

# FP2: Torso Pose Estimation on the HRP4 Humanoid Robot

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# Introduction

- Torso pose estimation on the HRP4 humanoid robot
- Extended Kalman Filter
- IMU measurements and RGBD camera
- Accelerometer and Gyroscope Integration
- Trilateration algorithm
- MPC loop closure
- Cartesian and Posture Regulation
- C++ and HRP4 model in V-REP

# Kinematic Model

Let's use the following kinematic model to describe the evolution of the state  $\mathbf{x}$  through time:

$$\dot{\mathbf{x}} = J(\mathbf{q}_s, \mathbf{o}_s) \dot{\mathbf{q}}_s$$

with  $\mathbf{o}_s$  orientation of  $\mathcal{F}_s$  and  $\dot{\mathbf{q}}_s$  velocities of the support joints acting as control inputs.

The robot is equipped with a RGBD camera and an IMU, used to measure the pose of the torso frame:

$$\mathbf{y} = h(\mathbf{x}, \mathbf{q}_n) = \begin{pmatrix} p_t \\ o_t \end{pmatrix}$$

It is now possible to define a discrete-time stochastic system:

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{x}_k + T J(\mathbf{q}_{S,k}, \mathbf{o}_S) \dot{\mathbf{q}}_{S,k} + \mathbf{v}_k \\ \mathbf{y}_k &= h(\mathbf{x}_k, \mathbf{q}_{n,k}) + \mathbf{w}_k\end{aligned}$$

with  $\mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{V}_k)$  and  $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{W}_k)$  zero-mean white Gaussian noises and  $\mathbf{V}_k \in \mathbb{R}^{6 \times 6}$ ,  $\mathbf{W}_k \in \mathbb{R}^{6 \times 6}$  their respective covariance matrices.

# Extended Kalman Filter: Prediction

At each timestep  $k$ , a prediction  $\hat{\mathbf{x}}_{k+1|k}$  is generated using  $\hat{\mathbf{x}}_k$ :

$$\hat{\mathbf{x}}_{k+1|k} = \hat{\mathbf{x}}_k + J(\mathbf{q}_{s,k}, \mathbf{o}_s) \Delta \mathbf{q}_{s,k}, \quad \Delta \mathbf{q}_{s,k} = \mathbf{q}_{s,k+1} - \mathbf{q}_{s,k}$$

with  $\Delta \mathbf{q}_{s,k} \approx T \dot{\mathbf{q}}_{s,k}$  obtained using encoder readings.

The covariance prediction matrix is updated accordingly:

$$\mathbf{P}_{k+1|k} = \mathbf{P}_k + \mathbf{V}_k$$

The predicted output associated to  $\hat{\mathbf{x}}_{k+1|k}$  is computed as well:

$$\hat{\mathbf{y}}_{k+1|k} = h(\hat{\mathbf{x}}_{k+1|k}, \mathbf{q}_{n,k+1})$$

## Extended Kalman Filter: Correction

It is possible to determine the corrected state estimate by computing:

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{G}_{k+1}\boldsymbol{\nu}_{k+1}$$

with  $\boldsymbol{\nu}_{k+1} = \mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1|k}$  innovation and  $\mathbf{G}_{k+1}$  Kalman gain matrix:

$$\mathbf{G}_{k+1} = \mathbf{P}_{k+1|k} \mathbf{H}_{k+1}^T (\mathbf{H}_{k+1} \mathbf{P}_{k+1|k} \mathbf{H}_{k+1}^T + \mathbf{W}_{k+1})^{-1}$$
$$\mathbf{H}_{k+1} = \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right|_{\mathbf{x}=\hat{\mathbf{x}}_{k+1|k}}$$

The corrected covariance matrix is updated as well:

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1|k} - \mathbf{G}_{k+1} \mathbf{H}_{k+1} \mathbf{P}_{k+1|k}$$

# Accelerometer Integration

Considering a constant acceleration  $\ddot{\mathbf{p}}_{t,k}$  in an interval  $[t_k, t_{k+1})$ :

$$\dot{\mathbf{p}}_{t,k+1} = \dot{\mathbf{p}}_{t,k} + \ddot{\mathbf{p}}_{t,k}T$$

$$\mathbf{p}_{t,k+1} = \mathbf{p}_{t,k} + \dot{\mathbf{p}}_{t,k}T + \frac{1}{2}\ddot{\mathbf{p}}_{t,k}T^2$$

At each timestep  $k$ , the accelerometer returns the linear acceleration  ${}^t\mathbf{a}_{t,k}$  expressed in  $\mathcal{F}_t$ . It must be transformed to  $\mathcal{F}_w$ :

$$\ddot{\mathbf{p}}_{t,k} = {}^wR_t(\hat{\mathbf{x}}_k){}^t\mathbf{a}_{t,k} - \mathbf{g}$$

with  $\mathbf{g} = [0, 0, -g]^T$ ,  $g = 9.81\text{m/s}^2$  and with:

$${}^wR_t(\hat{\mathbf{x}}_k) = R_z(\gamma)R_y(\beta)R_x(\alpha)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  respectively roll, pitch and yaw angles of  $\hat{\mathbf{x}}_k$ .

# Gyroscope Integration

Considering a constant angular velocity in an interval  $[t_k, t_{k+1})$ :

$$\mathbf{o}_{t,k+1} = \mathbf{o}_{t,k} + T\dot{\mathbf{o}}_{t,k}$$

At each timestep  $k$ , the gyroscope returns the angular velocity in  ${}^t\boldsymbol{\omega}_{t,k}$  expressed in  $\mathcal{F}_t$ . It must be related to  $\dot{\mathbf{o}}_{t,k}$ :

$$\dot{\mathbf{o}}_{t,k} = T(\hat{\mathbf{x}}_k){}^t\boldsymbol{\omega}_{t,k}$$

with:

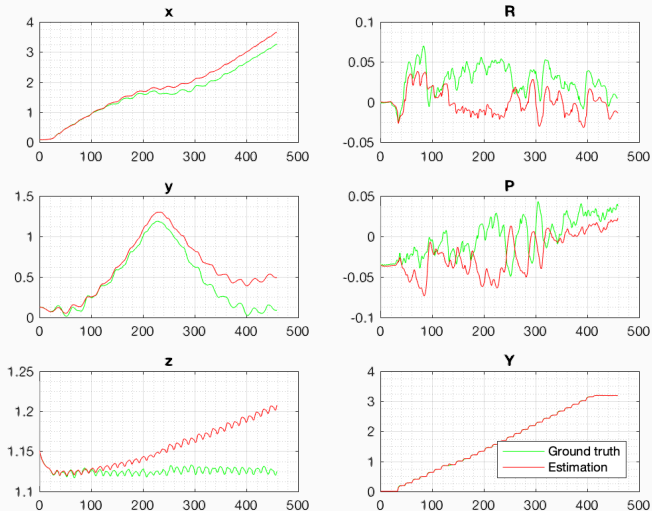
$$T(\hat{\mathbf{x}}_k) = \begin{pmatrix} 1 & \sin(\alpha)\tan(\beta) & \cos(\alpha)\tan(\beta) \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha)/\cos(\beta) & \cos(\alpha)/\cos(\beta) \end{pmatrix}$$

where  $\alpha$  and  $\beta$  respectively roll and pitch angles of  $\hat{\mathbf{x}}_k$ .



# EKF with IMU Integration

Figure 1: Estimates of the pose of the torso using IMU measurements.



# Filtering Linear Velocities

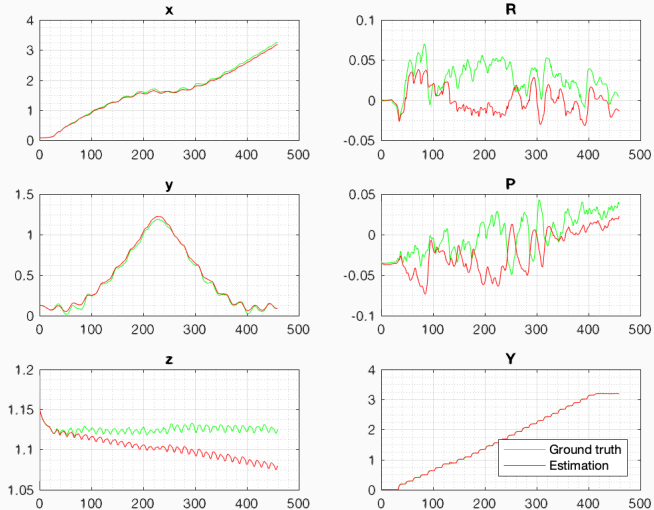
Add to the state an approximated estimation of the velocity in such a way to fix drift accumulation.

Velocity is obtained using estimated position of the robot in two successive frames:

$$\dot{\hat{p}}_{t,k} \approx \dot{\hat{p}}_{t,k} \approx \dot{\hat{p}}_{t,k-1} = \frac{\hat{p}_{t,k} - \hat{p}_{t,k-1}}{T}$$

# Filtering Linear Velocities

Figure 2: Improved estimate of  $(x, y)^T$  coordinates decreasing the error on  $z$ .



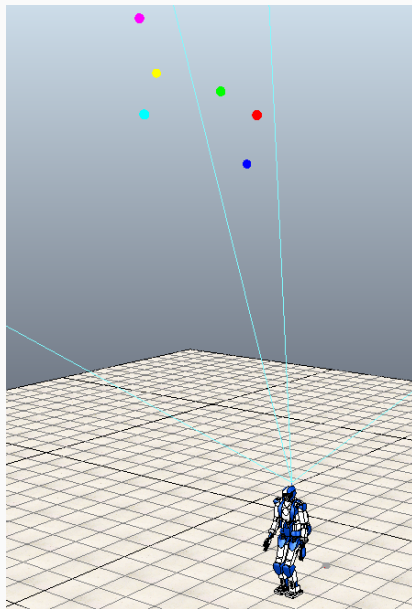
# Trilateration



# Trilateration

Removes integration and double integration from the calculus of the position.

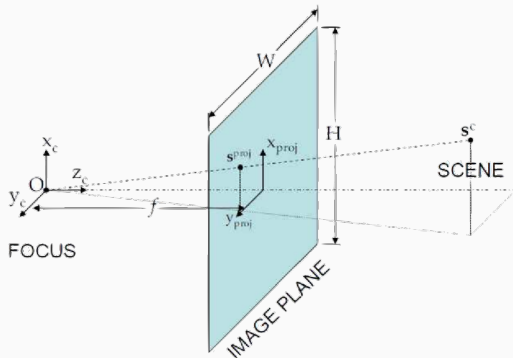
Requires RGBD camera positioned on the head of the robot, identifiable landmarks with known positions.



# Trilateration

Obtain the distance of the landmarks from the camera:

$$\begin{aligned} {}^{cam}x &= \frac{2p_x - w}{w} {}^{cam}z \cdot \tan\left(\frac{\phi}{2}\right) \\ {}^{cam}y &= \frac{2p_y - h}{h} {}^{cam}z \cdot \tan\left(\frac{\phi}{2}\right) \\ {}^{cam}z &= (\lambda_f - \lambda) \cdot p_z + \lambda \end{aligned} \quad r_i = \left\| \begin{pmatrix} {}^{cam}x_i \\ {}^{cam}y_i \\ {}^{cam}z_i \end{pmatrix} \right\|^2 + \bar{r} \quad (i = 1, 2, \dots, n)$$



# Trilateration

Knowing the position  $(x_i, y_i, z_i)^T$  of each landmarks in the world frame and its distance  $r_i$  from the camera, the position  $\mathbf{p}_h$  of the camera can be found by solving:

$$(p_{h,x} - x_i)^2 + (p_{h,y} - y_i)^2 + (p_{h,z} - z_i)^2 = r_i^2 \quad (i = 1, 2, \dots, n)$$

which can be rewritten as linear system  $\mathbf{Ax} = \mathbf{b}$ :

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ \vdots & \vdots & \vdots \\ x_n - x_1 & y_n - y_1 & z_n - z_1 \end{pmatrix}, \quad \mathbf{x} = \mathbf{p}_h - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{pmatrix}$$

$$b_{k1} = \frac{1}{2} [r_1^2 + r_k^2 + (x_k - x_1)^2 + (y_k - y_1)^2 + (z_k - z_1)^2] \quad (k = 2, 3, \dots, n)$$

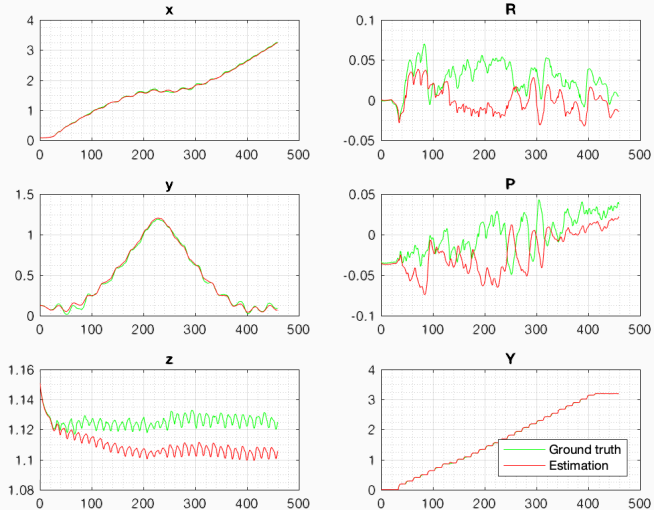
Hence, the position of the torso can be determined by computing:

$$\mathbf{p}_h = \mathbf{x} + \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad \mathbf{p}_t = \mathbf{p}_h - {}^w\mathbf{R}_t(\hat{\mathbf{x}}_k)({}^t\mathbf{p}_h - {}^t\mathbf{p}_t)$$

We need at least 5 landmarks: 4 to obtain a square matrix, and at least 1 more to compute the pseudoinverse that bring results also in case of noisy measurements.



Figure 3: Further improved estimation for  $x, y, z$ .



## MPC Loop Closure

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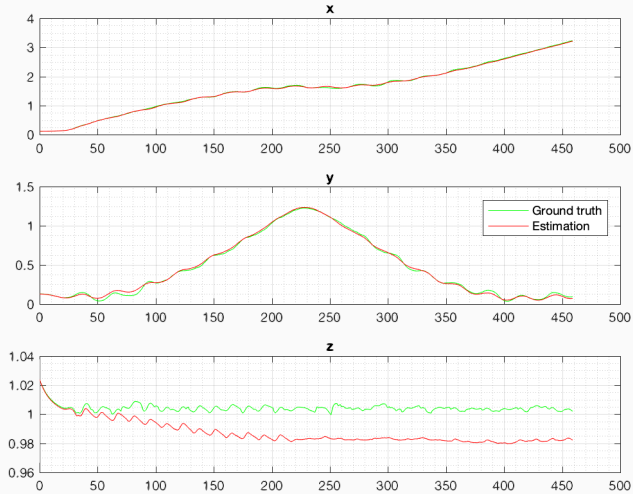
# MPC Loop Closure

The MPC needs the position of the CoM with respect to the support foot to close the loop.

We want to estimate the same position but with the support foot reference frame with roll and pitch (wrt world frame) equal to zero.

# MPC Loop Closure

Figure 4: Estimated position of the CoM with respect to the ground truth.



Given the estimate of the pose of the torso  $(\hat{\mathbf{p}}_t, \hat{\mathbf{o}}_t)^T$ , it is possible to obtain the estimate of the position of the support foot  $(\hat{\mathbf{p}}_s, \hat{\mathbf{o}}_s)^T$  and of the reference frame attached to the CoM  $(\hat{\mathbf{p}}_{CoM}, \hat{\mathbf{o}}_{CoM})^T$  using the kinematics relations.

The position of the CoM in a rotated reference frame of the support foot  $\mathcal{F}_{s'}$ , which has the z-axis orthogonal to the floor is:

$${}^{s'}\hat{\mathbf{p}}_{CoM} = \mathbf{R}_z^T(\gamma)(\hat{\mathbf{p}}_{CoM} - \hat{\mathbf{p}}_s)$$

This position can be used as a reference signal in the MPC in order to generate the gait of the humanoid robot.

# Regulation

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# Kinematic Model of the Unicycle

Let's model the robot as a unicycle:

$$\dot{x} = v \cos(\theta)$$

$$\dot{y} = v \sin(\theta)$$

$$\dot{\theta} = \omega$$

with  $v$  and  $\omega$  respectively linear and angular velocity.

# Proportional Controller

Let's consider the following control law:

$$v = k_1 \|\mathbf{p}_g - \hat{\mathbf{p}}_t\|$$

$$\omega = k_2 e_\theta$$

with  $e_\theta$  angle between the sagittal vector of the unicycle and the vector pointing from the unicycle towards the goal,  $k_1 = 0.18$  and  $k_2 = 0.014$ . Forcing  $v$  and  $\omega$  to zero when  $\|\mathbf{p}_g - \hat{\mathbf{p}}_t\| < 0.25$ . Desired and final configuration:

$$\mathbf{q}_g = (-3, 5, \cdot)^T$$

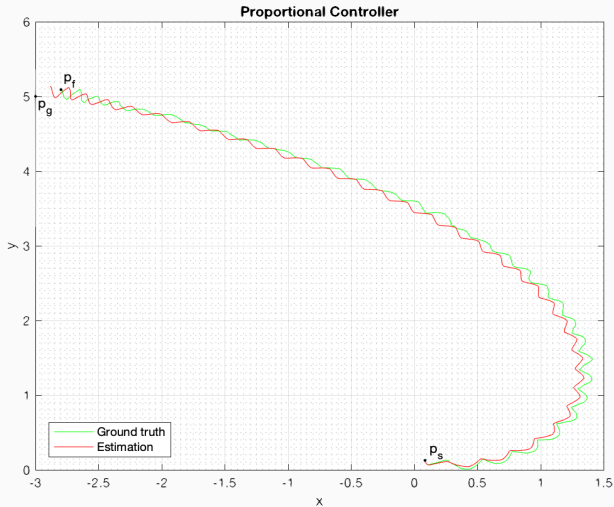
$$\mathbf{q}_f = (-2.798, 5.090, 0.98\pi)^T$$

$$\hat{\mathbf{q}}_f = (-2.885, 5.127, 0.979\pi)^T$$



# Proportional Controller: x-y Plot

Figure 5: Trajectory followed by the robot when using the controller defined above. Setting velocities to zero when  $\|p_g - \hat{p}_t\| < 0.25$ .



Let's express the coordinates of the unicycle in a reference frame  $\mathcal{F}_g$  fixed at a position  $(x_g, y_g)^T$  and rotated by  $\theta_g$  around  $\mathcal{F}_w$ :

$$\begin{pmatrix} {}^g x \\ {}^g y \\ {}^g \theta \end{pmatrix} = R_z^T(\theta_g) \begin{pmatrix} x - x_g \\ y - y_g \\ \theta - \theta_g \end{pmatrix}$$

# Cartesian Regulation

Let's consider the following control law:

$$v = -k_1({}^g x \cos({}^g \theta) + {}^g y \sin({}^g \theta))$$

$$\omega = k_2(\text{Atan2}({}^g y, {}^g x) - {}^g \theta + \pi)$$

with  $k_1 = 0.07$  and  $k_2 = 0.01$ . Forcing  $v$  and  $\omega$  to zero when  $\|\mathbf{p}_g - \hat{\mathbf{p}}_t\| < 0.2$ . Desired and final configuration:

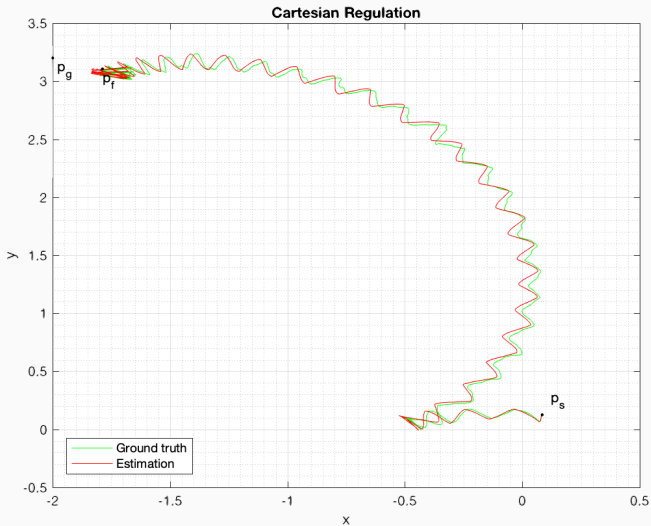
$$\mathbf{q}_g = (-2, 3.2, \cdot)^T$$

$$\mathbf{q}_f = (-1.788, 3.105, 1.562\pi)^T$$

$$\hat{\mathbf{q}}_f = (-1.823, 3.108, 1.562\pi)^T$$

# Cartesian Regulation: x-y Plot

**Figure 6:** Trajectory followed by the robot when using the controller defined above. Setting velocities to zero when  $\|p_g - \hat{p}_t\| < 0.2$ .



# Kinematic Model of the Unicycle in Polar Coordinates

Let's model the robot as a unicycle using polar coordinates:

$$\begin{aligned}\dot{\rho}_r &= -v \cos(\gamma_r) \\ \dot{\gamma}_r &= \frac{\sin(\gamma_r)}{\rho_r} v - \omega \\ \dot{\delta}_r &= \frac{\sin(\gamma_r)}{\rho_r} v\end{aligned}$$

Polar coordinates can be obtained from the generalized coordinates of the unicycle  $(x, y, \theta)^T$  by computing:

$$\begin{aligned}\rho_r &= \sqrt{{}^g x^2 + {}^g y^2} \\ \gamma_r &= \text{Atan2}({}^g y, {}^g x) - {}^g \theta + \pi \\ \delta_r &= \gamma_r + {}^g \theta\end{aligned}$$

# Posture Regulation

Let's consider the following control law:

$$v = k_1 \rho_r \cos(\gamma_r)$$

$$w = k_2 \gamma_r + k_1 \frac{\sin(\gamma_r) \cos(\gamma_r)}{\gamma_r} (\gamma_r + k_3 \delta_r)$$

with  $k_1 = 0.1$ ,  $k_2 = 0.007$ ,  $k_3 = 0.004$ . Forcing  $v$  and  $w$  to zero when  $\rho_r < 0.2$ . Desired and final configuration:

$$\mathbf{q}_g = (-2, 3.2, \pi)^T$$

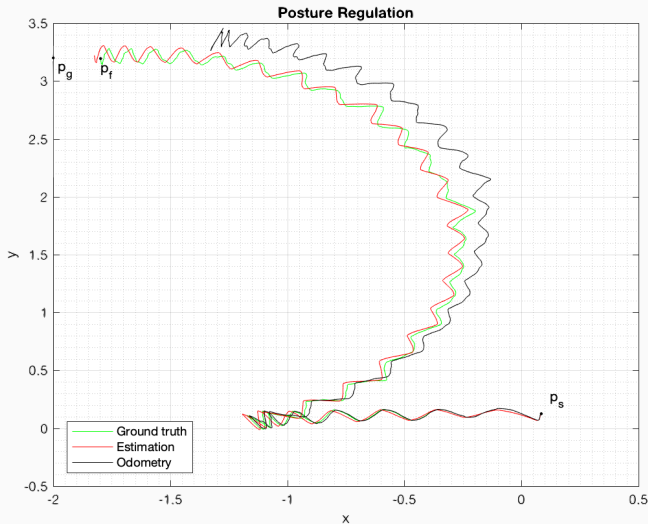
$$\mathbf{q}_f = (-1.797, 3.194, 1.024\pi)^T$$

$$\hat{\mathbf{q}}_f = (-1.824, 3.222, 1.024\pi)^T$$

$$\bar{\mathbf{q}}_f = (-1.282, 3.424, 0.912\pi)^T$$

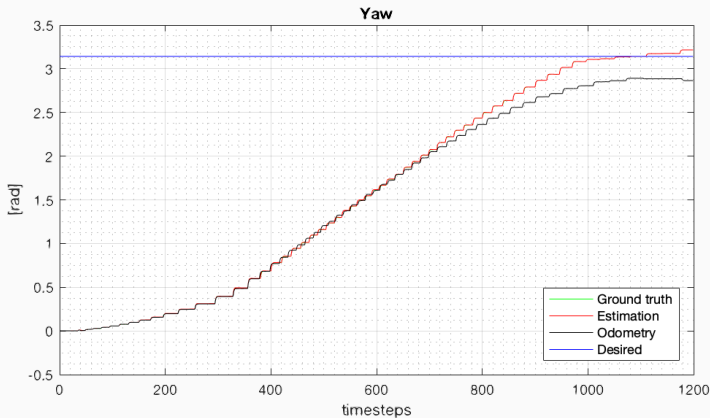
# Posture Regulation: x-y Plot

Figure 7: Trajectory followed by the robot when using the controller defined above. Setting velocities to zero when  $\rho_r < 0.2$ .



# Posture Regulation: Yaw Plot

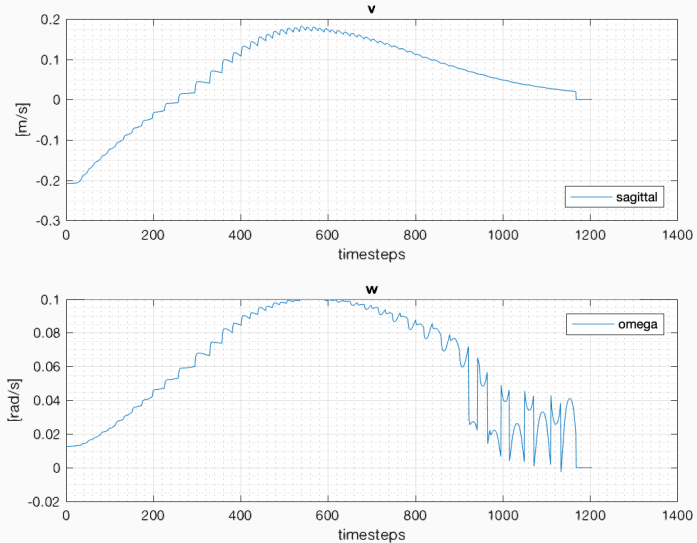
Figure 8: Evolution of the yaw while following the trajectory of the previous slide.





# Posture Regulation: Velocity Profile Plots

Figure 9: Profiles of the linear velocity  $v$  and the angular velocity  $w$  used to control the robot.






# Conclusion

- EKF correctly localizes the robot
- RGBD camera improves localization
- MPC gait generator does not work as desired
- VSLAM and sensor fusion

Q&A

# References

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