of the pose of the torso $(\hat{p}_t, \hat{o}_t)^T$, it is possible to obtain the estimate of the position of the support foot $(\hat{p}_s, \hat{o}_s)^T$ and of the COM $(\hat{p}_{COM}, \hat{o}_{COM})^T$ using the kinematics relations.

The position of the CoM in a rotated reference frame of the support foot $\mathcal{F}_{s'}$ which has the z-axis orthogonal to the floor is:

$$\mathbf{s}'\hat{\mathbf{p}}_{\mathsf{CoM}} = \mathbf{R}_{\mathsf{Z}}^{\mathsf{T}}(\gamma)(\hat{\mathbf{p}}_{\mathsf{CoM}} - \hat{\mathbf{p}}_{\mathsf{S}})$$

This position can be used as a reference signal in the MPC in order to generate the gait of the humanoid robot.

Regulation

Kinematic Model of the Unicycle

Brief description:

$$\dot{x} = v\cos(\theta)$$

$$\dot{y} = v\sin(\theta)$$

$$\dot{\theta} = \omega$$

with v and ω respectively linear and angular velocity.

Proportional Controller

Control law:

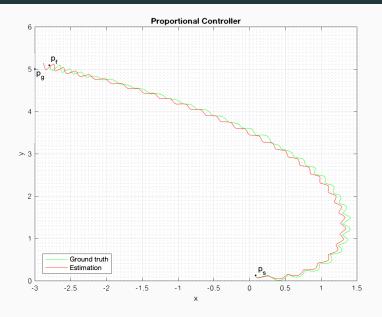
$$v = k_1 \| \mathbf{p}_g - \hat{\mathbf{p}}_t \|$$
$$\omega = k_2 e_{\theta}$$

with e_{θ} angle between the sagittal vector of the unicycle and the vector pointing from the unicycle towards the goal, $k_1=0.18$ and $k_2=0.014$. Forcing v and ω to zero when $\|\boldsymbol{p}_g-\hat{\boldsymbol{p}}_t\|<0.25$. Desired and final configuration:

$$\mathbf{q}_g = (-3, 5, \cdot)^{\mathsf{T}}$$

 $\mathbf{q}_f = (-2.798, 5.090, 0.98\pi)^{\mathsf{T}}$
 $\hat{\mathbf{q}}_f = (-2.885, 5.127, 0.979\pi)^{\mathsf{T}}$

Proportional Controller: x-y Plot



Cartesian Regulation

Let's express the coordinates of the unicycle in a reference frame \mathcal{F}_g fixed at a position $(x_g, y_g)^T$ and rotated by θ_g around \mathcal{F}_w :

$$\begin{pmatrix} g_X \\ g_Y \\ g_\theta \end{pmatrix} = R_Z^T(\theta_g) \begin{pmatrix} x - x_g \\ y - y_g \\ \theta - \theta_g \end{pmatrix}$$

Cartesian Regulation

Control law:

$$v = -k_1({}^{g}x\cos({}^{g}\theta) + {}^{g}y\sin({}^{g}\theta))$$

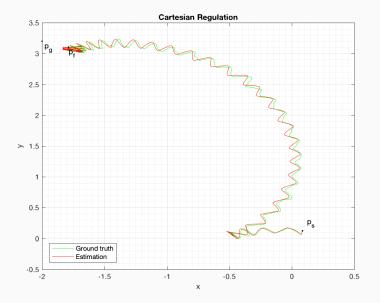
$$\omega = k_2(\operatorname{Atan2}({}^{g}y, {}^{g}x) - {}^{g}\theta + \pi)$$

with $k_1=0.07$ and $k_2=0.01$. Forcing v and ω to zero when $\|\boldsymbol{p}_g-\hat{\boldsymbol{p}}_t\|<0.2$. Desired and final configuration:

$$q_g = (-2, 3.2, \cdot)^T$$

 $q_f = (-1.788, 3.105, 1.562\pi)^T$
 $\hat{q}_f = (-1.823, 3.108, 1.562\pi)^T$

Cartesian Regulation: x-y Plot



Kinematic Model of the Unicycle in Polar Coordinates

Brief description:

$$\dot{\rho}_r = -v\cos(\gamma_r)$$

$$\dot{\gamma}_r = \frac{\sin(\gamma_r)}{\rho_r} v - \omega$$

$$\dot{\delta}_r = \frac{\sin(\gamma_r)}{\rho_r} v$$

Polar coordinates can be obtained from the generalized coordinates of the unicycle $(x, y, \theta)^T$ by computing:

$$\rho_r = \sqrt{{}^g x^2 + {}^g y^2}$$

$$\gamma_r = \text{Atan2}({}^g y, {}^g x) - {}^g \theta + \pi$$

$$\delta_r = \gamma_r + {}^g \theta$$

Posture Regulation

Control law:

$$v = k_1 \rho_r \cos(\gamma_r)$$

$$w = k_2 \gamma_r + k_1 \frac{\sin(\gamma_r) \cos(\gamma_r)}{\gamma_r} (\gamma_r + k_3 \delta_r)$$

with $k_1=0.1$, $k_2=0.007$ $k_3=0.004$. Forcing v and ω to zero when $\rho_r<0.2$. Desired and final configuration:

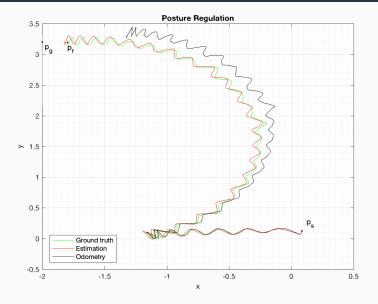
$$\mathbf{q}_g = (-2, 3.2, \pi)^{\mathsf{T}}$$

$$\mathbf{q}_f = (-1.797, 3.194, 1.024\pi)^{\mathsf{T}}$$

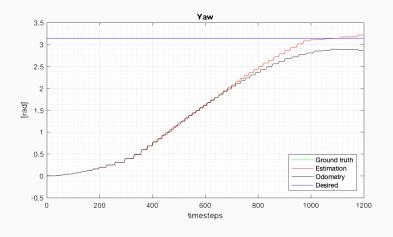
$$\hat{\mathbf{q}}_f = (-1.824, 3.222, 1.024\pi)^{\mathsf{T}}$$

$$\bar{\mathbf{q}}_f = (-1.282, 3.424, 0.912\pi)^{\mathsf{T}}$$

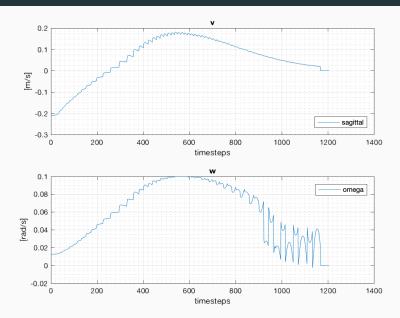
Posture Regulation: x-y Plot



Posture Regulation: Yaw Plot



Posture Regulation: Velocity Profile Plots



Conclusion

Conclusion.



References

- G. Oriolo, A. Paolillo, L. Rosa, and M. Vendittelli, "Humanoid odometric localization integrating kinematic, inertial and visual information," *Auton. Robots*, vol. 40, no. 5, pp. 867–879, 2016.
- W. Hereman and J. William S. Murphy, "Determination of a Position in Three Dimensions Using Trilateration and Approximate Distances," 1995.
- B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, Robotics:
 Modelling, Planning and Control.
 Springer Publishing Company, Incorporated, 1st ed., 2008.