ENGO 585 – Wireless Location

Winter 2021 – L01, B01
Part 1: Intro

Kyle O’Keefe, Instructor

11 January 2021

ENGO 585 – People

- Instructor: Kyle O'Keefe
  - CCIT 306 (or at home)
  - email: kpgokeef@ucalgary.ca
  - Office hours:
    - Generally available 8:30 to 16:30
- Teaching Assistant:
  - Luis Rodriguez Mendoza
  - email: larodrig@ucalgary.ca

Under Normal Circumstances

- 24 Lectures
  - Live, in person, some interaction, improvised examples
- 6 Labs
  - Live, in person, help from instructor, TAs and classmates
- Some notes posted online
- No technology in the lectures
- Lots of time for technology in the labs
- Closed book midterm and exam
  - Open ended questions to demonstrate knowledge rather than memorization

Fall 2020 Covid-19 Special Edition

- 24 Lectures
  - Live, on Zoom, some interaction, improvised examples (I hope)
- 6 Labs
  - Live, on Zoom, help from instructor, TAs and classmates
- Some notes, slides, white boards, etc. posted online
- Obviously technology in the lectures
- Lots of time for technology in the labs
- Online Midterm and Exam
  - Open ended questions to demonstrate knowledge rather than memorization
  - Considering having oral final exam

A bit more about me

- I'm originally from Vancouver
  - I studied physics at UBC, then Geomatics Engineering at Calgary
  - Faculty member since 2004
- First computer (when I was 8)
  - IBM PC – Intel 8088 4.77 MHz, 256k RAM, CGA graphics
    - $3000 in 1983 (**$6912 in 2020!)**
    - Why? My mother wanted me to learn what she thought would become the standard. (my school got 2 Apple 2e's 1985)
  - First smart phone
    - Samsung S3, open box return, $200 in 2015
- Languages
    - English (1976?), LOGO (1983), BASIC (1984), French (1986), C (1991), C++ (1994), IDL (1996), MATLAB (1997), Python (learning now)

What about you?

- Most of you are ENGO 4th years
- Some MEng students
- Pre-reqs
  - How do you feel about Least Squares (ENGO 361/363)?
  - What about Networks (ENGO 419)?
  - GPS (ENGO 465)?
  - Data Analysis (ENGO 563)?
- We are going to develop your LS skills and apply them to Wireless Location
- GPS will be used as an example

Tips to Succeeding in ENGO 585

- This is not meant to be a theoretically difficult course
  - It's your last term, you are busy and tired
  - But it is an important course
- Try to participate
  - Ask me questions in the lectures
  - Discuss amongst yourselves
  - This is whole point of University.
- Do the labs **on your own**
  - Ask questions of your classmates, but don't look at their code
  - Roadmaps may exist, ignore them, the point of the labs is to do them

Zoom

- Lectures will be in this Zoom meeting
- Labs are in a different Zoom meeting
- I would rather not record the lectures
  - Discussion: Recording is convenient for people in different time zones, but makes students less likely to participate
  - Question:
    - How many of you are telecommuting from a different time zone, and if so how many hours away?

D2L

- Course Outline, Notes, Labs and Midterm will be on D2L
- Let’s have look at the course outline

Next Class

- Intro to Wireless Location and a brief history

ENGO 585 – Lecture 2

- Today
  - Chapter 1: Overview of Wireless Location
    - Introduction and History
    - Historical Example – Build your own Wireless Telegraph
    - Applications
- Next Week
  - Chapter 2: Estimation for Wireless Location

ENGO 585 L2 - Introduction

- Wireless Location: What is it?
  - Location?
  - Wireless?
    - “Without wires”
      - Term has only recently returned to use in North American English
      - Formerly (and more recently in England) a “wireless” was the term used for any type of radio telegraph or telephone
      - Recently “wireless” has returned as a term to replace the word “cellular”, as in “wireless phone”, “wireless messaging”
      - Ironic: change in usage because the words “radio” and “cellular” seemed old-fashioned and low-tech
- So what is wireless location?
  - Determining the location of a wireless communication device
  - Locating and object using wireless means

ENGO 585 L2 - Introduction

- The most common example of a wireless location system today is GPS
  - Satellites transmit radio signals that allow a GPS receiver to position itself
  - Applications of GPS are well known
    - Vehicle navigation, aviation, surveying, geodesy, time transfer, recreation, etc.
  - GPS offers worldwide coverage but doesn't work well where most people in industrialized countries live and work: **In cities and indoors**

ENGO 585 L2 - Introduction

- This course will review the fundamentals behind all wireless location systems and then introduce some of the non-GPS technologies that can be used.
- This is a relatively new field: Not many textbooks.
- Notes will be posted on D2L
- The notes are mainly text and missing the diagrams
  - Come to class to see the diagrams
- A reading list will be provided online for each topic covered in this course

ENGO 585 L2 – (Pre-) History

- Pre-1900 – Maxwell develops theory of electromagnetic waves, Hertz produces them with a spark-gap
- 1896 – Marconi demonstrates first wireless telegraph in England
- 1905 – Fessenden invents continuous-wave voice transmission
- 1912 – Armstrong invents regenerative circuit – amplifier and continuous-wave transmitter
- 1924 – British begin radio-direction-finding research
- 1927 – First quartz clock
- 1935 – Radar proposed by Watson-Watt

ENGO 585 L2 – (Pre-) History

- 1940's – Loran and other radio-navigation systems developed
- 1947 – cellular telephone service is proposed, FCC allocates enough bandwidth for 32 channels (IMTS channels for car phones, each major city is a "cell", wait lists for channels).
- 1947 – First transistor
- 1949 – First atomic clock
- 1950s and 60s – Integrated circuits
- 1979 – First automated cellphone network (NTT in Japan)

ENGO 585 L2 – (Recent?) History

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- 1978 – First GPS satellite launch
- 1982 – AMPS Cellular phone service in North America ("1G")
- 1983 – TCP/IP selected as ARPANET protocol
- 1983 – Motorola DynaTAC (the Brick) https://www.youtube.com/watch?v=0WUF3yjgGf4
- 1989-1992 – Bluetooth developed
- 1991 – First "2G" or GSM in Finland
- 1995 – GPS fully operational
- 1997 – First WiFi, 801.11b in 1999
- 1999 – First location-based services patent
- 1999 – First LBS app: Weather.com for Palm

Michael Douglas as Gordon Gekko (http://www.thestar.com/entertainment/movies/2010/05/14/wells_why_gordon_gekko_is_still_at_home_on_wall_street.html)

http://www.pocket-lint.com

ENGO 585 L2 – (Recent?) History

- 2000 – GPS S/A turned off (May 1)
- 2000 – First Geocache (May 3)
- 2001 – E911 deadline
- 2002 – First "Integrated" Blackberry (pda, phone, text, email and web-browsing)
- 2003 – Ultra-wideband approved in US by FCC
- 2007 – 3G
- 2007 – iPhone
- 2007-2020 – Widespread smartphone adoption

ENGO 585 L2 – History

- *Electrical Review*, November 30, 1901, page 654:

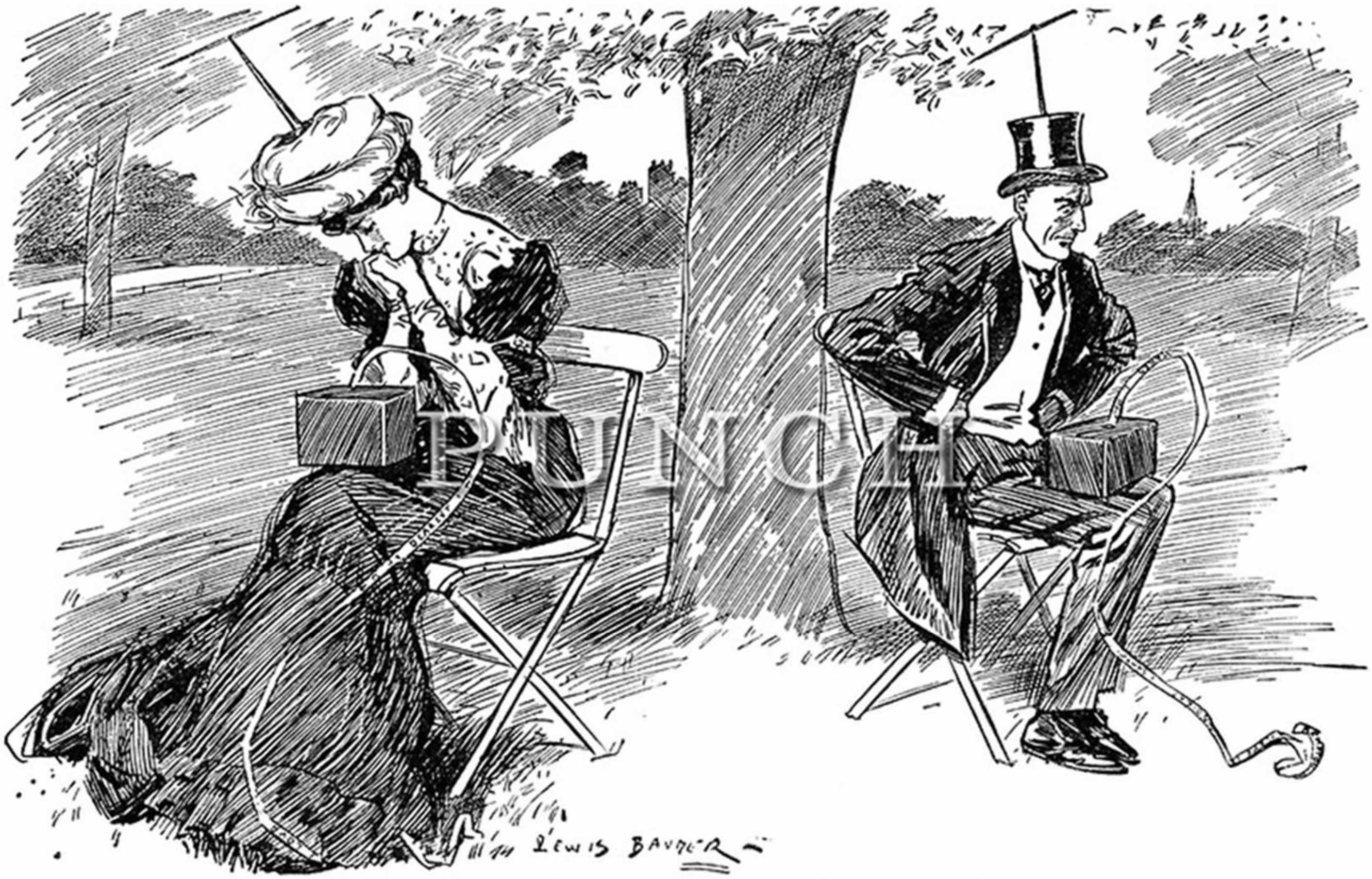
  WIRELESS TELEGRAPHY STOCK.

  For some time past the advertising columns of the daily papers in this country have been largely patronized by various wireless telegraph companies offering stock in their enterprises for sale. It is noticeable that none of these companies is offering to sell instruments or to transmit messages…

- The same thing happened again in 2000

ENGO 585 L2 – History

FORECASTS FOR 1907.



IV.—Development of Wireless Telegraphy. Scene in Hyde Park.
[These two figures are not communicating with one another. The lady is receiving an amatory message, and the gentleman some racing results.]

- Punch Magazine 1906 (British satire 1841-2002) cartoon by Lewis Baumer predicts Sexting and Online Gambling

ENGO 585 L2 – History

- *The Electrical Experimenter*, January, 1916, page 487:

THE WIRELESS 'PHONE WILL GET YOU.

Many a man in the spring of the weather and of youth has stood on his doorstep and longed to be able to shout his thoughts to all the world. Many have felt this, but few have ever thought they could do it. Wireless telephony has made such a scheme within the range of the possibilities. If your wife is cross, or your enemy is hot on your trail, or the partner wants to tell you that note falls due to-morrow, don't think you can go to San Francisco or China and get away from her or them. Gadzooks: You can't--the wireless 'phone will get you.

ENGO 585 L2 – History

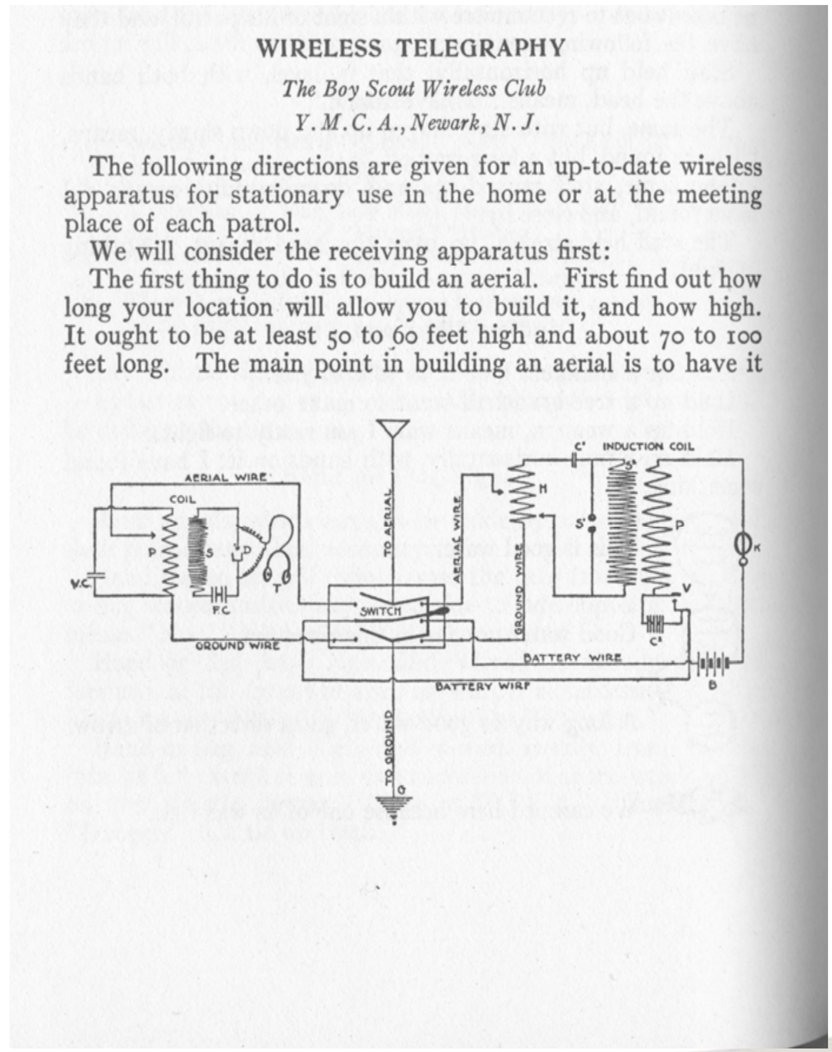
- *Electrical Experimenter*, August, 1919, page 372:

POCKET WIRELESS SOON,
PREDICTS MARCONI OFFICIAL.

… Isaacs foresees the day, not far distant, when pocket wireless telephones will be in wide use. A business man's secretary, walking along the street, Isaacs says, will hear a bell ring in his pocket, will put a receiver to his ear and hear "his master's voice" give him instructions, probably from an airplane hundreds of miles away.

ENGO 585 L2 – Wireless Example

- *Boy Scout's of America Official Handbook for Boys* (1911) contains instructions for building a wireless telegraph
- Don't try this at home today!
  - Dangerous
  - Unlawful to operate in Canada


WIRELESS TELEGRAPHY

The Boy Scout Wireless Club
Y.M.C.A., Newark, N.J.

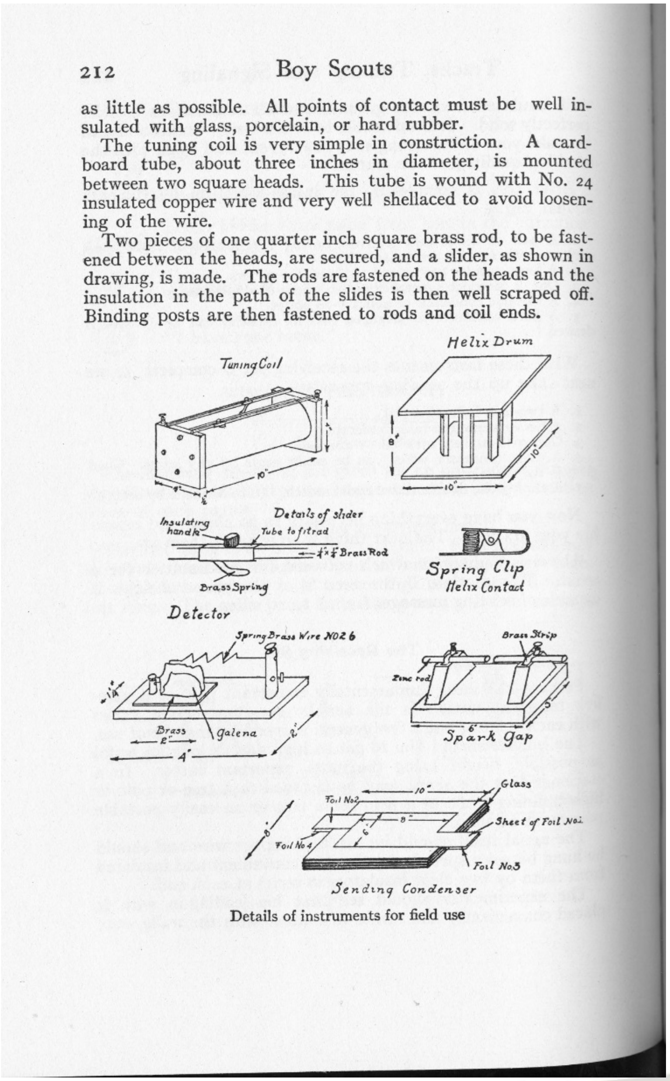
The following directions are given for an up-to-date wireless apparatus for stationary use in the home or at the meeting place of each patrol.

We will consider the receiving apparatus first:

The first thing to do is to build an aerial. First find out how long your location will allow you to build it, and how high. It ought to be at least 50 to 60 feet high and about 70 to 100 feet long. The main point in building an aerial is to have it

ENGO 585 L2 – Wireless Example

- Build your own parts!
  - Inductors: Copper wire and toilet paper tubes
  - Capacitors: Layers of wax-paper and tin foil
  - Spark gap: brass and zinc nails
  - Diode: Lead sulphide crystal
  - Need to purchase a battery and headphones



212 Boy Scouts

as little as possible. All points of contact must be well insulated with glass, porcelain, or hard rubber.

The tuning coil is very simple in construction. A cardboard tube, about three inches in diameter, is mounted between two square heads. This tube is wound with No. 24 insulated copper wire and very well shellaced to avoid loosening of the wire.

Two pieces of one quarter inch square brass rod, to be fastened between the heads, are secured, and a slider, as shown in drawing, is made. The rods are fastened on the heads and the insulation in the path of the slides is then well scraped off. Binding posts are then fastened to rods and coil ends.

Details of instruments for field use

ENGO 585 L2 – Wireless Example

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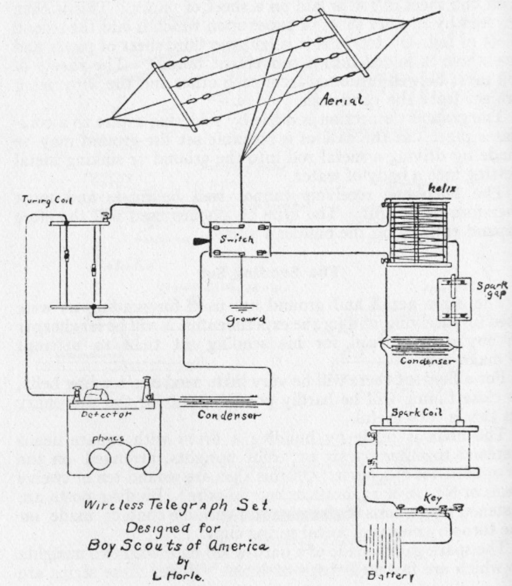
- How does it work? Transmitting
  1. Press telegraph key, causes current to flow through ignition coil – high voltage on secondary side
  2. Charge accumulates in capacitor, when voltage high enough, capacitor discharges across spark gap producing a noise-like signal
  3. Because gap is coupled in an LC circuit, the circuit resonates. (like a bell or tuning fork).
  4. This signal is radiated by the antenna

214 Boy Scouts

placed two pieces of battery zincs so as to make the gap between their ends variable. Binding posts are fastened to the strips for contact.

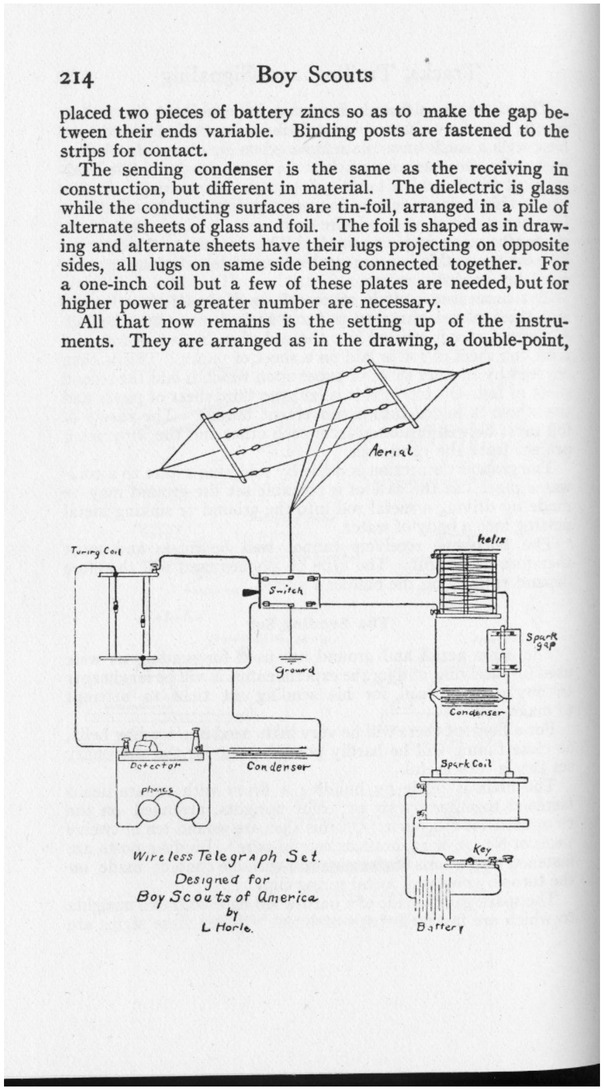
The sending condenser is the same as the receiving in construction, but different in material. The dielectric is glass while the conducting surfaces are tin-foil, arranged in a pile of alternate sheets of glass and foil. The foil is shaped as in drawing and alternate sheets have their lugs projecting on opposite sides, all lugs on same side being connected together. For a one-inch coil but a few of these plates are needed, but for higher power a greater number are necessary.

All that now remains is the setting up of the instruments. They are arranged as in the drawing, a double-point,



ENGO 585 L2 – Wireless Example

- How does it work? Receiving
  1. LC receive circuit is tuned to the same frequency and will resonate driven at that frequency
  2. Diode allows one-way current only, one-way current is the signal
  3. Signal converted to sound in headphones
- How would you use this "wireless" to:
  - Measure a range?
  - Measure a direction?
  - Find your location?


214 Boy Scouts

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ENGO 585 L2 – Applications

- Many more than the current “GPS applications”
- Current applications
  - E911
  - Asset management: healthcare, retail, transport (find a package, medical device, patient, etc.)
  - Gaming
  - Recreation/online communities (ie: where are my friends)
  - Location based advertising (Google isn’t free, you are the product)
- Future applications
  - Automatic indoor navigation
  - Law enforcement (already with GPS)

ENGO 585 L2 – Next lecture

- Friday
  - No Lab this week
- Next lecture
  - Chapter 2: Estimation for Wireless Location

ENGO 585 – Wireless Location

Winter 2021 – L01, B01
Part 2: Estimation for Navigation

Kyle O’Keefe, Instructor

11 January 2021

ENGO 585 Part 2 – Estimation for Navigation

a. Review of Parametric Least Squares
b. Math (Observaton) Models for Navigation
c. Accuracy Measures
d. Sequential LS and the Linear Kalman Filter
e. Non-linear Kalman Filters

ENGO 585 L3 – Review of Parametric LS

- Linear Parametric Least Squares
  - The idea: we have a *linear observation model* that relates observations l to the unknown parameters or states x
  - Linear observation model means that the relationship between x and l can be written as a matrix A
  - The model is not perfect and has errors r with covariance C_l

$$l = Ax + r \qquad\qquad C_l$$

  - If $l = Ax$ is uniquely determined (ie A is square), then

$$x = A^{-1}l$$

  - If overdetermined, use the pseudo-inverse

$$l = \qquad Ax$$

ENGO 585 L3 – Review of Parametric LS

- Linear Parametric Least Squares (cont.)
  - Without proof, the "optimal solution" if the measurements have different variances is

$$\hat{x} = \left(A^T C_l^{-1} A\right)^{-1} A^T C_l^{-1} l$$

  - Where $\hat{x}$ denotes the estimate of x.
  - Again without the proof, the estimated covariance of $\hat{x}$ is

$$C_{\hat{x}} = \left(A^T C_l^{-1} A\right)^{-1}$$

ENGO 585 L3 – Review of Parametric LS

- Non-Linear Parametric Least Squares
  - If the model is not linear then we can write it as a function $f(x)$ instead of as a matrix

$$l = f(x) + r \qquad C_l$$

  - Then we can replace the function with its Taylor series
    - This only works for small deviations δ from a point of expansion x_0
  - Let $x = x_0 + \delta$ then

$$f(x) = f(x_0) + \frac{\partial f}{\partial x}(x - x_0) + \ \ldots \cong f(x_0) + \frac{\partial f}{\partial x}\delta$$

  - So if we start at a point of expansion, then we can approximate the linear relationship with the partials

$$A = \frac{\partial f}{\partial x} \text{ or } A = \frac{\partial l}{\partial x}$$

ENGO 585 L3 – Review of Parametric LS

- Non-Linear Parametric Least Squares (cont.)
  - Then we can define the difference between the observations and the observations at the point of expansion as the misclosure w

$$w = l - f(x_0) \text{ or } w = f(x_0) - l$$

  - And then the solution is

$$\hat{\delta} = \left(A^T C_l^{-1} A\right)^{-1} A^T C_l^{-1} w$$

  - or

$$\hat{\delta} = -\left(A^T C_l^{-1} A\right)^{-1} A^T C_l^{-1} w$$

  - and

$$\hat{x} = x_0 + \delta \qquad C_{\hat{x}} = \left(A^T C_l^{-1} A\right)^{-1}$$

  - Red version common in UNB and UCalgary-derived texts

ENGO 585 L3 – Review of Parametric LS

- Non-Linear Parametric Least Squares (cont.)
  - For this to work, both w and δ need to be ***small***
  - To make sure, we iterate by moving the point of expansion by δ until δ converges to near-zero and the solution $\hat{x}$ doesn't change anymore
  - This misclosure can be thought of as the pre-adjustment residuals, ie how much the observations don't fit the p.o.e
  - On the last iteration, when δ is almost zero, the final estimate $\hat{x}$ is the same as x_0 and now the misclosure is equal to the residuals

$$w = l - f(x_0) \text{ and } r = l - f(\hat{x})$$

  - I prefer this notation because both w and r are defined as observed minus modelled. But many texts insist on switching the definition of w

ENGO 585 L2 – Next lecture

- Next 4 lectures
  - Chapter 2: Estimation for Navigation/Wireless Location
- Friday: Lab 1 and start of term project

ENGO 585 L4 – Math Models for Navigation

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- Location from Angle of Arrival
  - In 2D, AOA means we are either observing azimuth α_{ij} or direction, r_{ij}
  - Depends on if the orientation ω_i of the observer is known.

$$\alpha_{ij} = r_{ij} + \omega_i$$

  - Can write as a parametric observation equation (of x and y)

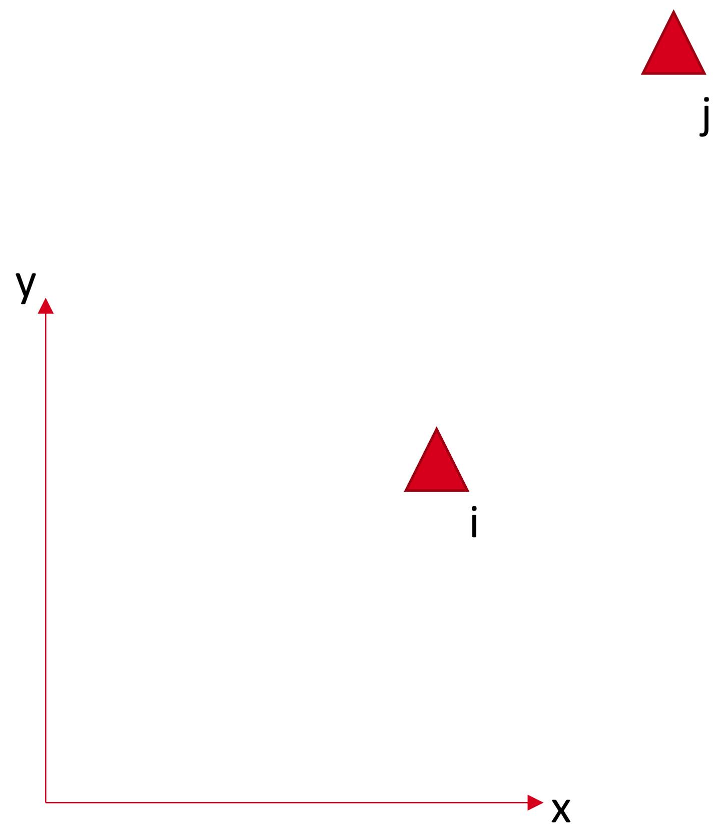
$$\alpha_{ij} = \arctan\left(\frac{x_j - x_i}{y_j - y_i}\right)$$

  - or

$$r_{ij} = \arctan\left(\frac{x_j - x_i}{y_j - y_i}\right) - \omega_i$$

ENGO 585 L4 – Math Models for Navigation

- Location from Angle of Arrival



- Location from Angle of Arrival
  - Differentiate w.r.t. the unknowns $[x, y]_j$ to get the design matrix row

$$A_i = \left[\frac{y_j - y_i}{r^2} \quad -\frac{x_j - x_i}{r^2}\right]$$

  - The azimuth at point j depends on the unknown coordinates of point j, the known coordinates of the other point, i, and the range, r, between, squared.
  - If you are measuring direction (ie your orientation is unknown) then unknowns are $[x, y, \omega]_j$ and the design matrix row is:

$$A_i = \left[\frac{y_j - y_i}{r^2} \quad -\frac{x_j - x_i}{r^2} \quad -1\right]$$

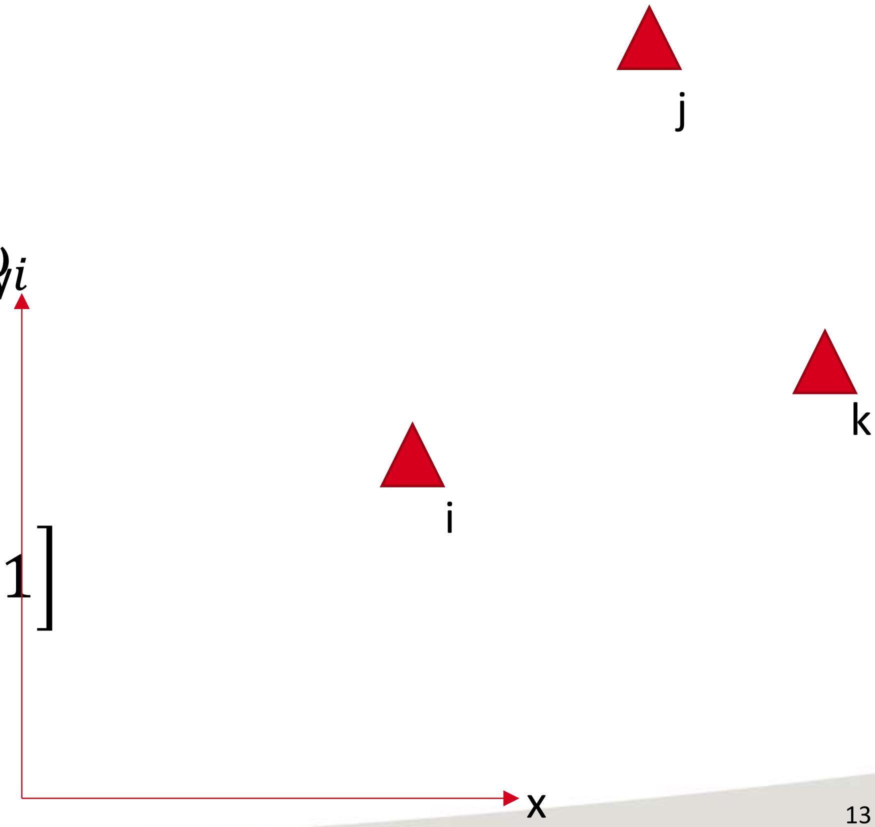
- Location from Angle of Arrival
  - Warning: Sign conventions depend on the convention for i,j, (from/to) and also x,y vs E,N (or N,E) so always draw the picture!
  - Important part is that the ability to observe x depends on the y-difference and the ability to observe y depends on the x-difference
  - Azimuth: angle from one point to another wrt North
  - Bearing: angle from one point to another wrt coordinate axes
  - Direction: an angle observed by a device wrt to the device
    - Think reading a total station horizontal circle

ENGO 585 L4 – Math Models for Navigation

- Location from Angle of Arrival
  - What about "Angle" as it's used in surveying
    - At-From-To
  - To get these, you need to difference two directions

$$r_{ij} - r_{ik} = \arctan\left(\frac{x_j - x_i}{y_j - y_i}\right) - \omega_i - \arctan\left(\frac{x_k - x_i}{y_k - y_i}\right) - \omega_i$$

$$A_i = \begin{bmatrix} \frac{y_j - y_i}{r^2} & -\frac{x_j - x_i}{r^2} & -1 \end{bmatrix} - \begin{bmatrix} \frac{y_k - y_i}{r^2} & -\frac{x_k - x_i}{r^2} & -1 \end{bmatrix}$$



ENGO 585 L4 – Math Models for Navigation

- Location from Angle of Arrival
  - Warning: If you subtract two measurements, you have made a linear combination – so do the same to the covariance
  - Sets for at-from-to angles are correlated

ENGO 585 L4 – Math Models for Navigation

- Time of Arrival: aka ranging or trilateration
  - This is the easy example

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

  - Where r_i is the range to the ith known point.
  - So the design matrix row is:

$$A_i = \begin{bmatrix} \frac{x - x_i}{r} & \frac{y - y_i}{r} \end{bmatrix}$$

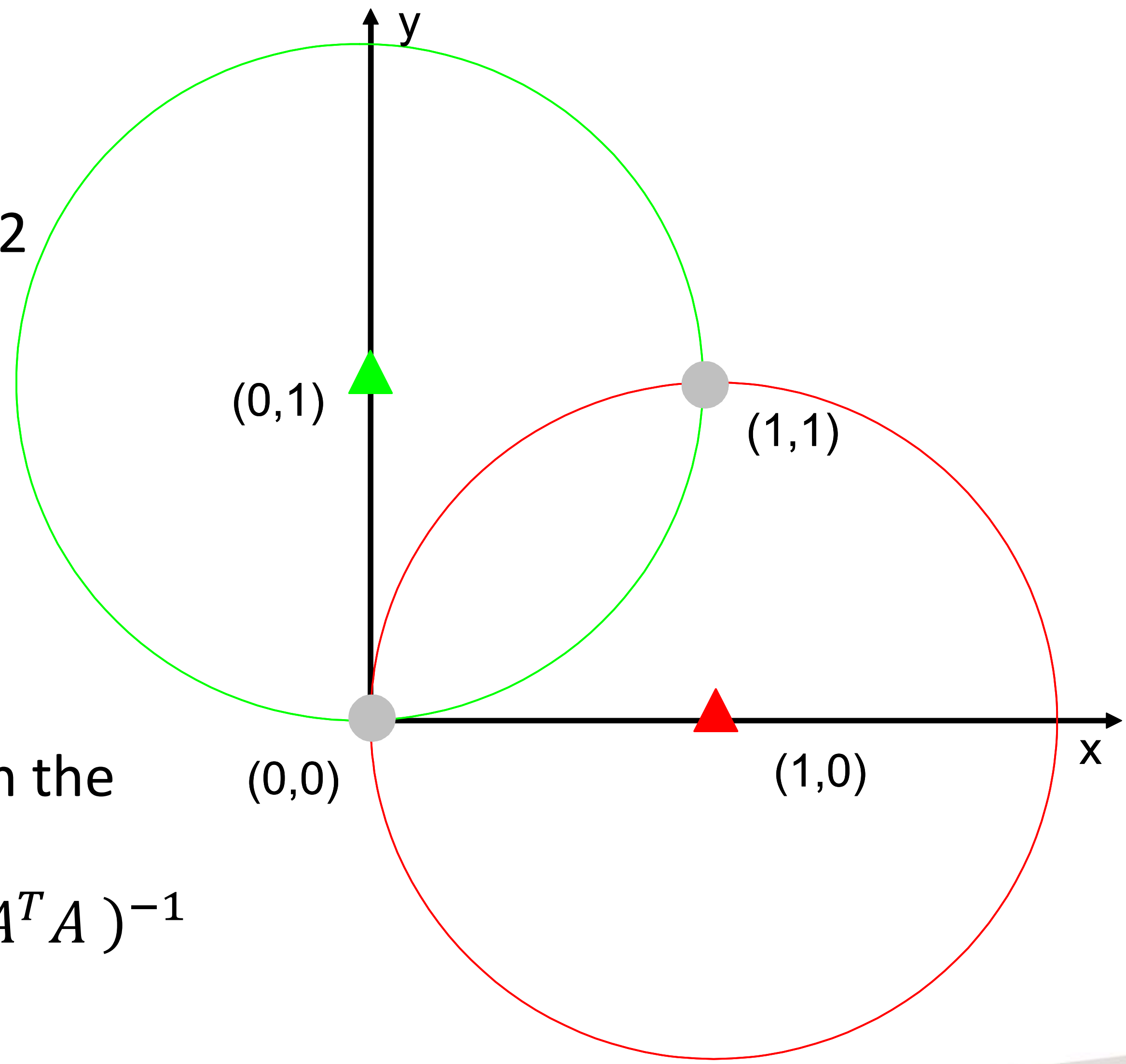
ENGO 585 L4 – Math Models for Navigation

- Time of Arrival:
  - Simplest example: 2 ranges

$$A = \begin{bmatrix} \frac{1-1}{1} & \frac{1-0}{1} \\ \frac{1-0}{1} & \frac{1-1}{1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

  - POE needs to be on the correct side
  - Covariance from $(A^T A)^{-1}$

$$C_{\hat{x}} = (A^T A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

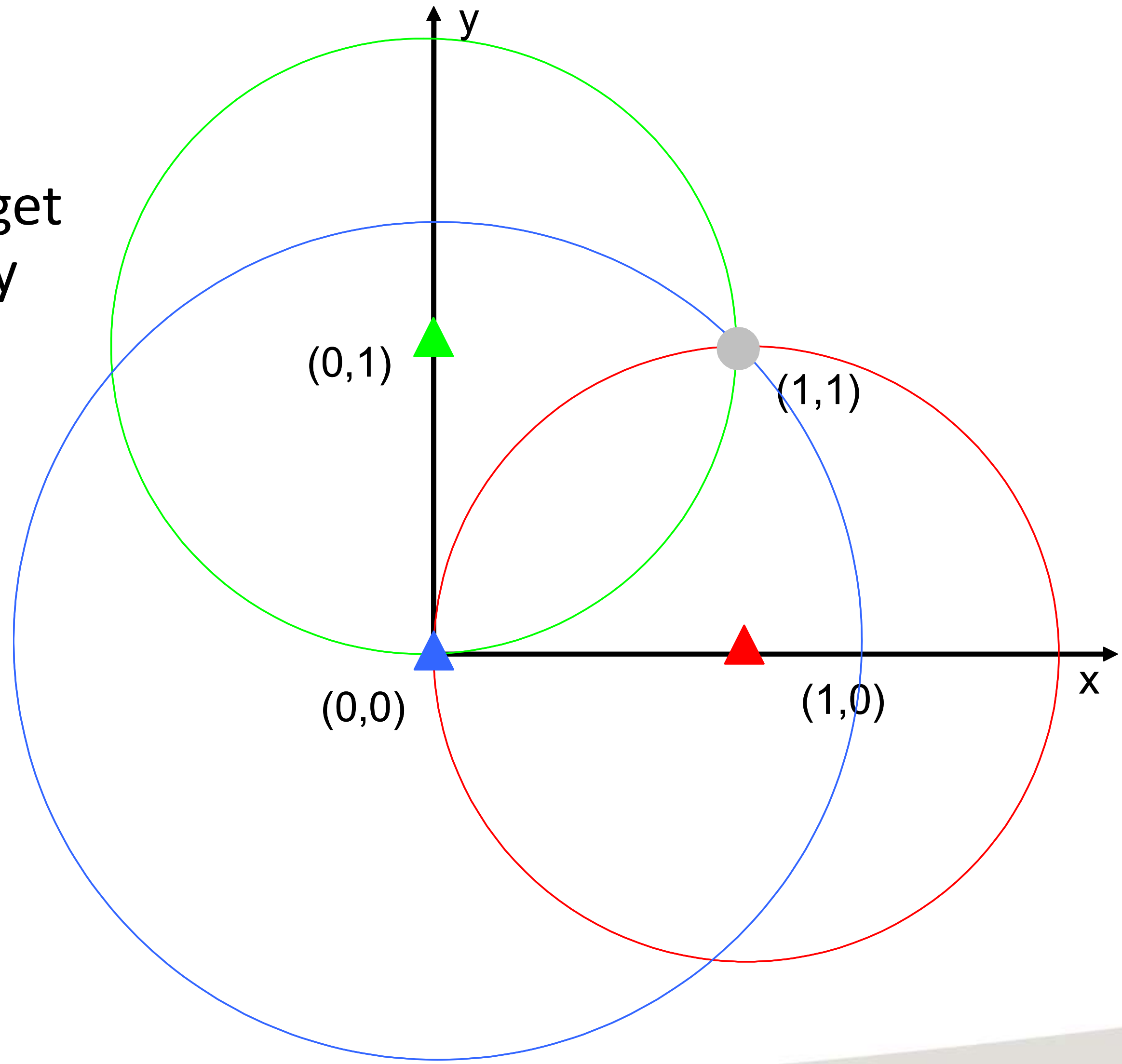


ENGO 585 L4 – Math Models for Navigation

- Time of Arrival:
  - Add a 3rd range to get rid of the ambiguity

$$A=\begin{bmatrix}\frac{1-1}{1} & \frac{1-0}{1}\\ \frac{1-0}{1} & \frac{1-1}{1}\\ \frac{1-0}{\sqrt{2}} & \frac{1-0}{\sqrt{2}}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & 0\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$$

$$C_{\hat{x}}=(A^T A)^{-1}=\begin{bmatrix}0.75 & -0.25\\ -0.25 & 0.75\end{bmatrix}$$



ENGO 585 L4 – Math Models for Navigation

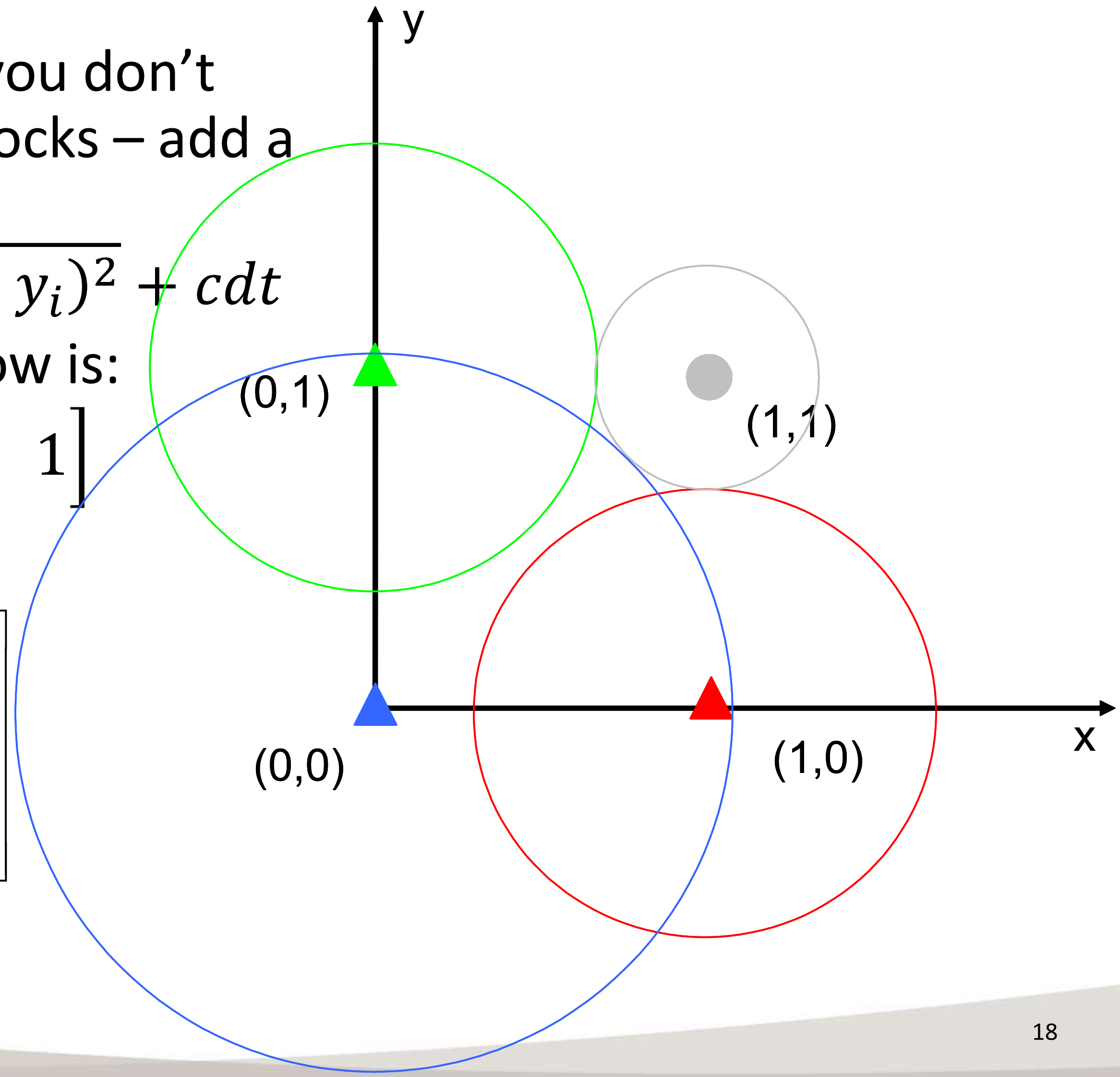
- Pseudoranging:
  - One-way ranging and you don't have a synchronized clocks – add a clock offset

$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} + cdt$$

  - So the design matrix row is:

$$A_i = \begin{bmatrix} \dfrac{x-x_i}{r} & \dfrac{y-y_i}{r} & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \dfrac{1-1}{1} & \dfrac{1-0}{1} & 1 \\ \dfrac{1-0}{1} & \dfrac{1-1}{1} & 1 \\ \dfrac{1-0}{\sqrt{2}} & \dfrac{1-0}{\sqrt{2}} & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ \dfrac{\sqrt{2}}{2} & \dfrac{\sqrt{2}}{2} & 1 \end{bmatrix}$$



ENGO 585 L4 – Math Models for Navigation

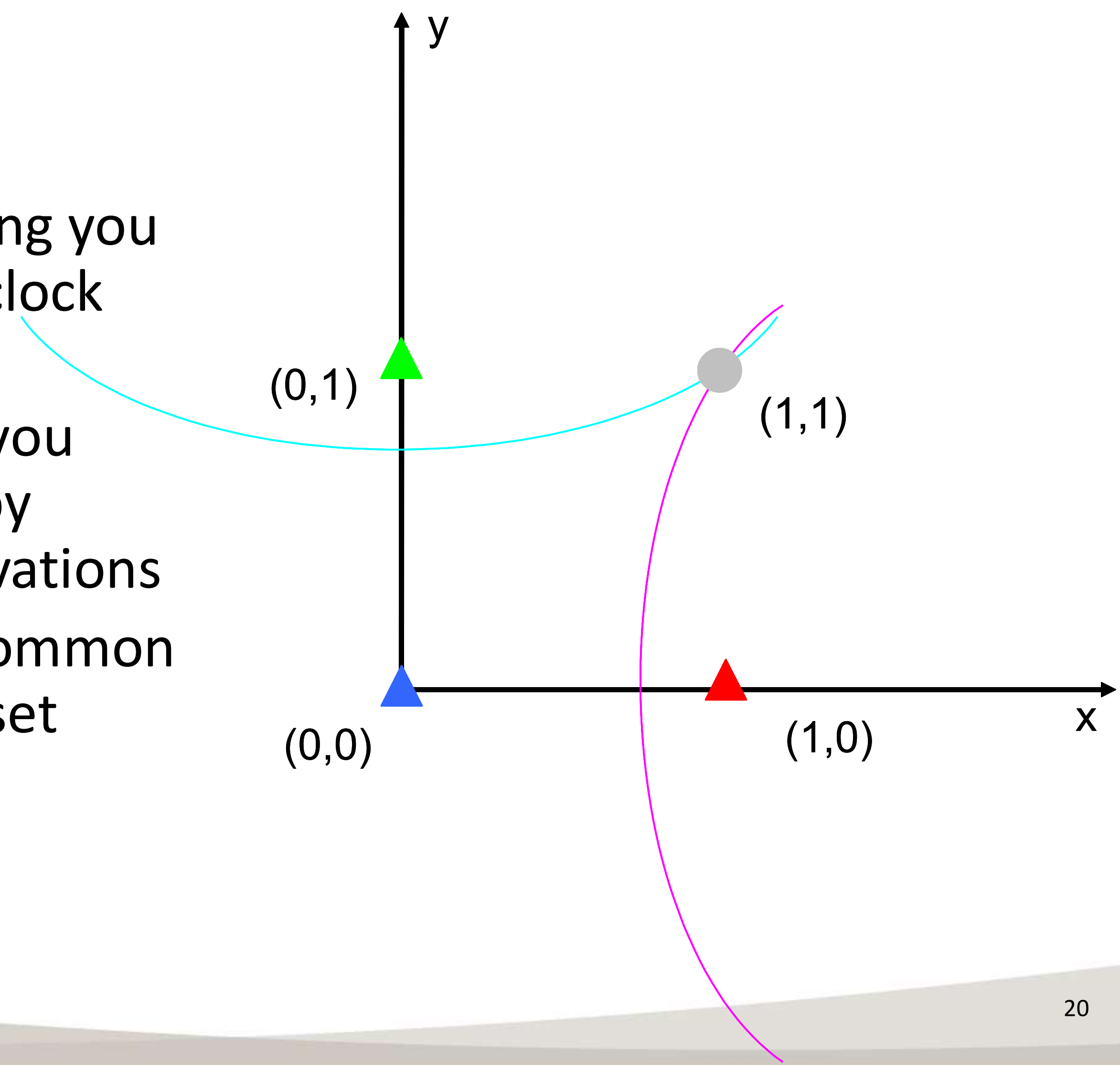
- Pseudoranging:
  - This design matrix can give us a C_x

$$C_{\hat{x}} = (A^T A)^{-1} = \begin{bmatrix} 9.242 & 8.242 & -9.950 \\ 8.242 & 9.242 & -9.950 \\ -9.950 & -9.950 & 11.657 \end{bmatrix}$$

  - Which is way worse that the 3 range case.

ENGO 585 L4 – Math Models for Navigation

- Time Difference of Arrival:
  - With pseudoranging you can estimate the clock
  - Or with TDOA or hyperbolic mode you just cancel it out by subtracting observations
  - This cancels the common bias, the clock offset

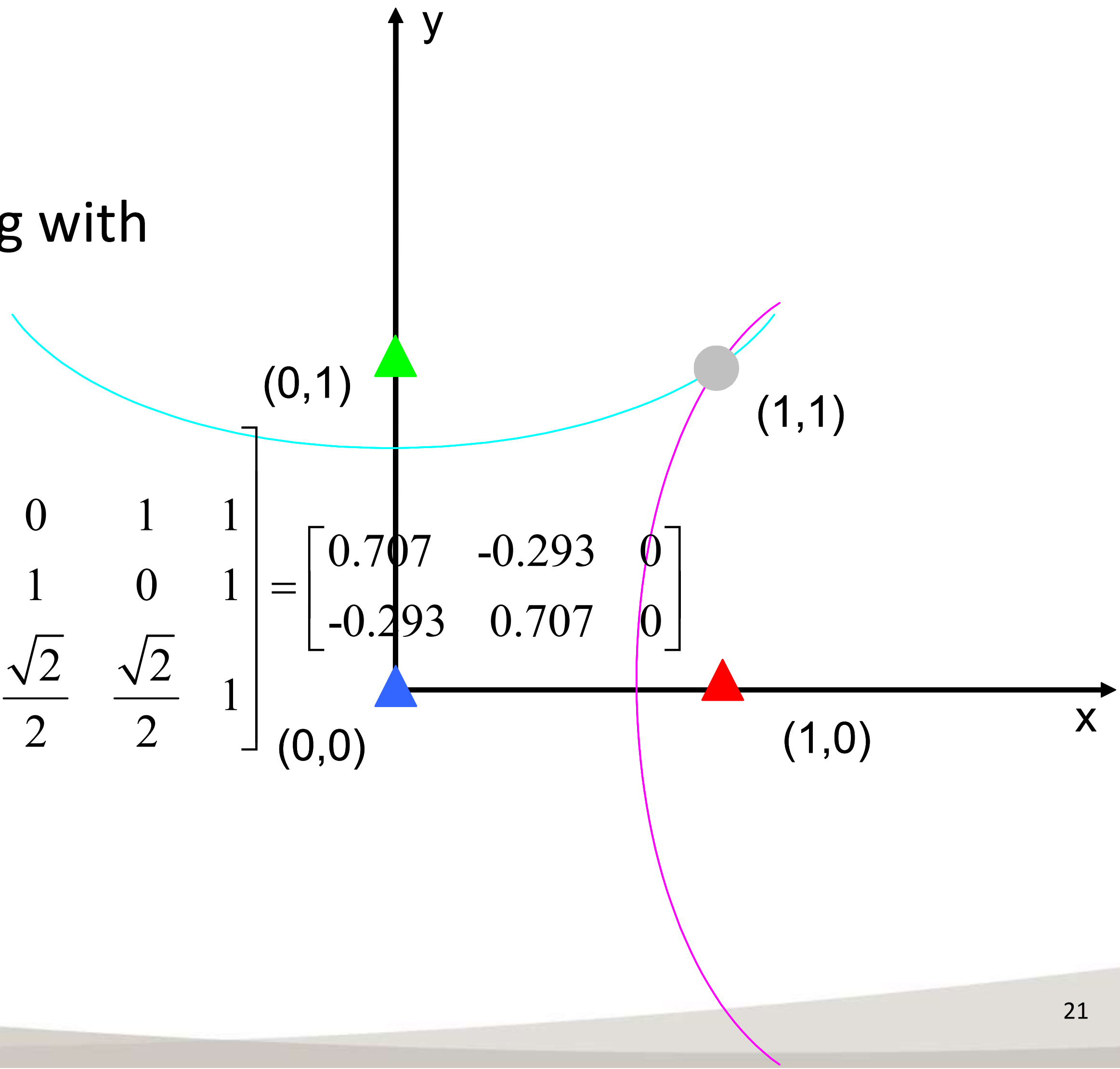


ENGO 585 L4 – Math Models for Navigation

- Time Difference of Arrival:
  - Can do differencing with linear algebra

$$B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A_{TDOA} = BA_{pseudoranging} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.293 & 0 \\ -0.293 & 0.707 & 0 \end{bmatrix}$$



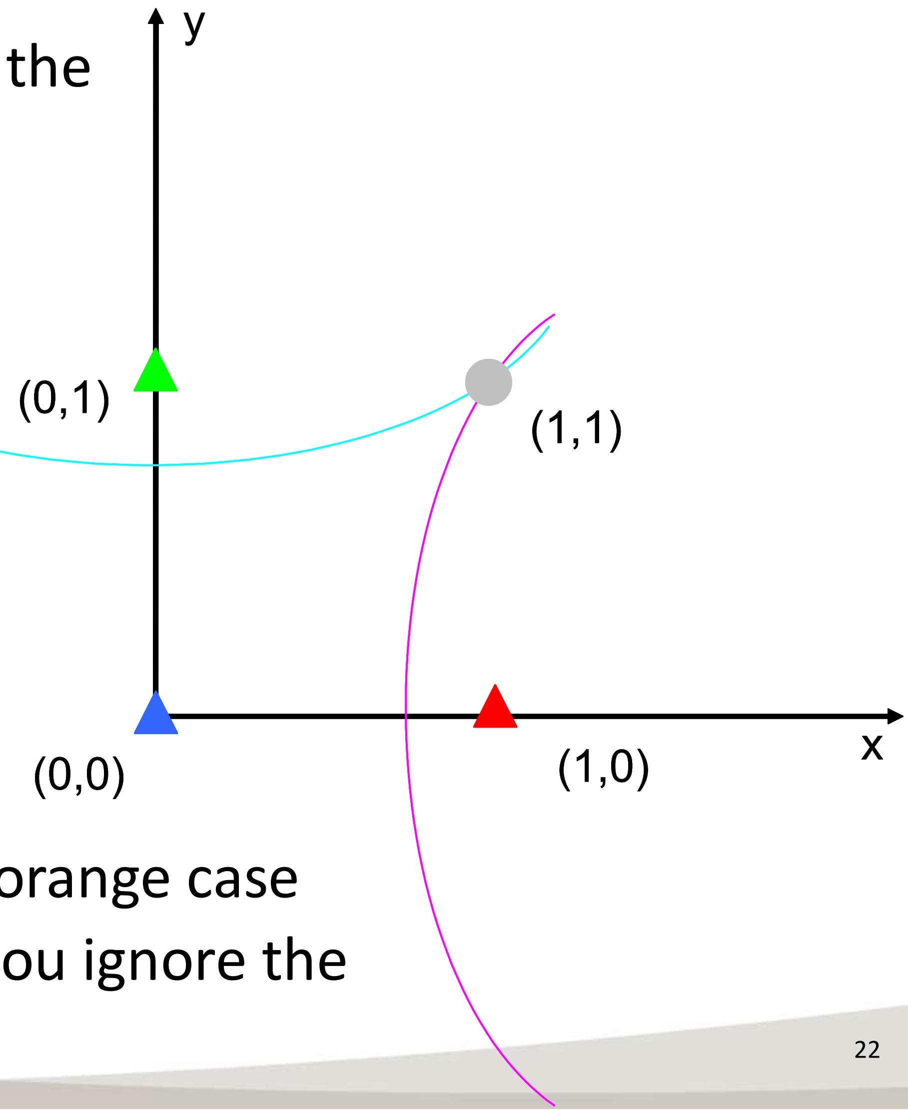
ENGO 585 L4 – Math Models for Navigation

- Time Difference of Arrival:
  - Do the same differencing to the covariance matrix

$$C_l = BIB^T = BB^T = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$(A^T B^T (BC_l B^T)^{-1} BA) = \begin{bmatrix} 0.529 & -0.471 & 0 \\ -0.471 & 0.529 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_{\hat{x}} = \begin{bmatrix} 9.242 & 8.242 \\ 8.242 & 9.242 \end{bmatrix}$$



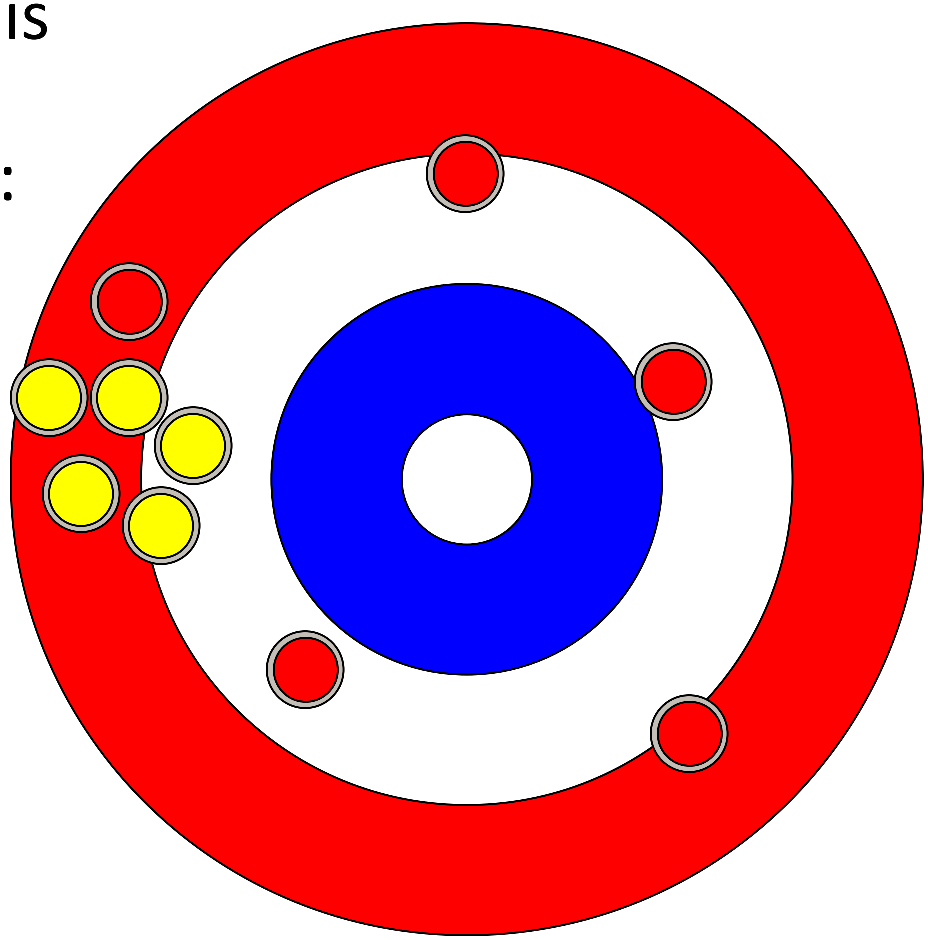
  - Same results as the 3 pseudorange case
  - You get a different result if you ignore the correlation

ENGO 585 L4 – Next lecture

- Friday: Lab 1 and start of term project
- Next 3 lectures
  - Chapter 2: Estimation for Navigation/Wireless Location
    - Accuracy measures
    - Kalman Filtering

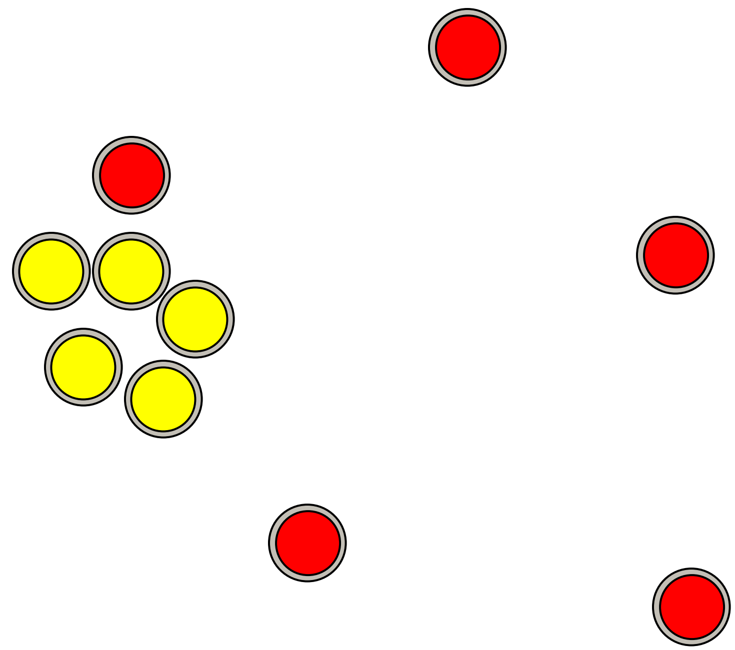
ENGO 585 L5 – Accuracy Measures

- When the true solution is known, accuracy can be assessed from a sample:
  - Yellow:
    - Precise: Yes
    - Accurate: No
  - Red:
    - Precise: No
    - Accurate: Yes



ENGO 585 L5 – Accuracy Measures

- When the true solution is not known, accuracy can not be assessed :
  - Yellow:
    - Precise: Yes
    - Accurate: ??
  - Red:
    - Precise: No
    - Accurate: ??



ENGO 585 L5 – Accuracy Measures

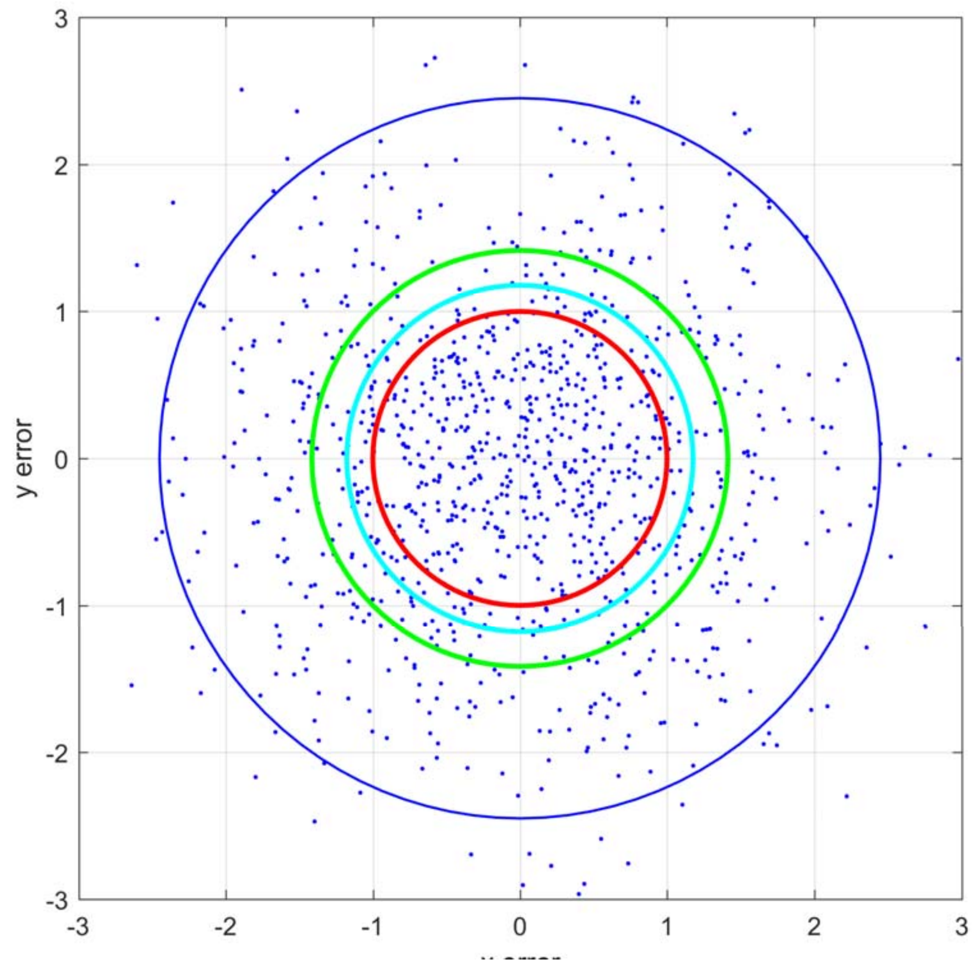
- Lecture 3 and 4: Review of LS, non-linear LS and observation 4+ different observation models

$$C_{\hat{x}} = \left(A^T C_l^{-1} A\right)^{-1}$$

- Covariance of the estimated states (parameters) for LS and non-linear LS can be used to define many different accuracy measures

ENGO 585 L5 – Accuracy Measures

- Accuracy Measures derived from $C_{\hat{x}}$
  - Error Ellipse
    - From eigenvalues and vectors of $C_{\hat{x}}$
  - 95% confidence region (2.45 x error ellipse)
  - "RMS circle"
    - RMS = $\sqrt{mean^2 + \sigma^2}$
    - Valid if mean error = 0
    - RMS = $\sqrt{\sigma_x^2 + \sigma_y^2}$
    - aka *DRMS*
  - Circular Error Probable (median 2D error)



ENGO 585 L5 – From Least Squares to the Kalman Filter

- So far: Parametric LS and non-linear parametric LS
  - One batch of measurements l, one C_l and one design matrix A
- What if you have multiple batches of measurements?
  - Three options:
    1. Put all in one big batch
    2. Summation of normals
    3. Sequential LS

ENGO 585 L5 – From Least Squares to the Kalman Filter

- All in one batch

$$l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \qquad C_l = \begin{bmatrix} C_{l_1} & 0 \\ 0 & C_{l_2} \end{bmatrix} \qquad A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$$

- Answer includes a lot of multiplying by zero

$$\hat{\delta} = (A)^{-1} A^T C_l^{-1} w$$

$$\hat{\delta} = \left(\begin{bmatrix} A_1^T & A_2^T \end{bmatrix} \begin{bmatrix} C_{l_1} & 0 \\ 0 & C_{l_2} \end{bmatrix}^{-1} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \right)^{-1} \begin{bmatrix} A_1^T & A_2^T \end{bmatrix} \begin{bmatrix} C_{l_1} & 0 \\ 0 & C_{l_2} \end{bmatrix}^{-1} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

- So don’t do the multiplying and you get summation of normals

ENGO 585 L5 – From Least Squares to the Kalman Filter

- Summation of normal can be for any number of groups of observations, provided they are uncorrelated, ie. C_l is block diagonal.

$$\hat{\delta}_2 = N_2^{-1}u_2 = \left(A_1^T C_{l_1}^{-1} A_1 + A_2^T C_{l_2}^{-1} A_2\right)^{-1}\left[A_1^T C_{l_1}^{-1} w_1 + A_2^T C_{l_2}^{-1} w_2\right]$$

- Add a 3rd set of observations:

$$\hat{\delta}_3 = \left(A_1^T C_{l_1}^{-1} A_1 + A_2^T C_{l_2}^{-1} A_2 + A_3^T C_{l_3}^{-1} A_3\right)^{-1}\left[A_1^T C_{l_1}^{-1} w_1 + A_2^T C_{l_2}^{-1} w_2 + A_3^T C_{l_3}^{-1} w_3\right]$$

- But you can also collect and store the “prior” N and u

$$\hat{\delta}_3 = \left(N_2 \ + A_3^T C_{l_3}^{-1} A_3\right)^{-1}\left[u_2 + A_3^T C_{l_3}^{-1} w_3\right]$$

ENGO 585 L5 – From Least Squares to the Kalman Filter

- So Summation of normals becomes an iterative method

$$\hat{\delta}^{(+)} = \left(N^{(-)} + A^T C_l^{-1} A\right)^{-1} \left[u^{(-)} + A^T C_l^{-1} w\right]$$

- Where the $^{(-)}$ and $^{(+)}$ mean before and after adding the current measurements

- $\left(N^{(-)}\right)^{-1}$ is the prior covariance $C_{\hat{x}}$ and $u^{(-)}$ contains effect of all the prior misclosures
- How the POE is defined is a bit murky
  - Usually this example is given for the linear case
  - We are improving the parameter estimate and assume that the prior estimate is close

ENGO 585 L5 – From Least Squares to the Kalman Filter

- Back to linear notation
  - Can always linearize again
- So Summation of normals becomes an iterative method

$$\hat{x}^{(+)} = \left(N^{(-)} + A^T C_l^{-1} A\right)^{-1} \left[u^{(-)} + A^T C_l^{-1} l\right]$$

- To implement, have to store an mxm matrix and an $mx1$ vector and invert an mxm (or uxu) matrix so this method works well when the number of states is small
- There is another way to formulate the problem so that the inverse is nxn

ENGO 585 L5 – From Least Squares to the Kalman Filter

- Summation of normals

$$\hat{x}^{(+)} = \left(N^{(-)} + A^T C_l^{-1} A\right)^{-1} \left[u^{(-)} + A^T C_l^{-1} l\right]$$

- Can be transformed to

$$\hat{x}^{(+)} = \hat{x}^{(-)} + \left(N^{(-)}\right)^{-1} A^T \left[A\left(N^{(-)}\right)^{-1} A^T + C_l\right]^{-1} \left(l - A\hat{x}^{(-)}\right)$$

- The proof is 2 pages long,
- Simplify by replacing $\left(N^{(-)}\right)^{-1}$ with $C_{\hat{x}(-)}$

$$\hat{x}^{(+)} = \hat{x}^{(-)} + C_{\hat{x}(-)} A^T \left[A C_{\hat{x}(-)} A^T + C_l\right]^{-1} \left(l - A\hat{x}^{(-)}\right)$$

ENGO 585 L5 – From Least Squares to the Kalman Filter

- Then define

$$K = C_{\hat{x}(-)} A^T \left[A C_{\hat{x}(-)} A^T + C_l \right]^{-1}$$

- then

$$\hat{x}^{(+)} = \hat{x}^{(-)} + K \left(l - A\hat{x}^{(-)} \right)$$

- and (again without proof)

$$C_{\hat{x}(+)} = C_{\hat{x}(-)} - KAC_{\hat{x}(-)} = (I - KA)C_{\hat{x}(-)}$$

- Now we have iterative formulas for $\hat{x}^{(+)}$ and $C_{\hat{x}(+)}$ that uses a single nxn inverse

ENGO 585 L5 – From Least Squares to the Kalman Filter

- What do they mean?

$$\hat{x}^{(+)} = \hat{x}^{(-)} + K\left(l - A\hat{x}^{(-)}\right)$$

  - If observation differs from the prior estimate, then change the prior estimate by K times the difference
  - K is a weighting factor for the new observations called the *gain*
  - New observations make $C_{\hat{x}^{(+)}}$ smaller
- The almost trivial example – batch vs iterative averaging

ENGO 585 L5 – From Least Squares to the Kalman Filter

- The almost trivial example – batch vs iterative averaging
  - “Observe” a number 3 times (maybe we’re measuring the length of ENGO 585 Zoom lectures in minutes

$$l = \begin{bmatrix}50 & 51 & 52\end{bmatrix}^T \qquad C_l = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \qquad A = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

  - Do batch LS

ENGO 585 L5 – From Least Squares to the Kalman Filter

- The almost trivial example – batch vs iterative averaging
  - "Observe" a number 3 times (maybe we're measuring the length of ENGO 585 Zoom lectures in minutes

$$l = \begin{bmatrix} 50 & 51 & 52 \end{bmatrix}^T \qquad C_l = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

  - Do sequential by assuming first obs is the "prior estimate" $\hat{x}^{(-)} = 50$ and $C_{\hat{x}^{(-)}} = 1$. For the 2nd and 3rd observations, $A = C_l = 1$

ENGO 585 L5 – From Least Squares to the Kalman Filter

- The almost trivial example – batch vs iterative averaging

ENGO 585 L5 – Next lecture

- Sequential LS only works with then states (parameters) are stationary
- Wednesday:
  - Intro to KF notation
  - Dynamics models
  - SLS + a dynamics model = Kalman Filter
- Friday:
  - Lab 2: Non-linear LS, SLS, EKF
- Monday:
  - Non-Linear KF: The Extended KF
  - Alternatives to the KF

ENGO 585 L6 – Kalman Filter Notation

- UCalgary LS Notation

$l = Ax + r$ or $l = f(x) + r$

C_l

- Batch

$$\hat{x} = \left(A^T C_l^{-1} A\right)^{-1} A^T C_l^{-1} l$$

$$C_{\hat{x}} = \left(A^T C_l^{-1} A\right)^{-1}$$

- Sequential

$$K = C_{\hat{x}(-)} A^T \left[A C_{\hat{x}(-)} A^T + C_l\,\right]^{-1}$$

$$\hat{x}^{(+)} = \hat{x}^{(-)} + K\left(l - A\hat{x}^{(-)}\right)$$

$$C_{\hat{x}(+)} = (I - KA) C_{\hat{x}(-)}$$

- KF Notation

$z = Hx + v$ or $z = h(x) + v$

R

- Batch

$$\hat{x} = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} z$$

$$P = \left(H^T R^{-1} H\right)^{-1}$$

- Sequential

$$K = P H^T [H P H^T + R\,]^{-1}$$

$$\hat{x}^{(+)} = \hat{x}^{(-)} + K\left(z - H\hat{x}^{(-)}\right)$$

$$P^{(+)} = (I - KH) P^{(-)}$$

ENGO 585 L6 – Dynamics Models

- Sequential LS works for stationary states but what if the states change?
- The Kalman filter adds a dynamics model to predict the state vector forward in time
- Idea: Model the state as a deterministic process with some uncertainty

$$\dot{x} = Fx + Gu$$

- This is called a Markov process
  - Future depends only on the current state x and some uncertainty Gu where u is a vector of white noise.
  - F is called the dynamics matrix and $\dot{x} = Fx$ is a linear system of first order ODEs

ENGO 585 L6 – Dynamics Models

- An example of

$$\dot{x} = Fx$$

- Consider 1D motion where $x = [x \quad v_x]$, [pos,vel]
- Then $\dot{x} = [v_x \quad a_x]$
- And the dynamics model relating them is for example

$$\begin{bmatrix} v_x \\ a_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ v_x \end{bmatrix}$$

- This defines a "constant velocity" model that given an initial position and velocity can predict future position and velocity for all time.
- How? By solving the ODEs

ENGO 585 L6 – Dynamics Models

- This is one of the simplest ODEs

$$\dot{x} = Fx$$

- Solve by choosing a test function, like $x(t) = e^{Ft}$
- Plug in and see if it works, but what is e to a matrix?
- Replace with Taylor series (or use numerical methods)

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots$$

or in this case

$$e^{Ft} = I + Ft + \ \dots$$

So

$$\begin{bmatrix} x \\ v_x \end{bmatrix}_t = I + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} t \begin{bmatrix} x \\ v_x \end{bmatrix}_0 = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v_x \end{bmatrix}_0$$

ENGO 585 L6 – Dynamics Models

- Or for small time steps Δt from $k-1$ to k (so that the Taylor series stays valid)

$$\begin{bmatrix} x \\ v_x \end{bmatrix}_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ v_x \end{bmatrix}_{k-1}$$

- Does this make sense?
  - Position equals prior position plus velocity times time
  - Velocity is constant (it's the constant velocity model)
- Can replace F with any matrix (not dependant on x).
  - ie. it is a linear system
- If the dynamics can't be represented by a matrix, or the time step is too big, use numerical methods to solve the ODEs

ENGO 585 L6 – Dynamics Models

- $e^{F\Delta t} = I + F\Delta t$ is called the transition matrix, Φ

$$x_k = \Phi x_{k-1}$$

is a discrete time way to describe the system of ODEs

- If we linearly transform a vector, what about its covariance matrix?

$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}}$$

- For a deterministic system this is great, but what about the uncertainty
- For that we add process noise, Q to represent the error in the dynamics model

$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q$$

ENGO 585 L6 – Dynamics Models

- The dynamics model is then defined by Φ and Q just like the measurement model is defined by H and R
- There are many different dynamics models
  - Random constant: $\Phi = I, Q = 0$
  - Random walk: $\Phi = I, Q > 0$
  - Constant velocity
    - Previous example, states and first derivates. Predicted state = prior state + derivative x time
  - First order Gauss-Markov
    - State that tends to zero over time $\Phi = e^{-\beta \Delta t}$, $\mathrm{Q} \geq 0$

ENGO 585 L6 – SLS + Dynamics Model = KF

- The Kalman Filter is a loop that predicts using a dynamics model then updates using SLS

$$x_k = \Phi x_{k-1}$$
$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q$$

$\hat{x}^{(-)}, P^{(-)}$

$\hat{x}^{(+)}, P^{(+)}$

z, R, H

$$K = PH^T [HPH^T + R\,]^{-1}$$
$$\hat{x}^{(+)} = \hat{x}^{(-)} + K\left(z - H\hat{x}^{(-)}\right)$$
$$P^{(+)} = (I - KH)P^{(-)}$$

ENGO 585 L6 – Next lecture

- Friday:
  - Lab 2: Non-linear LS, SLS, EKF
- Monday:
  - Non-Linear: The Extended KF
  - Alternatives to the KF

ENGO 585 L7 – Extended Kalman Filter

- The Kalman Filter has two linear models
  - Dynamics valid for small time steps
  - Observation for small deviation from a point of expansion

$$x_k = \Phi x_{k-1}$$
$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q$$

$\hat{x}^{(-)}, P^{(-)}$

$\hat{x}^{(+)}, P^{(+)}$

z, R, H

$$K = PH^T [HPH^T + R\,]^{-1}$$
$$\hat{x}^{(+)} = \hat{x}^{(-)} + K\left(z - H\hat{x}^{(-)}\right)$$
$$P^{(+)} = (I - KH)P^{(-)}$$

- $z - H\hat{x}^{(-)}$ is small, like a misclosure,
- Called the “innovation sequence”

ENGO 585 L7 – Extended Kalman Filter

- So how to linearize the observation model?
  - Same as with Least Squares
  - Replace Hx with $h(x)$ and H becomes $\frac{\partial z}{\partial x}$

$$x_k = \Phi x_{k-1}$$
$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q$$

$\hat{x}^{(-)}, P^{(-)}$

$\hat{x}^{(+)}, P^{(+)}$

z, R, H

$$K = PH^T [HPH^T + R\,]^{-1}$$
$$\hat{x}^{(+)} = \hat{x}^{(-)} + K\left(z - h(\hat{x}^{(-)})\right)$$
$$P^{(+)} = (I - KH)P^{(-)}$$

ENGO 585 L7 – Extended Kalman Filter

- Two approaches
  - Linearized: have ODEs for $x(t)$
    - Solve in advance: $x_0(t)$, then estimate δx where $x(t) = x_0(t) + \delta x$
    - Can evaluate Φ and $\frac{\partial z}{\partial x}$ for all expected measurements in advance

$$\delta x_k = \Phi \delta x_{k-1}$$
$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q$$
$$K = PH^T [HPH^T + R\,]^{-1}$$
$$\delta \hat{x}^{(+)} = \delta \hat{x}^{(-)} + K\left(z - H\delta \hat{x}^{(-)}\right)$$
$$P^{(+)} = (I - KH)P^{(-)}$$

  - This works because δx is small, very efficient for 1960s computers

ENGO 585 L7 – Extended Kalman Filter

- Two approaches
  - Extended: Use $\hat{x}^{(-)}$ as the POE, $H = \left.\frac{\partial z}{\partial x}\right|_{x^{(-)}}$
    - Solve in advance: $x_0(t)$, then estimate δx where $x(t) = x_0(t) + \delta x$
    - Evaluate Φ and $\frac{\partial z}{\partial x}$ in real time

$$x_k = \Phi x_{k-1}$$
$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q$$
$$K = PH^T [HPH^T + R\,]^{-1}$$
$$\delta\hat{x}^{(+)} = 0 + K\left(z - h(\hat{x}^{(-)})\right)$$
$$P^{(+)} = (I - KH)P^{(-)}$$

  - The update is often written as the small correction only
  - The "full" state $\hat{x}^{(+)}$ is still full $\hat{x}^{(-)} + \delta\hat{x}^{(+)}$
  - Prediction may or may not be linear

ENGO 585 L7 – Extended Kalman Filter

ENGO 585 L7 – Filter Tuning and convergence

- H, Φ deterministic, P, Q stochastic, P a function of both

$$P_k = \Phi P_{k-1} \Phi^{\mathrm{T}} + Q, K = PH^T [HPH^T + R]^{-1}, P^{(+)} = (I - KH) P^{(-)}$$

ENGO 585 L7 – Filter Tuning and convergence

- Observability (without Proof in ENGO 585)
  - If you observe a set of states x in a KF with observations z
    - x always has solution (because you start with $x^{(-)}$
    - If $H^T H$ is invertible, then you fully observe the states
  - But what if you have extra states?
    - Can be observable over time
    - $z_0 = Hx_0$
    - $z_1 = Hx_1 = H\Phi x_0$
  - So over two epochs could possibly observe position and velocity, even if only positions are fully observable
    - Do this in Lab 2
  - Without proof, if a state is fully observable in one epoch, then its time derivatives are observable over time

ENGO 585 L7 – Alternatives to EKF

- If either or both of the measurement model $h(x)$ or the dynamics model $F(x)$ cannot be linearized easily and/or have non-gaussian errors r or Gu then the formulas to update P will not work.
- Unscented Kalman Filter
  - If the probability density of the estimated states is not jointly Gaussian
    - Choose a set of sample points that are representative of the true pdf and propagate them through the full non-linear dynamics model.
- Particle Filter
  - Why stop at a few points?
    - Create a large sample of points to represent the pdf of x
    - Promote the ones that fit the measurements and resample

ENGO 585 L7 – Next lecture

- Wednesday:
  - Chapter 3 – Intro of radio
  - Intro to Lab 3, 5, 6 data collection hardware
- Friday:
  - Lab period is work time/Q&A
  - Kyle will be at ENE 031 Survey Stores to hand out mBeds

ENGO 585 – Wireless Location

Winter 2021 – L01, B01
Part 3: RF Measurements and Error Sources

Kyle O'Keefe, Instructor

03 February 2021

ENGO 585 Part 3 – RF Measurements and Error Sources

a. Fundamentals of Radio Frequency propagation
   i. Amplitude, power, frequency, phase
   ii. Signal structures

b. Measurements
   i. Received Signal Strength
   ii. Angle of Arrival
   iii. Time of Arrival
   iv. Time Difference of Arrival

c. RF Technologies and Examples
   a. Historical Systems
   b. GPS and A-GPS
   c. GPS Pseudolites
   d. Ultra-wideband
   e. Wi-Fi
   f. RFID

ENGO 585 L8 – Fundamentals of RF

- Electromagnetic radiation:
  - Wave model:
    - Oscillating electric and magnetic field travelling through space
    - By Maxwell's equations a change in electric field causes and change in magnetic field and a change in magnetic field causes a change in electric field.
    - 19th Century: The waves were carried by "luminiferous ether"
    - 20th Century: Couldn't find ether: New theories->Relativity etc.
  - Particle model:
    - Massless photons travelling through space
    - Photons are the particle that carries the electromagnetic force

ENGO 585 L8 – Fundamentals of RF

- How to make EM radiation
  - EM radiation arises when charged particles (electrons, protons etc) lose energy
  - Examples:
    - Changes in atomic nuclei: Gamma rays
    - High energy electrons collide with a metal: X-rays
    - Electrons jump between orbitals in molecule: Visible light
    - Electric current moving back and forth in a wire:
      - Radio-frequency (RF) radiation or radio waves

ENGO 585 L8 – Fundamentals of RF

- Type of EM depends on energy of photon/wave
  - All EM radiation travels at speed c and has no mass
  - Where is the energy?
    - Frequency corresponds to energy through Planck's constant, h

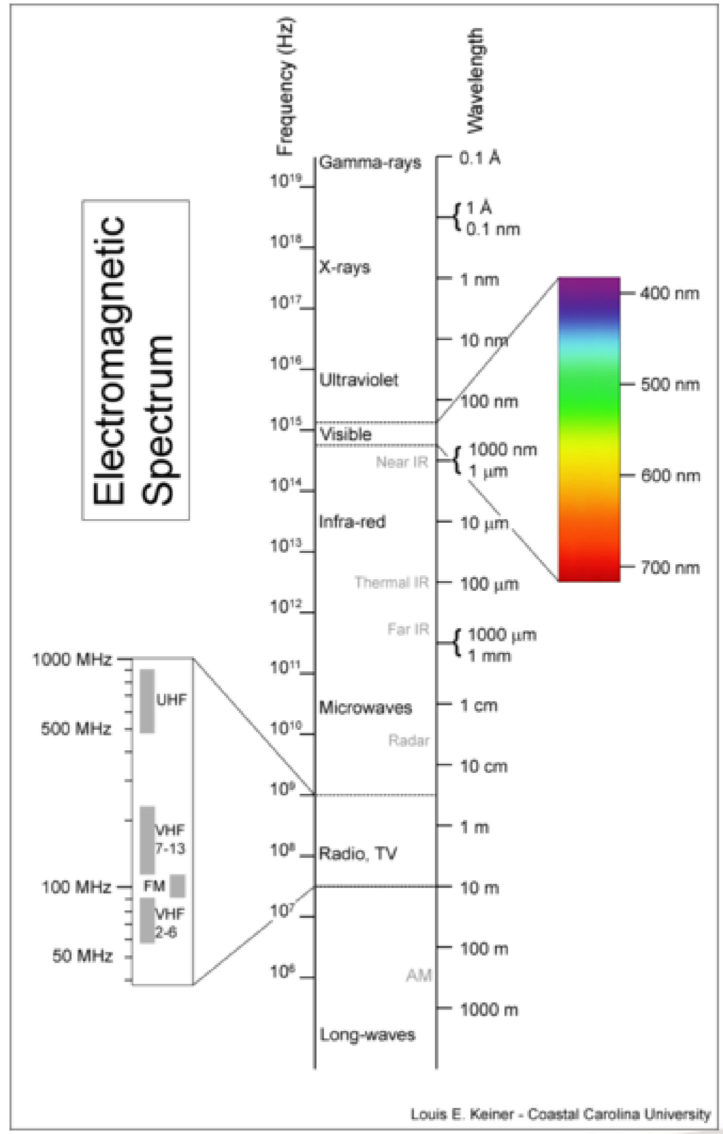
$$E = hf$$

$$h = 6.6260 \times 10^{-34}\,\mathrm{J \cdot S}$$

$$\lambda = \frac{c}{f}$$

ENGO 585 L8 – Fundamentals of RF

- EM spectrum includes everything from gamma rays to radio
- Note that microwaves really aren't that small
- In this course, we will be dealing mainly with the radio portion of the spectrum



ENGO 585 L8 – Fundamentals of RF

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- Classification schemes for radio frequencies
  - IEEE radar band names
  - Different from NATO band names

HF	3-30 MHz	
VHF	30-300 MHz	TV, marine radio
UHF	300-1000 MHz	aka P-band 900 MHz Cell phones
L-band	1-2 GHz	1800 MHz Cell phones
S-band	2-4 GHz	2.45 M/W ovens 802.11b/g (2.4 GHz)
C-band	4-8 GHz	802.11a (5 GHz)
X-band	8-12 GHz	
Ku-band	12-18 GHz	
K-band	18-26 GHz	
Ka-band	26-40 GHz	
V-band	40-75 GHz	
W-band	75-111 GHz	

ENGO 585 L8 – Next lecture

- Next lectures
  - Chapter 3: Fundamentals of RF cont. Amplitude, Power, Frequency, Phase
- Friday: Handing out equipment for Lab 3, 5, 6
  - ENE 031 Survey Stores from 9:00 to 11:00
  - Mbeds for everyone
  - Inertial for everyone (Lab 5)
  - Wifi boards (Lab 3) and barometers (Lab 6) for some
    - Form groups of 3 (but somehow stay away from each other for data collection
    - Some of you can use your phones.
  - Lab 4 (UWB on campus outside?)
    - Option B is I give you data (I don't like this option)

ENGO 585 L9 – Fundamentals of RF (cont)

- Last lecture: Super simplified overview of RF as a form of EM
- Today: Some math
  - EM radiation can be inferred from Maxwell's Equations
- Maxwell's equations:
  - Should have been in PHYS 259 or ENGG 225, but probably weren't
- Gauss' Law $\nabla \cdot E = \frac{1}{\varepsilon_0} \rho$
- Faraday's Law $\nabla \times E = -\frac{\partial B}{\partial t}$
- No name $\nabla \cdot B = 0$
- Ampere's Law (1) $\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$
  - (1 plus Maxwell's displacement current)

ENGO 585 L9 – Fundamentals of RF (cont)

- Gauss' Law $\nabla \cdot E = \frac{1}{\varepsilon_0} \rho$
  - Like the Gauss Law for gravity: Electric field is proportional to charge density
  - Also implies that electric charges are sources/sinks of field lines
- Faraday's Law $\nabla \times E = -\frac{\partial B}{\partial t}$
  - A change in B cause an E-field that curls
- No name $\nabla \cdot B = 0$
  - There are no magnetic monopoles: Magnetic field lines always close in loops
- Ampere's Law (1) $\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$
  - A current in a wire, or a change in E-field leads to a curling B-field

ENGO 585 L9 – Fundamentals of RF (cont)

- In a vacuum there are no charges or currents:
  - Gauss' Law $\nabla \cdot E = 0$
  - Faraday's Law $\nabla \times E = -\frac{\partial B}{\partial t}$
  - No name $\nabla \cdot B = 0$
  - Ampere's Law (1) $\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$
- 4 coupled PDEs
  - Can decouple them by applying $\nabla \times$ to Faraday's law and then the vector identity

$$\nabla \times (\nabla \times \mathrm{A} = \nabla(\nabla \cdot A) - \nabla^2 A$$

ENGO 585 L9 – Fundamentals of RF (cont)

- Faraday's Law $$\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$
- $$\nabla(\nabla \cdot E) - \nabla^2 E = \nabla \times \left(-\frac{\partial B}{\partial t}\right)$$
- Sub in Gauss Law on left and re-order the time and space derivatives on the right

$$0 - \nabla^2 E = -\frac{\partial}{\partial t}\nabla \times B$$

- The sub Ampere's law on the right

$$0 - \nabla^2 E = -\frac{\partial}{\partial t}\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

- Result is an equation that says 2nd derivative wrt t equals 2nd derivative wrt to space
- Can do the same with Ampere's Law for B-field

ENGO 585 L9 – Fundamentals of RF (cont)

- Result: Decoupled PDEs for E and B

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

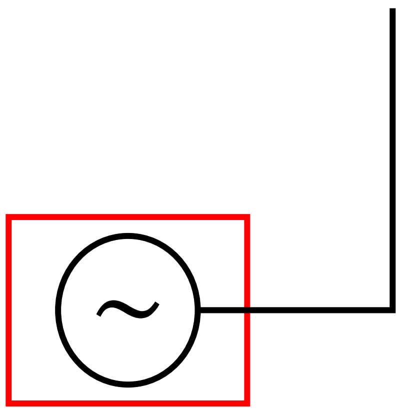
- So are there functions that solve these?
  - $\psi(x,t) = \cos(kx - \omega t + \phi)$ or $\psi(x,t) = \sin(kx - \omega t + \phi)$
  - Where $k = \frac{2\pi}{T}$, $\omega = \frac{2\pi}{\lambda}$, and $v = \frac{\omega}{k}$
    - k is the wave number, T is the period (in seconds)
    - ω is the angular frequency, λ is the wavelength (in metres)
    - v is the phase velocity (in ms^{-1}) if $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
  - If you plug in $v = \frac{1}{\sqrt{8.85418 \times 10^{-12} \cdot 4\pi \times 10^{-7}}} = 2.99 \times 10^8$

ENGO 585 L9 – Fundamentals of RF (cont)

- So Maxwells equations predict that currents in wires and charges will result in waves that happen to travel at the speed of light!
- This was “discovered” before we understood that light was the same thing as radio
- So how can we make a radio frequency EM wave?
  - Using a current in wire

ENGO 585 L9 – Fundamentals of RF (cont)

- Consider a wire connected to an alternating current



- Polarization refers to the plane of E-field
  - Vertical, Horizontal, Circular

ENGO 585 L9 – Amplitude, Frequency, Phase, Power

- $\psi(x,t) = Acos(kx - \omega t + \phi)$
  - Is just a wave
    - In general kx is a vector quantity, defines where the wave is going
    - ωt is the time part (how it varies in time at one place).
  - A is the amplitude, ω the frequency, ϕ the phase
  - All three of these can be modulated to transmit information

ENGO 585 L9 – Next lecture

- Wednesday:
  - Amplitude, Frequency and Phase modulation
  - Bandwidth and it’s relationship with time
- Monday:
  - Reading Week (early this year)!
- Next Monday:
  - Ch3 continues with each of the technology examples

ENGO 585 L10 – Modulation and Measurements

- Last lecture: Currents in wire make radio frequency waves described by $\psi(x,t) = Acos(kx - \omega t + \phi)$
- Sine or Cos doesn't include any information
  - How to send information? Modulation!
- Vary A, ω, or ϕ with the message you want to sent
- Amplitude modulation came first, then frequency, then phase

ENGO 585 L10 – Amplitude Modulation

- Replace A with the signal you want to send. The signal must have a much lower frequency than the carrier.

ENGO 585 L10 – Amplitude Modulation

- In the frequency domain?

ENGO 585 L10 – Bandwidth

- The amount that the carrier is spread across different frequencies is called the Bandwidth

ENGO 585 L10 – Bandwidth

- Shapes in the time domain have corresponding frequency distributions

ENGO 585 L10 – Frequency Modulation

- Similar to AM, FM uses small changes in the frequency to carry the data.
  - Advantage is that it uses half as much bandwidth, so leaves room for stereo

ENGO 585 L10 – Phase Modulation

- Phase Modulation uses large changes (180 degree) in phase to encode digital data
    - Example: Binary phase shift keying (BPSK) used in GPS and other digital radio
    - Just like in AM or FM, modulation adds bandwidth

ENGO 585 L10 – Phase Modulation

- BPSK with pseudorandom noise = Direct Sequence Spread Spectrum
- DSSS with a pseudorandom noise code allows for code-division multiple access
- CDMA is a popular form of multiple access (think GPS)
- Also TDMA and FDMA
- Cellphone use all three (codes, frequency channels and time slots).

ENGO 585 L10 – Bandwidth and Resolution

- The ability of a signal to measure time depends on the bandwidth
- High bandwidth means high frequency components
- High frequency components define sharp edges in the time domain

ENGO 585 L10 – Bandwidth and Resolution

- There is a limit to time resolution
  - Perfect edges, or narrower pulses in time require infinite bandwidth
- Time resolution related to bandwidth

ENGO 585 L10 – Power and Log units

- Logarithmic quantities very common in radio
- Often expressed in dB
- Bel "gain" defined as $\log_{10}\left(\frac{P}{P_{ref}}\right)$
- One decibel gain defined as $10\log_{10}\left(\frac{P}{P_{ref}}\right)$
- Note that a reference value is always required
  - Often 1 Watt
  - Sometimes 1 mW, EEs have made up units like dBW and dBm
  - Conversion between dbW and dBm is 30 (30 db = 1000)
- Power proportional to IV, V^2, or I^2, or area under frequency curve squared.

ENGO 585 L10 – Next lecture

- Friday:
  - Lab period to start Lab 3 – WiFi data collection
    - Don’t worry that we haven’t discussed how WiFi positioning works yet
- Next Week:
  - Reading Week – Early this year
- Monday after Reading Week
  - Ch3 continues with
    - Measurements
    - Technology examples: WiFi for lab 3, UWB for Lab 4, etc.

ENGO 585 L11 – Measurements

- We've seen what to do with measurements: AOA, TOA, TDOA and we've discussed the idea of signal amplitude and power
- What about making measurements:
  - Power: AKA received signal strength [indicator] (RSS)/[RSSI]
  - AOA
  - TOA
  - TDOA/Pseudoranging

ENGO 585 L11 – Measurements - RSSI

- RSSI
  - Free space loss should be $1/r^2$

ENGO 585 L11 – Measurements - AOA

- 3 AOA methods:
  - **Mechanical**, beamforming, estimation

ENGO 585 L11 – Measurements - AOA

- 3 AOA methods:
  - Mechanical, **beamforming**, estimation

ENGO 585 L11 – Measurements - AOA

- 3 AOA methods:
  - Mechanical, beamforming, **estimation**

ENGO 585 L11 – Measurements - TOA

- 3 TOA methods:
  - **Pulse**, FM, Correlation

ENGO 585 L11 – Measurements - TOA

- 3 TOA methods:
  - Pulse, **FM**, Correlation

ENGO 585 L11 – Measurements - TOA

- 3 TOA methods:
  - Pulse, FM, **Correlation**

ENGO 585 L11 – Measurements

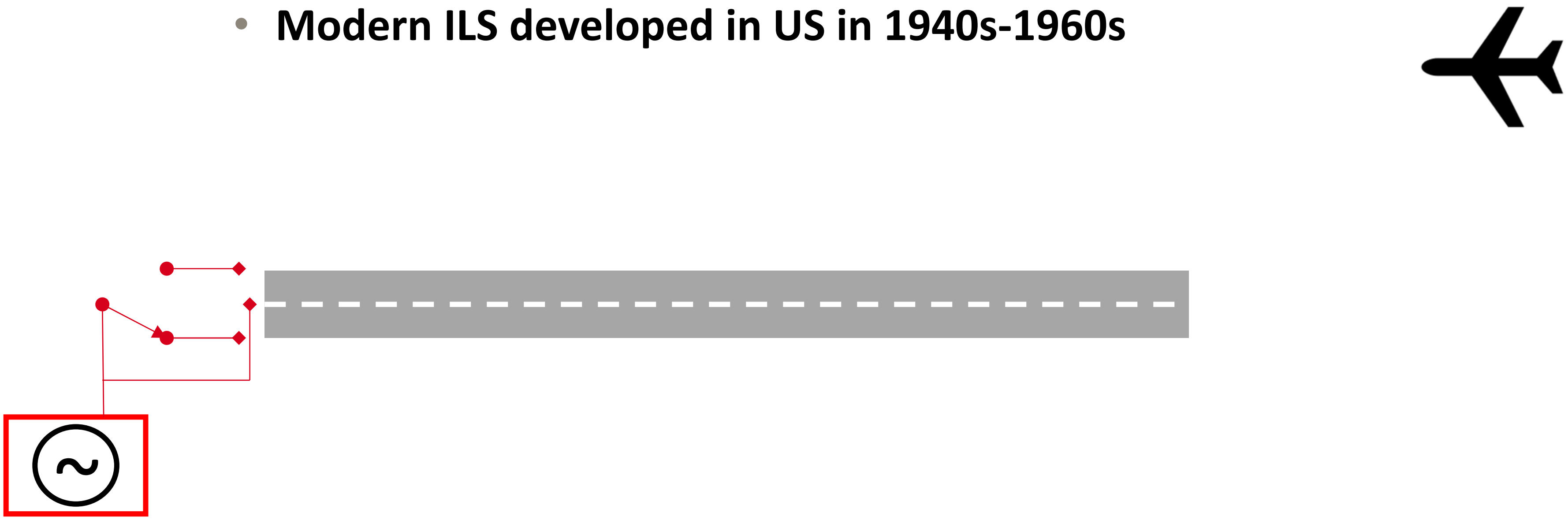
- End of Midterm material
- Next Lecture
  - Historical Methods
- Next Week

ENGO 585 L14 – Half-way Point!

- Lectures 11,12,13 were about measurements
- Lectures 14,15,16,17: Technology examples
  14. Historical Systems
      - 1940s
      - 1990s
  15. HSGPS, AGPS, Pseudolites
  16. Ultra-wideband
  17. WiFi, Bluetooth and RFID
- Lectures 18,19,20,21: Self-contained Sensors
- Lecture 22: E911, Lectures 23,24,25 Project presentations

ENGO 585 L14 – Historical Systems

- Instrument Landing System
  - **AOA in reverse**
    - **Create fixed beams and line up with them**
    - **First developed in Germany in late 1920s/early 1930s**
    - **Modern ILS developed in US in 1940s-1960s**



ENGO 585 L14 – Historical Systems

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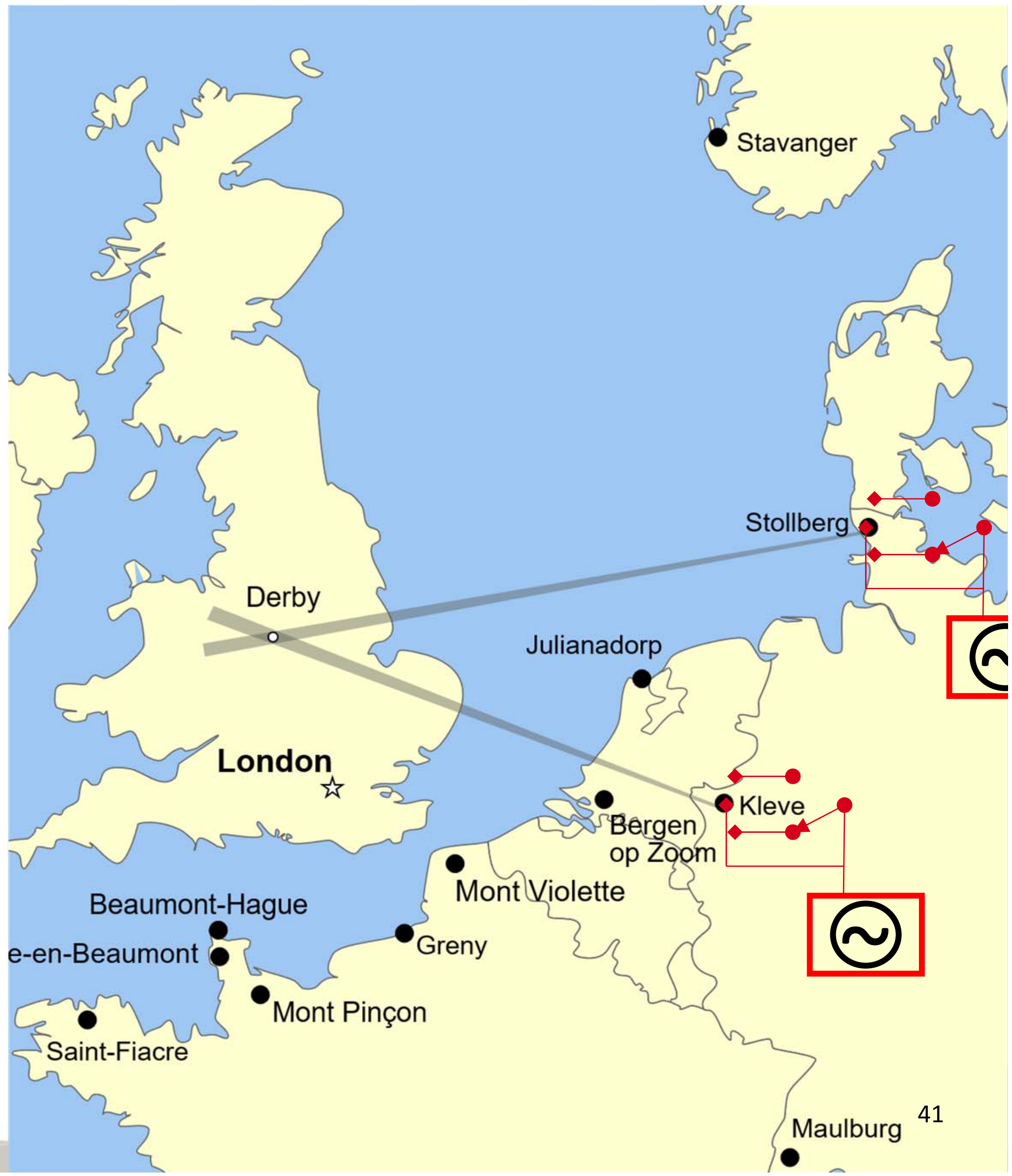
- Knickebein
  - Long range ILS projected from Germany to Britain
  - Pilots fly along one one beam until they meet the second
    - Drop bombs
    - AOA Dilution of precision is not bad
  - British found the signals
    - Broadcasting extra dots (spoofing)
    - Germans to drifted into the dash area
  - Map from Wikipedia
    - Battle of the Beams
      - User:Dahnielson



ENGO 585 L14 – Historical Systems

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- X-Gerat
  - More advanced version deployed from occupied France
    - Fly along one beam, across three cross beams
      1. Turn on system 30 km before target
      2. Start a count-up clock 10 km before target
      3. Reverse the clock 5 km before target
    - Only defeated after a downed aircraft was recovered

ENGO 585 L14 – Historical Systems

- Britain deployed a hyperbolic (TDOA) system called "GEE"
  - 3 transmitter
    - Master sends a plus
    - Waits 2 ms then sends 2 pulses
    - Slave 1 sends 1 pulse 1 ms after receiving the first
    - Slave 2 sends 1 puse 1 ms after receiving the 2^{nd}.

ENGO 585 L14 – Historical Systems

- LORAN (LOng RAnge Navigation)
  - US system for marine and air navigation
  - Very similar to GEE but lower carrier frequency (100 kHz)
  - Version A in the 1940s
  - LORAN-C 1970s until 2010
- Secondary Surveillance Radar
  - Identifcation Friend or Foe 1940s
  - Used in air traffic control
  - Very similar to RFID

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ENGO 585 – Historical Systems

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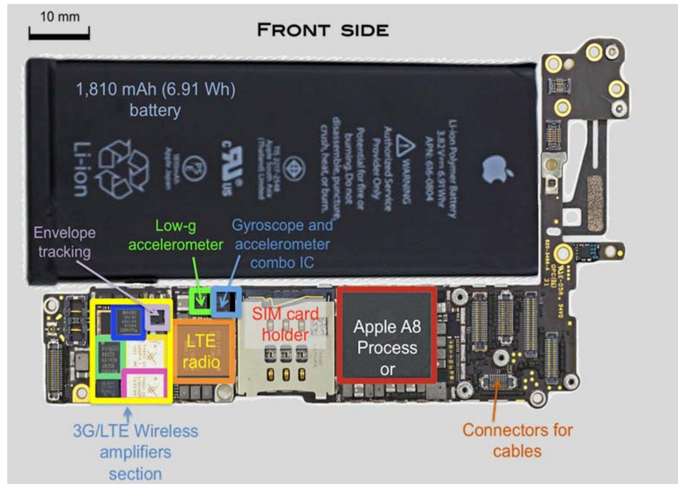
- Network-based cell-phone tracking
  - 1G AMPS Cell-loc (Calgary) for example
    - Install GPS receivers at the base stations and make precise TOA measurements of the cell signals
  - 2G GSM vs IS-95
    - GSM used TDMA (8 calls per channel)
      - Unsynchronized base-stations
      - Each phone had to sync with its base station
      - Possible to measure a rough range
      - AOA from directional base station antenna
    - IS-95 used CDMA
      - Synchronized base stations
      - Possible to measure range using correlation
      - Problems with near-far problem (we'll talk about that next week)
  - 3G and 4G: Mostly GNSS outside and fingerprinting inside

ENGO 585 L14 – Historical Systems

- Next Week
  - Handset GNSS (HS-GPS and A-GPS)
  - Pseudolites and repeaters
  - Ultra-wideband
  - UWB and Inertial Lab
- Week after
  - WiFi, Bluetooth, RFID
  - Ultrasonic if we have have time
  - Inertial
  - Inertial Lab and UWB
- After that
  - More inertial

ENGO 585 L15 – Inside a Smart Phone

- Smartphone
  - Battery
  - Computer
  - Touch Screen
  - Radios
    - GPS
    - Cellular
    - WiFi
    - Bluetooth
  - Sensors
    - Camera 1
    - Camera 2
    - Inertial
    - Magnetic
    - Pressure
    - Temperature
    - Fingerprint scanner
    - Light
  - Storage
    - Built-in
    - SD Card
    - SIM Card
  - Buttons, Jacks, Speakers, Microphones



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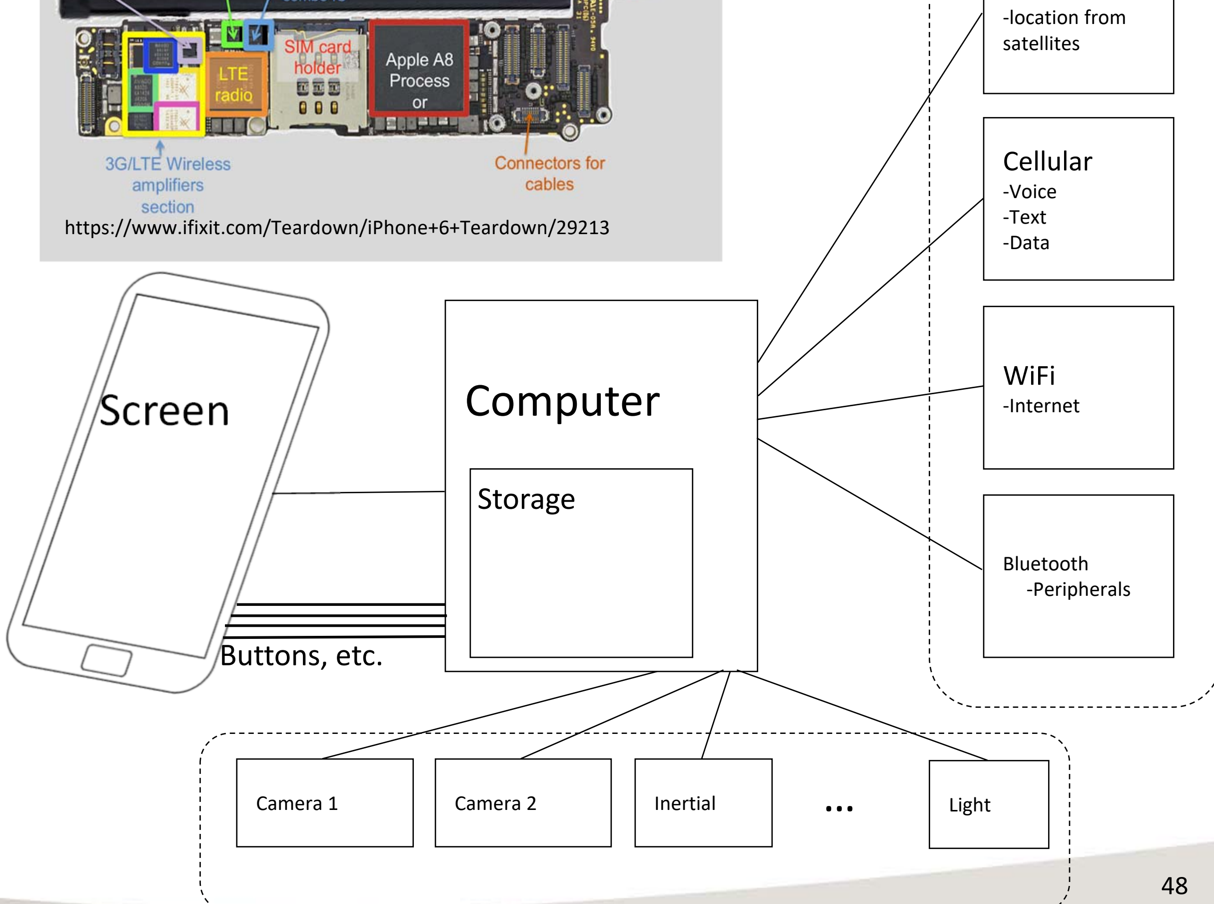
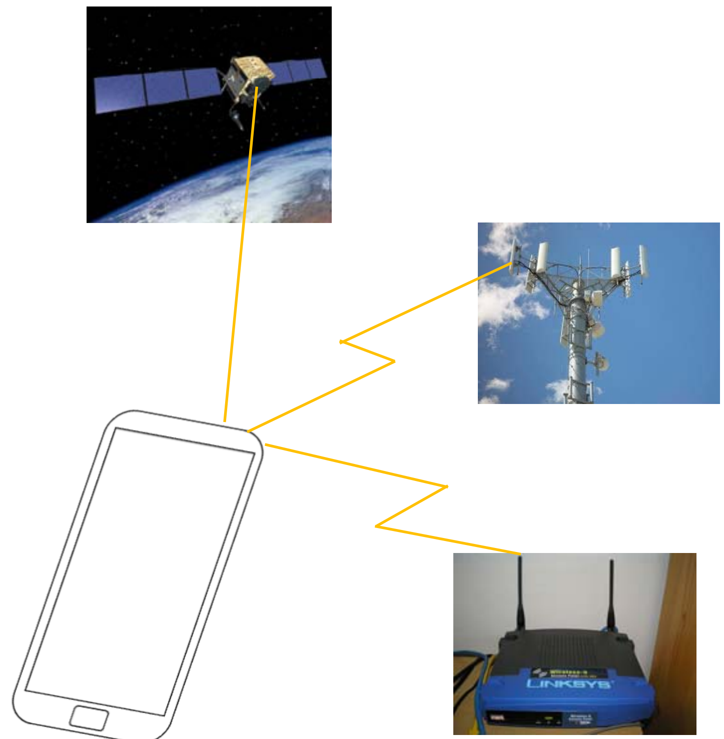


ENGO 585 L15 – Locating a Phone

- Cell phones can be located using radio (wireless) signals
  - Three main Wireless Location methods
    1. Cellular Network
    2. GPS
    3. WiFi™ and Bluetooth™ fingerprinting
  - Other sensors can assist
    - Step/heading using inertial and magnetic sensors
    - Matching photos using artificial intelligence



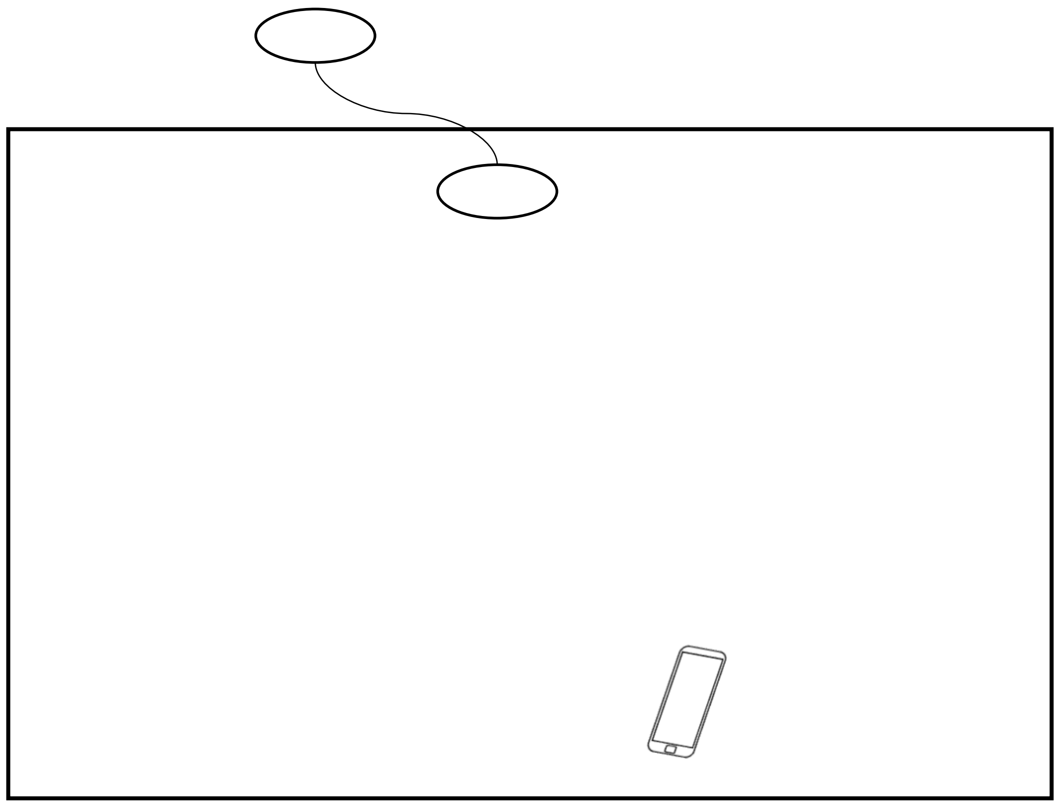
Images: gps.gov, Wikipedia.org

ENGO 585 L15 – Handset GNSS

- GNSS needs line of sight
- The signal level is very very low
- To work indoors need:
  - Some other source of GNSS: Pseudolites and Repeaters
  - Higher sensitivity: AGNSS, HSGNSS

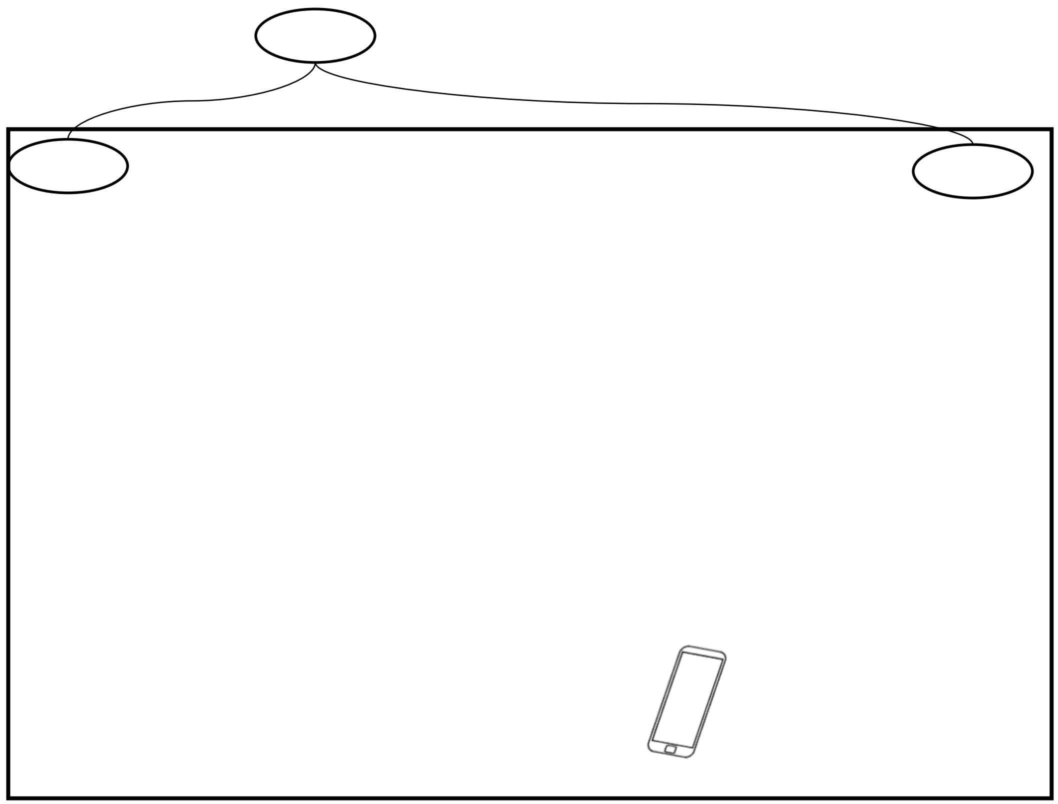
ENGO 585 L15 – Repeaters

- Rebroadcast GNSS indoors
  - Illegal in N. America
  - But done at GNSS trade shows



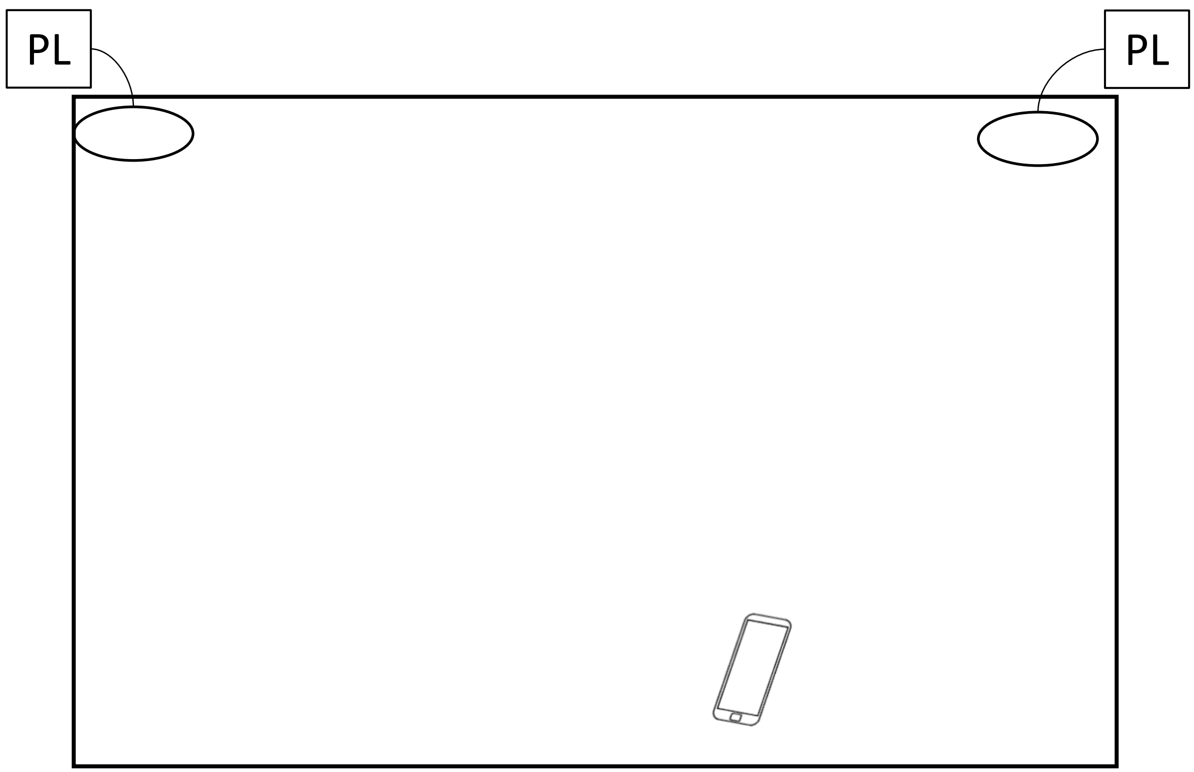
ENGO 585 L15 – Repeaters

- Rebroadcast GNSS indoors
  - What if you broadcast from 4 indoor antennas?



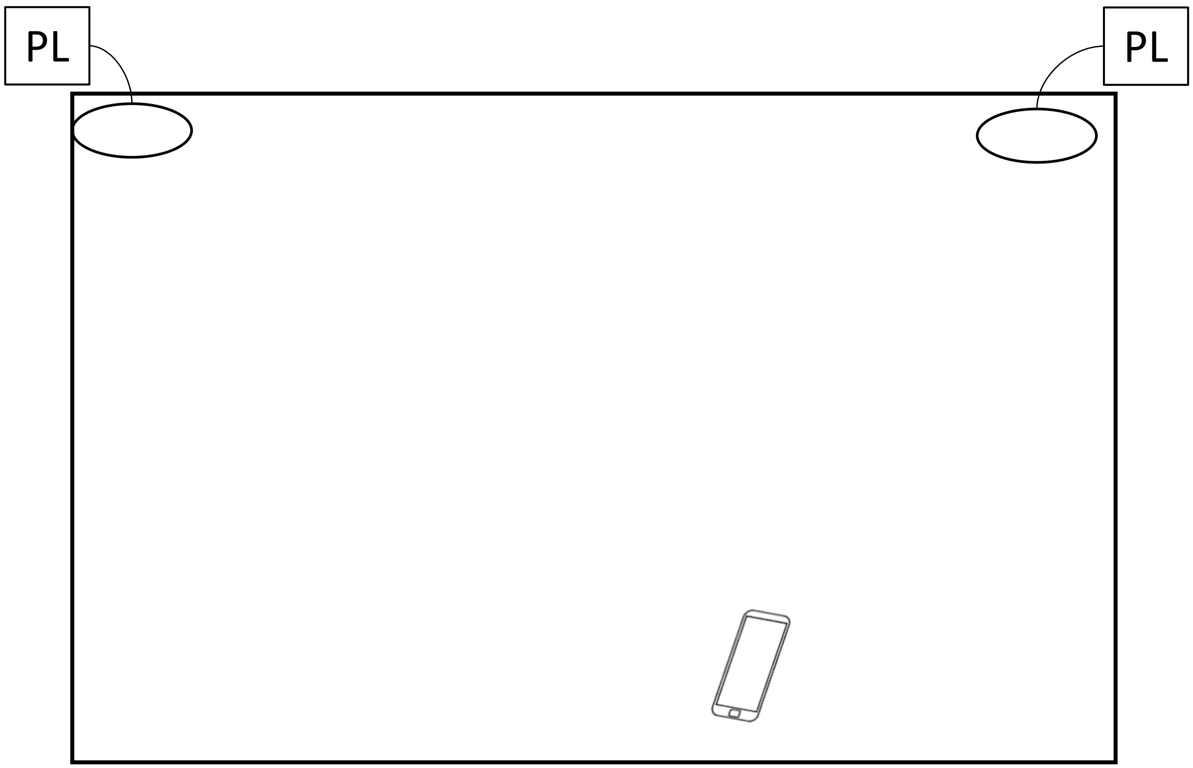
ENGO 585 L15 – Pseudolites

- Instead of repeating, generate GNSS-like signals
  - 4 indoor antennas each sending a pseudorange
  - Would it work?



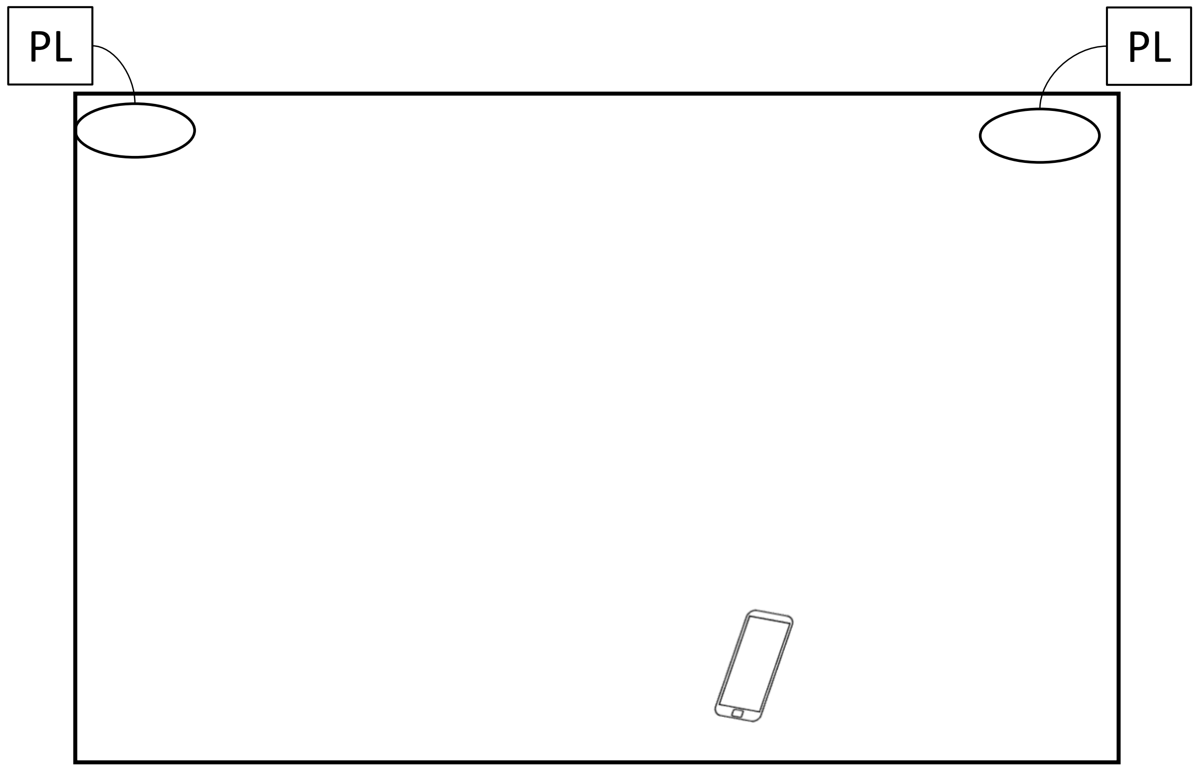
ENGO 585 L15 – Pseudolites

- Sort of…
  - Still illegal
  - Problems: Near-Far, multipath, receiver compatibility, time transfer/PL sync



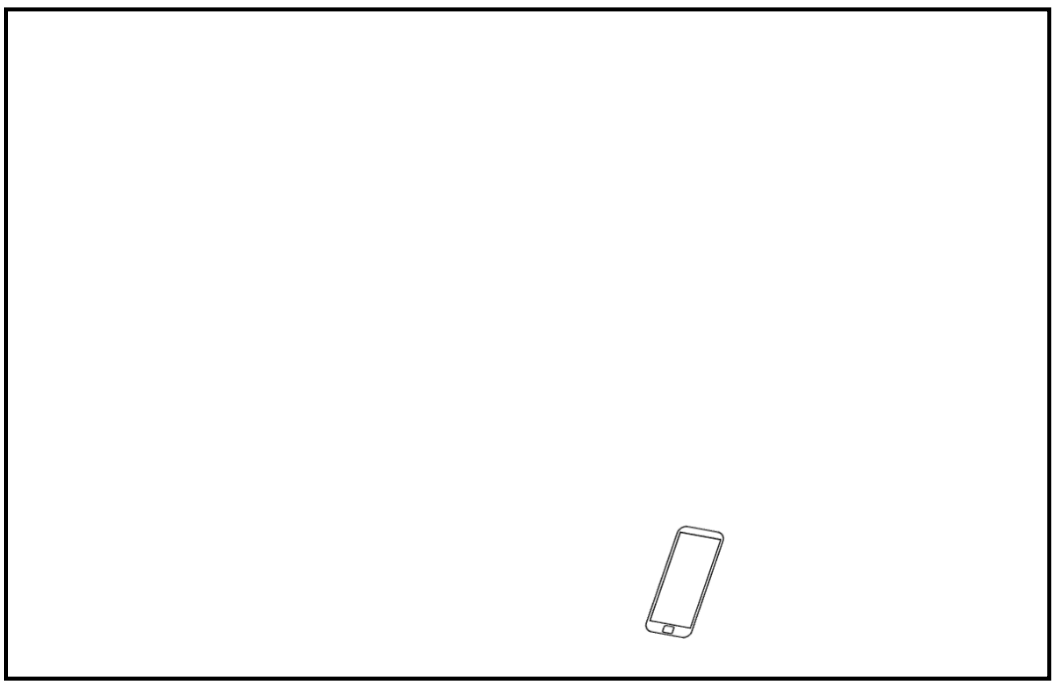
ENGO 585 L15 – Pseudolites

- Sort of…
  - One candidate was Japan’s IMES proposal



ENGO 585 L15 – AGPS and HSGPS

- Ok, so if this won't work, why not just receive GPS indoors?
  - Signal is very weak
  - Acquiring and Tracking takes lots of battery



ENGO 585 L15 – A and HS GPS/GNSS

- High Sensitivity
  - Lots of correlators to try different combinations
- Assisted GPS
  - Provide data to the receiver/phone through the network
    - The phone does less work, uses less battery or can try harder
  - What do you I mean try harder?
    - GPS C/A has 50 bps data. The code repeats every ms (1023 chips per millisecond)
    - One way to get more power is to integration over many milliseconds
    - But what happens if the data bit sign changes?
      - You start subtracting instead of added and your power goes down.
    - Two solutions:
      - Predict the data bits or try all combinations: HSGPS
      - Know the data bits: AGPS

ENGO 585 L15 – AGNSS

- AGPS can provide multiple types of assistance data
  - Data bits: To allow long integration
  - Ephemeris: for an instantaneous fix (instead of logging 30 seconds to get all of the subframes)
  - Approximate position: To make initial search for satellites faster
  - Almanac: To make initial search faster
  - Time and Frequency aiding: Again for faster/smaller search
- GNSS signals samples can also be logged or sent to the cloud.
  - GNSS Rx on phone is just a front-end, no signal processing

ENGO 585 L15 – HSGNSS and Modernized GNSS

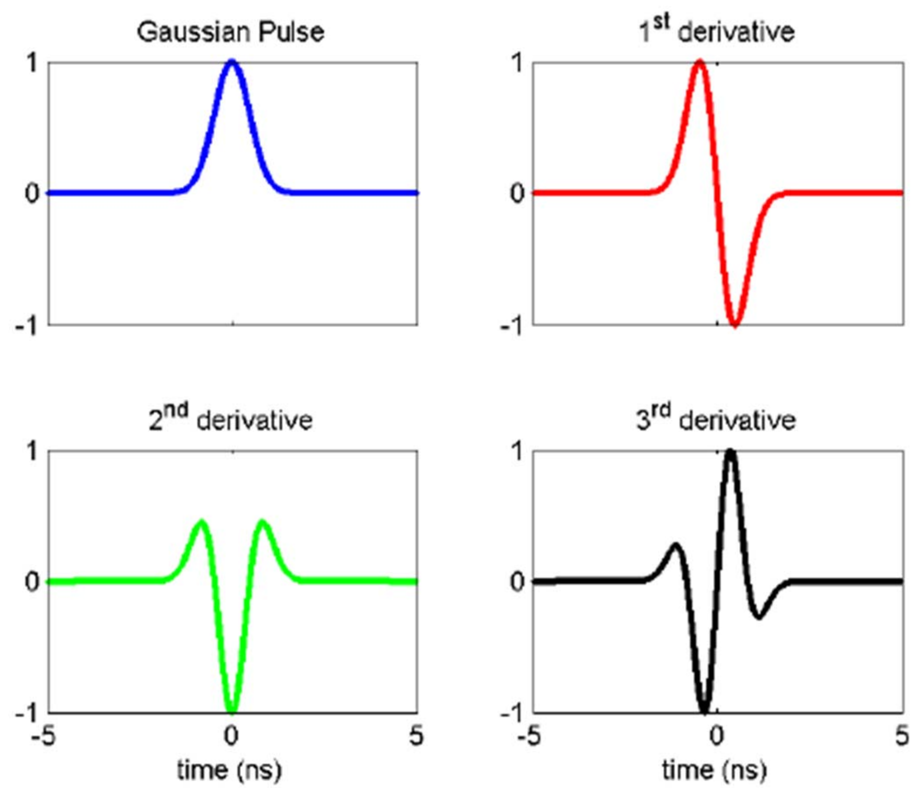
- Massively parallel correlators
  - To try different combinations of bits, or predicted bits
- Modernization programs
  - New GNSS and new GPS signals often have data bit free channels called pilot channels
  - These allow longer integration without assistance data
  - Example L2C, L5 each have a pilot channel
  - In theory: Infinite integration means eventually you'll get enough power
  - In practice: The receiver (phone) clocks are not precise enough for coherent integration over minutes

ENGO 585 L16 – Ultra-wideband Ranging

- Ultra-wideband
  - Also known as “carrier-free” or impulse radio, usually realized by having a radio switch on and off very quickly (on the order of pico- to nano-seconds)
  - By definition a system is UWB if
    - The fractional bandwidth is more the 20% of carrier frequency
    - The bandwidth is greater than 500 MHz (and maybe no carrier)
  - Initially used for radar in the 1960s
  - Renewed interest in late 1990s for high-bandwidth short-range data
    - To replace cables in home theatre systems, for example
    - This didn’t catch on, partly because of improvements in WiFi, partly improvements in compression for video streaming

ENGO 585 L16 – Ultra-wideband Ranging

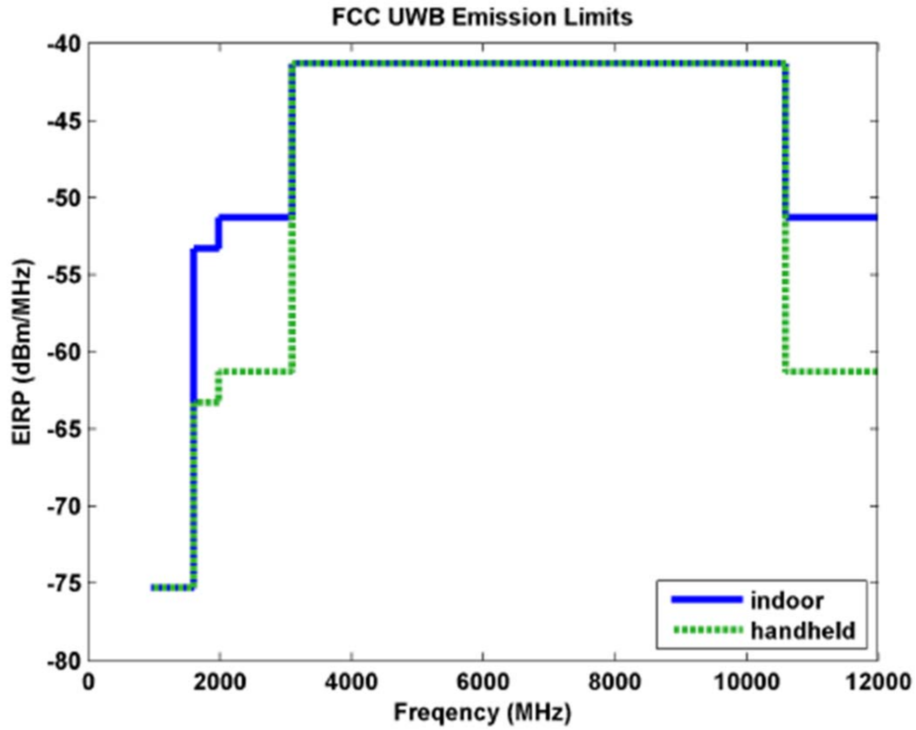
- Recall that square pulses have sidelobes but…
- A Gaussian pulse has Gaussian frequency distribution
- A narrow Gaussian pulse has a really wide frequency distribution that is also smooth.



from MacGougan (2009)

ENGO 585 L16 – Ultra-wideband Ranging

- FCC Licenced UWB in 2002
  - Power levels are very low
  - Not permitted on outdoor infrastructure
  - Even lower in the 2.4 GHz band and even lower in 1.5 GHz



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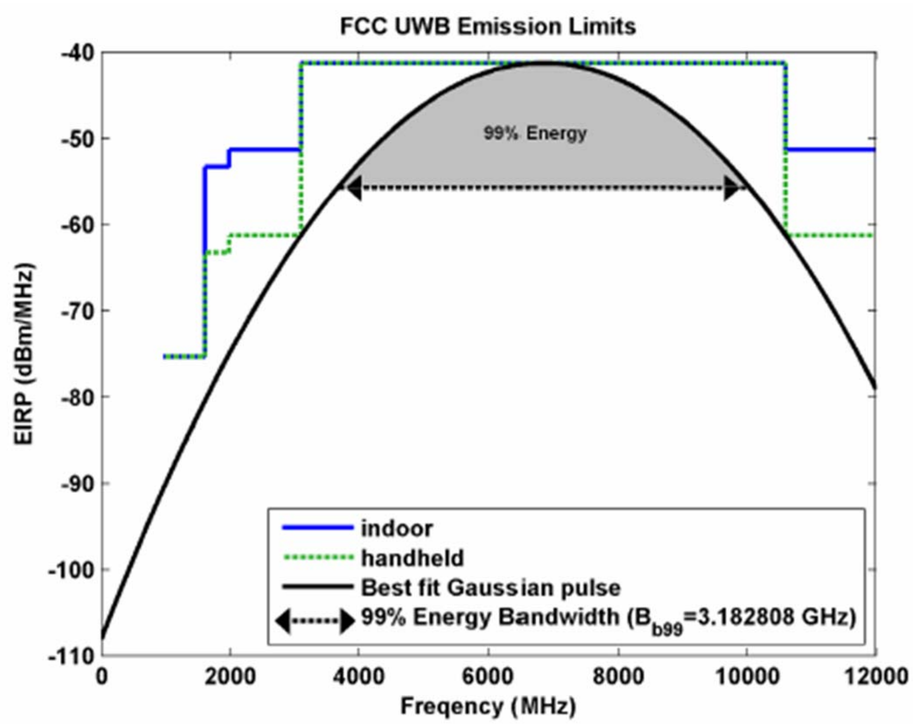
ENGO 585 L16 – Ultra-wideband Ranging

- FCC Licenced UWB in 2002
  - Power levels are very low
  - Not permitted on outdoor infrastructure
  - Even lower in the 2.4 GHz band and even lower in 1.5 GHz
- A Gaussian fits ok
- A 1st derivative of a Gaussian pulse will have more higher frequency components



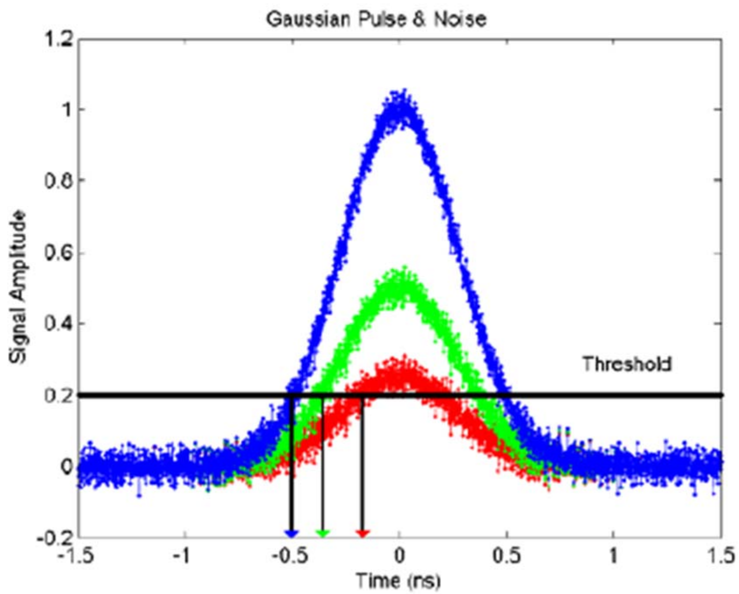
from MacGougan (2009)

ENGO 585 L16 – Ultra-wideband Ranging

- How does UWB transmit data?
  - Pulse position modulation and pulse amplitude modulation

ENGO 585 L16 – Ultra-wideband Ranging

- Time of Arrival
  - Usually measured when the pulse crosses a threshold
  - This can be a problem if the range changes after the threshold has been set
  - Can result in a scale factor error
- Lab 4
  - Test with one of the newer chips



from MacGougan and O'Keefe (2008)

ENGO 585 L16 – Ultra-wideband Ranging

- Main advantage of UWB
  - Multipath is almost eliminated?
  - Why?
    - Because the pulse width is so narrow in time that all the multipaths can be resolved
- A few companies developed development kits and products in the early 2000s
- Small chips developed in the early 2010s
- Added to Apple phones in Sept 2019
- UWB Android API not yet (some hints of it in 2020 apparently?)

- WiFi fingerprinting
  - Many methods, most are proprietary, most based on magnitude of the RSSI difference
  - Measure a vector of RSSI values $m = [m_1 \quad m_2 \quad \ldots]^T$
  - Find values in a look-up table $t = [t_1 \quad t_2 \quad \ldots]^T$
  - Find the magnitude of the difference $|m - t|$
    - m and t are vector in RSSI dB space
    - The result is a magnitude or distance in RSSI dB space
      - This is really arbitrary and the units don't make sense
      - But it works
  - Find k entries in the table with the smallest differences
  - Position is a weighted average of the "k nearest neighbours"
  - A between-access point differential method also exists

ENGO 585 L17 – WiFi, Bluetooth, RFID

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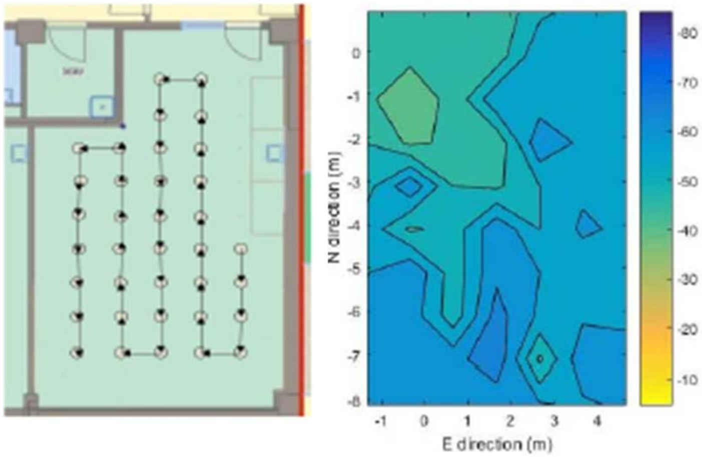
- Accuracy of the map depends on
  - Number of observation points
  - Number of access points
  - Age of the data
  - Does the environment change?



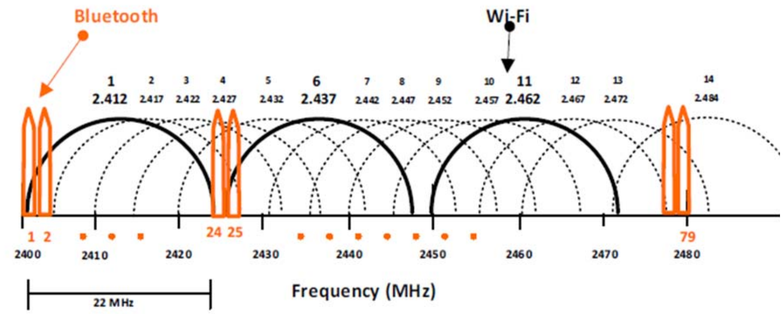
from Naghdi, S., Tjhai, C., and O'Keefe, K. (2018). "Assessing a UWB RTLS as a Means for Rapid WLAN Radio Map Generation." 2018 International Conference on Indoor Positioning and Indoor Navigation (IPIN), 1–5.

ENGO 585 L17 – WiFi, Bluetooth, RFID

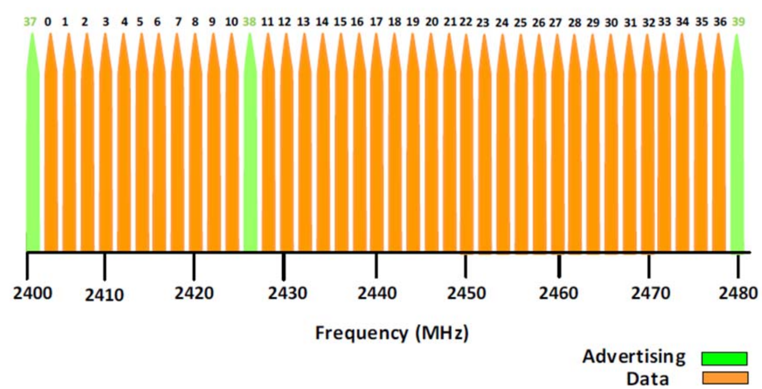
- WiFi – Other options:
  - Proximity only
  - AOA (from the access points)
  - Propagation modelling
    - Accuracy on the order of 10s of metres
    - WiFi range on the order of 50 m, so not really useful

ENGO 585 L17 – WiFi, Bluetooth, RFID

- Bluetooth and Bluetooth Low Energy
  - Classic Bluetooth has 79 1 MHz channels in the ISM band (with WiFi)
  - Bluetooth Low Energy defines 40 2 MHz channels
  - Both use Frequency Hopping as a form of spread spectrum and multiple access
    - Each pair has a different pseudorandom hopping sequence



WiFi and Bluetooth in the 2.4 GHz ISM band (from Naghdi, 2020)



Bluetooth Low Energy in the 2.4 GHz ISM band (from Naghdi, 2020)

ENGO 585 L17 – WiFi, Bluetooth, RFID

- BLE Location?
  - Similar strategies to WiFi
  - Range is shorter
  - Proximity most common
  - Fingerprinting possible
  - Path-loss modelling possible

ENGO 585 L17 – WiFi, Bluetooth, RFID

- RFID
  - Very short range version of secondary surveillance radar
  - Multiple standards
    - Different standards and frequencies (125 kHZ proximity cards, inventory, 13 MHz contactly payment, higher…)
    - Different physics: Near-field (tap pay at 13 MHz vs backscatter at higher frequencies/distances)
  - Applications to positioning
    - Readers have coordinates, users/goods get read, and you know where you are
    - Users carry readers, tags at locations, users read the tags and find out where they are

ENGO 585 – Wireless Location

Winter 2021 – L01, B01
Part 4: Inertial Navigation and Self-contained Sensors

Kyle O'Keefe, Instructor

17 March 2021

ENGO 585 Part 4 – Inertial Nav and Self-contained Sensors

a. Fundamentals Inertial Sensors and Navigation

   i. Sensors
   ii. Concepts of Inertial Navigaton
   iii. Coordinate Frames and Notation
   ~~iv. Navigation Equations~~
   v. Initialization ~~and Mechanization~~
   vi. INS Error Growth

b. GNSS-INS Integration

   i. Why Integration?
   ii. Loose Integration
   iii. Tight Integration
   iv. Ultra-tight Integration

c. Self-contained Sensors

   i. Magnetometers
   ii. Barometers
   iii. Map Matching
   iv. Non-holonomic Constraints

ENGO 585 L18 – Fundamentals of Inertial

- Sensors:
  - Measurements made in an inertial reference frame
    - ie. coordinate system that is neither accelerating nor rotating
  - So Newtonian mechanics can be used

$$x = \int v + const_1$$

$$v = \int a + const_2$$

  - Given initial position and velocity, and measurements of acceleration, can determine position and velocity at any time in the future
    - But errors in the initial conditions and observations will grow over time

ENGO 585 L18 – Fundamentals of Inertial

- Two types of “inertial” measurements
  - Accelerometers measure specific force
  - Gyroscopes measure rotation rates
- Usually an Inertial Measurement Unit (IMU) consists of
  - A triad of accelerometers
  - A triad of gyros

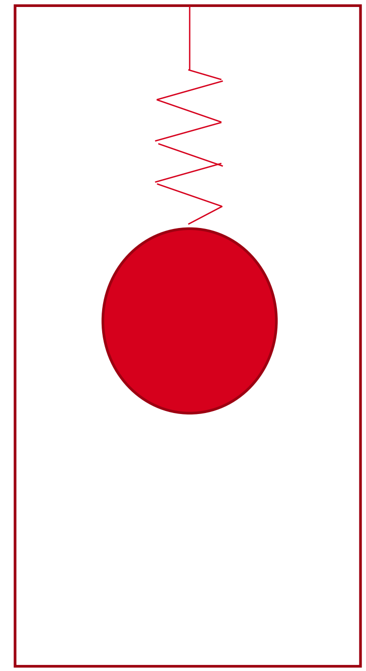
ENGO 585 L18 – Fundamentals of Inertial

- Accelerometers measure “specific force” not acceleration
  - Specific force is just a fancy way of saying “force per unit mass observed in the frame of the observer
  - Includes both acceleration of the user with respect to an inertial frame AND the force to oppose gravity
  - Textbooks define it as

$$f^i = a^i - g^i$$

  where a is the acceleration and g is the gravitational acceleration, both measured in the inertial frame
  - Always check the sign of this. A non-accelerating accelerometer should read “plus g” on the up-pointing-axis
  - If you accelerate up by 0.5 g, the output should be 1.5 g
  - If you are in free fall, the specific force should be zero

ENGO 585 L18 – Fundamentals of Inertial



- A typical model: Mass on a spring
  - Accelerometers don't actually sense gravitation, because gravitation acts on the proof mass and the case, and the spring
  - What they sense is the restoring force
  - If you are in free fall, the specific force is zero, but you are accelerating by g
- Accelerometers have lots of errors
  - Constant bias, temperature dependent bias, anisoelasticity, misalignment, scale factor, noise
  -

ENGO 585 L18 – Fundamentals of Inertial

- Gyroscopes
  - Original Mechanical Gyros measured orientation direction
    - A spinning top with lots of angular momentum will point in one direction
    - Put this in a gimbal and measure the angles
  - Modern gyros measure rotation rates ω_{ib}^{b}
    - The sub/superscripts mean: Rotation rate of b relative to i, expressed in the frame b
    - b refers to the body frame (of the gyro)
    - i is the inertial frame (not rotating w.r.t. to the universe)
  - Mechanisms:
    - Optical: Ring-laser Gyro and Fiberoptic Gyro
    - Vibrating beam: A driven string or tuning fork can sense the corriolis effect
  - Errors:
    - Similar to accels – bias, temp, misalignment, scale factor, noise

ENGO 585 L18 – Fundamentals of Inertial

- Basic idea:
    1. Gyroscopes used to keep track of the attitude of the sensors
    2. Accelerometer data is transformed into a navigation frame
    3. Integrate to get change in velocity, integrate again to get change in position
- IMU vs INS
    - An IMU is the device that contains the sensors
    - And INS is a system that does steps 1,2 and 3.

ENGO 585 L18 – Fundamentals of Inertial

- Coordinate Frames
  - Inertial, i (aka earth-centred inertial, ECI)
    - Earth orbit around the Sun can be neglected, rotation cannot
  - Earth-Centred Earth-Fixed, e (aka ECEF)
  - Local-level frame, l
    - strictly speaking a local-astro frame because we need to model and remove g
    - In practice a local-geodetic frame
  - Body-frame, b
    - The frame of the vehicle or IMU
    - Sometimes these are different
  - Navigation-frame, n
    - Where the solution is reported (could be i, l, b)
  - Danger: Reference use all kinds of different conventions
    - eg: for l frame: ENU, NEU, NED, etc.
    - For b frame: across-forward-up, forward-across-down, etc.

ENGO 585 L18 – Fundamentals of Inertial

- Lots of frames, easy to get confused
- Most inertial texts use this convention
  - Superscript means "expressed in the frame"
    - x^a means vector x in the a frame
  - Rotation matrices rotate from subscript to superscript
    - R_a^b means that $x^b = R_a^b x^a$
  - Position, Velocity, and Orientation expressed as:
    $\vec{r} = [x \quad y \quad z]^T$, $\vec{v} = [v_x \quad v_y \quad v_z]^T$, attitude $=$ $[\theta \quad \varphi \quad \psi]^T$
  - More Danger: Rotation Matrices do not commute
    - $R_l^b = R_2(\theta)R_1(\varphi)R_3(-\psi)$ if b-frame is across-forward-up
    - In some reference $R_l^b = R_1(\theta)R_2(\varphi)R_3(\psi)$ but body frame is forward-across-down

ENGO 585 L18 – Fundamentals of Inertial

- ~~Navigation Equations~~, Initialization, ~~Mechanization~~
  - Need integration constants if you want absolute positions
  - Position and Velocity
    - From GNSS
    - Static initialization (ie velocity at time = 0 is zero)
  - Attitude
    - Use GNSS, a magnetic compass, the attitude of the launch vehicle, or auto-alignment
    - Auto-allignment
      - Accelerometer data to get roll and pitch
      - Z-gyro to sense earth-rotation
        - Not possible with low-cost inertial

ENGO 585 L18 – Fundamentals of Inertial

- INS Error Growth
  - Consider the biases only
  - Without proof
    - Position error due to accel bias $\approx \frac{1}{2}bt^2$
    - Position error due to gyro bias $\approx \frac{1}{6}gbt^3$
  - What to do?
    - Zero velocity updates
    - Coordinate updates

ENGO 585 L19 – GNSS-INS Integration

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- GNSS Error Growth depends on IMU sensor grade

Sensor	Navigation Grade	Tactical Grade	Low-Cost
Gyro Bias	0.005–0.010 deg/h	0.1–10 deg/h	100 deg/hr to 100's of deg/s
Gyro Noise	0.002–0.005 deg/h/√Hz	0.2–0.5 deg/h/√Hz	N/A
Accel Bias	0.050–0.100 mm/s2	2-4 mm/s2	12+ mm/s2
Accel Noise	0.050–0.100 mm/s2/√Hz	2-4 mm/s2/√Hz	N/A

from Petovello (2013)

- The noises are power spectral densities used for the process noise in a Kalman filter when predicting the biases forward
- N/A means that there is no point predicting the biases

ENGO 585 L19 – GNSS-INS Integration

- So we need to update the INS with true velocity and true position in order to keep the biases estimated
  - GNSS was invented to do this
- Navigation and Tactical Grade allow for high rate strap-down INS + N/A integration with low rate GNSS updates
- For Low-Cost a continuous GNSS solution is more or less required and the INS can bridge small gaps
- The other Low-Cost option is to use the IMU for pedestrian dead reckoning.

ENGO 585 L19 – GNSS-INS Integration

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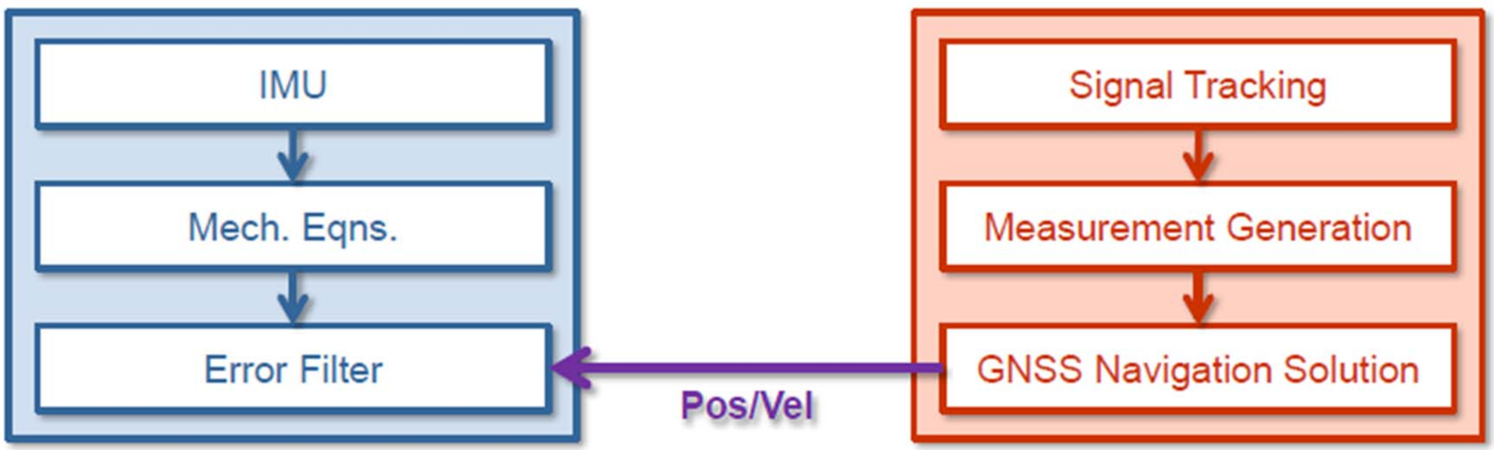
- GNSS and INS have very different characteristics

GNSS	INS
GNSS needs satellites	INS is self-contained
GNSS can be jammed or spoofed	INS is secure
GNSS has a low data rate (max 20 Hz)	INS 50 to 100 Hz
GNSS doesn't do attitude	INS does
GNSS errors are time invariant	INS has unbounded errors
GNSS position is absolute	INS position is relative

- Complementary advantages and dis-advanages
- INS errors are generally low frequency, GNSS high

ENGO 585 L19 – GNSS-INS Integration

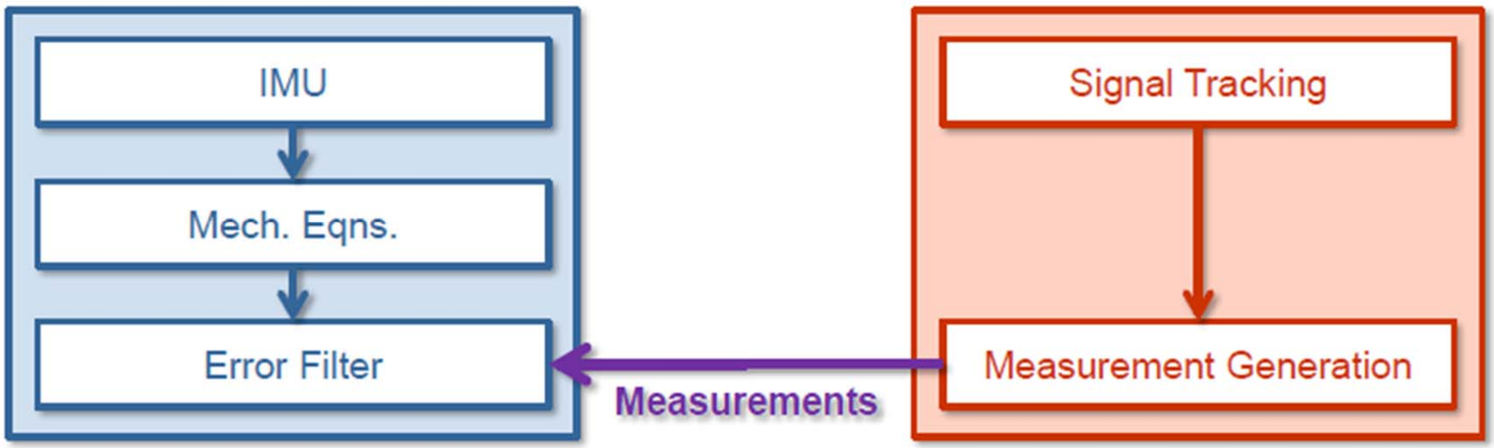
- Loose Integration
  - Most Common: GNSS is a black box whose output is fed to the INS



l, C_l sent to the INS are $\hat{x} = [x, v]^T$ and $C_{\hat{x}}$ from the GNSS

ENGO 585 L19 – GNSS-INS Integration

- Tight Integration
  - GNSS outputs observations (pseudorange and Doppler) and these are fed to the INS
    - Can provide GNSS updates with fewer than 4 GNSS obs
    - Potential for better GNSS outlier detection



l, C_l sent to the INS are $l = [\rho, \dot{\phi}]^T$ and C_l from the GNSS

ENGO 585 L19 – GNSS-INS Integration

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- Utra-tight (or Deep) Integration
  - IMU and GNSS are one device
  - The inertial solution can assist in GNSS signal acquisition and tracking while GNSS tracking limits growth of INS errors

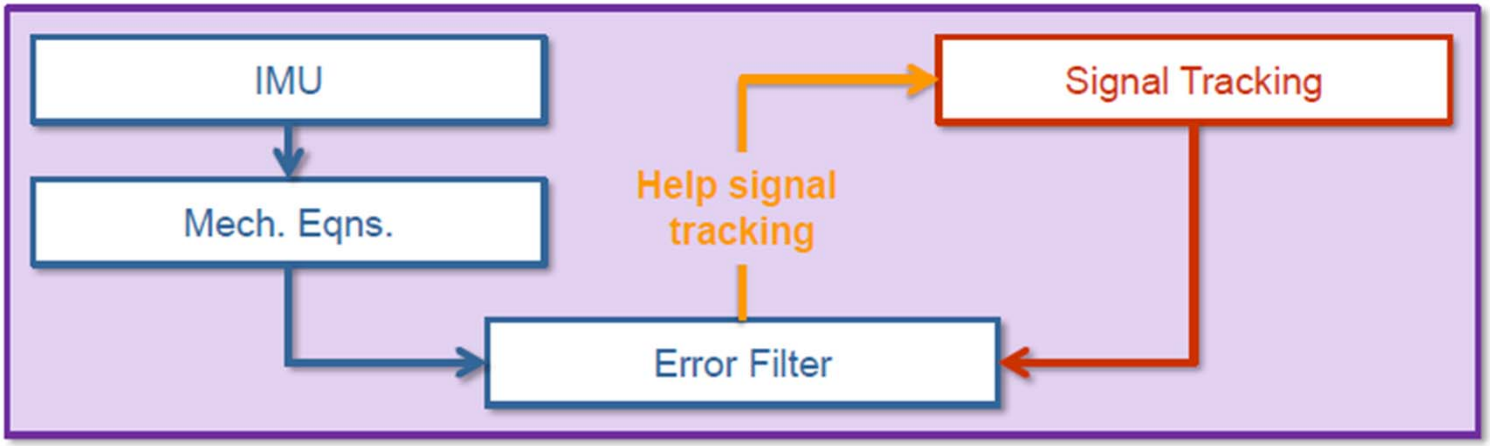

figure © 2013 Mark Petovello

ENGO 585 L19 – GNSS-INS Integration

- What about for low cost?
  - Two kinds of low cost: Automotive and Consumer
  - Automotive strapdown INS with tight coupling can work
    - with continuous GNSS
    - With aiding from other sensors: Odometer, non-holonomic constraints, visual odometry, visual gyro, …
    - Odo+NHC v forward from odometer, v up and sideways = 0
    - Visual: Optical flow for displacement (ie photogrammetry in reverse), straight lines in images will point to cardinal directions (to control gyro errors)
  - Mobile phones, not so much
    - Why to do?

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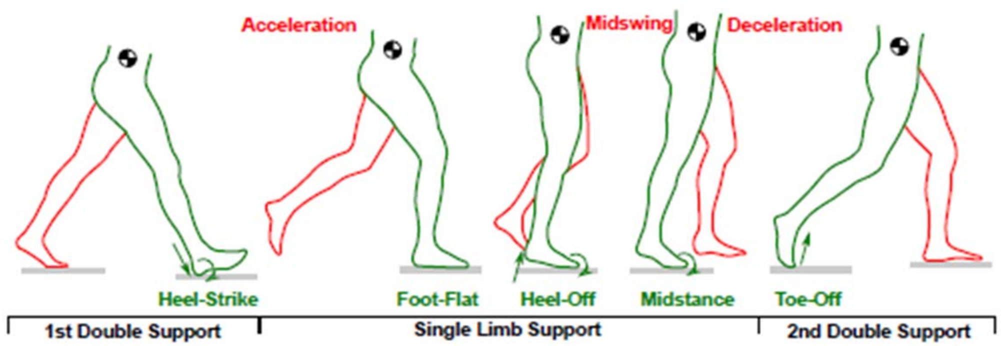
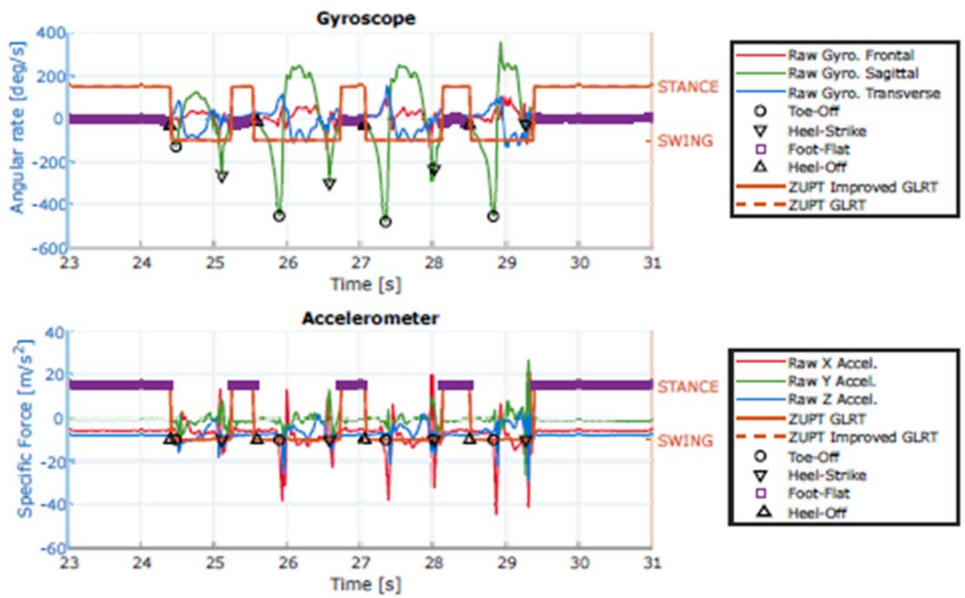
- Pedestrian Dead Reckoning
  - Use the accels or gyros to detect steps



from Tjhai (2020)

ENGO 585 L19 – GNSS-INS Integration

- Heel-strike and toe-off have signatures in the data



from Tjhai (2020)

- Foot flat events have zero velocity and zero angular rate
  - Reset the inertial solution for each step
  - Use accels and gyros to estimate single step length
  - Count the steps

ENGO 585 L19 – GNSS-INS Integration

- PDR works well to estimate distanced travelled but not heading
  - Use magnetometer for heading and/or GNSS updates
- Works best with a foot-mounted IMU
- More challenging in phones
  - Mode of the phone matters
    - pocket, compassing, phoning, hand-held
  - Detection of context is challenging
    - Walking, standing, stairs, elevators, in/out of vehicles etc
- Useful to combine with other sensors
  - RF positioning methods
  - Other sensors in the phone

ENGO 585 L20 – Barometers and Magnetometers

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- Both are self-contained sensors but both measure an external quantity that has to be used in a relative sense
- Barometers measure atmospheric pressure
  - Easy to convert into a height
- Magnetometers measure magnetic field
  - Can be used to determine attitude
- Both need to be calibrated with a reference value or model

ENGO 585 L20 – Baro and Mag

- Barometer measures pressure
  - Original analog kind uses a suspended column of mercury
  - The aneroid barometer uses a partially evacuated metal container and deforms under pressure
  - MEMS on a chip either resistive or capacitive
- Many units: inHg, mmHg ($\approx$ Torr), atm, mbar, hPa, kPa, psi…
- One standard atmosphere: 101.3 kPa (760 mmHg, etc.)

ENGO 585 L20 – Baro and Mag

- Relationship between pressure and height for short columns of dense liquids (like mercury or water)

$$P = \rho g h \ \frac{kg}{m^3}\frac{m}{s^2}m$$

  - But this equation is the pressure due to the height of the column of above us. So for example, 10 m water exerts approx. 100 kPA (1000 x 9.8 x 10 ≈ 100,000 Pa)

- In the atmosphere, the pressure decreases as you move up (because less air above you)

$$P(h) = P(h_0)e^{-\frac{\rho g h}{P(h_0)}}$$

- Decreases exponentially as you increase elevation

ENGO 585 L20 – Baro and Mag

- Two options to deal with this:
  - Calibrate at a known height
    - Example: Height at a trail head

$$P(h) = P(h_0)e^{-\frac{\rho g h}{P(h_0)}}$$

    - Decreases exponentially as you increase elevation
    - In Calgary (elevation 1000 m) about 89 kPA
    - Approx. 1.2 kPa per 100 m.
    - BMP 280's specs are ± 0.012 kPA relative accuracy (this is not a co-incidence)
  - Estimate $P(h_0)$ or h_0 as an extra state in a Kalman filter
    - When you have another source of height, use it to solve for the height offset.
    - Or use differential barometer to estimate change in height

ENGO 585 L20 – Baro and Mag

- Magnetometers
  - Like accelerometers and gyros suffer from biases
  - Can be calibrated by rotating them through many angles and fitting a sphere (see Renaudin et al 2010)
  - Once calibrated they sense the surrounding field
  - The geomagnetic field does not point north, in Calgary it points down and to the East.
    - Declination (East) and inclination (down) can be looked up on a map
    - Accel's to level the sensor and solve for the horizontal
    - Or estimate in run when you have another source of direction.
      - Vehicle nav system assumes you are driving a cardinal direction and corrects

ENGO 585 L20 – Baro and Mag

- Another note on odometers
  - Just like barometers and magnetometers, odometry also can be biased
  - Why?
    - Wheel size/wear and tire inflation
    - What to do about this?
    - What else: augment the state with a scale factor and estimate it when you have redundant measurements.
- With all of these: Baro, Mag, Odo:
  - Estimate the bias when you have enough observations to do so
  - If the bias changes slowly, give it a small process noise
  - Then when your extra observations go away, use the sensors.

ENGO 585 L20 – Baro and Mag

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ENGO 585 L21 – Map Matching and integration examples, SLAM

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- Map Matching
  - Simplest implementation: If you have a map use it as a constraint
    - Snap to map element
    - Across track constraint via pseudo-measurement
    - Eliminate impossible solutions (on the wrong side of the road or wall)
  - Really interesting example from NovAtel 2001-2005
    - Starts with doing carrier smoothing of GPS data
      - https://www.ion.org/publications/abstract.cfm?articleID=2203
    - Then adds INS and a track model
      - https://www.ion.org/publications/abstract.cfm?articleID=5225

ENGO 585 L21 – Map Matching and integration examples, SLAM

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- Multi-sensor fusion or integration
  - Very generic term used in navigation and robotics
  - In its simplist, GNSS-INS, or GNSS+WiFi+UWB ranging are all examples of multi-sensor fusion
  - Can use deterministic models + recursive estimators like EFK and use multiple sensors to estimate states
  - Also used to describe fusing of perception sensors on autonomous vehicles
    - Radar, Lidar, Vision map of obstacles
    - Fuse them to make a map of obstacles and then avoid those

ENGO 585 L21 – Map Matching and integration examples, SLAM

- What if you don't have a map?
- Simultaneous Location and Mapping (SLAM)
  - Particularly useful for robot/vehicle applications
  - Vehicle has a state $[x, y, \theta]^T$
  - Augment the state vector with the coordinates of map features $m = [m_{1x}, m_{1y}, m_{2x}, m_{2y} \dots]$
  - The vehicle state (or pose) has a dynamics model
  - The features are random constants (no dynamics)
  - There is some way to observe the features fully with one or two set of observations
    - For example range and bearing, stereophotogrammetry (either with two camera or between epochs)

ENGO 585 L21 – Map Matching and integration examples, SLAM

- When a feature is new, the first observation defines its location
- Then it becomes a target with covariance
- Next observations improves the pose and the map

- Many, many variations, optimizations, generalizations
- More Info
  - A really good introduction by Durant-Whyte https://ieeexplore.ieee.org/document/1638022
    - (also posted online on the author's webpage, ask Google)
  - Textbook: Probabilistic Robotics by Thrun et al.
  - Also check out the Wikipedia page
    - specifically the History and See Also sections at the bottom

ENGO 585 L21 – Schedule for Next 3 weeks

- Fri 2, Mon 5 Apr holidays
- Wednesday 7 April
  1. Alhassan - Location-based entertainment
  2. Izhan – Asset Tracking
  3. Sudam and Mikko - Location based services
  4. Chris and Rohit – Animal Tracking
- Friday 9 April
  5. Madison and Jacky - Intelligent Transportation
  6. WanJia and Zhe - Development and Applications of Wireless Location Techniques on Wildlife Tracking
  7. Amr - Positioning using 5G Networks
  8. Andrew and Nicholas - Wireless Location and Privacy Issues
  9. Gregory – Animal Tracking
  10. Carter and Torri - Animal Tracking
  11. Sam – A-GPS

ENGO 585 L21 – Schedule for Next 3 weeks

- Monday 12 April
  12. Wynand and Michael - Ultra-wide Band Positioning and Sensor Fusion
  13. Brendan – Animal Tracking
  14. Lingyi and Refeina - Wireless Technologies in Intelligent Transportation System
  15. Andreas - Indoor Positioning with Acoustic and Ultrasonic Technology
- Wednesday 14 April
  16. Mitchell and Zoe - The Importance of GNSS in Autonomous Vehicles
  17. Aaron and Eugene - Wireless location for law enforcement and privacy issue
  18. Kelly and Faith - Indoor RFID Navigation Technologies for Visually Impaired Pedestrians
  19. Zelin and Josh - Wireless Location in Intelligent Transportation

ENGO 585 L22 – E911

- Last topic: One example application: Enhanced 911
- Question: Have you ever called 911 from a mobile phone?
- When you do, they ask 2 questions
  - What is your emergency? (police, fire, ambulance)
  - Where are you?
- Landlines
  - Solved with a database of subscriber info and a second database of service areas
    - Connect to the correct 911 operator and tell the operator the address
    - This used to be much easier because the numbers weren't portable
    - VOIP Landlines had some problems

ENGO 585 L22 – E911

- What about mobile phones
  - Mobile connected to cell tower
  - Cell tower connects to a mobile switching centre
  - MSC assigns an identifier to the 911 call based on the tower location, so the right 911 operator answers
- But what about after the operator answers?
  - 911 operator has a copy of the local database of landlines
  - For mobiles this doesn't help
    - Instead either the phone or the network needs estimate the location
    - This gets transmitted during call

ENGO 585 L22 – E911

- E911 Phase 1
  - Wireless operator must provide the location of the tower within 6 minutes of the start of the call
- E911 Phase 2
  - 95% of handsets in service must be location capable by 2005
  - Horizontal location must be provided withing 6 minutes
- The initial plan laid out by FCC in 1996:
  - Handset-based technology:
    - 50 m for 67% of calls, 150 m for 90% of calls
  - Network-based technology
    - 100 m for 67% of calls, 300 m for 90% of calls
  - Deadline for Phase 2 kept getting delayed 2001, 2005, 2008, 2011
    - Why? 2 problems: 1 getting the coordinates, 2 mapping them