Chapter 2
Fundamentals of Satellite Positioning

Fundamentals of GNSS Positioning

Structure of a GNSS

A Brief History of GPS

An Overview of the other GNSS

Basic GNSS Position Computations

Characterizing Performance

GPS Reference and Time System

Brief Glimpse at the Rest of the Course

Relevant Sections in Misra and Enge:

1.1 – 1.4, 2.1 – 2.2, 2.3.2, 2.4 – 2.8, 3.1-3.9, 5.1.3, 6.1

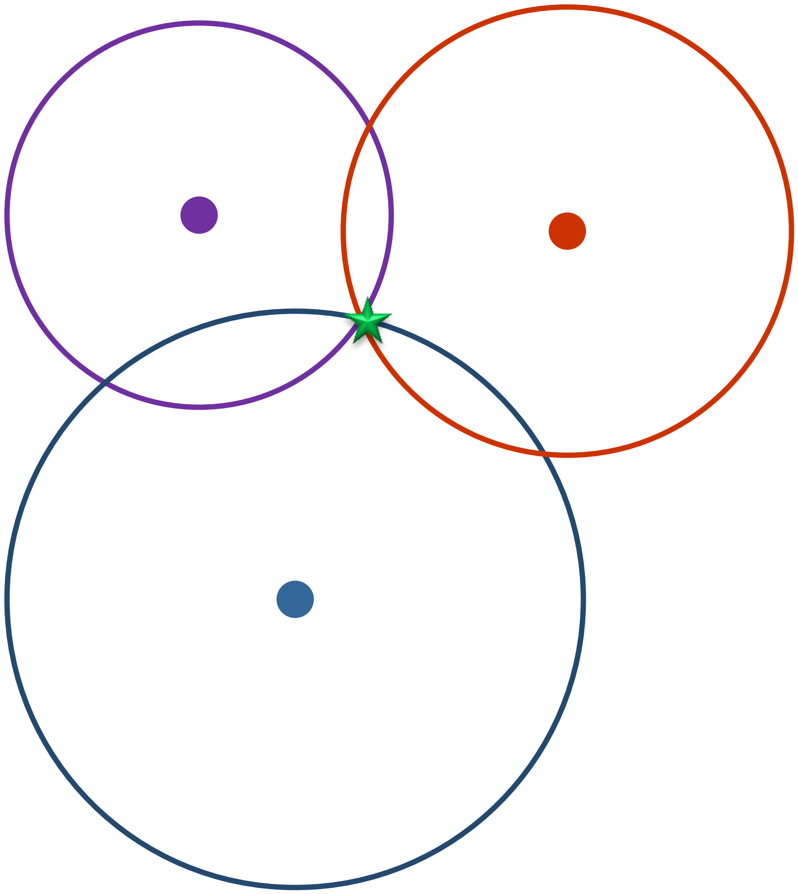
Introduction

- This chapter looks at the fundamentals of Global Navigation Satellite Systems (GNSS) in general
- Throughout the course, focus is given to the U.S. Global Positioning System (GPS) because it is the most mature system and the most common operational system
- All other GNSS – both current and future – have (or will have) much in common with GPS
- As the course progresses, the differences among the GNSS systems will be highlighted
    - Largest differences involve the signal structure, which we do not discuss in detail until later in the course

Fundamentals of GNSS Positioning

Satellite Positioning – The Basic Concept

- Satellite positioning is fundamentally based on the concept of trilateration
  - Measure distances to known points in order determine your position
- There are four unknown parameters (4D system)
  - Position (Y,X,Z or latitude, longitude, height) and time
  - Need four observations in order to compute a 3D position fix
- Biggest difference from terrestrial-based positioning is that the known/control points to which we are measuring are moving in space
  - Orbital velocity is approximately 4 km/s
  - Measurement geometry is continually changing



Why is Time the 4th Dimension?

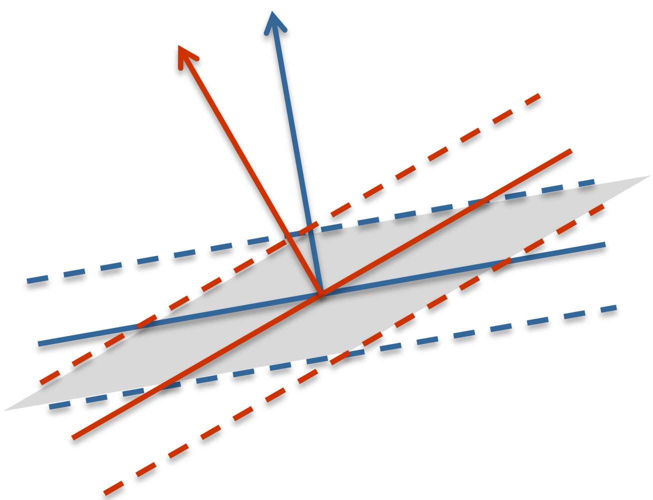
- The ranges measured by a GNSS receiver are made by determining the time is takes the signal to propagate from the satellite to the receiver
  - This propagation time is then converted to a range by multiplying by the speed of light
  - Given the above, small timing errors result in large ranging errors
    - Q: How big is a 1 ns timing error in terms of range?

- Although the satellites are highly synchronized (to the nanosecond level), GNSS receivers typically are not and this leads to a timing/clock error in the receiver
  - Error is common to all measurements from a given receiver at a given time
  - Clock error is therefore estimated along with the user position

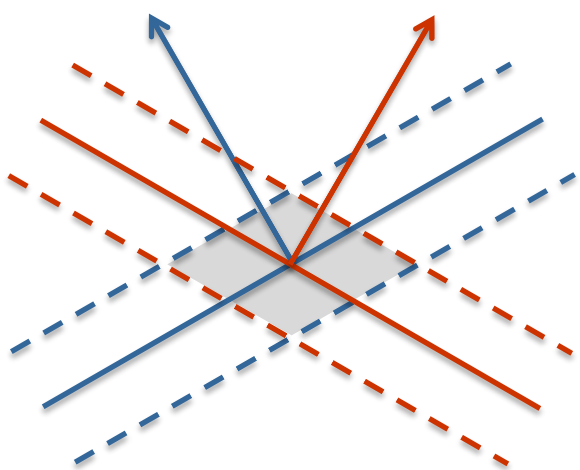
Satellite Geometry (1/2)

- Geometry is an important consideration in terms of achievable positioning accuracy
- To illustrate, consider a range measurement (solid line) with a given accuracy (region bounded by dotted line) from a satellite located in the direction of the arrow

For closely spaced satellites, the uncertainty area is highly elongated



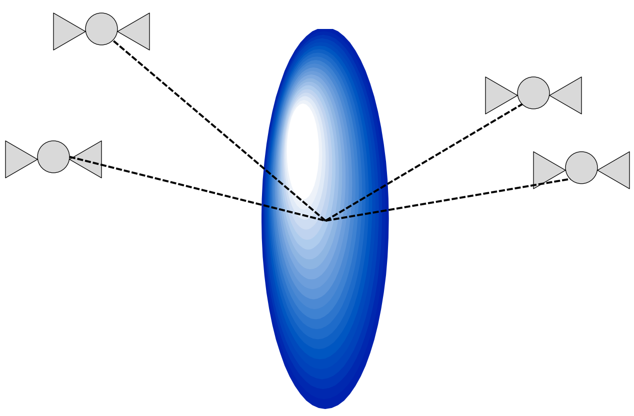
For more separated satellites, the uncertainty area is smaller



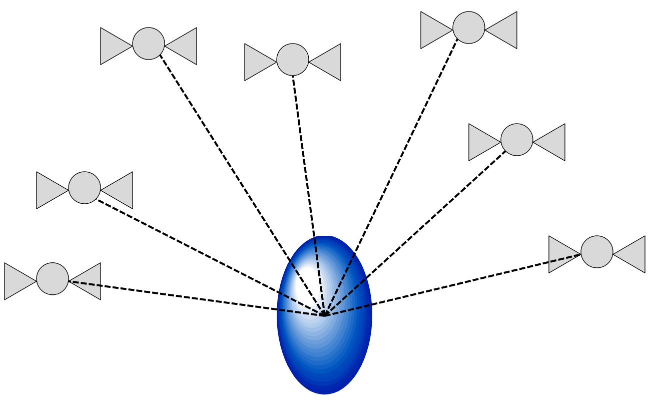
Satellite Geometry (2/2)

- More generally, as the satellite geometry improves, the uncertainty area becomes more spherical (vs. elliptical)

Good Horizontal Geometry, but Poor Vertical Geometry



Good Horizontal and Vertical Geometry



Satellite geometry must always be monitored to ensure that accuracy requirements can be met!

Structure of a GNSS

GNSS Segments

- Any GNSS consists of three main segments
  - The space segment, which consists of the satellites and the signals they broadcast
  - The control segment, which monitors and manages the satellite operations
  - The user segment, which covers receiver development

- In the following slides, the above are discussed in the context of GPS, but the general concepts will apply to other systems as well

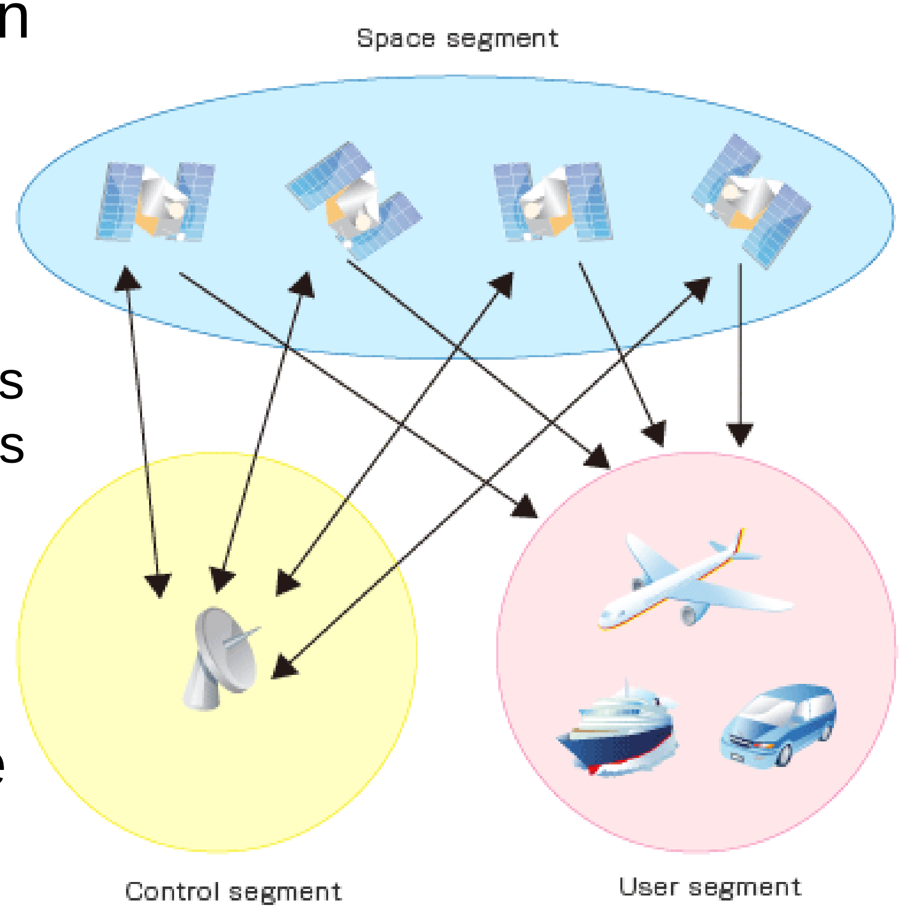


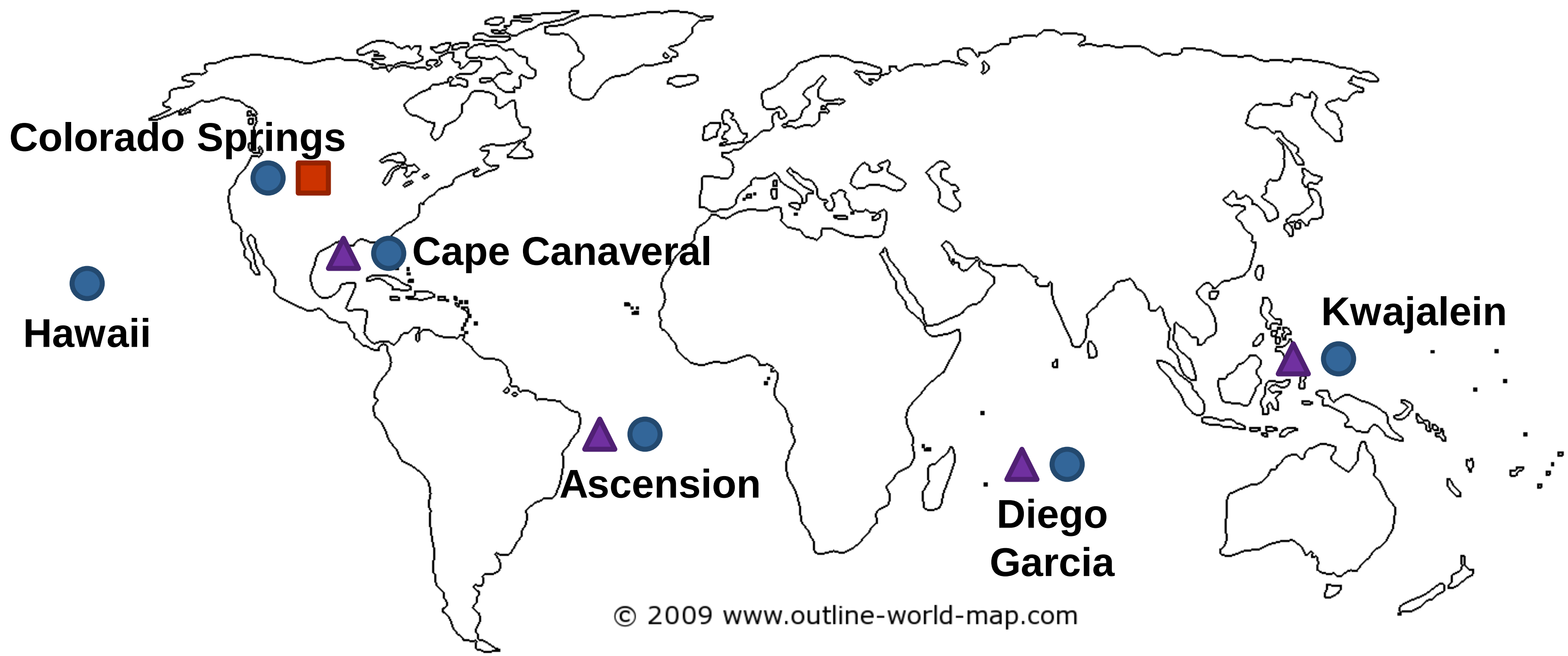
Image from Furuno, 2016

Ground/Control Segment

- The team and infrastructure to operate the GNSS system and its satellites
- Maintain the GNSS's system time
- Monitor the health of the satellites
- Measure and predict the orbits of the GNSS satellites
- Determine when satellites require maintenance/maneuvers to remain in the required constellation
- Upload the data which will ultimately be broadcast to users
  - The positions of the satellites
  - Corrections for the satellites' clocks and the ionosphere
  - Flags for the health of the satellite…

- Currently, each GPS satellite updated with new information at least once daily

GPS Ground/Control Segment – Stations



Master Control Station	Monitor Station	Ground Antenna

- Other stations are being added to improve satellite visibility from the ground
  - Q: What would be the main motivation for this?

Space Segment - Satellites

- The constellation of positioning satellites

- Satellite hardware consists of:
  - Electronics and software
  - Solar panels and batteries

    (to power the satellite)
  - Propulsion system

    (for satellite maneuvers)
  - Attitude control system
  - (to keep the antennas pointed at the earth and the solar panels at the sun)
  - Atomic clocks

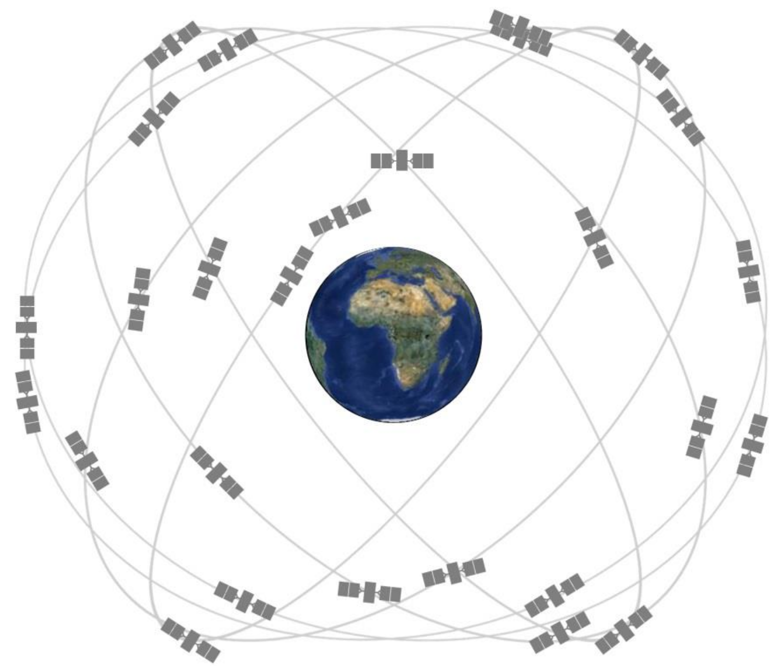
    (including backups)
  - Radio transceivers

    (to receive uploaded data and transmit navigation messages)

Picture: United States Government

Space Segments – Satellites

- The nominal GPS constellation consists of 24 satellites in six orbital planes, although there are actually closer to 30
- The Russian GLONASS consists of 24 satellites in 3 orbital planes, as will the European Galileo
- Most GNSS satellites are roughly 20000 km above the earth, in medium earth orbit (MEO)
- Some of the newer systems are regional systems, and make use of higher, geosynchronous orbits



Picture: United States Government

User Segment

- Consists of the user equipment – mainly receivers
- Receivers passively receive the signals transmitted from the GNSS satellites, and use them to calculate the user's position
- There are different categories of users
- Traditionally, for GPS:
  - Military
  - Civilian
- For some of the newer systems there are also
  - Commercial Services (paid subscription)
  - Publicly Regulated Services (available only to police, fire, government other than military)
  - Search and Rescue
  - Aviation Community

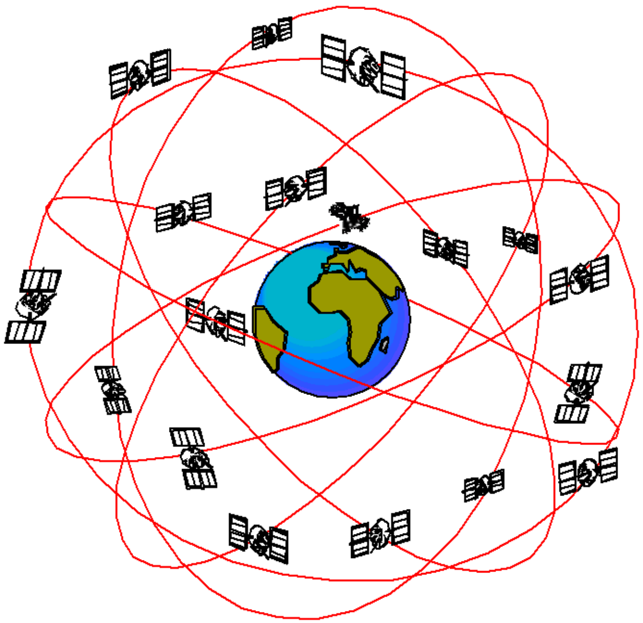
A Brief History of GPS

A Brief History of Satellite Positioning

- In many ways, the current GNSS are the culmination of several other projects
  - Predecessor to GPS was the US Navy's Transit system which was deployed in the 1960s – concept was to measure the integrated Doppler over time to form range differences
  - Following on the success of Transit, the U.S. Navy and Air Force began separate projects to improve upon Transit
    - Navy Timation project: Initiated in 1964 whereby precise clocks would be flown in space to provide both time and position information
    - Air Force 621B program: Lots of work put into development of signal structures with anti-jamming capability
  - In 1968, US DoD issued new requirements for precisely locating military forces worldwide – NAVSTAR GPS concept approved in 1973 (effectively the combination of Timation and 621B)
  - The Russian GLONASS system began development in the early 1980's but in the late 1990's and early 2000's was plagued with lack of satellites
  - The lessons learned from GPS and GLONASS have been used to develop newer GNSS systems in Europe, China, Japan and India

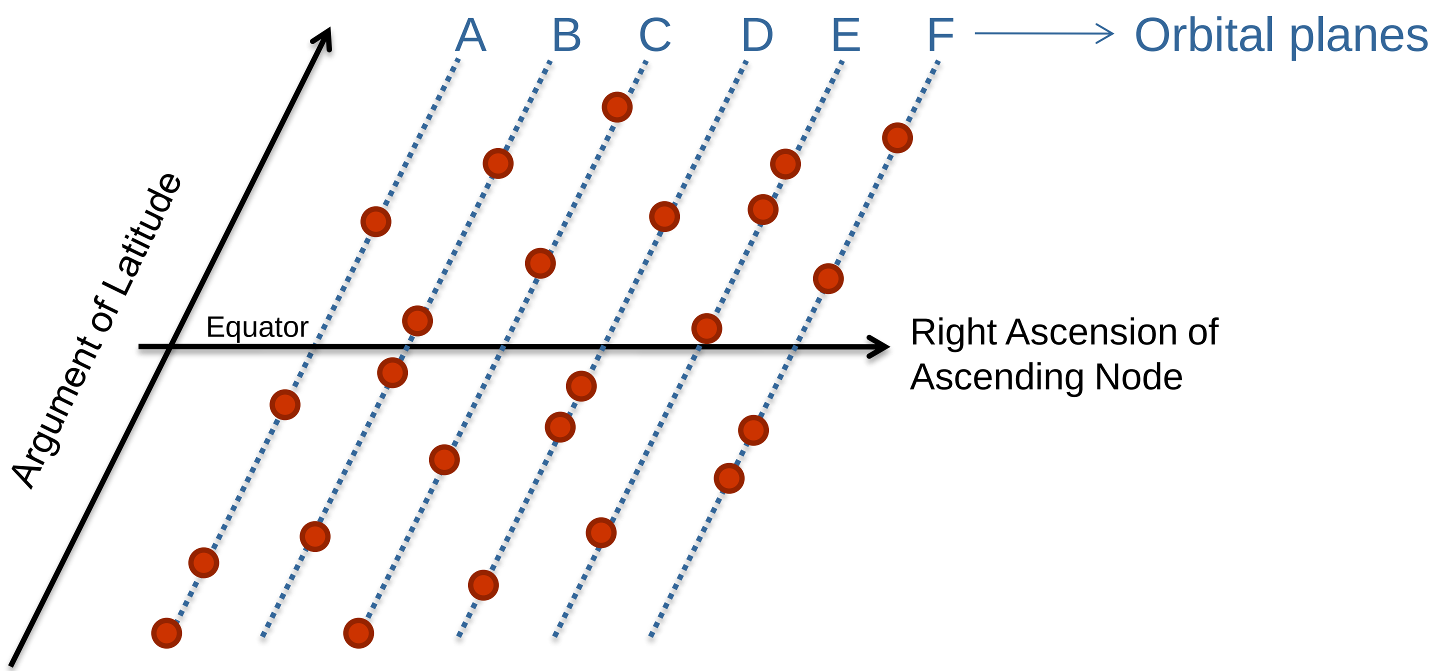
GPS Characteristics

- Nominal constellation:
  - 24 satellites
  - 6 orbital planes
  - 12-hour periods (sidereal)
  - 20,000 km altitude
- Worldwide, continuous coverage
- Line-of-sight, all weather system
- Currently transmits on three carrier frequencies:
  - L1 → f_{L1} = 1575.42 MHz → $\lambda_{L1} \cong 0.19$ m
  - L2 → f_{L2} = 1227.60 MHz → $\lambda_{L2} \cong 0.24$ m
  - L5 → f_{L2} = 1176.45 MHz → $\lambda_{L5} \cong 0.25$ m
- Frequency and time synchronized signals
- Two modulation codes for ranging
  - C/A and P codes
  - New ‘C’ and military codes on newer satellites (on L2)



GPS Space Segment – Constellation

- The nominal GPS constellation consists of 24 satellites with the following properties
  - Nearly circular orbits with repeating ground tracks
  - ~20,000 km altitude (~26,560 km orbital radius)
  - Orbital period of 12 sidereal hours
  - Six planes (A-F) with an inclination of 55° relative to the equator
  - Nominally four satellites per plane



GPS Space Segments – Satellites

- Each satellite is worth approximately $100M on orbit (cost of building and launching)
- Cost is borne entirely by the U.S. DoD
- Several generations of satellites have been launched to date
  - Block I (1978-1985)
  - Block II/IIA (1989-1995)
  - Block IIR/IIR-M (1997-present)
  - Block IIF (2010)
  - Block III (future)

	Block II/IIA	Block IIR	Block IIF
First Launch	1989	1997	2010
Weight (kg)	1000	1100	1700
Power (W)	700	1140	2900
Design Life (yrs)	7.5	10	12.5
Unit Cost (USD)	$43M	$30M	$28M

GPS Ground Segment – Reference Frame & Time

- GPS uses the World Geodetic System 1984 (WGS 84) as its reference frame; differences between WGS 84 and other global reference systems such as ITRF are at the centimetre level
  - All positions are computed relative to WGS84 and its ellipsoid
  - Q: What implications does this have in a practical sense?

- Timing is at the very heart of GPS since ranges are based on time of propagation of electro-magnetic waves
  - GPS has therefore defined its own time scale which is kept using a week count (GPS Week) and time into the week (TOW) in seconds
  - This atomic time scale agrees with UTC to within 100 ns (modulo 1 s) with the offsets being monitored by the OCS

GPS User Segment

- Military Users
  - For use by land, sea and airborne vehicles
  - Infantry location and navigation
  - Not a perfect military system since it is prone to jamming (relies on line-of-sight from antenna to satellite)

- Civil Users
  - Navigation was primary focus of GPS developers, but surveying community quickly adopted the system for high accuracy positioning
  - The use of GPS in the civilian community is expanding rapidly due to the decrease in receiver costs (i.e., 10's of thousands of dollars to a few dollars)
  - Civil community is much larger than military (in terms of number of devices)

Levels of Service

- The US DoD had always planned to provide service to the civil community

- Two different levels of services were defined

  - The *Precise Positioning Service (PPS)* was for military users and provides the best accuracy (without using special techniques)
    - Limited access to PPS is enforced through *anti-spoofing (AS)*

  - The *Standard Positioning Service (SPS)* was intended for civil use and has poorer performance
    - Poorer performance used to result from *selective availability (SA)*, but now is limited by the signal structure and measurement errors

Anti-Spoofing

- Anti-Spoofing (AS)
  - Prevents receivers from being spoofed by fake signals
  - Effected through encryption of P code
    - Encrypted P code becomes Y code
    - P code on L1 and L2 no longer possible with standard code correlation techniques

GPS Standard Positioning Service (SPS)

- Accuracy of the SPS has increased with removal of SA

- Global position accuracy, 95% (nominal conditions)
  - Horizontal ≤ 9 m
  - Vertical ≤ 15 m

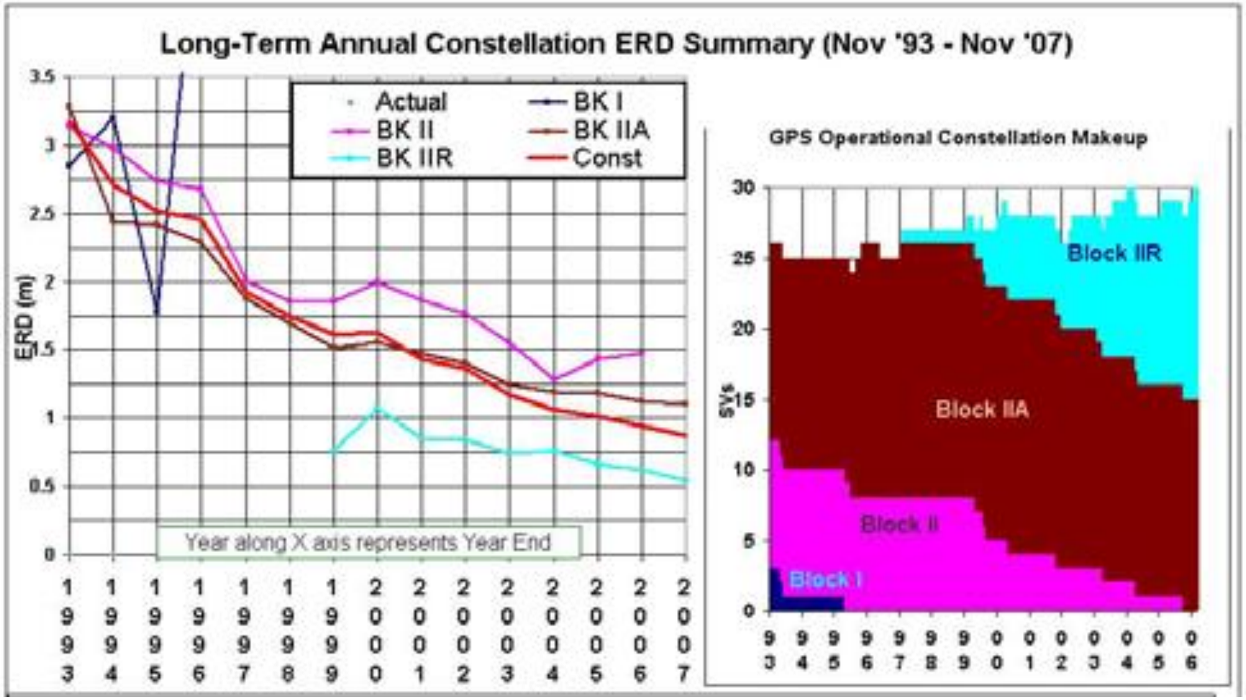
Assumes ≥ 98 % global PDOP of 6 or less

- Time transfer accuracy, 95% (nominal conditions)
  - ≤ 40 ns

Reference: **GPS Standard Positioning Service (SPS) Performance Standard**, 4th Edition, U.S. Department of Defense, September 2008. Available for download at: **http://www.navcen.uscg.gov/gps/geninfo/default.htm**

GPS User Equivalent Range Error

- The User Equivalent Range Error (UERE) describes ranging accuracy
  - Includes effect of signal-in-space (SIS: satellite orbit and clock errors), atmosphere, and noise and multipath
  - Estimated range deviation (ERD) plot below shows improvements of SIS errors over time



Reference: Taylor, J. et al. (2008), **GPS Control Segment Upgrade Goes Operational – Enhanced Phased Operations Transition Details**, Institute of Navigation NTM 2008 Conference.

GPS Management, Policy and Market

- Access is free for all users as documented in the 1996 Presidential Decision Document (March, 1996) and through a law passed by the US Congress (1998)
  - Both state that the US ‘will continue to provide the GPS Standard Positioning Service for peaceful civil, commercial and scientific use on a continuous, worldwide basis, free of direct user fees’

- US Government will give a 10 year notice of cancellation
- Level of accuracy has been guaranteed
- Day to day management is done by DoD
  - Input from the civil community through the Department of Transportation (DoT)
  - Concerns expressed from the international community that there should be more international input

GPS Management, Policy and Market

- Modernization
  - 2nd civilian frequency adopted in 1998: 1227.60 MHz (L2)
  - 3rd civilian frequency adopted in 1999: 1176.25 MHz (L5)

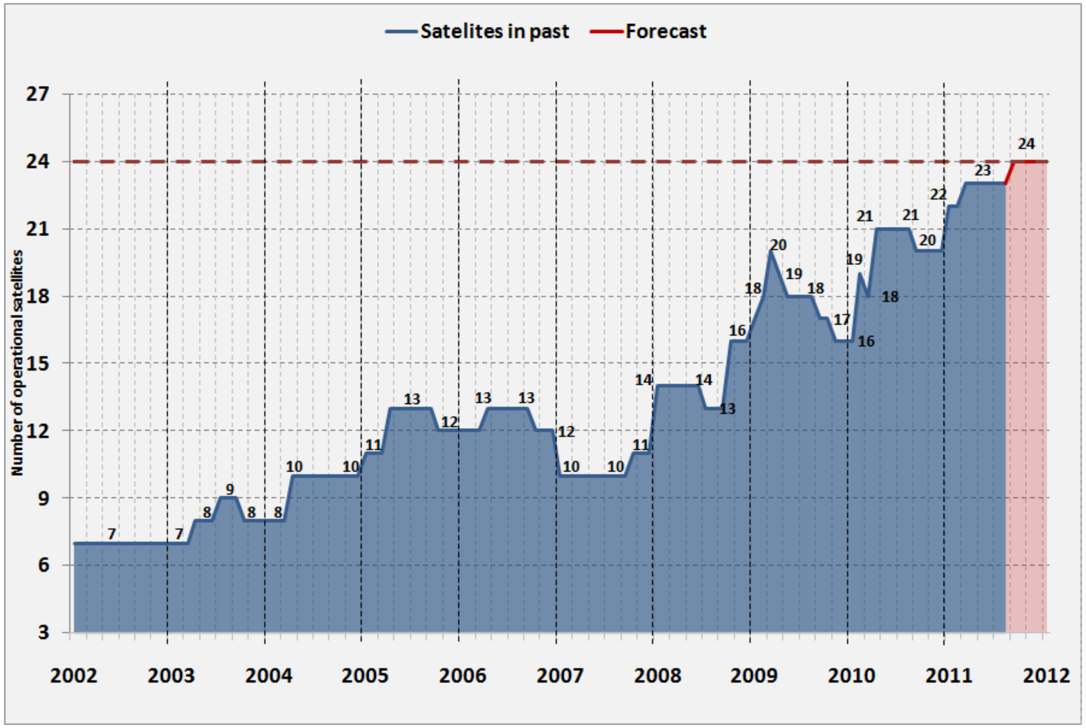
- New signal structures will provide better positioning accuracy

- GPS is currently a multi-billion dollar per year market…and growing
  - Main market drivers: automobile navigation, cellular telephones (location-based services), recreational use, aircraft landing, personal navigation devices (PND)

An Overview of the Other GNSS Systems

GLONASS

- GLONASS – Globalnaya navigatisionnaya sputnikovaya sistema (literally, GNSS)
- Russian system which is similar to GPS
- First launch in 1982 (three satellites launched at a time)
- Full Operational Constellation of 24 satellites achieved in 1995
- BUT... Due to a lack of funding the constellation dropped to 6 satellites in 2001
- It only returned to full operational capacity in October 2011



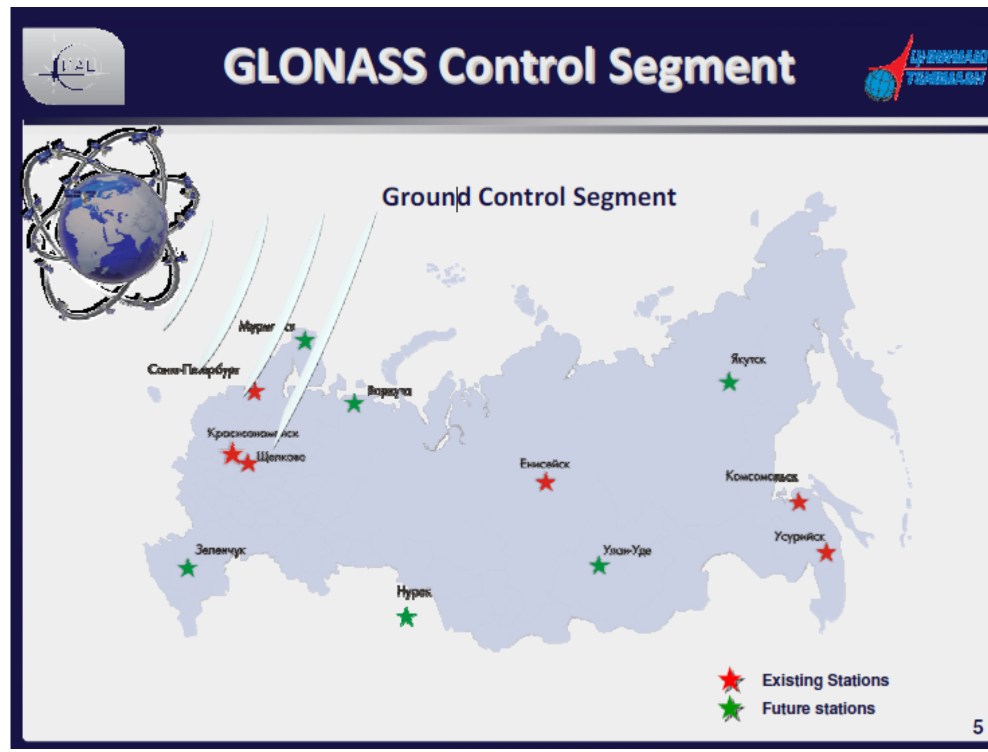
Graph from Revnivykh 2011

GLONASS

- GLONASS also transmits on L1 and L2 bands, but not on the same frequencies as GPS
- GLONASS uses frequency division multiple access (FDMA)
  - This means each satellite transmits on a slightly different frequency in order for a receiver to distinguish which signal is coming from which satellite
  - all other GNSS use code division multiple access – more on this later
- Signals:
  - L1 1602 MHz + K * 562.5 kHz
  - L2 1246 MHz + K * 437.5 kHz
  - K is a frequency number for the channel being transmitted by the GLONASS Satellite
  - K ranges from -7 to +6, defining a total of 12 possible frequencies
  - Satellites on the opposite sides of the earth, which are never visible at the same time, share the same frequency
  - Ultimately, this limits GLONASS to 24 operational satellites

GLONASS

- GLONASS differs from GPS in other small ways:
  - It uses a different parameterization to transmit the satellite locations to users
  - Traditionally the full ground segment is on Russian territory, which limits orbit accuracy for the GLONASS satellites, and consequently positioning accuracy for users
  - It uses a different time scale, aligned to Moscow
  - Uses PZ-90 coordinate system
- GLONASS Modernization
  - New GLONASS –K satellites
  - A newL3 CDMA signal
  - Future CDMA on L1 and L2
  - Possible L5
  - Extension of the ground segment



Graph from Revnivykh 2011

Galileo

- Galileo
  - Under development by the European Union
  - Similar to GPS with some additional signals and improvements in signal characteristics
  - Signals:
    - E1 1575.42 MHz (same as GPS L1, with a different signal structure)
    - E5a and E5b 1191.795 MHz (E5a is the same frequency as GPS L5)
    - E6 1278.750 MHz (for public regulated service/commercial service)
  - Initial validation is complete; now initially deployed
    - http://www.gsc-europa.eu/system-status/Constellation-Information
  - As of September 2015, 10 satellites had been launched, but 3 suffer from known problems
  - The modern signal structure should allow for less measurement noise than GPS, but the biggest advantage will likely be the compatibility of the two systems
  - Users will benefit from a dramatic increase in the number of measurements on the L1 and L5 frequencies

BeiDou System (BDS)

- China's GNSS
- Regional system declared fully operational December 27, 2012
  - 5 Geostationary (GEO) satellites, 5 Inclined geostationary (IGSO) and 4 Medium earth orbit (MEO)
- Full global system expected by 2020
  - 5 Geostationary (GEO) satellites, 3 Inclined geostationary (IGSO) 27 Medium earth orbit (MEO)
- Signals
  - B1 1561.098 MHz
  - B2 1207.140 MHz
  - Other signals may be designed for the global system

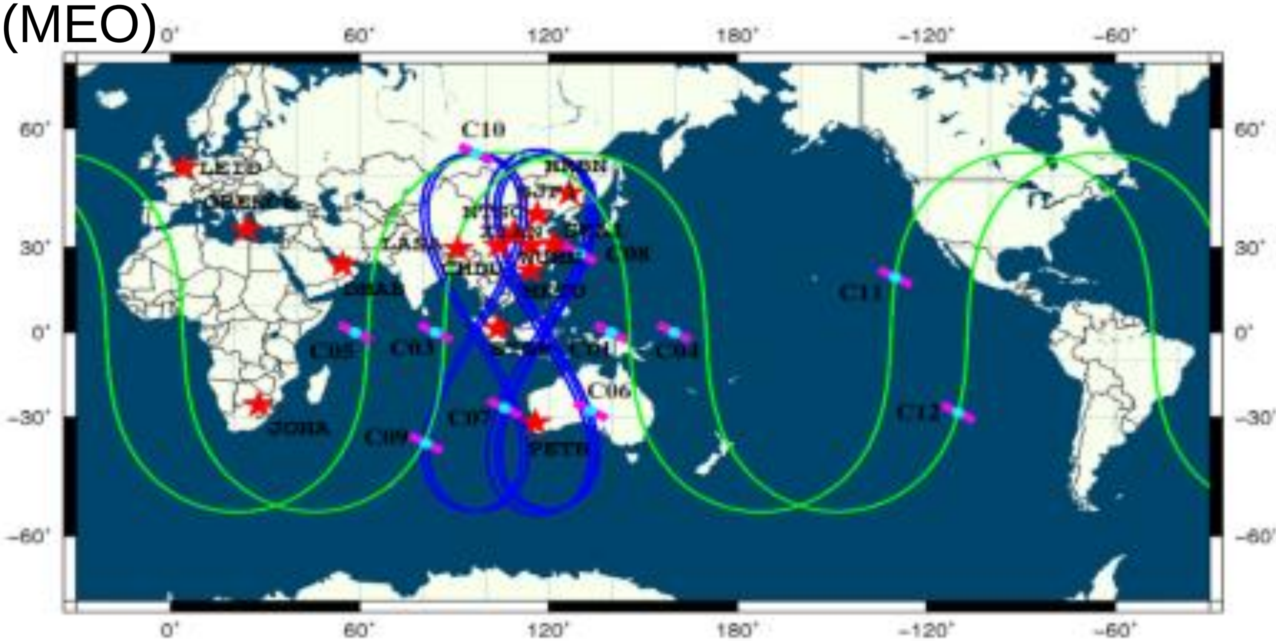


Image from Li et al 2014

Quazi-Zenith Satellite System

- Japan’s regional augmentation system
- Consisting of one (eventually 3 or more) inclined geosynchronous satellites
- Signals exactly match the frequency and structure of GPS – it is meant to augment GPS in the urban canyons of Asia rather than act as a stand-along GNSS
- Has an additional function as an SBAS system for integrity monitoring and corrections on a different PRN

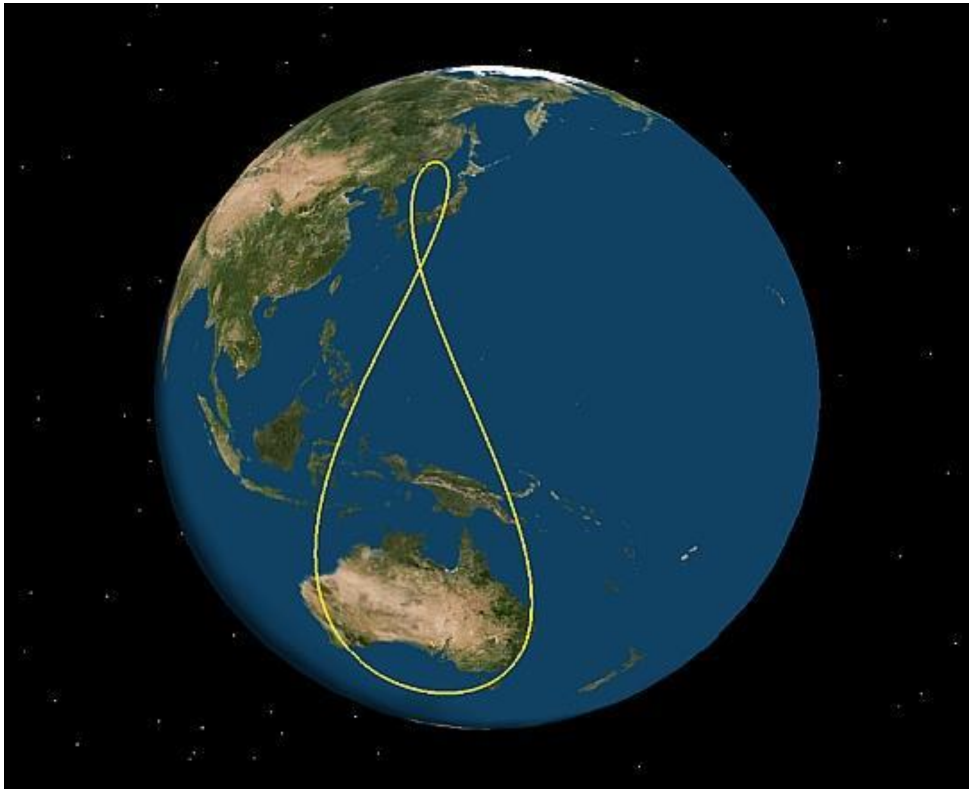
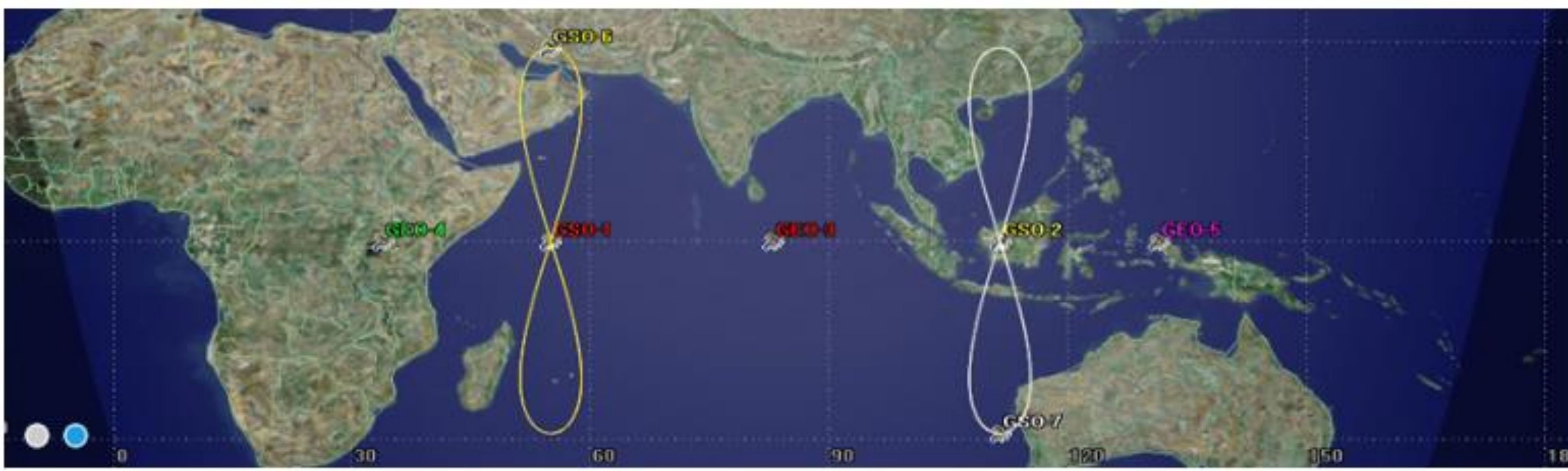


Image from Jaxa/ eoPortal

Indian Regional Navigation Satellite System – IRNSS

- IRNSS is a regional system being developed by India
- It will cover a region up to 1500 km from the Indian borders
- Operates on L5 (1176.45 MHz) and S-band (2492.028 MHz)
- Consists of 3 geostationary and 7 geosynchronous satellites, four of which had been launched by December 2015
- The IRNSS website is here: http://irnss.isro.gov.in/



Picture from http://irnss.isro.gov.in/

Augmentation Systems

- Several *augmentation systems* have been (and are being) developed, each of which consists of
  - A set of ground stations that record GNSS data
  - A processing station that computes errors in (corrections to) the GNSS
    - Errors originate from various sources and will be discussed in much detail later in the course
  - A geostationary satellite that provides an extra ranging signal as well as a means of broadcasting the corrections to users in order to improve their position accuracy

- Current and future augmentation systems
  - Wide Area Augmentation System – WAAS (USA)
  - European Geostationary Navigation Overlay Service – EGNOS (EU)
  - MTSAT Satellite-based Augmentation System – MSAS (Japan)
  - GPS-aided Geo Augmented Navigation – GAGAN (India)
  - System for Differential Corrections and Monitoring – SDCM (Russia)

Basic Satellite Positioning Computations

Types of Measurements

- As we will see in more detail in Chapter 5, there three main types of observations available from a GNSS receiver
  - Pseudorange
  - Doppler shift
  - Carrier phase (also called accumulated Doppler)

- The above measurements have different characteristics and purposes, but the immediate focus will be on the differences between the pseudorange and carrier phase measurements

Types of Measurements

- The simplified pseudorange (P) and carrier phase (Φ) measurement equations are (details in Chapter 5)

Pseudorange $$P = r + cdt + e^{P}$$

Carrier phase $$F = f\,l = r + cdt + Nl + e^{F}$$

ρ	geometric range (m)
c	speed of light (m/s)
dt	receiver clock error (s) – **common to all satellites at one epoch**
ε^{P}	combined pseudorange errors (m)
Φ	carrier phase measurement (m)
φ	carrier phase measurement (cycles)
λ	wavelength of the carrier signal (m/cycle)
N	ambiguous number of cycles between satellite and receiver (cycles) – usually called the *carrier phase ambiguity (“ambiguity”)*
ε^{Φ}	combined carrier phase errors (m)

General Measurement Characteristics

- For now, the two main differences between the pseudorange and carrier phase measurements are precision and ambiguity

- Precision
  - Pseudorange measurements have a precision of **several decimetres** to **several metres**
  - Carrier phase have a precision of **less than 0.25λ** (i.e., less than a quarter-wavelength)
    - Better than ~4.5 cm for L1

- Ambiguity
  - Pseudoranges are **absolute** in that they represent the range to the satellite (with a common bias across all satellites)
  - Carrier phases are **ambiguous** because the number of cycles between the receiver and the satellite is unknown and *differs* between all satellites

Single Point Positioning (1/2)

- Single point positioning is when you compute your solution using data from only one receiver and is (almost always) done using the pseudorange measurements. Expanding the measurement equation for the i-th satellite gives

$$\begin{aligned} P_i &= \rho_i + cdt + \varepsilon_P \\ &= \sqrt{\left(x_i^s - x^r\right)^2 + \left(y_i^s - y^r\right)^2 + \left(z_i^s - z^r\right)^2} + cdt + \varepsilon_i^P \\ &= f\left(\mathbf{x}\right) \end{aligned}$$

In this case, the unknowns are the position of the receiver (x^r,y^r,z^r) and the receiver clock bias (dt) which is common to all measurements at a given epoch:

$$\mathbf{x}^T = \begin{bmatrix} x^r & y^r & z^r & cdt \end{bmatrix}$$

These parameters will be estimated using least-squares

Single Point Positioning (2/2)

- The measurement is obviously non-linear with respect to our unknowns and so we have to linearize the system; taking the partial derivatives with respect to the unknowns gives the design matrix:

$$A_i = \left.\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}} = \left.\begin{bmatrix} \frac{\partial P}{x^r} & \frac{\partial P}{y^r} & \frac{\partial P}{z^r} & \frac{\partial P}{cdt} \end{bmatrix}\right|_{\mathbf{x}=\hat{\mathbf{x}}} = \begin{bmatrix} \frac{\hat{x}^r - x_i^s}{\hat{\rho}_i} & \frac{\hat{y}^r - y_i^s}{\hat{\rho}_i} & \frac{\hat{z}^r - z_i^s}{\hat{\rho}_i} & 1 \end{bmatrix}$$

Correspondingly, the least-squares solution – assuming equal variances across all observations – is given by

$$\delta\hat{\mathbf{x}} = \left(A^T A\right)^{-1} A^T \mathbf{w} \quad \Rightarrow \quad \hat{\mathbf{x}} = \hat{\mathbf{x}} + \delta\hat{\mathbf{x}}$$

The misclosure vector (**w**) is given by

$$\mathbf{w}_i = P_i - f\left(\hat{\mathbf{x}}\right)$$

A Closer Look at the Design Matrix

- Recall the design matrix from the least-squares algorithm

$$A_i = \begin{bmatrix} \dfrac{\hat{x}^r - x_i^s}{\hat{\rho}_i} & \dfrac{\hat{y}^r - y_i^s}{\hat{\rho}_i} & \dfrac{\hat{z}^r - z_i^s}{\hat{\rho}_i} & 1 \end{bmatrix}$$

What information does this contain?

Estimated Accuracy and DOPs

- The covariance matrix of the estimated parameters is given by

$$C_{\hat{x}} = \sigma_P^2 \underbrace{\left(A^T A\right)^{-1}}_{Q_x} = \sigma_P^2 \begin{bmatrix} q_{x,x} & q_{x,y} & q_{x,z} & q_{x,cdt} \\ q_{y,x} & q_{y,y} & q_{y,z} & q_{y,cdt} \\ q_{z,x} & q_{z,y} & q_{z,z} & q_{z,cdt} \\ q_{cdt,x} & q_{cdt,y} & q_{cdt,z} & q_{cdt,cdt} \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & \sigma_{x,y} & \sigma_{x,z} & \sigma_{x,cdt} \\ \sigma_{y,x} & \sigma_y^2 & \sigma_{y,z} & \sigma_{y,cdt} \\ \sigma_{z,x} & \sigma_{z,y} & \sigma_z^2 & \sigma_{z,cdt} \\ \sigma_{cdt,x} & \sigma_{cdt,y} & \sigma_{cdt,z} & \sigma_{cdt}^2 \end{bmatrix}$$

where Q is called the cofactor matrix and σ_P is the standard deviation of the pseudorange measurements. The diagonal elements of Q_x define what we call the Dilution of Precision (DOP) values, specifically

$$XDOP = \sqrt{q_{x,x}} \qquad YDOP = \sqrt{q_{y,y}} \qquad ZDOP = \sqrt{q_{z,z}} \qquad TDOP = \sqrt{q_{cdt,cdt}}$$

DOPs and Accuracy (1/2)

- The position accuracy can be then written as

$$\left.\begin{aligned}\sigma_x &= \sigma_P \times \mathrm{XDOP}\\ \sigma_y &= \sigma_P \times \mathrm{YDOP}\\ \sigma_z &= \sigma_P \times \mathrm{ZDOP}\\ \sigma_{cdt} &= \sigma_P \times \mathrm{TDOP}\end{aligned}\right\}$$

General formulation is:

$$\boxed{\text{Solution Accuracy} = \text{Measurement Accuracy} \times \text{DOP}}$$

Since the DOPs are computed from Q_x, which is related to the measurement geometry, the above equations show how the measurement geometry affects the solution accuracy

Mission planning (pre-planning) is the process of determining when the DOPs are best for a particular area. What is needed in order to do this?

DOPs and Accuracy (2/2)

- By extension from the previous slide, we can compute DOP values for the 3D and 4D solutions as well

$$\sigma_{3D}^2 = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

$$= \sigma_P \times \sqrt{XDOP^2 + YDOP^2 + ZDOP^2}$$

$$= \sigma_P \times PDOP$$ → **Position DOP**

$$\sigma_{4D}^2 = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 + \sigma_T^2}$$

$$= \sigma_P \times \sqrt{XDOP^2 + YDOP^2 + ZDOP^2 + TDOP^2}$$

$$= \sigma_P \times GDOP$$ → **Geometric DOP**

Horizontal and Vertical DOP (HDOP & VDOP)

- DOPs in the X, Y & Z directions are generally not very useful or intuitive and we generally prefer values in the horizontal (north & east) and vertical directions
- Assuming matrix R transforms a vector from the Cartesian frame to a local level (LL) frame (e.g., north, east, up) frame, then

$$x_{LL} = R x_{XYZ} \qquad \Rightarrow \qquad Q_{x_{LL}} = R Q_{x_{XYZ}} R^T$$

$$Q_{x_{LL}} = \begin{bmatrix} q_{NN} & q_{NE} & q_{NV} & q_{NT} \\ q_{EN} & q_{EE} & q_{EV} & q_{ET} \\ q_{VN} & q_{VE} & q_{VV} & q_{VT} \\ q_{TN} & q_{TE} & q_{TV} & q_{TT} \end{bmatrix} = \begin{bmatrix} NDOP^2 & \bullet & \bullet & \bullet \\ \bullet & EDOP^2 & \bullet & \bullet \\ \bullet & \bullet & VDOP^2 & \bullet \\ \bullet & \bullet & \bullet & TDOP^2 \end{bmatrix}$$

North, East and Vertical DOPs

$$HDOP = \sqrt{NDOP^2 + EDOP^2} \longrightarrow$$ Horizontal DOP

Alternate Computation of N/E/VDOP

- Recall that the X/Y/ZDOP values were obtained from the design matrix expressed in the Cartesian frame (denoted XYZ below)

$$A_i^{XYZ} = \begin{bmatrix} \dfrac{\hat{x}^r - x_i^s}{\hat{\rho}_i} & \dfrac{\hat{y}^r - y_i^s}{\hat{\rho}_i} & \dfrac{\hat{z}^r - z_i^s}{\hat{\rho}_i} & 1 \end{bmatrix} \quad \Rightarrow \quad Q_{\hat{x}}^{XYZ} \quad \Rightarrow \quad \text{X/Y/ZDOP}$$

- However, the design matrix is composed of unit vectors from the different satellites pointing to the receiver. If the unit vectors are expressed in the local level frame, the resulting Q_x matrix will contain N/E/VDOP values.

$$A_i^{LL} \quad \Rightarrow \quad Q_{\hat{x}}^{LL} \quad \Rightarrow \quad \text{N/E/VDOP}$$

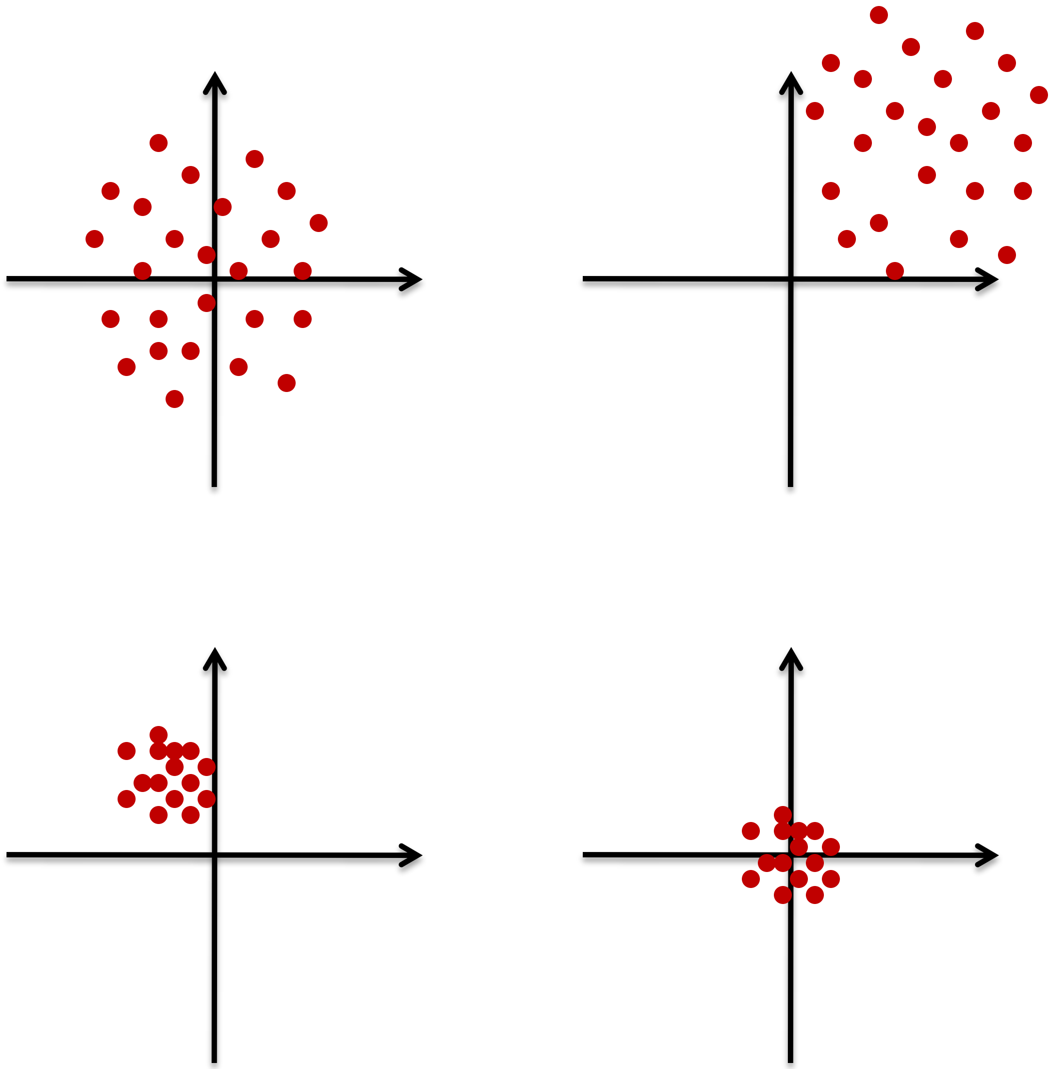
- Can you compute A^{LL} if you know the azimuth and elevation of the satellite? If so, how?

Characterizing Performance

Accuracy vs. Precision

- *Accuracy* is the degree of closeness of an estimate to its true (but unknown) value
- *Precision* is the degree of closeness of observations to their mean
  - Strictly, accuracy and precision are not the same
  - In practice, they are often assumed to be the same (used interchangeably)

- Which of the distributions on the right are accurate and which are more precise?



Position Covariance Matrix

- Accuracy is best defined by a covariance matrix
- Assuming 3D position estimation, the position covariance matrix might look like

$$C_x = \begin{bmatrix} \sigma_N^2 & \sigma_{N,E} & \sigma_{N,V} \\ \sigma_{E,N} & \sigma_E^2 & \sigma_{E,V} \\ \sigma_{V,N} & \sigma_{V,E} & \sigma_V^2 \end{bmatrix}$$

- From the above, you can define
  - A 2D error ellipse
    - Probability of 39.4%
    - Scaled by 2.447 gives a probability level of 95%
  - A 3D error ellipsoid
    - Probability of 19.9%
    - Scaled by 2.70 gives a probability level of 95%

Various Error Statistics

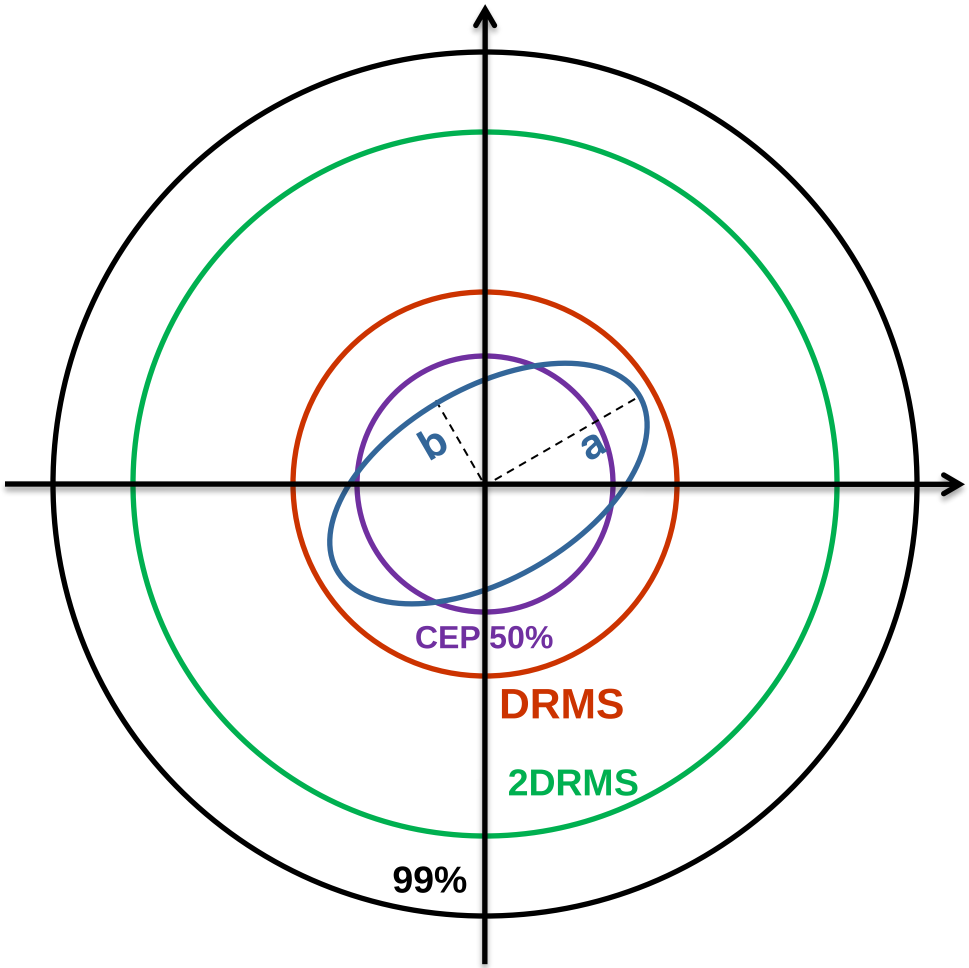
- DRMS (Distance Root Mean Squared) – 2D only

$$\text{DRMS} = \sqrt{\sigma_N^2 + \sigma_E^2}$$

  - One number to express 2D accuracy
  - Convenient but not as rigorous as error ellipse or full covariance matrix
  - Also called Radial Error (Circle), Mean Squared Position Error (MSPE), or Root Sum Square
  - Probability of circle with radius DRMS varies depending on N/S vs. E/W accuracy
    - If equal: probability is~ 63%
    - If N/S is ten times worse than E/W: probability is ~68%
- 2xDRMS (2DRMS) gives probability between 95.4% and 98%
- CEP (Circular Error Probable)
  - Circle with 50% probability

Comparison of Various Error Statistics

- DRMS
  - Probability of location within an area of constant radius
- Error ellipse
  - Constant probability, area varies
- 3D error metrics are also available
  - Error ellipsoid
  - Mean Radial Spherical Error – MRSE
    - 3D equivalent of DRMS
  - Spherical Error Probable – SEP
    - 3D equivalent of CEP



Various Error Statistics

- Conversion between different statistics can be complicated

- Great references include:

  - van Diggelen, F. (1998) **GPS Accuracy: Lies, Damn Lies and Statistics**. GPS World, Vol. 9, No. 1.

  - van Diggelen, F. (2007) **GNSS Accuracy: Lies, Damn Lies and Statistics**. GPS World, Vol. 18, No. 1.

Reference Frames

Reference System

- Recall that GPS uses WGS 84 as its reference system. This means that all of the ground stations – and thus satellites – have coordinates expressed in the WGS 84 frame.
- Correspondingly, since the satellites act as our "known" points for computing the user's position, the user's position is also computed in the WGS84 frame.
- What does this mean in terms of the "height" that is obtained from a GPS (or GNSS) receiver?

Parameter and Value	Description
a = 6378137.0 m	Semi – major axis
1/f = 298.257223563	Reciprocal of flattening
$\omega_e = 7292115 \times 10^{-11}$ rad s^{-1}	Angular velocity of the earth
GM = 3986004418×10^{8} m^3 s^{-2}	Earth's gravitational constant

History of WGS 84 (1/2)

- First GPS reference system was WGS 72 and was based on U.S. DoD Transit (‘Doppler’) stations (used until 1988)

- Original WGS 84 reference system was based on Doppler stations and produced a globally homogeneous reference frame with an accuracy of 1-2 m (reflecting the limitations of the Doppler system). No station velocities were taken into account.

- The current realization of WGS 84 for GPS is based on
  - 11 GPS tracking stations operated by the National Imaging and Mapping Agency (NIMA);
  - 5 DoD sites (part of the Operational Control System)
  - Some IGS stations

History of WGS 84 (2/2)

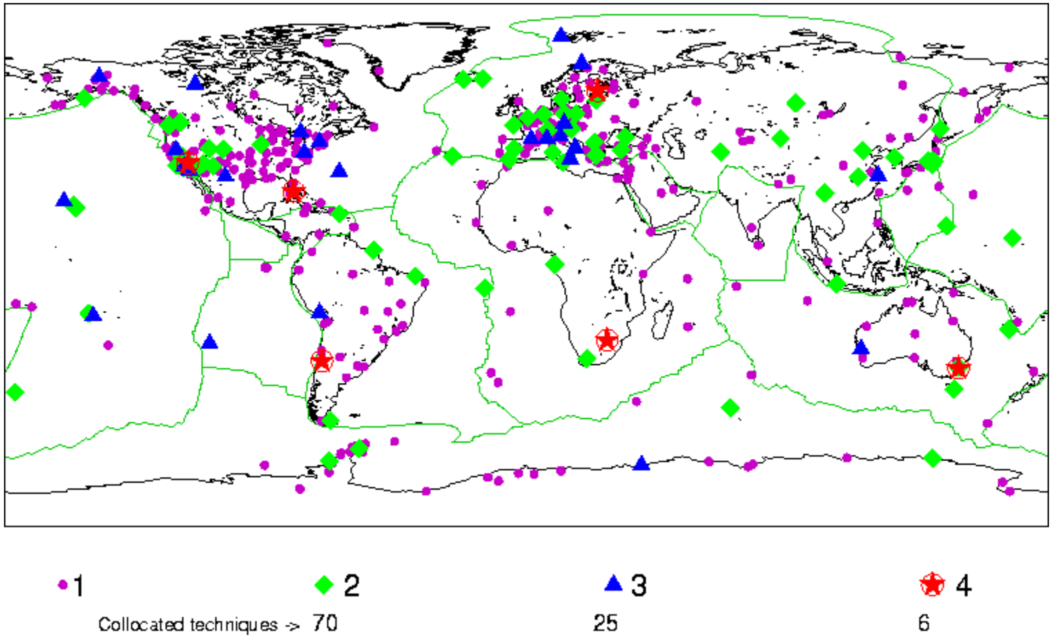
- Improved reference frame, WGS–84 (G1150), was developed using 15 days of GPS observations from NIMA/DoD stations plus 49 IGS stations
  - Refers to GPS week 1150, hence G1150 designation (epoch 2001.0)
  - Global consistency of about 1 cm
  - Considers station velocities
  - A subset of the IGS stations were constrained to ITRF values (to have agreement between ITRF and WGS84)
- GPS orbits are now with respect to WGS-84 (G1150)

Reference: Merrigan, M.J., E.R. Swift, R.F. Wong and J.T Saffel (2002), **A Refinement to the World Geodetic System 1984 Reference Frame**, Proceedings of ION GPS-02, Portland, OR, September 24-27, pp.1519-1529.

International Terrestrial Reference Frame (ITRF)

- International Earth Rotation and Reference Systems Service (IERS) was established in 1988 by the International Astronomical Union (IAU) and the International Union of Geodesy and Geophysics (IUGG).
  - Mission is to provide to the worldwide scientific and technical community reference values for Earth orientation parameters and reference realizations of internationally accepted celestial and terrestrial reference systems

- IERS leads the realization, use and promotion of the International Terrestrial Reference System (ITRS)
  - Latest realization is ITRF2008
  - Used VLBI, SLR, GPS, DORIS
  - ITRF is consistent with WGS84 to within 1-2 cm



http://lareg.ensg.ign.fr/ITRF/network.html

WGS 84 and Local Datums

- The generic transformation between any two datums consists of a scale, three translations (x,y,z) and three rotations (x,y,z). Transformation from WGS 84 to another datum may therefore involve any combination of these.
- In Canada, transformation to NAD 27 or NAD 83 datums is only accurate to the metre level
  - NAD 27 has local distortions that limit the usefulness of a 7-parameter transformation. Correspondingly, transformation parameters generally vary geographically.
  - Accuracy of the initial NAD 83 system
- NIMA (formerly DMA) gives relationship of WGS84 to some 83 datum around the world (see reference below)

Reference: DoD WGS84 – its definition and relationships with local geodetic systems, DMA (NIMA) TR 8350.2

GPS Time

GPS Time System

- GPS time is an atomic time scale that is steered to agree with UTC with an accuracy of 100 ns, modulo 1 s
  - UTC (USNO – U.S. Naval Observatory) is the reference for GPS time
- The time scale was defined to be coincident with UTC at the GPS standard epoch of January 6, 1980 (0 hours). No integer leap seconds are introduced into GPS time scale (unlike UTC). This implies that the offset between GPS and TAI is constant (at 32 s).
- Broadcast clock corrections give time corrections to obtain UTC (USNO) with an accuracy of 100 ns (10 – 30 ns in practice)
- GPS time is defined by a week number and time into the week
  - Time of Week (TOW) is in the range [0, 604800) seconds where 0 seconds is at Saturday midnight
  - Every Saturday at midnight, the TOW is reset to zero and the week counter is incremented by one

GPS System Time – Example

- A GPS time of 443210 seconds would occur on:

Day: $\dfrac{443210 \text{ seconds}}{86400 \text{ seconds per day}}$ = **5**.1297 days

Hour $\dfrac{(5.1297 - 5) * 86400}{3600 \text{ seconds per hour}}$ = **3**.114hours

Min: $\dfrac{(3.114 - 3) * 3600}{60 \text{ seconds per min}}$ = **6**.833 minutes

Sec: (6.833 - 6) * 60 = **50** seconds

Day 5 at 03:06:50 where Day 5 is a Friday

GPS Week Computation – Example

- To compute the GPS Week and time for a particular date, the number of weeks and seconds into week must be calculated accounting for leap years, etc.
- Example: GPS week and Time for January 25, 1993 at 10:00 hours
  - Days from January 5, 1980 to December 31, 1992 = 4749
    - 365 days per year x 13 years = 4745 days
    - Plus 1 for each leap year ** = 4 days
  - Days into 1993 = 24 (25th day is not over!)
  - Total days = 4749 + 24 = 4773
  - Total GPS Time (remove 5 days for start of Jan 6, 1980)
    - (4773 –5) days x 86400 sec/day + 10 h x 3600 sec/h = 411991200 seconds
  - GPS Week = 681
    - 4411991200 seconds = 681.2024 weeks
    - GPS Time of Week = (681.2024 – 681) x 604800 = 122400 seconds

** A leap year is one which is evenly divisible by four. In the case of centuries, the year must also be evenly divisible by 400 (i.e., 2000 is a leap year but 1900 is not)

A Brief Glimpse at the Rest of the Course

Chapter Breakdown

- Chapter 3
  - Satellite orbit representation
  - GPS orbit and clock representation
  - Dissemination of GPS orbit and clock data
- Chapter 5
  - Types of measurements available from a GNSS receiver and their basic characteristics
  - Concept of differential positioning
- Chapter 6
  - Satellite orbit and clock errors
  - Ionosphere & troposphere
  - Multipath & noise

Chapter Breakdown

- Chapter 7
  - Differential positioning
  - Ambiguity resolution
- Chapter 4
  - EM Propagation
  - GPS signal structure
  - Antenna characteristics
  - How a GNSS receiver works
- Chapter 8
  - Other GNSS and advanced topics
- Chapter 9
  - Augmentations and applications

Chapter 2 Figure References

- Furuno, “Technology: What is GPS?” [online http://www.furuno.com/en/gnss/technical/tec_what_gps] last accessed December 4, 2016.
- United States Government [online http://www.gps.gov/multimedia/images/] last accessed December 4, 2016
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- Revnivykh, S. (2011) “GLONASS Status and Modernization,” Presented at the 6th International Committee on GNSS, September 2011.
- Li, M. Qu, L. Zhao, Q., Guo, J. Su, X., and Li, X. (2014) “Precise Point Positioning with the BeiDou Navigation Satellite System,” *Sensors*, *14*,pp. 927-943; doi:10.3390/s140100927