Chapter 5
GNSS Observations

GNSS Measurements

Carrier Phase Ambiguity

Cycle Slips

Observation Combinations

Introduction to Differential Processing

GNSS Measurements

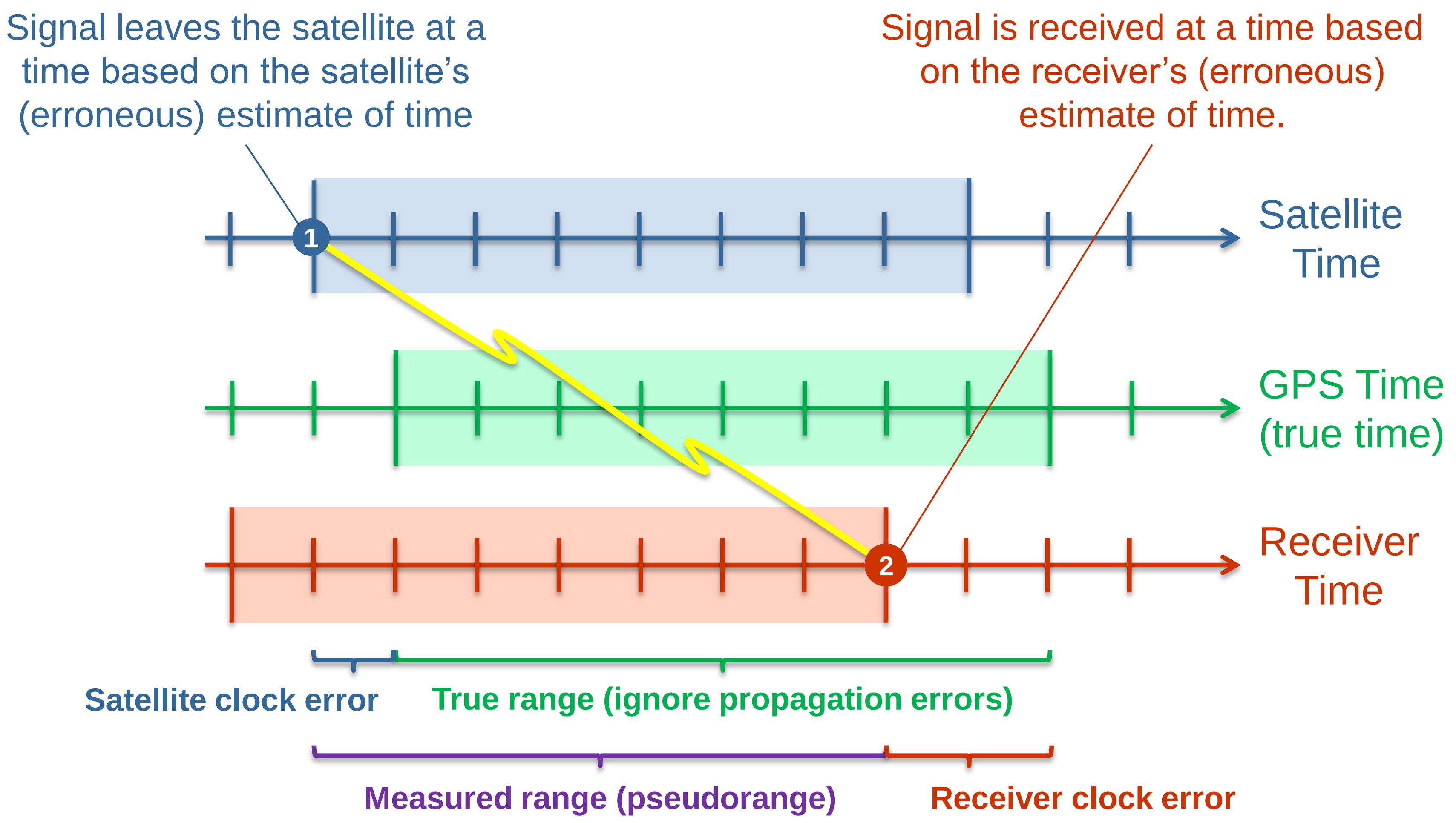
Review of GNSS Measurements

- Recall that there are three main types of observations available from a GNSS receiver
  - Pseudorange
  - Doppler shift
  - Carrier phase (also called accumulated Doppler)

- The pseudorange and carrier phase are proportional to the distance between the user and the satellite and the Doppler shift is proportional to the range rate between the user and the satellite

Effect of Clock Errors on Pseudorange and Carrier Phase Observations

Shaded areas represent the times between when the signal leaves the satellite and when it is received by the receiver as measured by different clocks



Pseudorange ("Code") Measurement

- The pseudorange measurement equation can be written as

$$P = \rho + d\rho + c(dT - dt) + d_{iono} + d_{trop} + m_P + n_P$$

$$\begin{aligned}
P &= \text{Pseudorange} \\
\rho &= \text{Geometric range} \\
&= \left|\vec{r}_{SV} - \vec{r}_{Rx}\right| = \sqrt{\left(x_{SV} - x_{Rx}\right)^2 + \left(y_{SV} - y_{Rx}\right)^2 + \left(z_{SV} - z_{Rx}\right)^2} \\
d\rho &= \text{Orbit error} \\
dT &= \text{Satellite clock error} \\
dt &= \text{Receiver clcok error} \\
d_{iono} &= \text{Ionosphere error (dispersive)} \\
d_{trop} &= \text{Troposphere error} \\
m_P &= \text{Pseudorange multipath error} \\
n_P &= \text{Pseudorange noise}
\end{aligned}$$

We will look at the different error sources in more detail in Chapter 6

Carrier Phase Measurement

- The carrier phase measurement equation can be written as

$$\Phi = \phi\lambda = \rho + \lambda N + d\rho + c(dT - dt) - d_{iono} + d_{trop} + m_{\Phi} + n_{\Phi}$$

Φ = Carrier phase in units of length (e.g., m)

ϕ = Carrier phase in units of cycles

λ = Carrier wavelength

ρ = Geometric range

N = Carrier phase ambiguity (integer)

$d\rho$ = Orbit error

dT = Satellite clock error

dt = Receiver clcok error

d_{iono} = Ionosphere error (dispersive)

d_{trop} = Troposphere error

m_{Φ} = Carrier phase multipath error

n_{Φ} = Carrier phase noise

We will look at the different error sources in more detail in Chapter 6

Pseudorange and Carrier Phase Differences

The main differences between the pseudorange and carrier phase measurements are

1. Carrier phase is ambiguous by the integer number of cycles ("ambiguity") between the receiver and satellite *when the signal was (re-)acquired* (NEW!). The pseudorange is "absolute".
2. The pseudorange measurement is less precise
   - Multipath
     - Pseudorange: Typically m-level to 10's of m for C/A code, depending on the receiver and environment
     - Carrier phase: Maximum of $\lambda/4$ (~4.5 cm for L1)
   - Noise
     - Pseudorange: cm- to m-level for C/A code, depending on the receiver
     - Carrier phase: Typically <1% of a cycle (mm level)
3. Sign of the ionosphere error (we will see why next chapter)

Doppler Measurement

- The Doppler measurement equation can be written as the time derivative of the carrier phase measurement (the carrier phase is the integrated Doppler)

$$\dot{\Phi} = \dot{\phi}\lambda = \dot{\rho} + d\dot{\rho} + c(d\dot{T} - d\dot{t}) - \dot{d}_{iono} + \dot{d}_{trop} + m_{\dot{\Phi}} + n_{\dot{\Phi}}$$

- The geometric range rate is given by

$$\dot{\rho} = \frac{\left(\vec{v}_{SV} - \vec{v}_{Rx}\right) \bullet \left(\vec{r}_{SV} - \vec{r}_{Rx}\right)}{\left|\vec{r}_{SV} - \vec{r}_{Rx}\right|}$$

- What can you estimate with Doppler measurements?

Carrier Phase Ambiguity

Carrier Phase Ambiguity (1/3)

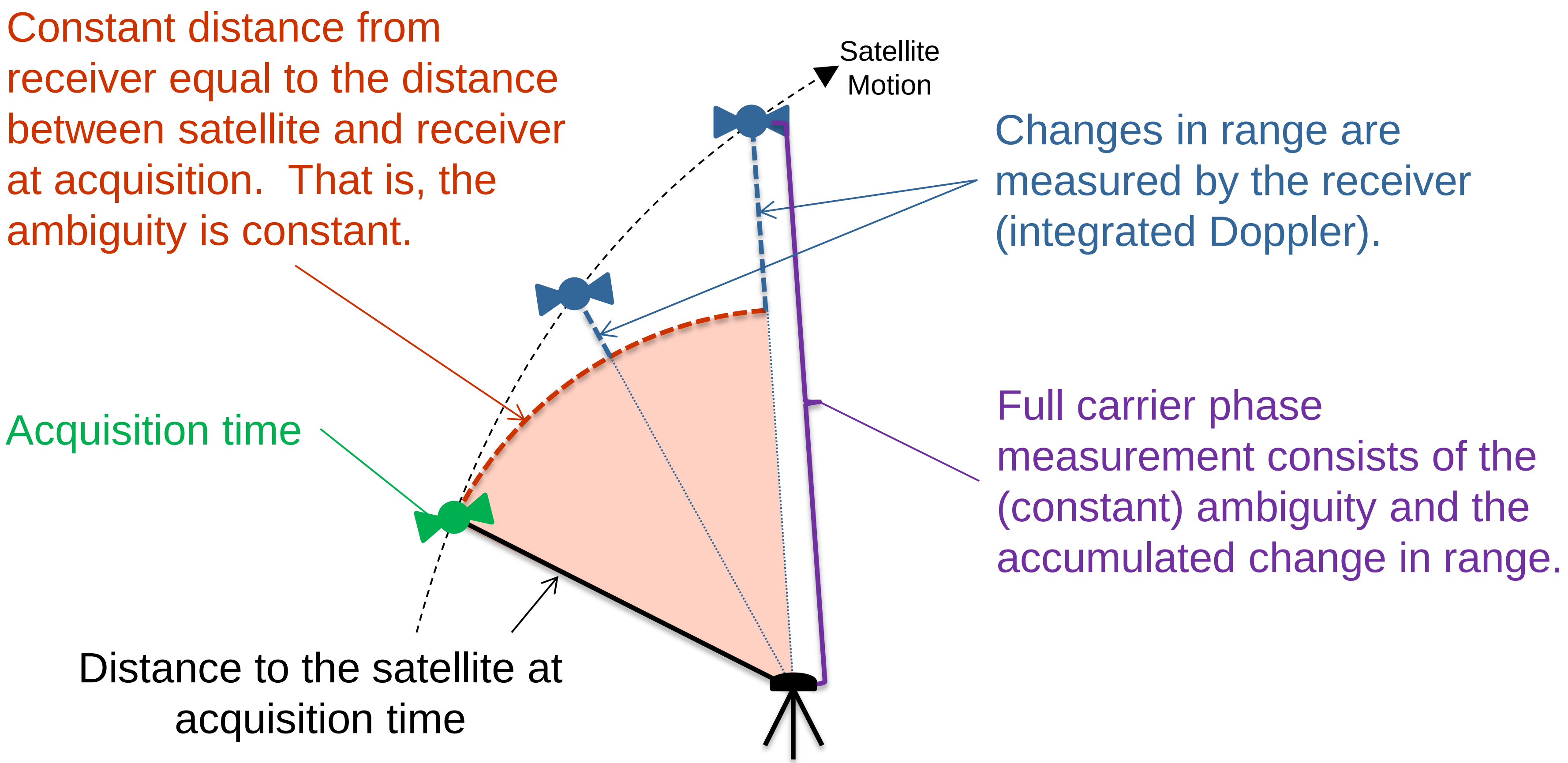
- The carrier phase is generated by integrating the Doppler within the receiver. Considering the geometric range term only, for convenience:

$$\begin{aligned}\Phi(t) &= \int_{t_0}^{t} \dot{\Phi}(t)\cdot dt \\ &\approx \int_{t_0}^{t} \dot{\rho}(t)\cdot dt \\ &= \Delta\rho(t_0, t) + \rho(t_0)\end{aligned}$$

- The integration constant, $\rho(t_0)$, is the distance between the satellite and the receiver *when the signal is first acquired* (i.e., at “lock on”). The integer number of cycles contained in this distance is the carrier phase ambiguity.

Carrier Phase Ambiguity (2/3)

- After acquisition, the integer number of cycles between the receiver and satellite is unknown. However, ***if no loss of lock occurs***, this value is ***constant***. Over time, the receiver continually measures the change in range.



Carrier Phase Ambiguity (3/3)

- Carrier phase ambiguities are integer by definition and are defined at acquisition. They are constant unless loss of lock occurs, even if only for a fraction of a second.
- The ambiguities are arbitrary (e.g., a few cycles or millions of cycles; positive or negative) and are different for each satellite-receiver measurement. In other words, they do not behave like the receiver clock error which is the same for all satellites. As such, estimating the ambiguities is much more difficult; more on this later in the course.
- The ***approximate*** ambiguity can be derived using pseudorange and carrier phase. Why is this only approximate?

GPS time (s)	Pseudorange (m)	Carrier phase (cycles)	Ambiguity (ϕ - P/λ)
387234	22441825.779	-975001.392	-118907592
387235	22441597.023	-976188.862	-118907577
387236	22441371.704	-977375.523	-118907580

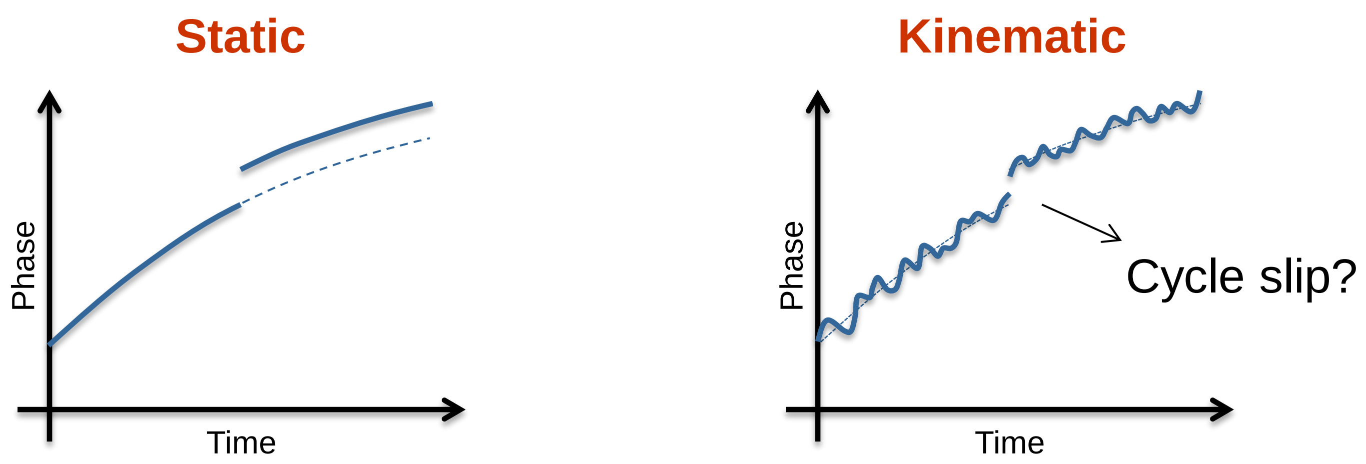
Cycle Slips

Definition of Cycle Slips

- If the receiver loses lock on the carrier (even if only briefly), the carrier phase ambiguity will change. A change in the carrier phase ambiguity over time is called a ***cycle slip***. Cycle slips occur for a variety of reasons
  - Weak signals
    - Signal attenuation or obstruction
    - Bad cable or cable connections
  - Multipath
  - High receiver dynamics (accelerations or jerks)
  - Scintillation effects due to ionosphere

Cycle Slips

- Detection of carrier phase cycle slips is critical for proper and effective use of carrier phase data. Furthermore, cycle slips are more difficult to detect in kinematic applications than in static applications because an additional Doppler shift is created by user motion which is difficult to predict. Options for cycle slip detection are dependent on operating mode (static vs. kinematic) and receiver type (single vs. dual frequency).



Single Frequency Cycle Slip Detection (1/2)

- One of the most common methods of detecting cycle slips is using the ***phase velocity trend*** method. In this case, the carrier phase measurement at a given epoch is predicted from a previous phase measurement and the predicted value is compared to the measured value. If the difference between the predicted and measured values exceeds a threshold, a cycle slip is declared to be present. This method can be applied to any frequency (i.e., L1, L2, L5, etc.).

Predict phase

$$\hat{\phi}_k = \phi_{k-1} + \frac{\dot{\phi}_k + \dot{\phi}_{k-1}}{2}\Delta t$$

Compare with measurement

$$\left|\hat{\phi}_k - \phi_k\right| > T \Rightarrow \text{Cycle slip}$$

- What parameters will affect the selection of T?

Single Frequency Cycle Slip Detection (2/2)

Example cycle slip calculations using phase velocity trend

GPS Time (s)	ϕ (cycles)	$\dot{\phi}$ (Hz)	$\hat{\phi}$ (cycles)	$\left\lvert \hat{\phi} - \phi \right\rvert$ (cycles)	Cycles Slip?
154426	4847.073	146.266	N/A	N/A	N/A
154427	4992.748	144.844	4992.628	0.120	no
154428	5136.899	143.688	5137.014	0.115	no
154429	5280.325	143.156	5280.321	0.004	no
154430	5452.985	142.032	5422.919	30.066	yes

Dual Frequency Cycle Slip Detection (1/3)

- If dual frequency data is available (e.g., L1 and L2), a different approach is possible. The method first forms the difference of the two phase measurements over time (denoted with δ)

$$\begin{aligned}\delta\Phi_{L1} &= \Phi_{L1,k} - \Phi_{L1,k-1} \\ &= \delta\rho + \delta d\rho + \delta cdT - \delta cdt - \delta d_{iono,L1} + \delta d_{trop} + \delta m_{L1} + \delta n_{L1} + \lambda_{L1}\delta N_{L1}\end{aligned}$$

$$\begin{aligned}\delta\Phi_{L2} &= \Phi_{L2,k} - \Phi_{L2,k-1} \\ &= \delta\rho + \delta d\rho + \delta cdT - \delta cdt - \delta d_{iono,L2} + \delta d_{trop} + \delta m_{L2} + \delta n_{L2} + \lambda_{L2}\delta N_{L2}\end{aligned}$$

- These two values are then differenced

$$\begin{aligned}\delta\Phi_{L1} - \delta\Phi_{L2} = &\left(\delta d_{iono,L1} - \delta d_{iono,L2}\right) + \left(\delta m_{L1} - \delta m_{L2}\right) \\ &+ \left(\delta n_{L1} - \delta n_{L2}\right) + \left(\lambda_{L1}\delta N_{L1} - \lambda_{L2}\delta N_{L2}\right)\end{aligned}$$

Dual Frequency Cycle Slip Detection (2/3)

- Over short time intervals (few seconds), the dispersive effect of the ionosphere *should* be negligible, that is

$$\left(\delta d_{iono,L1} - \delta d_{iono,L2}\right) \approx 0$$

- Furthermore, since phase multipath and noise are small, we can write

$$\delta\Phi_{L1} - \delta\Phi_{L2} \approx \left(\lambda_{L1}\delta N_{L1} - \lambda_{L2}\delta N_{L2}\right)$$

- Correspondingly testing the magnitude of this value is a valid means of detecting cycle slips. If a cycle slip is detected, however, you cannot determine whether it occurred on L1, L2, or both (unlikely, but possible).
- This method works equally well for static or kinematic data. Why?

Dual Frequency Cycle Slip Detection (3/3)

Example cycle slip calculations using dual frequency method

GPS Time (s)	ϕ_{L1} (cycles)	$\delta\Phi_{L1}$ (m)	ϕ_{L2} (cycles)	$\delta\Phi_{L2}$ (m)	$\delta\Phi_{L1} - \delta\Phi_{L2}$ (m)
154426	4847.073		3776.413		
		27.721		27.719	0.002
154427	4992.748		3889.917		
		27.431		27.431	0.000
154428	5136.899		4002.244		
		27.293		27.539	*-0.246*
154429	5280.325		4115.013		

Observation Combinations

Linear Phase Combinations

- The L1 and L2 measurements (pseudorange or carrier phase) can be linearly combined. For the carrier phase, the linear combination is denoted

$$\phi_{a,b} = a\phi_{L1} + b\phi_{L2}$$

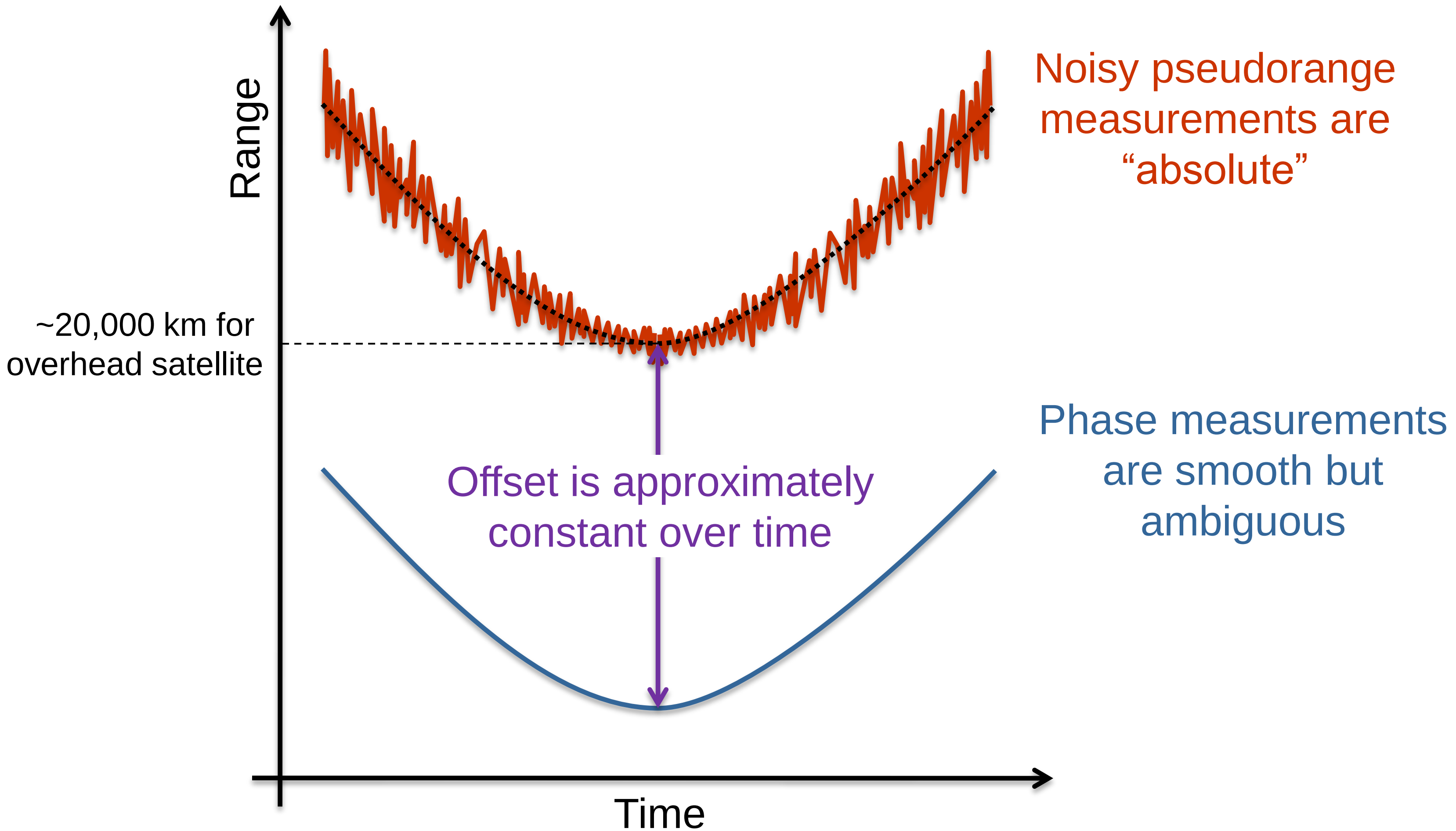
with a and b being selectable coefficients. The wavelength of above linear combination is

$$\lambda_{a,b} = \frac{c}{af_{L1} + bf_{L2}} = \frac{\lambda_{L1}\lambda_{L2}}{b\lambda_{L1} + a\lambda_{L2}}$$

- The general objective of such combinations is to improve the resulting measurement in some manner relative to the original measurement. This concept is often used when looking at different error sources and for ambiguity resolution.

Reference: Petovello, M.G. (2009) **What are Linear Carrier Phase Combinations and What are the Relevant Considerations?**, Inside GNSS, Vol 4, No 1, pp. 16-19. Available at: http://www.insidegnss.com/auto/janfeb-gnss-solutions.pdf.

Concept of Carrier Smoothing the Pseudorange



Carrier smoothing of the pseudoranges aims to get the best qualities of each measurement

Carrier Phase Smoothing (1/2)

- Carrier smoothing aims to merge the noisy but absolute pseudorange and the precise but ambiguous carrier phase. This approach provides an alternative to pure pseudorange observations and is used in virtually all receivers.
- The implementation is often a recursive algorithm wherein the "weight" given to the carrier phase is progressively increased over time. The algorithm starts with the pseudorange at the first epoch and at each epoch the smoothed pseudorange is given by

Smoothed pseudoranges
(initialized with first measurement)

Measured pseudorange

$$\tilde{P}_k = W_{P,k} \cdot P_k + W_{\Phi,k} \cdot \left[\tilde{P}_{k-1} + \left(\Phi_k - \Phi_{k-1} \right) \right]$$

Weights for carrier phase and pseudorange measurements

Measured carrier phase difference
(i.e., range difference, if cycle slip free)

Carrier Phase Smoothing (2/2)

- The weights for the pseudorange and phase measurements should be adjusted over time. A typical algorithm is

$$W_{P,k} = W_{P,k-1} - \Delta W \qquad W_{P,\min} \le W_{P,k} \le 1 \qquad W_{P,0} = 1$$

$$W_{\Phi,k} = W_{\Phi,k-1} + \Delta W \qquad 0 \le W_{\Phi,k} \le W_{\Phi,\max} \qquad W_{\Phi,0} = 0$$

$$W_{P,k} + W_{\Phi,k} = 1$$

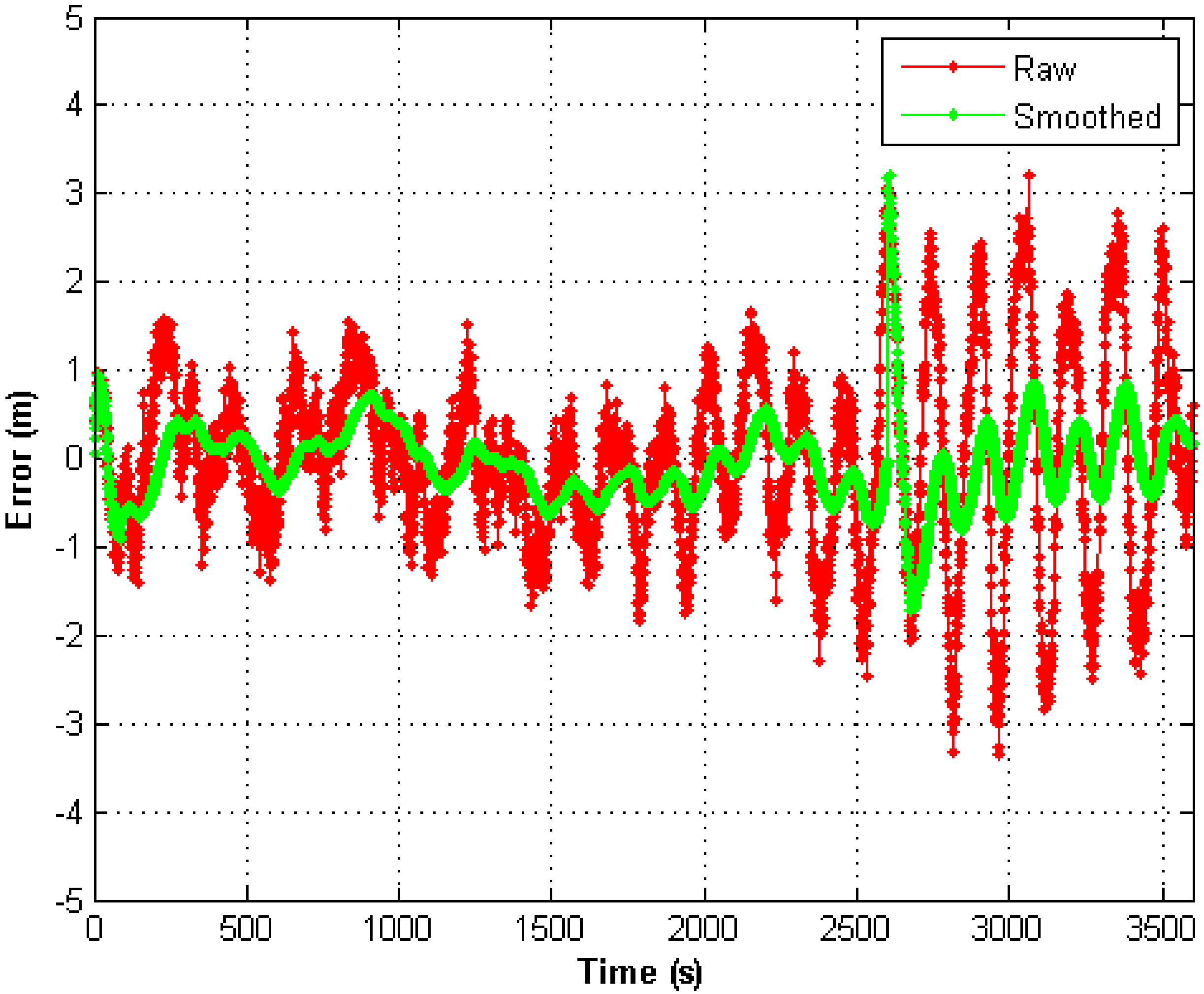
- The weight on the pseudorange is decreased over time as the weight on the carrier phase is increased. Generally, full weight is never given to the carrier phase data. Typically values for the above parameters are

$$DW = 1\% / s = 0.01 / s \qquad W_{P,\min} = 1\% = 0.01 \qquad W_{\Phi,\max} = 99\% = 0.99$$

- Of course, the entire algorithm assumes there are no cycle slips. The process restarts when a cycle slip is detected.

Reference: Lachapelle G., J. Hagglund, W. Falkenberg, P. Bellemare, M. Casey and M. Eaton (1986) *GPS Land Kinematic Positioning Experiments*. Proceedings of the 4th International Geodetic Symposium on Satellite Positioning, Austin, TX, 28 April-2 May, vol 2, pp 1327-1344.

Carrier Phase Smoothing Example



Is there a cycle slip in the data? How can you tell?

Assessing Code Multipath

- To assess the performance of code multipath and noise (with or without carrier smoothing), the ***code-minus-carrier*** combination is used

$$P - \Phi = 2d_{\mathrm{iono}} + m_P + n_P - m_\Phi - n_\Phi - \lambda N$$

- Since carrier phase multipath and noise are small in comparison to the other terms, they can be effectively ignored. The ionosphere and ambiguity terms are modeled by fitting a polynomial to the data since the ambiguity is constant (by definition, if no cycle slips occur) and the ionosphere error *tends* to vary slowly in time. Once this trend is removed, you are basically left with code multipath and noise.

Introduction to Differential Positioning

Differencing is Another Observation Combination

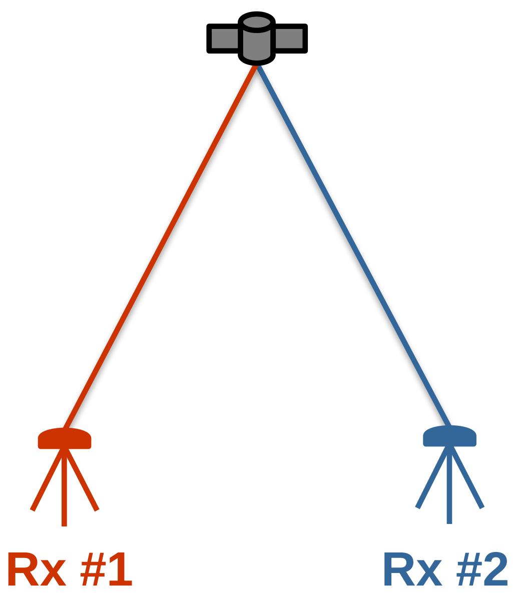
- Until now, we have only considered combinations of observations made from a single receiver to a single satellite. However, this concept can be expanded to include other satellites and/or other receivers. The main motivation for this approach is to reduce some of the errors in the measurements (we will discuss which errors in particular in the next chapter).
- For the time being, we will only consider differencing of pseudorange measurements, but the same concepts apply equally to carrier phase and Doppler measurements as well.
- We will revisit differencing again later in the course, but this initial introduction will help motivate why some of the information in the next chapter is so important.

Between-Receiver Single Differencing

- The first situation is when we have two receivers and one satellite. The pseudorange measurements from each receiver are then differenced as follows

$$
\begin{aligned}
\Delta P_{12} &= P_2 - P_1 \\
&= \Delta\rho + \Delta d\rho + c(\Delta dT - \Delta dt) + \Delta d_{iono} + \Delta d_{trop} + \Delta m_P + \Delta n_P
\end{aligned}
$$

- What happens if the errors are the *same* for both receivers?
- What happens if the errors are *similar* for both receivers?
- What happens if the errors are uncorrelated between the receivers?
- What "type" of position is obtained with this approach?

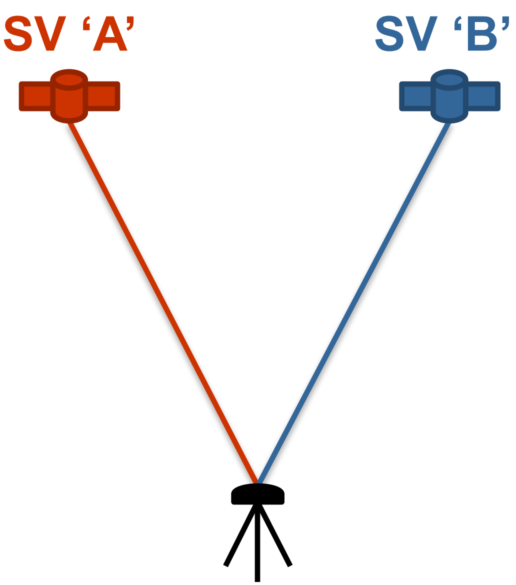


Between-Satellite Single Differencing

- The second situation is when we have one receiver and two satellites. The pseudorange measurements to each satellite are then differenced as follows

$$\begin{aligned}\nabla P^{AB} &= P^{B} - P^{A} \\ &= \nabla\rho + \nabla d\rho + c(\nabla dT - \nabla dt) + \nabla d_{iono} + \nabla d_{trop} + \nabla m_{P} + \nabla n_{P}\end{aligned}$$

- When more than two satellites are considered, only the independent differences are formed. For example, a “base satellite” is often used and all other satellites’ measurements are differenced with the base satellite.



Double Differencing

- The final situation involves two receivers and two satellites. By extension, we combine the between-receiver and between-satellite differences

$$\nabla\Delta P_{12}^{AB} = \Delta P_{12}^{B} - \Delta P_{12}^{A} = \nabla P_{2}^{AB} - \nabla P_{1}^{AB}$$
$$= \nabla\Delta\rho + \nabla\Delta d\rho + c(\nabla\Delta dT - \nabla\Delta dt) + \nabla\Delta d_{iono} + \nabla\Delta d_{trop} + \nabla\Delta m_{P} + \nabla\Delta n_{P}$$

- As one might expect, this combines the benefits of both single difference approaches. This will be investigated in more detail in chapter 7, but the concept will be important when looking at the different error sources in the next chapter.