

# Planning and Executing Humanoid Gaits in a World of Stairs

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Master Degree in Artificial Intelligence and Robotics

Department of Computer, Control and Management Engineering

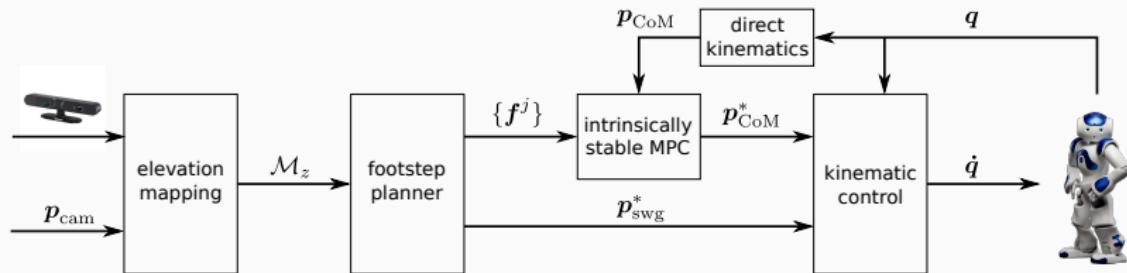
Sapienza University of Rome

# Introduction

- goal: humanoid robot locomotion in a *World of Stairs*
- assumption: humanoid is equipped with a depth sensor
- assumption: humanoid knows its location



# Proposed Approach



- **elevation map building**: autonomously build a map  $\mathcal{M}_z$
- **footstep planner**: generates a footstep sequence  $\{f^j\}$  on  $\mathcal{M}_z$  together with swing foot trajectories  $\{p_{swg}^*\}$
- **variable-height IS-MPC**: computes a stable gait along the planned footsteps  $\{f^j\}$

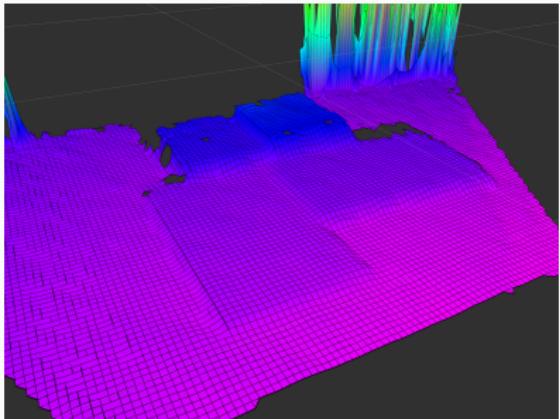
# Elevation Map Building: Framework

`elevation_mapping` [Fankhauser et al., 2018]

- robot-centric grid-based map:  $\mathcal{M}_z$
- height estimate  $\mathcal{N}(\hat{h}_i, \sigma_{h_i}^2)$  for each cell  $i$
- Kalman filter given new height and motion measurements
- map fusion:  $(\hat{h}_i, h_{i,\min}, h_{i,\max})$  such that  $h_i \in [h_{i,\min}, h_{i,\max}]$  with 95% confidence
- dynamic environments using visibility check based on ray tracing

# Elevation Map Building: Results

- ASUS Xtion Pro (**depth sensor**)
- working range: 0.5–3.5 m



# Footstep Planner

- requirements

R1 maximum footsteps height variation  $|z_f^j - z_f^{j-1}| \leq \Delta z_{\max}$

R2 footstep is fully in contact with the ground

R3 swing foot trajectory  $p_{\text{swg}}^j$  is collision free

- RRT-based planner iteration

1.  $p_{\text{rand}} \leftarrow \text{Rand}(\mathcal{M}_z)$
2.  $v_{\text{near}} \leftarrow \text{Nearest}(p_{\text{rand}}, \gamma, \mathcal{T})$
3.  $f_{\text{cand}} \leftarrow \text{Rand}(U)$
4. if  $f_{\text{cand}}$  feasible wrt R1-R2 then
5.    $p_{\text{swg}}^{\text{cand}} \leftarrow \text{BuildTrajectory}(\cdot)$
6.    $\mathcal{T}.\text{add}(v_{\text{new}}, v_{\text{near}})$  if  $p_{\text{swg}}^{\text{cand}}$  satisfies R3

- $U$  is the set of footstep primitives

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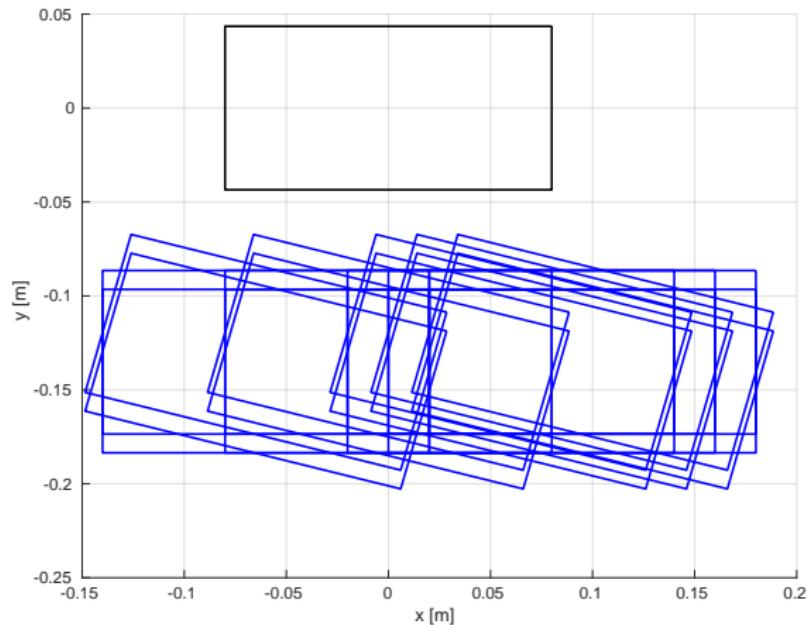
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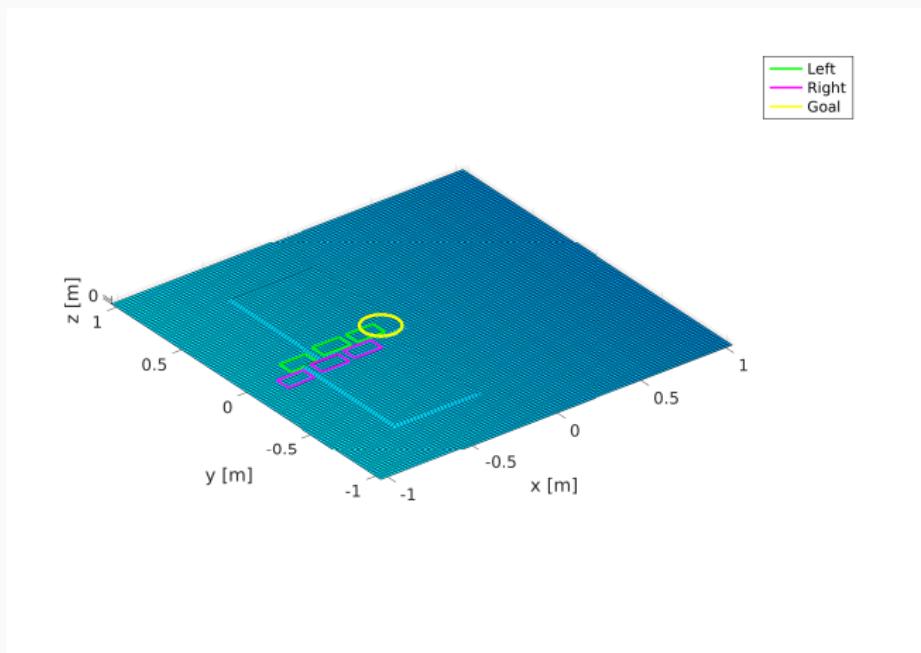
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# Footstep Planner: Primitives Catalogue $U$



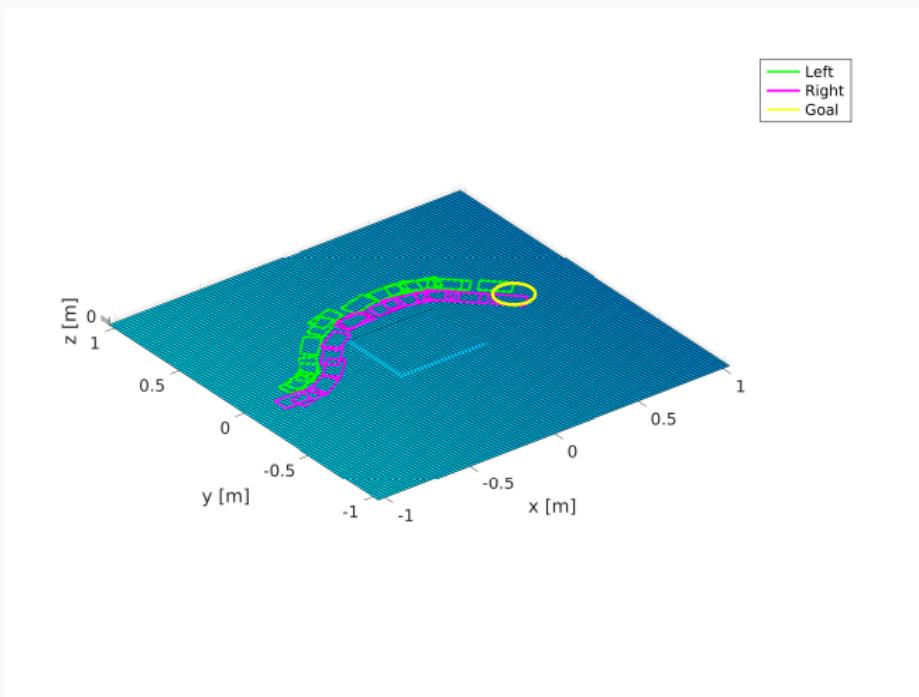
# Footstep Planner: Results (stair climbing)

tree size: 10 – solution size: 6 – runtime: 0.8 ms



# Footstep Planner: Results (obstacle avoidance)

tree size: 488 – solution size: 31 – runtime: 70 ms



# Variable Height CoM IS-MPC: 3D Motion Model

- LIP model not suitable for gait generation over uneven terrain
- constraint vertical motion such that

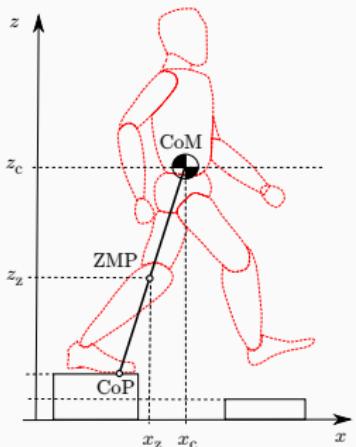
$$\frac{\ddot{z}_c + g}{z_c - z_z} = \omega^2$$

- CoM dynamics become

$$\ddot{x}_c = \omega^2(x_c - x_z)$$

$$\ddot{y}_c = \omega^2(y_c - y_z)$$

$$\ddot{z}_c = \omega^2(z_c - z_z) - g$$



# Variable Height CoM IS-MPC: MPC Formulation

- constrain ZMP into subregion of polyhedral cone (box)

$$R_{k+i}^T \begin{pmatrix} x_z^{k+i} - x_f^{k+i} \\ y_z^{k+i} - y_f^{k+i} \\ z_z^{k+i} - y_f^{k+i} \end{pmatrix} \leq \frac{1}{2} \begin{pmatrix} \tilde{d}_x^z \\ \tilde{d}_y^z \\ d_z^z \end{pmatrix}$$

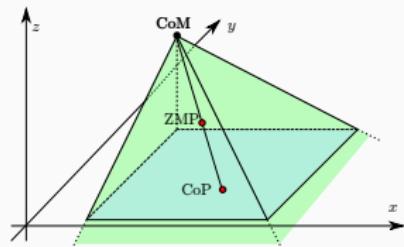
- bound CoM wrt ZMP (LIP stability)

$$\frac{1}{\omega} \frac{1 - e^{-\delta\omega}}{1 - e^{-N\delta\omega}} \sum_{i=0}^{N-1} e^{-i\delta\omega} \dot{x}_z^{k+i} = x_c^k + \frac{\dot{x}_c^k}{\omega} - x_z^k$$

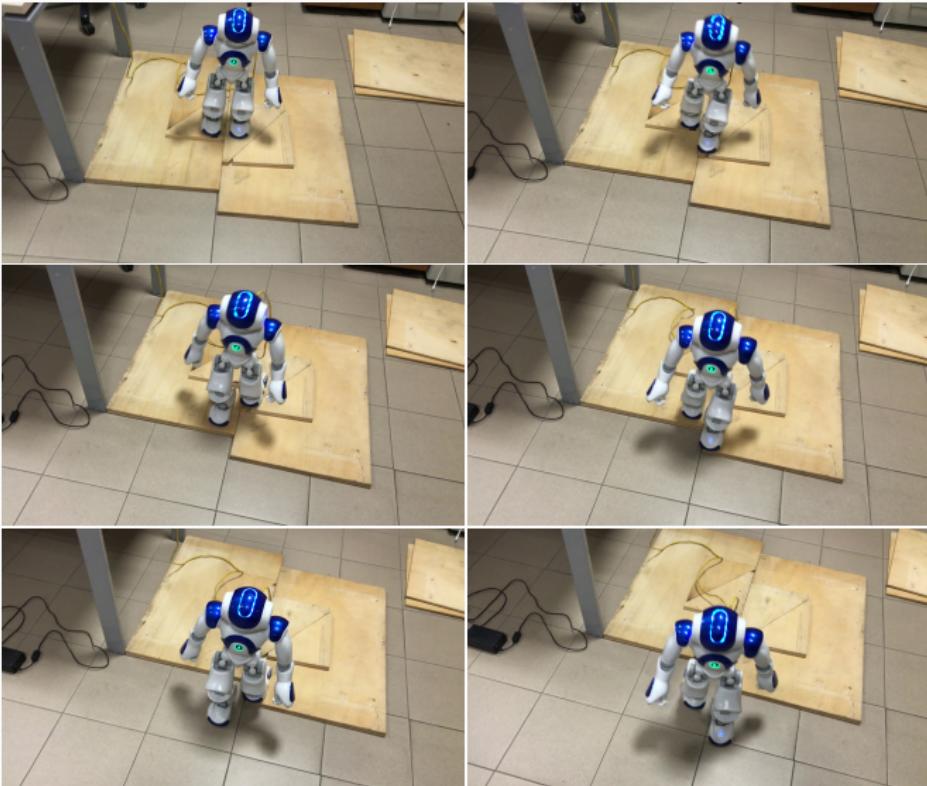
- solve QP problem using MPC scheme

$$\min_{\dot{x}_z^k, \dot{y}_z^k, \dot{z}_z^k} \|\dot{x}_z^k\|_2^2 + \|\dot{y}_z^k\|_2^2 + \|\dot{z}_z^k\|_2^2 + \beta(\|\Delta x_f^{k+1}\|_2^2 + \|\Delta y_f^{k+1}\|_2^2 + \|\Delta z_f^{k+1}\|_2^2)$$

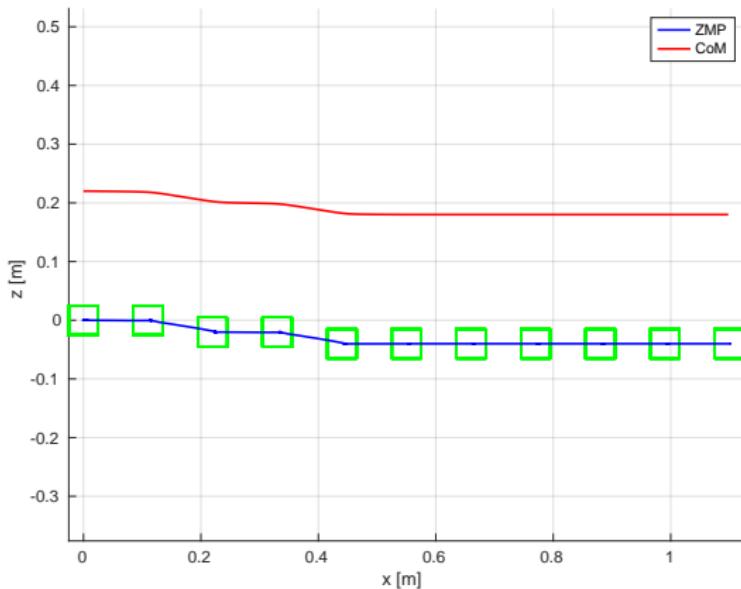
s.t. ZMP and stability constraints



# Variable Height CoM IS-MPC: Stair Climbing



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# RRT-based Footstep Planning

- Rapidly-Exploring Random Tree

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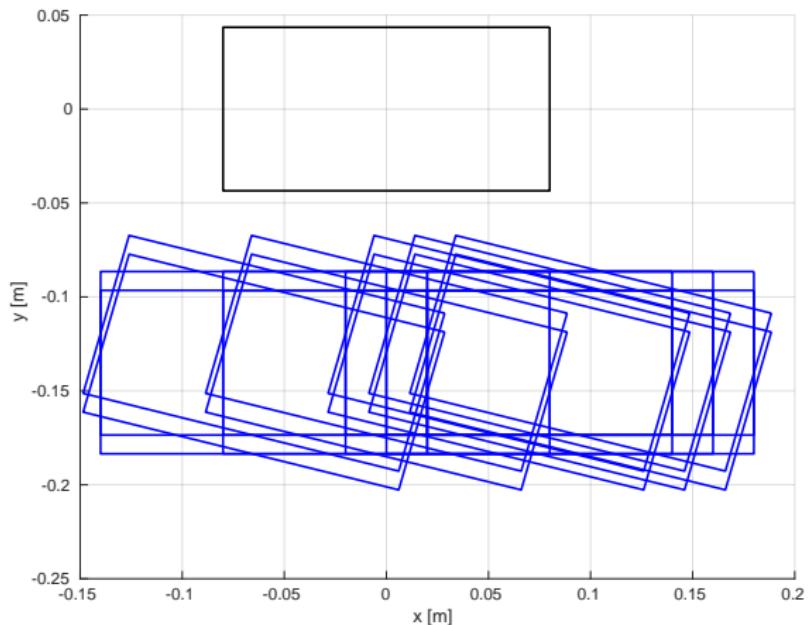
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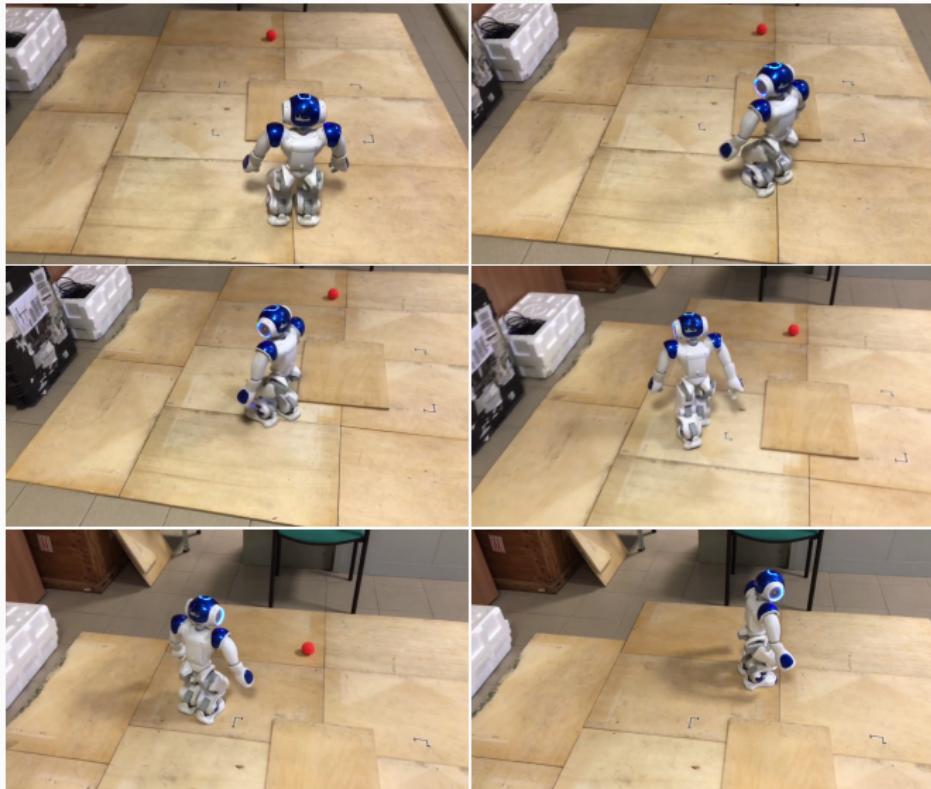
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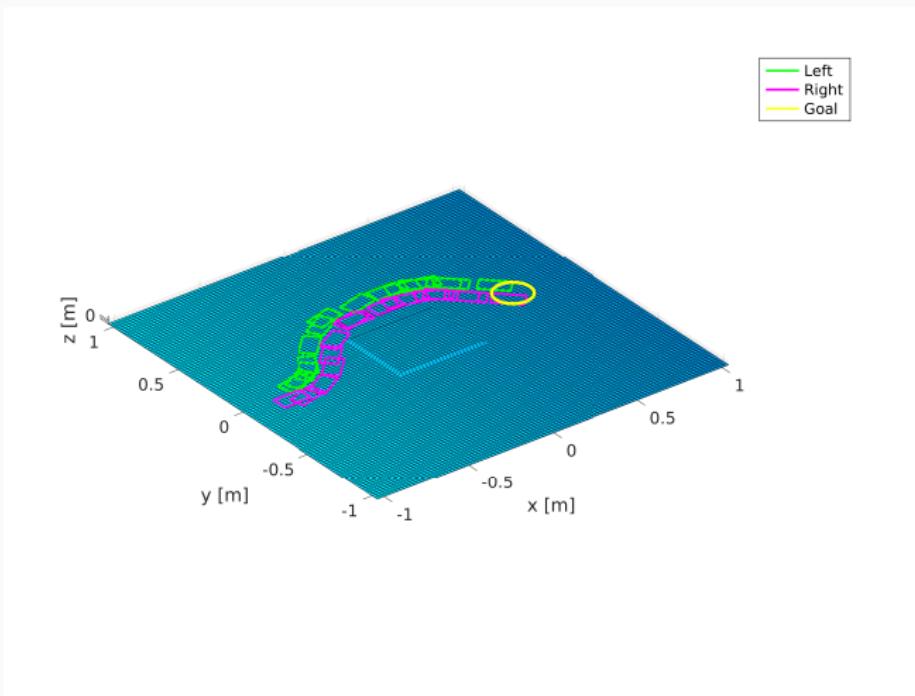


# RRT-based Footstep Planning: Obstacle Avoidance



# RRT-based Footstep Planning: Obstacle Avoidance

tree size: 488 – solution size: 31 – runtime: 70 ms



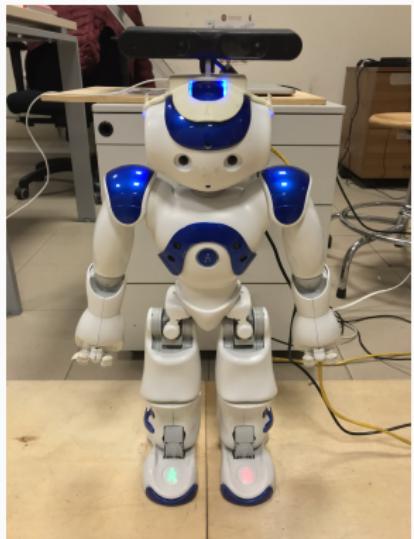
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## elevation\_mapping

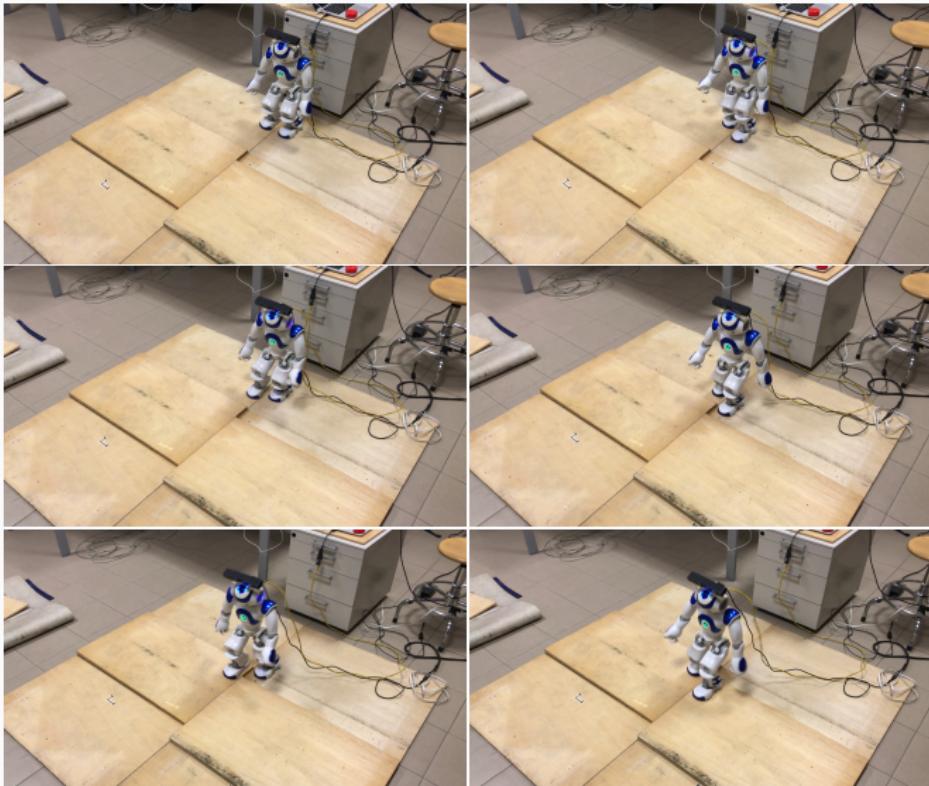
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- dynamic environments using visibility check based on ray tracing

# Elevation Map Generation: Settings

- NAO humanoid robot
- ASUS Xtion Pro (**depth sensor**)
- working range: 0.5–3.5 m
- elevation map extended with **safe zone**
- *unknown World of Stairs* environment

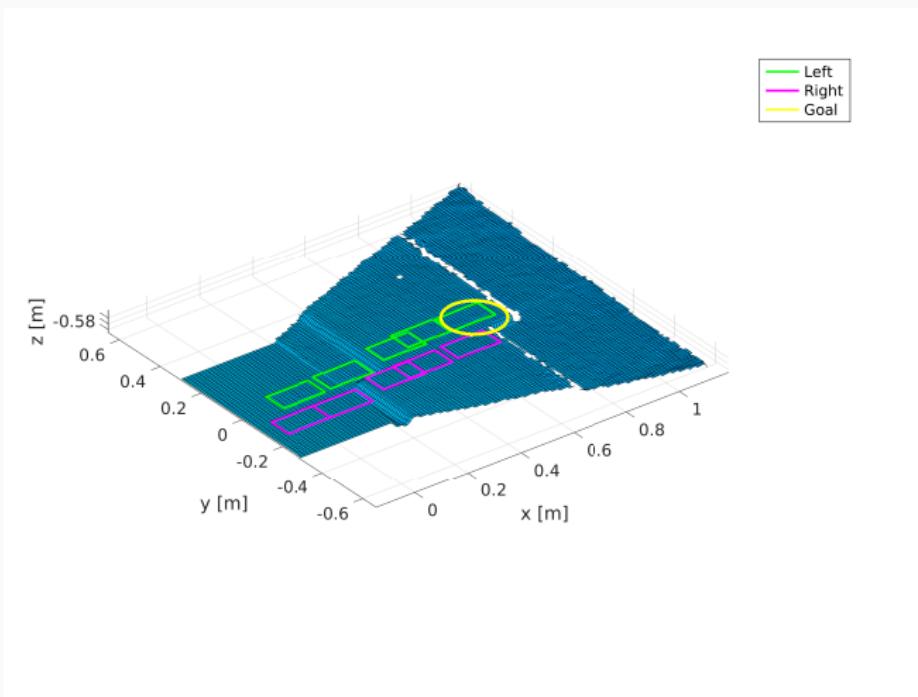


# Stair Climbing in Unknown Environment



# Stair Climbing in Unknown Environment: Footstep Plan

tree size: 454 – solution size: 10 – runtime: 331 ms



Video

## Conclusion: Results and Future Works

- NAO is able to autonomously climb the stairs in an unknown *World of Stairs* environment
- **localization** module and **continuous mapping**
- **replanning** phase
- dynamic and rough environments

Q&A

## References

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