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Chapter 4

GPS Observables and Differencing Concepts

GPS Observables

Code and carrier phase measurements. Timing issues. Linear combination of observations, loss of phase lock and cycle slips. Carrier phase smoothing of the code. Float (real number ambiguities) solutions.

Differencing Concepts

Point positioning methods & related equations. Between receiver, between satellite and between epochs single differencing, double & triple differencing. Derivation of associated design matrices

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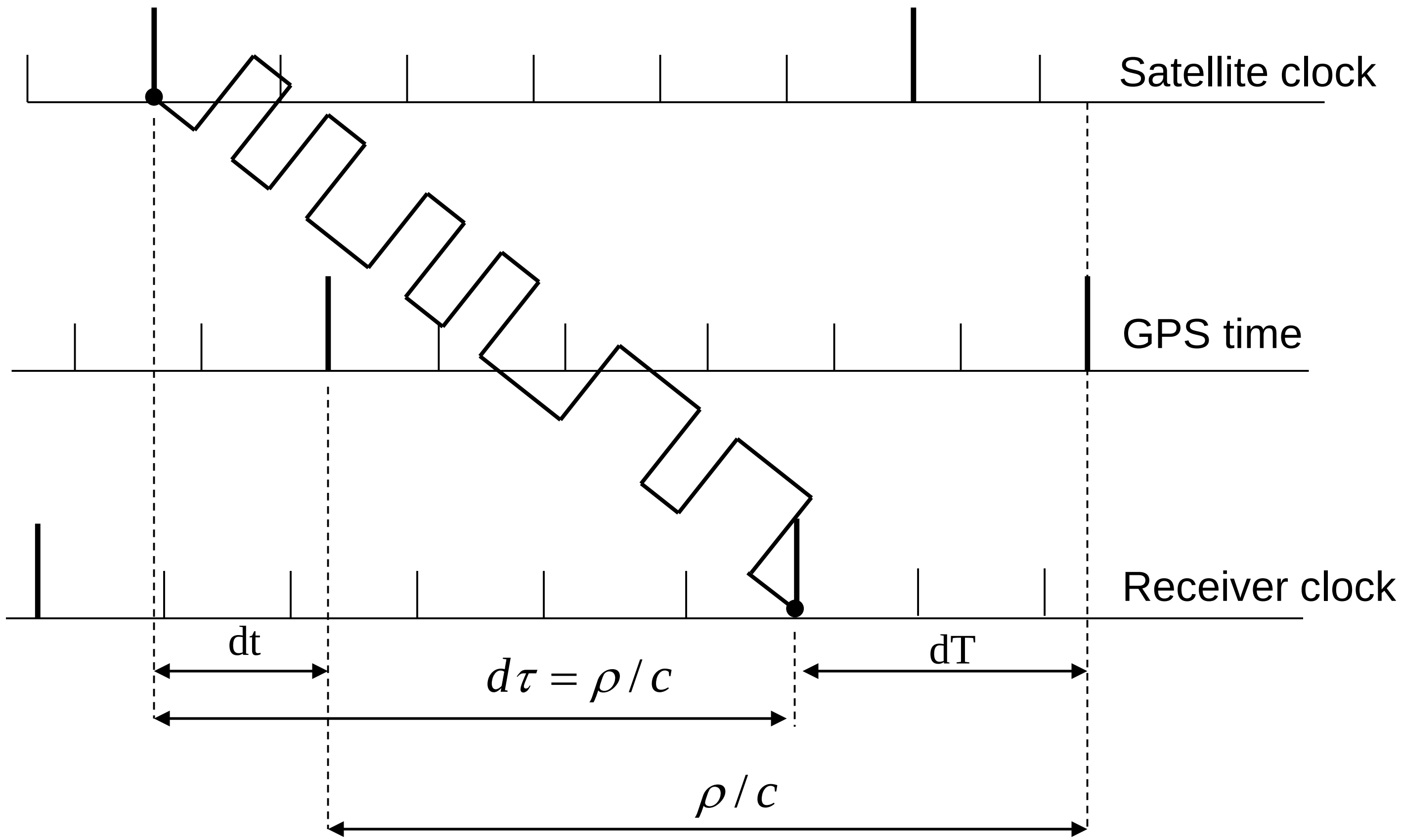
Pseudorange Concept



Pseudorange Observation Equation

- Observation equation (in units of length, (e.g. m)):

$$P = cd\tau = \rho + d\rho + c(dt - dT) + d_{ion} + d_{trop} + \varepsilon_P$$

$$\rho = \sqrt{(x^s - x_R)^2 (y^s - y_R)^2 + (z^s - z_R)^2}$$

where

P	***Pseudorange*** measurement
ρ	geometric range (i.e.$\lVert \mathbf{r}^s - \mathbf{R}_r \rVert$)
$d\rho$	orbital errors
x, y, z	position vector of SV (known) and rx (unknown)
dt, dT	satellite and receiver clock errors, respectively
d_{ion}	ionospheric delay
d_{trop}	tropospheric delay
ε_P	noise {function of ε(code noise) and ε(code multipath)}

ε (C/A code noise) ≈ 5 – 200 cm for LOS measurements
ε (P code noise) ≈ 10 cm
ε (code multipath) ≤ 1 chip (non-Gaussian)

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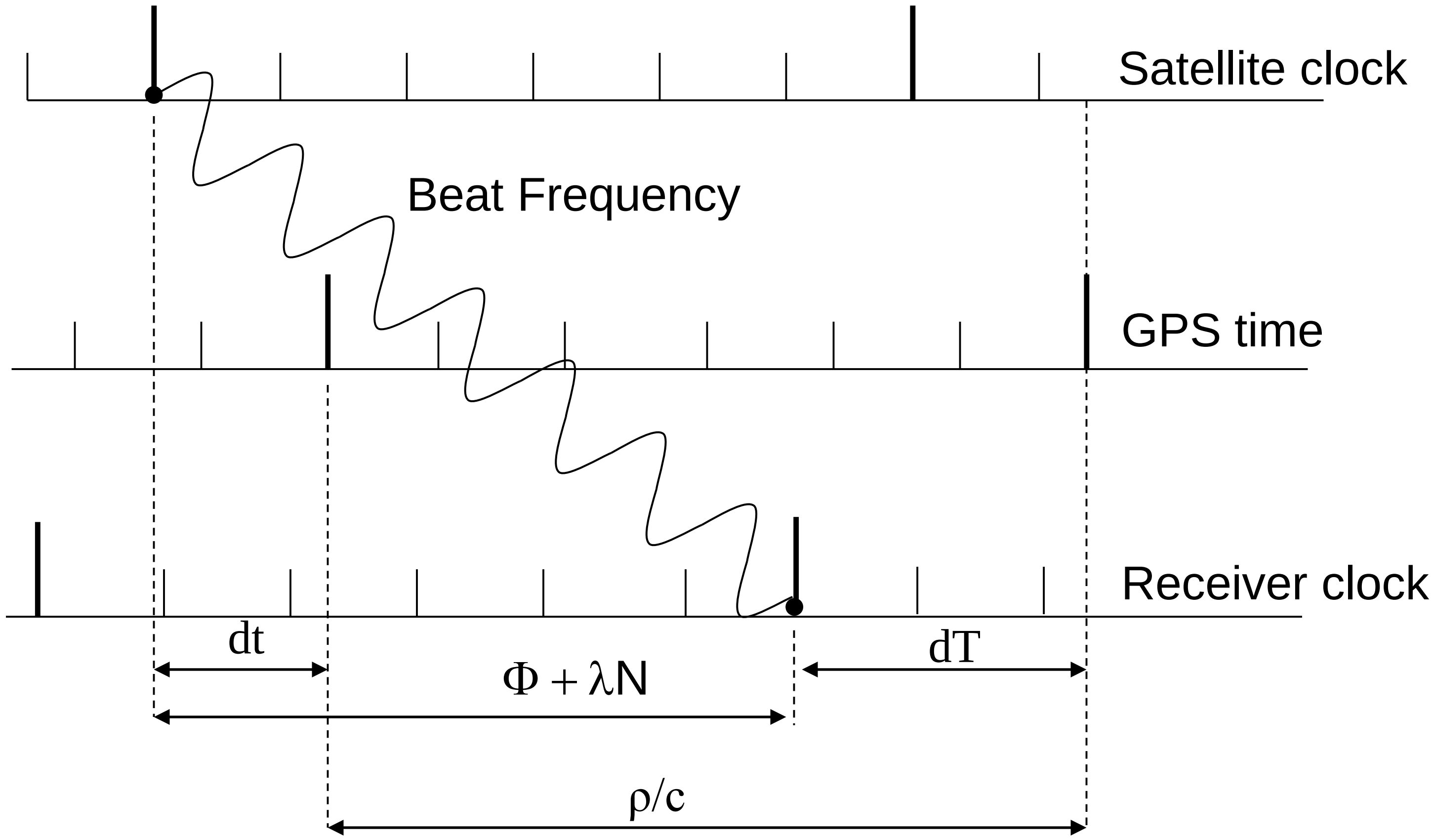
Carrier Phase Measurement Concept



Carrier Phase Observation Equation

- Observation equation (in unit of length (e.g. m)):

$$\Phi = \rho + d\rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_{\Phi}$$

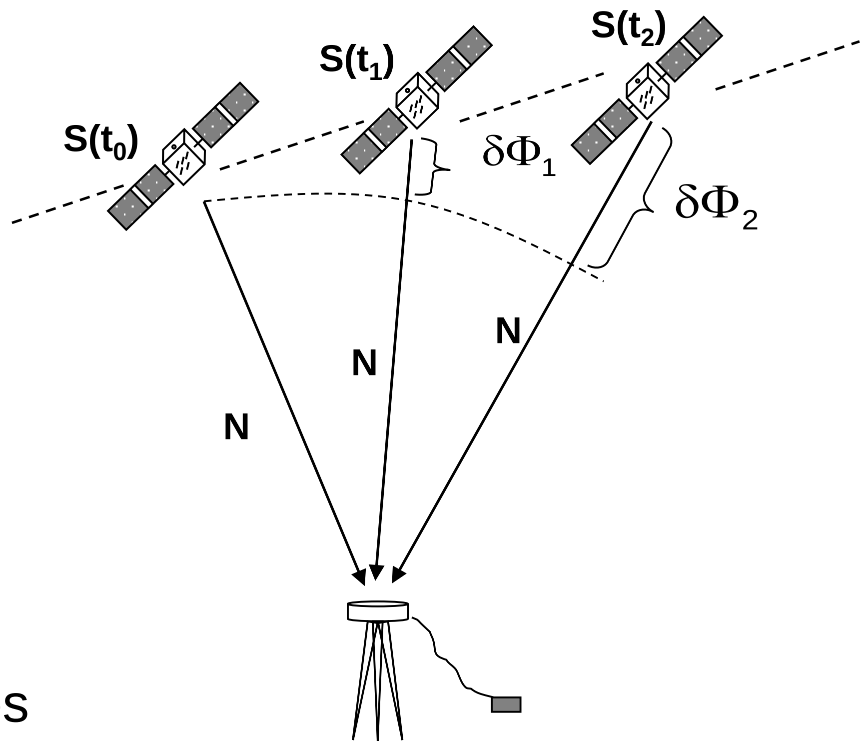
$$\rho = \sqrt{(x^s - x_R)^2 (y^s - y_R)^2 + (z^s - z_R)^2}$$

Φ	**<u>Phase</u>** measurement
ρ	geometric range (i.e.|| **r**^{s} – **R**$_r$ ||)
$d\rho$	orbital errors
x, y, z	position vector of SV (known) and rx (unknown)
dt, dT	satellite and receiver clock errors, respectively
λ, N	wavelength and ambiguity, respectively
d_{ion}	ionospheric delay
d_{trop}	tropospheric delay
ε_P	noise and multipath

noise ≈ 1-5 mm
multipath ≤ 0.25 λ

Carrier Phase Ambiguity *(1/2)*

- At lock-on, receiver measures fractional phase, integer number **N** (*integer ambiguity*) of cycles is unknown (one different ambiguity per satellite)
- A counter keeps track of accumulated number of cycles
- If no loss of lock occurs, the integer ambiguity remains constant
- Receiver accurately measures changes in range over time (due to satellite-receiver dynamics)



Carrier Phase Ambiguities (2/2)

- Receiver measures the fractional phase difference between the incoming signal and the reproduced signal
  - Number of integer cycles between epochs are counted by integrating the Doppler (beat) frequency (Hz * s)
  - Contains fractional + integer part (sometimes called accumulated phase or integrated Doppler)
- Carrier phase ambiguities are integer by definition and are defined at lock – on and are constant unless a cycle slip occurs
  - They are arbitrary (e.g. a few cycles or millions of cycles, + or -) and are different for each satellite-receiver measurement
- Example - approximate ambiguity derived from pseudorange (approximate since pseudorange is noisy – in reality it is constant)

GPS time (s)	Pseudorange (m)	Carrier phase (ϕ cycles)	Ambiguity (ϕ cycles – p/λ)
387234	22441825.779	-975001.392	-118907592
387235	22441597.023	-976188.862	-118907577
387236	22441371.704	-977375.523	-118907580

Doppler Frequency Overview

- Measure of the instantaneous phase rate (typically 1 Hz)
- Measurement is made on the phase lock loop
- Not affected by cycles slips and no phase ambiguity
- Observation equation (in m/s)

$$\dot{\Phi} = \dot{\rho} + d\dot{\rho} + c(d\dot{t} - d\dot{T}) - \dot{d}_{ion} + \dot{d}_{trop} + \varepsilon_{\dot{\Phi}}$$

where

$\dot{\Phi}$	**Doppler** measurement
$\dot{\rho}$	geometric range rate
$d\dot{\rho}$	orbital error drift
$d\dot{t}$	satellite clock error drift
$d\dot{T}$	receiver clock error drift
$\dot{d}_{ion}$	ionospheric delay drift
$\dot{d}_{trop}$	tropospheric delay drift
$\varepsilon_{\dot{\Phi}}$	noise (1-5 mm/s)

- Used for velocity estimation and cycle slip detection

Introduction to Differential Methods

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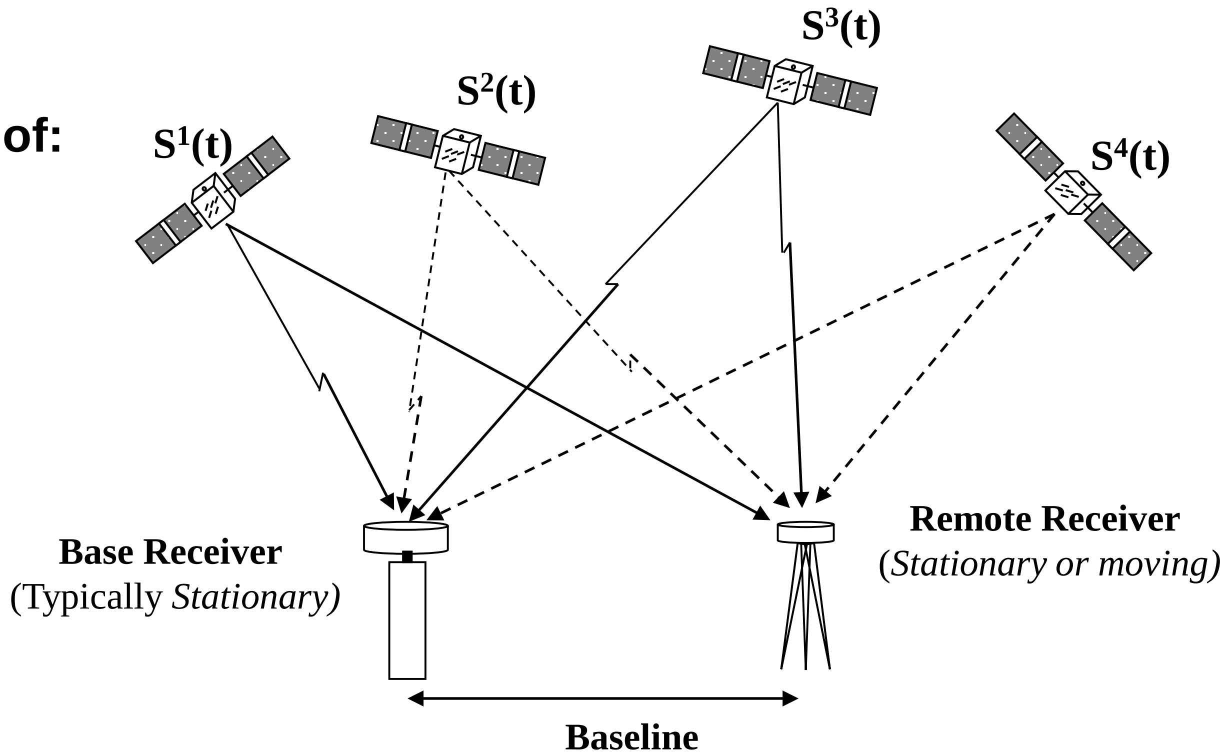
- Differential methods are commonly used eliminate **or** reduce errors that are spatially and/or temporally correlated
- For instance TDOA is a differential method in a sense as two pseudoranges (ranges with a common time bias – the receiver clock unknown) are differenced (one subtracted from the other)
- Differential methods are effective to remove timing biases, common propagation effects (e.g. Loran) and atmospheric effects (e.g. differential barometry to eliminate effects of atmospheric pressure variations)
- However, differential methods
  - Result in a loss of degrees of freedom
  - Thus, poorer geometry (higher Dilution of Precision)
  - Introduce serial correlations among derived measurements. The measurement covariance matrix of the derived measurements is a fully populated matrix.
- Differential methods apply to other systems, e.g. GPS-ground-based RF

Relative/Differential GPS Concept

- Measurements are assumed to be made same time - see previous discussion about measurement timing issues
- Position accuracy of 2nd receiver (Remote) with respect to that of 1st unit (Reference) is “better” because errors are reduced or eliminated
- 1st receiver can be mobile (relative GPS) or fixed (Differential GPS) {In sequel – no distinction is made between the two}

- **Reduction or elimination of:**
  - Orbital errors
  - Atmospheric errors
  - Satellite clock errors
- **Remaining errors**
  - Receiver noise
  - Multipath
  - Ionosphere (L_1)



DGPS Between-Receiver Single Difference (Δ) (1/2)

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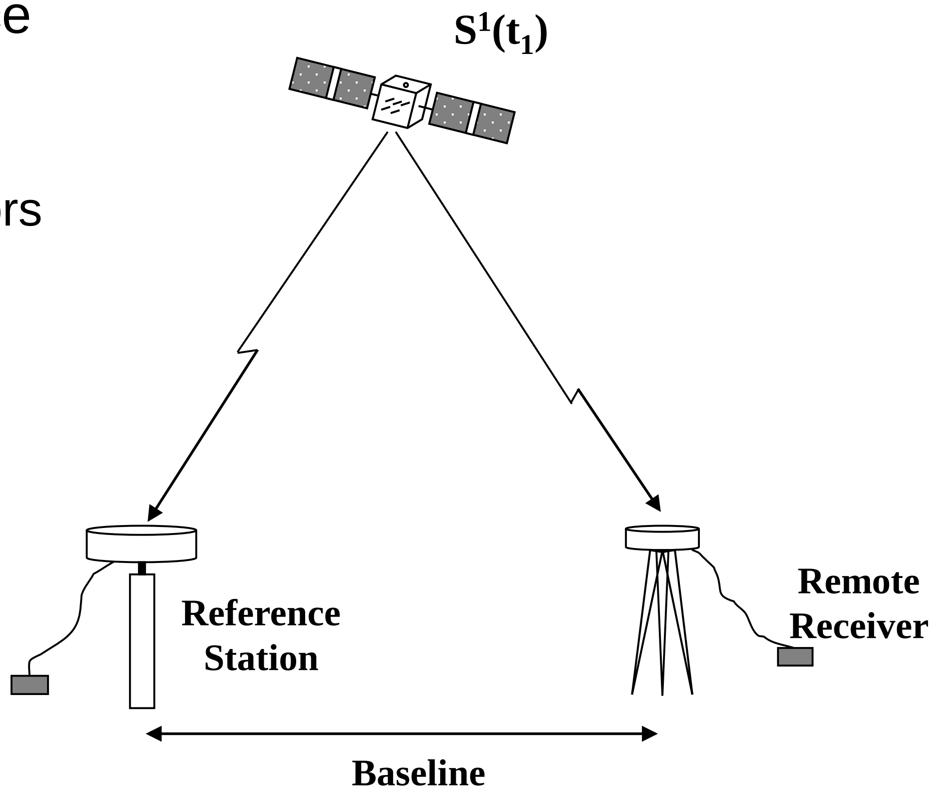
$$\Delta = (\bullet)_{rx_2} - (\bullet)_{rx_1}$$

$$\Delta P = \Delta\rho + \Delta d\rho - c\Delta dT + \Delta d_{ion} + \Delta d_{trop} + \varepsilon_{\Delta p}$$

$$\Delta\Phi = \Delta\rho + \Delta d\rho - c\Delta dT + \lambda\Delta N d_{ion} + \Delta d_{trop} + \varepsilon_{\Delta\Phi}$$

- Subtract the pseudorange at reference station from that at remote
  - Reduces orbital and atmospheric errors
  - Eliminates satellite clock error, dt
  - Does not reduce differential errors
- Method used for many GPS services



DGPS Between-Receiver Single Difference (Δ) (2/2)

- Unknown parameters:
  - x, y, z coordinates of REMOTE receiver contained in (for SV 1):

$$\Delta P_{s^1} = \left| r_{s^1} - r_{ref} \right| - \left| r_{s^1} - r_{rem} \right|$$

$$\textit{where}\ \mathrm{r}_{rem}\ \text{is unknown}$$

  - Relative receiver clock offset, cdT
  - All other terms assumed known (except noise)
- Partial derivatives of differential pseudorange:

$$\frac{\partial \Delta P_{s^1}}{\partial x_{r_{rem}}} = -\frac{(x_{s^1} - x_{r_{rem}})}{\rho_{r_{rem}-s^1}}, \quad \frac{\partial \Delta P_{s^1}}{\partial y_{r_{rem}}} = -\frac{(y_{s^1} - y_{r_{rem}})}{\rho_{r_{rem}-s^1}}, \quad \frac{\partial \Delta P_{s^1}}{\partial z_{r_{rem}}} = -\frac{(z_{s^1} - z_{r_{rem}})}{\rho_{r_{rem}-s^1}}, \quad \frac{\partial \Delta P_{s^1}}{\partial \Delta cdT} = -1$$

- Partial derivatives with respect to the known reference station 1 are 0

Carrier-Phase Differential Positioning

- Using between the basic carrier phase observation equation:

$$\Phi = \rho + d\rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_{\Phi}$$

  - Linear combinations of observations between receivers, satellites and time are generated to reduce the error budget so the full accuracy of the carrier phase observable can be exploited

- The quantities of interest are the coordinates of the receiver and we must isolate those parameters from the errors!

- Errors due to *orbit*, *satellite clock* and *atmospheric effects* are eliminated/reduced by differencing observations between:

  → ***RECEIVERS***

- Errors due to *receiver clock* can be eliminated by differencing observations between:

  → ***SATELLITES***

- *Ambiguities* are constant over time and can be eliminated by differencing observations between:

  → ***TIME EPOCH***

Between Receiver Single Difference Carrier Phase Design Matrix

- For the case of 4 satellites and 8 measurement epochs, the between receiver single difference design matrix **H** would have the following general structure:

$$\underbrace{x_r \qquad y_r \qquad z_r \qquad \Delta N^1 \cdots \Delta N^4 \qquad \Delta dT^1 \cdots \Delta dT^8}$$

$$H = \begin{bmatrix}
\frac{\partial \Delta\Phi_1^i}{\partial x_r} & \frac{\partial \Delta\Phi_1^i}{\partial y_r} & \frac{\partial \Delta\Phi_1^i}{\partial z_r} & 1 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial \Delta\Phi_1^l}{\partial x_r} & \frac{\partial \Delta\Phi_1^l}{\partial y_r} & \frac{\partial \Delta\Phi_1^l}{\partial z_r} & 0 & 0 & 0 & 1 & 1 & 0 & \cdots & 0 \\
\frac{\partial \Delta\Phi_2^{il}}{\partial x_r} & \frac{\partial \Delta\Phi_2^i}{\partial y_r} & \frac{\partial \Delta\Phi_2^i}{\partial z_r} & 1 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial \Delta\Phi_2^l}{\partial x_r} & \frac{\partial \Delta\Phi_2^l}{\partial y_r} & \frac{\partial \Delta\Phi_2^l}{\partial z_r} & 0 & 0 & 0 & 1 & 0 & 1 & \cdots & 0 \\
\frac{\partial \Delta\Phi_8^i}{\partial x_r} & \frac{\partial \Delta\Phi_8^i}{\partial y_r} & \frac{\partial \Delta\Phi_8^i}{\partial z_r} & 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial \Delta\Phi_8^l}{\partial x_r} & \frac{\partial \Delta\Phi_8^l}{\partial y_r} & \frac{\partial \Delta\Phi_8^l}{\partial z_r} & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 1
\end{bmatrix}$$

Note that: $\frac{\partial \Delta\Phi}{\partial x_r} = \frac{\partial \Delta P}{\partial x_r}$

Total unknowns = 3 + 4 + 8 = 15
Total observations = 32

Between Receiver Single Difference (Δ) Example

- Carrier phase for the reference and remote, between-receiver single differences for PRNs 9 and 13 are given as follows:

PRN 9			
GPS Time (s)	***Φ (Ref.) (cycles)***	***Φ (Rem.) (cycles)***	***ΔΦ(cycles)***
426916	627113.893	622753.863	4360.030
426924	643767.629	638833.948	4933.681
426932	660393.255	654918.473	5474.782
426940	676990.342	671012.888	5977.454

PRN 13			
GPS Time (s)	***Φ (Ref.) (cycles)***	***Φ(Rem.) (cycles)***	***ΔΦ(cycles)***
426916	-846502.672	-883957.417	37454.745
426924	-819350.117	-857378.431	38028.314
426932	-792211.933	-830781.263	38569.330
426940	-765088.594	-804160.483	39071.889

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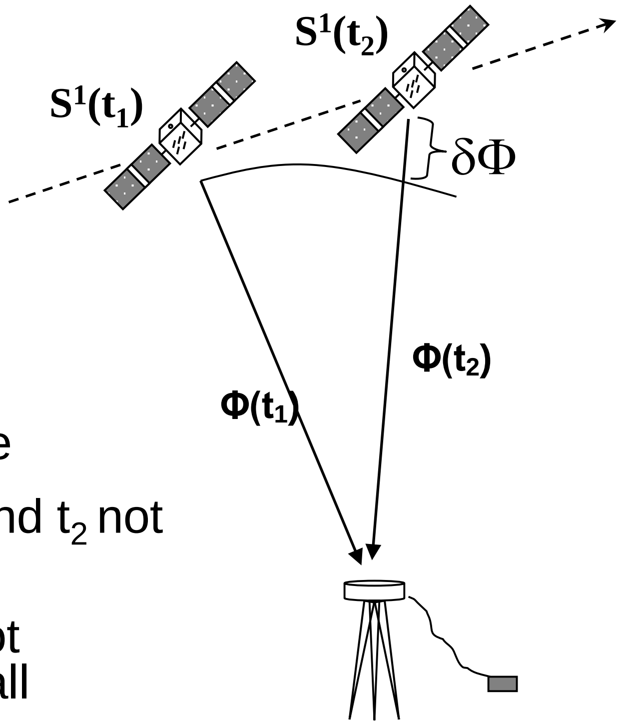
Between Time Epoch Single Difference (δ)

$$\delta(\bullet) = (\bullet)t_2 - (\bullet)t_1$$

$$\delta P = \delta\rho + \delta d\rho + c(\delta dt - \delta dT) + \delta d_{ion} + \delta d_{trop} + \varepsilon_{\delta p}$$

$$\delta\Phi = \delta\rho + \delta d\rho + c(\delta dt - \delta dT) - \delta d_{ion} + \delta d_{trop} + \varepsilon_{\delta\Phi}$$



- Carrier phase ambiguity N vanishes
  - Receiver remains phase-locked between t_1 and t_2
- This is a computation of the range difference over time
- Only able to compute position difference between t_1 and t_2 not absolute position
- Ambiguity term drops out so the range difference is not ambiguous and has an accuracy of < 1 cm over a small interval
- Impact of orbital and atmospheric errors increases over time [e.g. 1]
- Knowing error growth as a function of time is important if DGPS data link drops out for some time

[1] Olynink et al (2002) Temporal Impact of Selected Errors on Point Positioning. GPS Solutions, 6, 47-57

Between Time Epoch Single Difference Example

- Carrier phase for satellite PRN 9, between-epoch single differences at both the reference and remote are given as follows:

Reference			Remote		
GPS Time (s)	***Φ (cycles)***	***δΦ (cycles)***	***GPS Time (s)***	***Φ (cycles)***	***δΦ (cycles)***
426916	627113.893		426916	622753.863	
		16653.736			16080.085
426924	643767.629		426924	638833.948	
		16625.626			16084.525
426932	660393.255		426932	654918.473	
		16597.087			16094.415
426940	676990.342		426940	671012.888	

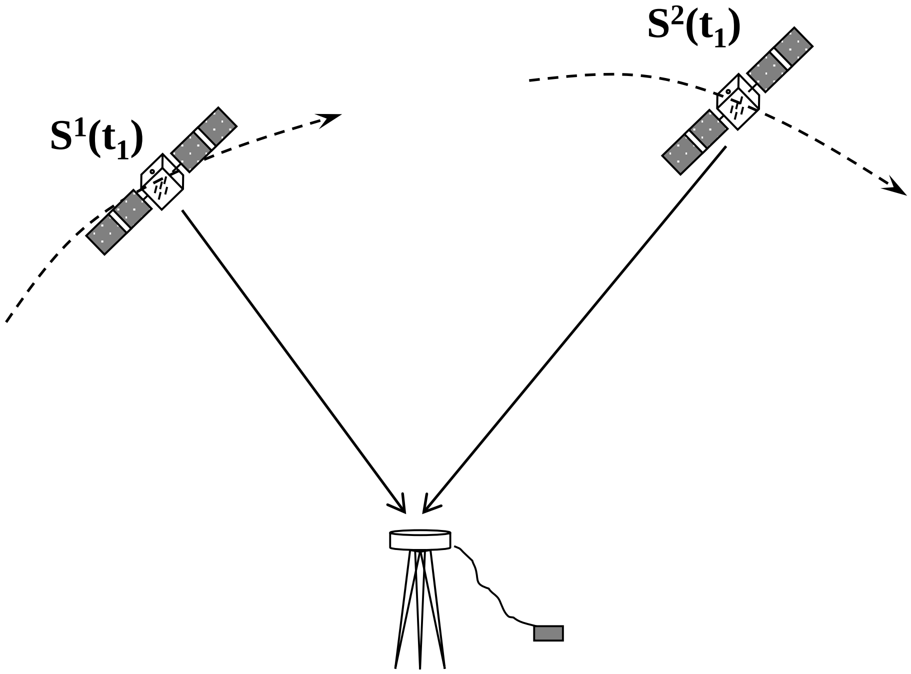
Between Satellite Single Differencing

$$\nabla = (\bullet)_{sat_2} - (\bullet)_{sat_1}$$

$$\nabla P = \nabla\rho + \nabla d\rho + c\nabla dt + \nabla d_{ion} + \nabla d_{trop} + \varepsilon_{\nabla p}$$

$$\nabla\Phi = \nabla\rho + \nabla d\rho + c\nabla dt + \lambda\nabla N - \nabla d_{ion} + \nabla d_{trop} + \varepsilon_{\nabla\Phi}$$

- <u>Eliminates</u> receiver clock error dT
- Does not reduce noise term
- Use to form double differences for carrier phase ambiguity resolution



Between Satellite Single Difference (∇) Example

- Carrier phase for PRN 9 and PRN 13, between-satellite single differences at both the reference and remote are given as follows:

Reference			
GPS Time (s)	***Φ(PRN 13) (cycles)***	***Φ(PRN 9) (cycles)***	***$\nabla\phi$ (cycles)***
426916	-846502.672	627113.893	-1473616.565
426924	-819350.117	643767.629	-1463117.746
426932	-792211.933	660393.255	-1452605.188
426940	-765088.594	676990.342	-1442078.936

Remote			
GPS Time (s)	***Φ(PRN 13) (cycles)***	***Φ(PRN 9) (cycles)***	***$\nabla\phi$ (cycles)***
426916	-883957.417	622753.863	-1506711.280
426924	-857378.431	638833.948	-1496212.379
426932	-830781.263	654918.473	-1485699.736
426940	-804160.483	671012.888	-1475173.371

Carrier-Phase Relative Positioning

Between Satellite-Receiver Double Difference ($\Delta\nabla$)

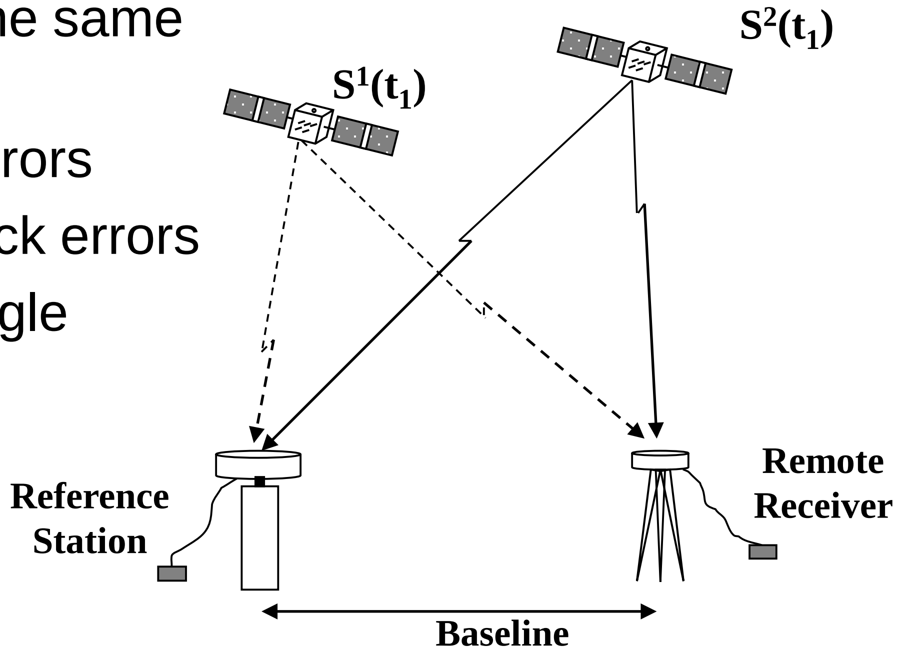
Used for Carrier Phase Ambiguity Resolution

$$\Delta\nabla = \{(\bullet)_{sat_2} - (\bullet)_{sat_1}\}_{rx_2} - \{(\bullet)_{sat_2} - (\bullet)_{sat_1}\}_{rx_1}$$

$$\Delta\nabla P = \Delta\nabla\rho + \Delta\nabla d\rho + \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \varepsilon_{\Delta\nabla p}$$

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\rho + \lambda\Delta\nabla N - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \varepsilon_{\Delta\nabla\Phi}$$

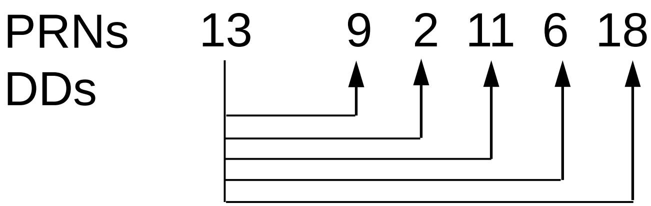
- Assumes that observations are at the same epoch
- Reduces orbital and atmospheric errors
- Eliminates satellite and receiver clock errors
- Increases noise 2x compared to single measurements



Carrier-Phase Relative Positioning

Between Satellite-Receiver Double Difference ($\Delta\nabla$) Example

- Given 6 satellites, the double differences (DDs) can be formed as follows:
  - (n-1) observations are formed where n is the number of satellites



- Satellite which is common to is called the 'base' satellite
  - Highest satellite is chosen as it is least susceptible to errors and typically the strongest signal
- An example of $\Delta\nabla\phi$ between PRN 13 and PRN 9 is given in the following table:

GPS Time (s)	***Reference***		***Remote***		***$\Delta\nabla\phi$ (13-9) (cycles)***
	Φ_{13} (cycles)	***Φ_9 (cycles)***	***Φ_{13} (cycles)***	***Φ_9 (cycles)***	
426916	-846502.672	627113.893	-883957.417	622753.863	33094.715
426924	-819350.117	643767.629	-857378.431	638833.948	33094.633
426932	-792211.933	660393.255	-830781.263	654918.473	33094.548
426940	-765088.594	676990.342	-804160.483	671012.888	33094.435

Between Satellite-Receiver Double Difference Usage

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- Method is used for precise static & kinematic DGPS
- Ambiguities resolved either as integer or real numbers
- For short distances, errors are small
  - Correct integer ambiguities CAN be determined!
- For long distance, residual errors remain and limit the achievable accuracy, depending if integer ambiguities are resolved and the type of ambiguity resolved
- Number of unknowns in static double difference processing is 3 + (n-1):
  - 3 coordinate components (for remote coordinate X, Y, Z)
  - + (n-1) ambiguities, where n is the number of satellites
- For example, if 6 SV are tracked for 1 hour at a 15 s rate, there are only 8 unknown parameters
  - Good observability
  - Number of observations is (n-1)*m, where m is the number of epochs
  - Number of epochs (in this case) = 1200
- Cycle slips and satellite changes affect the number of ambiguities

Measurement Timing Issues

- Carrier phase measurements are made at pre-determined epochs to be useful for precise DGPS
- Satellites are moving at a speed of 4 km/s
- Carrier phase rate can reach 5,000 cycles/s
- GPS time is estimated by the receiver with an accuracy of about 10-50 ns (1 ns = 10^{-9} s)
- But receivers do not re-adjust their clocks at every epoch and internal clocks may be readjusted only when the time misalignment is at the ms level (1 ms = 10^{-3} s)
- In 1 ms, the carrier phase can change by 5 cycles and the satellite positions by m along their orbits
- In differential mode, this is dealt with by calculating the satellite positions using the actual receiver times.
  - A timing error of 10 ns results in a satellite position error of less that 1 mm

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Between Satellite-Receiver Double Difference Carrier Phase Design Matrix

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- For the case of 4 satellites and 8 measurement epochs, the double difference design matrix **H** would have the following general structure:

$$H = \begin{array}{c} \overbrace{\begin{array}{cccccc} x_r & y_r & z_r & \Delta\nabla N^{ij} & \Delta\nabla N^{ik} & \Delta\nabla N^{il} \end{array}} \\ \begin{bmatrix} \frac{\partial\nabla\Delta\Phi_1^{ij}}{\partial x_r} & \frac{\partial\nabla\Delta\Phi_1^{ij}}{\partial y_r} & \frac{\partial\nabla\Delta\Phi_1^{ij}}{\partial z_r} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\nabla\Delta\Phi_1^{il}}{\partial x_r} & \frac{\partial\nabla\Delta\Phi_1^{il}}{\partial y_r} & \frac{\partial\nabla\Delta\Phi_1^{il}}{\partial z_r} & 0 & 0 & 1 \\ \frac{\partial\nabla\Delta\Phi_2^{ij}}{\partial x_r} & \frac{\partial\nabla\Delta\Phi_2^{ij}}{\partial y_r} & \frac{\partial\nabla\Delta\Phi_2^{ij}}{\partial z_r} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\nabla\Delta\Phi_2^{il}}{\partial x_r} & \frac{\partial\nabla\Delta\Phi_2^{il}}{\partial y_r} & \frac{\partial\nabla\Delta\Phi_2^{il}}{\partial z_r} & 0 & 0 & 1 \\ \frac{\partial\nabla\Delta\Phi_8^{ij}}{\partial x_r} & \frac{\partial\nabla\Delta\Phi_8^{ij}}{\partial y_r} & \frac{\partial\nabla\Delta\Phi_8^{ij}}{\partial z_r} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\nabla\Delta\Phi_8^{il}}{\partial x_r} & \frac{\partial\nabla\Delta\Phi_8^{il}}{\partial y_r} & \frac{\partial\nabla\Delta\Phi_8^{il}}{\partial z_r} & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Note that: $\frac{\partial\nabla\Delta\Phi^{ij}}{\partial x_r} = \frac{\partial\nabla\Delta P^{i}}{\partial x_r} - \frac{\partial\nabla\Delta P^{j}}{\partial x_r}$

Total unknowns = 3 + 3 = 6
Total observations = 24

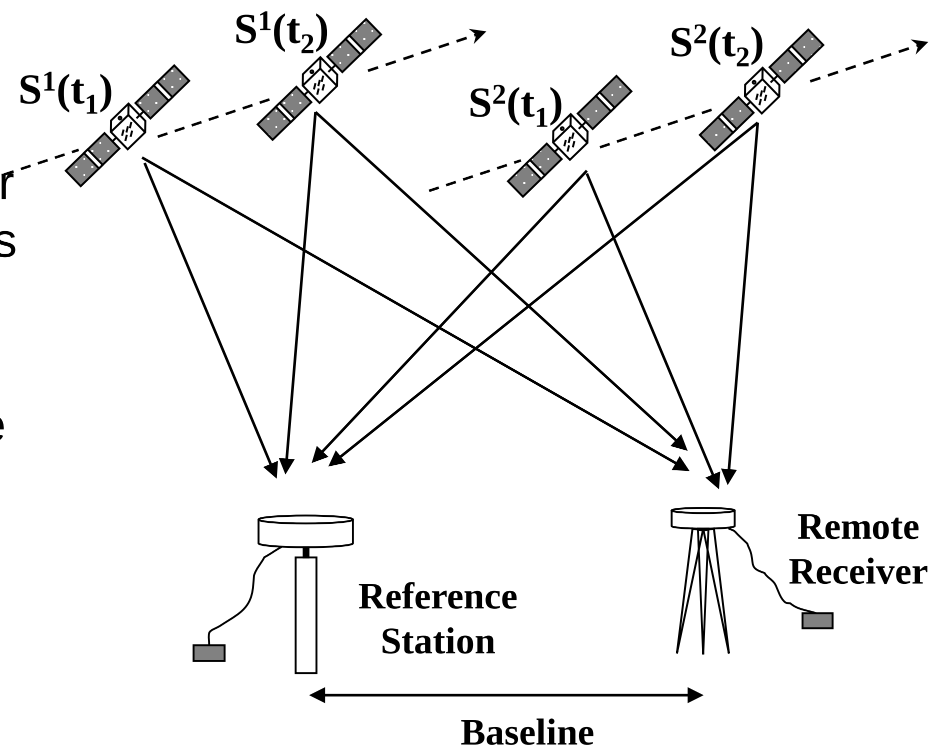
Between Time-Satellite-Receiver Triple Difference ($\delta\Delta\nabla$)

$$\delta\nabla\Delta = \left[\left\{(\bullet)_{sat_2} - (\bullet)_{sat_1}\right\}_{rx_2} - \left\{(\bullet)_{sat_2} - (\bullet)_{sat_1}\right\}_{rx_1}\right]_{t_2} - \left[\left\{(\bullet)_{sat_2} - (\bullet)_{sat_1}\right\}_{rx_2} - \left\{(\bullet)_{sat_2} - (\bullet)_{sat_1}\right\}_{rx_1}\right]_{t_1}$$

$$\delta\Delta\nabla P = \delta\Delta\nabla\rho + \delta\Delta\nabla d\rho + \delta\Delta\nabla d_{ion} + \delta\Delta\nabla d_{trop} + \varepsilon_{\delta\Delta\nabla P}$$

$$\delta\Delta\nabla\Phi = \delta\Delta\nabla\rho + \delta\Delta\nabla d\rho - \delta\Delta\nabla d_{ion} + \delta\Delta\nabla d_{trop} + \varepsilon_{\delta\Delta\nabla\Phi}$$

- Reduces orbital and atmospheric errors
- Eliminates satellite clock error (dt) and receiver clock error dT, as well as carrier phase ambiguity term if there are no loss of phase lock during the time interval
- Only usefull in kinematic solutions since time differencing eliminates the absolute positioning capability
- However, increases noise by $2\sqrt{2}$ as compared to a single measurement



$\delta\Delta\nabla$ *Triple Difference Example (1/2)*

- 15 triple differences (TDs) can be formed by observing 6 satellites for 4 epochs
  - $N_{TD} = (n-1)*(m-1) = (6-1)*(4-1) = 15$
    where n = number of satellites, and m = number of epochs)
- An example of a TD between PRN 13 and PRN 9 is given below

GPS Time (s)	***Reference***		***Remote***		$\delta\Delta\nabla\phi$ ***(13-9) (cycles)***
	Φ13 (cycles)	***Φ9 (cycles)***	***Φ13 (cycles)***	***Φ9 (cycles)***	
426916	-846502.672	627113.893	-883957.417	622753.863	
					-0.082
426924	-819350.117	643767.629	-857378.431	638833.948	
					-0.085
426932	-792211.933	660393.255	-830781.263	654918.473	
					-0.113
426940	-765088.594	676990.342	-804160.483	671012.888	

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$\delta\Delta\nabla$ *Triple Difference Example (2/2)*

- Example of TDs when a cycle slip occurs on PRN 9 at the Remote Receiver at 426932 s
- Note that only one TD is affected!

GPS Time (s)	***Reference***		***Remote***		$\delta\Delta\nabla\phi$ ***(13-9) (cycles)***
	Φ13 (cycles)	***Φ9 (cycles)***	***Φ13 (cycles)***	***Φ9 (cycles)***	
426916	-846502.672	627113.893	-883957.417	622753.863	
					-0.082
426924	-819350.117	643767.629	-857378.431	638833.948	
					4.915
426932	-792211.933	660393.255	-830781.263	*654923.473*	
					-0.113
426940	-765088.594	676990.342	-804160.483	*671017.888*	

$\delta\Delta\nabla$ *Triple Difference Usage*

Triple Differencing Limitations

- Used only for kinematic solutions using carrier phase observables
  - Maybe used to get a rapid approximate solution for subsequent cycle slip detection/correction
- Not used as often as double differences due to time correlation, kinematic restriction, increased noise and decreased observability
- Number of unknowns in triple difference processing is only
  - 3 coordinate components (for remote X, Y, Z)
- Number of observations is (n-1)*(m-1), where n is the number of satellites and m is the number of epochs
  - For example, 6 satellites tracked for 1 hour at a 15 second rate results in a total of 1195 observations
- A cycle slip only affects one triple difference
  - Usually such a cycle slip can be easily detected and rejected
- Triple difference measurements are time correlated – Kalman filters don't like this particular characteristic since they assume uncorrelated data

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$\delta\Delta\nabla$ *Triple Difference Carrier Phase Design Matrix*

- For the case of 4 satellites and 8 measurement epochs, the triple difference (TD) design matrix **H** would have the following general structure:

$$
H = \begin{array}{c}
\begin{array}{ccc} \quad x_r \quad & \quad y_r \quad & \quad z_r \quad \end{array} \\
\begin{bmatrix}
\frac{\partial\delta\nabla\Delta\Phi_{12}^{ij}}{\partial x_r} & \frac{\partial\delta\nabla\Delta\Phi_{12}^{ij}}{\partial y_r} & \frac{\partial\delta\nabla\Delta\Phi_{12}^{ij}}{\partial z_r} \\
\vdots & \vdots & \vdots \\
\frac{\partial\delta\nabla\Delta\Phi_{12}^{il}}{\partial x_r} & \frac{\partial\delta\nabla\Delta\Phi_{12}^{il}}{\partial y_r} & \frac{\partial\delta\nabla\Delta\Phi_{12}^{il}}{\partial z_r} \\
\frac{\partial\delta\nabla\Delta\Phi_{23}^{ij}}{\partial x_r} & \frac{\partial\delta\nabla\Delta\Phi_{23}^{ij}}{\partial y_r} & \frac{\partial\delta\nabla\Delta\Phi_{23}^{ij}}{\partial z_r} \\
\vdots & \vdots & \vdots \\
\frac{\partial\delta\nabla\Delta\Phi_{23}^{il}}{\partial x_r} & \frac{\partial\delta\nabla\Delta\Phi_{23}^{il}}{\partial y_r} & \frac{\partial\delta\nabla\Delta\Phi_{23}^{il}}{\partial z_r} \\
\frac{\partial\delta\nabla\Delta\Phi_{78}^{ij}}{\partial x_r} & \frac{\partial\delta\nabla\Delta\Phi_{78}^{ij}}{\partial y_r} & \frac{\partial\delta\nabla\Delta\Phi_{78}^{ij}}{\partial z_r} \\
\vdots & \vdots & \vdots \\
\frac{\partial\delta\nabla\Delta\Phi_{78}^{il}}{\partial x_r} & \frac{\partial\delta\nabla\Delta\Phi_{78}^{il}}{\partial y_r} & \frac{\partial\delta\nabla\Delta\Phi_{78}^{il}}{\partial z_r}
\end{bmatrix}
\end{array}
$$

Total unknowns = 3
Total observations = 21

Forming Differences in Matrix Form

- Forming differences can be done through a matrix transformation
- The benefit to doing it through a transformation is that the observation covariance matrix can easily be computed, taking into account correlation derived from the differencing
- Consider l is the observation vector and l' is the differences to be used in the estimation algorithm (e.g. least-squares or Kalman filter)
- The transformation has the form $l' = Bl$
- The transformation of the covariance matrix of the observations is then

$$C_{l'} = BC_l B^T$$

- The B matrix is composed of +1 and -1 that form the differences

Forming Differences in Matrix Form

Between-Receiver Single Difference (Δ)(using pseudoranges)

$$B = \begin{bmatrix} 1 & \cdots & 0 & -1 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & \cdots & 1 & 0 & \cdots & -1 \end{bmatrix}_{nx2n}$$

$$l'_{nx1} = \underbrace{\begin{bmatrix} 1 & \cdots & 0 & -1 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & \cdots & 1 & 0 & \cdots & -1 \end{bmatrix}}_{B}{}_{nx2n} \underbrace{\begin{bmatrix} P^1_{rx1} \\ \vdots \\ P^n_{rx1} \\ P^1_{rx2} \\ \vdots \\ P^n_{rx2} \end{bmatrix}}_{l}{}_{2nx1}$$

Note n is the number of satellites tracked by both receivers, thus $2n$ is the number of total pseudorange observations

$$C_{l'} = \underbrace{\begin{bmatrix} 1 & \cdots & 0 & -1 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & \cdots & 1 & 0 & \cdots & -1 \end{bmatrix}}_{B}{}_{nx2n} \underbrace{\begin{bmatrix} \sigma^2_{P^1_{rx1}} & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & \sigma^2_{P^n_{rx1}} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \sigma^2_{P^1_{rx2}} & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & \sigma^2_{P^m_{rx2}} \end{bmatrix}}_{C_l}{}_{2nx2n} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & -1 \end{bmatrix}}_{B^T}{}_{2nxn}$$

Forming Differences in Matrix Form

Double Difference $\Delta\nabla$(using pseudoranges)

$$B=\begin{bmatrix}1 & -1 & \cdots & 0 & -1 & 1 & \cdots & 0\\ 1 & 0 & \ddots & 0 & -1 & 0 & \ddots & 0\\ 1 & 0 & \cdots & -1 & -1 & 0 & \cdots & 1\end{bmatrix}_{(n-1)x2n}$$

Note the base satellite is the first element in the *l* observation vector

$$l'_{nx1}=\begin{bmatrix}1 & -1 & \cdots & 0 & -1 & 1 & \cdots & 0\\ 1 & 0 & \ddots & 0 & -1 & 0 & \ddots & 0\\ 1 & 0 & \cdots & -1 & -1 & 0 & \cdots & 1\end{bmatrix}\begin{bmatrix}P^1_{rx1}\\ P^2_{rx1}\\ \vdots\\ P^n_{rx1}\\ P^1_{rx2}\\ P^2_{rx2}\\ \vdots\\ P^n_{rx2}\end{bmatrix}$$

$$C_{l'}=\underbrace{\begin{bmatrix}1 & -1 & \cdots & 0 & -1 & 1 & \cdots & 0\\ 1 & 0 & \ddots & 0 & -1 & 0 & \ddots & 0\\ 1 & 0 & \cdots & -1 & -1 & 0 & \cdots & 1\end{bmatrix}}_{B}\underbrace{\begin{bmatrix}\sigma^2_{P^1_{rx1}} & \cdots & 0 & 0 & \cdots & 0\\ 0 & \ddots & 0 & 0 & \cdots & 0\\ 0 & \cdots & \sigma^2_{P^n_{rx1}} & 0 & \cdots & 0\\ 0 & \cdots & 0 & \sigma^2_{P^1_{rx2}} & \cdots & 0\\ 0 & \cdots & 0 & 0 & \ddots & 0\\ 0 & \cdots & 0 & 0 & \cdots & \sigma^2_{P^m_{rx2}}\end{bmatrix}}_{C_l}\underbrace{\begin{bmatrix}1 & 1 & 1\\ -1 & 0 & 0\\ \vdots & \ddots & \vdots\\ 0 & 0 & -1\\ -1 & -1 & -1\\ 1 & 0 & 0\\ \vdots & \ddots & \vdots\\ 0 & 0 & 1\end{bmatrix}}_{B^T}$$

Why Use Linear Combinations?

- Linear combinations can be used to minimize the effect of some error sources (e.g. ionosphere)
  - This can be particularly useful for carrier phase processing where errors can be minimized relative to the wavelength, thus facilitating ambiguity resolution
- Use a widelane approach to resolve carrier phase ambiguities
- Basic formulation

$$\phi_{m,n} = m \cdot \phi_{L1} + n \cdot \phi_{L2}$$

$$\lambda_{m,n} = \frac{\lambda_{L1}\lambda_{L2}}{m \cdot \lambda_{L2} + n \cdot \lambda_{L1}}$$

Linear Combination Example

If the following linear carrier phase combination is formed

$$\phi_{m,n} = m \cdot \phi_{L1} + n \cdot \phi_{L2}$$

its effective frequency is given by

$$f_{m,n} = m \cdot f_{L1} + n \cdot f_{L2}$$

and its effective wavelength is given by

$$\lambda_{m,n} = \frac{c}{f_{m,n}} = \frac{c}{m \cdot f_{L1} + n \cdot f_{L2}}$$

$$= \frac{\lambda_{L1} \cdot \lambda_{L2}}{m \cdot \lambda_{L2} + n \cdot \lambda_{L1}}$$

Satellite and Receiver Position Errors

The double differencing mode is used to eliminate clock errors and reduce atmospheric effects

Error / Meas.	Satellite Error		Receiver Error	
	Cycles	**Metres**	**Cycles**	**Metres**
L1 Error	$\frac{1}{\lambda_1}\nabla\Delta d\rho_{SV}$	$\nabla\Delta d\rho_{SV}$	$\frac{1}{\lambda_1}\nabla\Delta d\rho_{Rx}$	$\nabla\Delta d\rho_{Rx}$
Linear Combination	$\frac{1}{\lambda_{m,n}}\nabla\Delta d\rho_{SV}$	$\nabla\Delta d\rho_{SV}$	$\frac{1}{\lambda_{m,n}}\nabla\Delta d\rho_{Rx}$	$\nabla\Delta d\rho_{Rx}$
Ratio	$\frac{\lambda_1}{\lambda_{m,n}}$	1	$\frac{\lambda_1}{\lambda_{m,n}}$	1

$\nabla\Delta$ *Troposphere Error T*

Meas \ Error	Troposphere Error (T)	
	Cycles	**Metres**
L1 Error	$\frac{1}{\lambda_1}\nabla\Delta T$	$\nabla\Delta T$
Linear Combination	$\frac{1}{\lambda_{m,n}}\nabla\Delta T$	$\nabla\Delta T$
Ratio	$\frac{\lambda_1}{\lambda_{m,n}}$	1

$\nabla\Delta$ *Ionosphere Error I*

Meas \ Error	Ionosphere Error (I)	
	Cycles	**Metres**
L1 Error	$\frac{1}{\lambda_1}\frac{\nabla\Delta I}{f_1^{\,2}}$	$\frac{\nabla\Delta I}{f_1^{\,2}}$
Linear Combination	$\frac{1}{\lambda_1}\frac{\nabla\Delta I}{f_1^{\,2}}\left(\frac{nf_2+mf_1}{f_2}\right)$	$\frac{\nabla\Delta I}{f_1^{\,2}}\left(\frac{nf_2+mf_1}{nf_1+mf_2}\cdot\frac{f_1}{f_2}\right)$
Ratio	$\left(\frac{nf_2+mf_1}{f_2}\right)$	$\left(\frac{nf_2+mf_1}{nf_1+mf_2}\cdot\frac{f_1}{f_2}\right)$

$\nabla\Delta$ *Measurement Noise*

Meas \ Error	Measurement Noise**	
	Cycles	**Metres**
L1 Error	$\frac{1}{\lambda_1}\nabla\Delta\varepsilon$	$\nabla\Delta\varepsilon$
Linear Combination	$\frac{\sqrt{n^2+m^2}}{\lambda_1}\nabla\Delta\varepsilon$	$\frac{\lambda_2\sqrt{n^2+m^2}}{n\lambda_2+m\lambda_1}\nabla\Delta\varepsilon$
Ratio	$\sqrt{n^2+m^2}$	$\frac{\lambda_2\sqrt{n^2+m^2}}{n\lambda_2+m\lambda_1}$

** Assumes error on L1 and L2 is the same when expressed in units of cycles

Combinations of GPS L1 & L2 Phase Observables

- L1 and L2 data can be linearly combined to generate a new measurement
- Most common are
  - Widelane (m = 1, n = -1)
  - Narrowlane (m = 1, n = 1)
  - Ionospheric-free (m = 1, n = $-f_2/f_1$)

$$\phi_{m,n} = m\phi_{L1} + n\phi_{L2}$$

Measurement	m	n	λ(m)	Ambiguity
Widelane WL	1	-1	0.8619	$\nabla\Delta N_{WL} = \nabla\Delta N_{L1} - \nabla\Delta N_{L2}$
Narrowlane NL	1	1	0.1070	$\nabla\Delta N_{NL} = \nabla\Delta N_{L1} + \nabla\Delta N_{L2}$
Ionosphere-free IF	1	$-f_2/f_1$	0.4844	$\nabla\Delta N_{IF} = \nabla\Delta N_{L1} - f_2/f_1 \nabla\Delta N_{L2}$
Geometry-Free GF	λ_1	$-\lambda_2$	∞	$\nabla\Delta N_{GF} = \lambda_1\nabla\Delta N_{L1} - \lambda_2\nabla\Delta N_{L2}$
L1 only	1	0	0.1903	$\nabla\Delta N_{L1}$
L2 only	0	1	0.2442	$\nabla\Delta N_{L2}$

Comparison of GPS L1 and WL Errors

Error	L1 Error (m)	WL Error (m)	Ratio $\frac{\text{WL}}{\text{L1}}$
SV Position	$\nabla\Delta\delta\rho_{sv}$	$\nabla\Delta\delta\rho_{sv}$	1
Receiver position	$\nabla\Delta\delta\rho_{rx}$	$\nabla\Delta\delta\rho_{rec}$	1
Troposphere	$\nabla\Delta\delta\rho_{tropo}$	$\nabla\Delta\delta\rho_{tropo}$	1
Ionosphere	$-\frac{1}{f_1^{\,2}}\nabla\Delta_{iono}$	$\frac{-\lambda_{WL}\left(f_1-f_2\right)}{cf_1 f_2}\nabla\Delta_{iono}$ $=\frac{1}{f_1 f_2}\nabla\Delta_{iono}$	-1.283
Multipath	$\nabla\Delta_m$	$\frac{\lambda_{WL}}{\lambda_1}\nabla\Delta_m\sqrt{2}$	6.405
Noise	$\nabla\Delta_\varepsilon$	$\frac{\lambda_{WL}}{\lambda_1}\nabla\Delta_\varepsilon\sqrt{2}$	6.405

Comparison of GPS L1 and NL Errors

Error	L1 Error (m)	NL Error (m)	Ratio $\frac{\textbf{NL}}{\textbf{L1}}$
SV Position	$\nabla\Delta\delta\rho_{sv}$	$\nabla\Delta\delta\rho_{sv}$	1
Receiver position	$\nabla\Delta\delta\rho_{rx}$	$\nabla\Delta\delta\rho_{rec}$	1
Troposphere	$\nabla\Delta\delta\rho_{tropo}$	$\nabla\Delta\delta\rho_{tropo}$	1
Ionosphere	$-\frac{1}{f_1^{\,2}}\nabla\Delta_{iono}$	$\frac{-\lambda_{NL}\left(f_1+f_2\right)}{cf_1 f_2}\nabla\Delta_{iono}$ $=\frac{1}{f_1 f_2}\nabla\Delta_{iono}$	1.283
Multipath	$\nabla\Delta_m$	$\frac{\lambda_{NL}}{\lambda_1}\nabla\Delta_m\sqrt{2}$	0.795
Noise	$\nabla\Delta_\varepsilon$	$\frac{\lambda_{NL}}{\lambda_1}\nabla\Delta_\varepsilon\sqrt{2}$	0.795

Comparison of GPS L1 and IF Errors

Error	L1 Error (m)	IF Error (m)	Ratio $\frac{IF}{L1}$
SV Position	$\nabla\Delta\delta\rho_{sv}$	$\nabla\Delta\delta\rho_{sv}$	1
Receiver position	$\nabla\Delta\delta\rho_{rx}$	$\nabla\Delta\delta\rho_{rec}$	1
Troposphere	$\nabla\Delta\delta\rho_{tropo}$	$\nabla\Delta\delta\rho_{tropo}$	1
Ionosphere	$-\frac{1}{f_1^2}\nabla\Delta_{iono}$	0	0
Multipath	$\nabla\Delta_m$	$\frac{\lambda_{IF}}{\lambda_1}\nabla\Delta_m\sqrt{\frac{f_1^2+f_2^2}{f_1^2}}$	3.227
Noise	$\nabla\Delta_\varepsilon$	$\frac{\lambda_{IF}}{\lambda_1}\nabla\Delta_\varepsilon\sqrt{\frac{f_1^2+f_2^2}{f_1^2}}$	3.227

GPS L1+L2 vs. Narrowlane

- For short baselines, the formation of the narrowlane combination is typically considered best because it reduces the effect of noise and multipath
- However, using the L1 and L2 phase data independently is actually slightly preferred, as shown in the table below
- Other advantages include
  - Easier to resolve L1 and L2 ambiguities than to resolve narrowlane ambiguities
  - Less susceptible to cycle slips (particularly on L2 semicodeless)
  - Solution is possible with either L1 or L2 phase data (unlike for narrowlane where one needs both frequencies to form the narrowlane)

Error	Improvement of L1+L2 over Narrowlane
Geometric (tropo & orbit)	0 %
Ionospheric	3.0 %
Stochastic (noise & multipath)	1.5 %

Petovello, M.G. (2006) Narrowlane - Is It Worth It?. GPS Solutions, John Wiley & Sons, Inc., Volume 10, Number 3, pp. 187-195

Geometry-Free (GF) Combination

- Used to obtained the double-difference ionospheric error. It is expressed in unit of length as

$$\Phi_{GF} = \lambda_1\phi_1 - \lambda_2\phi_2 = \Phi_1 - \Phi_2$$

- Neglecting carrier phase noise and multipath, one obtains the double difference ionospheric effect on L1

$$\nabla\Delta IS_{L1} = \left(\frac{f_2^{\,2}}{f_2^{\,2} - f_1^{\,2}}\right)(\nabla\Delta\Phi_{GF} - \lambda_1\nabla\Delta N_1 + \lambda_2\nabla\Delta N_2)$$

- The measurement is free from tropospheric and orbital errors. It is slightly more than twice as noisy as L1 measurements.

- Used to assess the magnitude of the ionospheric effect on positioning (provided that the ambiguities can be resolved as integer values)

Fortes, L (2002) Optimising the use of GPS multi-reference stations for kinematics positioning. PhD Thesis, ENGO Report No. 20158, Dept of Geomatics Engineering, Univ. of Calgary

GPS L1/L2 Linear Combination - Interpretation (1/2)

4

- The longer the wavelength of the derived ambiguities, the easier to find the correct integers
- WL is therefore the easiest to determine, but
  - WL errors (ionosphere, multipath and noise) are amplified
- Therefore if only WL ambiguities are determined, the position solution will be affected by these errors. The effect of the ionosphere on a WL solution is great than that on a L1solution!
- The IF ambiguities are free from the ionosphere but they are not integer. If the IF ambiguities are estimated directly, they therefore remain stochastic quantities and part of the state vector (referred to as real number or float solution – Excellent choice if the differential ionosphere is significant and integer ambiguities cannot be determined)
- However if WL and either L1 or L2 integer ambiguities can be determined, the other single frequency ambiguities can be derived using

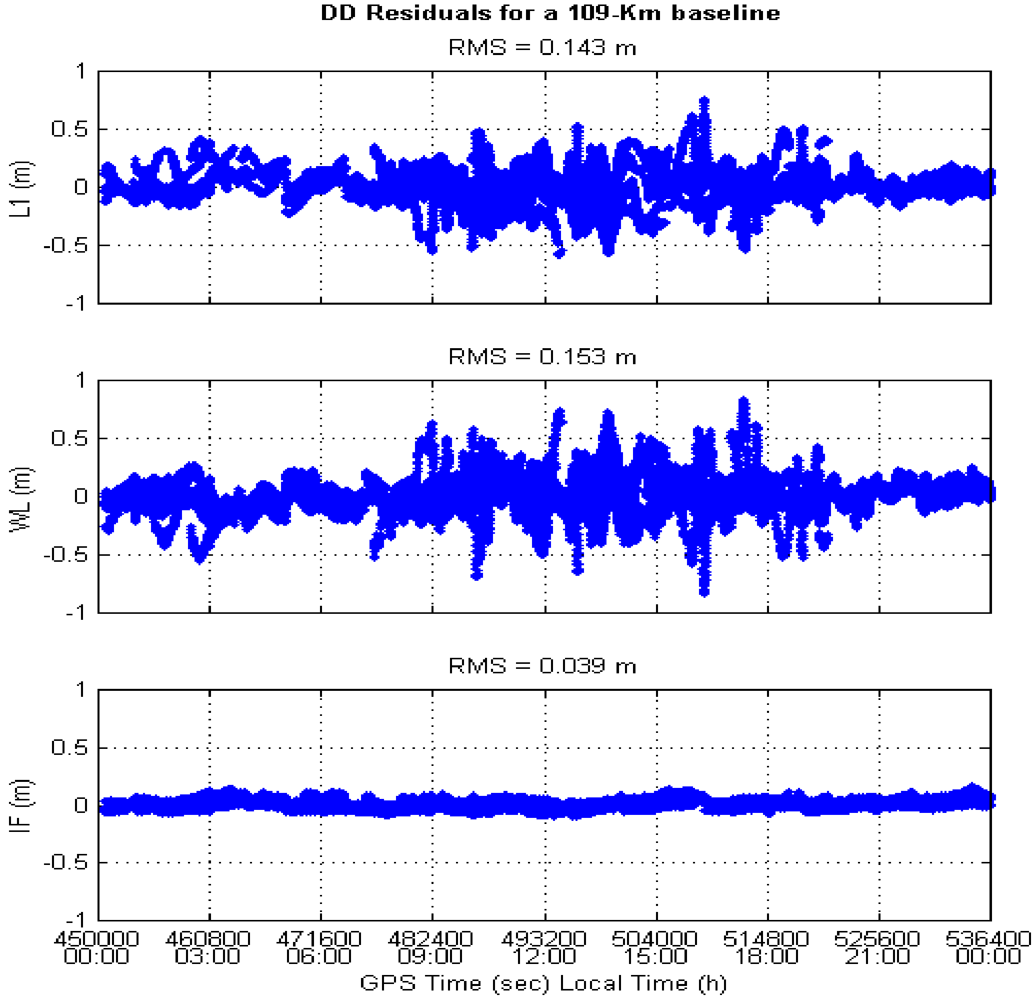
$$\nabla\Delta N_{WL} = \nabla\Delta N_{L1} - \nabla\Delta N_{L2}$$

GPS L1/L2 Linear Combination - Interpretation (2/2)

- The IF ambiguities can then be derived using the equation below. These ambiguities are not longer integer but it does not matter as they are derived from L1 and L2 integer ambiguities and they are still deterministic and are no longer part of the state vector – system observability improves
- The above is the best integer ambiguity solution that can be obtained when the effect of the ionosphere is significant – It is usually accurate to a few cm
- The others are affected by the ionosphere and can be in error by decimetres
- Remember: A position solution is calculated using the same carrier phase observable linear combination as that of the ambiguities
- CONCLUSION: Beware - A “fixed ambiguity” solution does not mean high accuracy until the type of fixed ambiguities resolved is known

$$\nabla\Delta N_{IF} = \nabla\Delta N_{L1} - {}^{f_2}\!/_{f_1}\, \nabla\Delta N_{L2}$$

Residual Errors for Various Observable Combinations - Example



L.P. Fortes et al (1999) Testing of a Multi-Reference GPS Station Network for Precise 3D Positioning in the St. Lawrence Seaway. Presented at ION GPS99, Nashville (September 13-17).

Loss of Phase Lock

- Phase lock loops provide continuous carrier phase measurements
- However in case of signal interference or blockage, they are the first to malfunction
- Occasionally carrier phase measurement continuity is lost, resulting in a new unknown ambiguity on a satellite or satellite pair
  - It rarely will occur on both the base and remote station
- A discontinuity of phase measurements is called a cycle slip
- Cycle slips occur when a satellite signal is obstructed or attenuated by a certain amount and, in certain cases, when the unpredictable Doppler effect due to the antenna motion is too large
- In order to maintain the integrity of a carrier phase-based solution, cycle slips must be detected and, if possible, corrected. Otherwise the DGPS accuracy will degrade.
- Detection and correction can be done in undifferenced or differenced mode, depending on the approach

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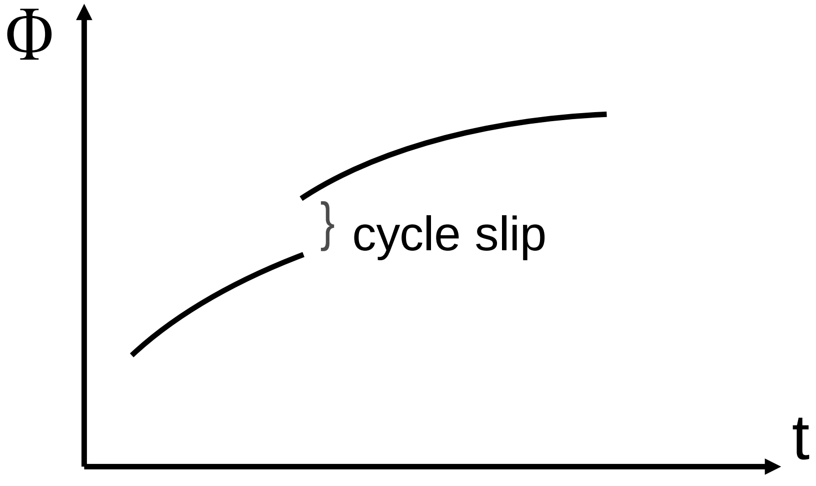
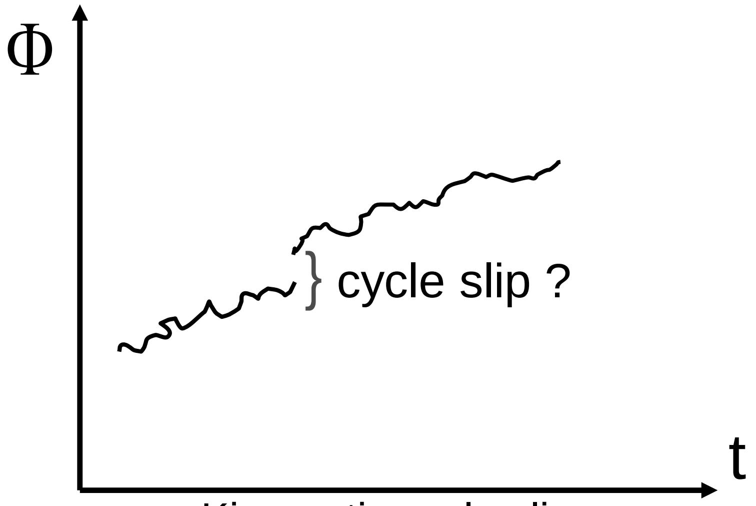
Cycle Slip Overview

- Detection of carrier phase cycling slips is critical for cm – level positioning and for carrier smoothing
- More difficult for kinematic positioning than static, as additional Doppler shift is created in kinematic mode (difficult to predict)
- Options for cycle slip detection are dependent on mode (static vs kinematic) and receiver type (single vs dual frequency)



Static cycle slip



Kinematic cycle slip

Double Difference Static Cycle-Slip Detection (1/2)

- Difference between the computed double difference and the measured double difference can be formed as follows at epoch k:

$$w_k = \nabla\Delta\rho_k - \nabla\Delta\Phi_k \approx \lambda\nabla\Delta N$$

- Since N is constant over time (assuming no cycle slips), at epoch k+1

$$w_{k+1} = \nabla\Delta\rho_{k+1} - \nabla\Delta\Phi_{k+1} \approx \lambda\nabla\Delta N$$

- Any difference between w_k and w_{k+1} would therefore indicate the presence of cycle slips (i.e. N has changed!)
- If for example $|w_k - w_{k+1}|$ = 0.19 m, this would indicate that there has been a slip of 1 cycle (0.191/λ)
  - It does not indicate on which satellite (base or non-base) or which receiver (monitor or remote)
  - However, it is not important to isolate the satellite or receiver for detection

Double Difference Static Cycle-Slip Detection (2/2)

- The method assumes that $\nabla\Delta\rho$ is known with a relatively good accuracy which implies that the remote's position has been determined in the first place !
- This issue can be resolved by first performing a triple difference solution to generate a good approximation of the remote's position – triple differences are not greatly affected by cycle slips
- The triple difference solution is then used to compute $\nabla\Delta\rho$
- Used in many commercially available software packages
- Only feasible for static mode

Kinematic Single Frequency Cycle Slip Detection (1/2)

- **Phase Velocity Trend Method:**

- Φ_k at t_k is predicted using Φ_{k-1} and the phase rates at k and k-1:

$$\hat{\phi}_k = \phi_{k-1} + \frac{\dot{\phi}_k + \dot{\phi}_{k-1}}{2} \Delta t$$

- If $\left| \hat{\phi}_k - \phi_k \right|$ < preset threshold, no cycle slip has occurred
- Threshold is a function of Δt and vehicle dynamics (assumes that phase velocity constant over Δt)
- Usually valid within 1 cycle if $\Delta t \leq 1$ s
- Usable in kinematic mode but threshold may exceed 1 cycle in high dynamics (e.g., survey launch)

Kinematic Single Frequency Cycle Slip Detection (2/2)

- Example: Cycle slips on PRN 12 at 154430 s

GPS Time (s)	ϕ (cycles)	$\dot{\phi}$ (Hz)	$\widehat{\phi}$ (cycles)	$\lvert\phi-\widehat{\phi}\rvert$ (cycles)	Cycles Slip?
154426	4847.073	146.266	----	----	----
154427	4992.748	144.844	4992.628	0.120	no
154428	5136.899	143.688	5137.014	0.115	no
154429	5280.325	143.156	5280.321	0.004	no
154430	5452.985	142.032	5422.919	30.066	yes

Kinematic Dual Frequency Cycle Slip Detection (1/2)

- Need L1 and L2 data (through P code or cross-correlation)
- Over short interval (t_k, t_{k+1}), form the following difference:

$$\delta\Phi_{L1} - \delta\Phi_{L2} = \delta d_{ion_{L1}} - \delta d_{ion_{L2}}$$

- Over a short time interval (e.g. 1 s), $\delta\Phi_{L1} - \delta\Phi_{L2}$ will be << 1 cycle because L1 and L2 are close and the dispersive effect of the ionosphere << 1 cycle
- $If\ \delta\Phi_{t_k,t_{k+1}} > 1\,\text{cycle}$, a cycle slip has occurred
- Valid to detect a cycle slip within one L1 or L2 cycle
- Cannot determine whether slip has occurred on L1 or L2
- Works in both static and kinematic mode

Kinematic Dual Frequency Cycle Slip Detection (2/2)

- Example PRN 12 – cycle slip on L2 at 154429 s

GPS Time (s)	Φ L1 (cycles)	δΦ L1 (m)	Φ L2 (cycles)	δΦ L2 (cycles)	δΦL1- δΦL2 (m)
154426	4847.073		3776.413		
		27.721		27.719	0.002
154427	4992.748		3889.917		
		27.431		27.431	0.000
154428	5136.899		4002.244		
		27.293		27.539	*-0.246*
154429	5280.325		*4115.013*		

Goad, KIS85 Symp Banff, 1985; Cannon Dept of Geomatics Eng Report 20019, UofC, 1987

Carrier Smoothed Pseudoranges (1/2)

MOTIVATION

- Merge ‘absolute’ pseudorange capability and ‘relative’ carrier phase capability – pseudorange is not ambiguous but NOISY, carrier phase is AMBIGUOUS but accurate
- Provides an alternative to pure pseudorange observations and is used in virtually all receiver’s firmware

METHODOLOGY

- Recursive filter to progressively increase weight on carrier while decreasing weight on pseudorange
- Smoothed pseudorange, $\breve{P}_k$, at time k

$$\breve{P}_k = W_{P\,k} P_k + W\phi_k \{\breve{P}_{k-1} + (\phi_k - \phi_{k-1})\}$$

$\breve{P}_k$ ↓ computed smoothed pseudorange

P_k ↓ measured pseudorange

$\breve{P}_{k-1}$ ↓ previously smoothed pseudorange

$(\phi_k - \phi_{k-1})$ ↓ range difference from measured carrier phase

Carrier Smoothed Pseudoranges (2/2)

- W_{Pk} and W_{ϕ_k} are the weights on the measured pseudorange and carrier phase components

$$W_{Pk} = W_{Pk-1} - 0.01\{e.g. 0.01 \leq W_{Pk} \leq 1.00\}$$
$$W_{\phi_k} = W_{\phi_{k-1}} + 0.01\{e.g. 0.00 \leq W_{\phi_k} \leq 0.99\}$$

- At initialization (t_1)

$$\breve{P}_1 = P_1 \left\{ W_{Pt1} = 1.0; W_{\phi_{t1}} = 1.0 - W_{P_t 1} = 0.0 \right\}$$

HATCH, R. (1982) The Synergism of GPS Code and Carrier Measurements. Proceedings of Third International Geodetic Symposium on Satellite Doppler Positioning, DMA/NGS, pp. 1213 – 1232, Washington, D.C.

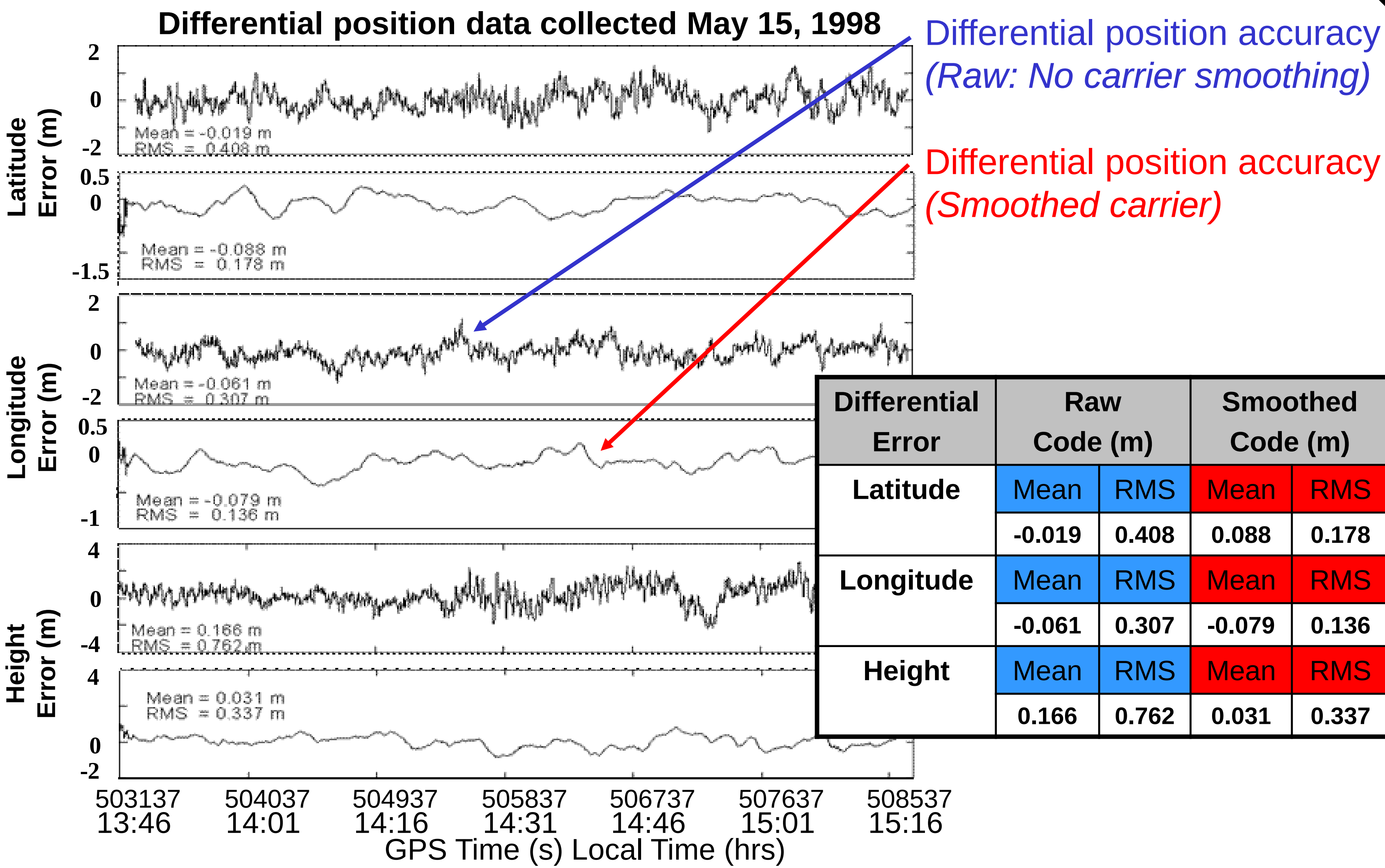
Carrier Smoothed Pseudoranges- Example

- Data collected on SV13

K	W_{P_k}	$P_k(m)$	$W_{P_k} P_k$ (m)	W_{Φ_k}	$\breve{P}_{k-1}(m)$	$\phi_k - \phi_{k-1}$ (m)	$W_{\Phi_k}\{P_{k-1} + (\phi_k - \phi_{k-1})\}$ (m)	$\breve{P}_k$
1	1.00	22441825.779	22441825.779	0.00	-----	-----	0.000	22441825.779
2	0.99	22441597.023	22217181.053	0.01	22441825.779	-225.968	22415.998	22441597.051
3	0.98	22441371.704	21992544.270	0.02	22441597.051	-225.814	448827.425	22441371.695
4	0.97	22441148.237	21767913.790	0.03	22441371.695	-225.697	673234.380	22441148.170
5	0.96	22440921.426	21543284.569	0.04	22441148.170	-225.640	897636.901	22440921.470
:	:	:	:	:	:	:	:	:
98	0.02	22420082.009	448401.640	0.98	22420300.228	-217.722	21971680.855	22420082.496
99	0.01	22419862.495	224196.450	0.99	22420082.496	-217.657	22195666.190	22419864.815
100	0.01	22419644.996	224194.294	0.99	22419864.815	-217.599	22195450.774	22419647.194

Pseudorange Relative Positioning

Raw vs. Smoothed Pseudorange DGPS



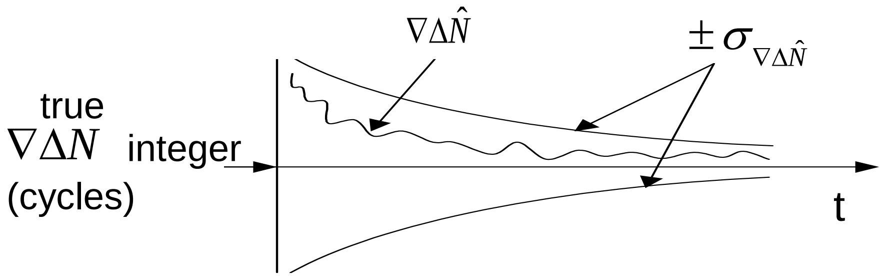
Differential Error	Raw Code (m)		Smoothed Code (m)	
Latitude	Mean	RMS	Mean	RMS
	-0.019	0.408	0.088	0.178
Longitude	Mean	RMS	Mean	RMS
	-0.061	0.307	-0.079	0.136
Height	Mean	RMS	Mean	RMS
	0.166	0.762	0.031	0.337

Float (Real Number) Ambiguity Filter

- Filter states (can also estimate velocity states)

$$x^T = \{\underbrace{\delta\varphi, \delta\lambda, \delta h}_{\substack{position \\ components}}, \underbrace{\nabla\Delta N_1, \nabla\Delta N_2, ..., \nabla\Delta N_n}_{\substack{floating\,(real-valued) \\ ambiguities}}\}$$

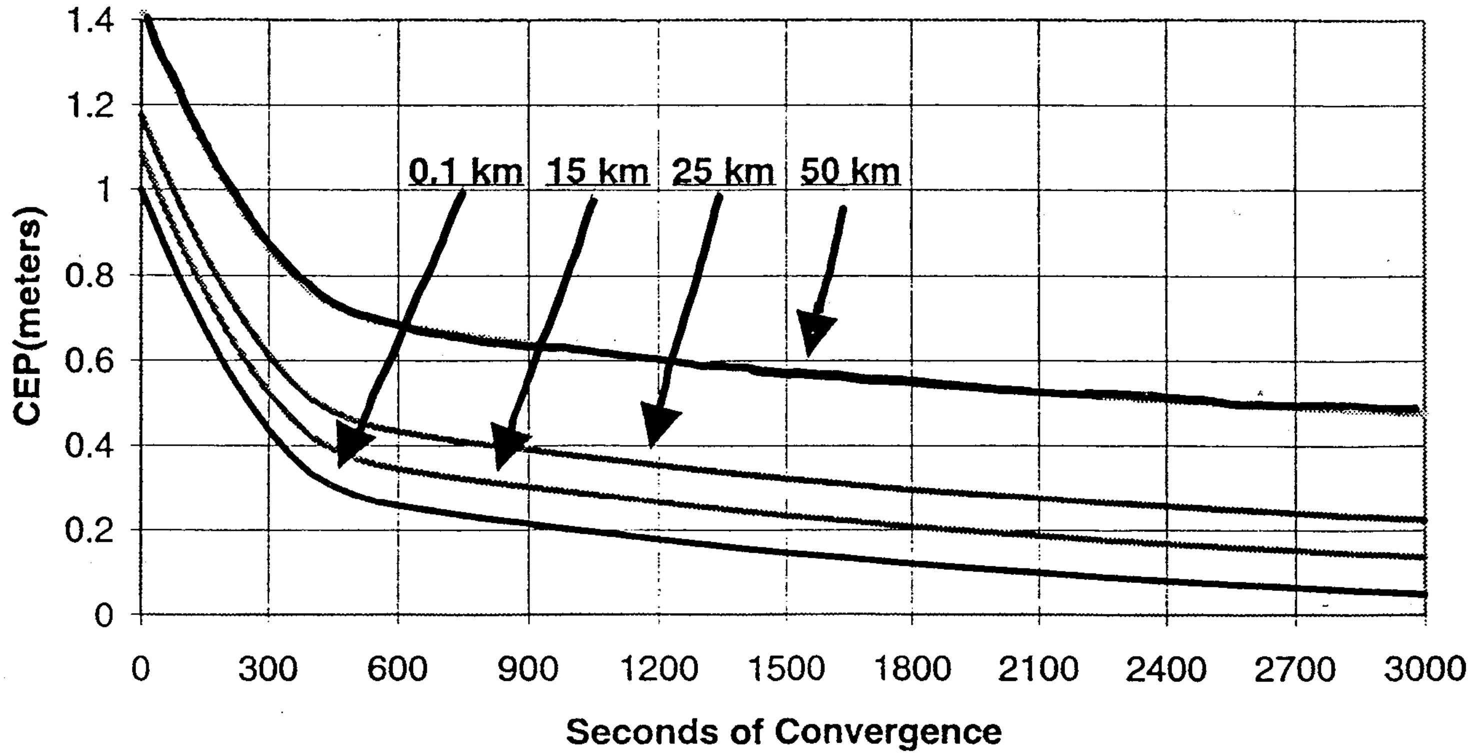
- Estimates of float ambiguities improve with time as the geometry accumulates



- Correlation between ambiguities is generally high
- Unmodelled systematic errors (e.g. from multipath and atmosphere) will bias the float solution
- Cycle slips cause a reset in the estimation process and of accuracy
- Numerous examples presented later

Example of Float Solutions: RTK Positioning Performance – NovAtel RT20 (L1)

- Typical horizontal accuracy convergence time



NovAtel RT20 Manual