

# Planning and Executing Humanoid Gaits in a World of Stairs

---

Candidate: Michele Cipriano

Thesis Advisor: Prof. Giuseppe Oriolo

January 21, 2020



SAPIENZA  
UNIVERSITÀ DI ROMA

Master Degree in Artificial Intelligence and Robotics

Department of Computer, Control and Management Engineering

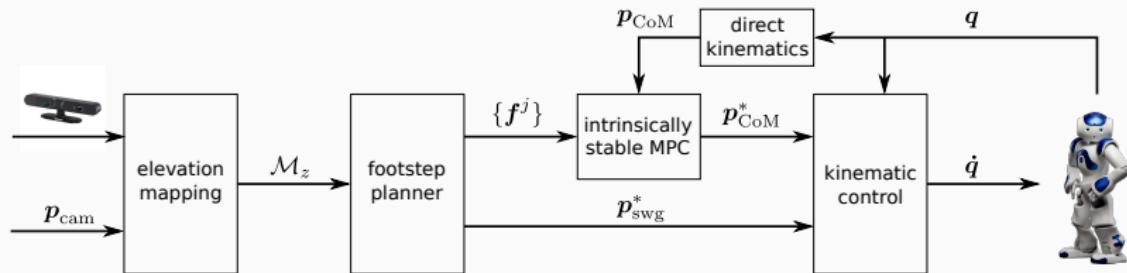
Sapienza University of Rome

# Introduction

- goal: humanoid robot locomotion in a *World of Stairs*
- assumption: humanoid is equipped with a depth sensor
- assumption: humanoid knows its location



# Proposed Approach



- elevation map building: autonomously build a map  $\mathcal{M}_z$
- footstep planner: generates a footstep sequence  $\{f^j\}$  on  $\mathcal{M}_z$  together with swing foot trajectories  $\{p_{swg}^*\}$
- variable-height IS-MPC: computes a stable gait along the planned footsteps  $\{f^j\}$

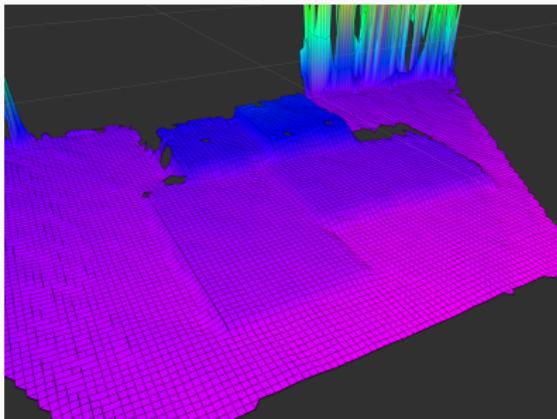
# Elevation Map Building: Framework

`elevation_mapping` [Fankhauser et al., 2018]

- robot-centric grid-based map:  $\mathcal{M}_z$
- height estimate  $\mathcal{N}(\hat{h}_i, \sigma_{h_i}^2)$  for each cell  $i$
- Kalman filter given new height and motion measurements
- map fusion:  $(\hat{h}_i, h_{i,\min}, h_{i,\max})$  such that  $h_i \in [h_{i,\min}, h_{i,\max}]$  with 95% confidence
- dynamic environments using visibility check based on ray tracing

# Elevation Map Building: Results

- ASUS Xtion Pro (**depth sensor**)
- working range: 0.5–3.5 m



# Footstep Planner: Algorithm

- requirements

R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

R2 footstep is fully in contact with the ground

R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$

2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$

3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$

4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then

5.  $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$

6.  $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$

2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$

3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$

4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then

5.  $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$

6.  $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

# Footstep Planner: Algorithm

- requirements

- R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

- R2 footstep is fully in contact with the ground

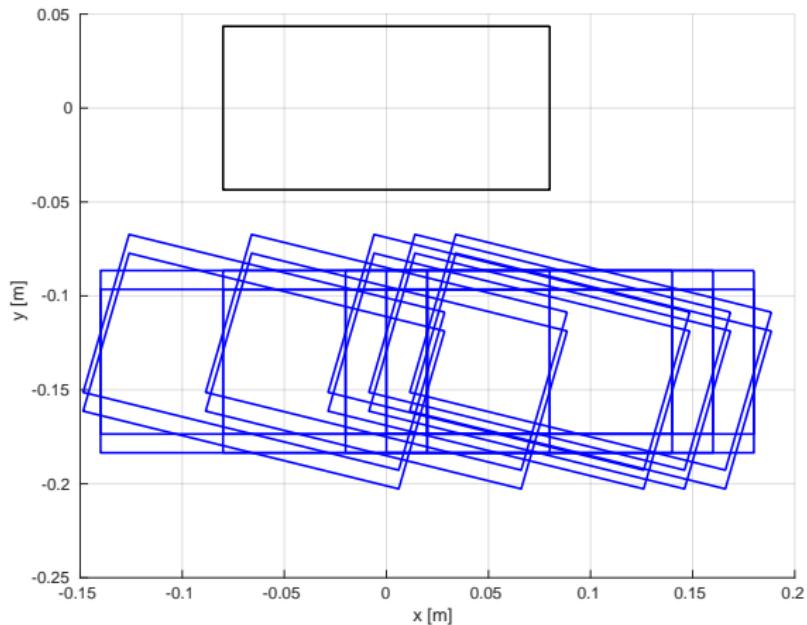
- R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

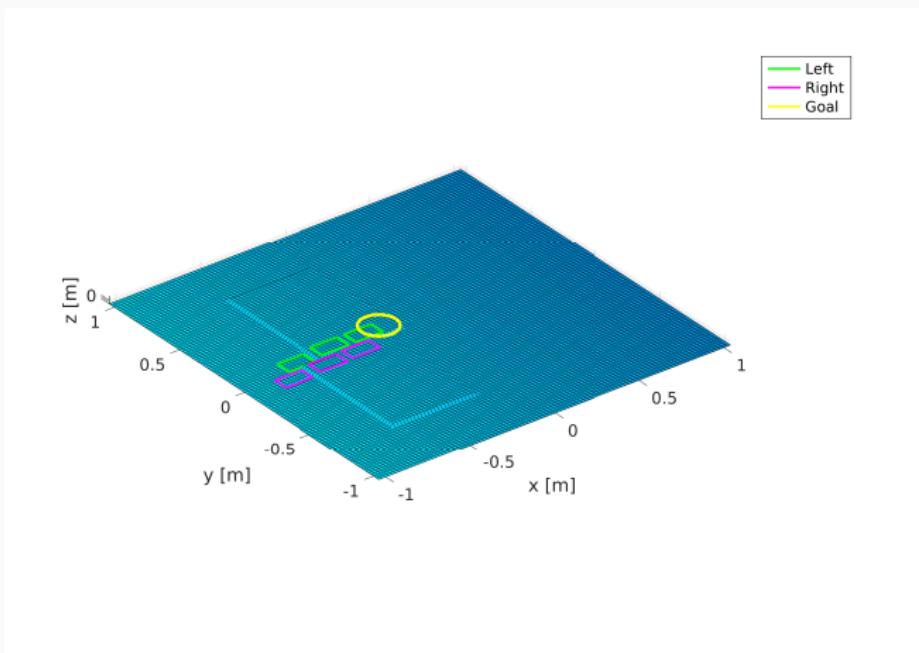
- $U$  is the set of footstep primitives

# Footstep Planner: Primitives Catalogue $U$



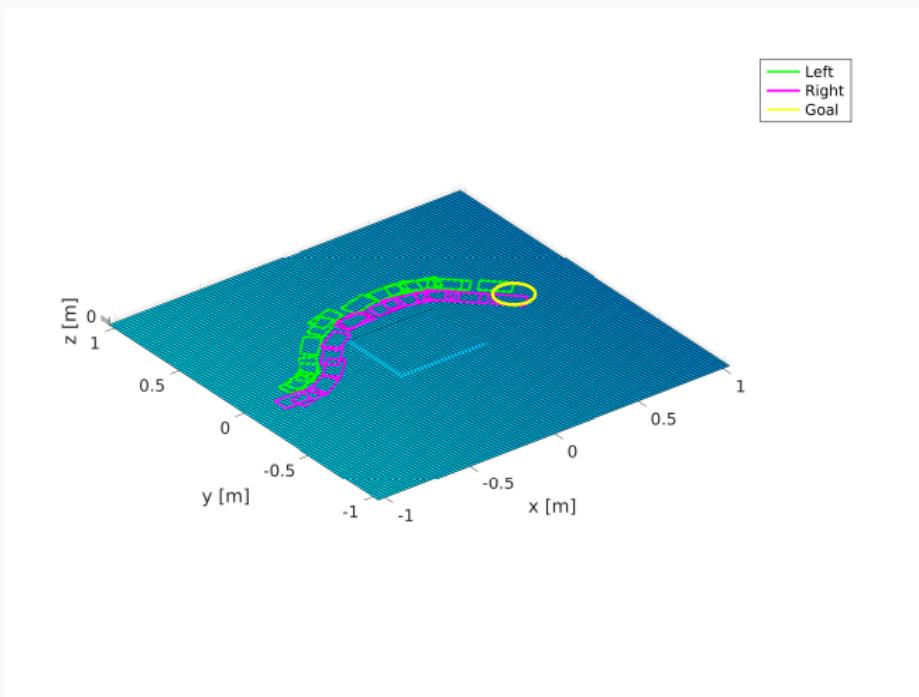
# Footstep Planner: Results (stair climbing)

tree size: 10 – solution size: 6 – runtime: 0.8 ms



# Footstep Planner: Results (obstacle avoidance)

tree size: 488 – solution size: 31 – runtime: 70 ms



# Variable Height IS-MPC: 3D Motion Model

- LIP model not suitable for gait generation over uneven terrain due to constant height assumption
- linearity can be maintained by constraining vertical motion such that

$$\frac{\ddot{z}_c + g}{z_c - z_z} = \omega^2$$

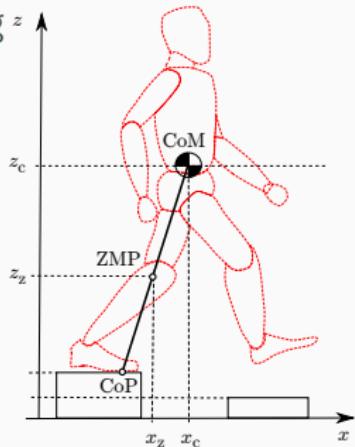
- CoM dynamics become

$$\ddot{x}_c = \omega^2(x_c - x_z)$$

$$\ddot{y}_c = \omega^2(y_c - y_z)$$

$$\ddot{z}_c = \omega^2(z_c - z_z) - g$$

- $x_c, y_c, z_c$ : CoM coordinates
- $x_z, y_z, z_z$ : ZMP coordinates

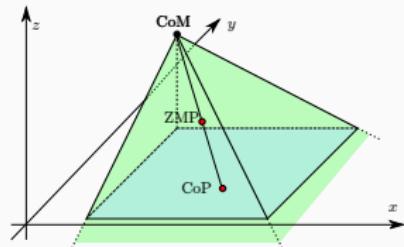


# Variable Height IS-MPC: MPC Formulation

- constrain ZMP into subregion of polyhedral cone (box)

$$R_{k+i}^T \begin{pmatrix} x_z^{k+i} - x_f^{k+i} \\ y_z^{k+i} - y_f^{k+i} \\ z_z^{k+i} - y_f^{k+i} \end{pmatrix} \leq \frac{1}{2} \begin{pmatrix} \tilde{d}_x^z \\ \tilde{d}_y^z \\ d_z^z \end{pmatrix}$$

- bound CoM wrt ZMP (LIP stability)



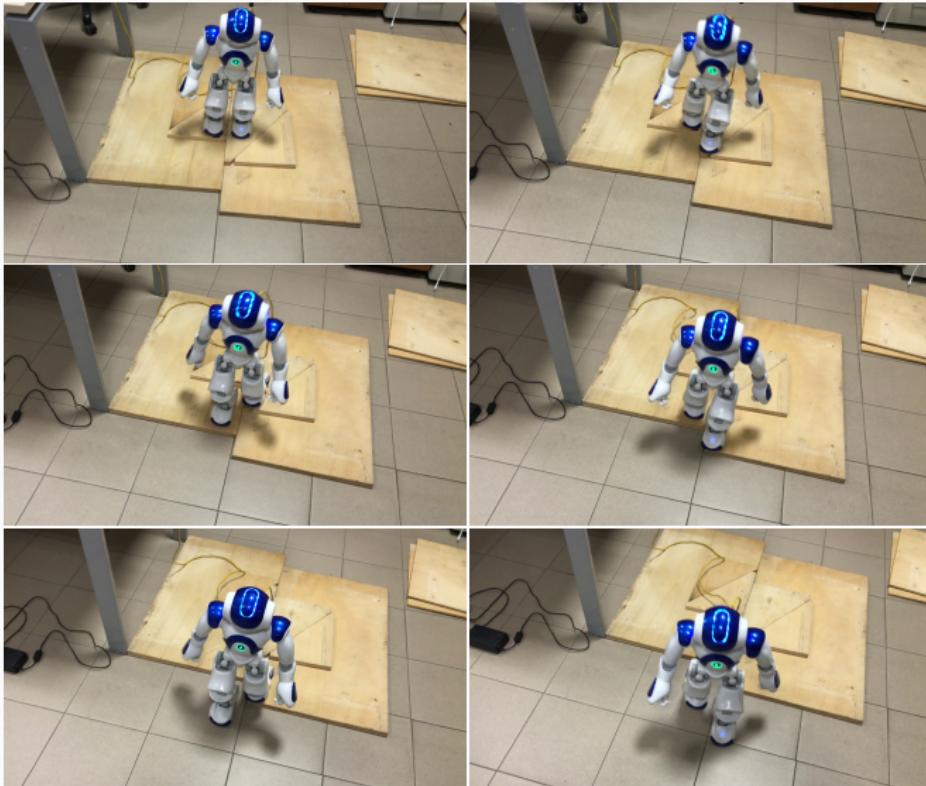
$$\frac{1}{\omega} \frac{1 - e^{-\delta\omega}}{1 - e^{-N\delta\omega}} \sum_{i=0}^{N-1} e^{-i\delta\omega} \dot{x}_z^{k+i} = x_c^k + \frac{\dot{x}_c^k}{\omega} - x_z^k$$

- solve QP problem using MPC scheme

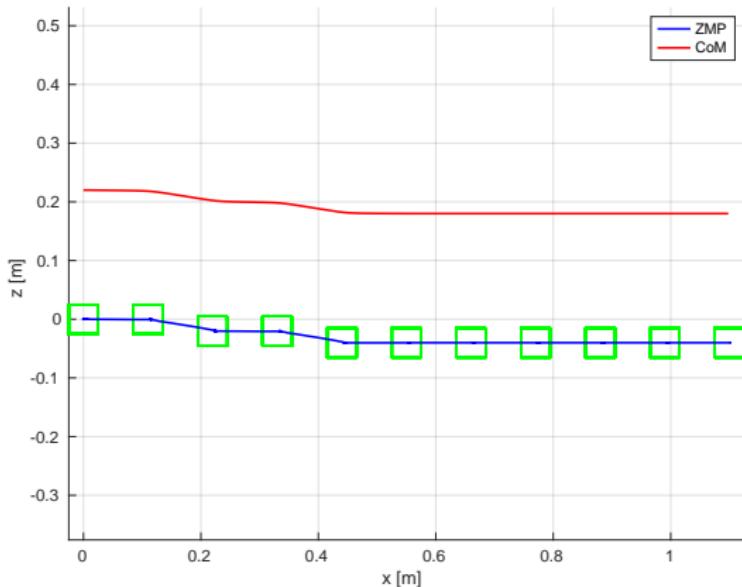
$$\min_{\dot{x}_z^k, \dot{y}_z^k, \dot{z}_z^k} \|\dot{x}_z^k\|_2^2 + \|\dot{y}_z^k\|_2^2 + \|\dot{z}_z^k\|_2^2 + \beta(\|\Delta x_f^{k+1}\|_2^2 + \|\Delta y_f^{k+1}\|_2^2 + \|\Delta z_f^{k+1}\|_2^2)$$

s.t. ZMP and stability constraints

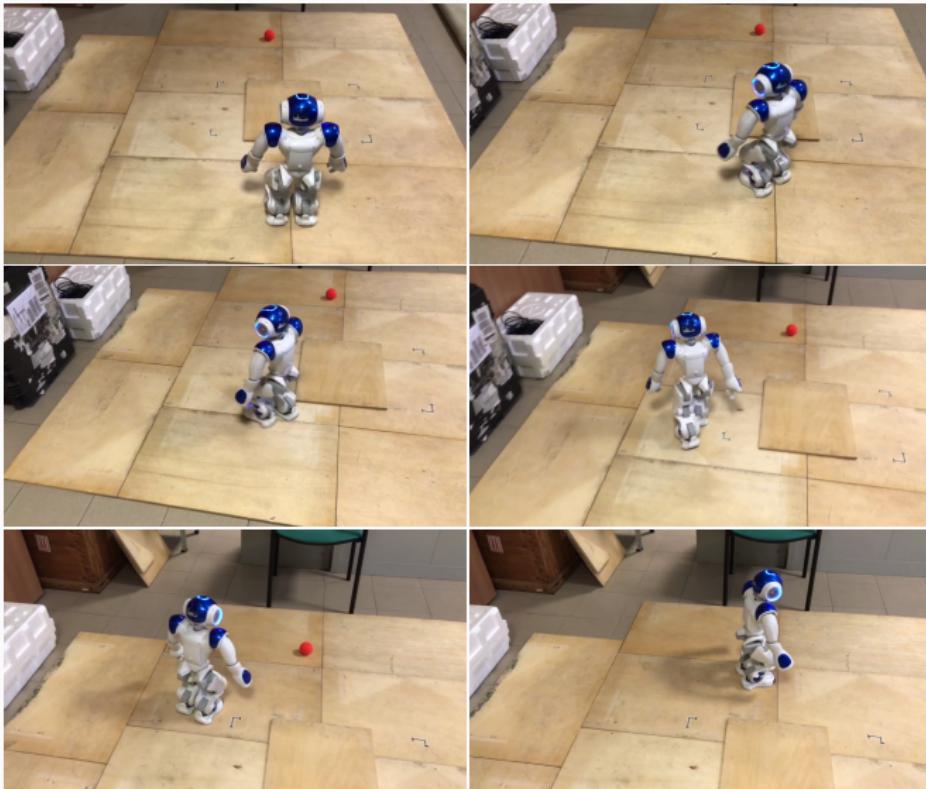
# Experiment: Downstairs



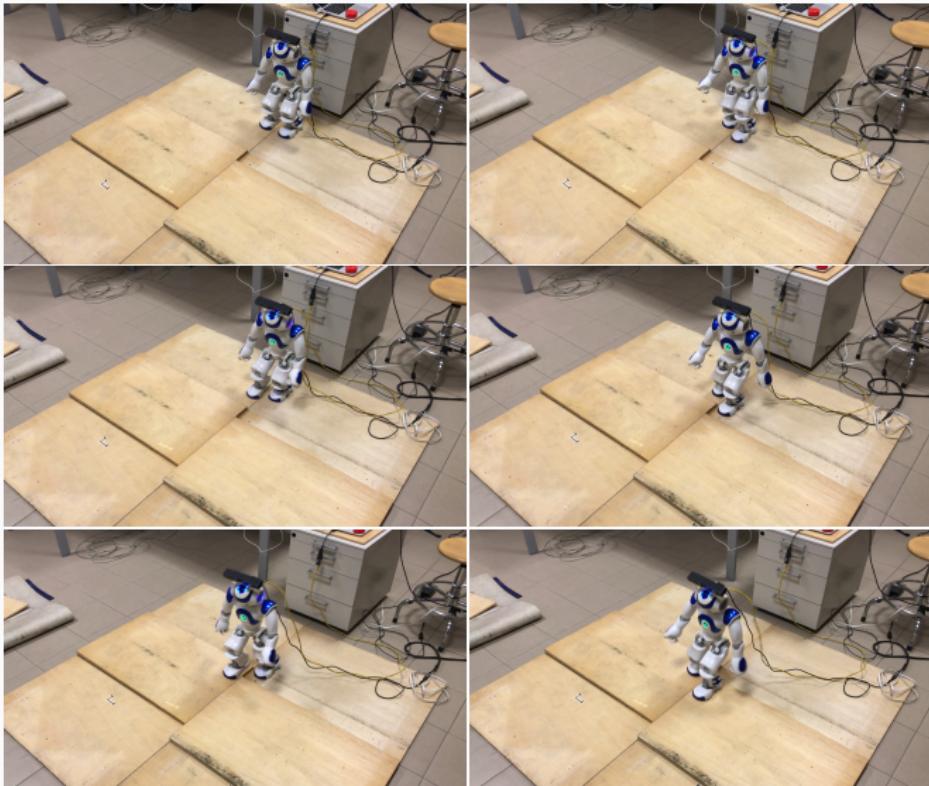
# Experiment: Downstairs



# Experiment: Obstacle Avoidance

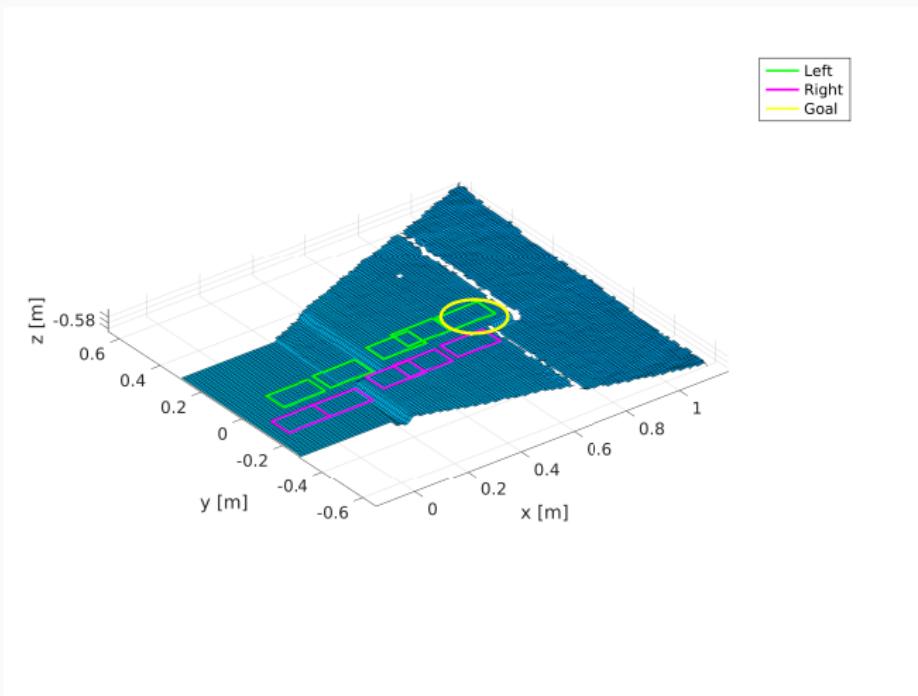


# Experiment: Unknown Environment



# Experiment: Unknown Environment

tree size: 454 – solution size: 10 – runtime: 331 ms



Video

# Conclusions

- NAO is able to autonomously climb the stairs in an unknown *World of Stairs* environment
- future works
  - localization module and continuous mapping
  - replanning phase
  - dynamic and rough environments

Q&A

## References

-  Fankhauser, P., Bloesch, M., and Hutter, M. (2018).  
Probabilistic terrain mapping for mobile robots with uncertain localization.  
*IEEE Robotics and Automation Letters (RA-L)*, 3(4):3019–3026.
-  Ferrari, P., Scianca, N., Lanari, L., and Oriolo, G. (2019).  
An integrated motion planner/controller for humanoid robots on uneven ground.  
In *18th European Control Conference, ECC 2019, Naples, Italy, June 25-28, 2019*, pages 1598–1603.
-  Zamparelli, A., Scianca, N., Lanari, L., and Oriolo, G. (2018).  
Humanoid Gait Generation on Uneven Ground using Intrinsically Stable MPC.  
*IFAC-PapersOnLine*, 51:393–398.