Chapter 3
Satellite Orbit and Clock Representation and Computation

Introduction to Satellite Orbits

Keplerian Orbits

Non-Keplerian Orbits

GPS Orbit Representation and GPS Navigation Message

GPS Almanac

Other Sources of GPS Orbit Information

Some material for this Chapter was derived from:

Gao, Y. (2004), **Geodetic Positioning Lecture Notes**, Winter 2004, Department of Geomatics Engineering, University of Calgary.

Introduction to Satellite Motion

- Knowing the motion of a satellite is critical to numerous applications including satellite-based positioning
  - Understanding orbital/celestial mechanics
  - Determining the gravity field of the Earth (e.g. Champ and GRACE missions)
  - Determining orientation parameters of Earth in space

- In the context of satellite-based positioning, the satellites are the control to which we make measurements in order to compute a position. The coordinates (and velocities) of the satellites must therefore be known to a high level of accuracy.

Introduction to Satellite Orbits

Gravitational Potential of the Earth

- Gravitational potential of the Earth (V) is expressed as

$$V = \frac{GM}{r} + R$$

Potential of a Central field

Potential of disturbing field

where

- G is the Universal Gravitational Constant = 6.673×10^{-11} $m^3 kg^{-1} s^{-2}$
- M is the mass of the Earth
- r is the distance of the satellite from the Earth's centre of mass

- Disturbing field is caused by mass irregularities

Orbital Motion in a Central Field

- Orbit in a potential central field

Earth's gravitational constant (3986004.418e8 m^3/s^{-2})

$$\vec{f} = m\vec{a} = m \cdot \text{grad}(V) = m \cdot \text{grad}\left(\frac{GM}{r}\right) = -m \cdot \frac{GM}{r^3}\vec{r} = -m \cdot \frac{\mu}{r^3}\vec{r}$$

substituting

$$\vec{a} = \ddot{\vec{r}} \qquad \Rightarrow \qquad \ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r}$$

- Above differential equation defines Keplerian motion. It consists of three second-order differential equations requiring six constants of integration for a solution. By extension, *a Keplerian orbit is described by six parameters*.

History of Orbits

- Johannes Kepler (1571-1630) formulated three laws of planetary motion using Tycho Brahe’s (1546-1601) astronomic observations
  - Laws describe planetary motion (geometric and empirical), but no explanation for the underlying forces
  - Kepler’s laws were a breakthrough for Copernicus’ heliocentric hypothesis

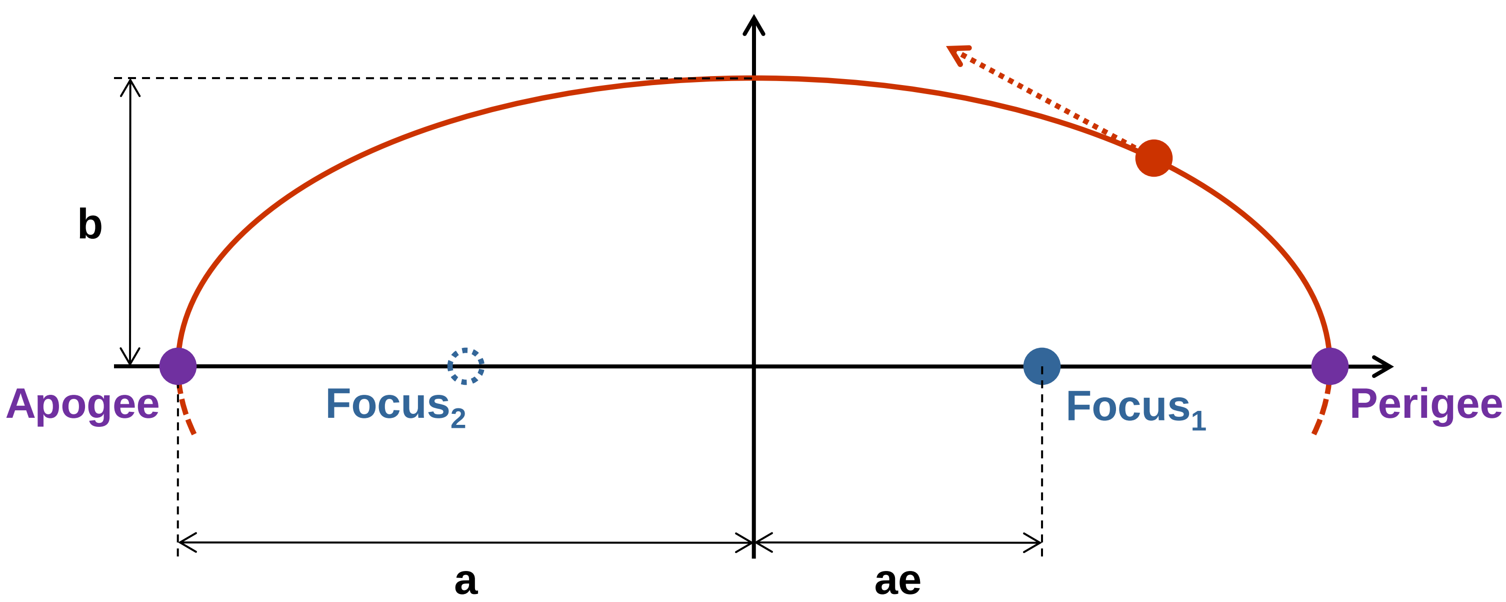
Johannes Kepler
(1571 - 1630)

Kepler’s Laws

- Kepler used a purely geometrical approach because he had no knowledge of the gravitational field
  - He derived laws defining the size and shape of the “satellite” orbit and the relation between geometry and motion
  - Kepler’s work was originally based on observations of planets but the laws are equally applicable to artificial satellites (e.g. GNSS, space shuttle, etc.)
- Newton refined Kepler’s work to include a gravitational field, thus explaining why the planets orbited the way they did

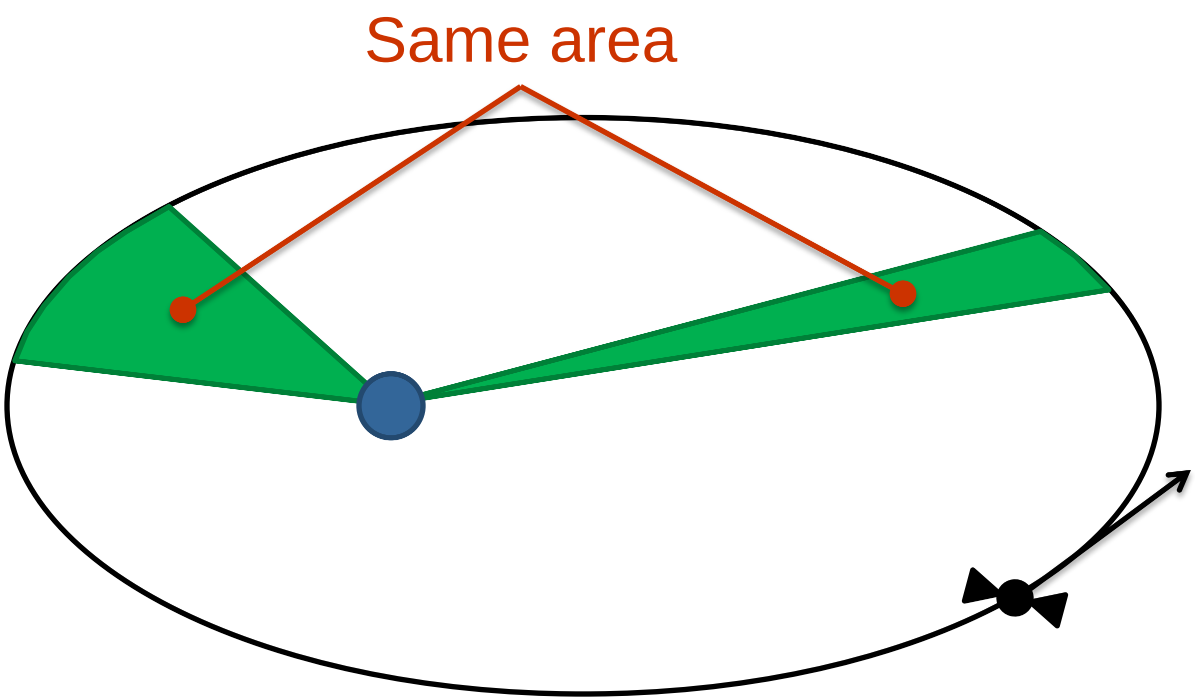
Kepler's First Law

- The first law says that a satellite orbits a body in an elliptical manner with one of the ellipse's focii centred on the body around which the satellite is orbiting (e.g., Earth, Sun, etc.); the other is "empty"
- Ellipse defines the geometry of the orbital motion
  - Semi-major axis (a), semi-minor axis (b), eccentricity (e)



Kepler's Second Law

- Kepler's second law states that a satellite sweeps out equal areas in equal amounts of time
  - Useful for predicting the position of a satellite as a function of time



Kepler's Third Law

- The third law states that the cube of the semi-major axis is related to the square of the orbital period

$$T^2 = \frac{4\pi^2}{GM} a^3$$

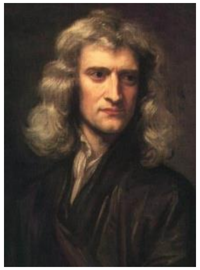
$$\sqrt{\frac{GM}{a^3}} = \frac{2\pi}{T} = n$$

Mean motion
(mean angular rate of orbit)

- Collectively, Kepler's laws define the size, shape and motion of a satellite acting *only under gravitational forces*, that is, in the absence of external perturbing forces
  - In practice, other forces are present and must be accounted for. We therefore reserve the term *Keplerian orbit* as a first order approximation of what actually happens.

Newton

- Sir Isaac Newton later provided his law of gravitation which explained Kepler's laws with physics
  - Furthermore, Newtonian mechanics is the basis of modern celestial mechanics



Sir Issac Newton
(1642-1727)

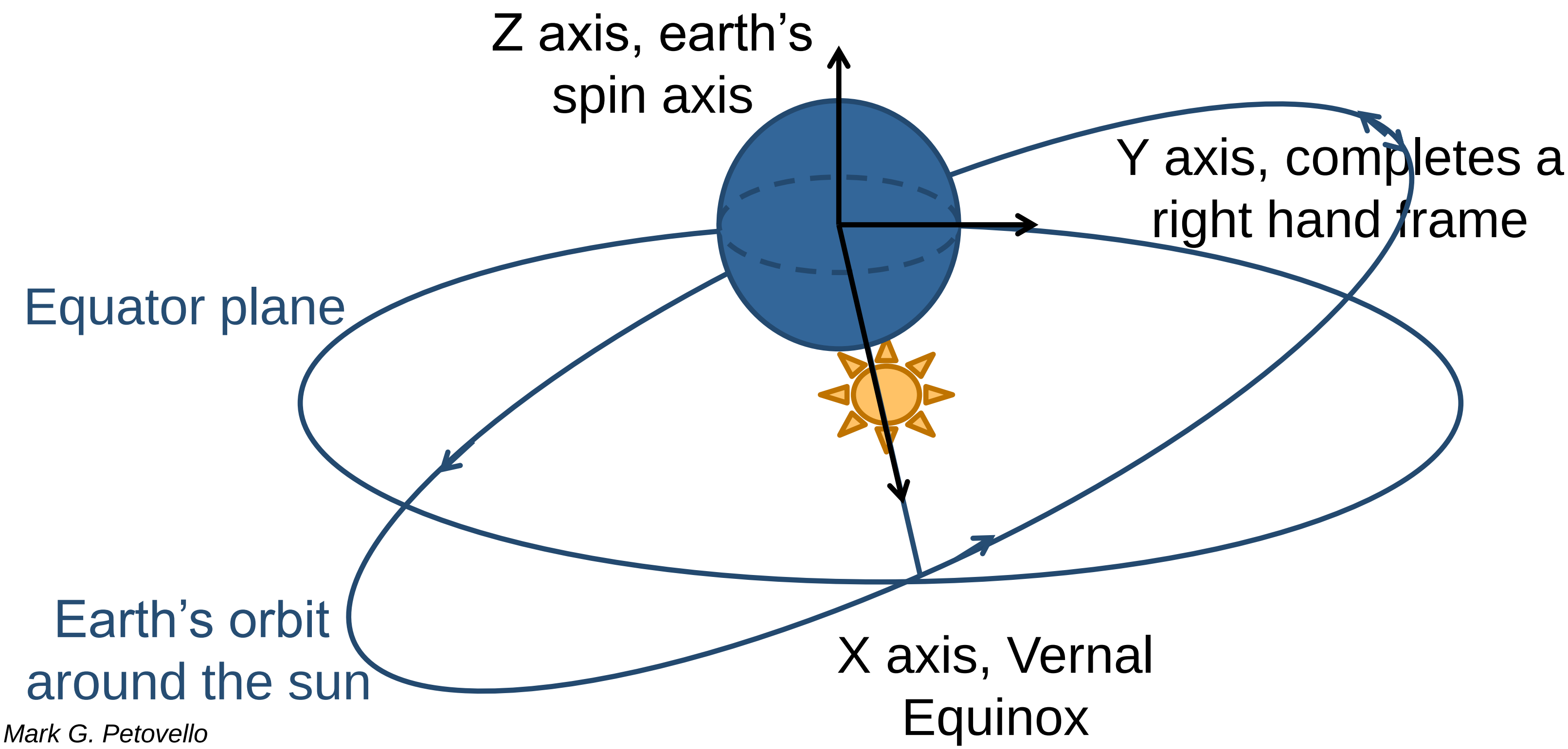
- Newton's laws of motion:
  - F = ma
  - An object in motion stays in motion and an object at rest stays at rest unless acted upon by an outside force
  - For every action there is an equal and opposite reaction

- All of Kepler's laws can be derived from Newton's laws

Some Relevant Coordinate Systems

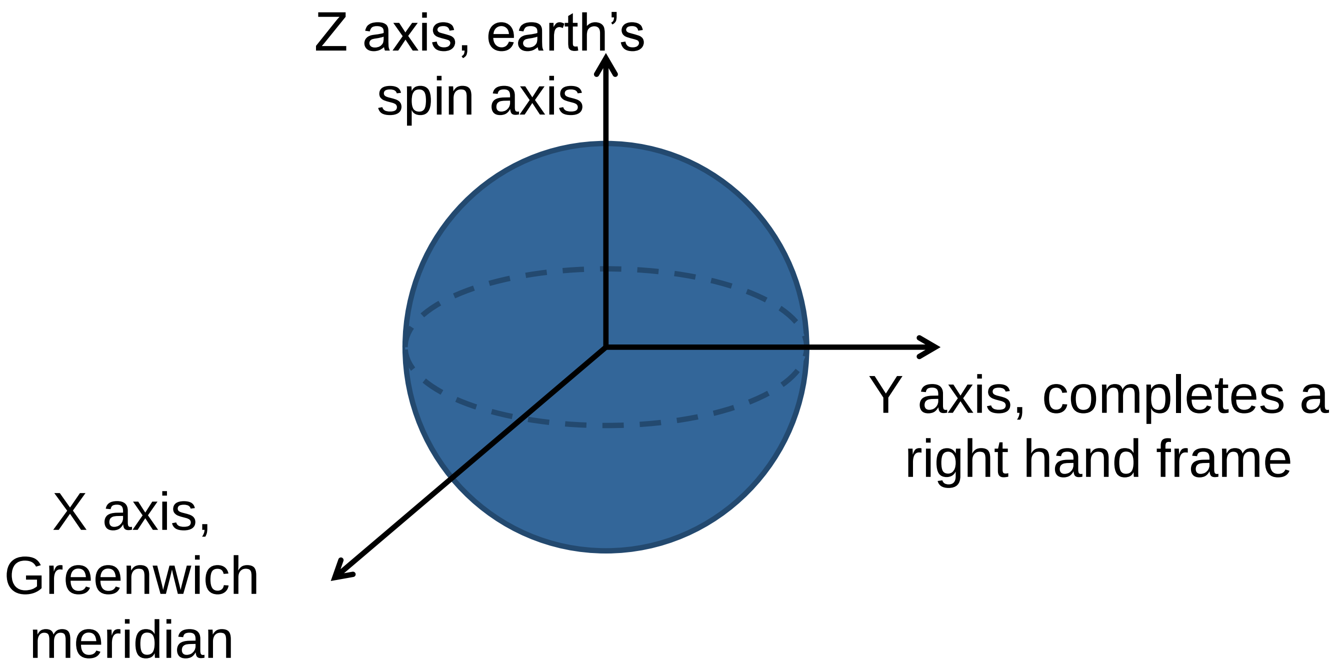
Coordinate systems

- Earth Centered Inertial (ECI)
  - The most intuitive frame for understanding orbital motion
  - The frame in which orbits form an ellipse
  - The parameters of a Keplerian orbit are therefore defined in this frame
  - Most orbit determination and orbit propagation is done in the ECI frame



Coordinate systems

- Earth Centered Earth Fixed (ECEF)
  - Coordinate frame rotates with the earth
  - GPS orbital parameters are defined in the ECEF frame because it is more useful for positioning on the earth's surface
  - realized by WGS84 for GPS, PZ90 for GLONASS, ITRF for the precise GPS orbits, …



Keplerian Orbits

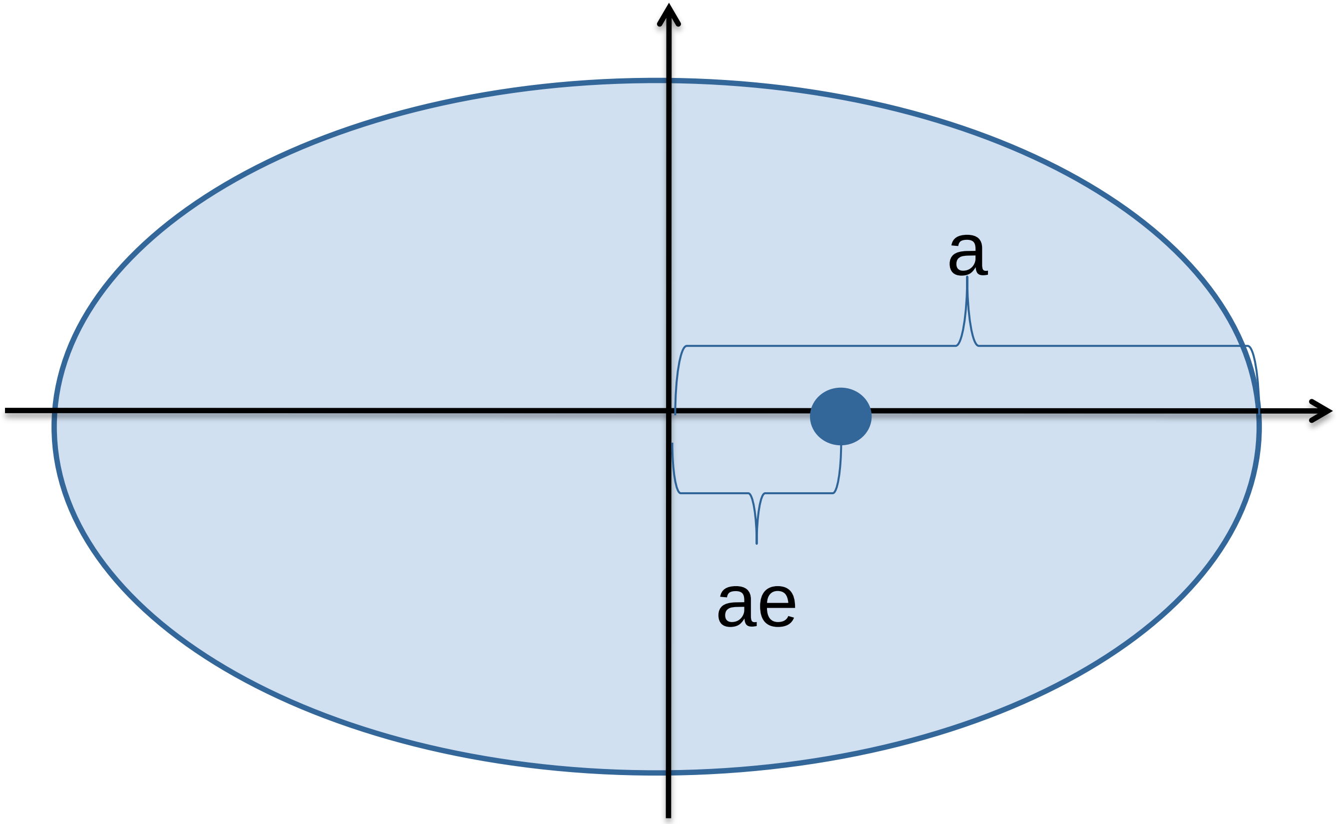
Six Parameters

- It takes six parameters to represent a Kepler orbit, they can be in the form of either:
  - A 3D position vector and it’s corresponding velocity vector
  - Keplerian Elements

- Assuming perfect Keplerian motion, this is enough information to determine the satellite’s position at any time in the future
- In reality, there are perturbing forces

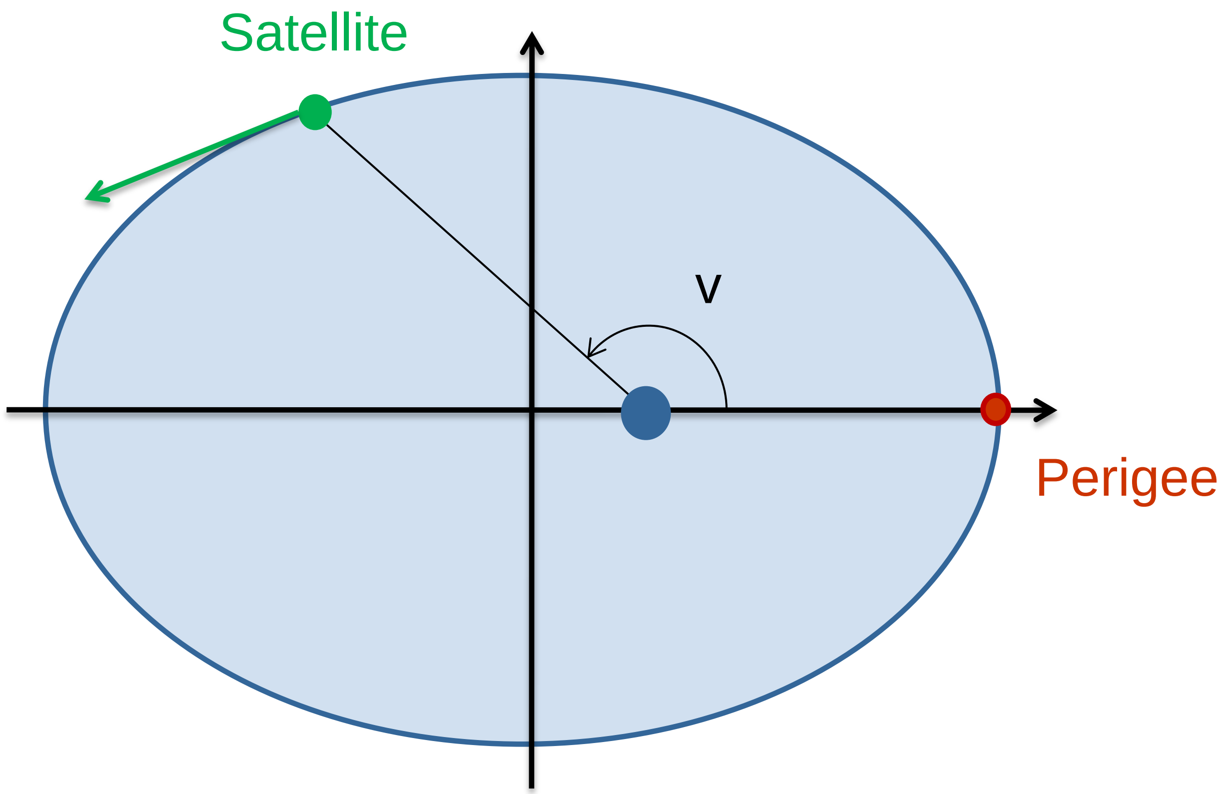
Keplerian Orbits – In Plane Motion

- Let us first consider motion within the orbital plane. The *size* and *shape* of the ellipse is respectively defined by its ***semi-major axis*** (a) and its ***eccentricity*** (e).



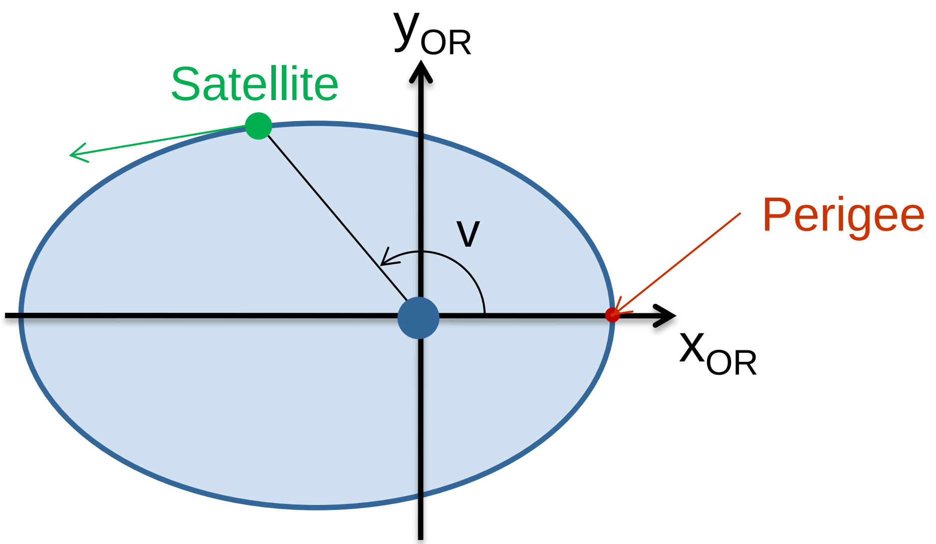
Keplerian Orbits – In Plane Motion

- In two dimensions (i.e., within the plane of the orbit), a satellite's position is determined by its ***true anomaly*** (v), which is the angular distance along the orbit measured from perigee. ***Perigee*** is the point along the orbit closest to the body being orbited.



Keplerian Orbits – Orbital Frame (OR)

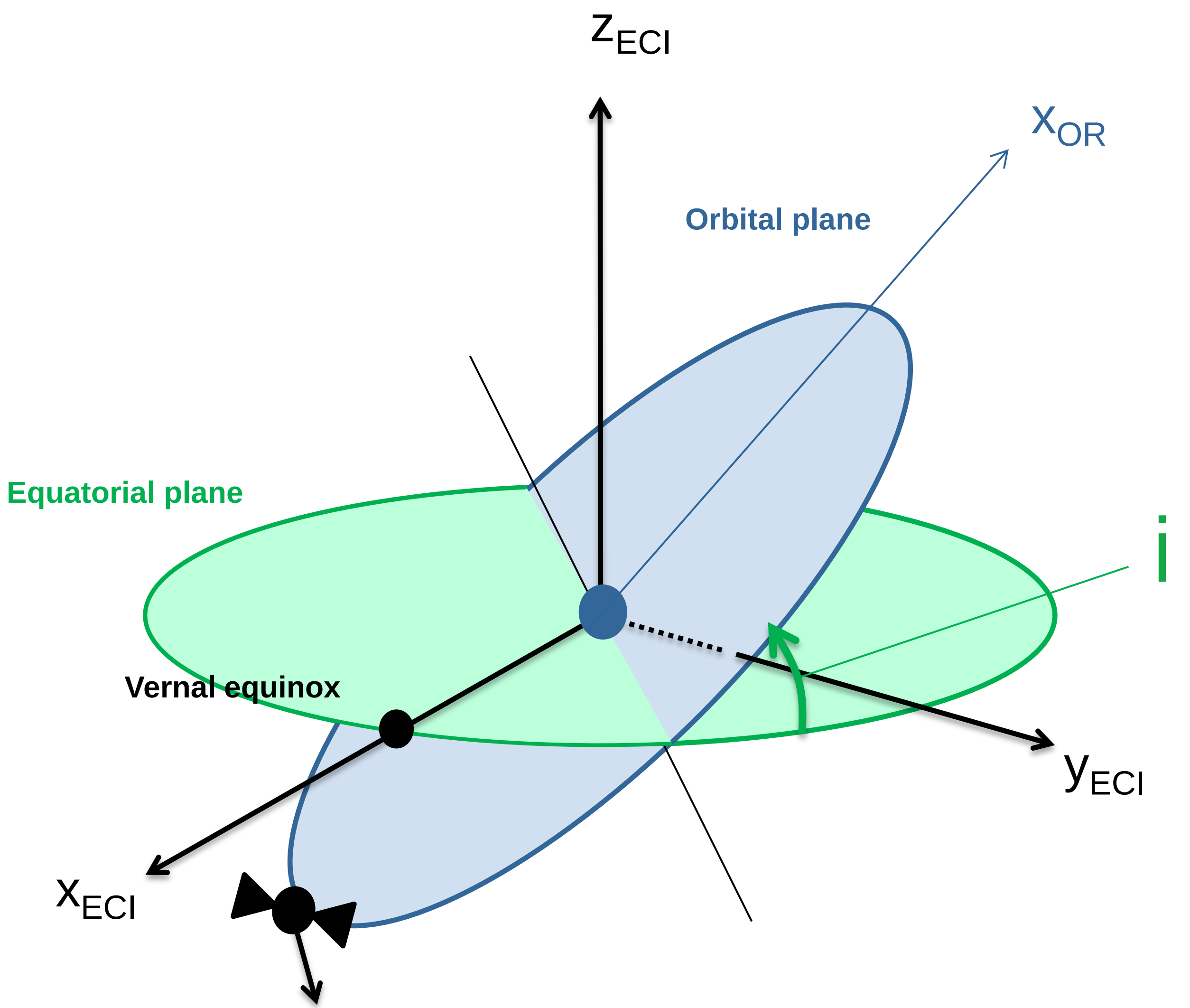
- We define the orbital frame (OR) as
  - Origin: Centre of mass of the Earth (or other orbited body)
  - X-Axis: Origin to perigee
  - Z-Axis: Normal to orbital plane
  - Y-Axis: Orthogonal to X & Z axes



- For a Keplerian orbit (i.e., in a central field), the z coordinate is always zero (i.e., in-plane motion only)

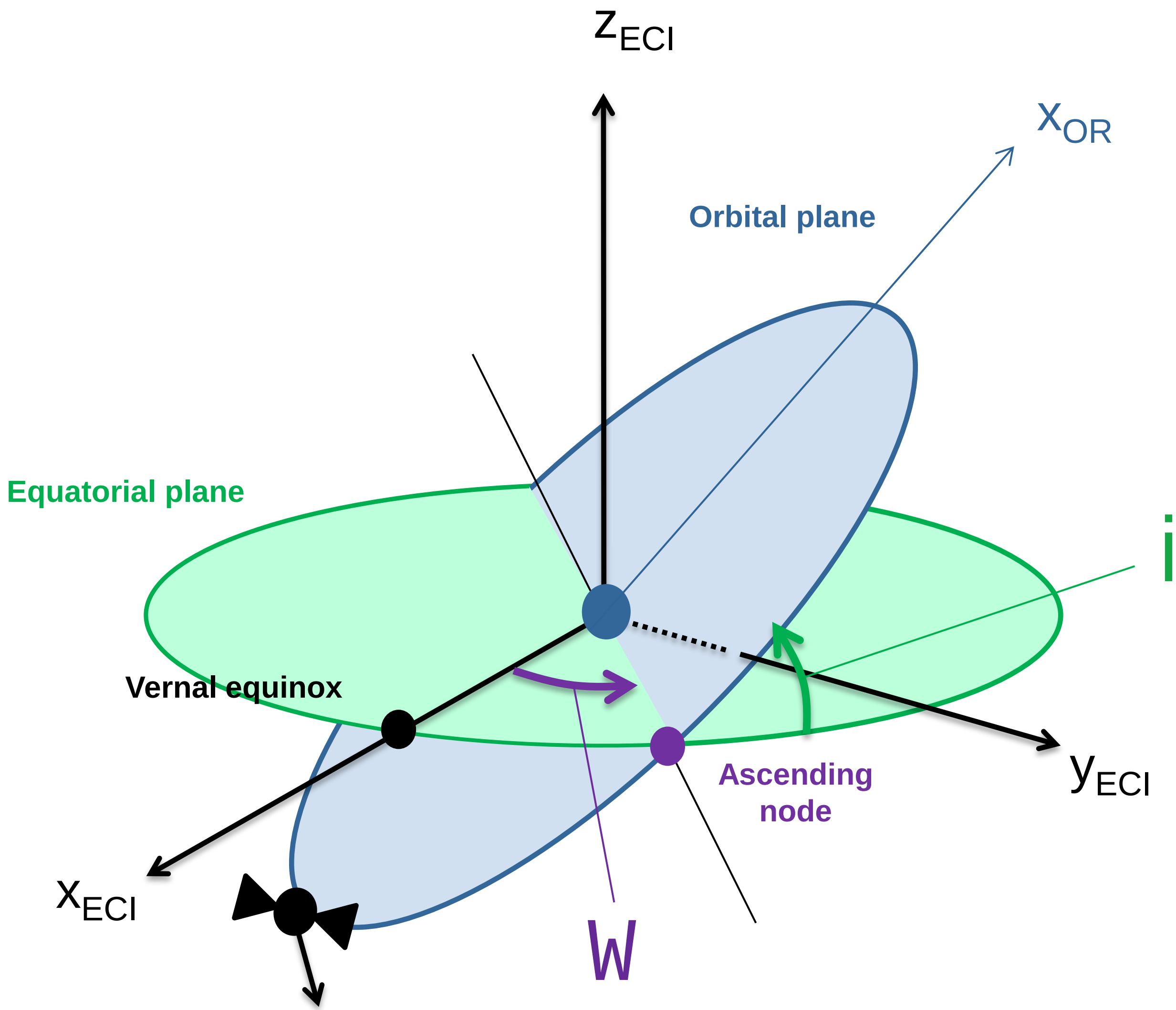
Keplerian Orbital System

- The ***inclination*** (i) defines the angle of the orbital plane with respect to the plane of the celestial equator



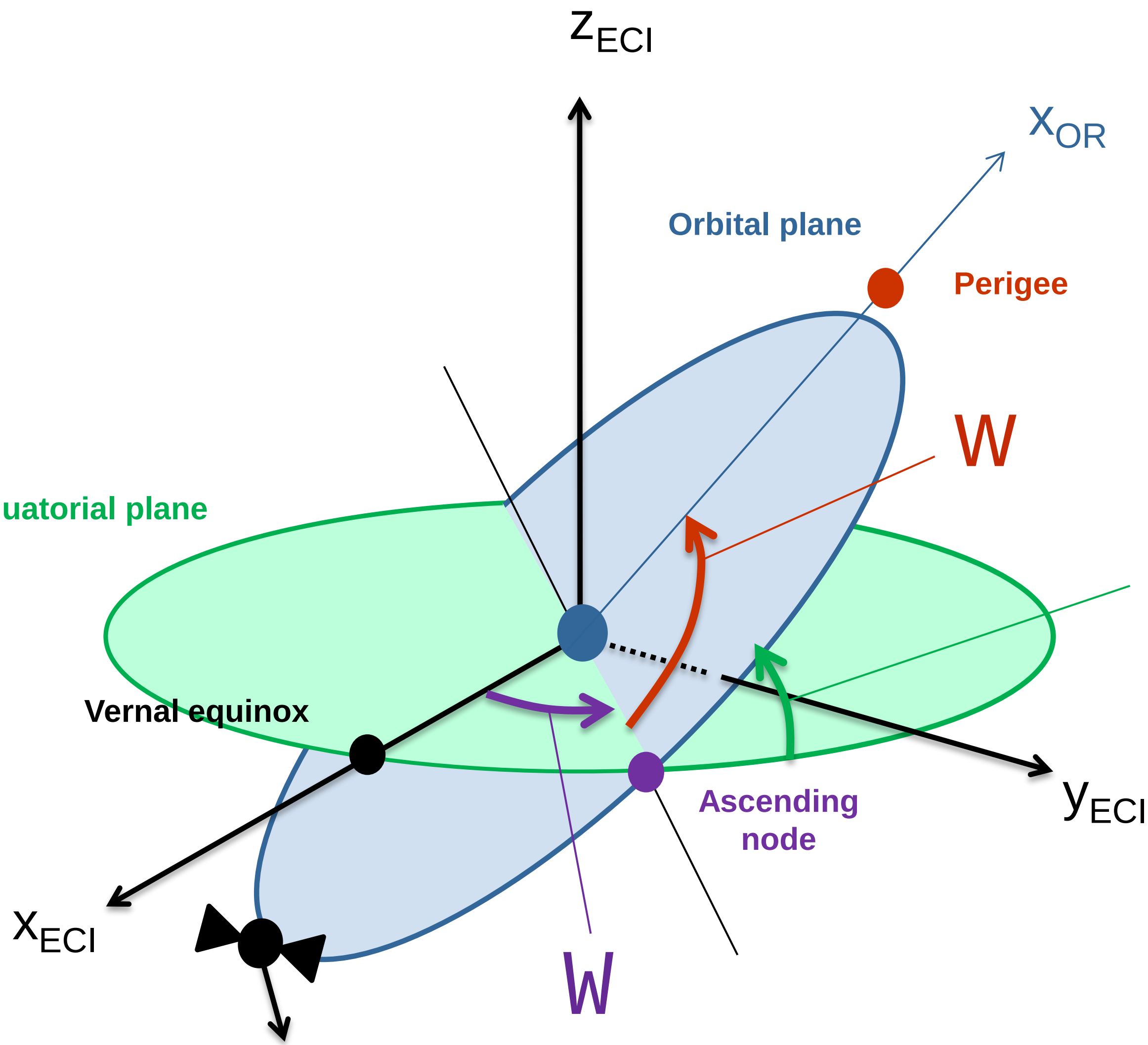
Keplerian Orbital System

- The ***right ascension of the ascending node*** (Ω) defines the angular distance of the ascending node from the vernal equinox



Keplerian Orbital System

- The ***argument of perigee*** (ω) is the angular distance of perigee from the ascending node (measured within the orbital plane)



Keplerian Elements - Angles

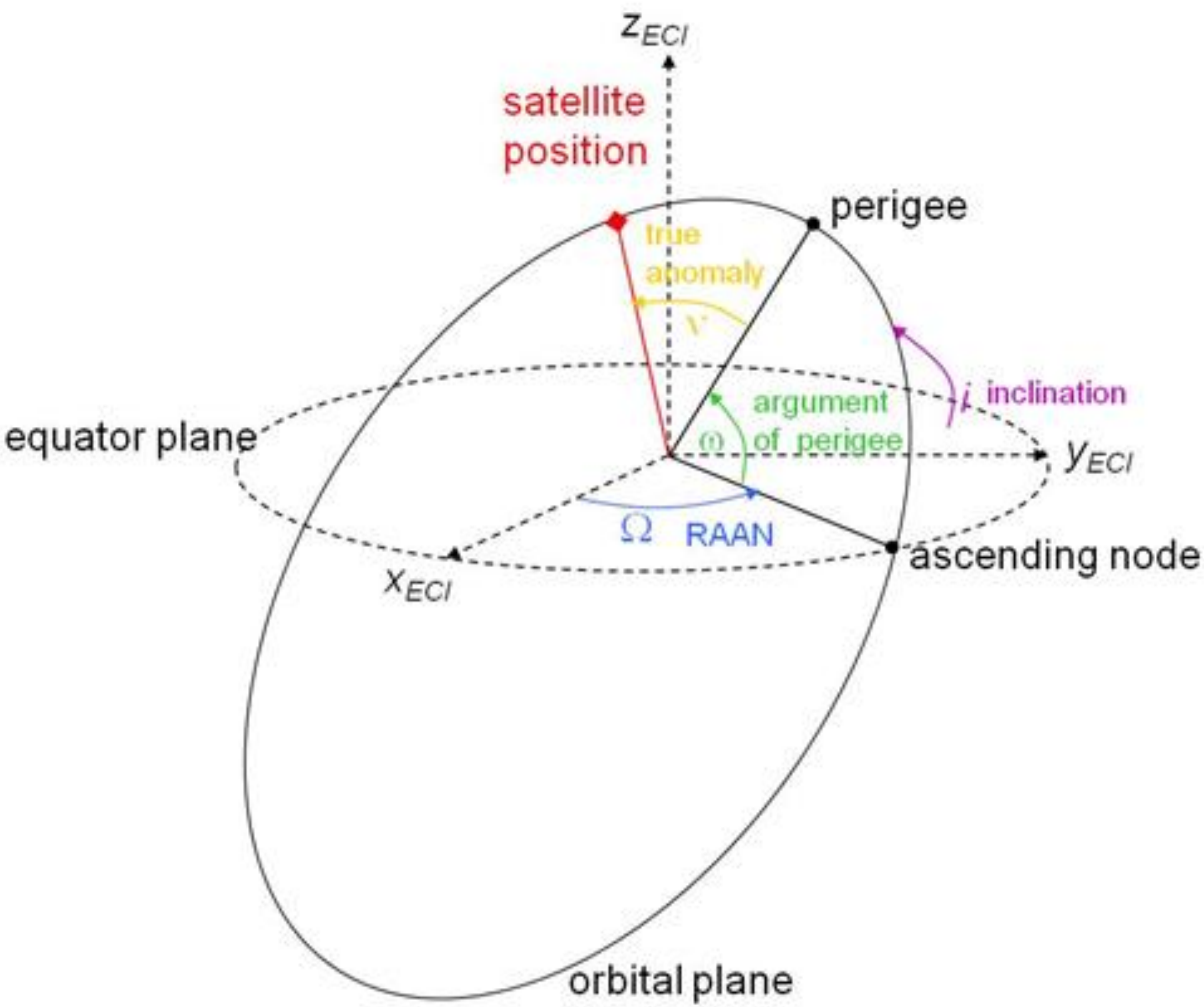


Image from http://www.ucalgary.ca/engo_webdocs/KOK/11.20329_erinkahr.pdf

Polar Coordinates of a Satellite

- Orbital eccentricity

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

- Range to satellite is given by (defined in any calculus text)

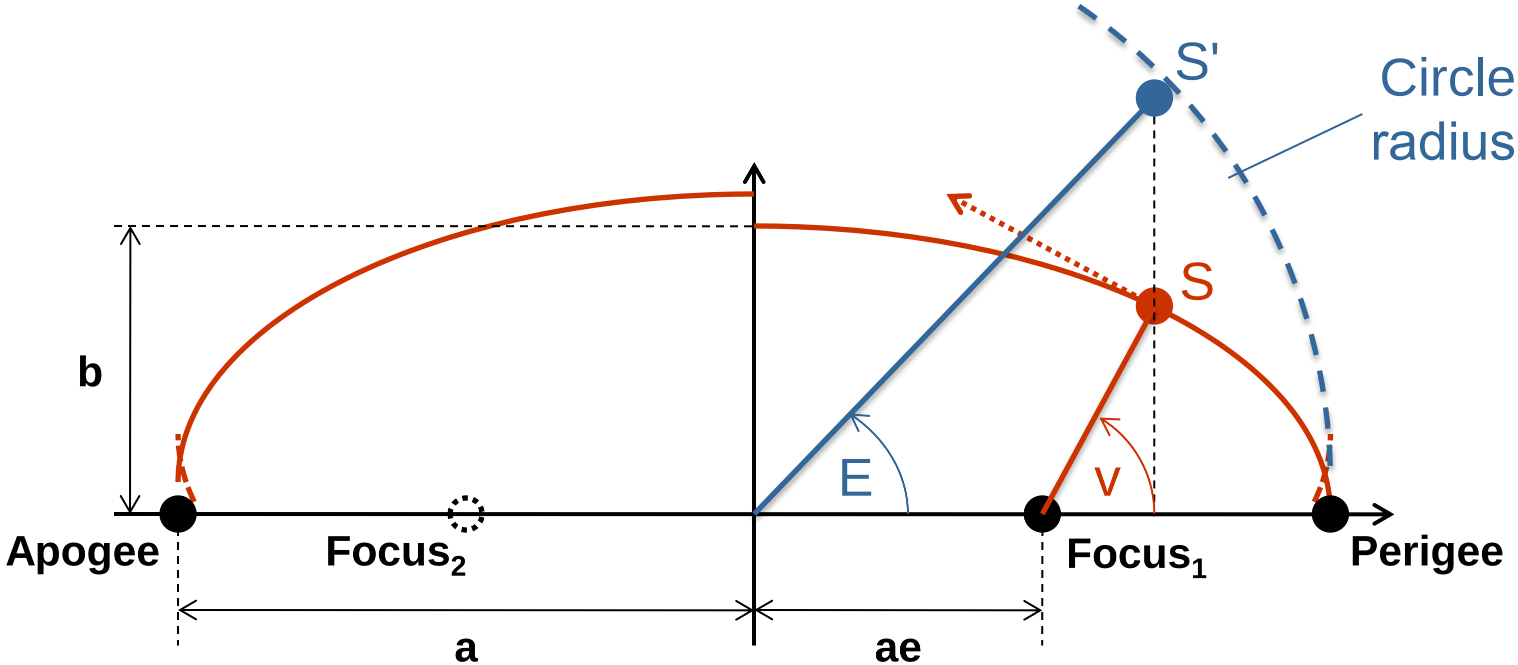
$$r = \|\mathbf{r}\| = \frac{a\left(1 - e^2\right)}{1 + e\cos v}$$

- Polar coordinates of satellite are given by range and true anomaly

$$\mathbf{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{OR} = \begin{pmatrix} r\cos v \\ r\sin v \\ 0 \end{pmatrix}$$

Anomalies of the Keplerian Orbit

- Required to relate the true anomaly to time
- The instantaneous position of the satellite in the plane is described by an angular quantity called an anomaly. There are three different "types" of anomaly that are used; mean (M), eccentric (E) and true (v), all of which are related.
- Mean anomaly is a mathematical abstraction to describe uniform satellite motion derived from Kepler's 2nd law.



Eccentric Anomaly (E)

- The eccentric anomaly is a non-linear function of time and determines the instantaneous position of S', which is a projection of the satellite on a circle of radius 'a' concentric with the orbital ellipse

$$\mathbf{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{OR} = \begin{pmatrix} r\cos v \\ r\sin v \\ 0 \end{pmatrix} = \begin{pmatrix} a(\cos E - e) \\ b\sin E \\ 0 \end{pmatrix} = a\begin{pmatrix} \cos E - e \\ \sqrt{1-e^2}\sin E \\ 0 \end{pmatrix}$$

- This gives an alternative relationship between v and E

$$\frac{Y^{OR}}{X^{OR}} = \tan v = \frac{b\sin E}{a(\cos E - e)} = \frac{\sqrt{1-e^2}\sin E}{\cos E - e}$$

Mean and Eccentric Anomalies

- Relationship between M and E is

$$M(t) = E(t) - e\sin E(t)$$

For near circular orbits, both values are nearly the same.

- In reverse, eccentric anomaly is computed iteratively from mean anomaly

From above: $$E(t) = M(t) + e\sin E(t)$$

1st step: $$E_0(t) = M(t) + e\sin(M(t) + e\sin E(t))$$
$$\approx M(t) + e\sin M(t) + O(e^2)$$

Iteration: $$E_i(t) = M(t) + e\sin E_{i-1}(t)$$

Mean Anomaly

- Mean anomaly assumes uniform motion of a satellite along its orbit. Correspondingly, mean anomaly determines the position of a *fictitious* satellite uniformly moving along the orbital ellipse

$$M = n\left(t - t_p\right)$$

where

- n is the mean motion of satellite
- t is the time at which position is required
- t_p is the time of satellite passage at perigee

$$n = \frac{2\pi}{T} = \sqrt{\frac{GM}{a^3}}$$

Vis-Viva Equation and Energy Transfer

- Explains the relationship between semi-major axis, instantaneous radius, and speed
- $\mathrm{v}^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$

- A few interesting conclusions
  - A satellite with increasing semi-major axis travels more slowly
  - As the satellite gets higher, it travels more slowly
  - As the satellite gets lower, it's speed increases

- Energy is conserved… when at satellite is higher it has more potential energy, when it gets closer to the earth it gets faster and has more kinetic energy – the vis-viva equation explicitly describes the trade-off

Non-Keplerian Orbits

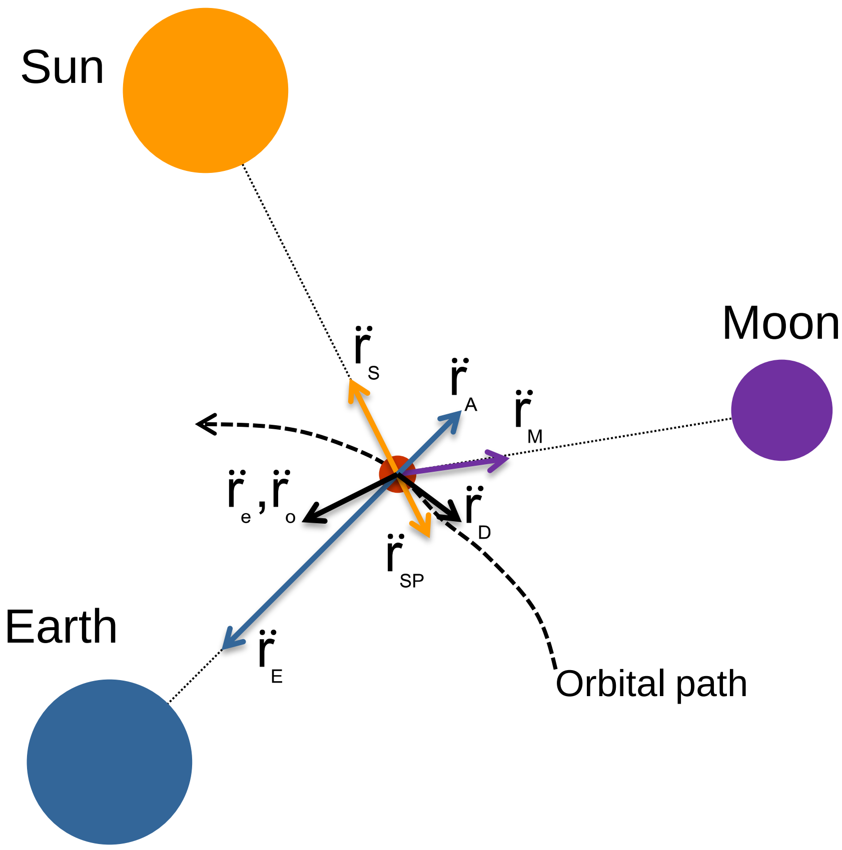
Perturbed Motion

- Until now, we considered orbits in the presence of a central field only (i.e., satellites orbiting a point mass). However, there are “extra” forces acting on the satellite that will cause the satellite motion to differ from a true ellipse.

- There are several forces that need to be considered including
  - Gravitational forces
  - Atmospheric drag
  - Solar radiation
  - Tidal loading

Forces Acting on a Satellite

- Gravitational attraction of the Earth (f_E) is the dominant force, but other *perturbing forces* cause secondary effects which need to be modeled for precise orbit determination



$\ddot{r}_S$ = Solar Gravitation

$\ddot{r}_M$ = Lunar Gravitation

$\ddot{r}_E$ = Earth Gravitation

$\ddot{r}_e$ = Earth tides

$\ddot{r}_o$ = Ocean tides

$\ddot{r}_D$ = Atmospheric Drag

$\ddot{r}_{SP}$ = Solar Radiation Pressure

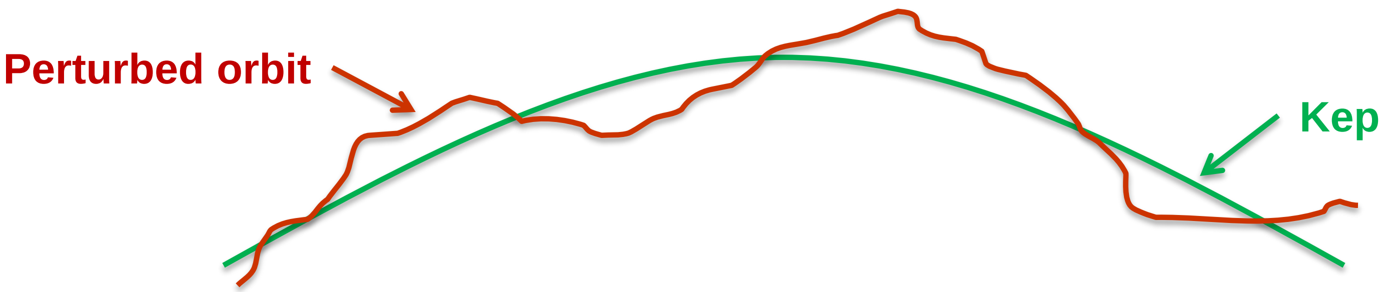
$\ddot{r}_A$ = Reflected solar pressure
= Albedo effect

From: Seeber, G. (1993) **Satellite Geodesy: Foundations, Methods and Applications**, Walter de Gruyter.

Perturbed Orbit

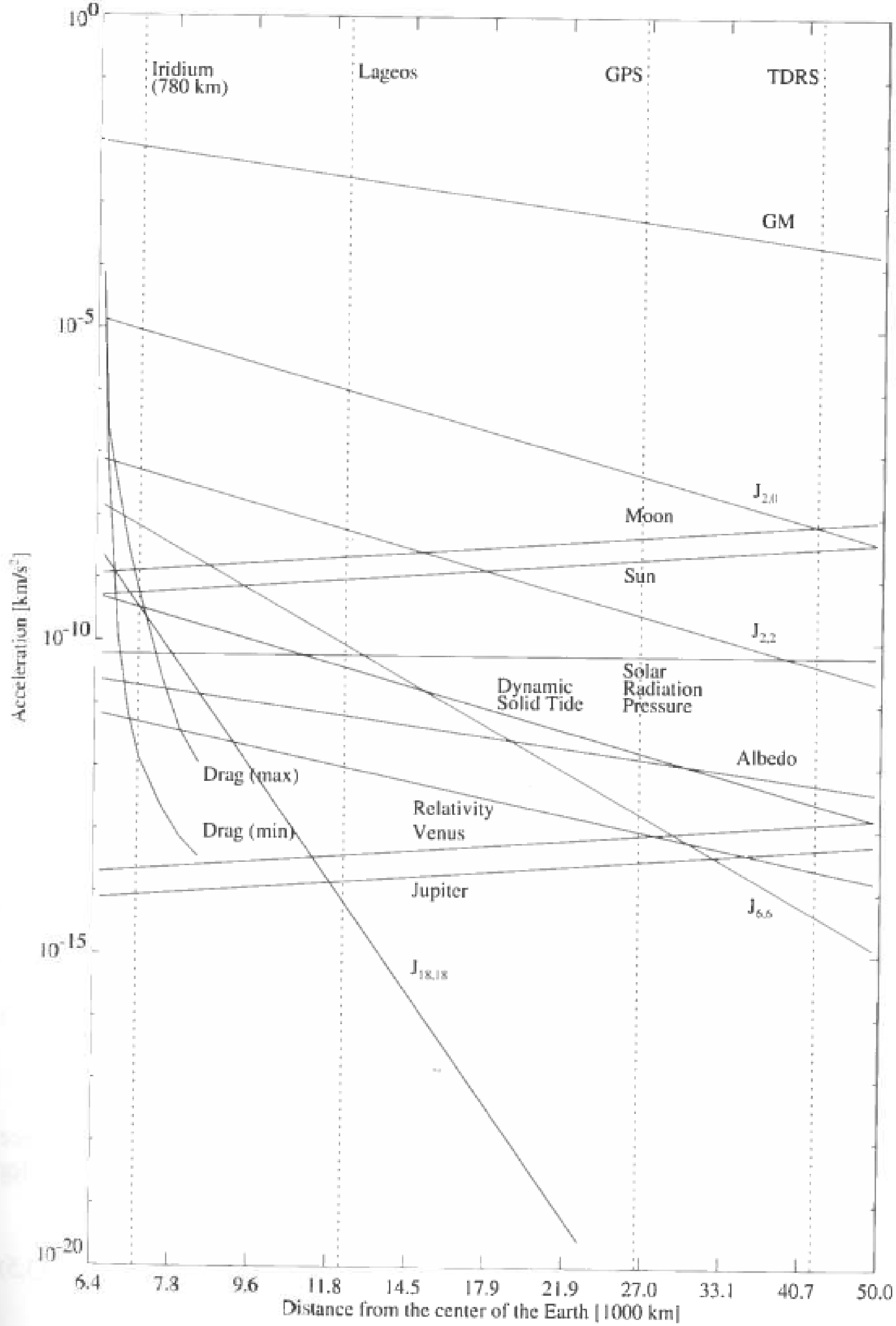
- Motion caused by total gravitational potential and other forces is called *perturbed motion.* The effect of perturbed motion is that the Keplerian elements change with time. We call the resulting orbit an *osculating ellipse*, which is the best fitting Keplerian ellipse at a given instance of time.

- To account for the above, GNSS satellites transmit basic Keplerian elements (6) plus time dependent parameters to serve as “corrections” to the Keplerian model.



Magnitudes of perturbing forces

Plot from Montenbruck and Gill, 2000, "Satellite Orbits" book



Orbit Representation

- Closed form “analytical” orbit models
  - Some initial parameters are input to a closed form equation or algorithm, and you can calculate the satellite’s position at any point in the past or future
  - GPS (and most GNSS) use closed form orbit models
- Numerical integration
  - You start with an initial position and velocity, and slowly step through time, calculating the incremental changes in acceleration, position and velocity at each epoch as you go
  - Can be extremely precise over the short term, but degrade as you propagate forward in time
  - GLONASS uses numerical integration

Orbit Determination

- Essentially a least squares process, or a Kalman filter for onboard orbit determination
- Reduced Dynamic Orbit Determination
  - Combines knowledge of orbital dynamics with kinematic measurements from satellite laser ranging, radar tracking, optical measurements, GNSS…
  - The unknowns can be an initial position and velocity or orbital elements, as well as other dynamic or measurement parameters such as:
    - Drag and solar radiation pressure coefficients
    - Empirical accelerations
    - Atmospheric effects on the measurements
    - Etc

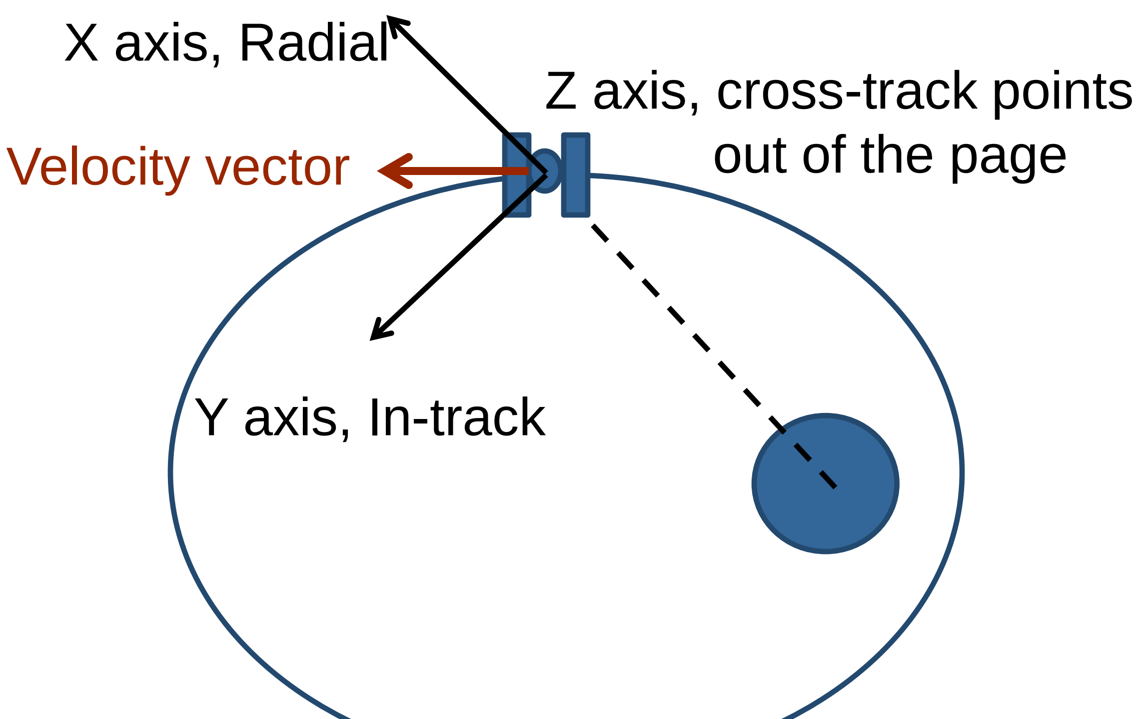
A few significant orbits

- Low earth orbit (LEO)
  - The international space station, scientific satellites, imaging satellites
  - Ranging from ~250 km to ~1200 km above the earth's surface
- Medium earth orbit (MEO)
  - Generally navigation satellites ~20000 km above the earth
- Geostationary (GEO) and Geosynchronous
  - Primarily used for communications satellites ~36000 km above the earth's surface
  - The orbit's semi-major axis is chosen to get the period T = 24 hours
- Sun-synchronous
  - The inclination is chosen such that the perturbing forces keep the satellite's ascending node at the same local time
  - Good for earth imaging, because the lighting conditions stay consistent

Some More Relevant Coordinate Systems

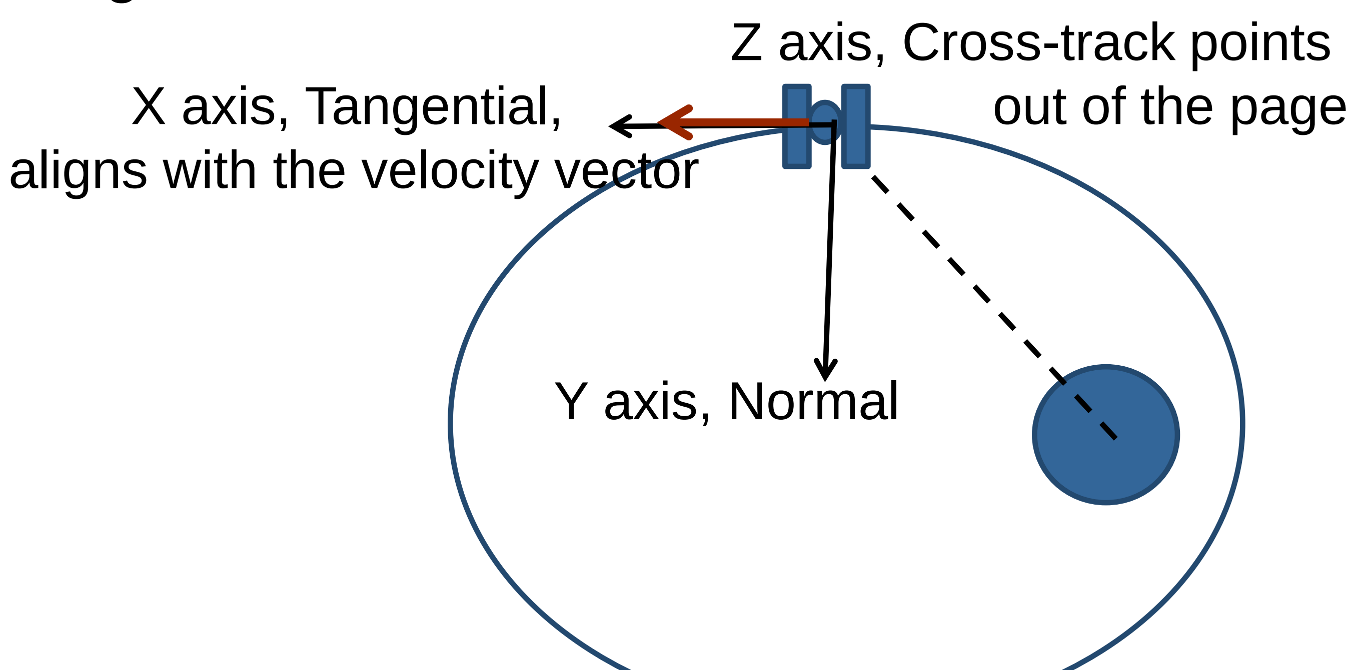
Coordinate Systems Centered on the Satellite

- The Radial, In-track, Cross-track frame (RIC)
  - Radial is defined by the vector from the center of the earth (or other central body)
  - Cross-track is defined as the cross-product of the position and velocity vector (note this is also the angular velocity vector)
  - In-track completes a right-hand frame and is approximately aligned with the forward velocity direction



Coordinate Systems Centered on the Satellite

- The tangential, normal, cross-track frame (TNC)
  - Tangential is defined as the forward velocity direction
  - Cross-track is defined as the cross-product of the position and velocity vector (as for RIC)
  - Normal completes a right-hand frame, and points approximately toward the central body (down)
- Note, for perfectly circular orbits the axes of the RIC and TNC frames are aligned



Computing the ECEF to RIC Rotation matrix

1. Reduce the position (radial vector) to a unit vector

$$e_{radial} = \frac{r}{|r|}$$

2. Adjust the velocity for the earth's rotation

$$v' = v + \omega \times r$$
$$\omega = [0,0,\omega]$$

3. Reduce the corrected velocity to a unit vector

$$e_{velocity} = \frac{v'}{|v'|}$$

4. Compute the cross track unit vector

$$e_{cross-track} = e_{radial} \times e_{velocity}$$

5. Compute the in-track unit vector

$$e_{in-track} = e_{cross-track} \times e_{radial}$$

6. Create the rotation matrix

$$R = \begin{bmatrix} e^{radial^T} \\ {e_{in-track}}^T \\ {e_{cross-track}}^T \end{bmatrix}$$

GPS Orbit Representation and GPS Navigation Message

Satellite Orbits in GNSS

- In GNSS, the ***broadcast ephemeris*** is continually transmit by each satellite to inform users of their position.
  - Consists of a set of *ephemeris parameters* that that are computed by the ground station and uplinked to the satellites
  - Several sets of parameters are uploaded at time to cover an extended period
    - Each set of parameters is valid over a certain time interval (with a given level of accuracy)
    - Using the parameters beyond the specified time interval will introduce larger errors

- If the satellite orbits cannot be determined perfectly, what happens to the computed position?

GPS Orbit and Clock Computations

- Recall that the control segment is responsible for monitoring and predicting the satellite orbits and clock errors
  - To accommodate the perturbations from a Keplerian orbit, GPS satellite orbits are modeled with a total of 16 parameters instead of the six Keplerian elements, with the extra parameters accounting for orbit perturbations.
  - GPS clocks are modeled with four parameters

- Since it impractical to compute a GPS satellite position and broadcast it to users in real time, GPS orbits and clocks are instead *predicted* over successive four hour intervals. The parameters for each four hour segment are broadcast to users as part the *GPS navigation message*. New parameters are broadcast typically every two hours.

GPS Navigation Message Overview

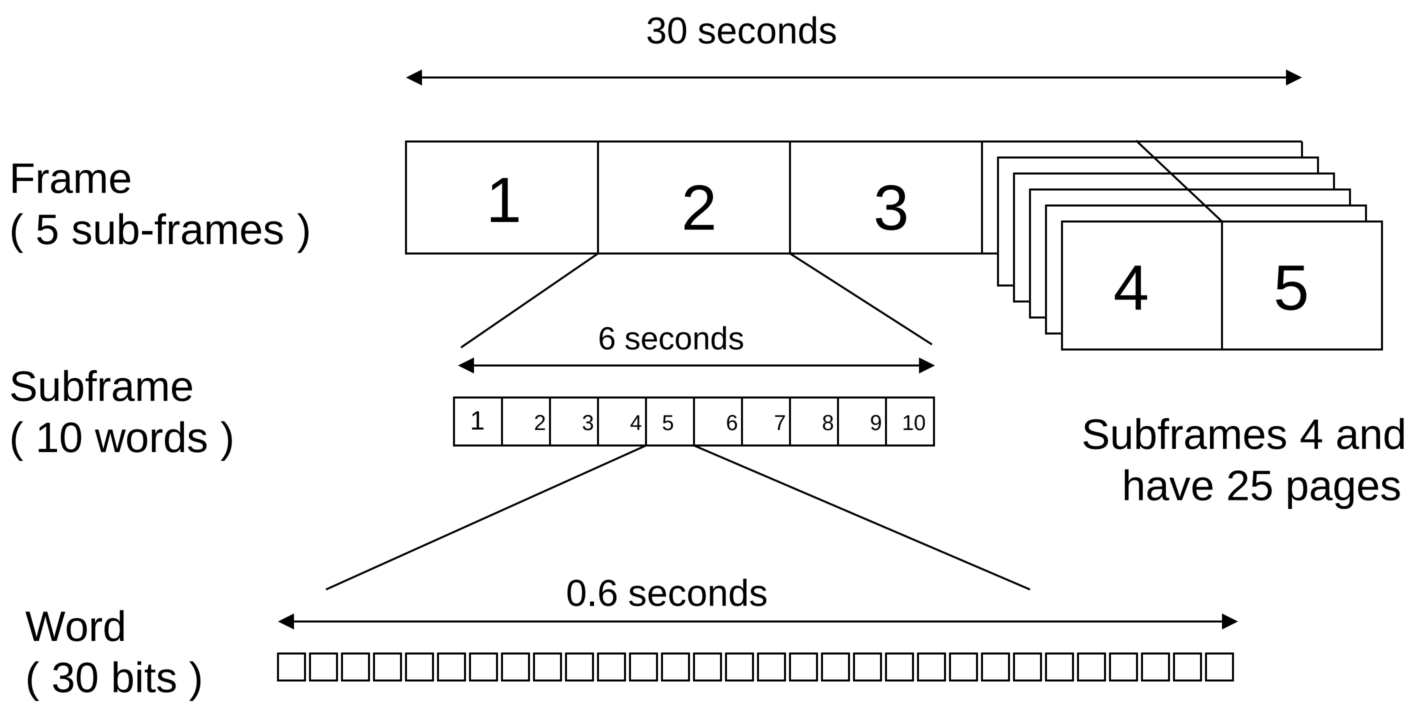
- The GPS navigation message contains information on satellite health, satellite clock, orbital parameters, other satellites in the constellation and other things. The orbit and clock information broadcast as part of the message is called the *broadcast ephemeris*. Each satellite broadcasts its *own* broadcast ephemeris parameters. Information about other satellites in the constellation (less accurate than the ephemeris) is called the *almanac* and will be discussed in more detail later.

- The basic structure of the navigation message is
  - Total message consists of 5 sub-frames/page and 25 pages
  - Each frame contains 1,500 bits of data which is transmitted at 50 bits per second (bps) (i.e., 30 seconds for transmittal)
  - Each frame consists of five sub-frames, which are 300 bits long (6 s)
  - Each sub-frame has ten words of 30 bits (0.6 s)
  - Each bit last 20 ms (0.02 s), i.e., it is transmit at 50 bits per second (bps)

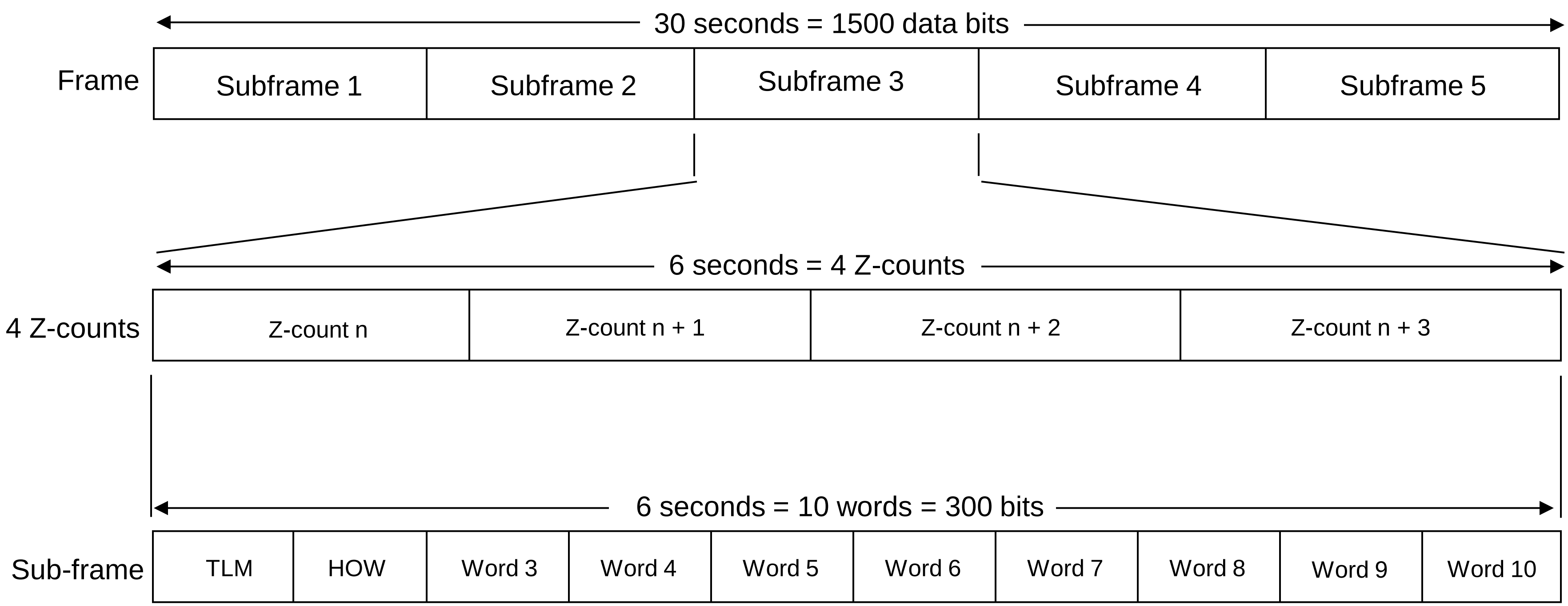
- A receiver requires at least 30 seconds to acquire one frame, which is the minimum needed to compute satellite position and clock.

GPS Navigation Message Structure

- The full message, or master frame, contains 25 *pages* of sub-frames 4 and 5
  - Total of 37,500 bits (25 frames x 5 sub-frames/frame x 10 words/sub-frame x 30 bits/word)
  - At 50 bps, 12.5 minutes is required for a receiver to extract the full message

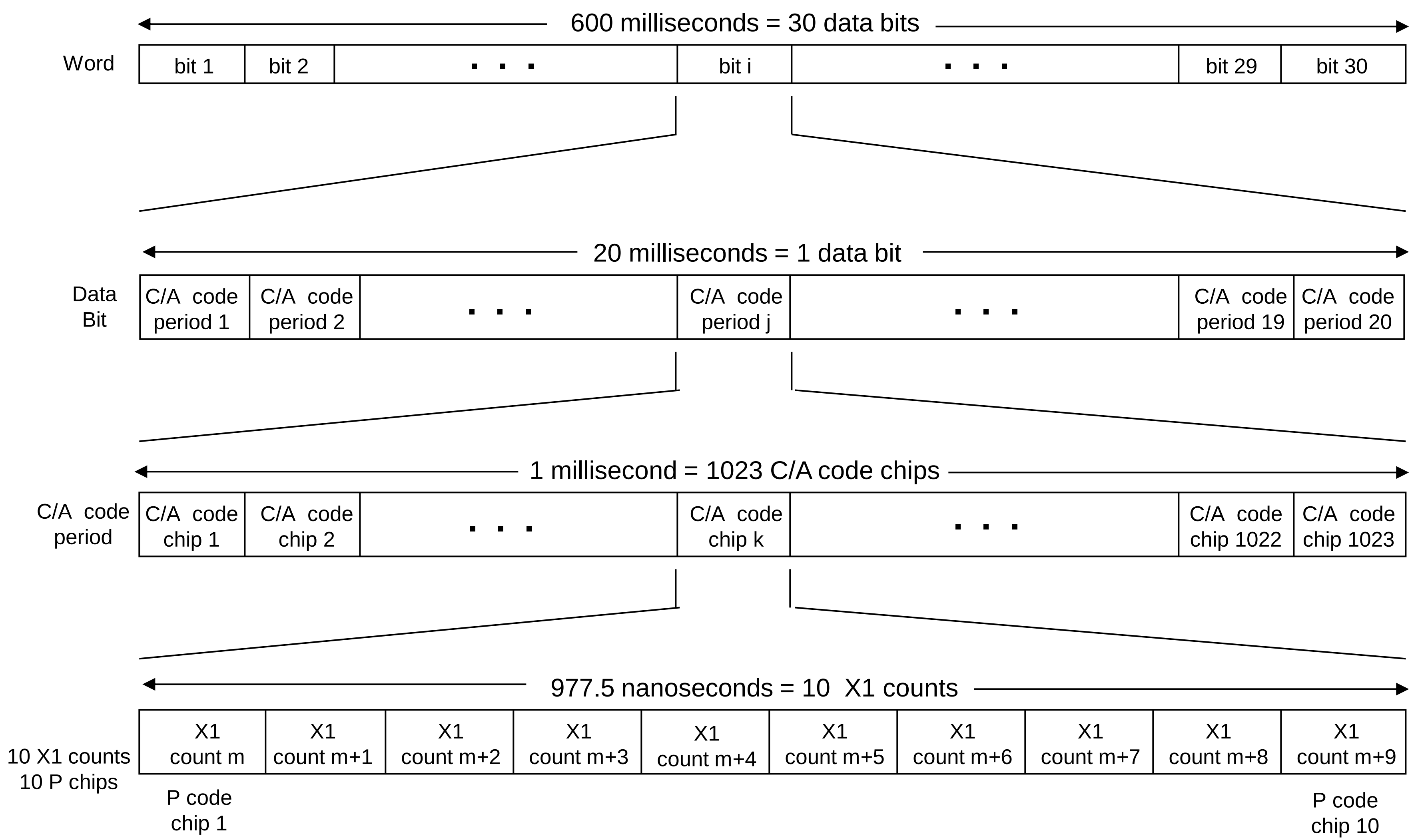


GPS Navigation Data Timing (1/2)



Source: Kaplan, E.. (1996), **Understanding GPS Principles and Applications**, Artech-House Publishers, Boston - London

GPS Navigation Data Timing (2/2)

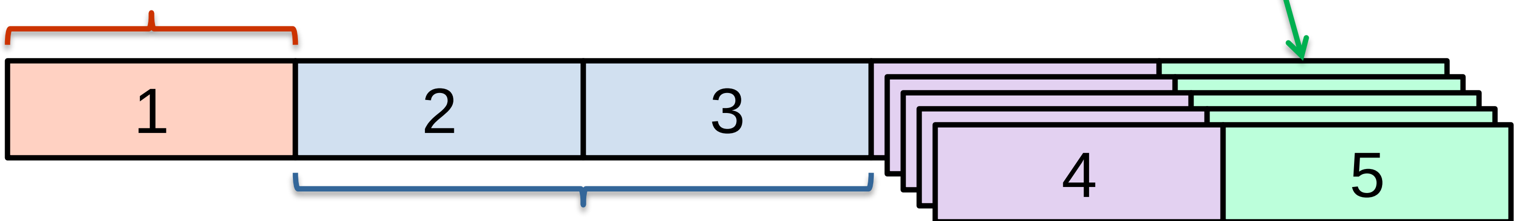


Source: Kaplan, E.. (1996), **Understanding GPS Principles and Applications**, Artech-House Publishers, Boston - London

GPS Navigation Message Content Overview

Sub-frame 1
- GPS week #
- Satellite accuracy and health
- Age of data
- Satellite clock corrections

Sub-frame 5
- Almanac for all satellites in the constellation satellites (pages 1-24)
- Health of other satellites (page 25)



Sub-frames 2 & 3
- Broadcast ephemeris (i.e., orbit parameters)

Sub-frame 4
- Almanac for satellites 25 – 32 (pages 2, 3, 4, 5, 7, 8, 9, 10)
- Ionospheric model, and UTC data (page 18)
- Antispoof flag – 32 satellites (page 25)
- Satellite configuration – 32 satellites (page 25)
- Health of satellites 25 – 32 (page 25)

Summary of Navigation Message Data

Information Requirement	Information from GPS Navigation Data
Precise satellite position at time of transmission	Satellite ephemeris using a modified Kepler model in a Earth-centered inertial frame with transformation to an ECEF
Precise satellite time at time of transmission	SV clock error models and relativistic correction
Time transfer information	GPS time to UTC time conversion data
Ionospheric corrections for single frequency users	Approximate model of ionosphere vs. time and user location
Quality of satellite signals/data	User range accuracy (URA) – a URA index 'N' is transmitted which gives a quantized measure of space vehicle accuracy available to the civil user

Reference: Parkinson, B.W. and J.J. Spilker, eds. (1996), **GPS Theory and Applications**, AIAA, Washington, DC.

Clock & Ephemeris Parameters (1/2)

The units specified below are as broadcast by the satellites. Users must convert the values to the appropriate units for orbit computation.

Parameter	Description
t_{oe}	Reference time for ephemeris parameters [s]
t_{oc}	Reference time for clock parameters [s]
a_{f0}, a_{f1}, a_{f2}	Polynomial clock parameters (bias [s], drift [s/s], drift rate [s/s^2])
$A^{1/2}$	Square root of the semi-major axis [$m^{1/2}$]
e	Eccentricity [dimensionless]
i_0	Inclination angle at reference time [semicircles]
Ω_0	Longitude of ascending node at weekly epoch [semicircles]
ω	Argument of perigee [semicircles]
M_0	Mean anomaly at reference time [semicircles]

Clock & Ephemeris Parameters (2/2)

The units specified below are as broadcast by the satellites. Users must convert the values to the appropriate units for orbit computation.

Parameter	Description
Δn	Mean motion difference from computed value [semicircles/s]
$d\Omega/dt$	Rate of change of right ascension [semicircles/s]
di/dt	Rate of change of inclination [semicircles/s]
C_{uc} & C_{us}	Amplitude of the sine (“s”) and cosine (“c”) harmonic correction term to the argument of latitude [rad]
C_{ic} & C_{is}	Amplitude of the sine (“s”) and cosine (“c”) harmonic correction term to the inclination angle [rad]
C_{rc} & C_{rs}	Amplitude of the sine (“s”) and cosine (“c”) harmonic correction term to the orbital radius [m]

Overview of GPS Ephemeris Calculations

- Satellite coordinates need to be computed at the time the signal left the satellite. This is known as transmit time and can be computed from

$$t_{Tx} = t_{Rx} - \frac{\rho}{c}$$

- In order to use the satellite coordinates for computing the receiver position, the satellite coordinates in the Earth Centred, Earth Fixed (ECEF) system are required first. For GPS, the ECEF is the WGS 84 frame (details to follow). However, the ECEF system is rotating with respect to inertial space, so the Earth rotation during the transit time must be taken into account. (Transit time is the time it takes for the signal to travel from the satellite to the antenna and averages about 70 ms).

GPS Constants

- The following values must be used without any approximations (i.e., no approximations or other values should be used, e.g., for π)

  - Gravitation constant ("GM") $\mu = 3.986005 \text{X} 10^{14} \text{m}^3 / \text{s}^2$
  - Mean earth rotation rate $\omega_e = 7.292115147 \text{x} 10^{-5} \text{rad/s}$
  - Pi $\pi = 3.1415926535898$
  - c (speed of light) $c = 299{,}792{,}458 \text{ m/s}$

GPS Ephemeris Calculations (1/6)

In the following, parameters in red are broadcast in the navigation message and parameters in blue are constants (see earlier slides)

- Step 1: Compute mean anomaly and mean motion

$t_k = t - t_{oe}$	Compute the relative time, where t is the time of transmission
$n_o = \sqrt{\mu / a^3} = \dfrac{2\pi}{T}$	Compute the mean motion
$n = n_o + \Delta n$	Correct the mean motion
$M_k = M_0 + n t_k$	Compute the anomaly at time of transmission

GPS Ephemeris Calculations (2/6)

- Step 2: Solve eccentric anomaly

$M_k = E_k - e \sin E_k$	Iteratively solve for eccentric anomaly

- Step 3: Compute true anomaly

$v_k = \arctan\left[\dfrac{\sqrt{1-e^2}\sin E_k}{\cos E_k - e}\right]$	Direct computation of true anomaly

- Step 4: Compute argument of latitude (of the satellite)

$\Phi_k = \omega + v_k$	Argument of latitude is the sum of argument of perigee plus true anomaly

GPS Ephemeris Calculations (3/6)

- Step 5: Compute corrections to Keplerian orbit

$\delta u_k = C_{uc}\cos(2\Phi_k) + C_{us}\sin(2\Phi_k)$	Correction to argument of latitude (along track correction)
$\delta r_k = C_{rc}\cos(2\Phi_k) + C_{rs}\sin(2\Phi_k)$	Correction to radial direction (across track correction)
$\delta i_k = C_{ic}\cos(2\Phi_k) + C_{is}\sin(2\Phi_k)$	Correction to inclination (out of plane correction)

GPS Ephemeris Calculations (4/6)

- Step 6: Compute corrected values

$u_K = \Phi_k + \delta u_k$	Corrected argument of latitude
$r_k = a\left(1 - e\cos E_k\right) + \delta r_k$	Corrected radius
$i_k = i_0 + \frac{di}{dt} t_k + \delta i_k$	Corrected inclination

- Step 7: Correct longitude of ascending node

$\Omega_k = \left(\Omega_0 - \omega_e t_{oe}\right) + \left(\frac{d\Omega}{dt} - \omega_e\right) t_k$	Correct longitude of ascending node

GPS Ephemeris Calculations (5/6)

- Step 8: Compute position at time of transmission

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = R_3(-\Omega_k) R_1(-i_k) R_3(-u_k) \begin{bmatrix} r_k \\ 0 \\ 0 \end{bmatrix}$$

- Step 9: Account for Earth rotation during propagation time

$$\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}' = R_3(\omega_e \cdot \underset{\substack{\uparrow \\ \text{Transit time} \\ \text{(propagation time)}}}{\Delta t}) \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}$$

GPS Ephemeris Calculations (6/6)

- In some applications, the satellite velocity is also required. This can be computed by taking the derivative of the previous equations. The following reference summarizes the final equations:

Remondi, B.W. (2004) **Computing satellite velocity using the broadcast ephemeris**, GPS Solutions, 8(3), pp. 181-183.

GPS Almanac

Almanac vs. Ephemeris

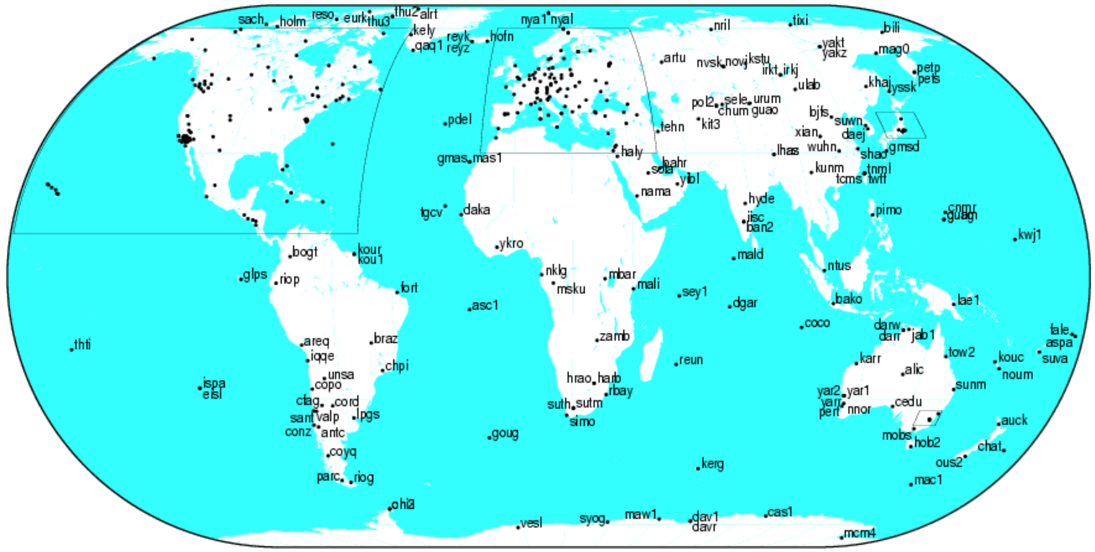
- We have already seen how the ephemeris parameters are used to compute a satellite's position. As we see later, the computed satellite positions are accurate to 1-3 m.
- In contrast to the ephemeris, the ***almanac data*** for a particular satellite consists of a subset of the ephemeris parameters. The limited number of parameters means that the ability of the almanac to predict the position of the satellite is reduced but is still useful for
  - Determining the approximate location of satellites for missing planning
  - Determining which satellites are in view (helps receiver find satellites when it is turned on)
- Each satellite broadcasts the ephemeris data for itself *only* but it broadcasts the almanac data for *all* satellites in the constellation

Almanac Parameters

Ephemeris Parameters	Almanac Parameters	Comments
t_{oe}, t_{oc}	t_{oa}	Reference time for almanac (may differ from (t_{oe} & t_{oc})
a_{f0}, a_{f1}, a_{f2}	a_{f0}, a_{f1}	Clock parameters
$A^{1/2}$, e, Ω_0, ω, M_0	$A^{1/2}$, e, Ω_0, ω, M_0	Same parameters as for ephemeris (may be different values)
i_0	δi	Change in inclination relative to 0.3 semi-circles
Δn	(zero)	Not included in almanac
$d\Omega/dt$	$d\Omega/dt$	Same parameter (may be different value)
di/dt	(zero)	Not included in almanac
C_{uc} & C_{us} C_{ic} & C_{is} C_{rc} & C_{rs}	(zero)	Not included in almanac

Other Sources of GNSS Satellite Orbit and Clock Information

Post-Mission Orbits from IGS



GMT Dec 29 17:25:19 2004

IGS network (see http://igscb.jpl.nasa.gov/)

- IGS = International GPS Service
  - Consists of 383 stations worldwide (as of December 30, 2004)
- Post-mission orbits (available after a few days) may be required for higher accuracy single point or differential applications
- Data from global tracking networks are combined to give more accurate post-mission orbits since little (or no) prediction is involved
- Other agencies provide similar data services

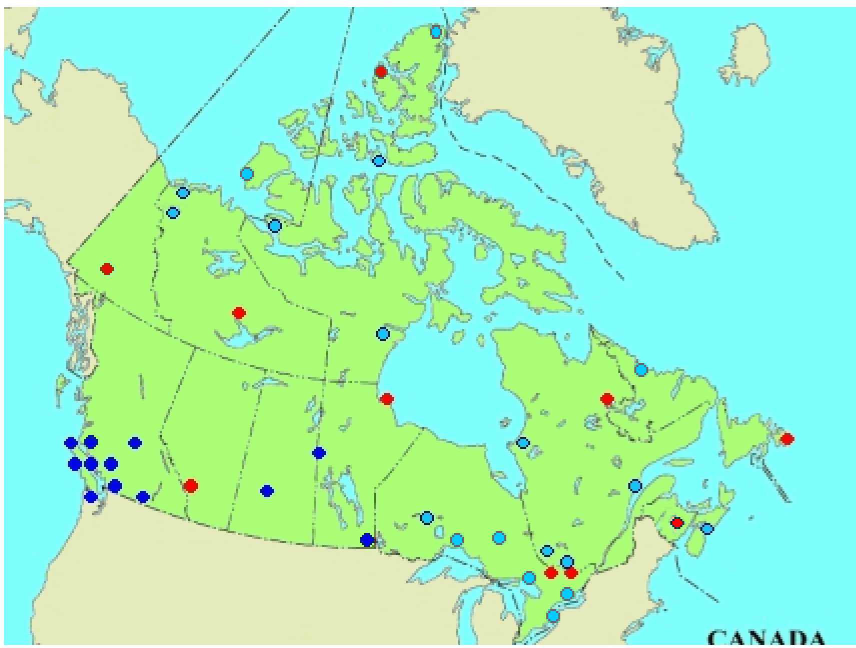
IGS GPS Products and Accuracy

IGS Product Table [GPS Broadcast values included for comparison]						
		Accuracy	**Latency**	**Updates**	**Sample Interval**	**Archive locations**
GPS Satellite Ephemerides/ Satellite & Station Clocks						
Broadcast	orbits	~100 cm	real time	--	daily	CDDIS(US-MD) SOPAC(US-CA) IGN(FR)
	Sat. clocks	~5 ns				
Ultra-Rapid (predicted half)	orbits	~5 cm	real time	four times daily	15 min	CDDIS(US-MD) SOPAC(US-CA) IGN(FR) IGS CB(US-CA)
	Sat. clocks	~3 ns				
Ultra-Rapid (observed half)	orbits	~3 cm	3-9 hours	four times daily	15 min	CDDIS(US-MD) SOPAC(US-CA) IGN(FR) IGS CB(US-CA)
	Sat. clocks	~0.15 ns				
Rapid	orbits	~2.5 cm	17-41 hours	daily	15 min	CDDIS(US-MD) SOPAC(US-CA) IGN(FR) IGS CB(US-CA)
	Sat. & Stn. clocks	0.075 ns			5 min	
Final	orbits	~2.5 cm	12-18 days	weekly	15 min	CDDIS(US-MD) SOPAC(US-CA) IGN(FR) IGS CB(US-CA)
	Sat. & Stn. clocks	0.075 ns			5 min	
Note 1: IGS accuracy limits, except for predicted orbits, based on comparisons with independent laser ranging results. The precision is better. Note 2: The accuracy of all clocks is expressed relative to the IGS timescale, which is linearly aligned to GPS time in one-day segments.						

http://igscb.jpl.nasa.gov/components/prods.html

Canadian Active Control System (CACS)

- Developed by NRCan – Geodetic Survey Division
- Functions
  - Fiducial sites (contributors to the IGS)
  - Precise orbits in ITRF and 30-s satellite clock corrections (< 1 ns)
  - GPS performance & integrity through continuous tracking
  - Supports the Canadian DGPS (CDGPS) service



Permanent CACS tracking sites

Western Canada Deformation array

Regional Active Control Sites

http://www.geod.nrcan.gc.ca/index_e/products_e/activeNetwork_e/acp_e.html#drop