6

# Chapter 6

## GPS Biases and Errors

**Effect of orbital errors on DGPS. Satellite and receiver clock errors. Code and carrier phase noise. Multipath occurrence, estimation, detection, correlation, reduction. Antenna phase centre. Inter-channel biases. Effect of station coordinate errors.**

**Atmospheric structure and characteristics. Ionospheric effects. Sunspot number and ionospheric activity. L1/L2 Corrections. Code/carrier phase ionospheric divergence. Tropospheric effects and modelling**

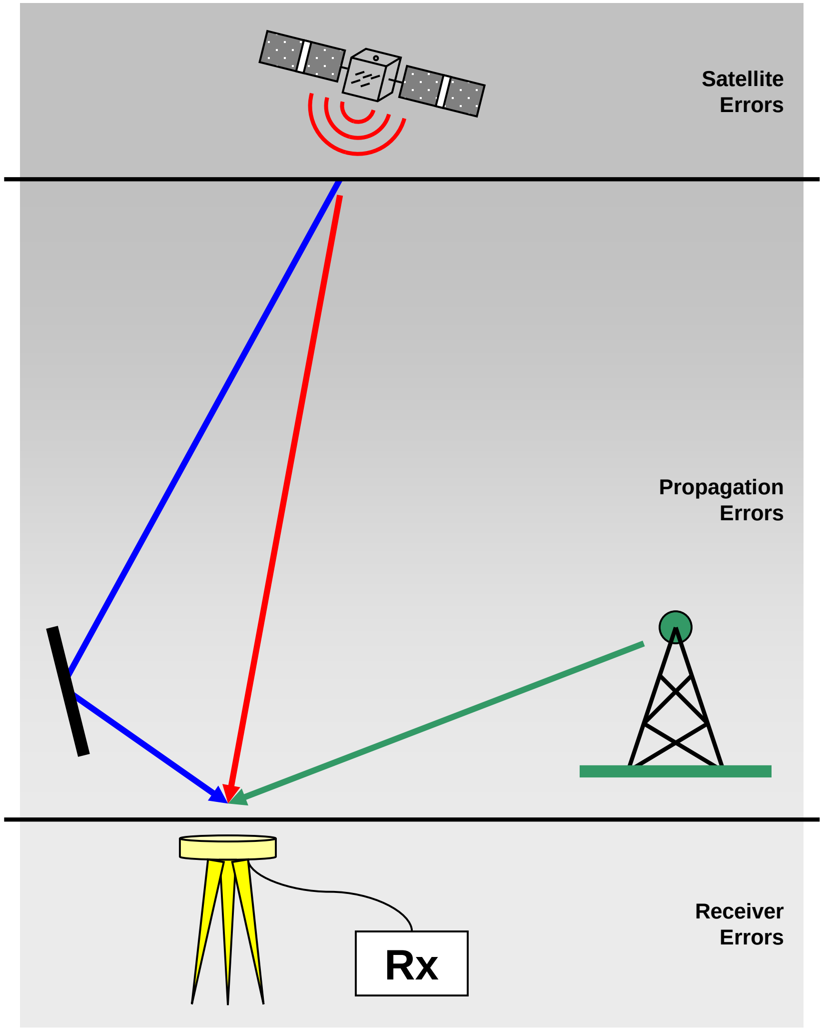
# GPS Errors

| | |
|---|---|
| Introduction | Phase Centre Variations |
| Orbital Error | Receiver Clock Error |
| Satellite Clock Error | Inter-channel Biases |
| Satellite Group Delay | Noise |
| Multipath | Processing Errors |
| Interference | Linear Combinations |
| | |

*With contributions from Dr. Mark Petovello*

# Sources of Error

- Three basic sources
  - Satellite-based errors
    - Orbit errors
    - Satellite clocks
    - Group delay
  - Propagation errors
    - Ionosphere
    - Troposphere
    - Multipath
    - Interference
  - Receiver-based errors
    - Antenna errors
    - Receiver clock
    - Inter-channel biases
    - Noise
    - Timing/Tracking errors



# Error Characteristics

- The two most important characteristics of any given error are
  - Magnitude of the error
    - Undifferenced (e.g. single point)
    - Differenced (e.g. double difference)
  - Variability of the error
    - Temporal correlation
    - Spatial correlation (relates to magnitude of differential error)
- Also of importance is the manner in which the error sources can be observed
  - Some applications see the error as the “signal” to be detected (instead of position)
    - Ionospheric modeling, water vapour estimation, etc.
  - Necessary for error separation and/or estimation
    - Compute magnitude of errors

# *Analysis Procedure*

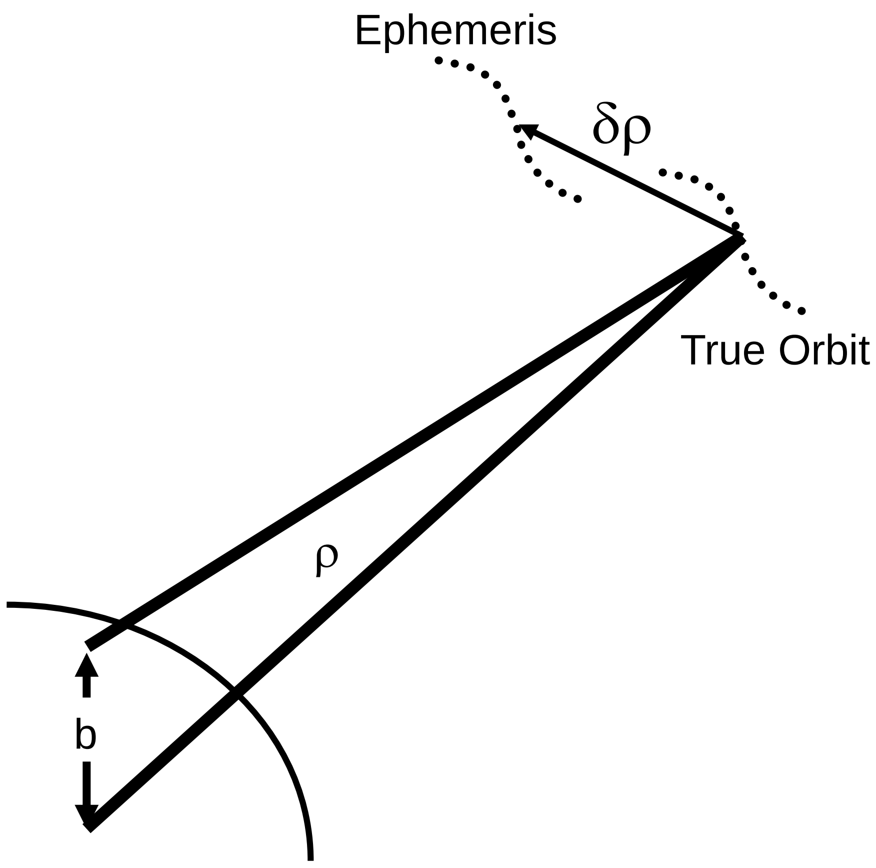
- If and when applicable, the following analysis tools will be used to characterize an error source
  - Theoretical induction
    - Using known physical properties, the errors can be characterized
  - Direct observation
    - Some errors can be directly “observed” using independent information, special linear combinations, or special processing techniques
  - Auto-correlation (ACF)
    - The auto-correlation function gives a direct estimate of the temporal variance and correlation of an error
    - The power spectral density (PSD) can also be used (since it is the Fourier transform of the autocorrelation function)
  - Sample data results
    - Some errors are difficult to “observe” except in certain data sets

## Orbital Error

### *Source of Error*

- Ephemeris information in navigation data is ***predicted*** from previous measurements of satellite motion and knowledge of the Earth's gravity field
    - Maximum of three satellite uploads per day means that the prediction is at least eight-hours long
- Combined affect of the above is erroneous satellite position (and velocity) information

# Differential Orbital Error




- Differential effect is purely geometrical
  - Changes as a function of orbital error components, satellite azimuth and elevation, and baseline length
- Typical error is expressed as:

$$\frac{\delta b}{b} = \frac{\delta \rho}{\rho}$$

For $\delta\rho \approx 5$ m ($1\sigma$) and

$\rho \approx 20{,}000$ km,

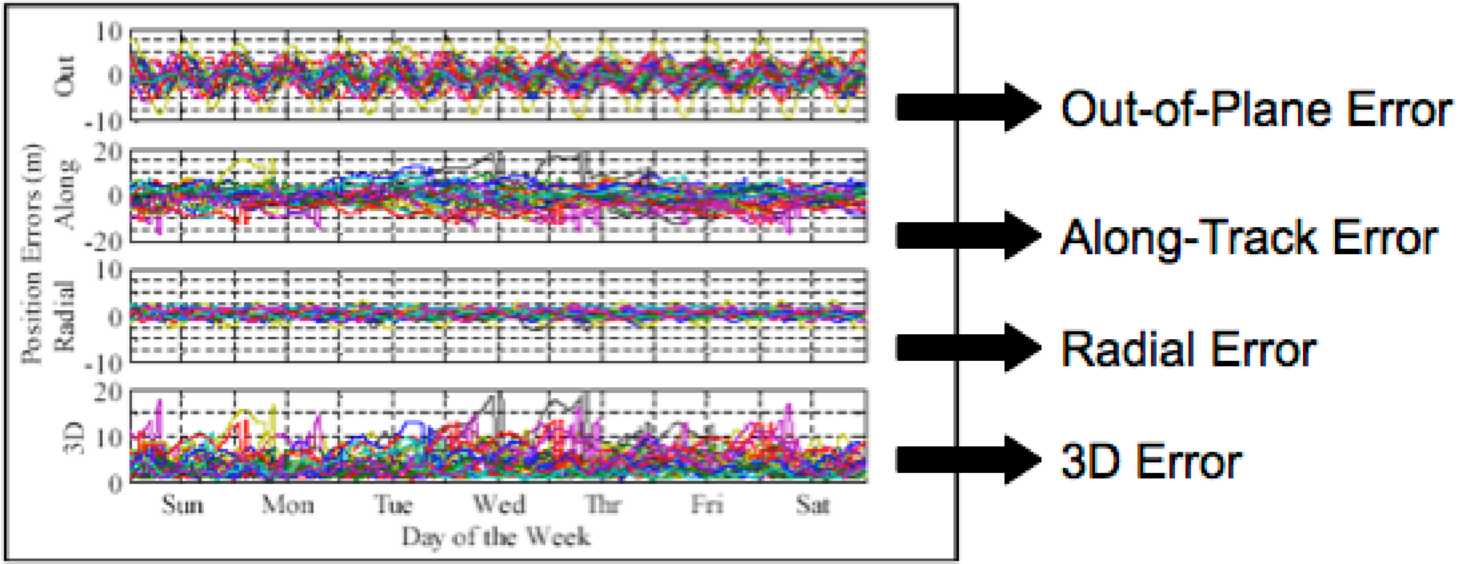
the error on the inter - rx

distance b is about 0.25 ppm

# Orbital Error

## *Broadcast Orbit Performance (c 2000)*

- Differences between broadcast and precise orbits (all satellites)

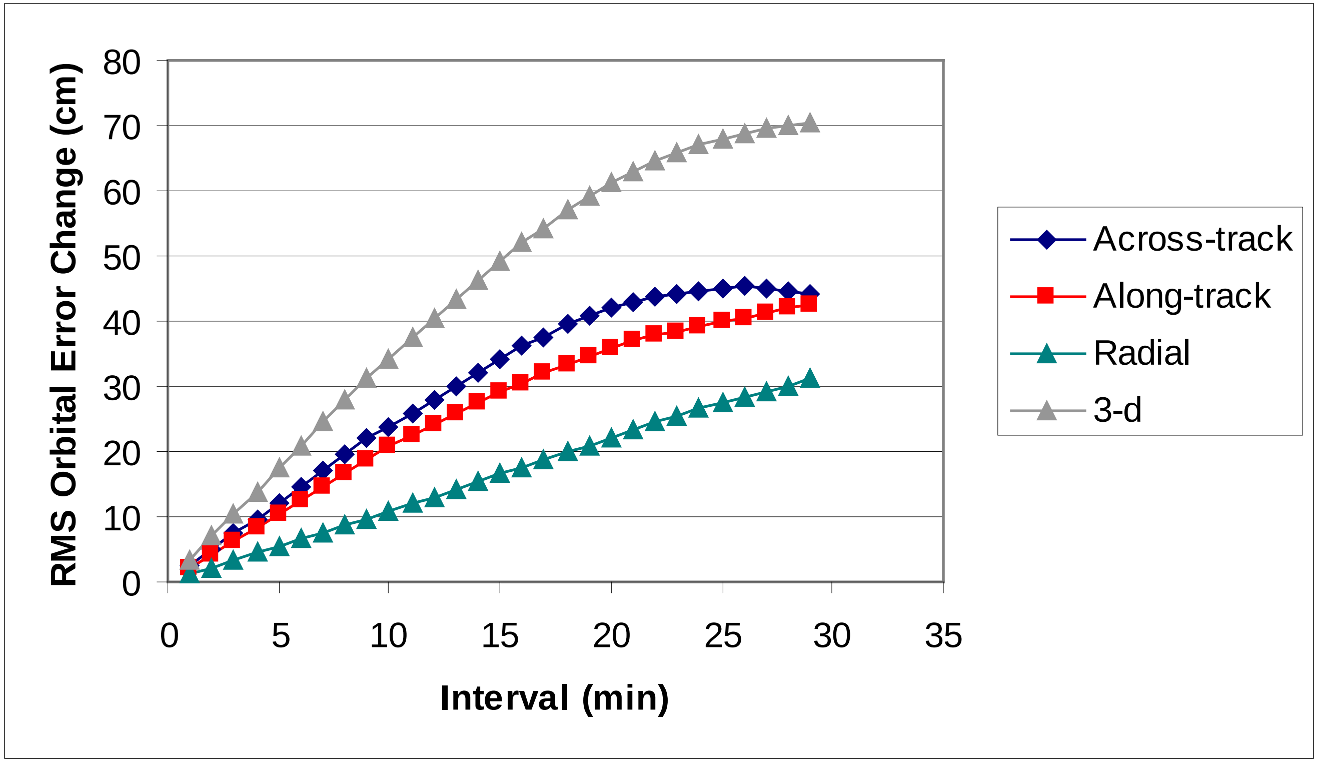



| Error | Percentile | | | | |
|---|---|---|---|---|---|
| | 50th | 68th | 95th | 99th | Max |
| 3D | 3.5 m | 4.7 m | 9.2 m | 12.7 m | 24.2 m |


- Ryan, S.J. (2002), **Augmentation of DGPS for Marine Navigation**, PhD Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 2016

# *Broadcast Orbit Performance (c 2001)*

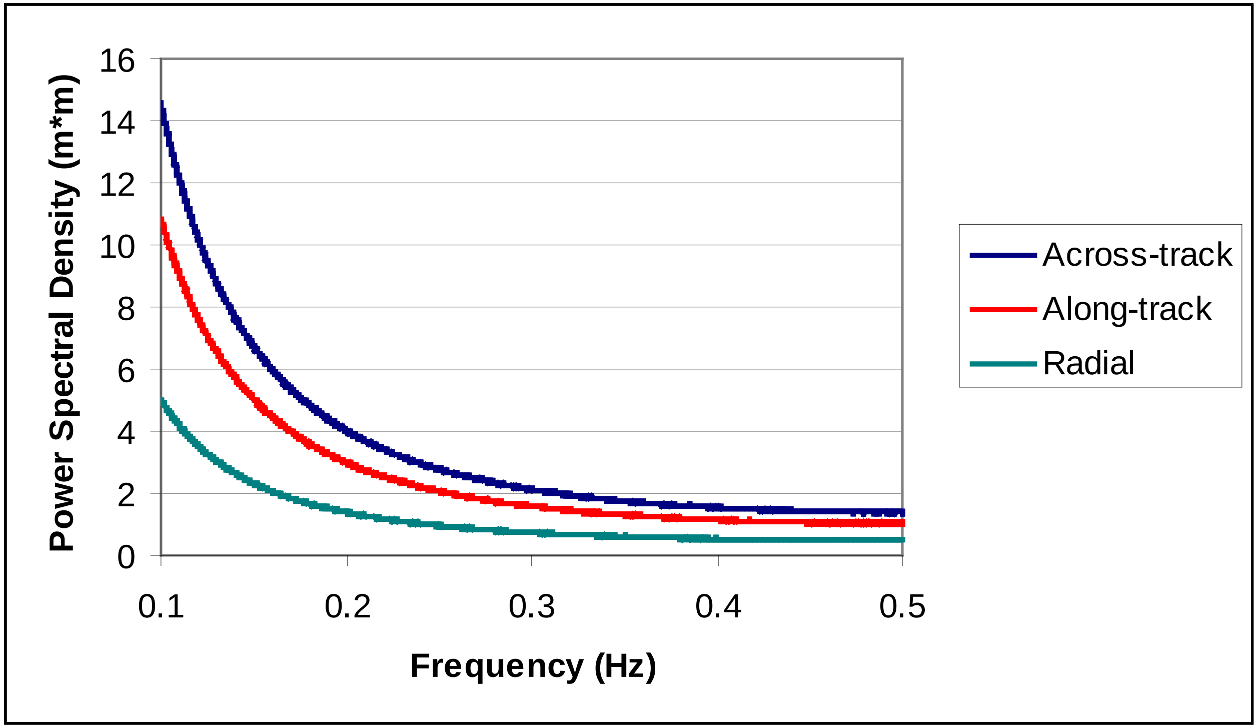
- Compute the RMS ***change*** in orbital error as a function of time and take the average across all satellites




Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164 (available at: http://geomatics.ucalgary.ca/links/GradTheses.html)

# Broadcast Orbit Performance

- Power spectral density of orbital error
  - Radial error is most constant but is the easiest to observe due to the measurement geometry




Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164 (available at: http://geomatics.ucalgary.ca/links/GradTheses.html)

# Other Sources of Orbit Information

| Description | Orbital Accuracy | Latency |
|---|---|---|
| Broadcast | ~100 cm | real-time |
| Ultra-Rapid (IGS) (Predicted Half) | ~5 cm | real-time |
| Ultra-Rapid (IGS) (Observed Half) | ~3 cm | 4x/day (0300, 0900,1500 2100 UTC) |
| Rapid (IGS) | ~ 2.5 cm | ~17 hours after observation (UTC) day |
| Final Precise (IGS) | ~2.5 cm | 12 -18 days (available weekly) |

http://igscb.jpl.nasa.gov/   (Updated July,2013)

# Source of Error



- Error source is similar to that of the orbital error
  - Clock behaviour is ***predicted*** from previous measurements of the satellite clock error
  - Broadcast clock error model assume quadratic error growth

$$dt_{SV} = a_0 + a_1\left(t - t_{oe}\right) + a_2\left(t - t_{oe}\right)^2$$

  - The coefficients $a_0$, $a_1$, $a_2$ are broadcast in the navigation message
    - Referenced to time $t_{oc}$
- Quality of onboard oscillators will have a direct impact on the accuracy of the prediction

# *Other Sources of Satellite Clock Information (IGS - 2013)*

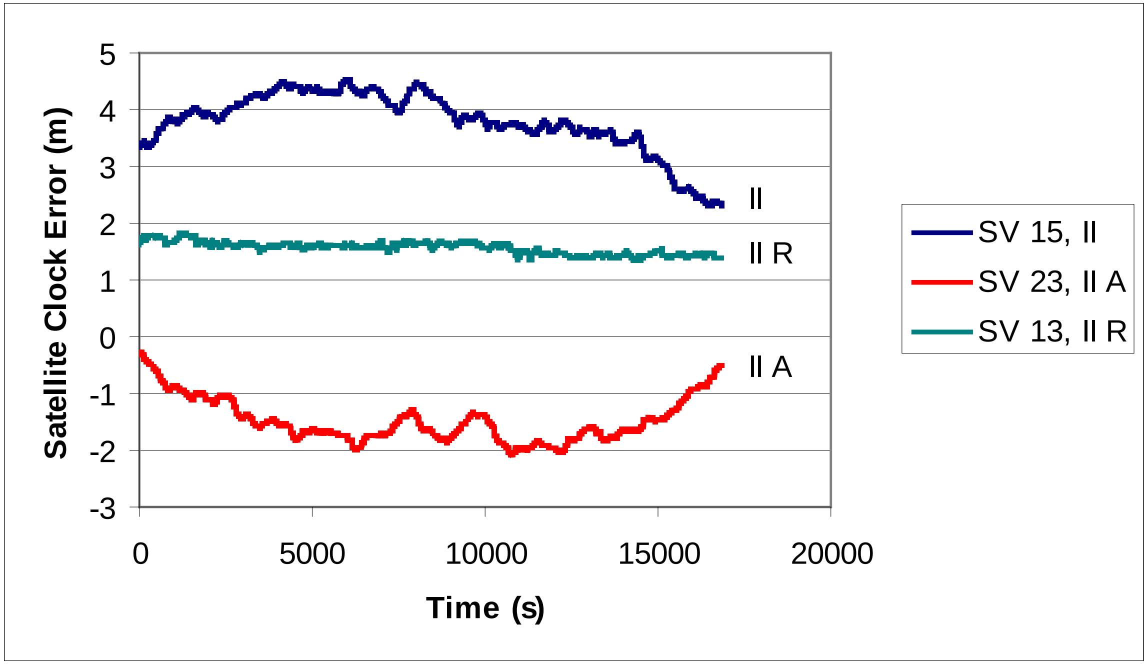
| Description | Accuracy (RMS) | Latency |
|---|---|---|
| Broadcast | ~5 ns | real-time |
| Ultra-Rapid (IGS) (Predicted Half) | ~3 ns | real-time |
| Ultra-Rapid (IGS) (Observed Half) | ~150 ps | 4x/day (0300, 0900,1500 2100 UTC) |
| Rapid (IGS) | ~75 ps | ~17 hours after observation (UTC) day |
| Final Precise (IGS) | ~75 ps | 12 -18 days (available weekly) |

http://igscb.jpl.nasa.gov/   (Updated July,2013)

# Broadcast Clock Model Performance



- Comparison of broadcast model and precise clock errors
  - Sample of one satellite for each satellite block
    - Block IIR shows very little drift (slope) and appears to be more or less random with a constant bias
    - Block II and IIA have more systematic errors

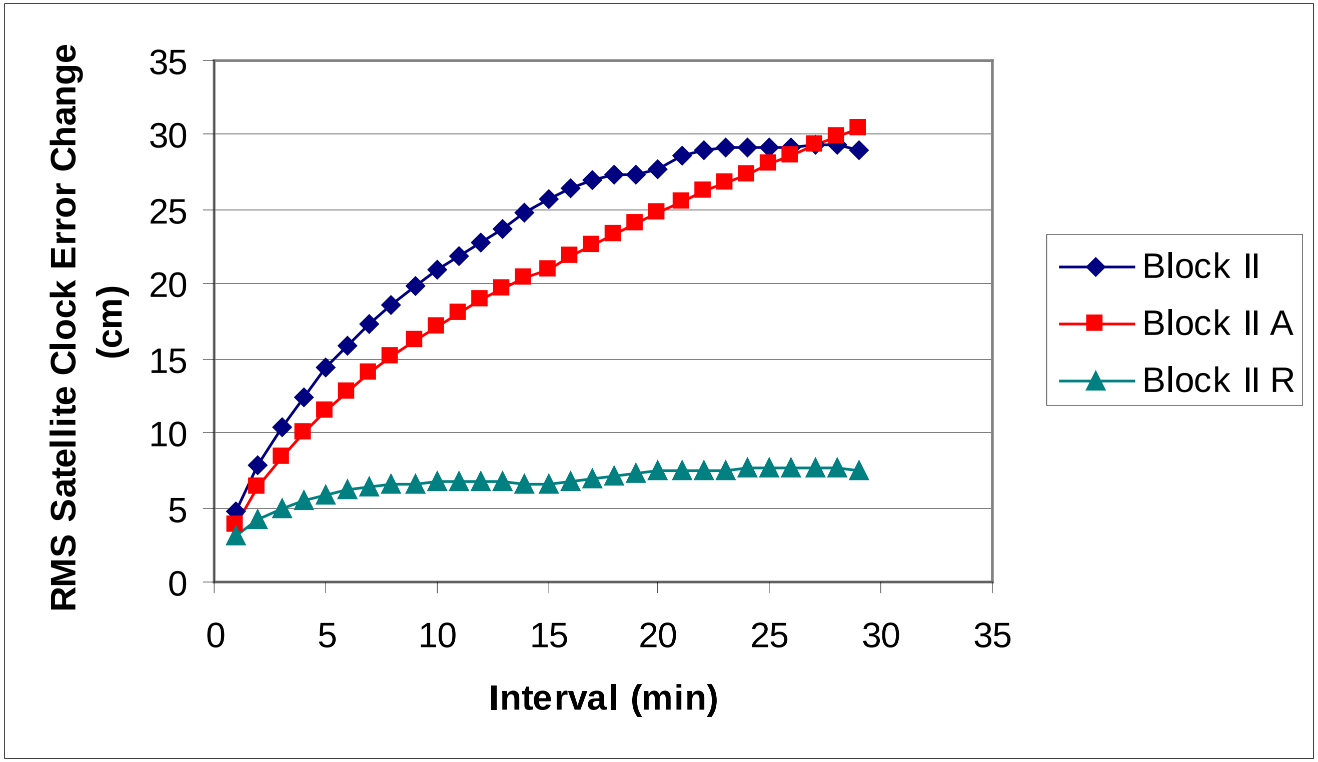



Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164

# Broadcast Clock Model Performance



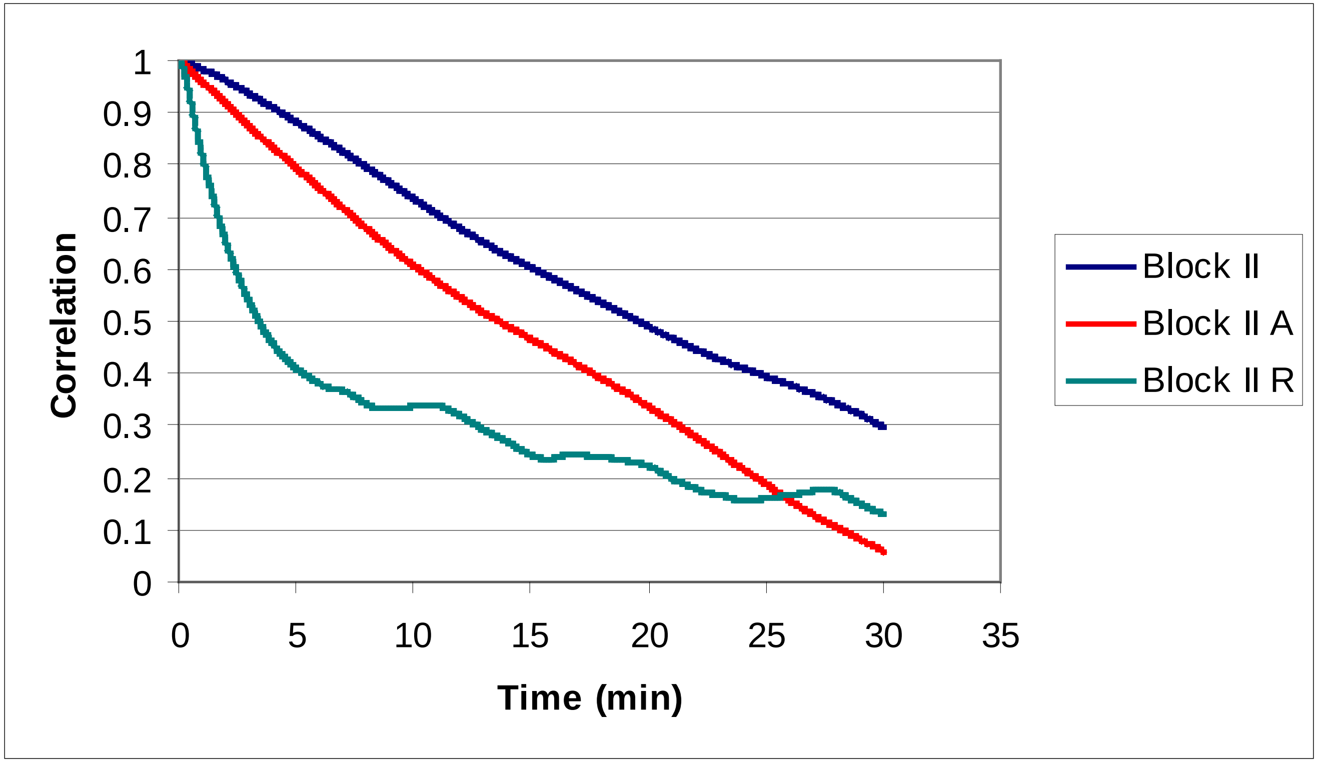
- Compute the RMS ***change*** in clock error as a function of time and take the average across all satellites
  - Clear difference in performance clock newer and older satellites




Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164

# Broadcast Clock Model Performance

- Compute the ACF for the satellite clock errors
  - Block II and IIA satellites have more systematic variations than do the Block IIR satellites and therefore show a slower decorrelation




Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164

# *Relativistic Effects*

- Special and general relativity effects must be taken into account
- This is done in two ways
  - The satellites' clock frequency is offset from the 10.23 MHz default to partially account for relativistic effects
  - The remaining effect is caused by the satellites' elliptical orbit which causes them to travel through different levels of gravitational potential, and causes their velocity to change. It can be compensated by adding the following term to the polynomial clock correction

$$\Delta t_{Rel} = -4.442807633 \times 10^{-10} \cdot e\sqrt{a}\,\sin(E_k)$$

- a is the semi-major axis of the orbit
- e is the eccentricity of the orbit
- $E_k$ is the eccentric anomaly of the orbit

Leva, J.L., M.U. de Haag, K. Van Dyke (2006) **Performance of Standalone GPS**, Chapter 7 of Understanding GPS Principles and Applications, 2nd Edition, Kaplan & Hegarty, Editors

## Satellite Group Delay

# *Total Group Delay (TGD)*

- Different delays are experienced in the satellites on the L1 and L2 channels
  - L1 and L2 signals are therefore not necessarily synchronized as they leave the satellite
- Single-receiver applications should take this effect into account
  - Broadcast message contains a scaled version of group delay [4]

$$T_{GD} = \frac{t_{L1} - t_{L2}}{1 - \Gamma} \qquad \Gamma = \frac{f_{L1}^2}{f_{L2}^2}$$

  - Correction is applied as follows, but is unnecessary if using the ionosphere-free linear combination

$$\begin{aligned} dt_{L1} &= dt_{SV} - T_{GD} \\ dt_{L2} &= dt_{SV} - \Gamma T_{GD} \end{aligned}$$

Polynomial correction

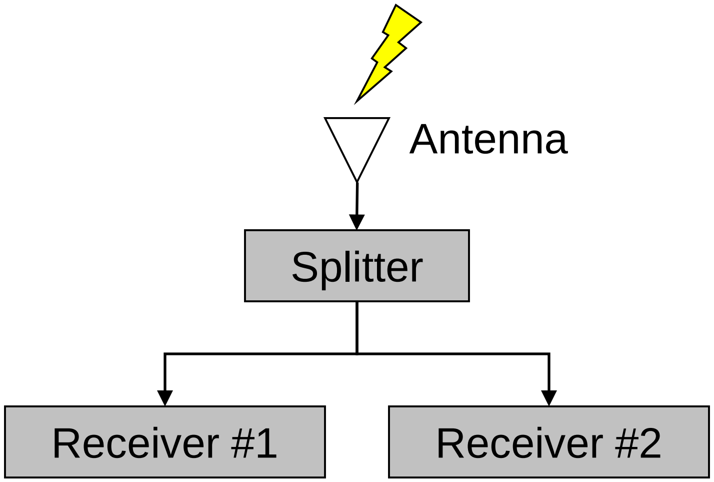
Spilker, J.J., Jr. (1996) **GPS Navigation Data**, B.W. Parkinson and J.J. Spilker, Jr. eds., Global Positioning System: Theory and Applications, Volume I, Chapter 4, American Institute of Aeronautics and Astronautics, Inc., Washington, DC, USA

# Quantifying Noise (1 of 2)

- Noise is typically estimated using a zero-baseline test, wherein two receivers are connected to a single antenna
  - By forming the double difference, *all* systematic errors are removed (including multipath)
  - Only the stochastic errors (i.e. noise), the receiver-satellite geometry, and the ambiguity term (for carrier phase only) remain
  - Method can be used on pseudorange, Doppler and carrier phase measurements

$$\nabla\Delta P = \nabla\Delta\rho + \varepsilon_{\nabla\Delta P}$$

$$\nabla\Delta\Phi = \nabla\Delta\rho + \lambda\nabla\Delta N + \varepsilon_{\nabla\Delta\Phi}$$



# Quantifying Noise (2 of 2)

- Zero-baseline test con't
  - By removing the geometric ranges using the known receiver position (and the integer ambiguity in the carrier phase case), only the (double difference) noise term remains

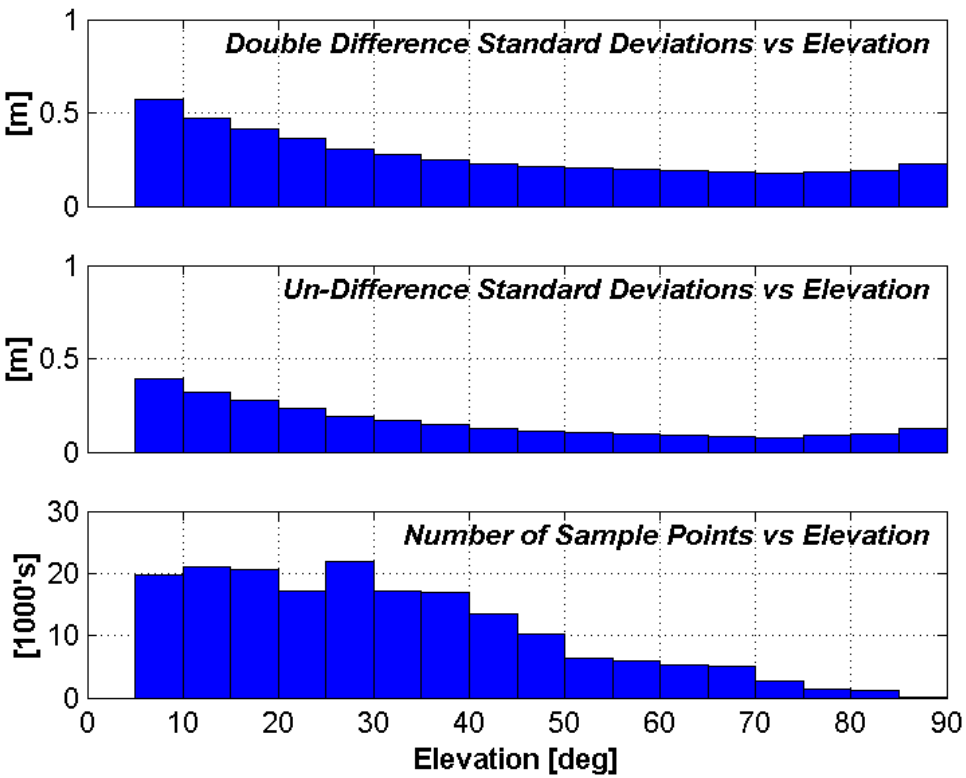
$$\varepsilon_{\nabla\Delta P} = \nabla\Delta P - \nabla\Delta\rho$$
$$\varepsilon_{\nabla\Delta\Phi} = \nabla\Delta\Phi - \nabla\Delta\rho - \lambda\nabla\Delta N$$

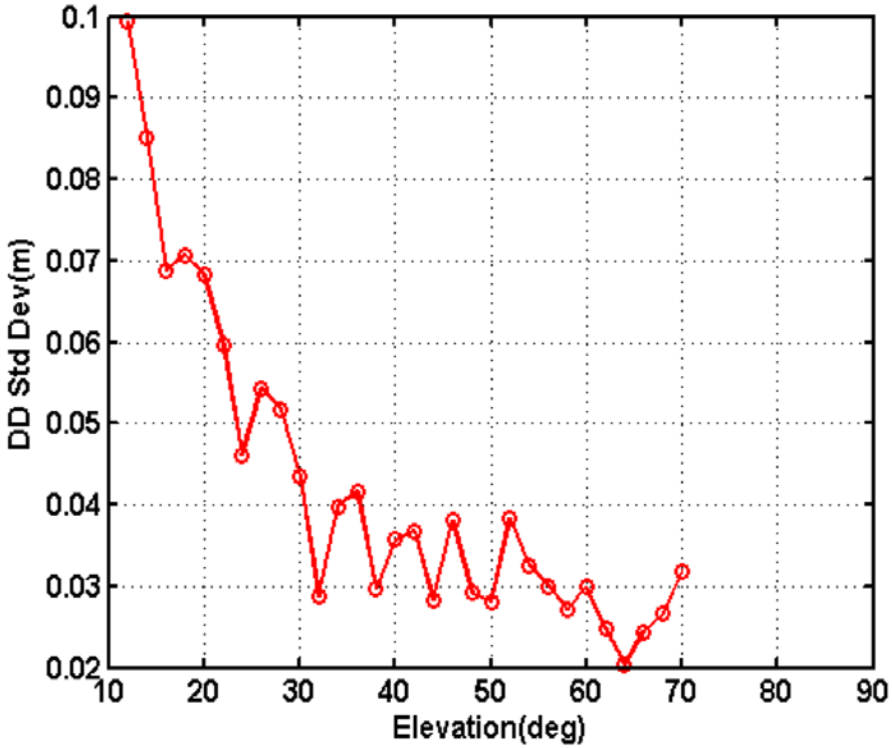
  - Compute the variance of the double difference noise terms
  - To determine the undifferenced noise variance, divide the above result by **four**
    - Since noise is signal strength dependent, a more sophisticated method of computing the undifferenced noise can be used

## Receiver Noise

# C/A Code Noise Example

- NovAtel OEM4 (left) and OEMV1 (right) DD C/A Code Noise

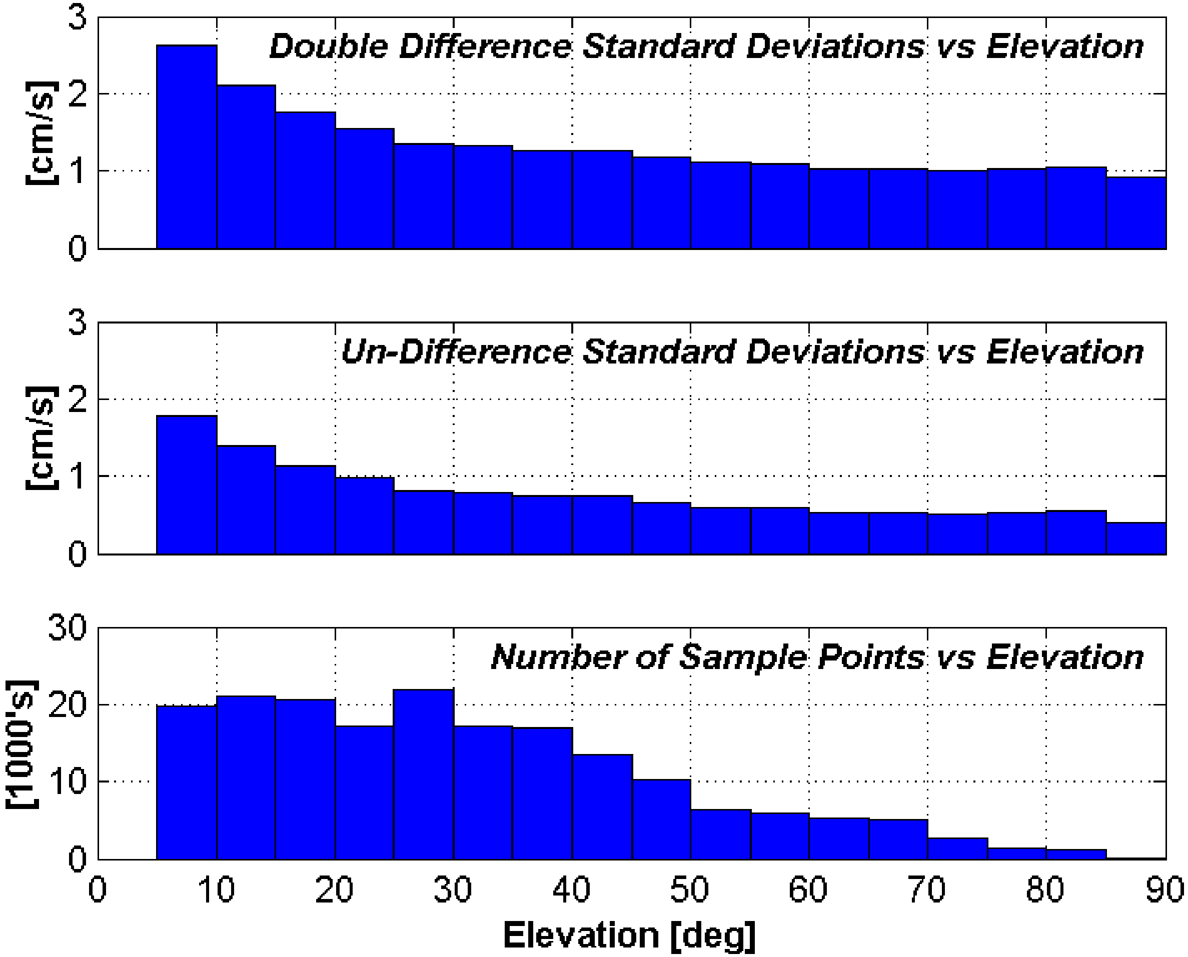


# L1 Doppler Noise Example



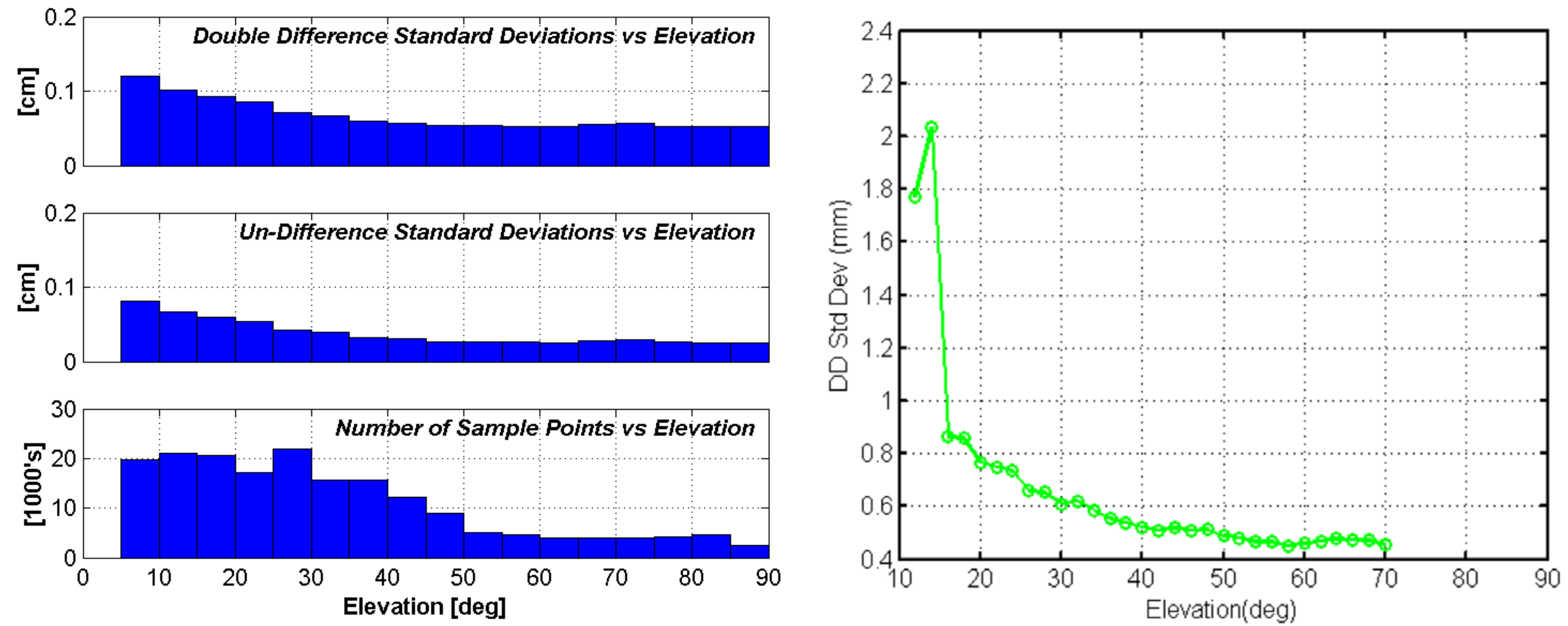
- NovAtel OEM4 L1 Doppler Noise



# L1 Carrier Phase Noise Example



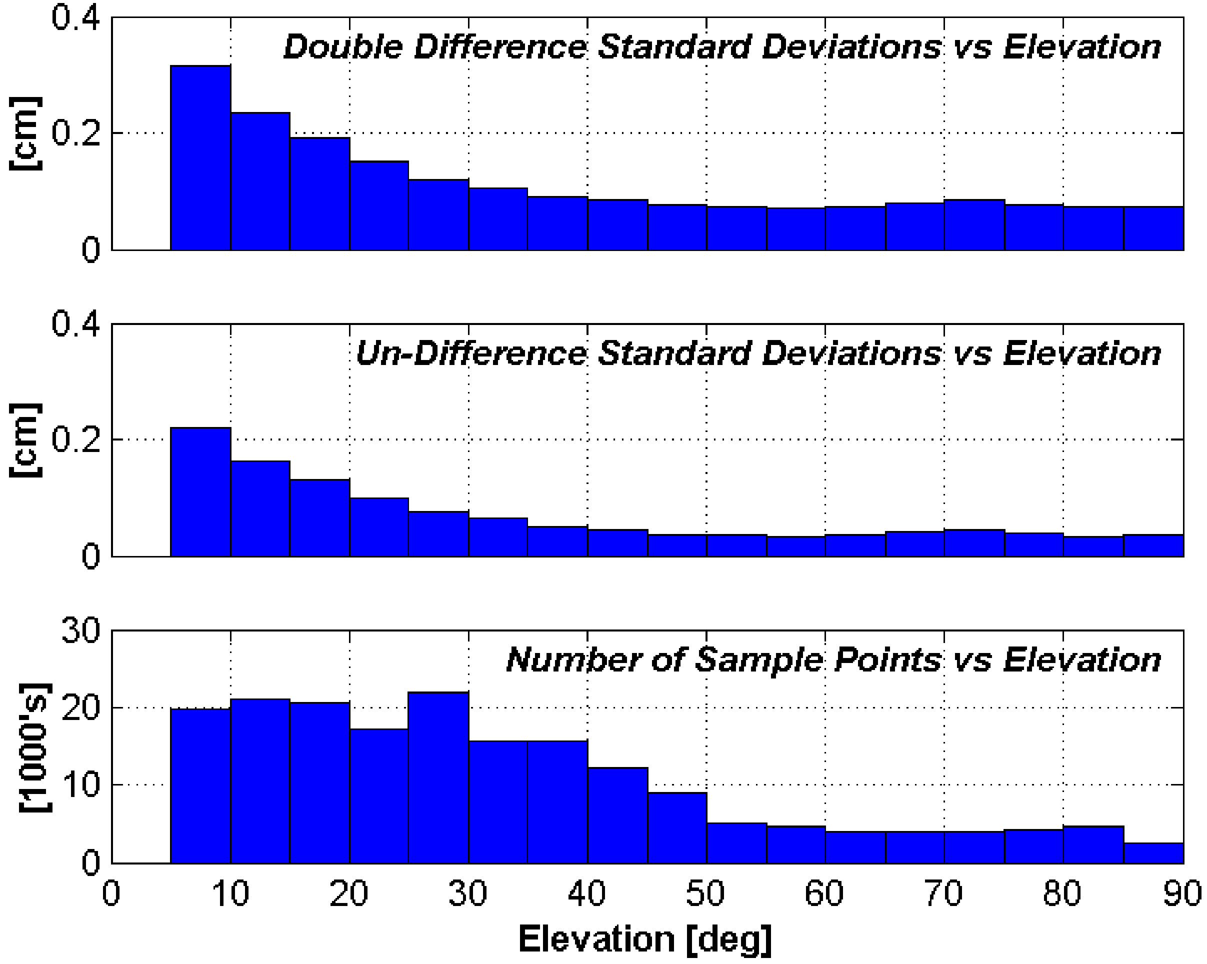
- NovAtel OEM4 (left) and OEMV1 (right) L1 Carrier Phase Noise



# L2 Semicodeless Carrier Phase Noise Example

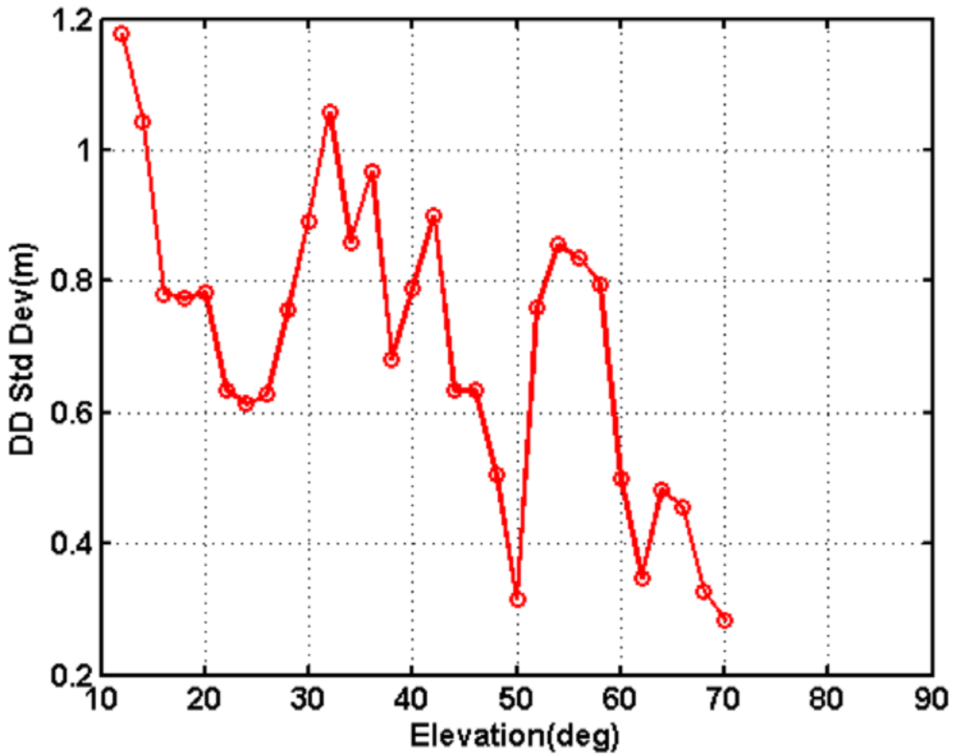


- NovAtel OEM4 L2 Carrier Phase Noise



# U-blox ANTARIS, Static Mode

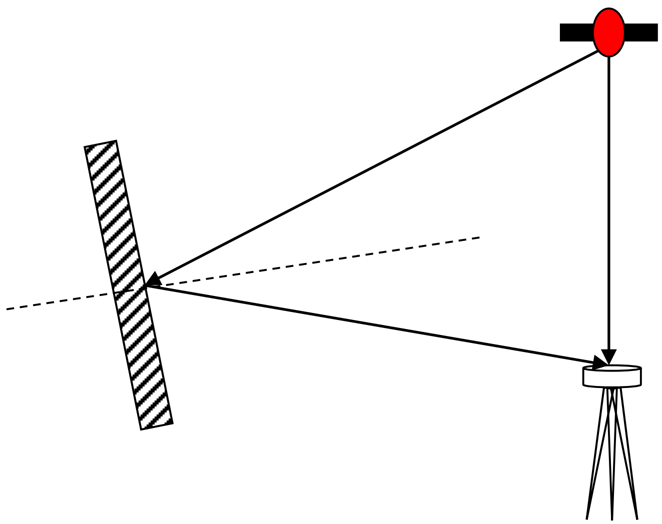
- Using a NovAtel 702 antenna
- Noise varies with elevation due to antenna vertical response pattern



## Multipath

### *General Concept*

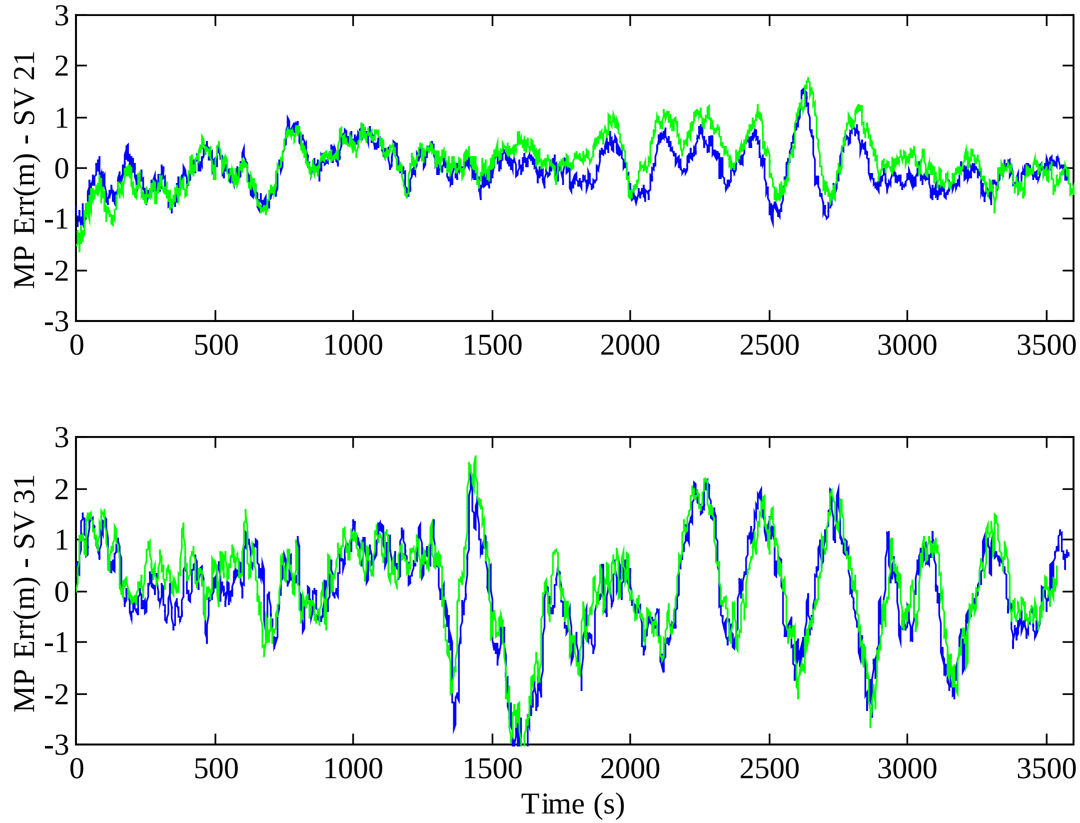
- Multipath occurs when the signal reaches the receive antenna via multiple paths
- Specular Multipath
  - Parallel incident rays remain parallel after reflection (smooth surfaces)
- Diffuse Multipath
  - Due to a rough surface, the incident wave is reflected in many directions
  - Causes loss of field strength in direction of antenna



# Multipath Correlation

- Decorrelates spatially very rapidly
- Correlated from day-to-day for a given location
  - If reflecting geometry is constant over time, the periodic nature of the satellite orbits will cause the reflection geometry to repeat every sidereal day
  - Need to offset day-to-day results by 3m 56s to account for difference between solar time and sidereal time

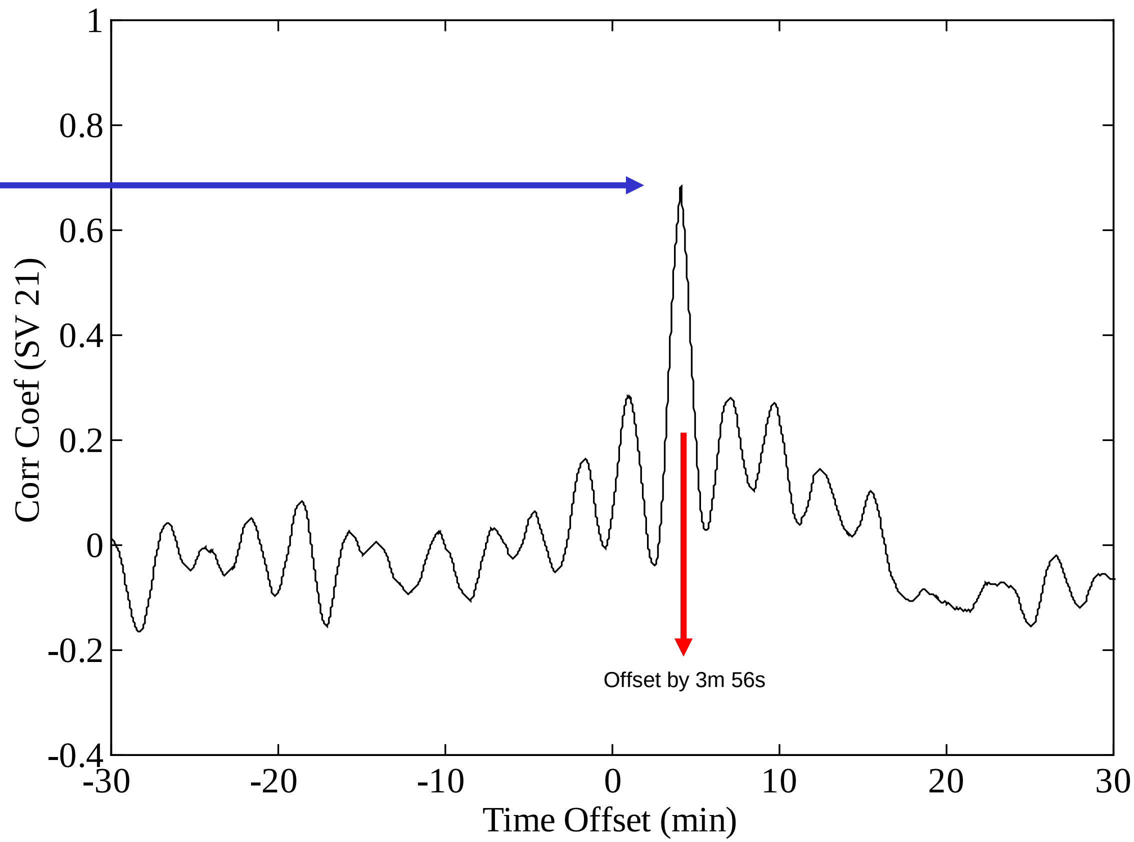
**Day-to-Day Multipath For PRNs 21 and 31**
Day 1 in blue (dark)
Day 2 in green (light)



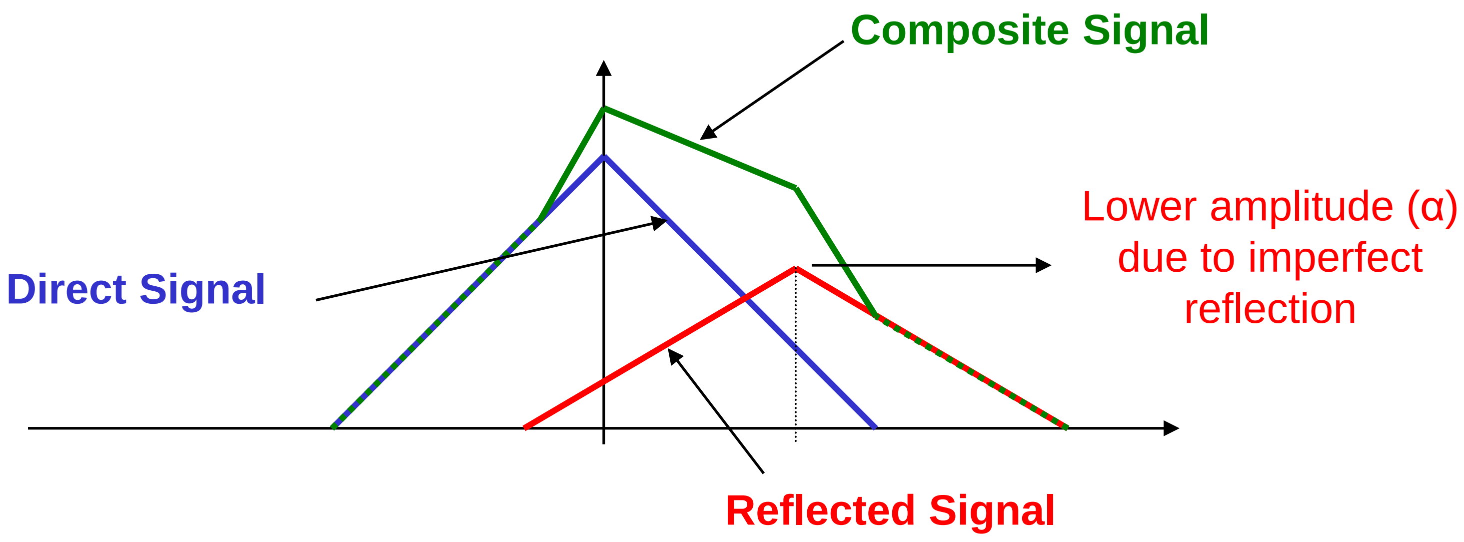
# Multipath Correlation

## Correlation Coefficient Day-to-Day Multipath for PRN 21

Not perfectly correlated due to noise



# Pseudorange Multipath Characteristics 1




- Not reduced through differential processing (possibly amplified)
- Can reach magnitudes of about 0.5 of a code chip (150 m)
  - For a wide correlator receiver, lower values for narrow correlator receivers

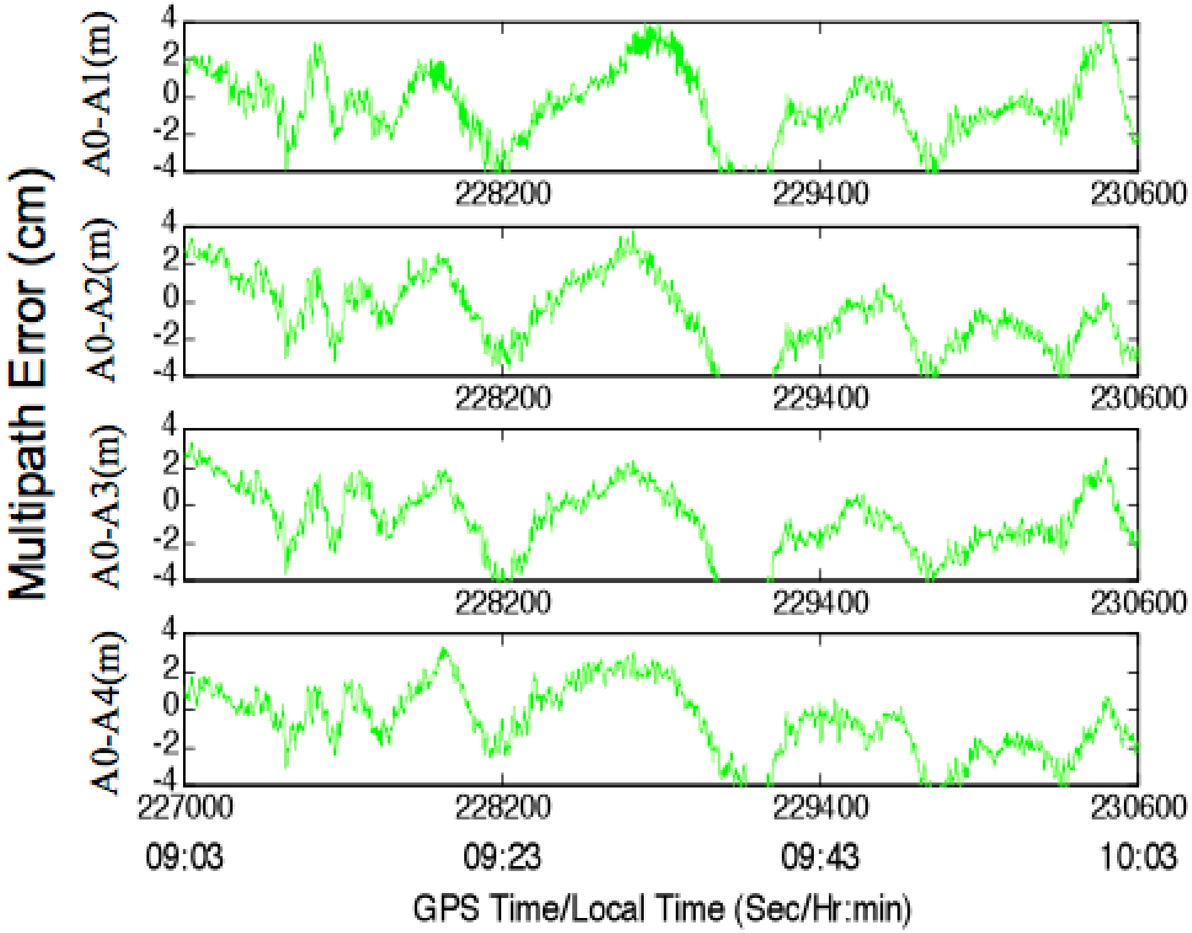
# *Pseudorange Multipath Characteristics 2*



- Completely dependent on reflecting geometry
- Generally causes <u>a systematic error</u> in the measurements
  - Close to sinusoidal in nature with periods of up to several minutes (for static receiver and reflectors)
- Can cause the measured range to be larger <u>or smaller</u>, depending on the phase of the reflected signal(s)
  - <u>Pseudorange multipath error is not zero mean</u>
- Generally no effect for reflectors more than 150 m away under LOS conditions
- NLOS conditions (e.g. indoor and urban canyons): unlimited
- Limiting factor for many LOS DGPS applications

## Relative Pseudorange Multipath For Closely Spaced Antennas

L1 C/A pseudorange single difference on SV09 with 6 antennas about 6 cm apart

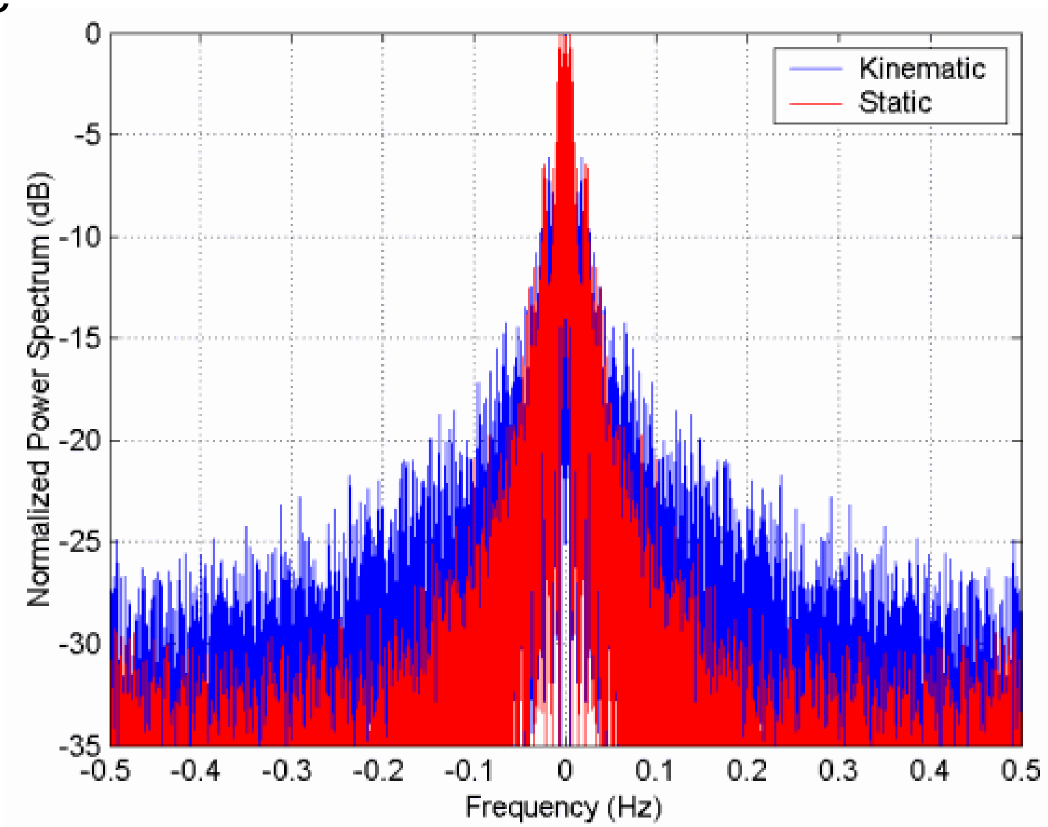



J. Ray (2000) Mitigation of GPS Code and Carrier Phase Multipath Effects Using a Multi-Antenna System. PhD Thesis, Report 20136, Dept of Geomatics Engineering, Univ. of Calgary.

# *Variability of Pseudorange Multipath*



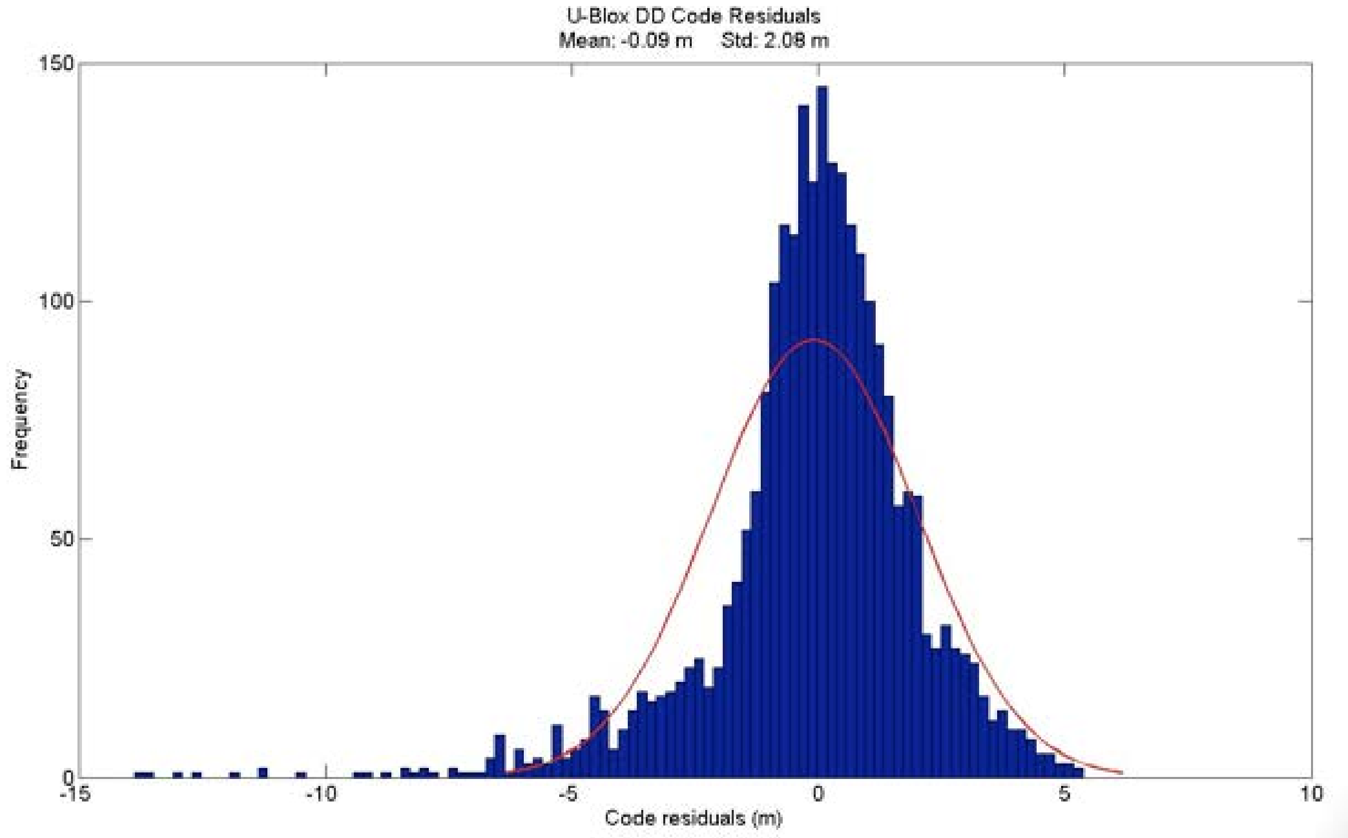
- Power spectrum of pseudorange multipath in static and kinematic (marine) environments
  - The more rapid change in reflection geometry for the kinematic case means that the error has higher frequency effects
  - In a static environment, the only parameter affecting the reflection geometry is satellite motion




Lachapelle G. (2003), O. Julien, G. MacGougan, M. E. Cannon, S. Ryan. **Ship GPS Multipath Detection Experiments**. Proceedings of the Institute of Navigation 59th Annual Meeting, Session C2, June 23-25, Albuquerque, NM.

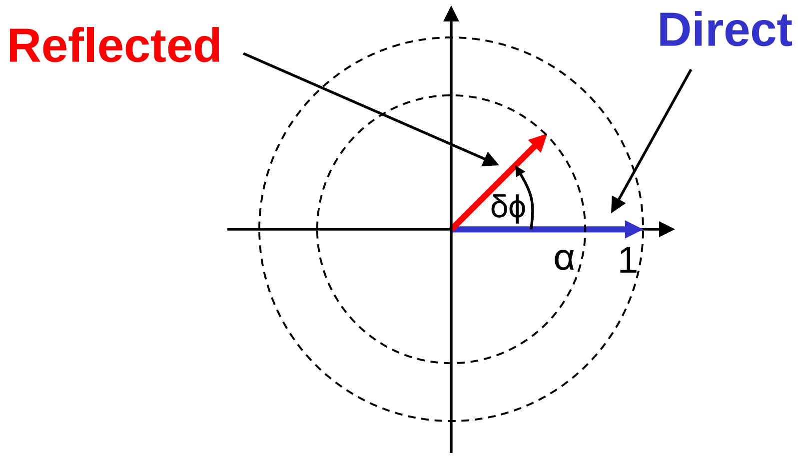
# *Example: u-blox Code Noise + Multipath*

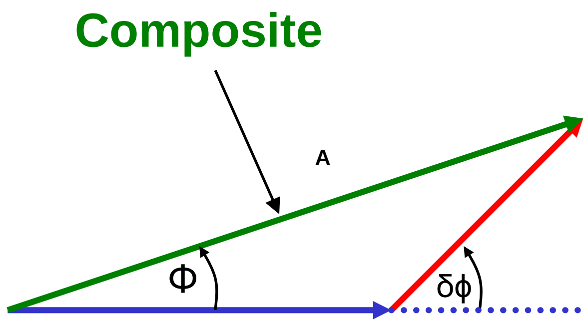
- U-blox ANTARIS, vehicular mode, patch antenna mounted 20 cm above vehicle roof
- Effect of noise and multipath on single code measurement: 1 m at 1 sigma level



# Carrier Phase Multipath Characteristics 1






**Multipath Amplitude**

$$A = \sqrt{1 + \alpha^2 + 2\alpha\cos\delta\phi}$$

**Phase Multipath Error**

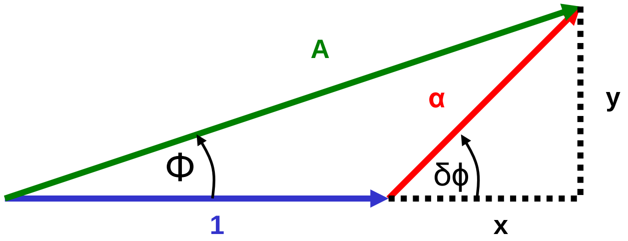
$$\Phi = \tan^{-1}\left(\frac{\alpha \cdot \sin\delta\phi}{1 + \alpha \cdot \cos\delta\phi}\right)$$

For α = 1 (perfect reflector), the phase error reaches a maximum value of 90º (¼ cycle) when δΦ is 180º but then the composite signal has zero amplitude! Therefore the maximum multipath error is ***theoretically*** ¼ cycle.

# Carrier Phase Multipath Characteristics 2

1) **With reference to the diagram, from the right most triangle on the right the following relations are found:**

$$y = \alpha \cdot \sin \delta\phi \qquad x = \alpha \cdot \cos \delta\phi$$




2) **The multipath error (i.e., the phase of the composite signal) is then given by:**

$$\phi = \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$= \tan^{-1}\left(\frac{\alpha \sin\delta\phi}{1+\alpha \sin\delta\phi}\right)$$

**Note that assuming α = 1, the value of δϕ that maximizes ϕ is 180° (prove this using standard calculus techniques). The resulting maximum phase multipath error is 90° (1/4 cycle).**

# Carrier Phase Multipath Characteristics 3

**3) The amplitude of the composite signal is then given by:**

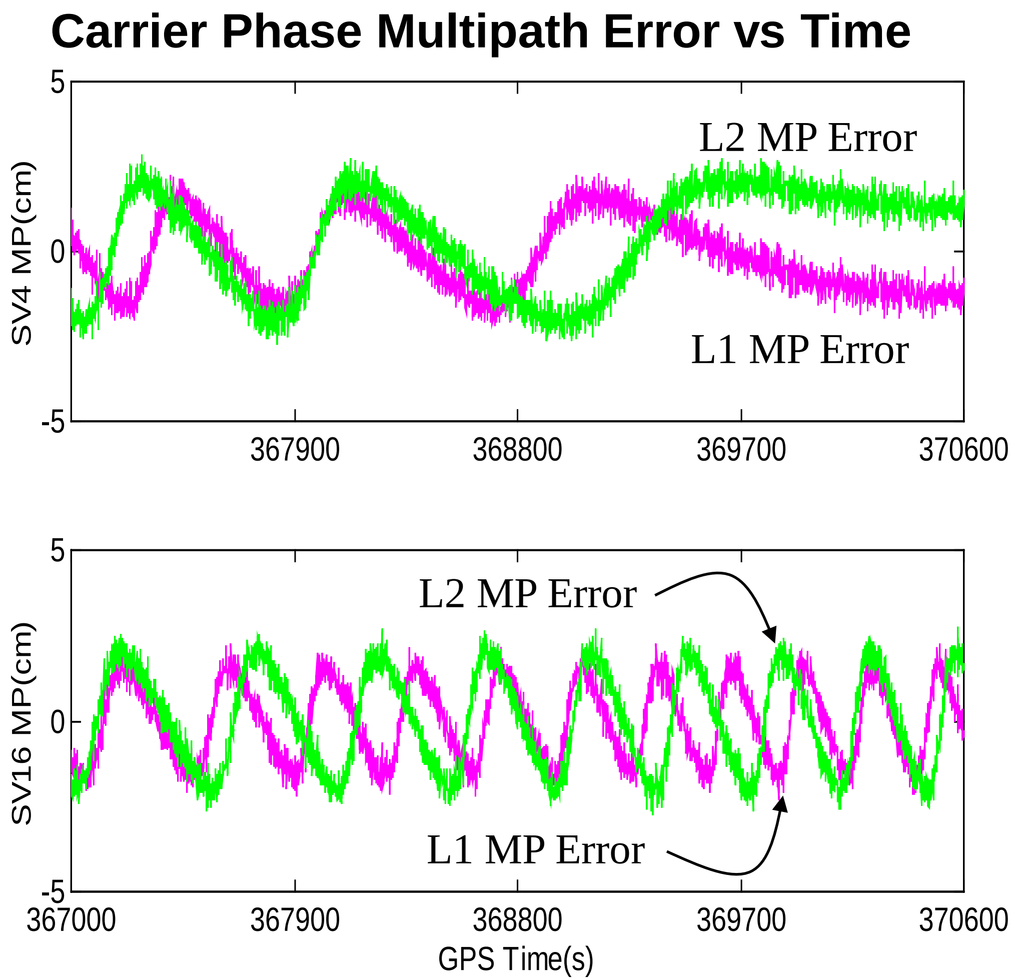
$$
\begin{aligned}
A &= \sqrt{(1+x)^2 + y^2} \\
&= \sqrt{(1+\alpha \cdot \cos\delta\phi)^2 + (\alpha \cdot \sin\delta\phi)^2} \\
&= \sqrt{\left(1+2\alpha\cos\delta\phi + \alpha^2 \cdot \cos^2\delta\phi\right) + \left(\alpha^2 \cdot \sin^2\delta\phi\right)} \\
&= \sqrt{1+\alpha^2 + 2\alpha\cos\delta\phi}
\end{aligned}
$$

Note that if α is unity and δϕ is 180° then A is zero! In other words, the maximum multipath error is never actually 90°, since for this to happen, there would be no signal left to track! As such a multipath error of 90° is purely theoretical. Practically, maximum values are closer to about 20% of a cycle.

# Carrier Phase Multipath Characteristics 4



- Errors in L1 & L2 carriers due to multiple reflectors for SV 4 & 16
- L1 multipath has higher frequency
- L1 and L2 have the same phase error amplitude
- Highly dependent on antenna-reflector and line-of-sight vectors




J. Ray (2000) Mitigation of GPS Code and Carrier Phase Multipath Effects Using a Multi-Antenna System. PhD Thesis, Report 20136, Dept of Geomatics Engineering, Univ. of Calgary.

# Carrier Phase Multipath Characteristics 5



- Not reduced through differential processing (highly localized)
- Can reach magnitudes of 0.25λ (nearly 5 cm at L1)
  - Measured phase can be too large or too small
  - Carrier phase multipath is zero mean
- Generally causes a systematic error in the measurements
  - Sinusoidal in nature
- Amplitude inversely proportional to distance (closer reflecting objects cause higher multipath)
- Affect the accuracy of positioning and attitude determination
- Can significantly affect the ability to resolve integer ambiguities

# *What To Do About Multipath 1*

- Site selection
  - Minimum obstructions
  - Roofs are particularly poor
  - Watch for surrounding medium (pooled water makes an excellent reflector)
- Detection
  - Analyze measurement residuals
  - Day-to-day multipath correlation
- Receiver and Antenna Design
  - Some receiver hardware/firmware is better able to mitigate multipath effects (mostly for code)
    - Narrow correlator, Strobe correlator, PAC correlator
    - NovAtel's Vision Correlator technology
    - Use of chokering (Static applications/Reference stations)

# *Narrow vs Wide Correlator and 2D Chokering*

- Field test example

| Antenna | PRN | Narrow Correlator | | | Wide Correlator | | |
|---|---|---|---|---|---|---|---|
| | | Max (m) | Min (m) | RMS (m) | Max (m) | Min (m) | RMS (m) |
| No Chokering | 17 | 2.2 | -2.4 | 0.7 | 5.2 | -5.2 | 1.6 |
| | 23 | 0.9 | -1.4 | 0.4 | 2.9 | -2.5 | 0.8 |
| With 2D Chokering | 17 | 0.7 | -0.6 | 0.2 | 2.8 | -2.1 | 0.8 |
| | 23 | 0.5 | -0.7 | 0.1 | 1.2 | -1.7 | 0.4 |

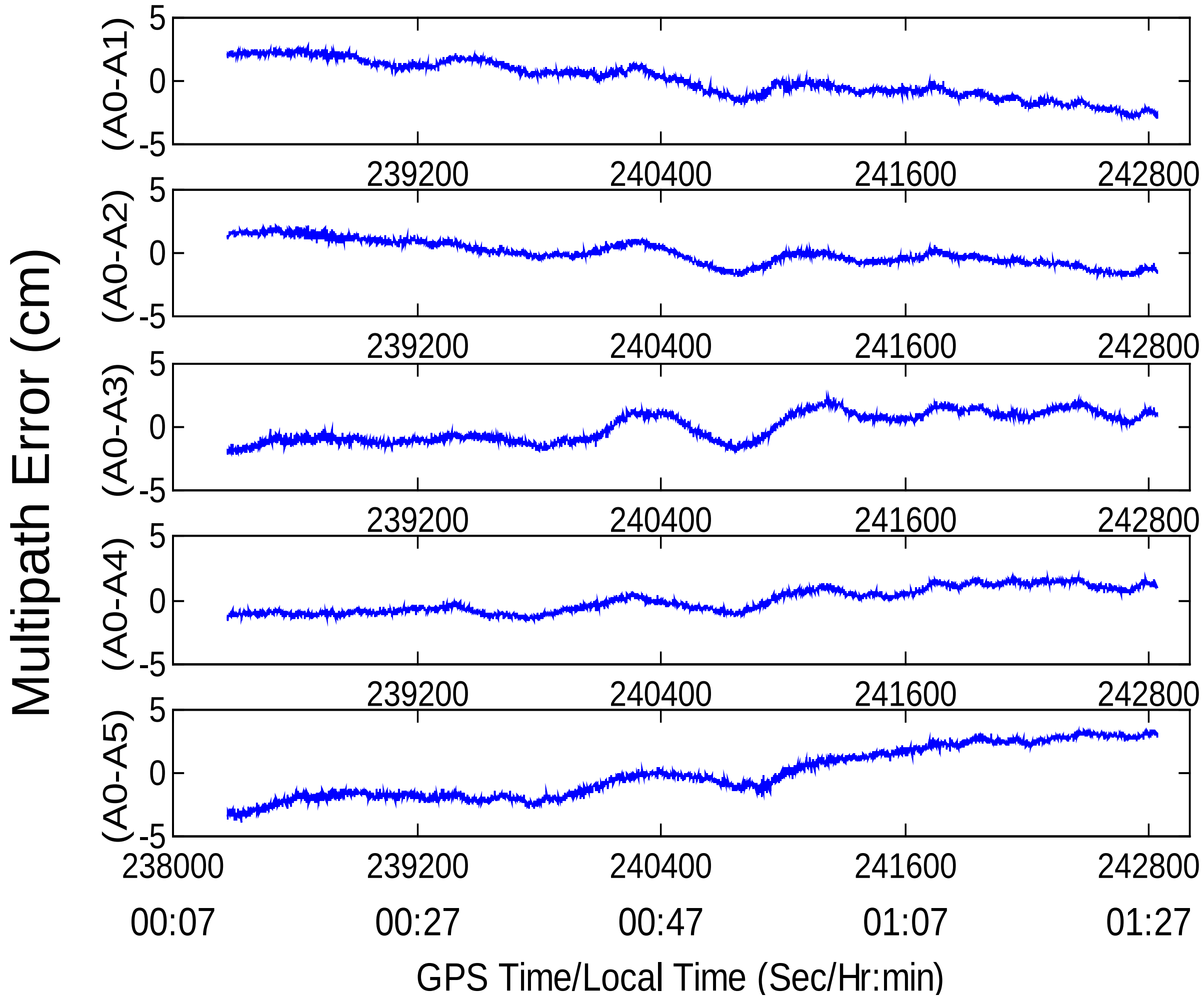
# What To Do About Multipath 2



- Modeling
    - Try to model the surrounding environment in terms of multipath reflectors
- Estimation and Elimination (J. Ray 2000, PhD thesis, UofCalgary)
    - Uses the concept that multipath phase at an antenna is related to the phase at another antenna by
        - Their relative geometry
        - Direction of the multipath signal
    - Use multiple antennas (at least 4) to estimate multipath over time

**Carrier Phase Multipath Reduction**

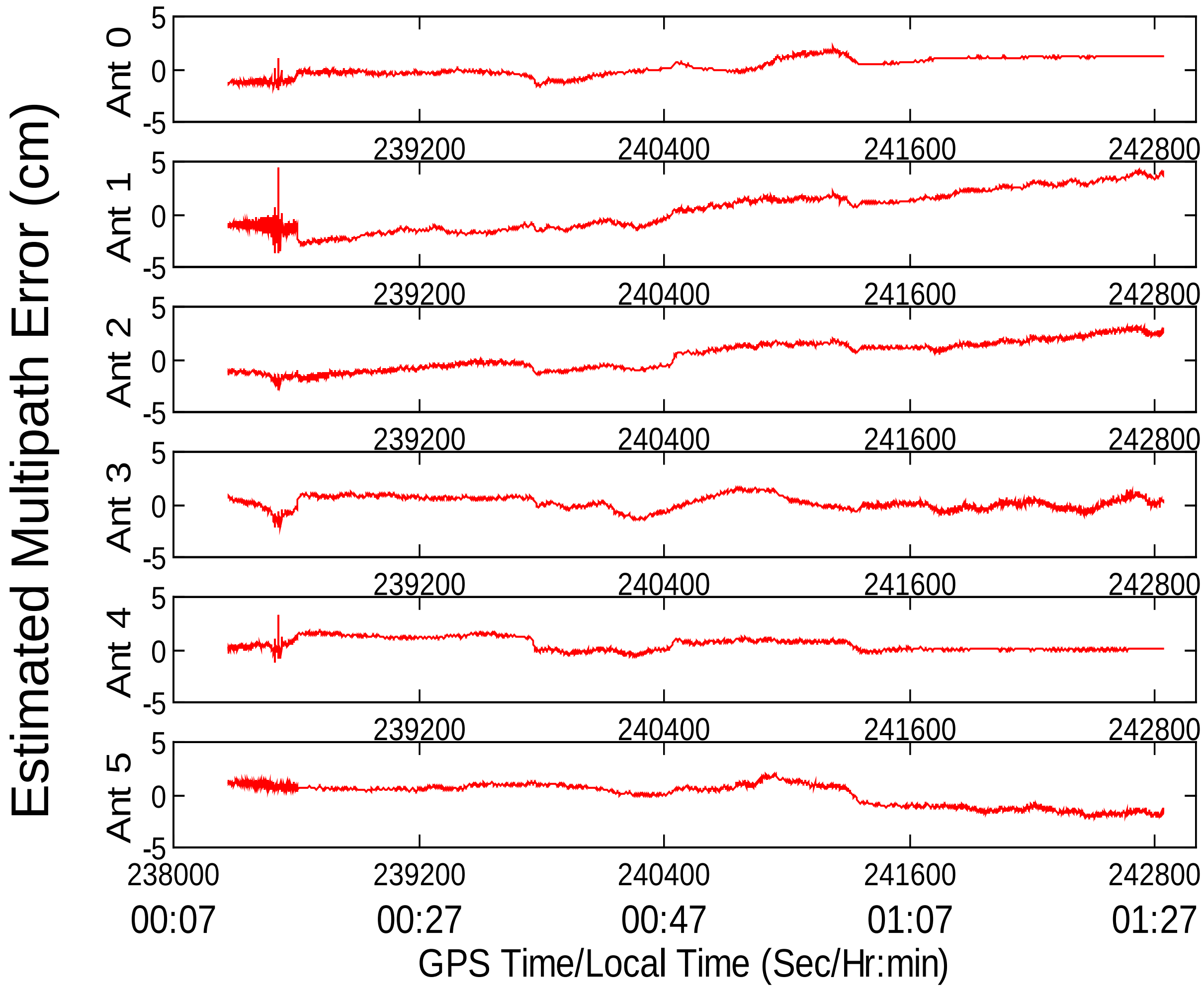
## *Relative Carrier Phase Multipath For Closely Spaced Antennas – Example (1/3)*




J. Ray (2000) Mitigation of GPS Code and Carrier Phase Multipath Effects Using a Multi-Antenna System. PhD Thesis, Report 20136, Dept of Geomatics Engineering, Univ. of Calgary.

## *Estimated Carrier Phase Multipath Error Example (2/3)*

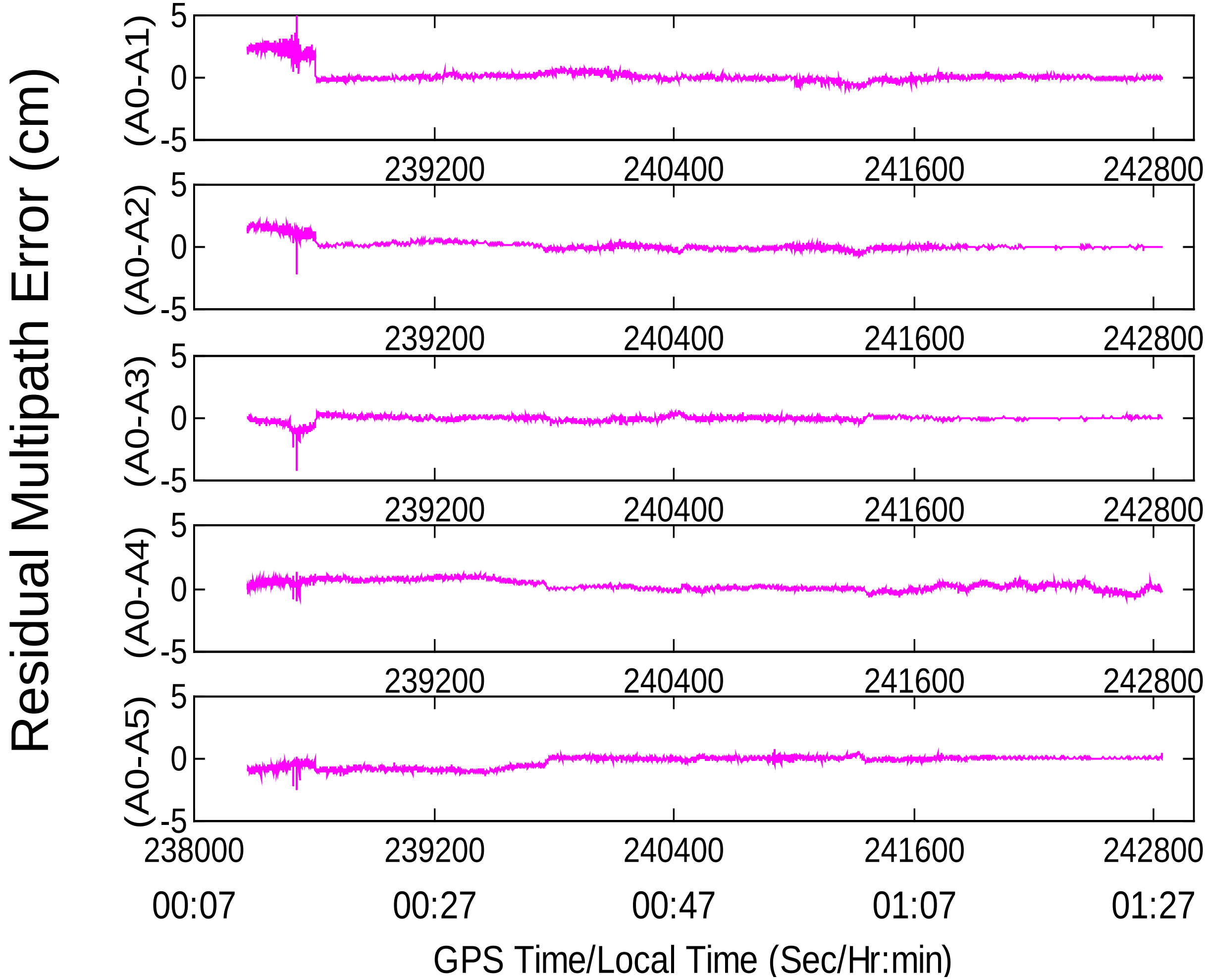
- Estimation and Elimination con't



# Residual Multipath Errors (3/3)

- Estimation and Elimination con't




**76.2% Improvement**

# *Measuring Multipath (and Noise)*

- Pseudorange multipath – Method One
  - Use two known stations separated by one code chip (to decorrelate multipath) and conduct a DGPS test
  - Difference the double difference code and phase measurements to estimate the combined effect of code noise and multipath (phase noise and multipath are assumed negligible) and compute the variance
  - To obtain multipath-only statistics, substract noise-only variance from above value

# Measuring Multipath (and Noise)

- Pseudorange multipath – Method Two
  - Use one receiver and substract the carrier from the code
  - Ambiguity and ionosphere are estimated using a polynomial

$$MP_{Pseudorange} = p - \phi - \lambda N - 2d_{Iono}$$

- Carrier phase multipath
  - Use method where multiple, closely spaced, antennas are used to estimate multipath

# *Interference Overview*

**Interference** consists of any unwanted disturbance within a useful frequency band, including distortions of phase and amplitude (e.g. atmospheric effects, multipath), and noise

Any effect which causes degradation in $C/N_0$ can be considered as interference

- Classification and description of deterioration types
- Interference effects
- Interference sources
- GPS Jammers
- Radio frequency interference (RFI) mitigation methods
- Interference analysis setup
- Receiver testing with a GNSS simulator and sample results

# *GPS Interference*

- Purpose
  - Degrade the GPS receiver performance (accuracy and reliability)

- GNSS vulnerable to interference
  - Dependence on external RF signals from GNSS satellites, which makes it vulnerable to unintentional or intentional interference
  - Uses RF signals with received power levels typically 20dB below the ambient noise floor

- Increasingly important to assess
  - Impact of other RF systems and services on GNSS (e.g. shared spectrum services such as the Mobile Satellite Service, and Ultra-Wide Band services)
  - GNSS capability for indoor location and use in urban canyon and under foliage

- Interference can result in poor geometry and large position errors, or in system unavailability

# GPS Interference Classification



- Interference has many origins
  - Dynamic effects (Doppler effects)
  - Power fluctuations
    - Distance between receiver and satellite and antenna gain variations
  - Masking
    - e.g. buildings and foliage
  - Spectrum interference (Cross-correlation effects)
  - Atmospheric effects (ionosphere scintillation)
  - Pseudo-evil waveforms
    - e.g. code and carrier waveforms not synchronized at signal generation
  - Signal aliasing
    - Overlapping of signal spectrum during sampling in receiver front-end
  - Multipath fading
  - Unintentional interference
  - Intentional interference (jamming and spoofing)

# Jamming Types and Sources



| *Type* | *Typical Sources* |
|---|---|
| Wideband-Gaussian | Noise, Intentional Noise Jammers |
| Wideband phase/frequency modulation | TV transmitter harmonics |
| Wideband spread spectrum | Pseudolites, Intentional spread spectrum jammers |
| Wideband pulse | Radar transmitters |
| Narrowband phase/frequency modulation | AM station transmitters harmonics, CB radio harmonics |
| Narrowband swept continuous wave (CW) | Intentional CW jammers, FM station harmonics |
| Narrowband CW | Intentional CW jammers, near-band unmodulated transmitter's carriers |

Ward, P. (1999) Effects of RF Interference on GPS Satellite Signal Receiver Tracking, Chapter 6 of Understanding GPS Principles and Applications, Kaplan, E.D. editor.

# *Impact of RFI*

- Direct impact
  - Signal attenuation (lower $C/N_0$)
  - Acquisition failure
  - Code and/or carrier tracking loss
  - Tracking error (e.g. multipath)

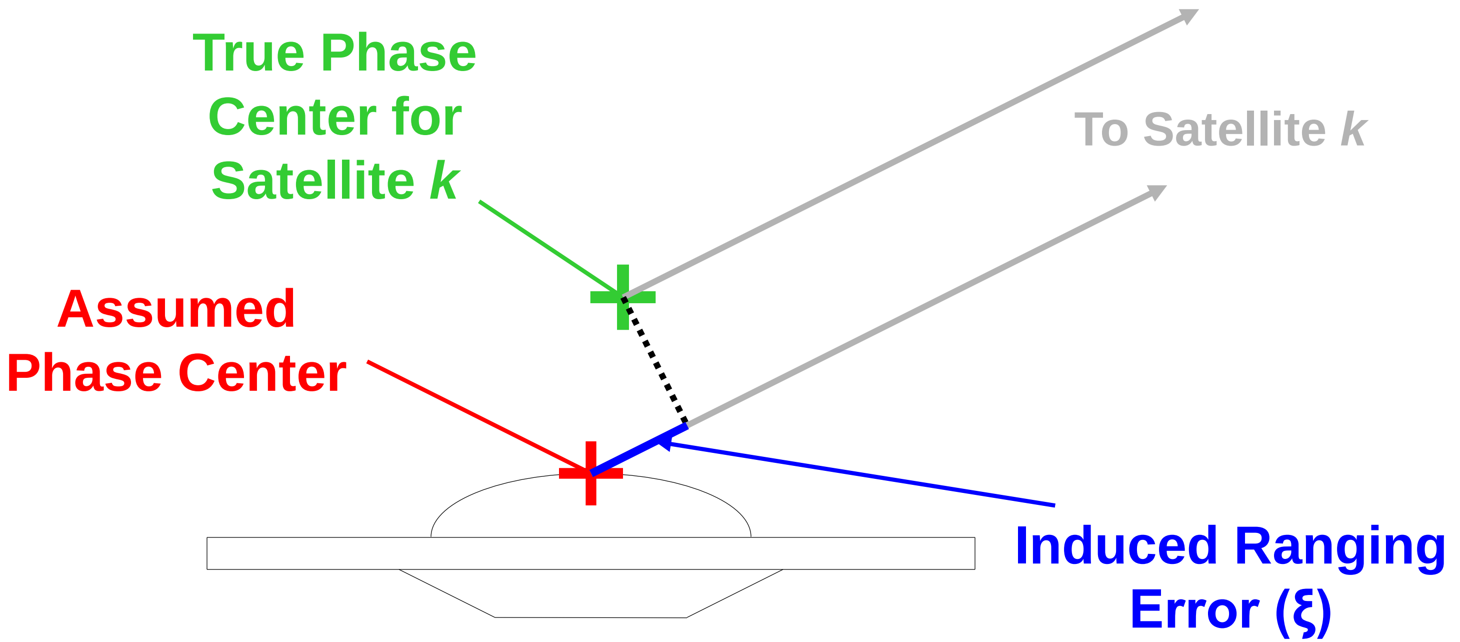
- Indirect impact
  - Large measurement errors
  - Poor geometry (due to satellite failures)
  - Navigation failure (insufficient number of satellites)

# *What is Phase Centre Variation?*

- All GPS measurements are assumed to relate to a known *phase centre* that is consistent over time and among all satellites being tracked
- Practically, this does not happen
  - Phase center is dependent on frequency
  - Phase center is dependent on angle of incidence of the signal relative to the antenna
  - These dependencies (variations) are collectively known as phase center variation (PCV)
- Calibration models
  - Elevation-dependent models
    - Tabular values to be interpolated
    - Polynomial coefficients
  - Azimuth and elevation-dependent models
    - Surface parameterization
    - Spherical harmonics

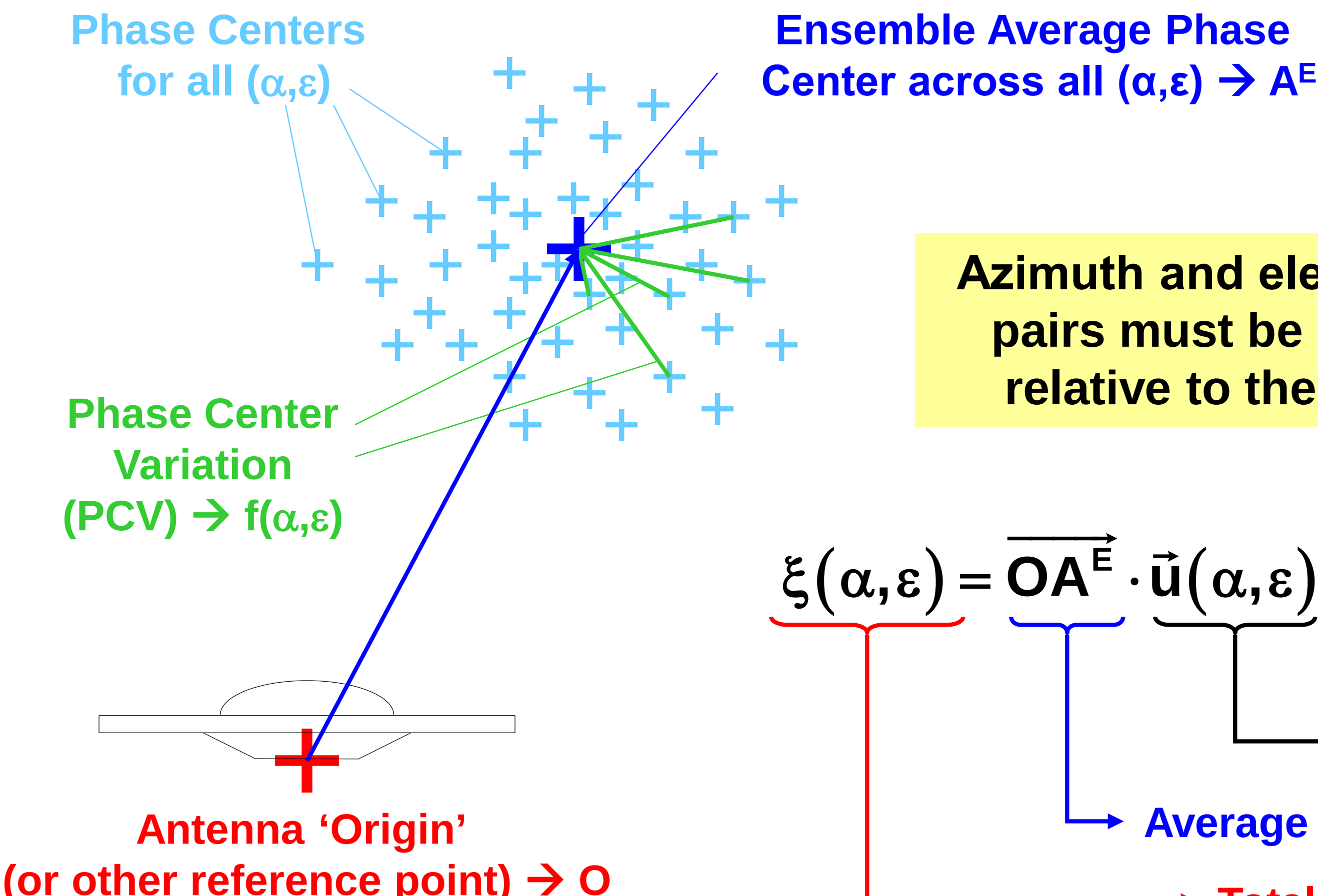
## Phase Centre Variations

# *Physical Interpretation*




Although the 'true' phase center is shown as being outside the antenna, this is not necessary the case. The true phase center can also be inside body of the antenna.

# *Phase Centre Correction*




**Azimuth and elevation (α,ε) pairs must be measured relative to the antenna!**

$$\underbrace{\xi(\alpha,\varepsilon)}_{\text{Total Phase center Error}} = \underbrace{\overrightarrow{OA^E}}_{\text{Average Phase Center Error}} \cdot \underbrace{\vec{u}(\alpha,\varepsilon)}_{\text{Unit Vector to SV}} + \underbrace{f(\alpha,\varepsilon)}_{\text{PCV}}$$

# Observation Equations



- Phase center effect is dependent on the frequency (f) and may differ between the code and carrier phase measurements
  - The antenna can be considered a filter that affects each of the PRN codes differently, resulting in a PRN-specific delay

$$P(\alpha, \varepsilon, f) = \rho + d_{ion,f} + d_{tropo} + d_{orbit} + d_{clock} + \xi_{P,f}(\alpha, \varepsilon) + m_P + n_P$$

$$\phi(\alpha, \varepsilon, f) = \left(\rho - d_{ion,f} + d_{tropo} + d_{orbit} + d_{clock} + \xi_{\phi,f}(\alpha, \varepsilon) + m_\phi + n_\phi\right) \cdot \frac{1}{\lambda_f} + N_f$$

# Code vs Carrier Effects



- If the antenna is considered as a filter, then it affects incoming signals based on their spectrum
  - Spectrum of the phase and code are different
- There is a different effect on code than on the carrier
  - Code spectrum may be distorted which may create a distorted correlation peak
  - Phase is more likely to be simply shifted
- Previous studies have shown PRN-specific delays due to their different spectral lines

# Linear Code Combinations



- Consider a generic linear carrier phase combination of the form

$$\mathbf{P_{m,n} = m \cdot P_{L1} + n \cdot P_{L2}}$$

- Consider the effect of L1 and L2 phase center errors

$$\xi^{[m]}_{P,(m,n)} = \mathbf{m} \cdot \xi^{[m]}_{P,L1} + \mathbf{n} \cdot \xi^{[m]}_{P,L2}$$

- The error is directly dependent on the coefficients of the linear combination

# Linear Phase Combinations



- Consider a generic linear carrier phase combination of the form

$$\phi_{m,n} = m \cdot \phi_{L1} + n \cdot \phi_{L2} \quad \Rightarrow \quad \lambda_{m,n} = \frac{\lambda_{L1} \cdot \lambda_{L2}}{m \cdot \lambda_{L2} + n \cdot \lambda_{L1}}$$

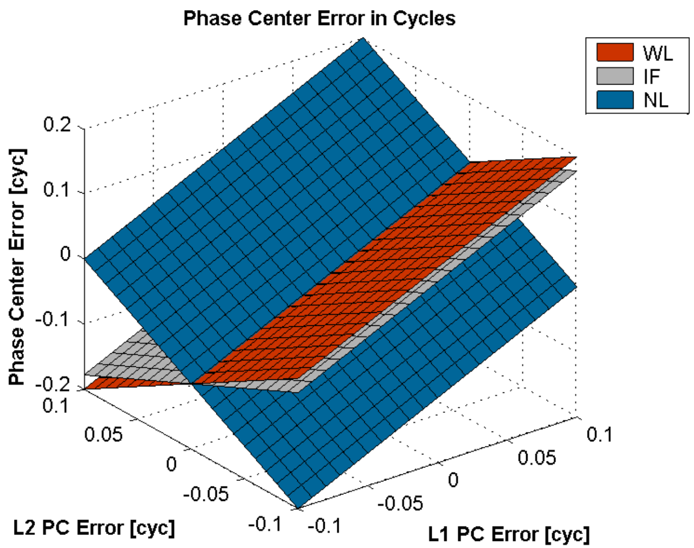
- Consider the effect of L1 and L2 phase center errors

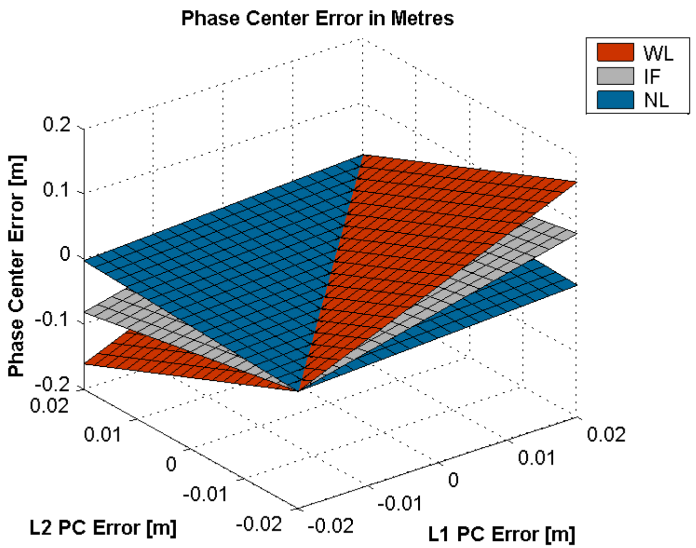
**Units of cycles** $\Rightarrow$ $\xi^{[cyc]}_{\phi,(m,n)} = m \cdot \xi^{[cyc]}_{\phi,L1} + n \cdot \xi^{[cyc]}_{\phi,L2}$

**Units of metres** $\Rightarrow$ $\xi^{[m]}_{\phi,(m,n)} = \lambda_{m,n} \cdot \xi^{[cyc]}_{\phi,(m,n)}$

# PCV Errors on Phase Combinations

Maximum error is approximately the same in units of cycles




Maximum error is considerably different in units of metres

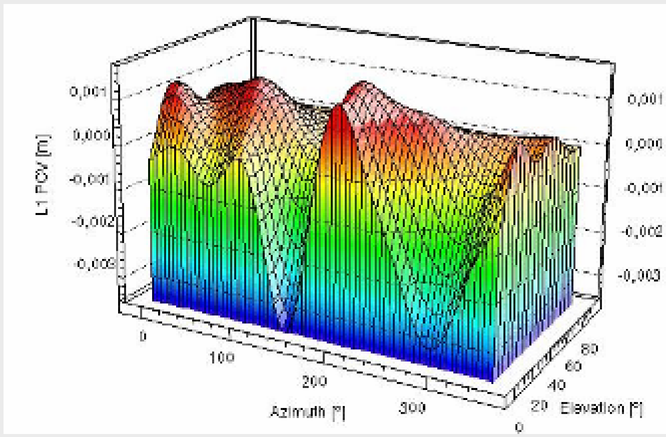
**Often, a linear combination is used to reduce the magnitude of an error in units of cycles in order to improve ambiguity resolution. However, for PCV errors, this does not necessarily happen!!!**

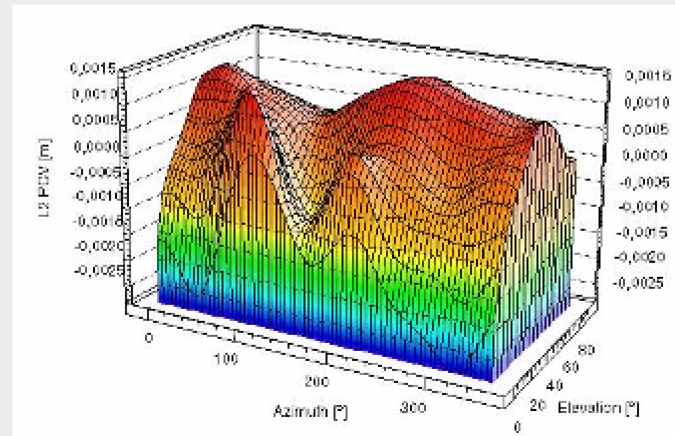
# Phase Centre Variations

## *Sample PCV Values*

**NovAtel 702 Antenna**

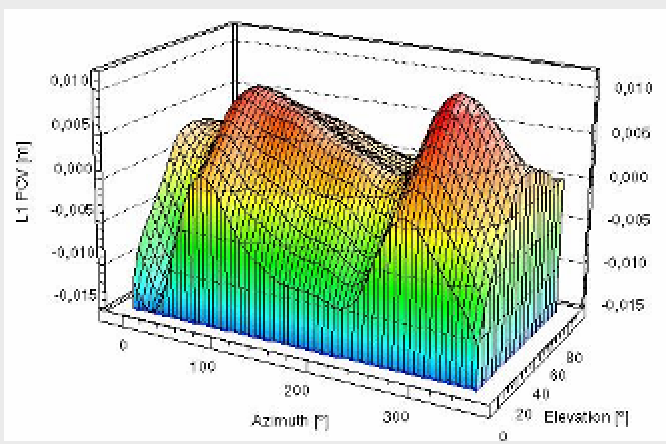
**L1 PCV: [-3.0 mm → +1.0 mm]**

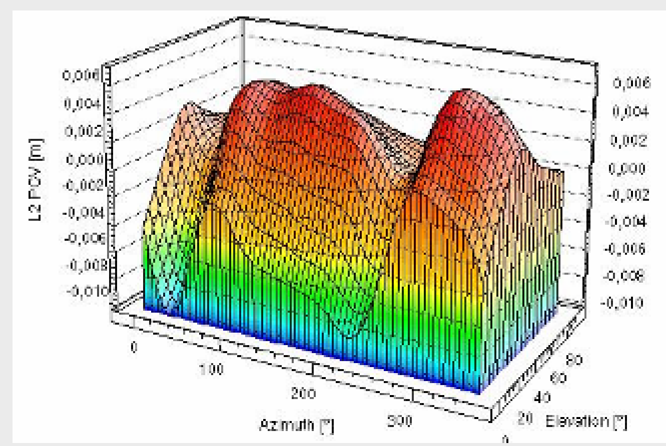



**Carl Zeiss Jena Geodetic Antenna**

**L1 PCV: [-15.0 mm → +10.0 mm]**




**Reference: [http://gnpcvdb.geopp.de/]**

## Considerations



- Variations can be highly variable between antenna models/manufacturers
  - Errors will not cancel in differential mode
- If same antennas are used in differential mode, their orientation must be the same
  - Cannot compensate for different attitudes unless the PCV is actually known
- Linear combinations could be more susceptible to PCV effects
  - Widelane (WL), Narrowlane (NL), Iono-Free (IF)
- PCV effect is proportional to its contribution to the overall error budget
  - Noise → mm-level
  - Multipath → mm to cm-level (phase)
  - Orbit → mm-level
  - Atmosphere → distance dependent (≤ dm-level)
- Average error is ‘absorbed’ by clock term

# *The “Real” Uncorrected Effect*

- If multiple satellites are tracked, the average of all PCV errors will be inseparable from the clock error

$$\phi(\alpha,\varepsilon,f) = \left(\rho - d_{iono,f} + d_{tropo} + d_{orbit} + \rlap{$\displaystyle\underbrace{\phantom{d_{clock} + \overline{\xi_{\phi,f}(\alpha,\varepsilon)}}}_{d'_{clock}}$} d_{clock} + \overbrace{\overline{\xi_{\phi,f}(\alpha,\varepsilon)} + \delta\xi_{\phi,f}(\alpha,\varepsilon)}^{\xi_{\phi,f}(\alpha,\varepsilon)} + m_{\phi} + n_{\phi} + \lambda_f N\right) \cdot \frac{1}{\lambda_f}$$

$$\phi(\alpha,\varepsilon,f) = \left(\rho - d_{iono,f} + d_{tropo} + d_{orbit} + d'_{clock} + \delta\xi_{\phi,f}(\alpha,\varepsilon) + m_{\phi} + n_{\phi} + \lambda_f N\right) \cdot \frac{1}{\lambda_f}$$

- Changes in number of visible satellites will cause discontinuities in the computed position

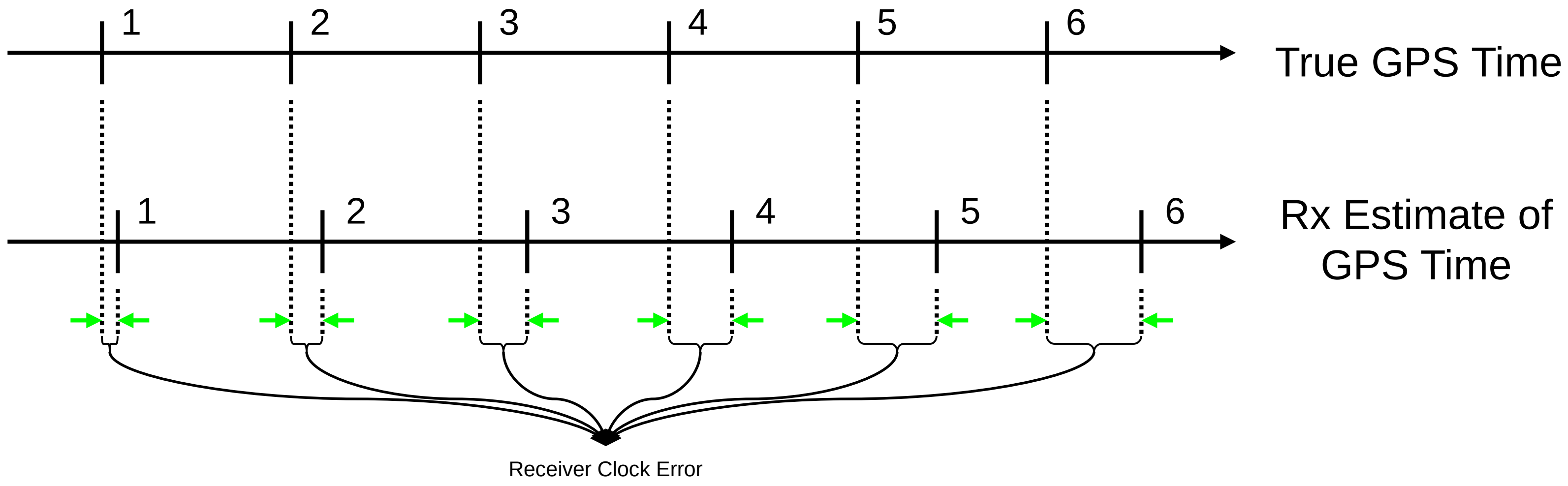
# Effect on Positioning

- There is no difference in positioning accuracy between the geometry-free and geometry-based methods
  - You always need to implement the latter to get any position information
  - Only exception is if the ambiguity resolution performance is different between the two approaches
- Positioning accuracy will be dependent on the magnitude of the errors and on satellite geometry
- Previous studies have shown the error to appear mostly in height [19]
  - Height errors of up to 10 cm
- Horizontal errors are also possible
- Relative PCV is most important
  - Not having absolute information introduces an error of about 0.015 ppm
    - 1.5 mm per 100 km of receiver separation

# Receiver Clock Error

## *What Is It?*

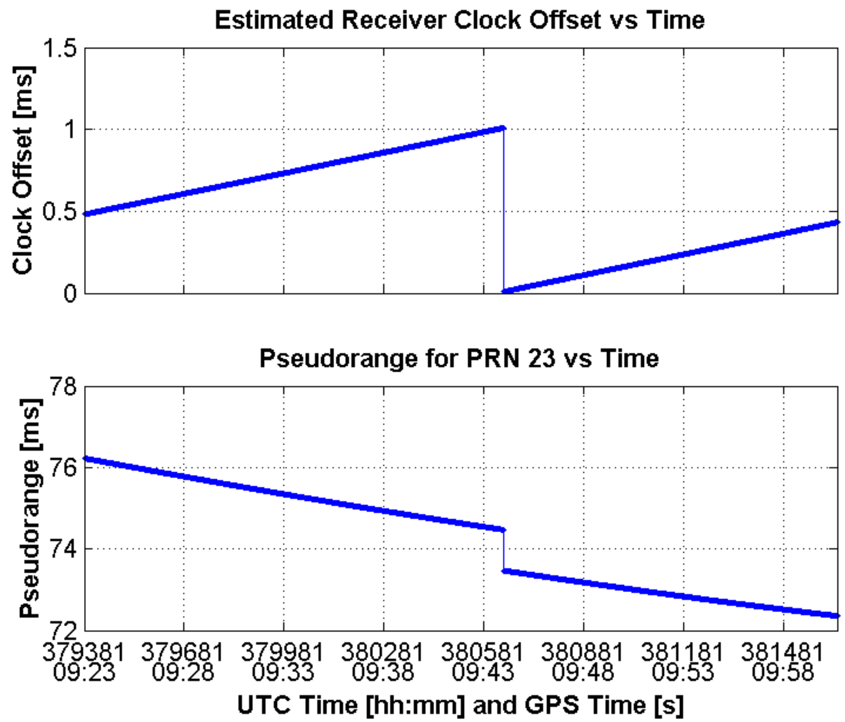
- Difference between true GPS time and the receiver's estimate of GPS time




- Function of time due to clock drift (which is a function of quality of oscillator used in the receiver)

# *Practical Considerations 1*

- The receiver clock error is theoretically unbounded but receivers typically adjust/reset their estimate of GPS time to bound the error to within a certain range
  - This “resetting” of the GPS time estimate introduces discontinuities in the measurements
  - Most receivers limit the clock offset to ±1 ms which leads to “millisecond jumps” in the output data
  - Effect can be seen on the code data, and sometimes carrier phase data, depending on the receiver




  - In DGPS mode, above results in asynchronous measurements between receivers

# Practical Considerations 2



- Differential processing
  - Large differences in the clock offset between two receivers mean that the data from the two receivers are not synchronized
  - Special processing considerations are required in this case to handle this difference

- Single point processing
  - The magnitude of the clock offset is essentially irrelevant since it must be estimated as part of the measurement adjustment process

## Inter-Channel Biases

### *Concept*

- Each satellite is tracked on a different channel inside the receiver but each channel will have slightly different delays depending on the frequency of the incoming signal
  - This is analogous to the frequency response of an electrical system

- These relative delays between channels do not cancel in a between-satellite single difference or in a double difference

- Biases are usually very stable over time and can thus be calibrated out
  - Repeated testing in controlled outdoor environments
  - Hardware simulator testing

# *Considerations*



- Inter-channels biases are important for the most accurate applications

- Very important for ionospheric modeling applications
  - Inter-channel biases delay L1 and L2 signals differently
  - This extra delay will be interpreted as an ionospheric signal unless otherwise accounted for

- Delays are receiver/hardware specific
  - All receivers must be calibrated separately even if all receivers are of the same type

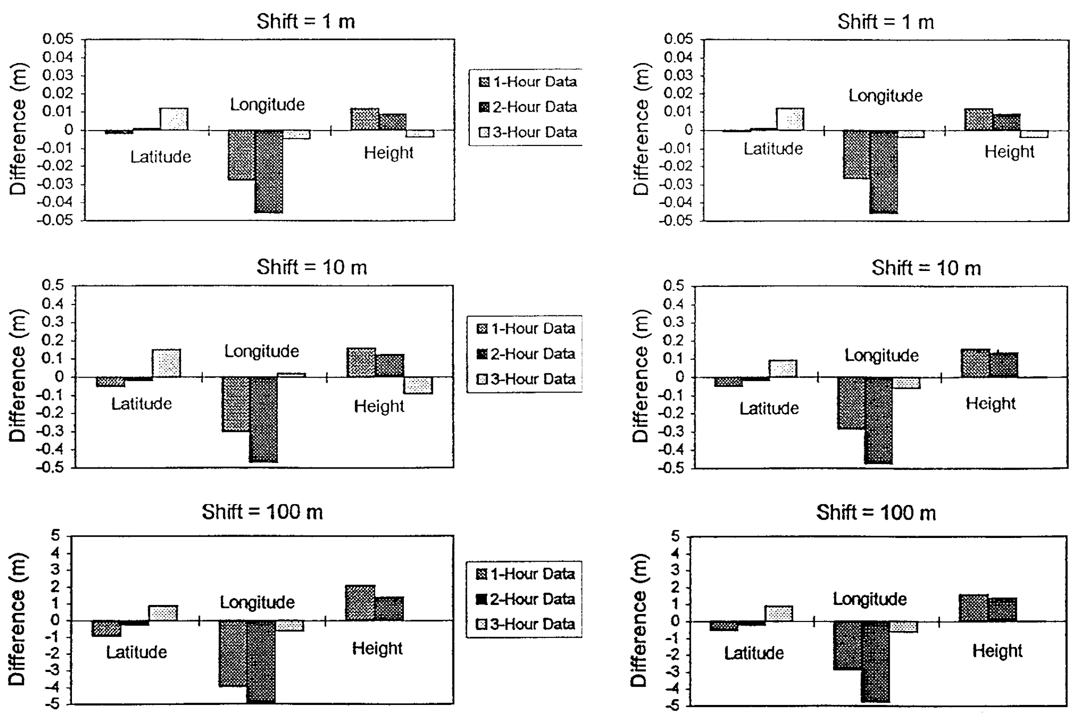
# *Reference Station Coordinate Errors 1*

- Not all errors are caused by propagation or receiver effects; some are the result of incorrect data processing

- Incorrect reference station coordinates
  - Introduces second order errors in design matrix
  - Effect is a function of
    - Satellite geometry
    - Duration of observations (for static case)
    - Distance between reference and remote antennas
    - Magnitude of reference station coordinate error

# Reference Station Coordinate Errors 2

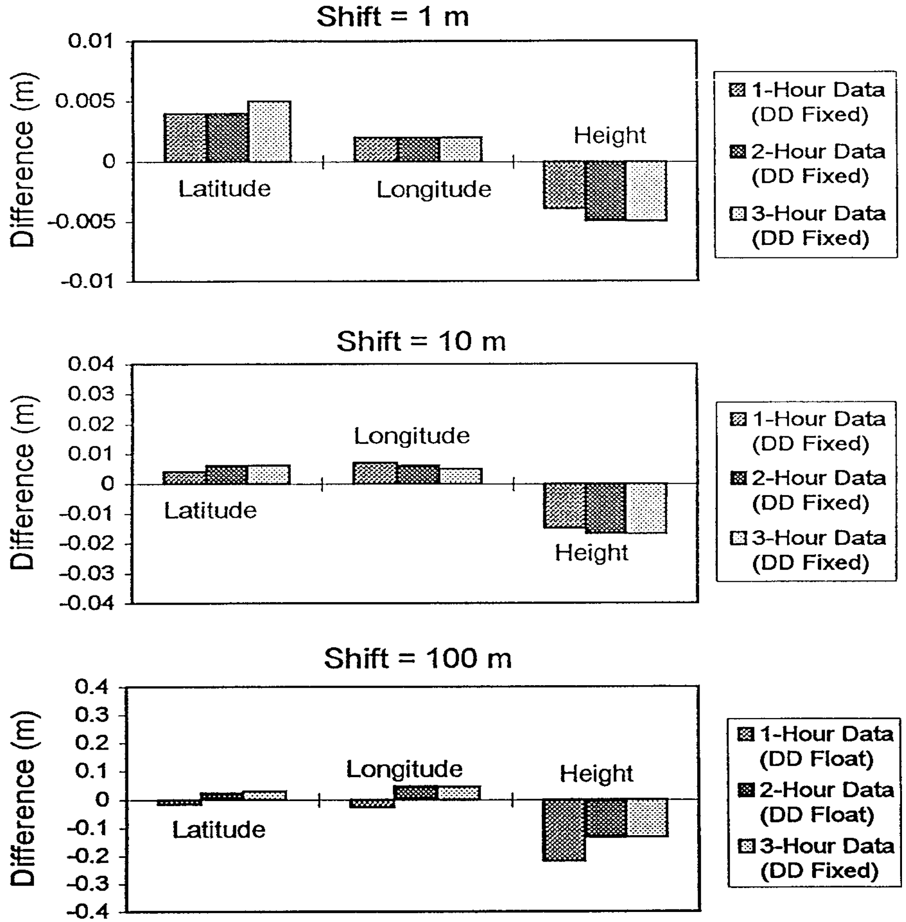
- Incorrect reference station coordinates con't
  - Compare results with correct and shifted reference station coordinates over a 424 km baseline (two-day test)




Tang, C. (1996) Accuracy and Reliability of of Various DGPS Approaches. MSc Thesis, published as Report No. 20095, Department of Geomatics Engineering, The University of Calgary

# Reference Station Coordinate Errors 3

- Incorrect reference station coordinates con't
  - 16 km baseline



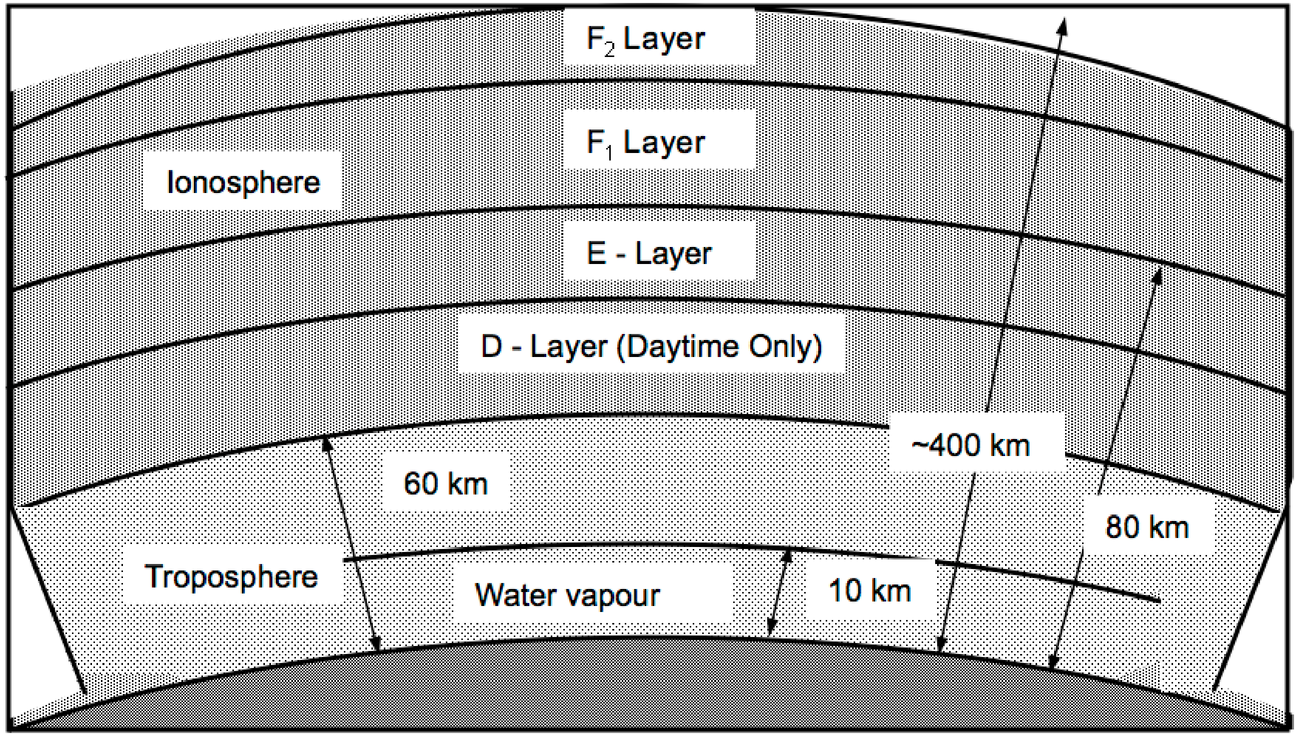
## *Possible Sources*



- Incorrectly time tagged data
  - See case study on using carrier phase data for precise velocity determination
- Inappropriate dynamics model when filtering
  - Will introduce systematic errors during high-dynamic maneuvers

## Atmosphere

# *Overview*

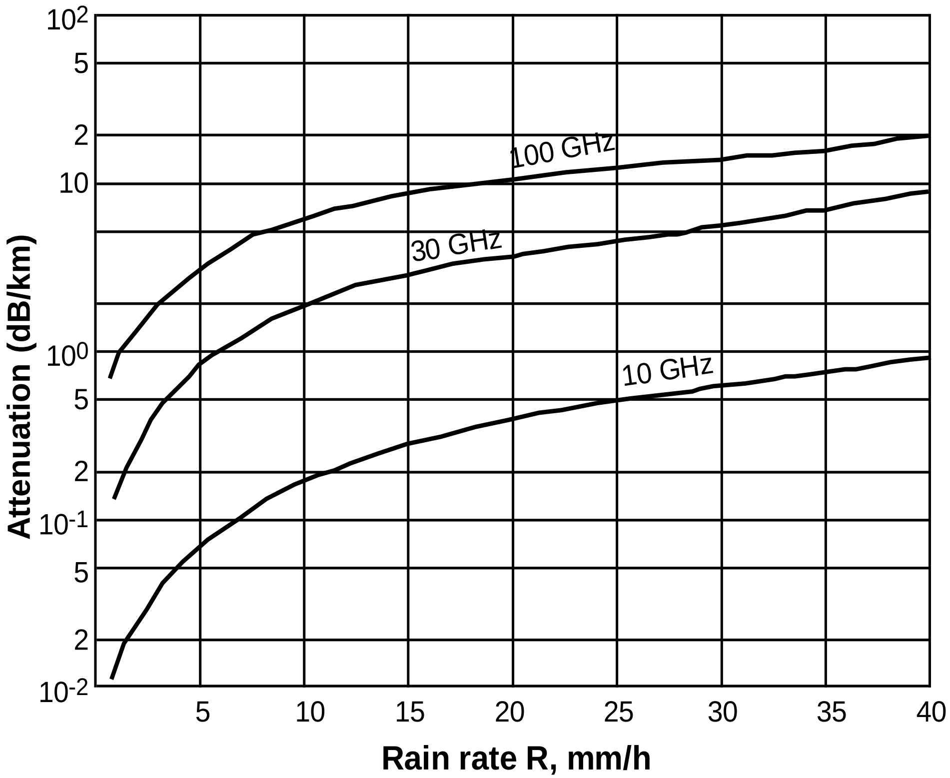



- Ionospheric effect  is due to the presence of ionized gas molecules
  - Dispersive at RF frequencies (index of refraction varies with frequency): Vg ( group velocity )     vΦ  (phase velocity) {vg = - vΦ}
- Tropospheric effect
  - Usually referred to part of part from ground level to 60 km
  - Lumped effect in the stratosphere and troposphere
  - Non-dispersive at RF frequencies

# Atmospheric Attenuation 1



- Ionosphere + troposphere
  - Attenuation at 1.5 GHz < 2 dB
- Rain
  - Attenuation in dB/km as a function of rain rate is given in the adjacent diagram for 10, 30 and 100 GHz
- In the case of GPS, the attenuation is even lower
  - Satellite–based waves cross only a few km of rain (rain clouds are below a few thousand m), so the attenuation is negligible



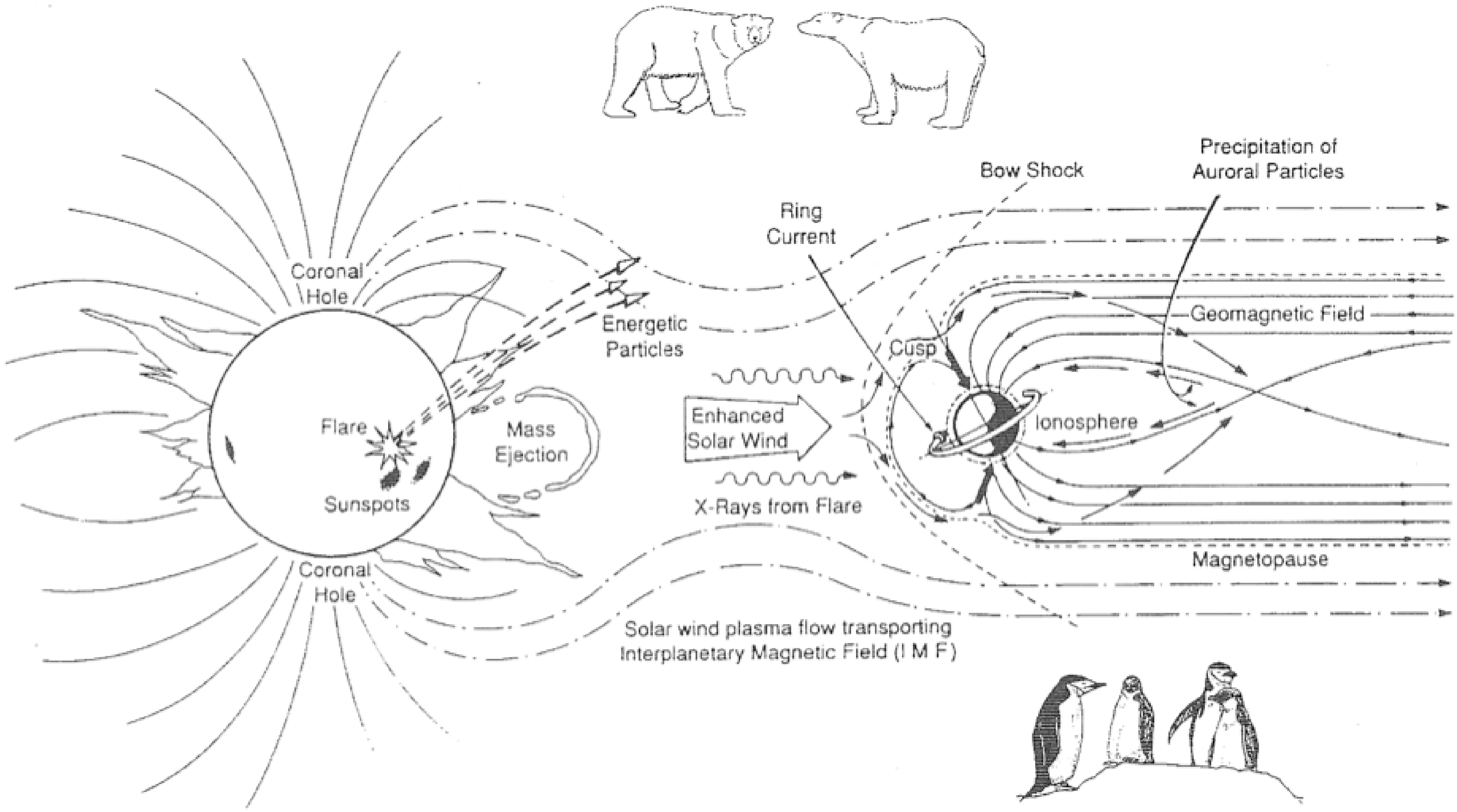
# Atmospheric Attenuation 2

- Fog
  - Suspended mist
  - Heavy fog with a 400 ft visibility has a water content of about 0.32 gr/m$^3$
  - At 7 and 75 GHz, attenuation is 0.1 and 1 dB/km
- Ice crystals
  - Relatively low attenuation
- Dry snow
  - At least one order of magnitude less than wet snow
- Wet snow
  - Higher than rain, e.g., 2 dB/km at 35 GHz for a precipitation of 5 mm/h
- Ice on antennas – can be significant (especially wet ice covered by a thin layer of water – see reference below)

O'KEEFE, J. STEPHEN, G. LACHAPELLE, and R. GONZALES (1999) Effect of Ice Loading on a GPS Antenna. Proceedings of National Technical Meeting, The Institute of Navigation (January 25-27, San Diego, CA), 861-869.

# Ionosphere

## *Overview*




The high-latitude zones within the solar–terrestrial environment. After *Synoptic Data for Solar–Terrestrial Environment*, The Royal Society (September 1992). Wildlife by J. C. Hargreaves.

R.D. Hunsucker and J.K. Hargreaves (2003) The high-latitude ionosphere and its effects on radio propagation. Cambridge University Press

# General Structure and Characteristics

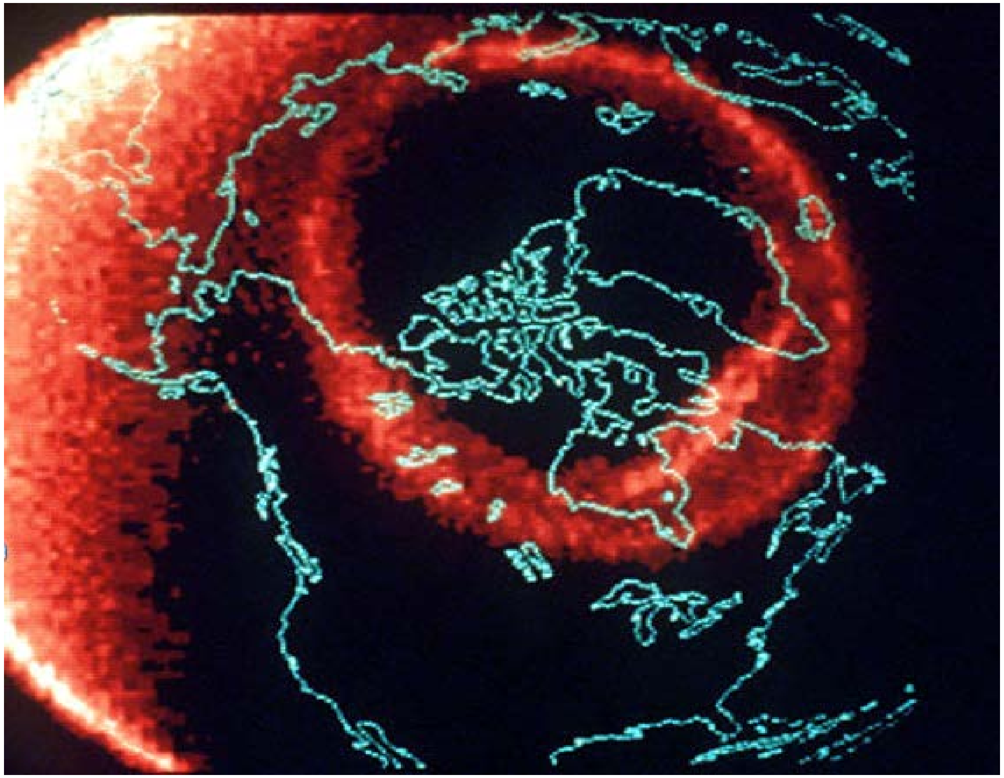
- Extends from about 50 to over 1,000 km above the Earth
- Characterized by free electrons (and ions) in the atmosphere
  - The amount of electrons (called the electron density) is directly related to the effects on RF signal propagation in the medium
  - Diurnal effect due to Sun
- Dispersive medium for GPS frequencies
  - Effects on RF propagation are a function of the frequency of the signal
- Pseudorange measurements are delayed and carrier phase measurements are advanced
- Activity varies by a factor of 3 due to 11-year solar cycle
- Temporal and spatial correlation
  - Average correlation time: 3 hours
  - Average correlation distance: 1,000 km
  - These correlation characteristics cause a major limitation for carrier phase ambiguity resolution
- Zenith errors increase by a factor of three near the horizon

## *Regional Effects 1*



**Auroral Oval**

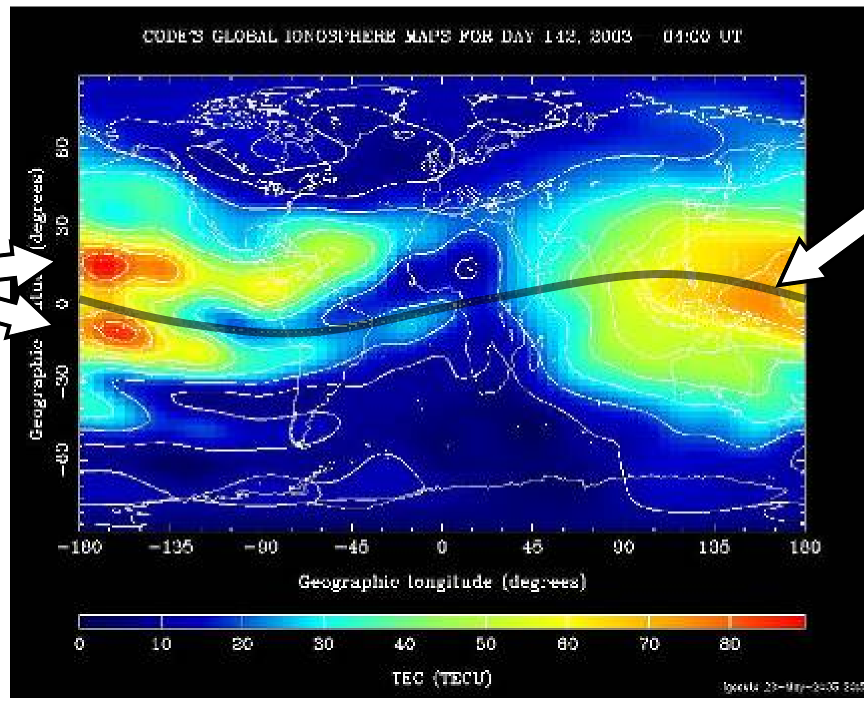
small-scale irregularities, transient sub-storm events, scintillation



# Ionosphere

## *Regional Effects 2*

- Equatorial anomaly
  - Several forces involved
  - Net result is an increase in ionization at ±10º *geomagnetic* latitude
  - Increased scintillation




http://www.cx.unibe.ch/aiub/ionosphere.html

# Summary of Ionospheric Effects on GPS



- Group delay/phase advance
- Absorption
  - ~0.2 dB for GPS
- Faraday rotation
  - Change in plane of wave polarization
- Doppler shift
- Dispersion
  - Relative delay of signal across bandwidth
  - Causes reduction in coherence
- Refraction
- Scintillation
  - Localized irregularities in electron density

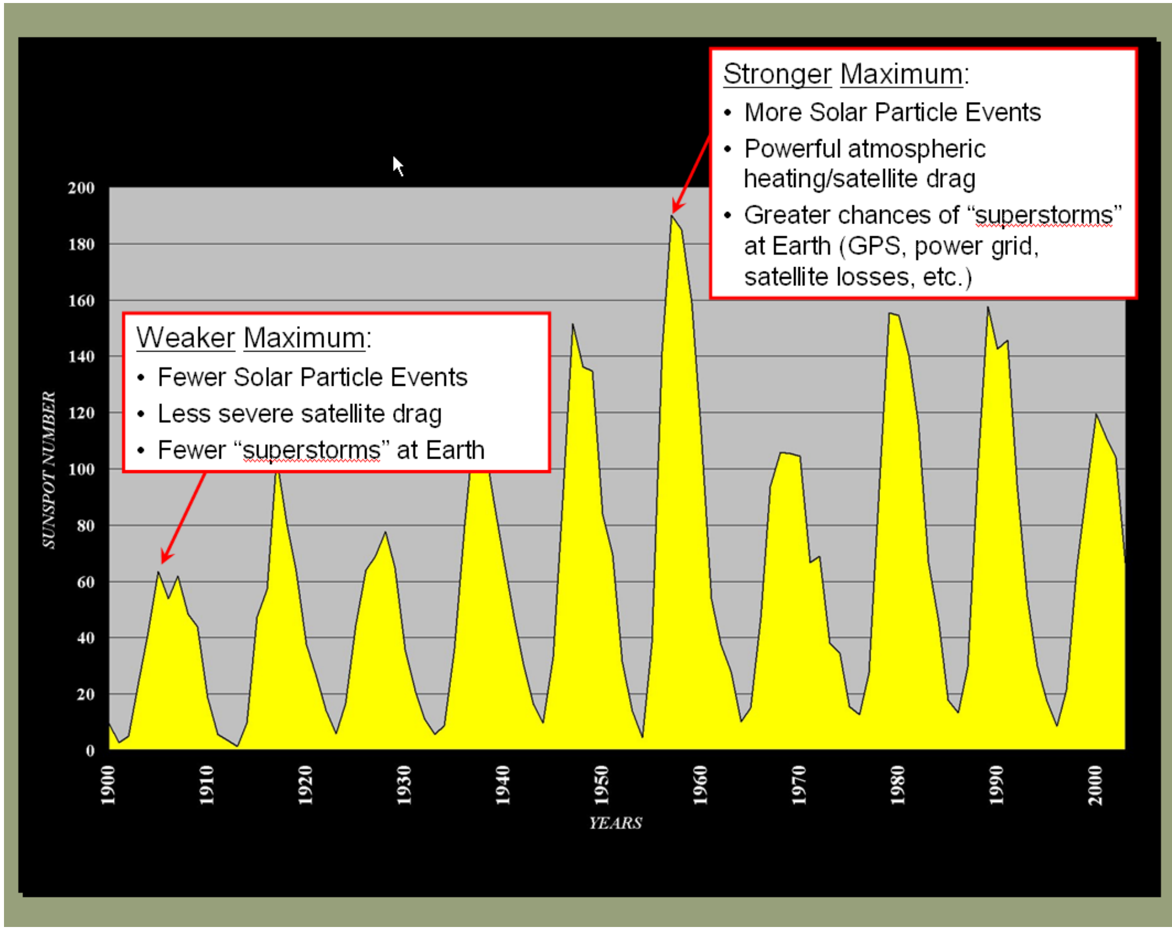
# Source: Role of the Sun

- The Sun emits an ionized plasma at supersonic speeds called the solar wind
  - Excellent conductor (approximately infinite conductivity)
  - Sun’s magnetic field is “frozen” in the solar wind
  - When the solar wind reaches Earth, the Sun’s “frozen” magnetic field interacts with the Earth’s magnetic field
  - Certain conditions can cause the Earth’s magnetic field to “open”, thus allowing the ionized solar wind to enter the upper atmosphere
- Solar radiation can cause the ionization of some neutral particles (H, He, $O_2$, etc.)
  - Amount of ionization in the atmosphere is a function of the rate of separation and recombination

# Solar Cycle and Sun Spots

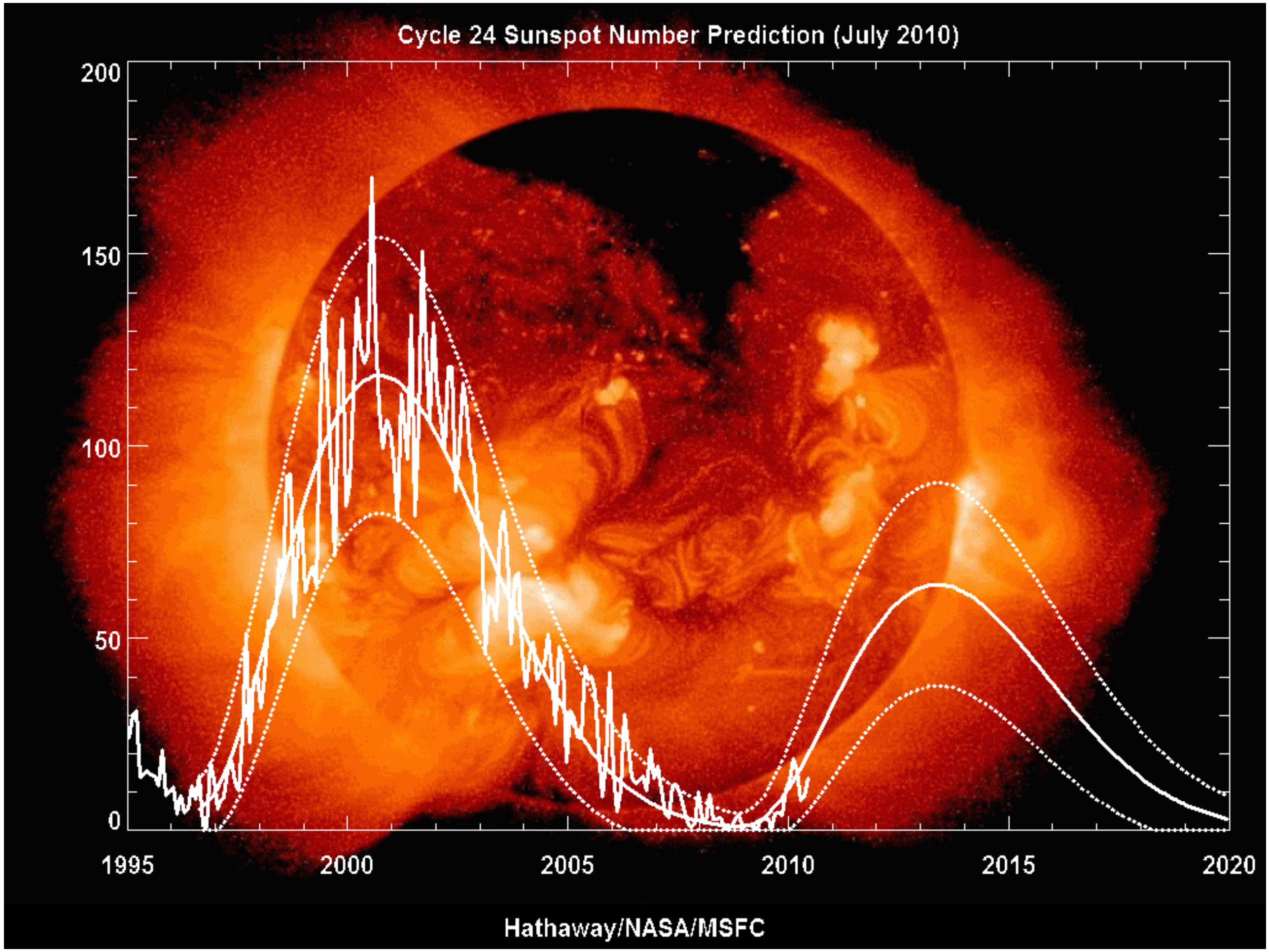


- Changes in solar activity directly influence level of ionospheric activity and thus GPS performance
- Solar activity reaches a maximum every 11 years (solar cycle)
- During solar maximums, the number of sun spots increases and used as an indicator of solar activity



# Current Solar Cycle

- Solar Cycle 24




- http://solarscience.msfc.nasa.gov/images/ssn_predict_l.gif

# Phase and Group Velocities 1

- Phase velocity of a single wave is $v_p = \lambda\ f$
- For a group of waves (pseudoranges) with slightly different frequencies, the group velocity is defined as

$$v_g = -\frac{df}{d\lambda}\lambda^2$$

- One can also write

$$dv_p = f d\lambda + \lambda df \quad \text{or} \quad \frac{\mathrm{df}}{\mathrm{d}\lambda} = \frac{1}{\lambda}\frac{dv_p}{d\lambda} - \frac{f}{\lambda}$$

$$v_g = -\lambda\frac{dv_p}{d\lambda} + f\lambda$$

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}(\mathit{Rayleigh}\ \text{equation})$$

Hofmann-Wellenhof et al (2008) GNSS – GPS, GLONASS, Galileo & More. Springer,

# Phase and Group Velocities 2

- Velocity can also be written as a function of the index of refraction n

$$v_p = \frac{c}{n_p} \qquad \mathrm{v_g} = \frac{\mathrm{c}}{\mathrm{n_g}}$$

- One can also write

$$\frac{\mathrm{dv_p}}{\mathrm{d}\lambda} = -\frac{c}{n_p^2}\frac{dn_p}{d\lambda}$$

$$n_g = n_p - \lambda\frac{dn_p}{d\lambda} = n_p + f\,\frac{dn_p}{df}$$

# *Indices of Refraction*

- Phase index of refraction is given as

$$n_p = 1 + \frac{k_1}{f^2} + \frac{k_2}{f^3} + \frac{k_3}{f^4} + \cdots \text{ with } \mathrm{k}_1 \approx -40.3\,\mathrm{N}_e \text{ (see next slide)}$$

- Group (pseudoranges) index of refraction is given as

$$n_g = n_p + f \cdot \frac{dn_p}{df} = 1 - \frac{k_1}{f^2} - \frac{2k_2}{f^3} - \frac{3k_3}{f^4} - \cdots$$

- $k_i$'s are frequency independent but electron density dependent
- ($N_e$-nbr of electrons per $m^3$) along the propagation path
  - First order approximation accounts for most of the error
    - Higher order effects are at most 3-4 orders of magnitude smaller (<1% of first order term)

Hofmann-Wellenhof et al (2008) GNSS – GPS, GLONASS, Galileo & More. Springer,

## Indices of Refraction

- $k_1$ is a physical quantity
  - $q_e$ is the charge of an electron
  - m is the mass of an electron
  - $\varepsilon_o$ is the permittivity of free-space

$$k_1 = \frac{q_e^2}{8\pi^2 \cdot m \cdot \varepsilon_o} \cdot N_e = \underbrace{\frac{\left(-1.602 \cdot 10^{-19} C/el\right)^2}{8\pi^2 \left(9.107 \cdot 10^{-31} kg/el\right)\left(8.854 \cdot 10^{-12} F/m\right)}}_{-40.3\ m^3\ Hz^2\ el^{-1}} \cdot N_e$$

$n_p \leq 1 \rightarrow$ Phase advance
$n_g \geq 1 \rightarrow$ Group delay

Number of electrons varies with level of ionospheric activity

# Group Delay and Phase Advance

- To first order, the group delay (m) is given as

$$\Delta S_{g_{[m]}} = -\int_S \frac{k_1}{f^2} ds = \frac{40.3}{f^2} \int_S N_e ds = +\frac{40.3}{f^2} TEC \quad \text{where} \quad \text{TEC} = \int_{SV}^{User} N_e ds$$

- where TEC is the total electron content in a 1 m x 1 m column (given in TECU units (1 TECU = $10^{16}$ electrons/m$^2$)
- Similarly, the phase advance (in m) is given as

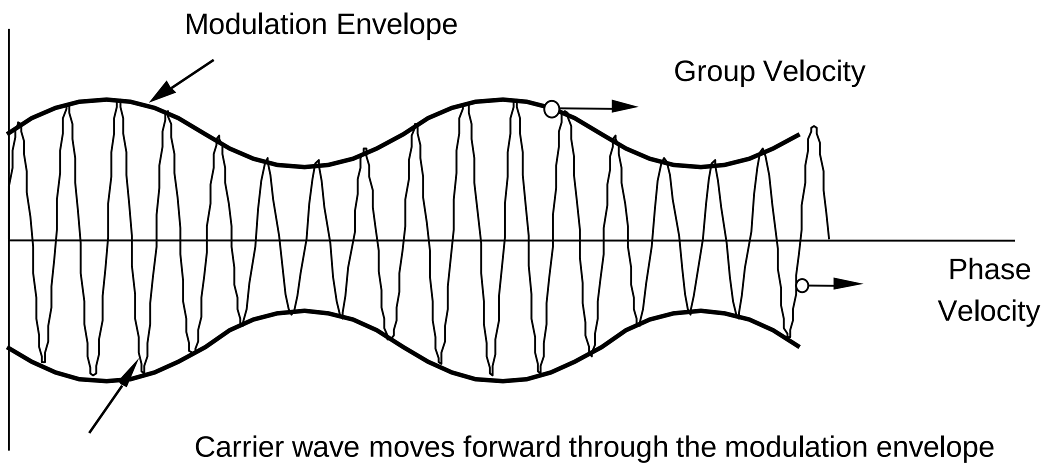
$$\Delta S_{p_{[m]}} = -\int_S \frac{k_1}{f^2} \cdot ds = -\frac{40.3}{f^2} TEC$$

  - To calculate the advance in cycles, divide by wavelength

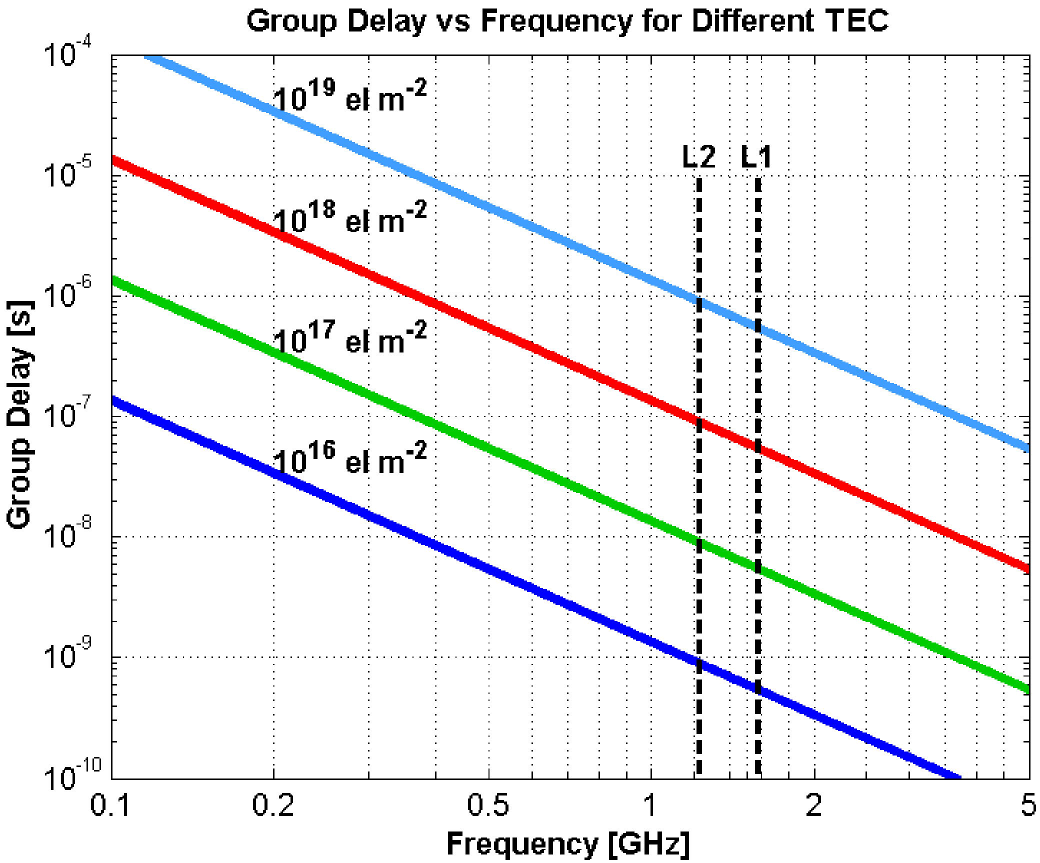
$$\Delta S_{p_{[cyc]}} = \frac{\Delta S_{p_{[m]}}}{\lambda} = \frac{\Delta S_{p_{[m]}}}{c} f = -\frac{40.3}{cf} TEC$$

# Group Delay and Phase Advance

- The group delay applies to the modulation envelope (i.e. the group of frequencies)
  - Refers to the information/energy carried by a signal
  - Travels slower than the speed of light
- The phase advance applies to the wave of the individual carrier
  - This appears to travel faster than the speed of light
  - Since all measurements are made on the group, the speed of light is ***not*** actually exceeded



# Group Delay Versus Frequency




**NOTE**: Some people use TEC Units (TECU) to measure TEC levels

1 TECU $\equiv 10^{16}$ el m$^{-2}$

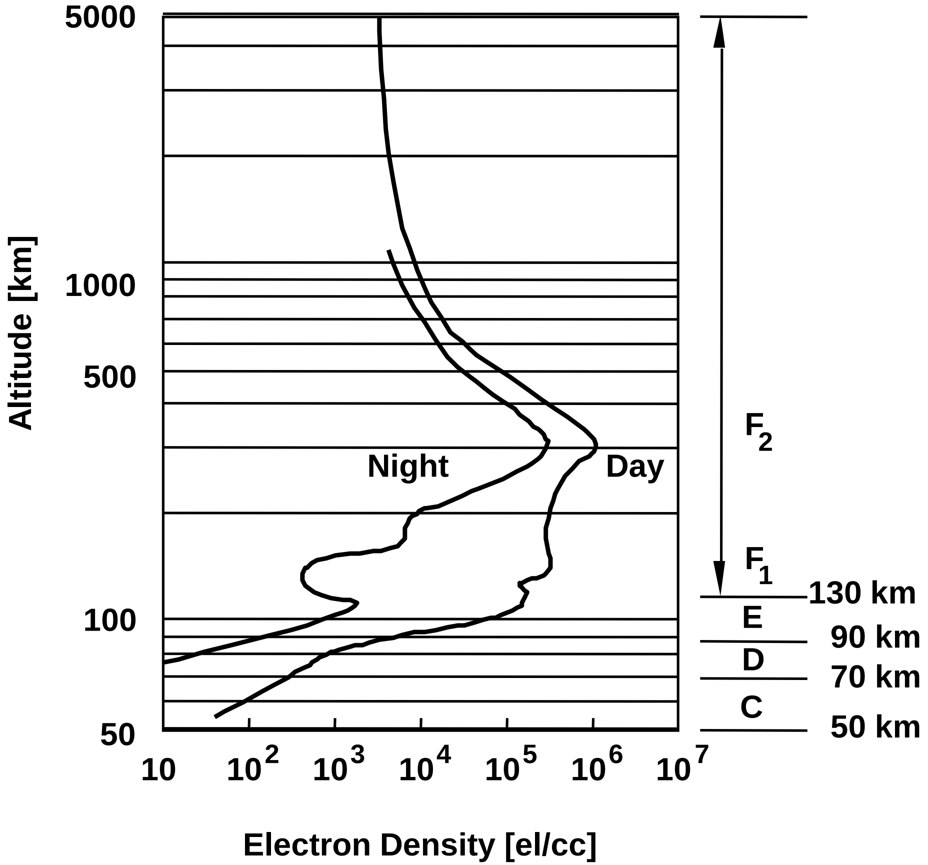
$\equiv$ 0.16 m @ L1

- Phase delay has same magnitude as above, but opposite sign

# Electron Distribution



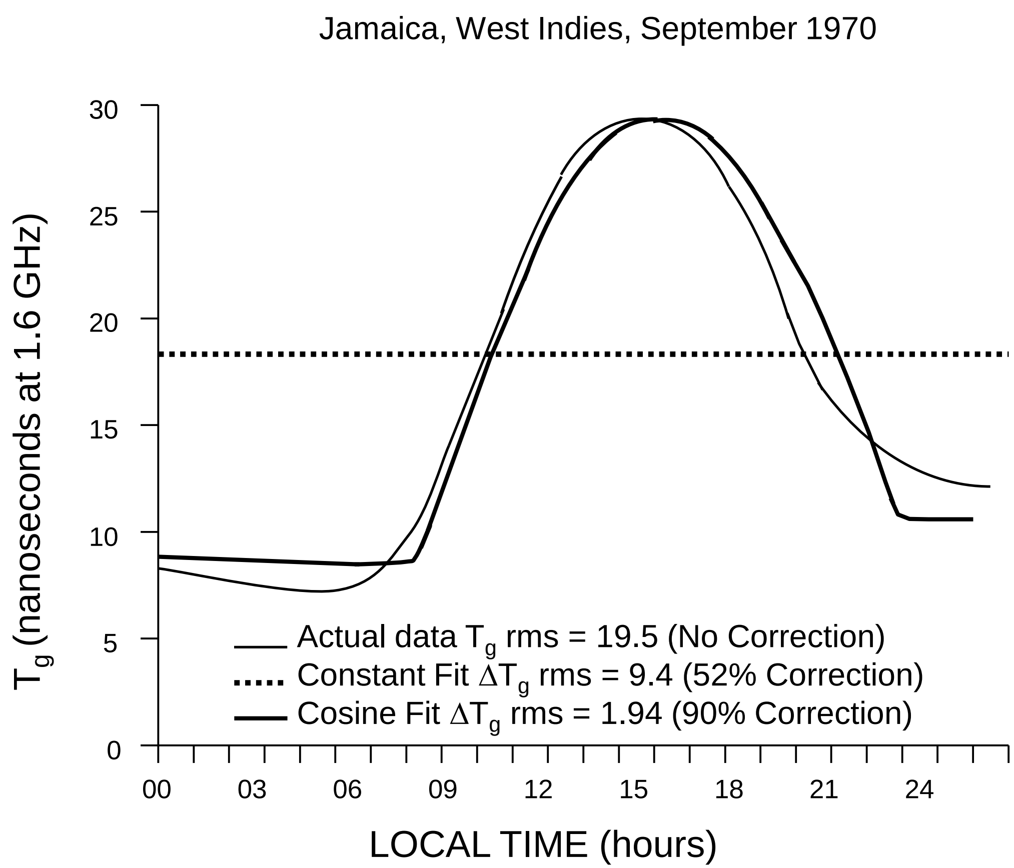
- Electron distribution is a function of altitude
- Differences between night and day illustrate role of the sun




Ionosphere is broken up into different layers

# Ionospheric Effect Trend at 1.5 GHz

- Diurnal maximum at 14:00 h, local time
  - Function of solar ionization rate and recombination rate
- Broadcast ionosphere model uses a half-cosine approximation during the day and DC term at night
- Secondary peak near 2200 is also possible



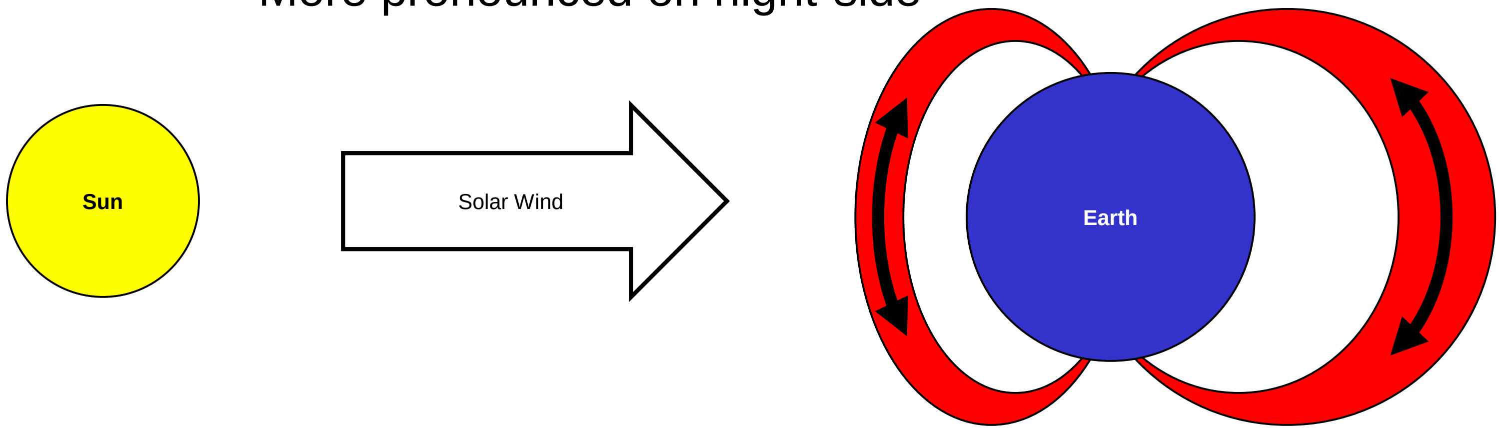
# Ionospheric Characteristics



- Highest effects at Spring equinox (except near solar minimum)
- Activity varies by a factor of 3 due to solar cycle
- Mid-latitude effect: variability of 25% from monthly mean
- Temporal and spatial correlation
  - Average correlation time: 3 hours
  - Average correlation distance: 1,000 km
- Undifferenced group delay at zenith averages about 50 ns (~15 m)
  - Zenith errors increase by a factor of three near the horizon

# Geographic Dependence in TEC

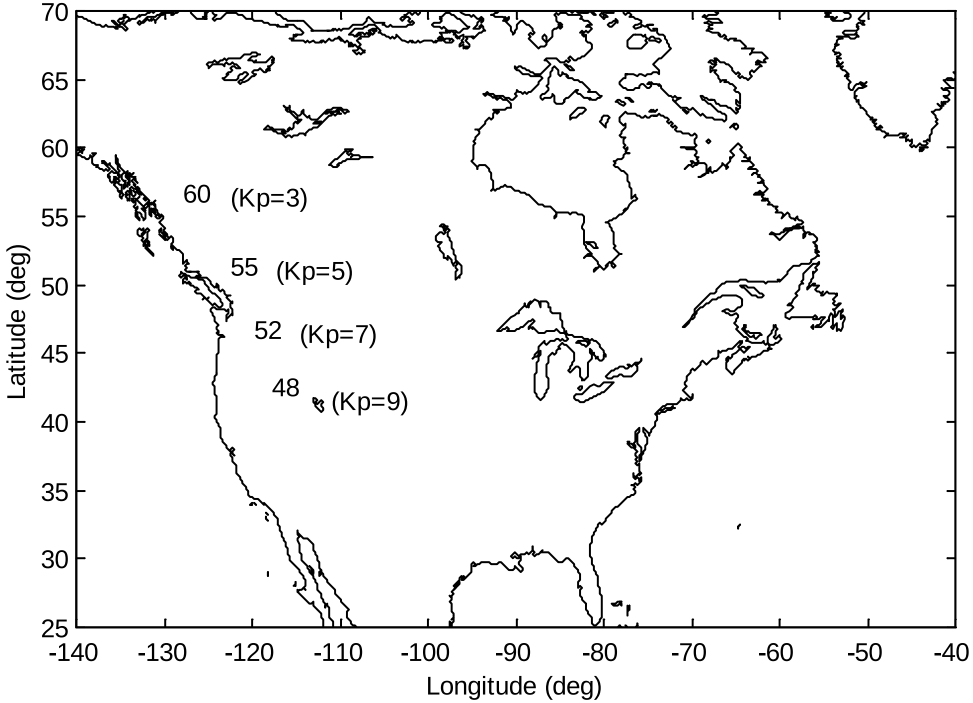
- Auroral zone
  - Energetic particles are mapped along the Earth's magnetic field lines to upper latitudes (55º -70º)
  - Region of increased ionization is referred to as the Auroral Oval
    - More pronounced on night-side




- Large gradients in TEC at equatorial-edge of oval
  - High-latitude trough ("Canadian border effect")

# Geographic Dependence in TEC

- Auroral zone con't
  - Plot shows lower boundary for different levels of auroral activity




| Kp | Level of Activity |
|---|---|
| 3 | Low |
| 5 | Moderate |
| 7 | Major |
| 9 | Intense |

# *Ionospheric Phenomena*

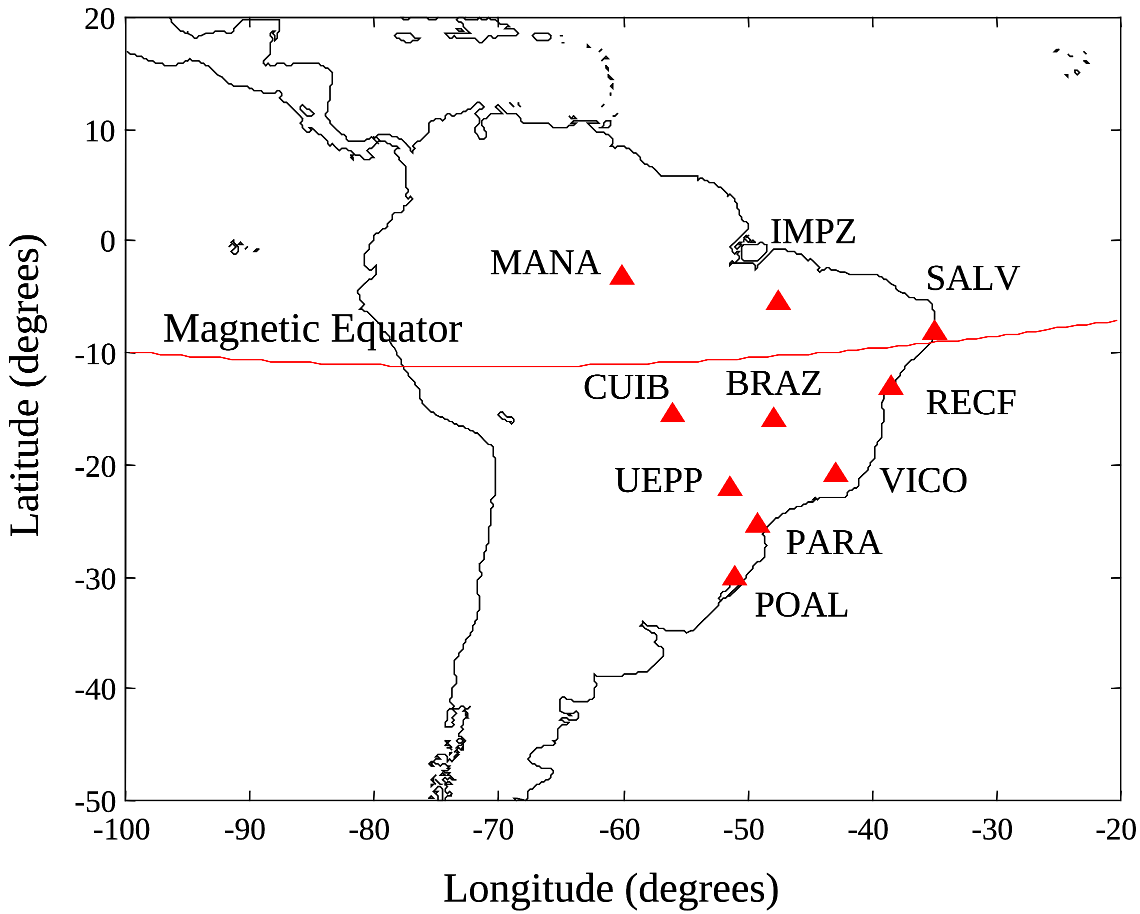
- Geomagnetic/Magnetospheric Sub-Storm
  - High-latitudes
  - Precipitation of energetic particles into the night-side ionosphere
  - Southward extension of auroral oval
- Magnetic Storm
  - Low-latitudes
  - Global event
- Ionospheric Storm
  - Mid-latitudes
  - Characterized by variations in TEC
- Traveling Ionospheric Disturbance
  - Travels from high to low latitudes
  - Significant variation in F-layer TEC at mid-latitudes

# *Ionospheric Phenomena*

- Scintillation
  - Low (equatorial anomaly) and high (auroral oval) latitudes
  - Electron density irregularities
  - Rapid variations of both the amplitude and phase of a traversing signal/wave
  - Correlated with level of geomagnetic activity
  - Impact on GPS includes
    - Lower SNR (higher measurement noise)
    - Cycle slips (especially on L2 when AS is on, and on codeless receivers)
    - Possible loss of lock (especially on L2 when AS is on, and on codeless receivers)
  - Best countermeasure is to use geomagnetic predictions to avoid collecting data during highly active periods

# Scintillation Example

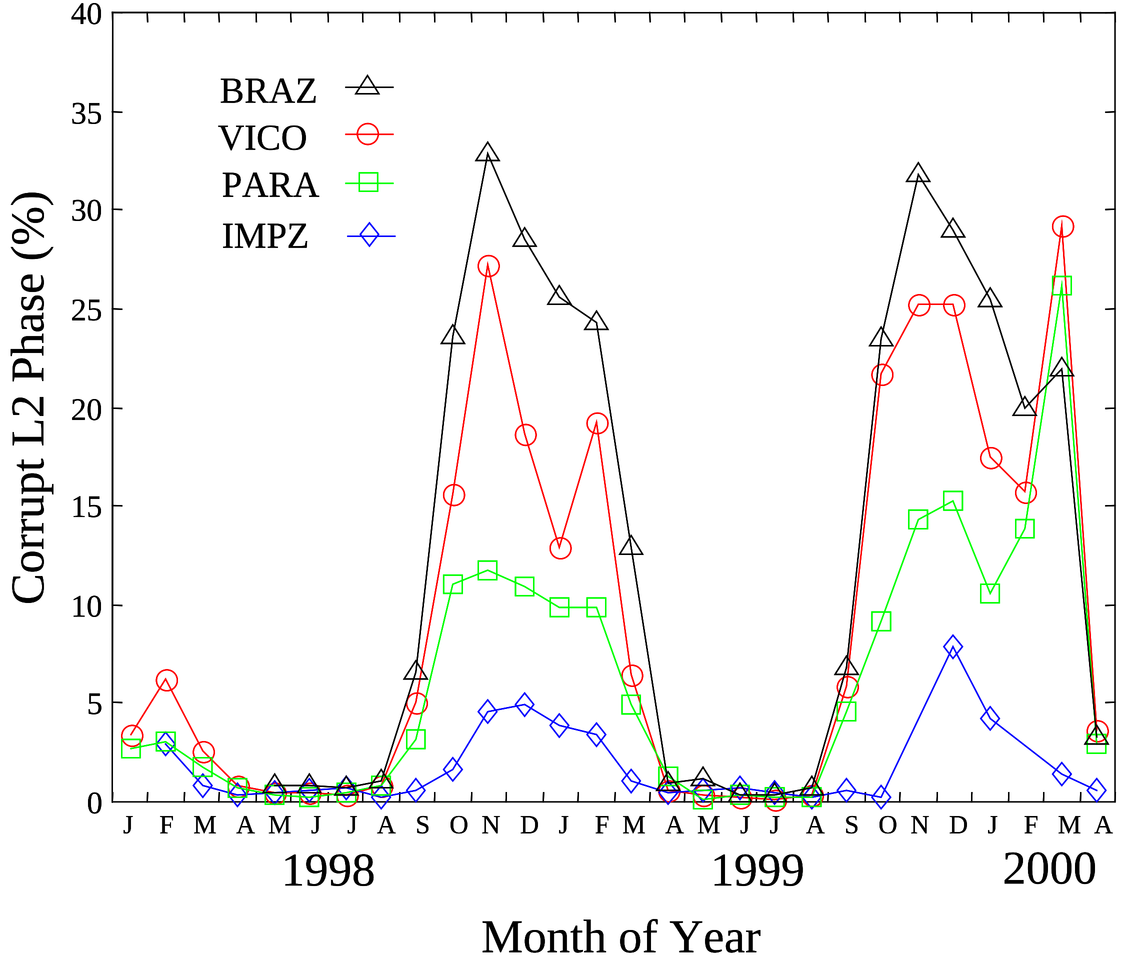
- Brazilian stations of the RBMC network
- Data available 1998-2000
- Trimble 4000SSi receivers




Plots from S. Skone

# Scintillation Example – Corrupted L2 Codeless Phase Measurements

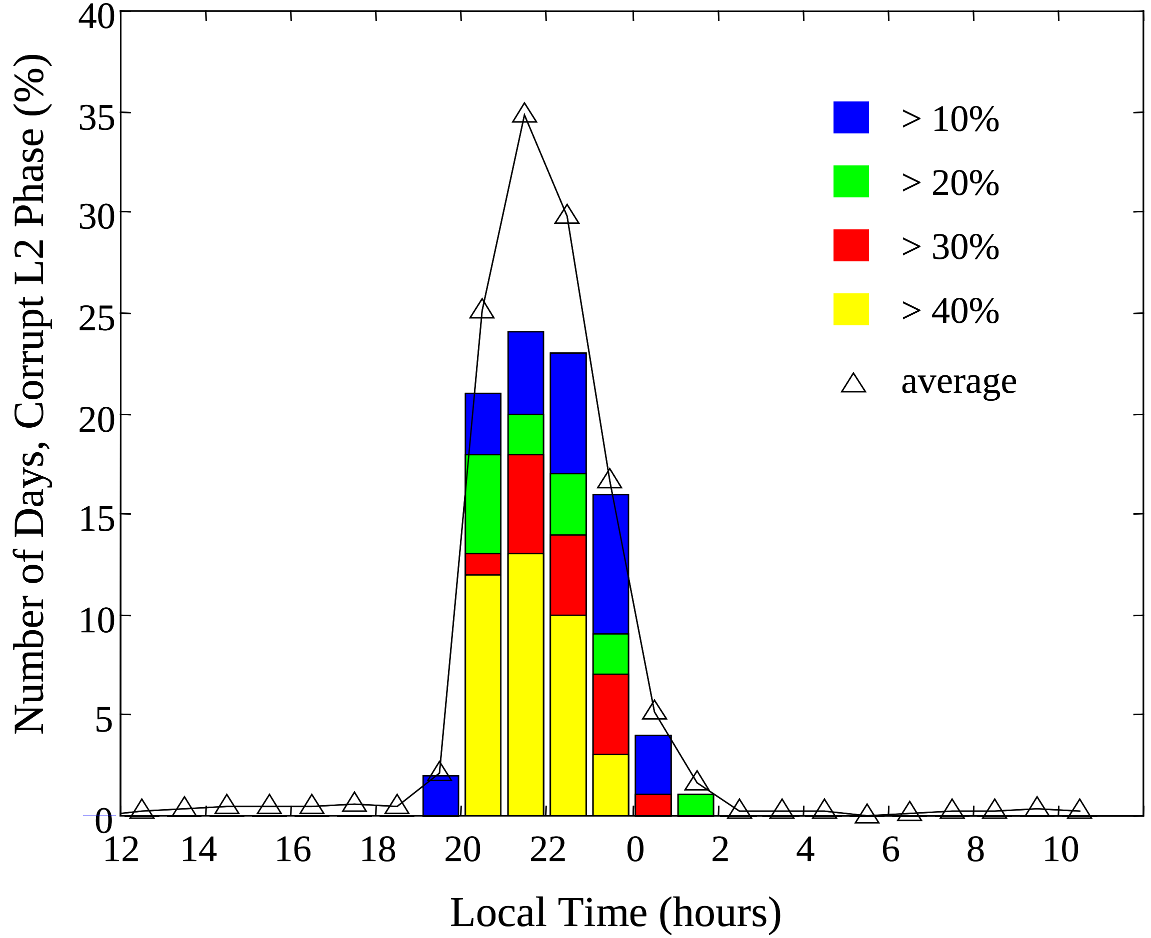
- Seasonal variation in L2 phase tracking
- Mean monthly %
- Local Times 2000-2300




Plots from S. Skone

# Scintillation Example - Corrupted L2 Codeless Phase Measurements

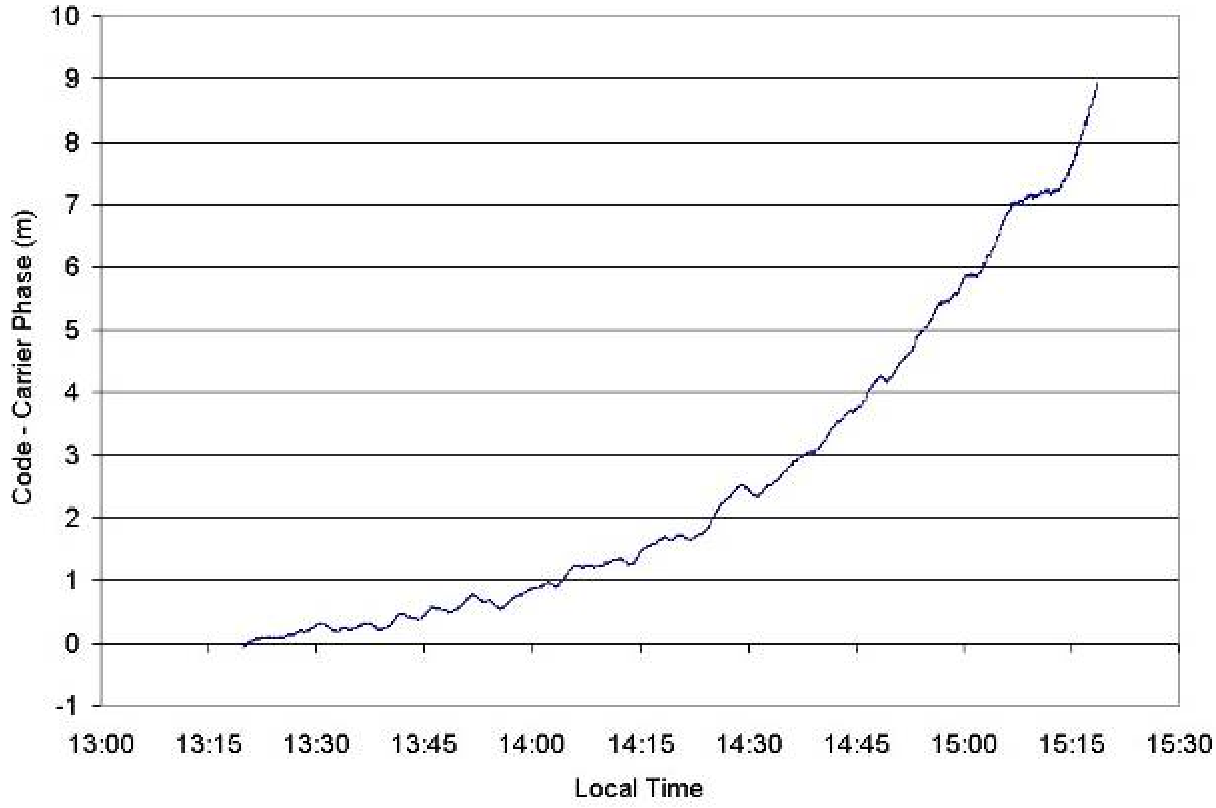
- Diurnal variation in L2 phase tracking
- Vicosa station
- March 2000




Plots from S. Skone

# Code-Carrier Divergence

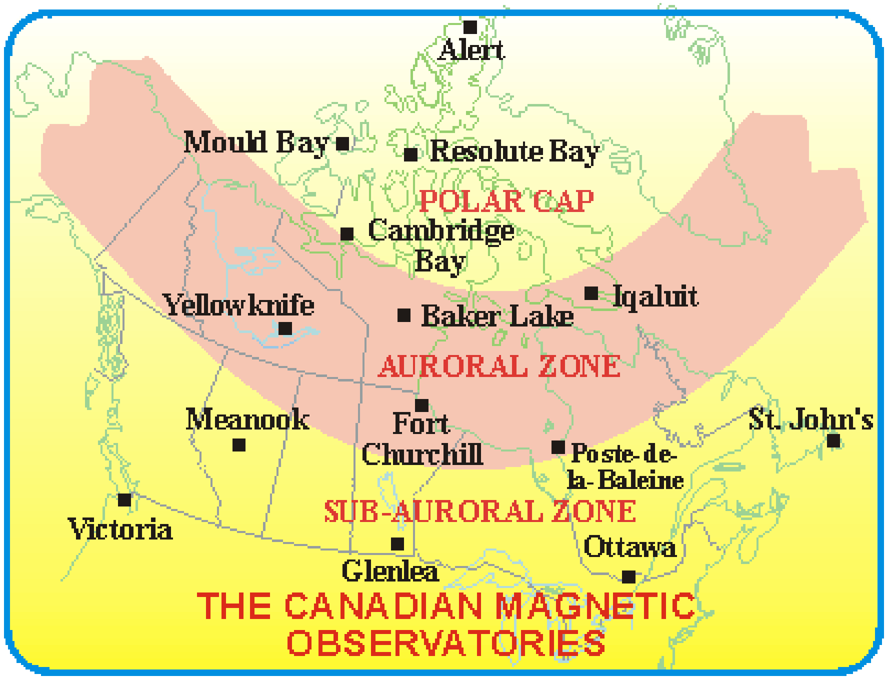
- Code and carrier can diverge over time when they are mixed
- Magnitude is dependent on the ionospheric activity
- Example for data collected on May 7, 2001 at the UofC
    - 2-hour data set (10 Hz)



# *Predicting the Ionosphere using Geomagnetic Observatories - Example*

- There are 14 geomagnetic observatories in Canada operated by the Geological Survey of Canada (NRCan)
- Observatories make observations of the Sun to predict the behaviour of the Earth's magnetic field (and thus indirectly predict ionospheric activity)
  - Forecasts are available for 27-day periods and are available at http://gsc.nrcan.gc.ca/geomag/index_e.php
- Activity is reported as a DRX index
  - Average of the hourly ranges (max minus min) in the northward component of the magnetic field
- DRX is reported for three regions
  - Polar cap ($\varphi > 70^{o}$N - "above" the auroral oval)
  - Auroral zone ($55^{o} > \varphi > 70^{o}$N)
  - Subauroral zone ($\varphi < 55^{o}$N)

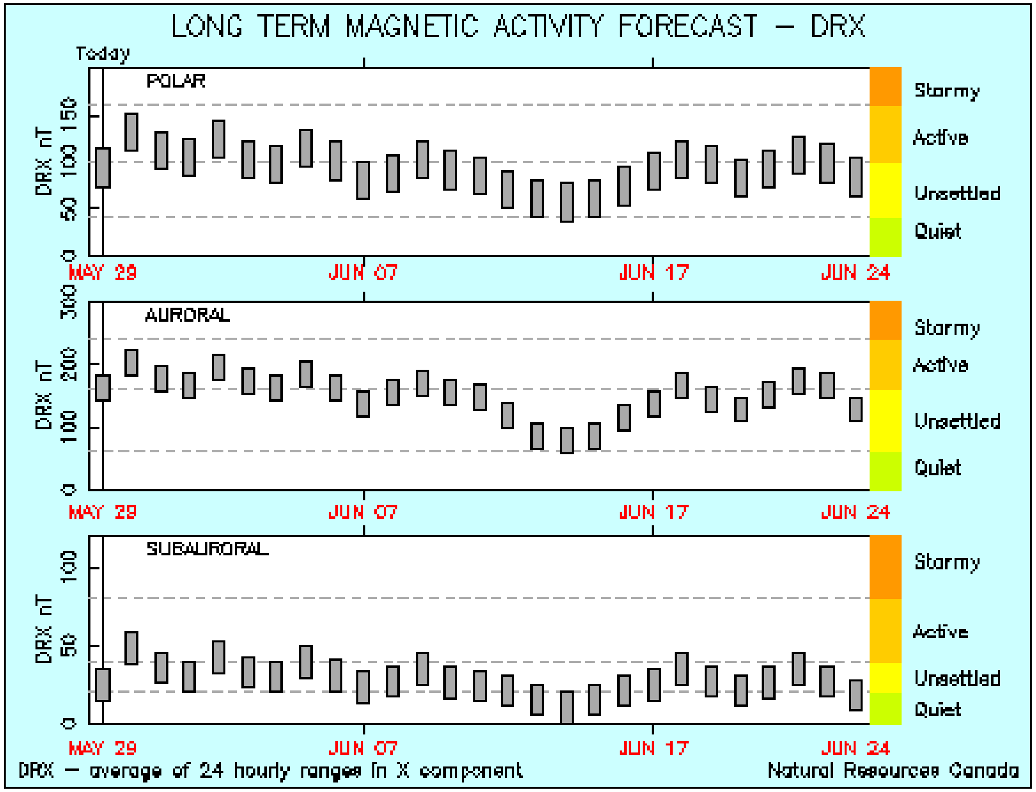
# *Predicting the Ionosphere using Geomagnetic Observatories*




| Zone/DR Index | Quiet | Unsettled | Active | Stormy |
|---|---|---|---|---|
| Polar Cap | 0 – 40 nT | 41 – 100 nT | 100 – 160 nT | > 160 nT |
| Auroral | 0 – 60 nT | 61 – 160 nT | 160 – 240 nT | > 240 nT |
| Sub-Auroral | 0 – 20 nT | 21 – 40 nT | 40 – 80 nT | > 80 nT |

# Predicting the Ionosphere using Geomagnetic Observatories

- Typical 27-day geomagnetic forecast
  - Short-term (24-hour) forecasts are also available



# *Ionospheric Compensation 1*

- Ionosphere-Free (IF) Linear Combination
  - Removes the first order effects of the ionosphere
    - For code measurements:

$$P_{IF} = P_{L1} - \frac{f_{L2}^2}{f_{L1}^2} \cdot P_{L2}$$

$$= \rho + \frac{40.3 \cdot TEC}{f_{L1}^2} + \varepsilon_{L1} - \frac{f_{L2}^2}{f_{L1}^2} \cdot \left( \rho + \frac{40.3 \cdot TEC}{f_{L2}^2} + \varepsilon_{L2} \right)$$

$$= \left( 1 - \frac{f_{L2}^2}{f_{L1}^2} \right) \cdot \rho + \underbrace{\varepsilon_{L1} - \frac{f_{L2}^2}{f_{L1}^2} \cdot \varepsilon_{L2}}$$

Non-dispersive errors are reduced to 39.3% of their original value

# Ionospheric Compensation 2



- Ionosphere-Free (IF) Linear Combination con't
  - For phase observations

$$\Phi_{IF} = \Phi_{L1} - \frac{f_{L2}}{f_{L1}} \cdot \Phi_{L2}$$

$$= \frac{\rho}{\lambda_{L1}} + \frac{40.3 \cdot TEC}{cf_{L1}} + \frac{\varepsilon_{L1}}{\lambda_{L1}} - \frac{f_{L2}}{f_{L1}} \cdot \left( \frac{\rho}{\lambda_{L2}} + \frac{40.3 \cdot TEC}{cf_{L2}} + \frac{\varepsilon_{L2}}{\lambda_{L2}} \right)$$

$$= \left( 1 - \frac{f_{L2}^2}{f_{L1}^2} \right) \cdot \rho + \varepsilon_{L1} - \frac{f_{L2}}{f_{L1}} \cdot \varepsilon_{L2}$$

# Ionospheric Compensation 3



- Broadcast model
- Also called “Klobuchar model” or “half-cosine model”
  - Parameters (8) are transmit in the GPS navigation message
  - Assumes two components
    - Constant (DC) nighttime component with half-cosine during day

$$d_{iono}^{Night} = DC = 5\,ns$$

$$d_{iono}^{Day} = DC + A\cos\left[2\pi\left(t - 50400\right)/\,P\right]$$

    - A and P are functions of the broadcast parameters
    - t is local time (50400 ≡ 14:00)
  - Intended for single-frequency users in continental US (CONUS)
    - Removes ~50% of ionospheric delay at mid-latitudes

Klobuchar, J.A. (1996) **Ionospheric Effects on GPS**, B.W. Parkinson and J.J. Spilker, Jr. eds., Global Positioning System: Theory and Applications, Volume I, Chapter 4, American Institute of Aeronautics and Astronautics, Inc., Washington, DC, USA

# *Ionospheric Compensation 4*

- Wide Area Models (2D)
  - Example: NRCan Model using data from Canadian Active Control System (CACS) network
    - Uses dual-frequency data to estimate the ionospheric effect
    - Assumes TEC is concentrated on a thin spherical shell at an altitude of 350 km
    - Must account for inter-frequency (SV & Rx) and inter-channel (Rx) biases to obtain unbiased estimate of ionospheric delay
    - Slant delays at the “pierce point” of the receiver-satellite range with the shell are mapped to vertical and used to generate a grid of delays (in latitude and longitude)
    - User interpolates between grid points and maps error to a slant delay
    - Accuracy of 10 cm at zenith to 1 m at the horizon

# *Ionospheric Compensation 5*

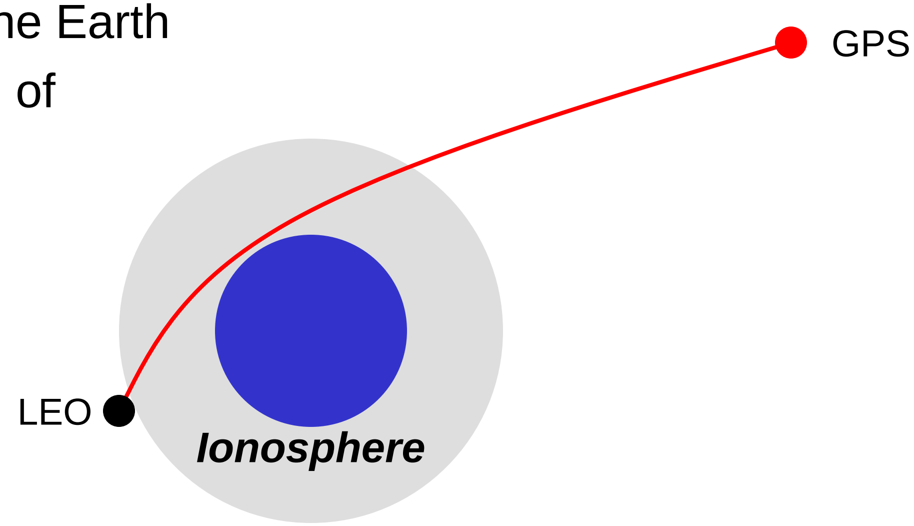
- Global Ionospheric Models
  - Grid-based models
    - IONosphere map EXchange (IONEX) format
  - Spherical harmonic expansion
    - Bernese ION format

- Three-dimensional models
  - Ground-based tomography
    - Represent electron density as a finite function of latitude and longitude
    - Better than 2D approach by up to 1 m

# Ionospheric Compensation 6



- Three-dimensional models con't
  - Voxel approach
    - Divide ionosphere into separate boxes (voxels) and estimate vertical TEC in each voxel
    - Accuracies of better than 10 cm (vertical) in various conditions
  - Occultation
    - Use LEO satellites to measure GPS signals as they occult the Earth
    - Provides vertical profile of ionosphere



## Ionospheric Indices



- K Index
  - Local indicator
  - Measured in eight three-hour intervals daily
  - Ranges from zero (low) to nine (high)
- Kp Index
  - Global indicator of geomagnetic (ionospheric) variation
  - Average of local K indices
- Disturbed Storm-Time (DST) Index
  - Magnetic index from ~5 low-latitude stations
  - Similar to Kp index, but in one-hour intervals

# Measuring the Ionosphere with Two Frequencies 1

- Exploit the dispersive nature of the ionosphere by making pseudorange measurements on two frequencies

$$\Delta P_{[m]} = P_{L1} - P_{L2} \cong I_{L1}^{P} - I_{L2}^{P}$$

$$= 40.3 \cdot TEC \cdot \left[ \frac{1}{f_{L1}^2} - \frac{1}{f_{L2}^2} \right]$$

$$= \frac{f_{L2}^2 - f_{L1}^2}{f_{L2}^2} \cdot I_{L1}$$

$$I_{L1}^{P} = \left(P_{L1} - P_{L2}\right) \cdot \frac{f_{L2}^2}{f_{L2}^2 - f_{L1}^2}$$

$$P_{IF} = P_{L1} - I_{L1}^{P}$$

$$= P_{L1} - \left(P_{L1} - P_{L2}\right) \cdot \frac{f_{L2}^2}{f_{L2}^2 - f_{L1}^2}$$

$$= \frac{-f_{L1}^2}{f_{L2}^2 - f_{L1}^2} \cdot P_{L1} + \frac{f_{L2}^2}{f_{L2}^2 - f_{L1}^2} \cdot P_{L2}$$

$$= \frac{f_{L1}^2}{f_{L1}^2 - f_{L2}^2} \cdot P_{L1} - \frac{f_{L2}^2}{f_{L1}^2 - f_{L2}^2} \cdot P_{L2}$$

$$= P_{L1} + 1.54 \{P_{L1} - P_{L2}\} \text{ for GPS}$$

Correct $P_{L1}$ for first order ionospheric effects (>99% of total) at the cost of increased noise

$$\Rightarrow \quad \sigma_{IF}^2 = (2.546)^2 \sigma_{L1}^2 + (1.546)^2 \sigma_{L2}^2$$

$$\approx (2.978)^2 \sigma_P^2$$

# Measuring the Ionosphere with Two Frequencies 2

- Same approach can be taken with carrier phase measurements

Scale to L1 cycles

$$\Delta\phi_{[cyc]} = \phi_{L1} - \frac{f_{L1}}{f_{L2}} \cdot \phi_{L2} \cong I^{\phi}_{L1} - \frac{f_{L1}}{f_{L2}} \cdot I^{\phi}_{L2} + N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2}$$

$$= \frac{-40.3 \cdot TEC}{c} \cdot \left[ \frac{1}{f_{L1}} - \frac{f_{L1}}{f_{L2}} \cdot \frac{1}{f_{L2}} \right] + N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2}$$

$$= I^{\phi}_{L1} \cdot \frac{f_{L2}^2 - f_{L1}^2}{f_{L2}^2} + N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2}$$

"Geometry-free" combination for phase

$$I^{\phi}_{L1} = \left[ \left( \phi_{L1} - \frac{f_{L1}}{f_{L2}} \cdot \phi_{L2} \right) - \left( N_{L1} - \frac{f_{L1}}{f_{L2}} \cdot N_{L2} \right) \right] \cdot \frac{f_{L2}^2}{f_{L2}^2 - f_{L1}^2}$$

- Requires knowledge of carrier phase ambiguities
  - If no cycle slips occur a given time interval, the differential ionospheric error over the interval can be computed

# Measuring the Ionosphere with Two Frequencies 3

- Can apply the correction to the L1 phase
  - Instead, form the ionosphere-free (IF) linear combination for carrier phase measurements

$$\phi_{IF} = \phi_{L1} - \frac{f_{L2}}{f_{L1}} \cdot \phi_{L2}$$

  - IF ambiguities are not integer in nature

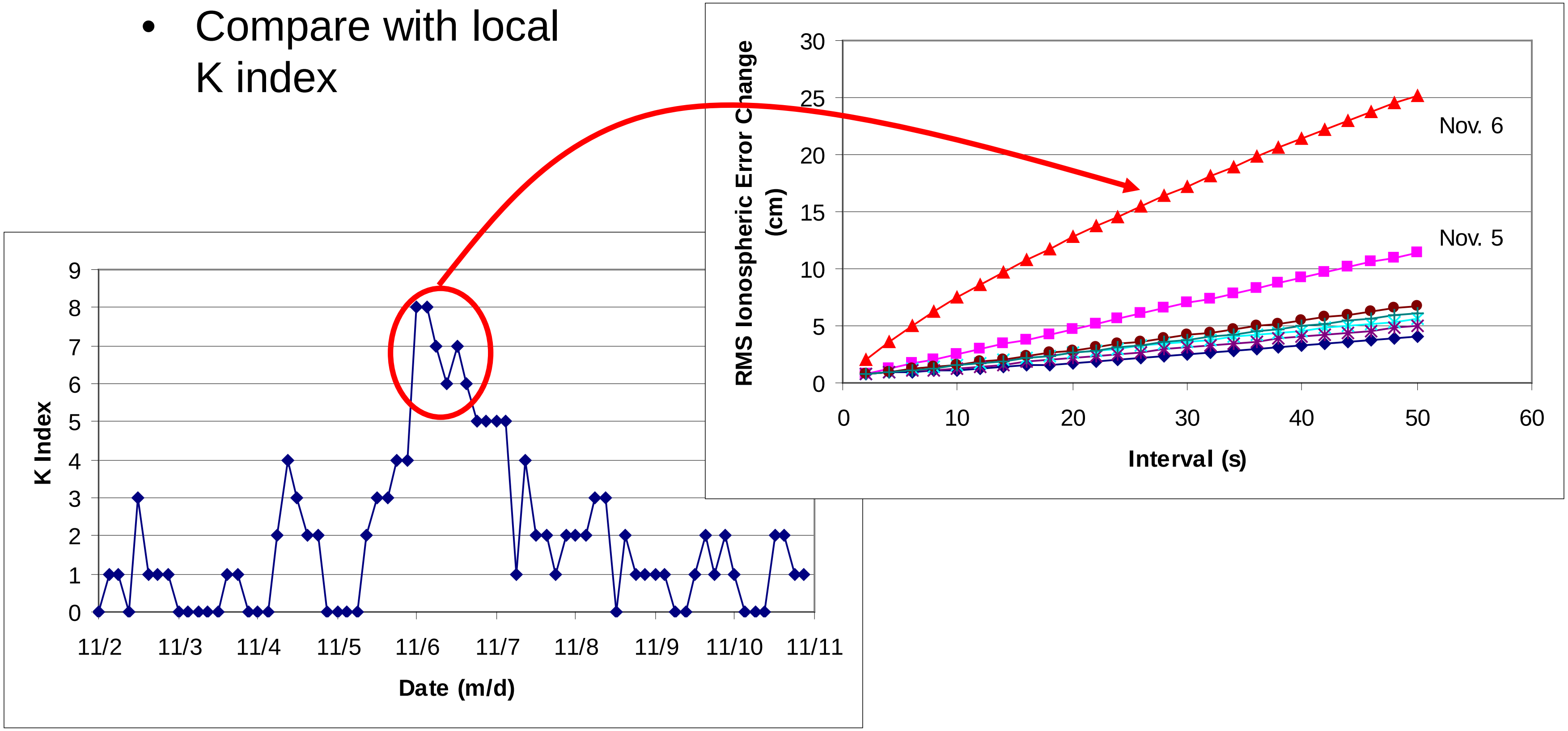
$$N_{IF} = N_{L1} - \frac{f_{L2}}{f_{L1}} \cdot N_{L2}$$

    - Must estimate ambiguities as real-valued parameters
    - Slow convergence (several hours)
  - Alternatively, if L1 and L2 ambiguities are known, an “IF fixed” solution is possible

# Undifferenced Errors



- Compute the RMS ***change*** in orbital error as a function of time and take the average across all satellites
  - Compare with local K index

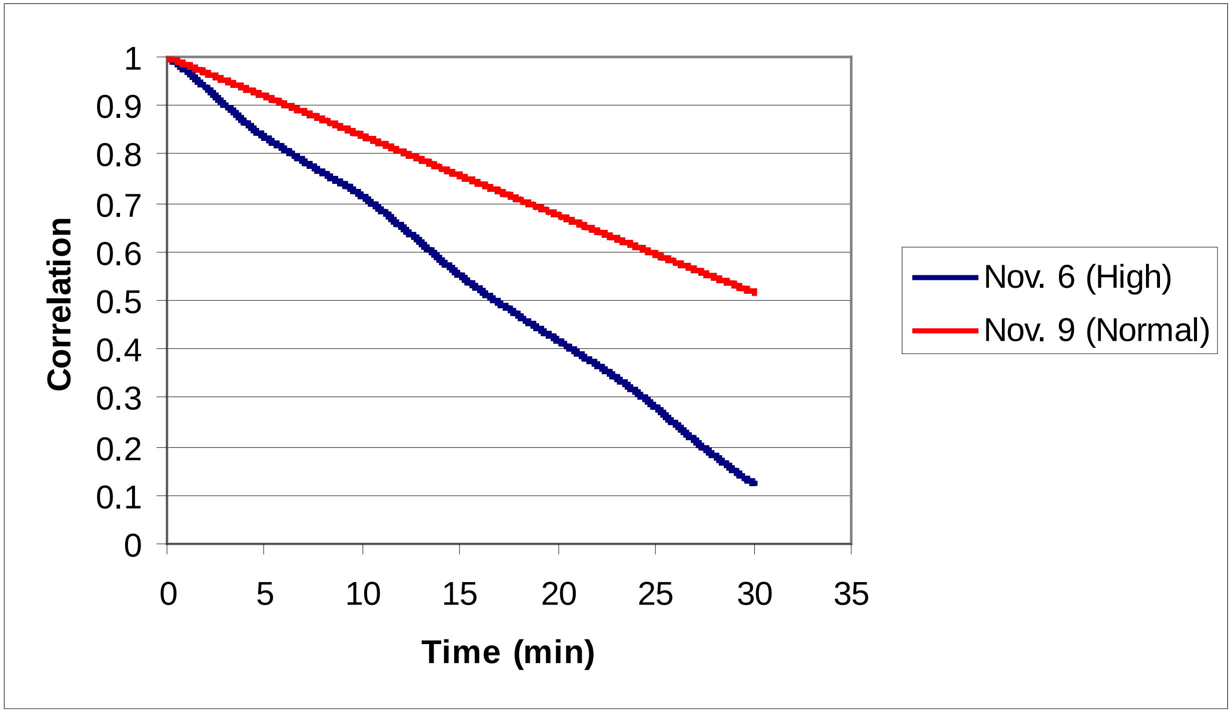



Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164

# Undifferenced Errors



- Autocorrelation function shows a much more rapid error decorrelation during periods of high ionospheric activity

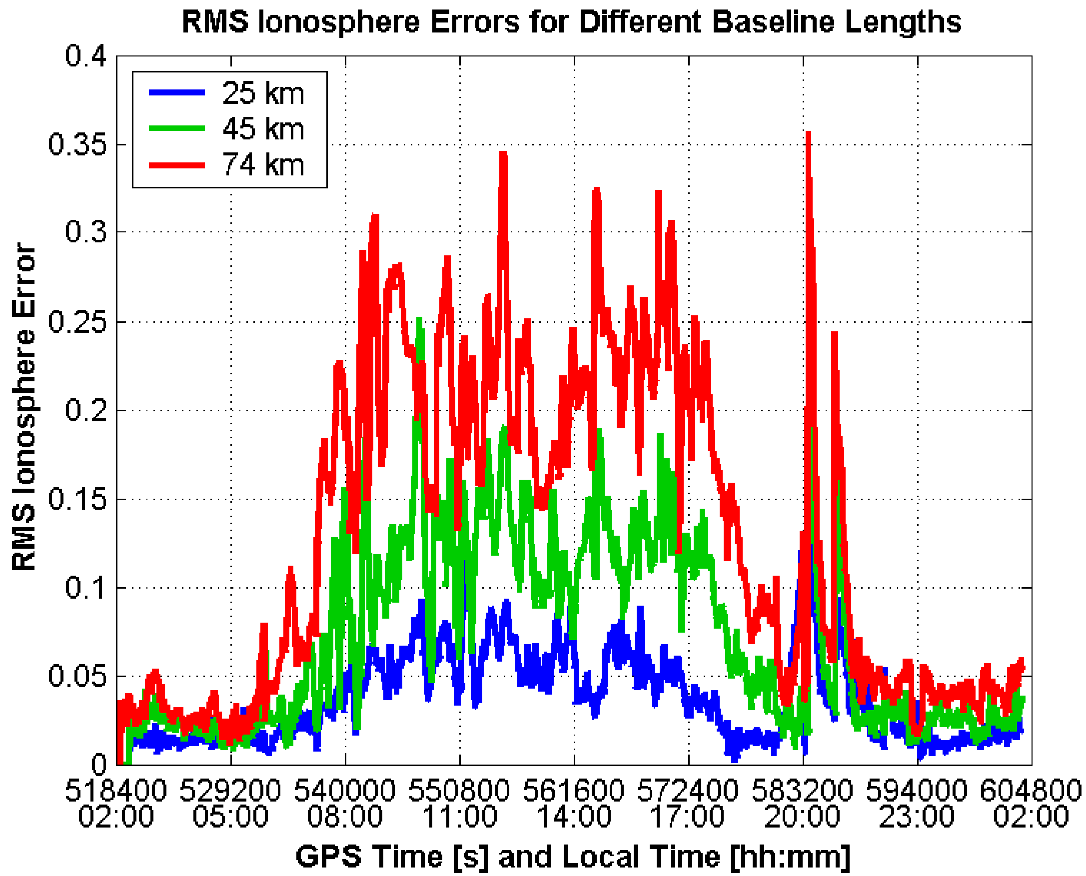



Olynik, M.C. (2002), **Temporal Characteristics of GPS Error Sources and Their Impact on Relative Positioning**, MSc Thesis, Department of Geomatics Engineering, University of Calgary, UCGE Report Number 20164

# Differential Error Example



- RMS (across satellites) of double difference ionosphere errors as a function of time
  - Diurnal effect is obvious
  - Strong dependence with baseline length
  - Jumps near 20:00 and 21:00 caused by new low elevation satellites




Data from TUBITAK Marmara Continuous GPS Network – MAGNET (Turkey)

# *Observed Differential Error Magnitudes*

| Geographic Location | Error Magnitudes | |
|---|---|---|
| | Typical (RMS) | Extreme |
| **Auroral Zone** | 1-3 ppm | 10 ppm<br>10 ppm |
| **Mid-Latitude** | 1-3 ppm | 8 ppm<br>10 ppm |
| **Low Latitudes** | 1-3 ppm | 17 ppm<br>30 ppm |

Troposphere

## *General Structure and Characteristics 1*

- Proper troposphere extends 9 km over the pole to 16 km over the equator. It includes most of the water vapour. In GPS applications, the neutral (**non-dispersive**) part of the atmosphere, up to about 50-70 km, is called troposphere.
- The troposphere is non-dispersive for GPS frequencies (i.e. it affects L1 and L2 the same)
- Delay is about 2.4 m at the zenith to 9.3 m at a 15 deg elevation angle
- Simple models can estimate the effect to within 20 cm – not a problem for low accuracy positioning
- Problems exist for high accuracy positioning – a tropospheric modeling error is amplified in the vertical component

# *General Structure and Characteristics 2*

- Tropospheric delay is subdivided into dry (hydrostatic) and wet components, with the latter being harder to model
- Not a large problem for short baselines with similar heights, but can be otherwise if a high accuracy is required
- Methods to compensate for the troposphere include
  - Apply a tropospheric model (e.g. Saastomoinen)
  - Model a residual zenith delay as part of the estimation process - available as an option in many GPS post-processing packages
  - Use a water vapour radiometer (expensive)

# Summary of Tropospheric Effects



- Range Delay
  - Group and phase index of refraction is greater than unity
- Ray Bending
  - Refraction effects
- Absorption
  - 0.03 dB at zenith
  - 0.3 dB at lower elevations
- Scintillation
  - Caused by small scale variations in atmosphere particularly at lower altitudes (e.g. clouds)

## *Refractivity*



- For tropospheric effect, use of the index of refraction is often replaced by the refractivity, N

$$N = (n-1)\cdot 10^6 = \delta n \cdot 10^6$$

  - Can be interpreted as a parts-per-million error
  - Typical global average is 320
- Refractivity is usually broken into dry (hydrostatic) and wet parts

$$N = N_d + N_w$$

# Refractivity

- Dry (hydrostatic) refractivity accounts for 80-90% of the total errors and is a function of
  - Surface pressure (P)
  - Surface temperature (T)
- Wet refractivity accounts for 10-20% of the total errors and is a function of
  - Partial pressure of water vapour (e)
  - Surface temperature (T)
- Sample formulation

$$N = \underbrace{776.4\frac{P}{T}}_{\text{Dry}} + \underbrace{3.73x10^{6}\frac{e}{T^{2}}}_{\text{Wet}}$$

  - P and e in kPa
  - T in Kelvin

# Troposphere Models

- Models try to account for the height dependence of the tropospheric effects
- Models usually consist of two parts
  - Dry and wet refractivity modeling at the zenith
  - Dry and wet mapping function

$$\begin{aligned} N &= N_d + N_w \\ &= N_d^z \cdot m_d(\varepsilon) + N_w^z \cdot m_w(\varepsilon) \end{aligned}$$

  - Mapping functions assume the atmosphere is isotropic
- Typical model accuracies
  - Dry term accuracy is better than 1%
  - Wet term accuracy is 10-20%

# Troposphere Models

- Hopfield's model assumes the refractivity varies with altitude

$$N_d^z(h) = N_{d_{Surf}}^z \cdot \left( \frac{H_d - h}{H_d} \right)^4 \qquad N_w^z(h) = N_{w_{Surf}}^z \cdot \left( \frac{H_w - h}{H_w} \right)^4$$

where

$$\mathrm{H_d} = 40136 + 148.72 \cdot (\mathrm{T} - 273.16) \qquad \mathrm{H_w} = 11\,\mathrm{km}$$

- Integrating with respect to height gives the following approximations

$$N_d^z \approx \frac{N_{d_{Surf}}^z H_d}{5} \qquad N_w^z \approx \frac{N_{w_{Surf}}^z H_w}{5}$$

Hopfield, H. S., "Tropospheric effect on electromagnetically measured range: Prediction from surface weather data." Radio Science, vol. 6, no. 3, pp. 357-367, (1971)

# Troposphere Models

- Hopfield model con't
  - Mapping functions

$$m_d\left(\varepsilon\right)=\frac{1}{\sin\sqrt{\varepsilon^2+6.25}} \qquad m_w\left(\varepsilon\right)=\frac{1}{\sin\sqrt{\varepsilon^2+2.25}}$$

- Other tropospheric models
  - Saastamoinen
  - Modified Hopfield
  - Black and Eisner
  - Baby et al.
  - Ifadis (wet term only)

Black, H. D., “An easily implemented algorithm for the tropospheric range correction.” Journal of Geophysical Research, vol. 83, no. B4, pp. 1825-1828, (1978)

Saastamoinen, J., “Contributions to the theory of atmospheric refraction.” Bulletin Géodésique, no. 107, pp. 13-34, (1973).

# Measuring the Troposphere

- Use the ionosphere-free linear combination
  - Need to use carrier phase data since code is too noisy
  - Remaining differential errors are
    - Troposphere (sought)
    - Orbital error
    - Phase multipath
    - Phase noise

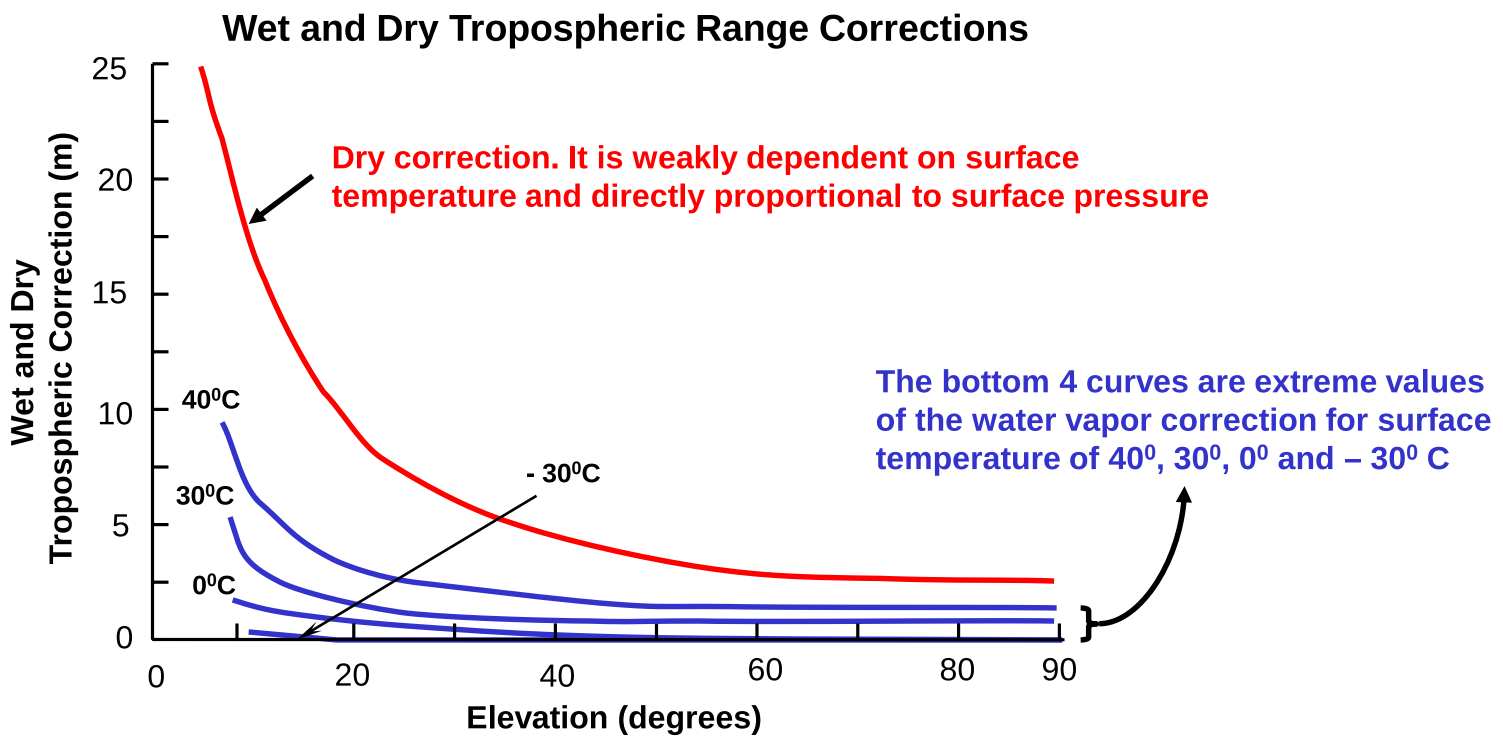
Need to mitigate these effects (precise orbits, good site selection, high quality receivers, etc.)

- Water Vapour Radiometer (WVR)
  - Measures frequencies that are dispersive in the troposphere
  - Uses an approach similar to that used with L1 and L2 for the ionosphere
  - Very expensive
  - Does not work when it rains

# Undifferenced Errors



- Zenith error is approximately 2.4 m
  - Total effect is proportioned approximately as 80-90% for the dry term and 10-20% for the wet term




Black, H.D. and A. Eisner (1984), **Correcting Satellite Doppler Data for Tropospheric Effects**, Proc. Of the International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Nat. Tech. Univ. of Athens, pp. 1-31.

**Troposphere**

# *Differential Errors*

- Once a troposphere model is applied, the differential errors are rarely larger than about 3 ppm
- Typical differential error magnitudes are less than 1 ppm
- Increased differential errors occur mainly in areas of high humidity and variations thereof (e.g. land-see) and/or areas of unstable weather (e.g. storms)