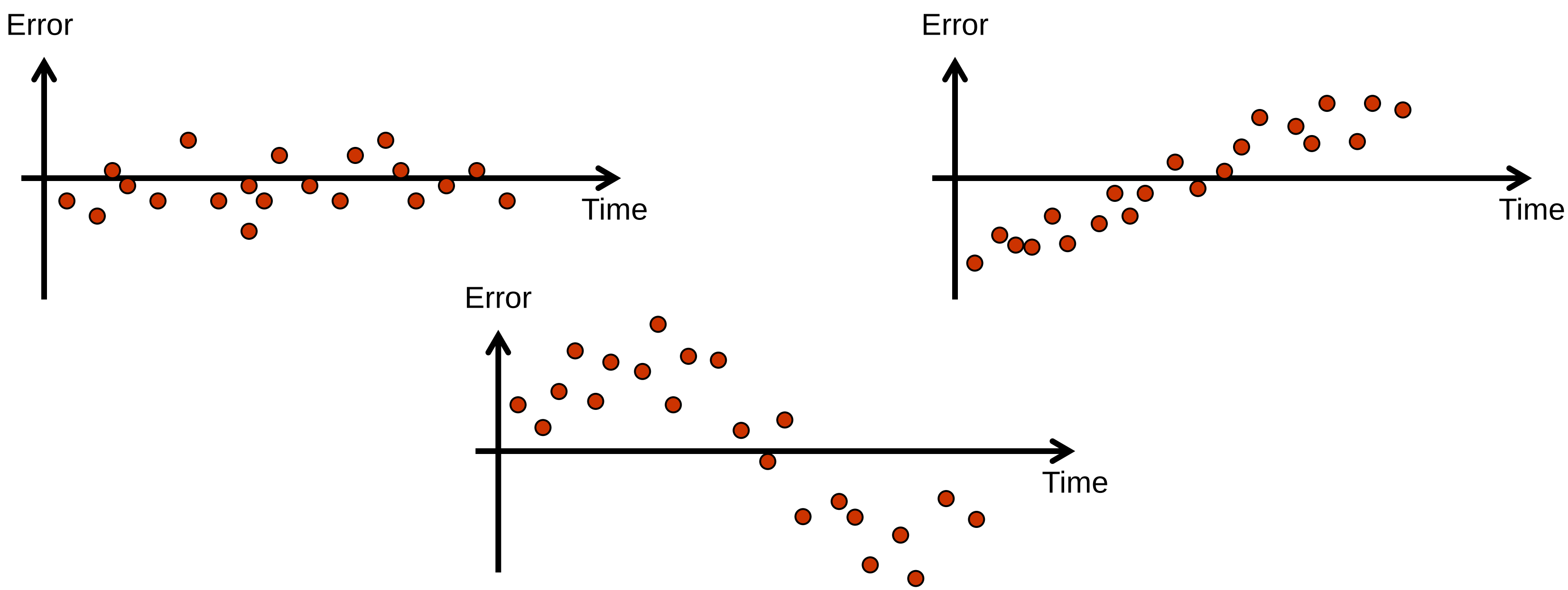
Chapter 6
GNSS Errors

Error Fundamentals

Types of Errors

- There are generally two types of errors; systematic and stochastic. The latter encompasses errors whose values can only be modeled in a statistical manner. How would you classify the following errors?



- How can these different types of errors be handled?

Systematic Errors

- Systematic errors can arise from a variety of sources including operating principles of the system, temperature variations, shock/vibration, etc. There are three basic methods of dealing with systematic errors; modeling, calibration and measurement differencing.
  - Modeling tries to parameterize the error as a function of one or more parameters (e.g., excess delay as a function of the elevation of a satellite).
  - Calibration is the process of using a specific set of tests/measurements to estimate the magnitude of an error (e.g., using pre-defined distances to measure scale factor and bias in an EDM).
  - Measurement differencing tries to cancel errors that are common to two or more measurements. This is only effective to the extent that the measurements being differenced share the same error (i.e., the level of correlation of the measurements).

Stochastic Errors

- Stochastic errors can only be modeled in a statistical manner (more on this in the Data Analysis course). The key point is that linear combinations of stochastic errors tend to *increase* the level of error in a statistical sense. This is well known from the law of error propagation:

$$y = f(x) \qquad \Rightarrow \qquad C_y = \left(\frac{\partial f(x)}{\partial x}\right) C_x \left(\frac{\partial f(x)}{\partial x}\right)^T$$

- Since the errors cannot be predicted in a deterministic sense, stochastic errors are dealt with using estimation techniques such as least-squares and Kalman filtering.

Error Characteristics

- In the context of GNSS, the two most important characteristics of any given error are the magnitude of the error and the variability of the error.
  - In terms of the magnitude, we are concerned with the “absolute” error as well as the relative/differential errors. The latter are usually expressed as part per million (ppm), with 1 ppm = 1 mm per km.
  - In terms of variability, we want to know how the errors change spatially (i.e., with user location) as well as temporally (i.e., in time).

- Also of importance is the manner in which the error sources can be observed/computed. Some applications see the error as the “signal” to be detected (instead of position). This is helpful to separate the different errors.

Error Variability

- ***Temporal variability*** refers to how much and how quickly an error changes over time, at a given location
  - Related to how your computed position will change over time
  - Errors that change quickly (e.g., noise) can be averaged out in a short period of time, but slowly changing errors will take longer to average out

- ***Spatial variability*** refers to how much and how quickly an error changes with distance between two locations
  - Important in the context of measurement differencing/cancellation
  - If errors at two locations are the same or similar, the relative position of the two points can be estimated very well because the common errors will largely cancel in the difference
  - If an error changes very quickly with distance, the level of common errors will be the lower compared to if the error varies slowly with distance; the result will be a less accurate relative position

Differential Errors

- In the context of GNSS, ***differential (or relative) errors*** refer to the difference in the errors at two locations/receivers
- Consider pseudorange measurements made at two points

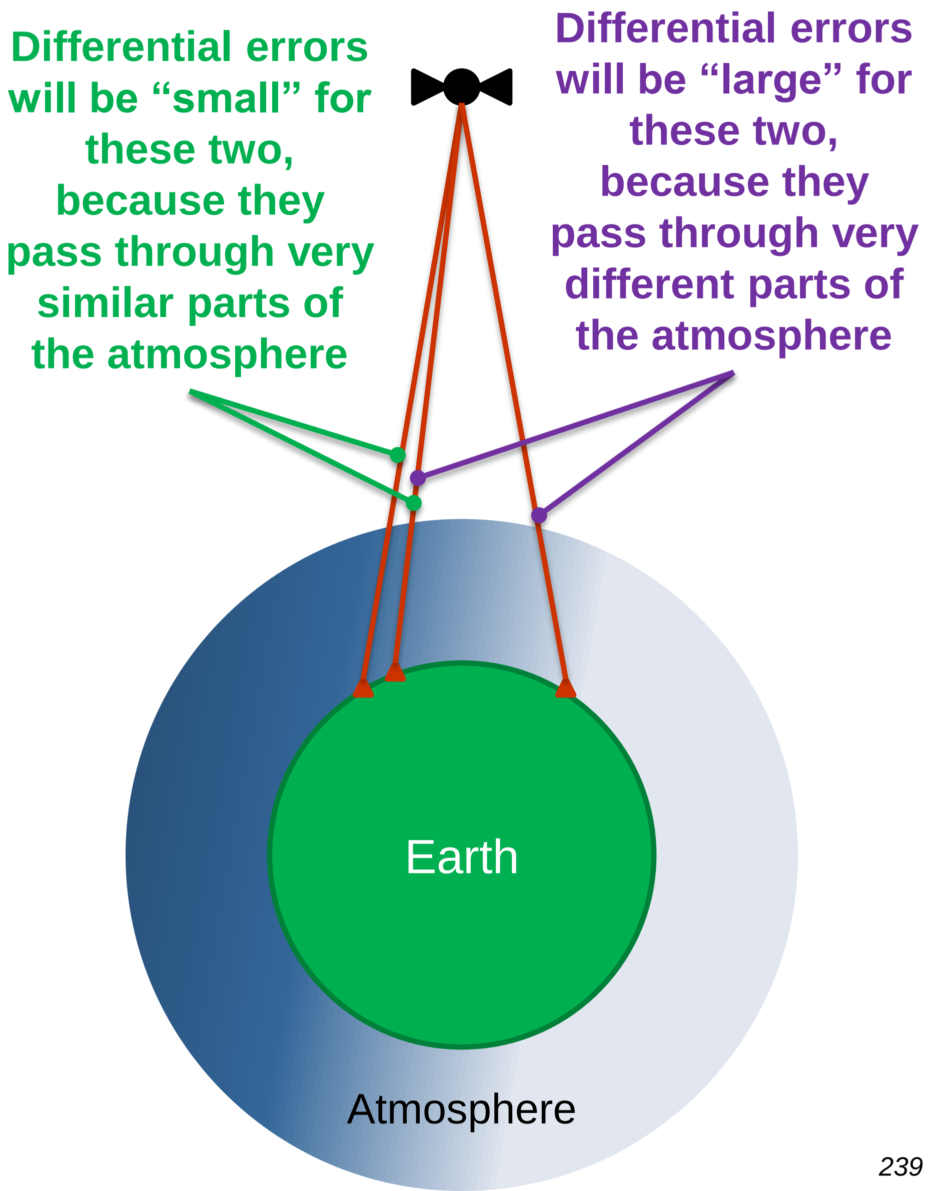
$$P_A = \rho_A + Error_A$$

$$P_B = \rho_B + Error_B$$

- The differential error is the difference between errors at the two points

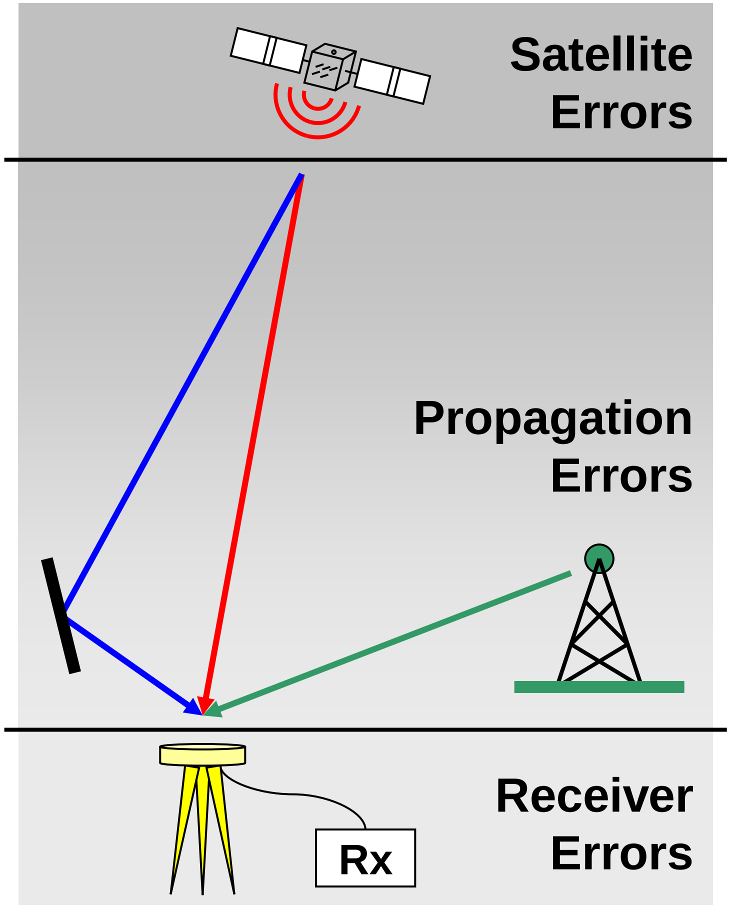
$$\Delta Error = Error_A - Error_B$$

- Often expressed in parts per million (ppm)
  - 1 ppm = 1 mm of (differential) error per 1 km (10^6 m) of separation



Overview of GNSS Errors

GPS Error Sources at a Glance



- Three basic sources of error
  - Satellite-based errors
    - Orbit errors
    - Satellite clocks
    - Group delay (not discussed)
  - Propagation errors
    - Ionosphere
    - Troposphere
    - Multipath
    - Interference
  - Receiver-based errors
    - Noise
    - Receiver clock (estimated)
    - Antenna errors (Chapter 4)
    - Inter-channel biases
    - Timing/Tracking errors

Typical GPS SPS Errors

Error	Uncompensated	Compensated
Broadcast satellite clock	• <1 ms (300 km)	• ~5 ns or ~1.5 m (1σ)
Broadcast orbit	• 1 m (1σ)	• N/A
Ionosphere	• 1-30 m at zenith (scale by ~3 near horizon)	• ~0 for dual frequency receiver • 50% reduction using basic model
Troposphere	• 2.4 m at zenith (scale by ~10 near horizon)	• ~0.2 m
Code multipath	• dm to m level depending on Rx • Max of 0.5 chip	• N/A
Carrier phase multipath	• ≤0.25λ, but typically 0.1λ (1σ)	• N/A
Code Noise	• dm to m level depending on Rx	• N/A
Carrier Phase Noise	• Typically ≤1 mm	• N/A

User Equivalent Range Error (UERE)

- UERE is the quadratic sum of errors affecting the measurements. The value may be different for different type of measurements (i.e., pseudorange, Doppler or carrier phase). May also depend on
  - Receiver and antenna characteristics, since these impact the quality of measurements (e.g., in terms of multipath)
  - L1 vs L1/L2 (for the ionosphere)
  - Operating environment
  - Available error models and their accuracy

$$\mathrm{UERE} = \sqrt{\sigma_{d\rho}^2 + \sigma_{cdT}^2 + \sigma_{iono}^2 + \sigma_{trop}^2 + \sigma_{m}^2 + \sigma_{n}^2}$$

- UERE excludes effects of estimated parameters (i.e., effect of receiver clock is not included since this is estimated)

References

- The following references give a nice characterization of many of the key GNSS/GPS error sources

  - Olynik, M.C. (2002) **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164 http://plan.geomatics.ucalgary.ca/papers/02.20162.molynik.pdf

  - Olynik, M.C., M.G. Petovello, M.E. Cannon and G. Lachapelle (2002) Temporal impact of selected GPS errors on point positioning, GPS Solutions, Vol 6, No 1-2, pp. 47-57. http://plan.geomatics.ucalgary.ca/papers/olynik2002.pdf

Satellite Clock Errors

Broadcast Satellite Clock Model

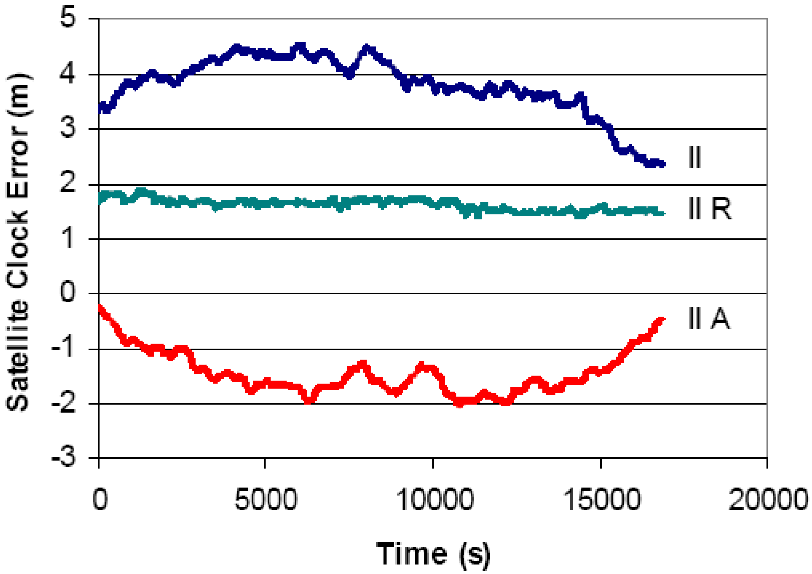
- Due to the high quality of the oscillators in the satellites, the satellite clock errors change smoothly and slowly with time. They are easily modeled using a simple polynomial, whose coefficients (a_{f0}, a_{f1} & a_{f2}) are broadcast in the GPS navigation message. The correction is given by

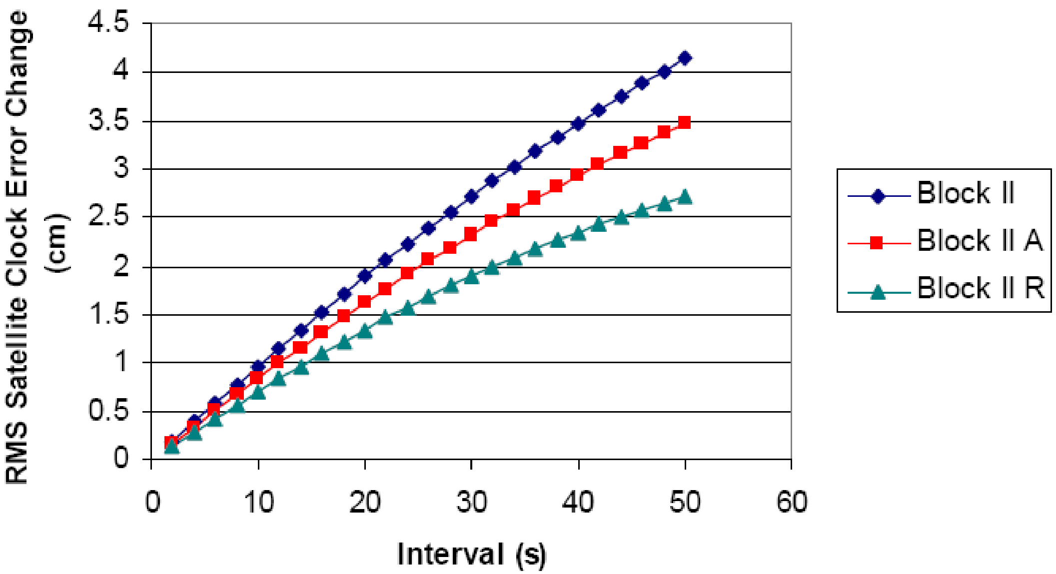
$$dT = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2$$

- The magnitude of the correction is less than 1 ms, and just like the broadcast ephemeris (for satellite orbit), the satellite clock parameters are predicted ahead of time. The error in the prediction is very small; typically 5 ns, or ~1.5 m (http://igscb.jpl.nasa.gov/components/prods.html).
- The error is the ***same*** for all receivers tracking the same satellite at the same time. What does this mean in terms of differential error?

Broadcast Clock Correction Errors

- By comparing the broadcast correction to precise clock corrections (post-mission), the error in the former can be obtained.
  - Newer satellites (Block IIR) tend to have smaller and more linear errors
  - Error changes very slowly with time (< 4 cm / 50 s)



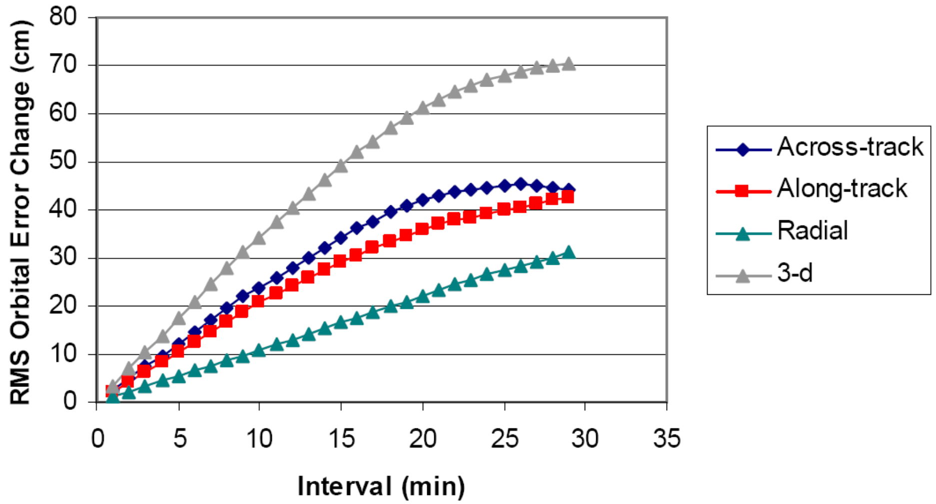


Source: Olynik, M.C. (2002) **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary

Satellite Orbit Errors

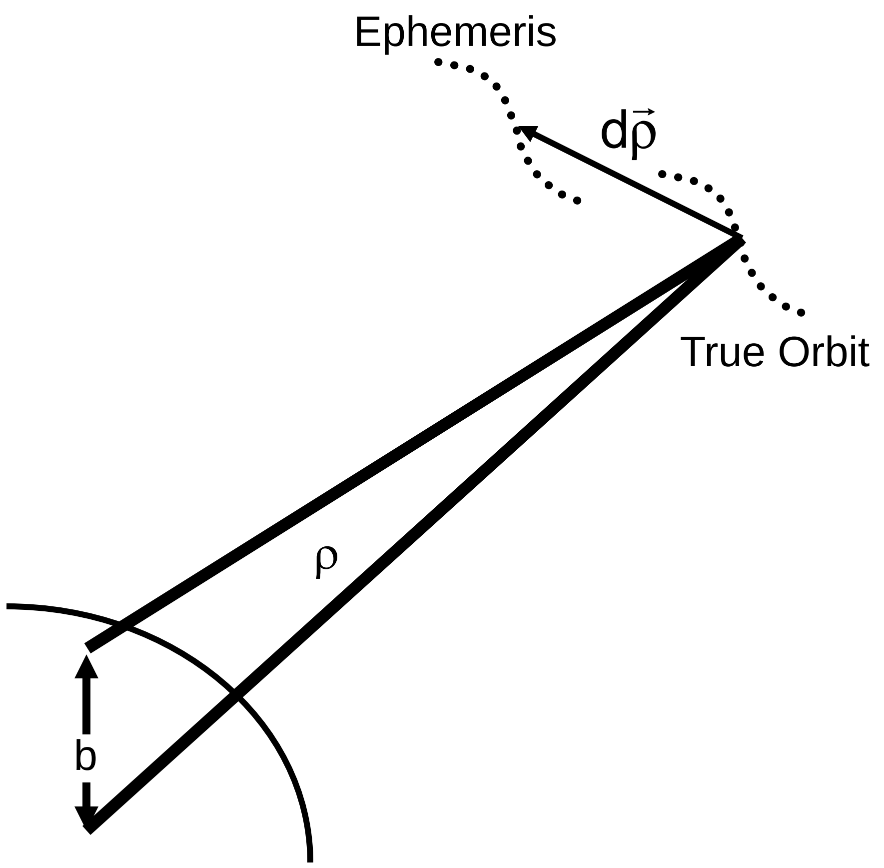
Broadcast Orbital Errors

- We saw back in Chapter 3 that the broadcast orbital errors are on the order of 1 m (1σ).
- The errors also change very slowly in time. This is primarily because of the high altitude of the satellites (little effect of atmosphere and mass anomalies within the Earth).
  - Plot below shows that the 3D error changes by 0.7 m over 30 min, or at a rate of ~0.4 mm/s



Source: Olynik, M.C. (2002) **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary

Differential Orbital Errors



- The differential error is purely geometrical and depends on the orbital error ***vector***, and the line of sight to the different receivers. The worst-case error is approximately given by

$$d\rho_{max} = \frac{\left|d\vec{\rho}\right|}{\rho} b$$

  where b is the baseline length, ρ is the range to the satellite and dρ is orbital error.

- Given that the broadcast orbital error is ~1 m and the range to the satellites is ~20,000 km, the differential error is 0.05 ppm

$$d\rho_{max} = \left(0.05 \times 10^{-6}\right) b$$

Ionosphere

Ionosphere Overview

- The ionosphere is the part of the atmosphere (50 – 1,000 km above the Earth) that contains free electrons. The existence of the ionosphere is due to UV radiation from the Sun. The Sun therefore plays a key role in the level of ionospheric activity. To this end, there are two main variations in the ionospheric effect:
  - Daily (diurnal) variations due to Earth rotation
  - The amount of energy released by the Sun varies with an 11-year solar cycle.

- Largest effects occur in polar regions (auroral zone) and near the geomagnetic equator. Ionospheric scintillation is a major concern, especially in auroral zones and near the geomagnetic equator.

Ionosphere Error Overview (1/2)

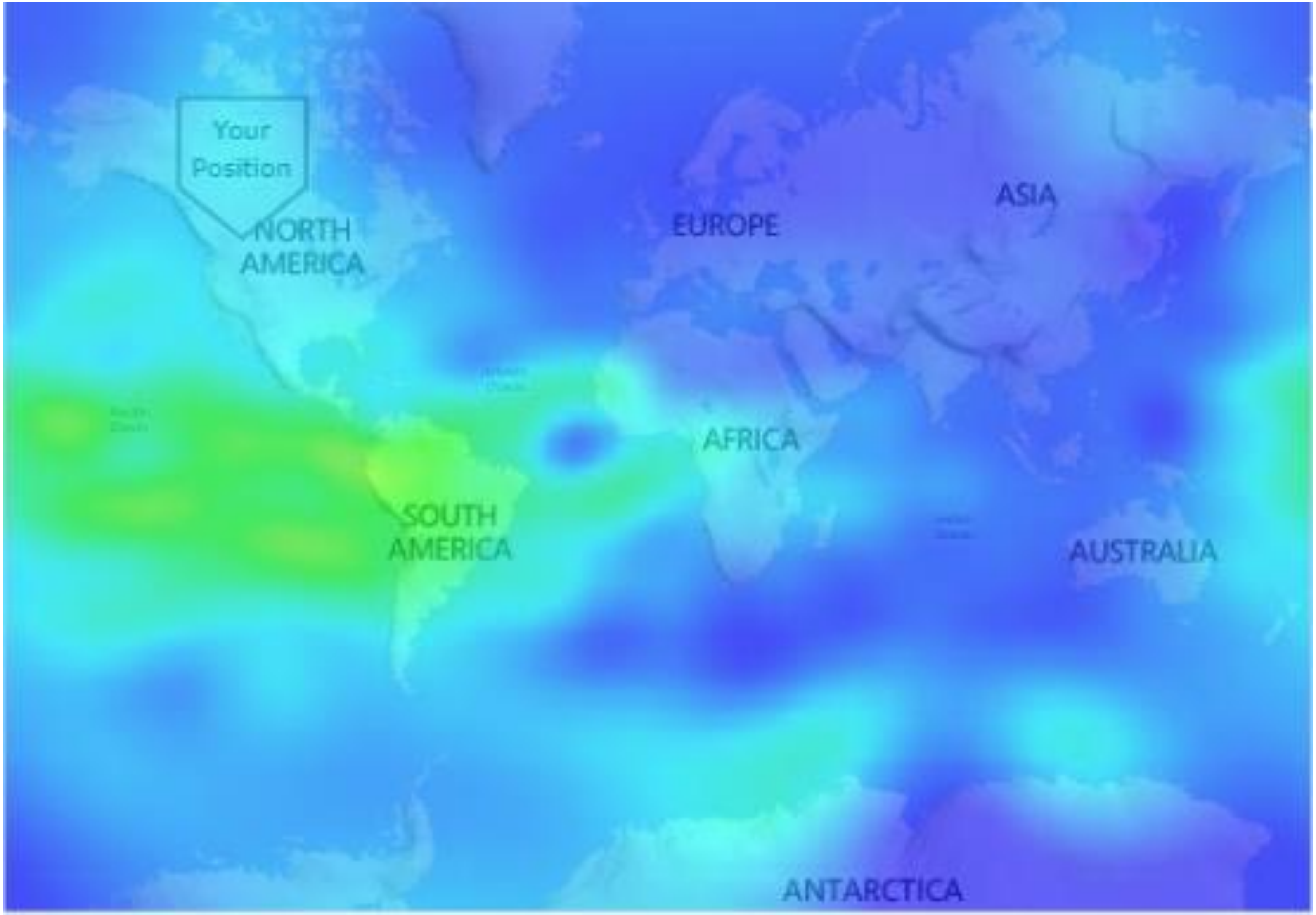
- Ionosphere is dispersive at GNSS frequencies meaning that the magnitude of the error depends on frequency of signal passing through the medium. The magnitude of the error is proportional to the density of free electrons in the atmosphere. The electron density is quantified by counting the number of electrons in a column with a cross-sectional area of 1 m^2, and is called ***Total Electron Content (TEC)***.

- An interesting phenomenon involving the ionosphere is that the pseudorange is *delayed* ("group delay") while the carrier phase is *advanced* ("phase advance") – an effect called ***code-carrier divergence***. Both effects have the same magnitude (i.e., opposite sign).

Ionosphere Error Overview (2/2)

- Some of the key ionosphere error properties are as follows
  - The delay/advance ranges from 1 m to 100 m depending on time of day, solar cycle and satellite elevation (relative to the user)
  - The diurnal maximum occurs around 1400 local time
  - The solar cycle varies the magnitude of the error by a factor of ~3 over a period of 11 years
  - Total delay in the zenith can range from a from ~1 m to ~30 m and increases approximately three-fold at the horizon
  - Increased effects are often encountered in the *auroral zone* near the poles and at ±20° latitude from the *geomagnetic* equator

TEC Map for January 19, 2016, 2:00 pm Calgary

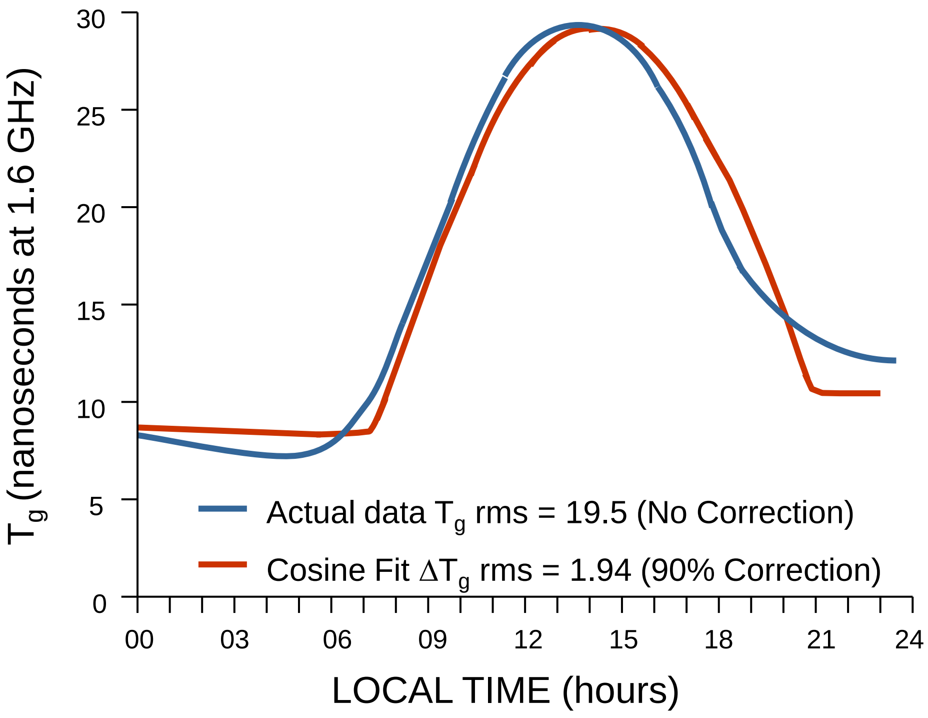


http://www.trimble.com/GNSSPlanningOnline/#/IonoMap

Diurnal Ionosphere Variation

- The diurnal maximum occurs at approximately 1400 hours local time. It is a function of the ionization rate and ion recombination rate.

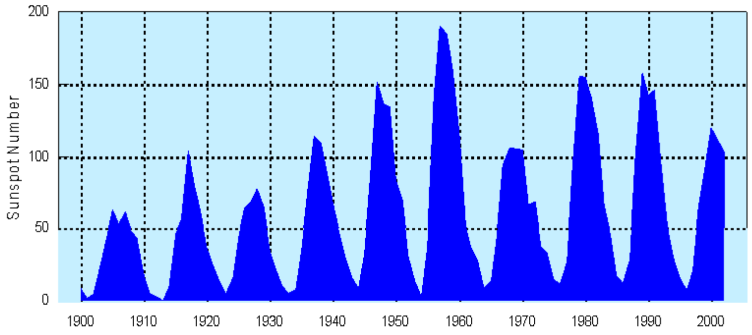
- The GPS broadcast ionosphere model ("Klobuchar model") uses a half-cosine approximation during the day and constant term at night



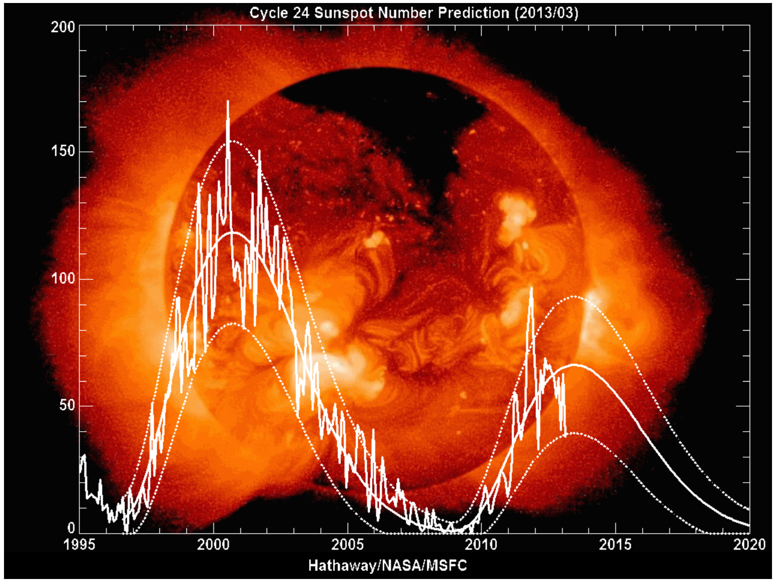
Solar Cycle

- Every 11 years the sun undergoes a period of activity called the "solar maximum”. At this time, there are more sunspots and solar flares. The number of sunspots is a quantification of solar flares, which are the rapid release of large amount of energy.
- Plot to the right shows actual and predicted solar activity

Annual Sunspot Number Since 1900 (from: NOAA)



Current Solar Cycle
(http://solarscience.msfc.nasa.gov/images/ssn_predict_l.gif)



References: Kunches, J. (2007) **GNSS & Space Weather: Making the Least Out of Solar Max**, Inside GNSS, Vol 2, No. 6, 7 pages.

Kintner, P.M., T. Humphreys and J. Hinks (2009) **GNSS and Ionospheric Scintillation: How to Survive the Next Solar Maximum**, Inside GNSS, Vol 4, No. 4, 9 pages.

Indices of Refraction (1/2)

- Before giving equations for the indices of refraction, we first consider a simplified model for a GNSS signal

$$s(t) = \underbrace{\cos(\omega_m t)}_{\text{Modulation Wave}} \underbrace{\cos(\omega_c t)}_{\text{Carrier Wave}}$$

- The modulation wave is a simplified version of the ranging code (recall that the Fourier transform allows us to express any signal as a sum of sinusoids; here, we consider one sinusoid) and is usually called the "group". Correspondingly it travels at the ***group velocity***. By extension, the phase travels at the ***phase velocity***.
- In a dispersive medium, the group velocity and the phase velocity are not equal. Section 5.3 of Misra and Enge (2006) explain this very well.

Ref: Misra, P. and P. Enge (2006) **Global Positioning System Signals, Measurements, and Performance**, Second Edition, Ganga-Jamuna Press.

Indices of Refraction (2/2)

- Phase index of refraction is given as

First order error

$$n_p = 1 - \frac{k_1}{f^2} - \frac{k_2}{f^3} - \frac{k_3}{f^4} - \cdots$$

Higher order terms generally neglected

- Group index of refraction (pseudorange) is given as

First order error

$$n_g = n_p - f \cdot \frac{dn_p}{df} = 1 + \frac{k_1}{f^2} + \frac{2k_2}{f^3} + \frac{3k_3}{f^4} + \cdots$$

Higher order terms generally neglected

- k_i is frequency independent but is a function of the electron density, n_e, along the propagation path (k_1 = -40.3 TEC)
  - First order approximation accounts for most of the error
  - Higher order effects are at most 3-4 orders of magnitude smaller (<1% of first order term)

Klobuchar, J.A. (1996) Ionospheric Effects on GPS, B.W. Parkinson and J.J. Spilker, Jr. eds., Global Positioning System: Theory and Applications, Volume I, Chapter 4, American Institute of Aeronautics and Astronautics, Inc., Washington, DC, USA.

Group Delay and Phase Advance

- To first order, the group delay ionosphere error (in m) is

$$\Delta S_{g_{[m]}} = \int_S \left(n_{g_{[m]}} - 1\right) \cdot ds \approx -\int_S \frac{k_1}{f^2} \cdot ds = +\frac{40.3 \cdot TEC}{f^2}$$

Integral along the signal path

- Similarly, the phase advance (in m) is given as

$$\Delta S_{p_{[m]}} = \int_S \left(n_{p_{[m]}} - 1\right) \cdot ds \approx \int_S \frac{k_1}{f^2} \cdot ds = -\frac{40.3 \cdot TEC}{f^2}$$

Same error magnitude but opposite sign

- To calculate the advance in cycles, divide by wavelength

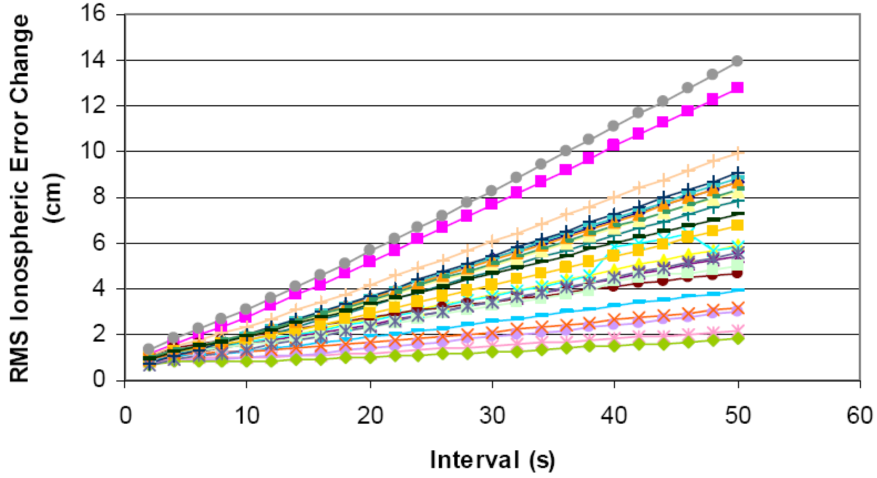
$$\Delta S_{p_{[cyc]}} = \frac{\Delta S_{p_{[m]}}}{\lambda} = \frac{\Delta S_{p_{[m]}} \cdot f}{c} = -\frac{40.3 \cdot TEC}{cf}$$

- TEC ≡ Total Electron Content $\quad TEC = \int_S n_e \cdot ds$

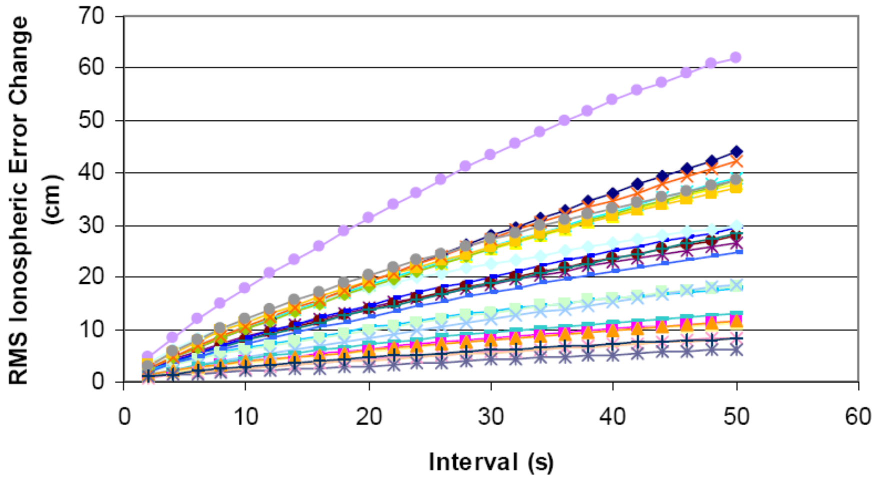
Electron density

Temporal Variability of the Ionosphere

- Top plot shows the typical change of the ionosphere for each satellite under a quiet ionosphere. On average, the ionosphere changes 6 cm/min.



- Bottom plot below shows change in the ionosphere on a very active day. The errors change considerably faster.



Source: Olynik, M.C. (2002) **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary

Ionospheric Scintillation

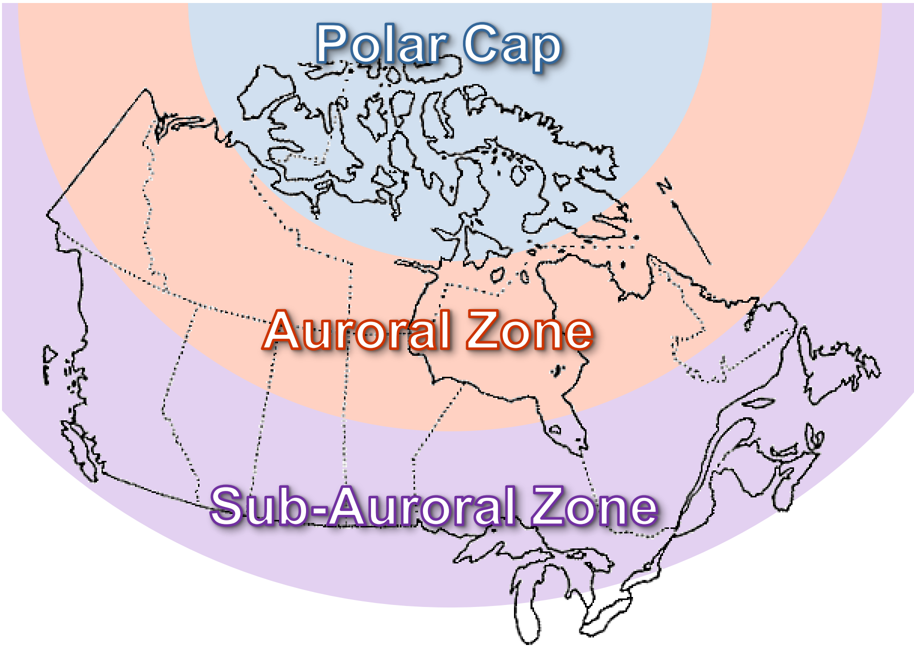
- Scintillation is the rapid variation in the phase and amplitude of the received signal. It is characterized by depth of fading and fading period. Scintillation is caused by electron density irregularities, which, in turn, is typically correlated with level of geomagnetic activity.

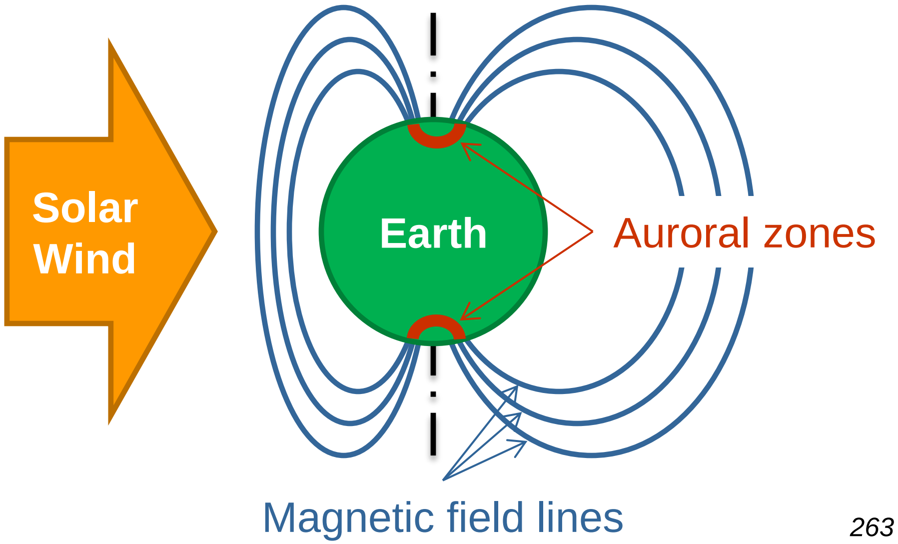
- The effects on GNSS measurements
  - Loss of phase lock due to lower SNR, especially receivers trying to track the L2 P(Y) code (recall that tracking this signal results in worse performance)
  - Loss of phase lock due to phase variations outside the tracking loop bandwidth; in some cases it may prevent the receiver from tracking the signal at all.
  - It can make data more difficult or even impossible to process

Auroral Zone

- The ***auroral region*** is typically 55 to 70 degrees North latitude (it also occurs in the southern hemisphere). It roughly correlates with where the magnetic field lines enter the Earth. However, during intense ionospheric activity, it slides towards the equator.
- Ionospheric activity and scintillation are larger and/or more common in the auroral oval. Care must therefore be taken when working in this area.

Typical Auroral Zone





Mitigating the Ionosphere

- The following options are available for mitigating the effect of the ionosphere
  - Use the Broadcast ionospheric model
  - Using dual frequency data to remove first order ionospheric effects
  - Use a grid model supplied by third party
  - Observe when the ionosphere is quiet. Geomagnetic activity is predicted up to ~1 month in advance and is correlated with ionospheric activity.

- Each of these are discussed in more detail on the following slides.

Broadcast Ionosphere Model

- As the name implies, this model is broadcast in the GPS navigation message (it was developed by J. Klobuchar and is therefore also called the Klobuchar model). It consists of eight parameters that describe a half-cosine whose peak value occurs at approximately 1400 local time.

- The model removes over 50% of ionospheric delay at mid-latitudes.

- Details of how to implement the model are available in the GPS Interface Control Document (ICD).

Dual-Frequency Pseudorange Correction

- Exploit the dispersive nature of the ionosphere by making pseudorange measurements on two frequencies

$$
\begin{aligned}
\Delta P_{[m]} &= P_{L1} - P_{L2} \cong I_{L1}^{P} - I_{L2}^{P} \\
&= 40.3 \cdot TEC \cdot \left[\frac{1}{f_{L1}^{2}} - \frac{1}{f_{L2}^{2}} \right] \\
&= \frac{f_{L2}^{2} - f_{L1}^{2}}{f_{L2}^{2}} \cdot I_{L1}
\end{aligned}
$$

$$
I_{L1}^{P} = \left(P_{L1} - P_{L2}\right) \cdot \frac{f_{L2}^{2}}{f_{L2}^{2} - f_{L1}^{2}}
$$

Correct the measurement

$$
\begin{aligned}
P_{L1}^{IF} &= P_{L1} - I_{L1}^{P} \\
&= P_{L1} - \left(P_{L1} - P_{L2}\right) \cdot \frac{f_{L2}^{2}}{f_{L2}^{2} - f_{L1}^{2}} \\
&= \frac{-f_{L1}^{2}}{f_{L2}^{2} - f_{L1}^{2}} \cdot P_{L1} + \frac{f_{L2}^{2}}{f_{L2}^{2} - f_{L1}^{2}} \cdot P_{L2} \\
&= \frac{f_{L1}^{2}}{f_{L1}^{2} - f_{L2}^{2}} \cdot P_{L1} - \frac{f_{L2}^{2}}{f_{L1}^{2} - f_{L2}^{2}} \cdot P_{L2}
\end{aligned}
$$

- This approach removes >99% of the ionosphere error.
  - Is there a downside to this?

Dual-Frequency Carrier Phase Correction

- The same basic approach can theoretically be taken with carrier phase measurements

Scale to L1 cycles

$$\Delta\phi = \phi_{L1} - \frac{f_{L1}}{f_{L2}} \cdot \phi_{L2} \cong I^{\phi}_{L1} - \frac{f_{L1}}{f_{L2}} \cdot I^{\phi}_{L2} + N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2}$$

$$= I^{\Phi}_{L1} \cdot \frac{f^2_{L2} - f^2_{L1}}{f^2_{L2}} + N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2}$$

"Geometry-free" combination for phase

$$I^{\phi}_{L1} = \left[\left(\phi_{L1} - \frac{f_{L1}}{f_{L2}} \cdot \phi_{L2}\right) - \left(N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2}\right)\right] \cdot \frac{f^2_{L2}}{f^2_{L2} - f^2_{L1}}$$

- In this case, we require knowledge of the carrier phase ambiguities (difficult!). By extension, if no cycle slips occur over a given time interval, the change in the ionospheric error can be computed. Can we combine this with the pseudorange estimate from the previous slide?

Ionosphere-Free (IF) Linear Phase Combination

- Instead of computing and applying the error as on the previous slide, the ionosphere-free (IF) linear combination for carrier phase measurements is usually formed

$$\phi_{IF} = \phi_{L1} - \frac{f_{L2}}{f_{L1}} \cdot \phi_{L2}$$

- However, in this case, the carrier phase ambiguity is also not integer. The importance of this will become obvious in the next Chapter when we discuss how to estimate the ambiguities.

$$N_{IF} = N_{L1} - \frac{f_{L2}}{f_{L1}} \cdot N_{L2}$$

Geomagnetic Observatories & Predictions (1/2)

- There are 14 geomagnetic observatories in Canada which are operated by the Geological Survey of Canada (NRCan). Furthermore, the Canadian Magnetic Observatory Network is used to predict and monitor geomagnetic activity.
    - Geomagnetic forecasts are computed for 27 day periods http://www.spaceweather.gc.ca/sf-eng.php

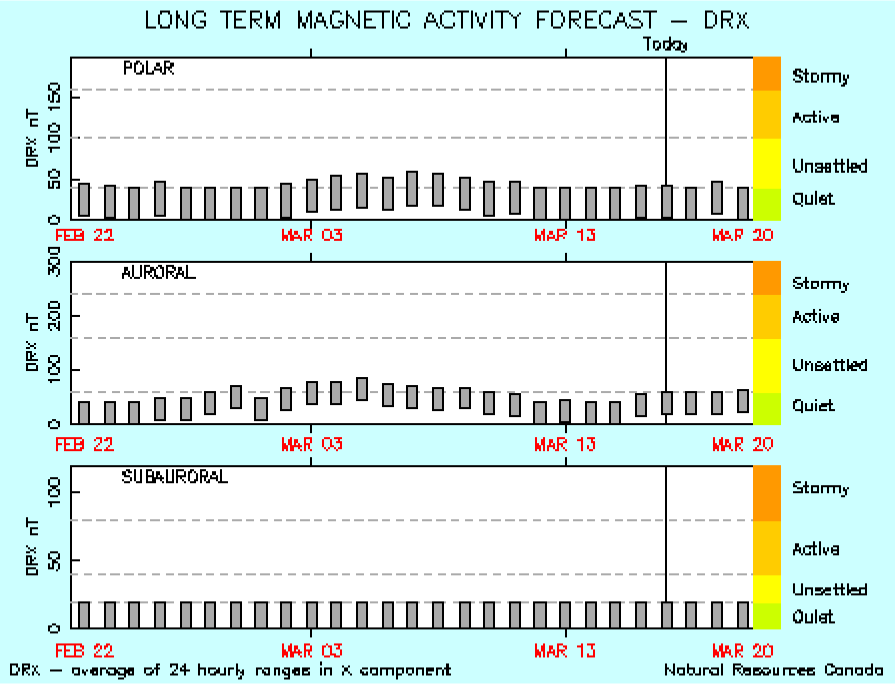
- Activity is reported in nanoteslas (nT) for three regions:
    - Polar cap ($|\phi| > 70°$ N)
    - Auroral Zone ($55° < |\phi| < 70°$ N)
    - Subauroral Zone ($|\phi| < 55°$ N)

- Daily Range (DRX) Index used
    - nT – nanotesla [unit of magnetic induction: 1 tesla = 1 weber/m^2]

Geomagnetic Observatories & Predictions (2/2)

- Another metric that can be useful is the K-index. We will consider two such indices:
  - Kp index: This is a global indicator of geomagnetic activity. It is computed as the global average value of the disturbance levels in the (horizontal) magnetic field components
  - Kr index: This is similar to the Kp index but is available in read-time and can be predicted ahead over short time periods (few hours). It is also often provided on a regional basis.
- Both of the above indices vary from 0 (quiet) to 9 (major storm)

Sample Geomagnetic Predictions :: DRX

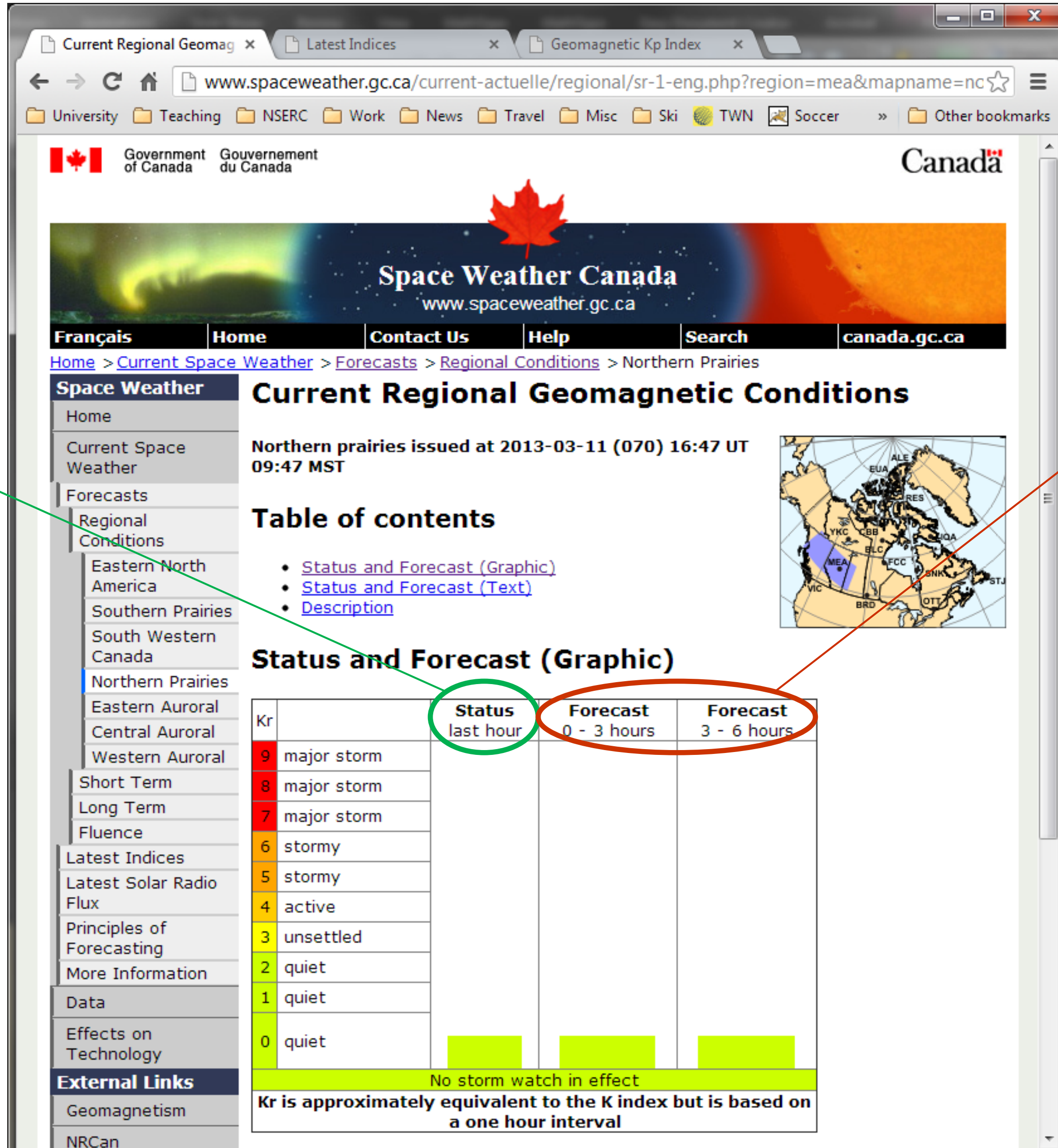


http://www.spaceweather.gc.ca/sf-eng.php

Zone/DR Index	DRX values for forecast classification in nT			
	Quiet	Unsettled	Active	Stormy
Polar Cap	0-40	40-100	100-160	>160
Auroral Zone	0-60	60-160	160-240	>240
Sub-Auroral Zone	0-20	20-40	40-80	>80

Sample Geomagnetic Predictions :: Kr Index

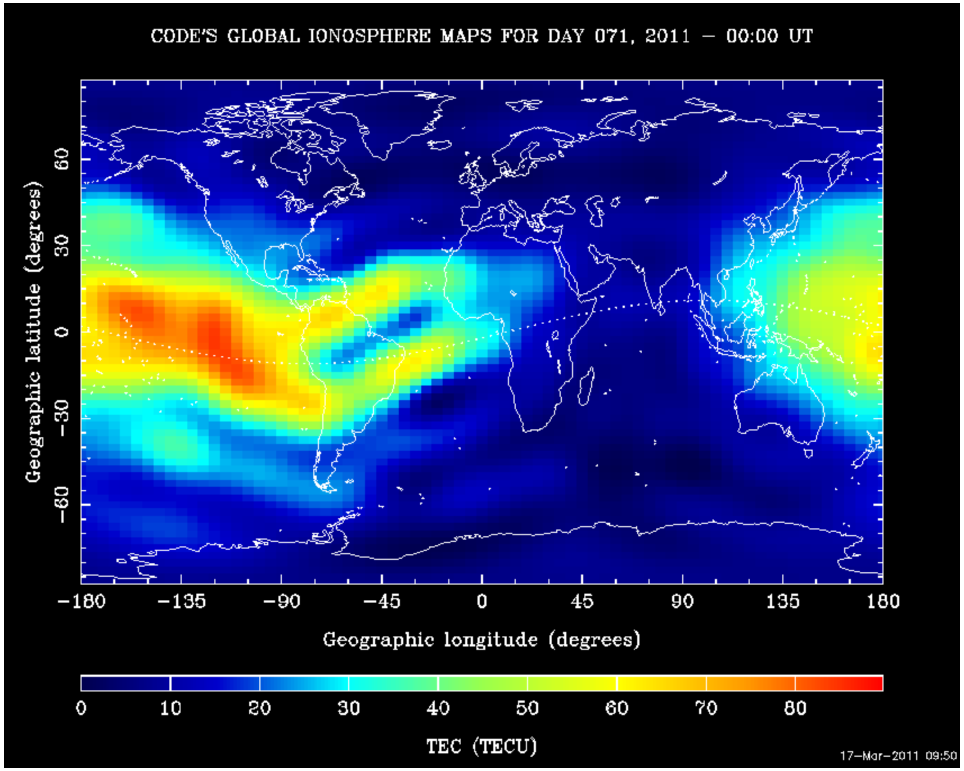
Recent Activity



Forecasts

Global Ionospheric Maps

- Several civilian groups have developed global or regional ionospheric maps based on the IGS (or similar) data. This is generally done in conjunction with Wide Area Differential GPS (e.g., WAAS) processing.



http://aiuws.unibe.ch/ionosphere/gim.gif

This can be used to interpolate errors for your location. More specifically, the errors are interpolated to where the line-of-sight signal intersects with the upper atmosphere (which is typically modeled as a thin shell).

Observed Differential Ionosphere Errors

- The magnitude of the differential error is a function of geographic location

Geographic Location	Error Magnitudes	
	Typical (RMS)	Extreme
Auroral Zone	1-3 ppm	10 ppm [1] 10 ppm [2]
Mid-Latitude	1-3 ppm	8 ppm [3] 10 ppm [1]
Low Latitudes	1-3 ppm	17 ppm [4] 30 ppm [5]

[1] Skone, S. (1998) Wide Area Ionosphere Grid Modelling in the Auroral Region, PhD Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20123 (available at: http://geomatics.ucalgary.ca/links/GradTheses.html).

[2] Lachapelle, G., P. Alves, L.P. Fortes and M.E. Cannon (2000) DGPS RTK Positioning Using a Reference Network, Proceedings of ION GPS 2000, Institute of Navigation, pp. 1165-1171.

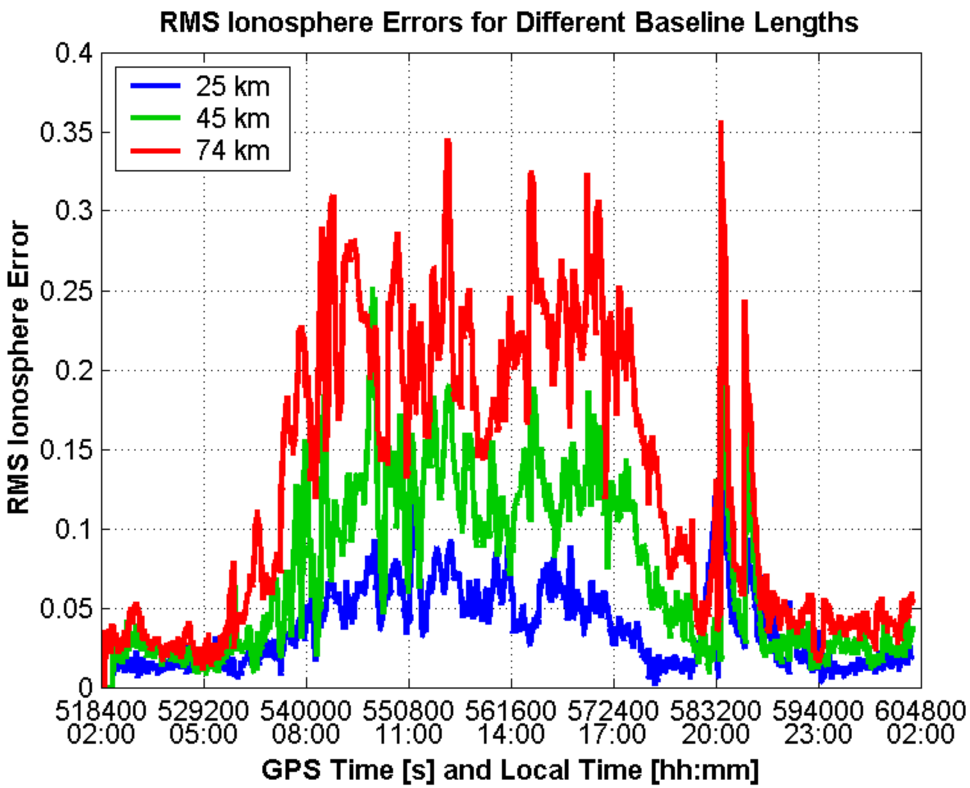
[3] Klobuchar, J.A., P.H. Doherty, M.B. El-Arini (1995) Potential Ionospheric Limitations to GPS Wide-Area Augmentation System (WAAS), Navigation: Journal of the Institute of Navigation, Vol 42, No 2, Alexandria, VA, pp. 353-370.

[4] Fortes, L.P., M.E. Cannon and G. Lachapelle (2000) Testing a Multi-Reference Station Network for OTF Positioning in Brazil, Proceedings of ION GPS 2000, Institute of Navigation,
pp. 1133-1142.

[5] Wanninger, L. (1993) Effects of Equatorial Ionosphere on GPS, GPS World, July 1993, pp. 48-54.

Differential Ionospheric Error Example

- RMS (across satellites) of double difference ionosphere errors as a function of time
  - Diurnal effect is obvious
  - Strong dependence with baseline length
  - Jumps near 20:00 and 21:00 caused by new low elevation satellites

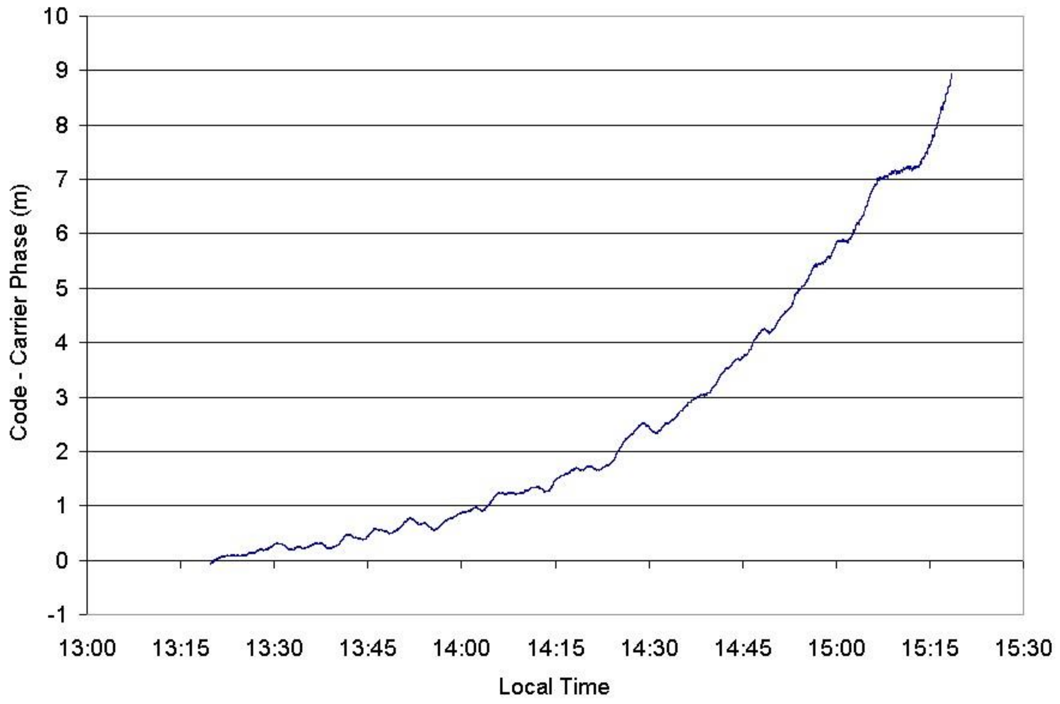


Data from TUBITAK Marmara Continuous GPS Network – MAGNET (Turkey)

Code-Carrier Divergence

- Code and carrier diverge over time as a byproduct of the group delay and phase advance. The magnitude is dependent on the level of ionospheric activity.

Example for data collected on May 7, 2001 at the UofC over a 2 hour period



Troposphere

Troposphere and GNSS (1/2)

- The term “troposphere” is a bit ambiguous. The atmospheric science community defines it as extending ~9 km over the poles to ~16 km at the equator and includes most of the water vapour. *For GNSS, however, the neutral (non-dispersive) part of the atmosphere, up to about 50-70 km, is called troposphere.* Unlike the ionosphere, the troposphere is non-dispersive. That is, it affects L1 and L2 the same.

- The magnitude of the error is about 2.4 m at the zenith and increases by approximately a factor of four for an elevation angle of 15º. Fortunately, relatively simple models can estimate the effect to within about 20 cm and thus the error is not a significant problem for low accuracy positioning. Rather, problems can arise for high accuracy positioning.

Troposphere and GNSS (2/2)

- The tropospheric delay is subdivided into dry (hydrostatic) and wet components, with the latter being harder to model accurately.
- In general, differential error is not a large problem for short baselines with similar heights. Conversely, if high accuracy is required or if receivers have different heights, the differential error may be important.
- Methods to compensate for the troposphere:
  - Apply a tropospheric model (e.g., Hopfield or Saastomoinen)
  - Model a residual zenith delay as part of the estimation process
  - Use a water vapour radiometer

Brunner, F.K. and W.M. Welsch (1993), ‘Effect of the Troposphere on GPS Measurements, GPS WORLD, January.

Refractivity

- Tropospheric models usually replace the index of refraction with the refractivity, N, defined as

$$N = (n-1)\cdot 10^6 = \delta n \cdot 10^6$$

- Given its definition, the refractivity can be interpreted as a parts-per-million error. To this end, the typical global average is 320 ppm (i.e., for 100 m of propagation, the troposphere introduces 3.2 cm of error).

- Analogous to the separation of the tropospheric error is divided into dry and wet parts, the total refractivity is also separated into dry and wet parts

$$N = N_d + N_w$$

Dry refractivity

Wet refractivity

Dry and Wet Refractivity

- The dry (hydrostatic) refractivity accounts for 80-90% of the total errors and is a function of surface pressure (P) and surface temperature (T). Fortunately, these parameters are able to be well predicted with altitude and thus a ‘dry’ model can remove ~99% of the error.
- In contrast, the wet refractivity accounts for 10-20% of the total errors and is a function of partial pressure of water vapour (e) and surface temperature (T). Since the water vapour content is difficult to predict with altitude, a ‘wet’ model can only remove 80-90% of the error.

Term	Error Magnitude (Typ.)	Correction Accuracy (Typ.)	Error After Correction (Typ.)
Dry	1.9 m (80%)	99%	0.02 m
Wet	0.5 m (20%)	85%	0.08 m
Total	2.4 m	96% (computed)	0.10 m

Troposphere Models

- Troposphere models usually try to model the dry and wet refractivity in the zenith and then map this error to a particular elevation angle (of the satellite). Note that the mapping function is usually different for the dry and wet errors.

$$\begin{aligned} N &= N_d + N_w \\ &= N_d^z \cdot m_d(\varepsilon) + N_w^z \cdot m_w(\varepsilon) \end{aligned}$$

- A couple notes
  - The mapping functions, $m(\varepsilon)$, assume the atmosphere is isotropic. That is, they assume the atmosphere is the same in all directions. Is this reasonable?
  - Ideally, the zenith refractivity values should account for the height of the user (above h=0). Otherwise, the algorithm may over-compensate the error.

Example: Hopfield Model (1/2)

- The refractivity at the surface is modeled as

$$N^z_{d,Surf} = 77.64 \frac{P}{T} \qquad N^z_{w,Surf} = 3.73 \times 10^5 \frac{e}{T^2}$$

P & e in mbar
T in Kelvin

- The model then assumes the refractivity varies with altitude (h) to a maximum height (H_d or H_w) above which the refractivity is assumed to be zero:

$$N^z_d(h) = N^z_{d,Surf} \cdot \left(\frac{H_d - h}{H_d} \right)^4 \qquad N^z_w(h) = N^z_{w,Surf} \cdot \left(\frac{H_w - h}{H_w} \right)^4$$

where, typically

$$H_d = 40136 + 148.72 \cdot (T - 273.16)$$

$$H_w = 11{,}000 \text{ m}$$

Example: Hopfield Model (2/2)

- Integrating from zero along a vertical path gives

$$
\begin{aligned}
d_{trop} &= 10^{-6}\int\left[N_d^z(h)+N_w^z(h)\right]\cdot dh \\
&= 77.6\times10^{-6}\frac{P_{Surf}}{T_{Surf}}\frac{H_d}{5}+0.0373\frac{e_{Surf}}{T_{Surf}^2}\frac{H_w}{5}
\end{aligned}
$$

  - How do you accommodate the situation where the user is not at sea level?

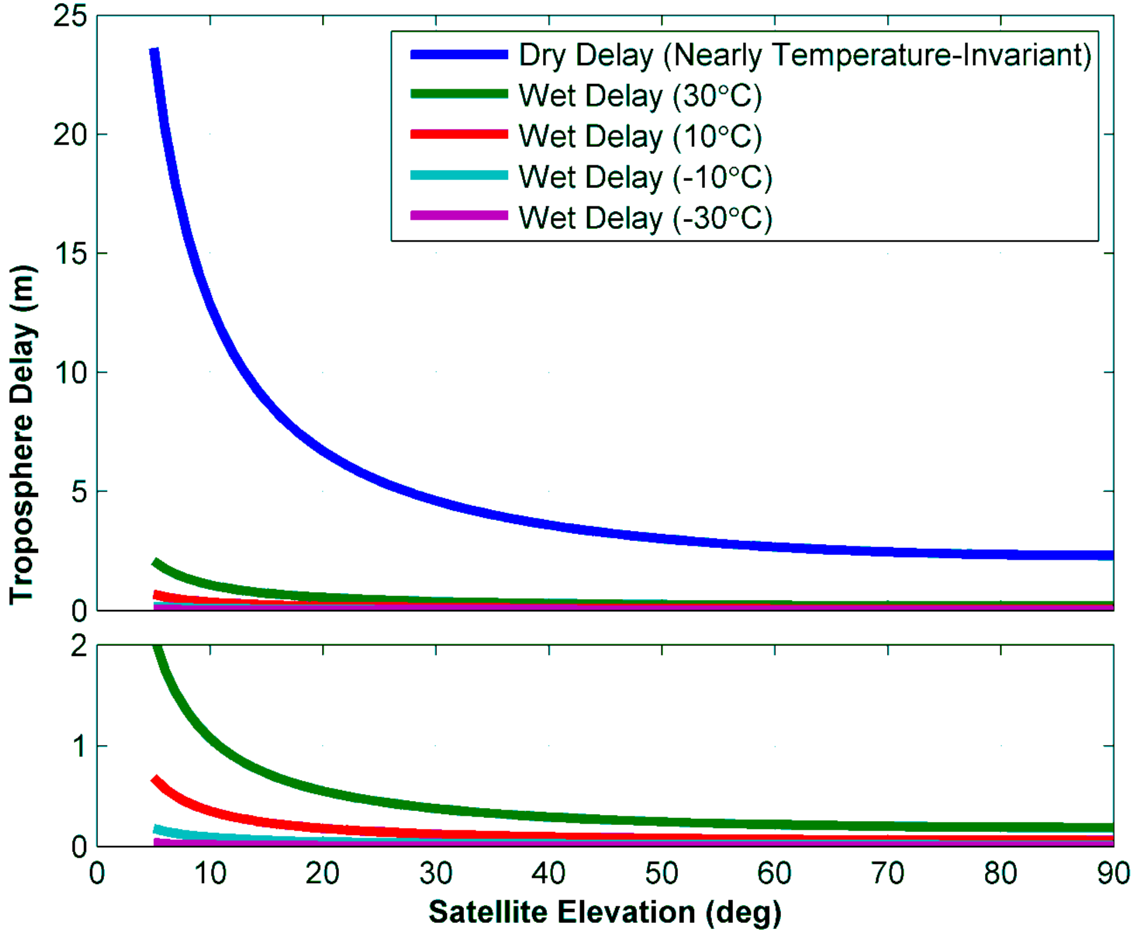
- Some possible mapping functions include

$$
m_d(\varepsilon)=\frac{1}{\sin(\varepsilon)+\dfrac{0.00143}{\tan(\varepsilon)+0.0445}} \qquad m_w(\varepsilon)=\frac{1}{\sin(\varepsilon)+\dfrac{0.00035}{\tan(\varepsilon)+0.017}}
$$

- Other tropospheric models include Saastamoinen, Modified Hopfield, Black and Eisner, Baby et al., Ifadis (wet term only)

Effect of Temperature

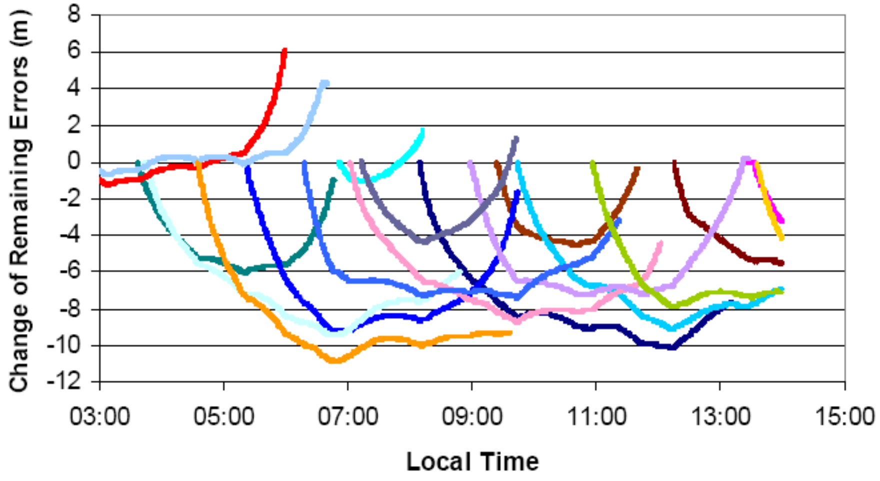
- Temperature has a significant impact on the troposphere error. As the temperature decreases, the wet component tends to zero. Why?



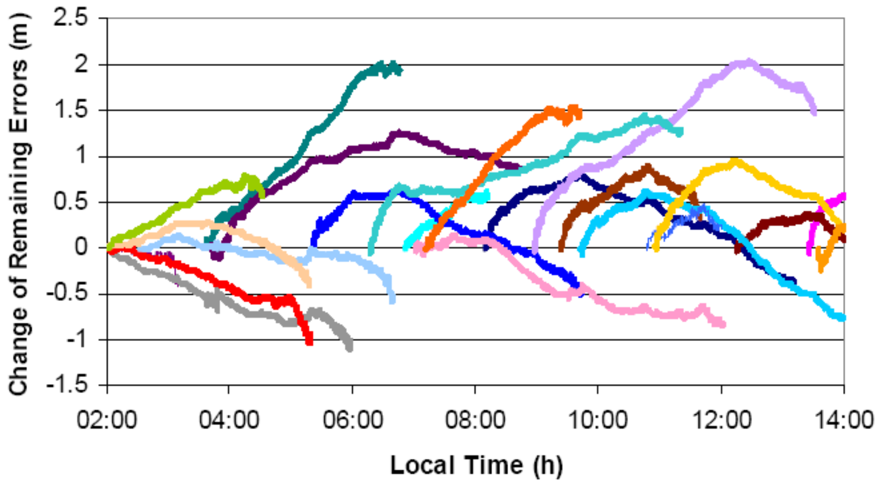
Temporal Variability of Troposphere Error

- Plots below show the change in range errors after compensating for orbit, satellite clock and ionosphere errors
  - Result contain troposphere, multipath and noise errors
  - Large effect due to satellite elevation
- Applying a troposphere model is quite effective at reducing the errors

Before Applying Troposphere Model



After Applying Troposphere Model



Source: Olynik, M.C. (2002) **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary

Differential Tropospheric Error

- Once a troposphere model is applied, the differential errors are typically less than 1 ppm.

- Differential errors larger than about 3 ppm are very rare. These occur mainly in areas of high humidity and/or areas of unstable weather (e.g., storms).
  - Large height differences at both receivers (e.g., airplane and airport) combined with a troposphere model that does not consider user height would also introduce large errors.

Receiver Clock Error

GNSS Receiver Clock Error

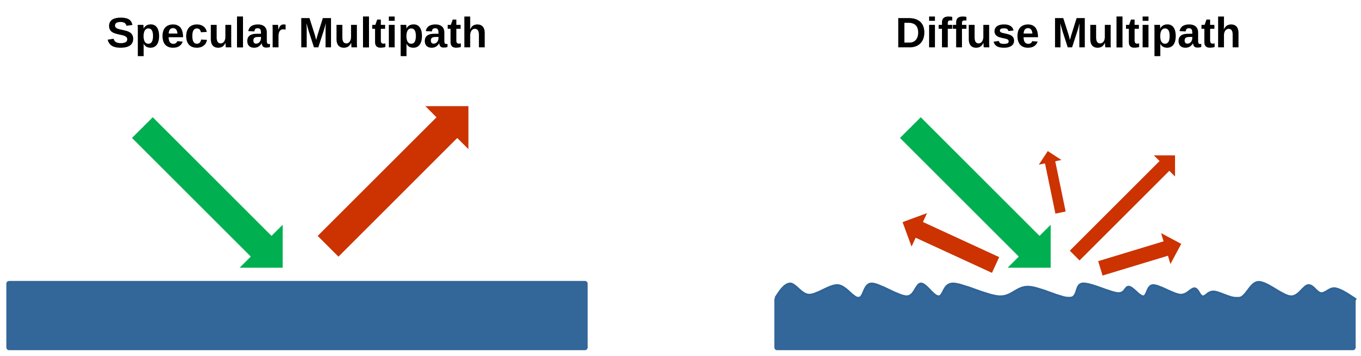
- We have already seen that a GNSS receiver's estimate of GNSS time is generally incorrect and thus introduces a bias into *all* of the measurements at a given instant.
  - What does this say about what will happen when a between-satellite difference is formed?

- Although the clock error could theoretically grow without bound, GNSS receiver manufacturers usually limit it to within predefined levels (typically ±1 ms). This can be enforced by periodically resetting the receiver's clock estimate (called a millisecond jump) or by "steering" the receiver's oscillator such that the clock error remains approximately constant (i.e., no clock drift).

Multipath

Multipath Error Overview

- Multipath is when the signal is received by a user via more than one path (usually the line of sight path and one or more reflected paths).
  - Specular multipath is when parallel incident rays remain parallel after reflection. This occurs when reflecting off of smooth surfaces such as glass and standing water.
  - Diffuse multipath occurs when a signal get reflected in different directions. This occurs with rough reflecting surfaces.



- Which type of multipath do you think is worse?

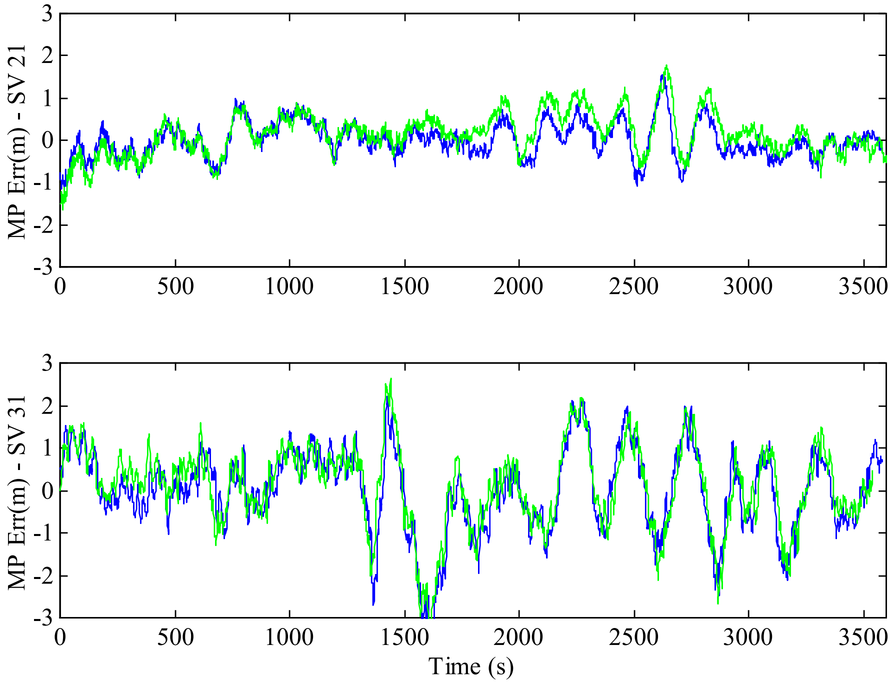
Multipath Correlation

- At a given (static) location, multipath is typically correlated over 10's of seconds. Beyond this time, the error appears uncorrelated until one full sidereal day has passed (see graph to the right).
  - Why does this happen? What assumptions are implicit in this statement?
- Spatially, however, multipath decorrelates over a few cm. In other words, for all practical purposes, multipath errors at different receivers is independent.

Day-to-Day Multipath For PRNs 21 and 31

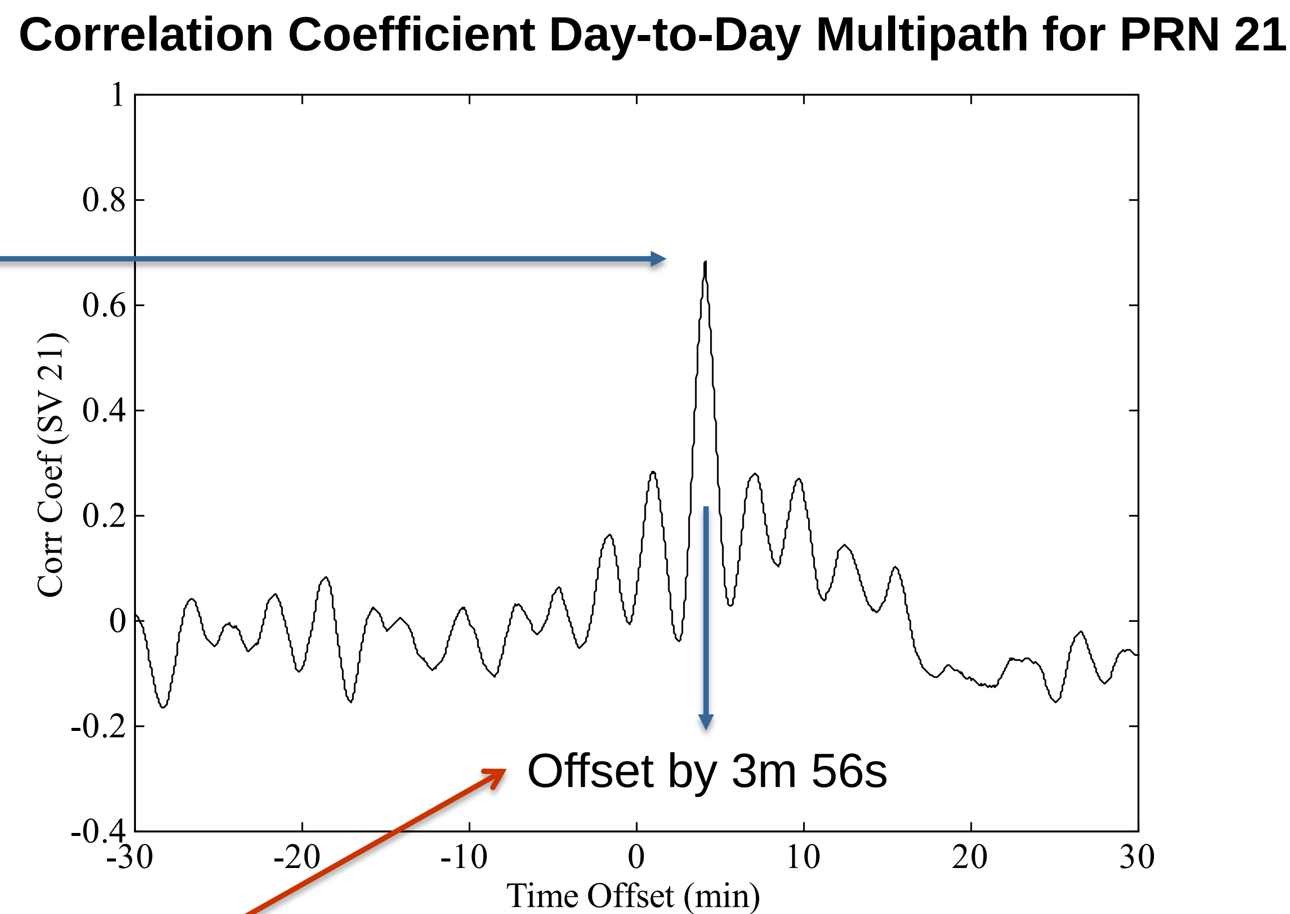
Day 1 in blue (dark)

Day 2 in green (light)



Day-to-Day Multipath Correlation

Not perfectly correlated due to noise



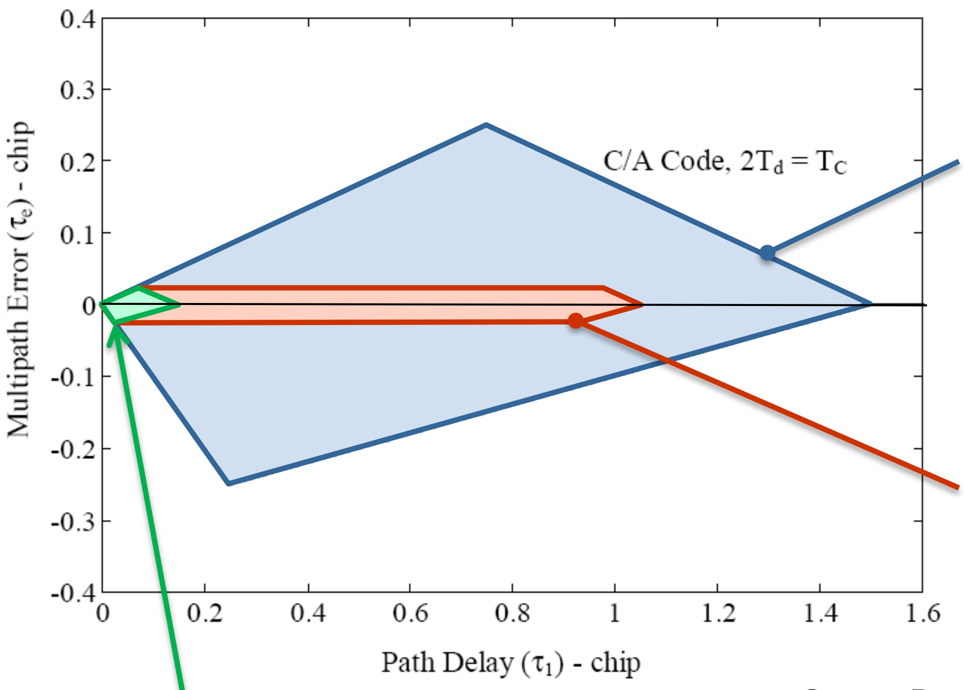
Why is the offset 3m 56s?
Where does this come from?

Multipath Error Summary

Characteristic	Pseudorange	Carrier Phase
Presence of Error	Completely dependent on reflecting geometry	
Magnitude	Maximum of 0.5 chips	Maximum of 0.25 λ
Zero mean?	No	Yes
Temporal variability	Approximately sinusoidal over a few 10's of seconds	
Temporal correlation	Over a few 10's of seconds	
Spatial correlation	Zero (beyond ~10 cm)	
Reduced by differential?	No (amplified in terms of a standard deviation)	

Pseudorange Error Envelope

- Plot shows the error envelope for different “types” of correlators, assuming the reflected signal has half the power of the direct signal
  - Upper curve is for in-phase reflection
  - Bottom curve is for out of phase reflection
  - Reflections not perfectly in or out of phase fall in shaded area



“Wide Correlator”
This term is used to describe the type of code tracking that is used in typical consumer-grade GNSS receivers. As shown, large multipath errors are possible, but the processing is efficient and saves pawer.

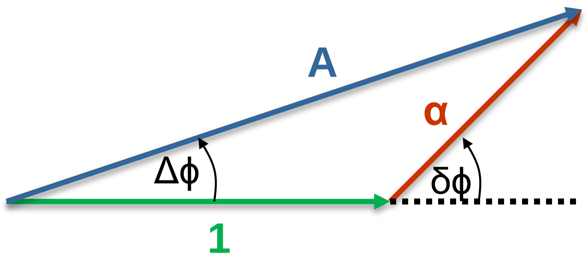
“Narrow Correlator”
This is a technology developed to help mitigate the effect of multipath. Every manufacturer of geodetic-grade GNSS receivers will have a similar type of technology (but called something different, e.g., strobe correlator, double delta correlator, vision correlator, etc.)

P-Code w/ Wide Correlator

Source: Ray, J.K. (2002) **Mitigation of GPS Code and Carrier Phase Multipath Effects Using a Multi-Antenna System**, PhD Thesis, Department of Geomatics Engineering, University of Calgary.

Carrier Phase Multipath

- Carrier phase multipath is best envisioned using a phasor diagram
  - The direct signal has amplitude of unity and we will assume zero phase
  - A reflected signal has amplitude α and is offset from the direct signals by δϕ
  - As with code multipath, the receiver only sees the sum of the direct and reflected signals (composite signal) and this is what it tracks



- The composite signal now has a phase offset relative to the direct signal → multipath
- It can be shown that the *maximum* error is theoretically limited to λ/4, but in practice it is closer to λ/5
- A typical error (standard deviation) is between λ/10 and λ/6, depending on the environment

Estimating Multipath

- In general, the standard deviation of multipath can only be estimated using special test setups. To this end, there are two options:

- Short baseline test
  - Use two receivers at known locations. The receivers should be located nearby each other so that the differential effects of the atmosphere are effectively zero.
  - After collecting data, form the double difference, realizing that the geometric range terms are known. What remains should be noise and multipath. Show the equations to support this statement.

- Code-minus-Carrier
  - This approach was introduced in Chapter 5 and is used to estimate the change in the pseudorange multipath. Why can we not estimate the full magnitude?

What do to about Multipath?

- Site selection
  - Select site with minimum obstructions.
  - Roofs are particularly poor
  - Watch surrounding medium (e.g. water)
- Antenna selection
  - Select antenna which minimizes multipath (e.g., use ground plane or choke ring)
- Modeling
  - Requires modeling of conductivity of surrounding environment, which is very difficult and mathematically cumbersome.
- Use receivers with multipath mitigation technologies
  - Includes some of the advanced correlator techniques discussed in Chapter 4

TOWNSEND, B., and P. FENTON [1994] A Practical Approach to the Reduction of Pseudorange Multipath Errors in a L1 GPS Receiver. ION GPS-94, Salt Lake City, 21 – 23 Sept.

VAN NEE, R.D.J., and J. SIERECELD, FENTON, P., and B. TOWNSEND [1994] The Multipath Estimating Delay Lock Loop – Approaching Accuracy Limits. Proceedings of PLANS-94, pp. 246-251.

Noise

Measuring/Quantifying Noise

- The most common method of measuring noise is to use a zero-baseline test. As the name suggests, you use two collocated receivers (i.e., one antenna and split the signal). In this way, all of the errors except for noise are the same in both receivers.

- By forming the double difference, the only term that remains is the double difference noise, and, in the case of carrier phase measurements, the double difference ambiguities.
  - Write out the measurement equations to support these comments.

- How does the level of noise in the above procedure compare with the level of noise on any given measurement?