Chapter 7

Float and On-The-Fly Ambiguity Resolution I

Math models. Linear phase combinations, widelane, narrow lane, ionospheric-free ambiguities and interpretation. Review of early methods. Advanced methods (FASF, LAMBDA decomposition)

On-The-Fly Ambiguity Resolution II

Ambiguity resolution strategies. Partial ambiguity fixing. Combination of GPS with other GNSS (Galileo, GLONASS). Triple frequency approaches, cascading schemes. Examples.

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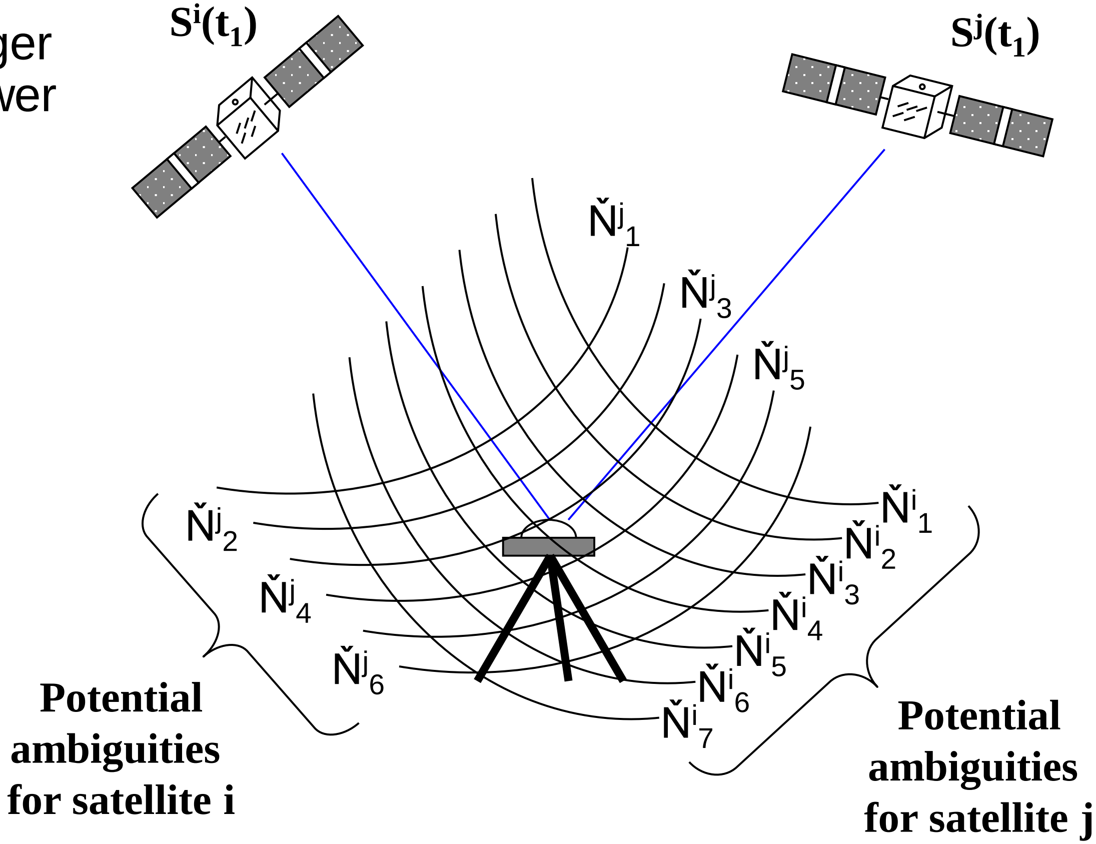
Carrier Phase (CP) Ambiguity Resolution (AR)

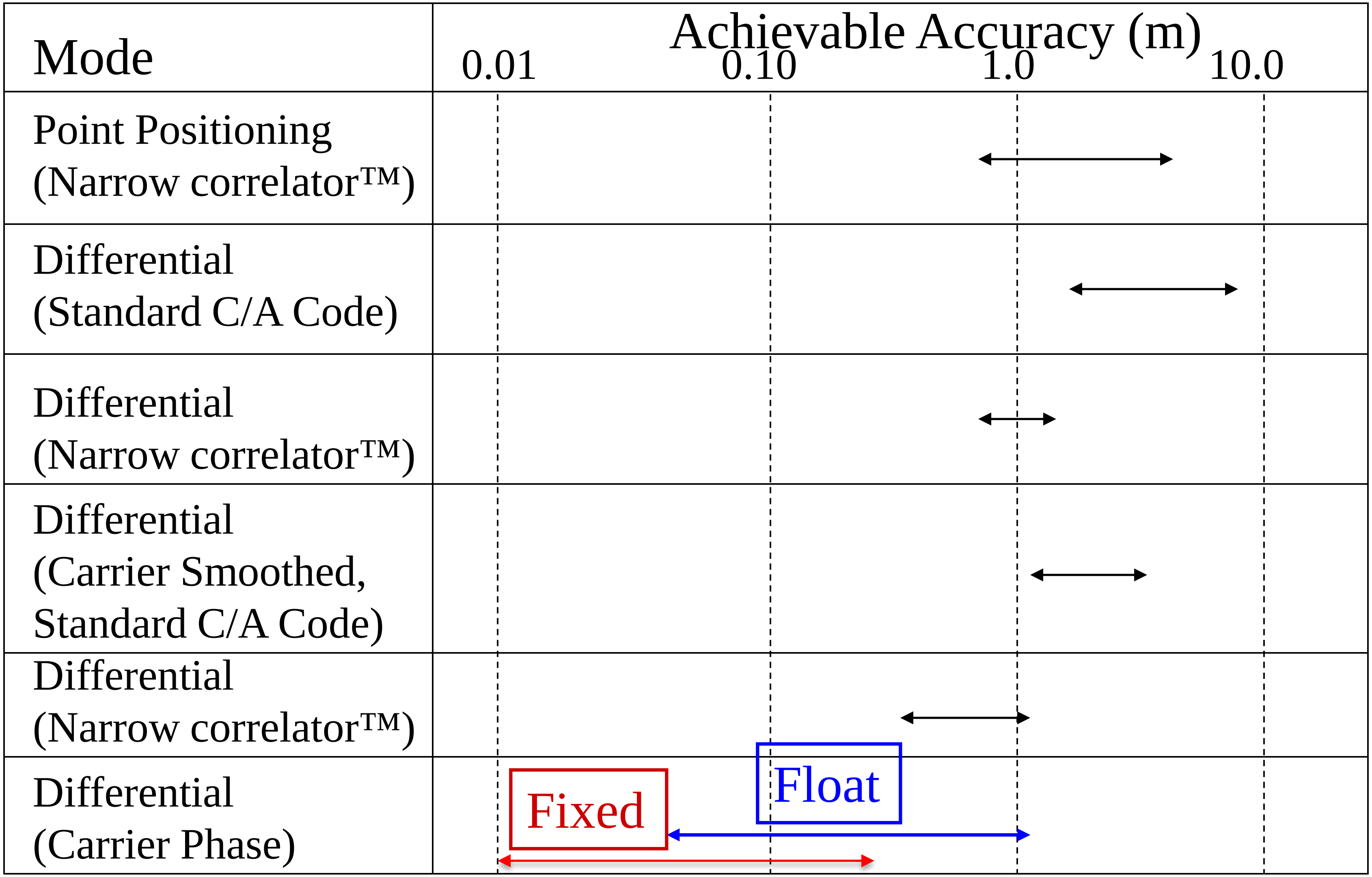
Ambiguity Resolution (AR) Overview

- As more satellites are observed, there are fewer crossover points (i.e. valid combinations) to test
- As the wavelength gets longer (i.e. widelane), there are fewer crossover points to test



Ref: P. Alves, ENGO 625 Lead Discussion (2000)

Why Resolve Ambiguities?



Why is Accuracy Better when Ambiguities are Resolved as Integer Numbers?

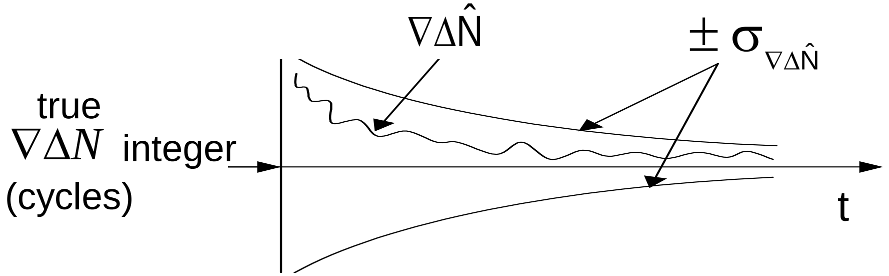
- When carrier phase measurements are used, ambiguities occur and have to be resolved either as real or integer values
- Initially, the ambiguities are part of the state vector. Thus, for 8 SVs, the state vector increases by 7 double differences ambiguities (from a minimum of 3 for the three position differences)
- If the correct integers are found (They always exist. The question is – Can they be found?), the state vector collapses, improving system observability substantially
- If in doubt, resolve as real numbers (Float solution)
- **Caution**: Resolving the integer values does not necessarily yield a cm-level accuracy. Accuracy depends on the type of ambiguities resolved - How the ionosphere is dealt with…

Float (Real Number) Ambiguity Filter

- Filter states (can also estimate velocity states)

$$x^T = \{\underbrace{\delta\varphi, \delta\lambda, \delta h}_{\substack{position \\ components}}, \underbrace{\nabla\Delta N_1, \nabla\Delta N_2, ..., \nabla\Delta N_n}_{\substack{floating\,(real-valued) \\ ambiguities}}\}$$

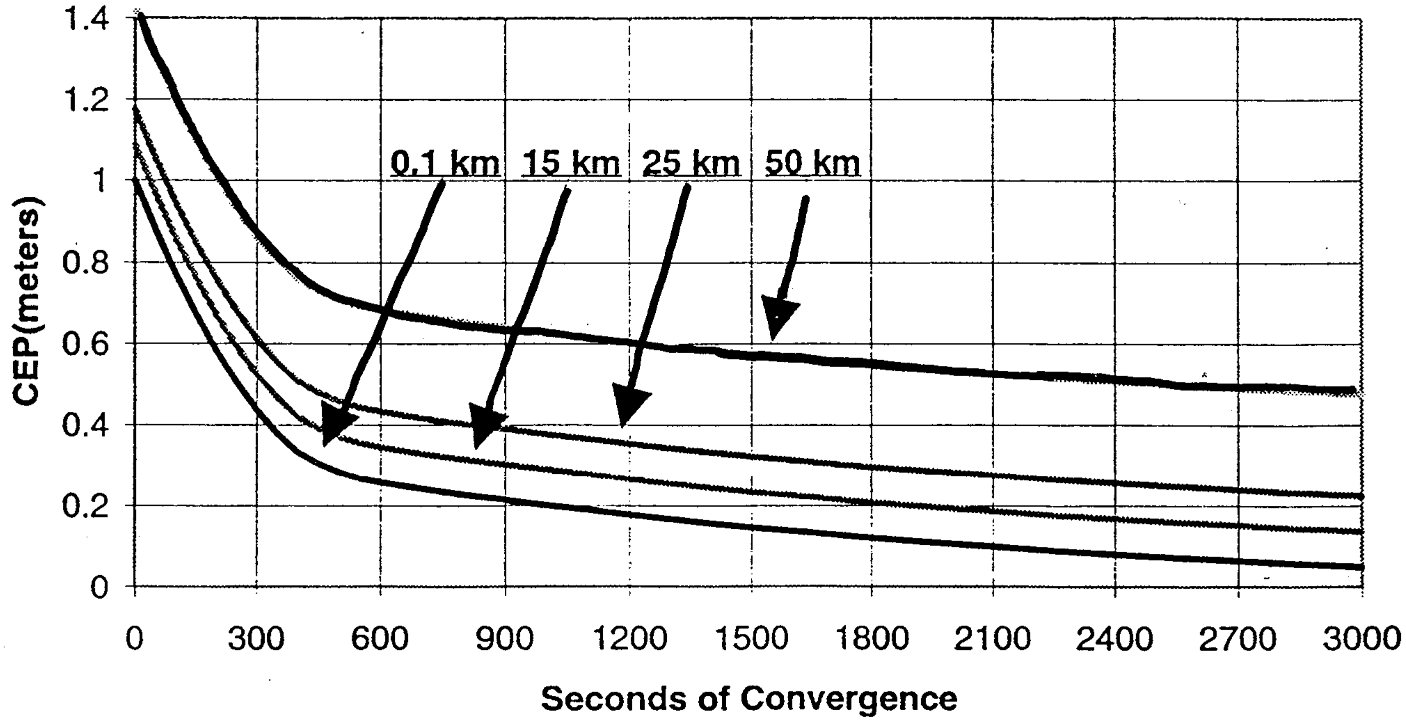
- Estimates of float ambiguities improve with time as the geometry accumulates



- Correlation between ambiguities is generally high
- Unmodelled systematic errors (e.g. from multipath and atmosphere) will bias the float solution
- Cycle slips cause a reset in the estimation process

Example of Float Solutions: RTK Positioning Performance – NovAtel RT20 (L1)

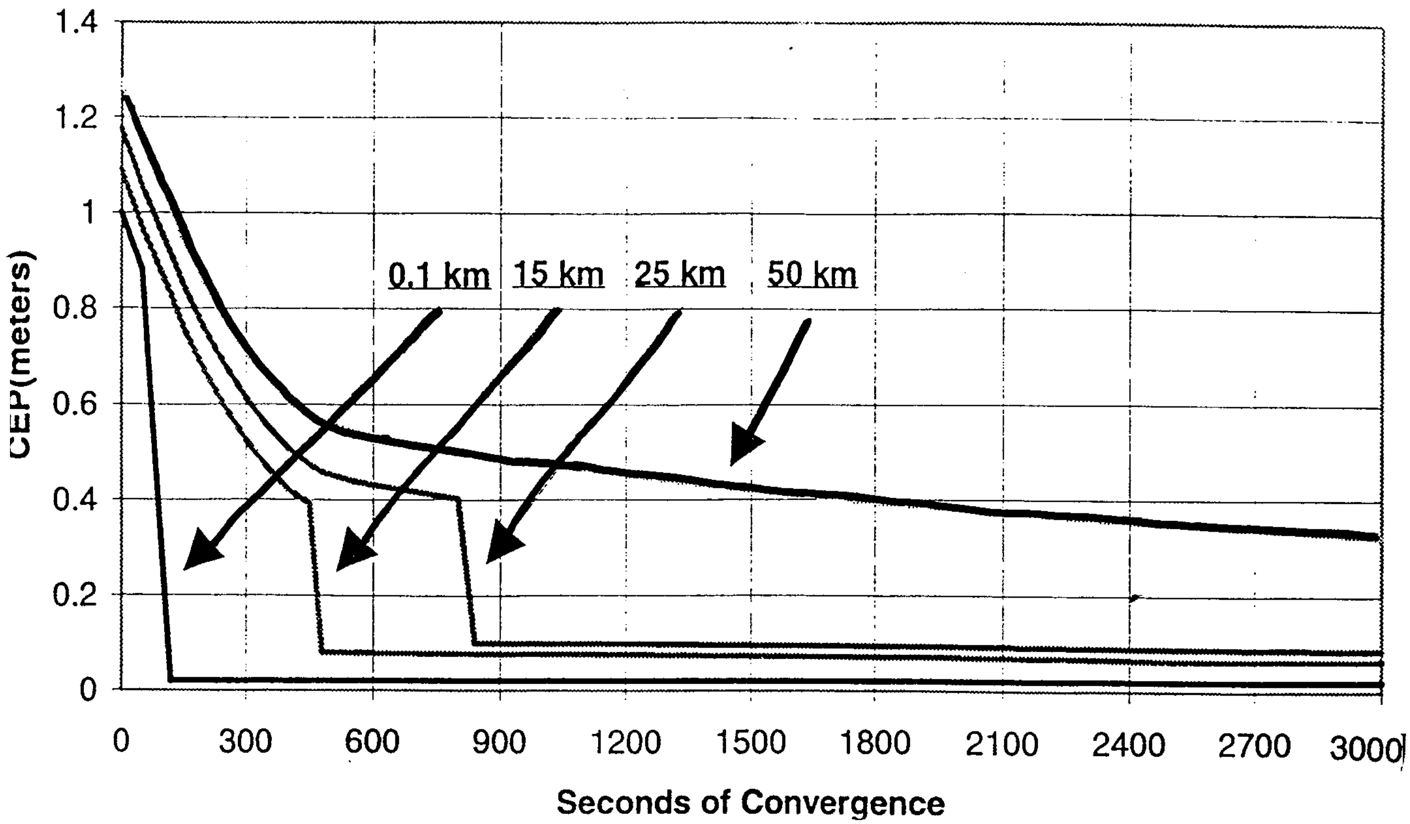
- Typical horizontal accuracy convergence time



NovAtel RT20 Manual

Example (Float/Fixed): RTK Performance – NovAtel RT2 (L1/L2)

- Typical horizontal accuracy convergence



NovAtel RT2 Manual

Baseline Dependence

Short baselines (< 20 km): $\nabla\Delta\Phi = \nabla\Delta\rho + \lambda\nabla\Delta N$

- For long baselines correlated errors become significant:

$$\nabla\Delta\Phi = \nabla\Delta\rho + \lambda\nabla\Delta N + \nabla\Delta d\rho - \nabla\Delta d_{ion} + \nabla\Delta d_{trop}$$

- Correlated errors must be modeled or reduced to resolve ambiguities on long baselines

- Most strategies test various integer ambiguity combinations and select the one the lowest sum-of-squares of residuals
- Two problems exist which reduce effectiveness
  1) Determination of the ambiguity set which minimizes the sum-of-squares of residuals
  2) Statistically validating the ambiguity set

Factor Affecting Integer Ambiguity Resolution

- Static versus kinematic
  - Ambiguities are generally easier to resolve in static mode
- Baseline separation (correlated errors)
  - The closer the user is to the reference, the easier it is to resolve ambiguities because the correlated errors are reduced
- Multipath
  - Even if two points are close together, ambiguities will not be resolved if there is significant pseudorange multipath
- Length of data set and geometry
  - Information is gained through satellite geometry change – longer observation times give better opportunity for resolution
  - The more satellites tracked, the better
- Type of receiver
  - L1/L2 receivers can resolve ambiguities much faster than single frequency systems due to widelaning (WL) combination of L_1 and L_2
- Ambiguity algorithm
  - Some differences in performance are due to the algorithm used

Single Point Math Model

$$\Phi = \rho + c(dt - dT) + \lambda N - d_{ion} + d_{trop} + \varepsilon_{multipath} + \varepsilon_{noise}$$

$$\begin{bmatrix} -\frac{(X_1 - X)}{\rho_1} & -\frac{(Y_1 - Y)}{\rho_1} & -\frac{(Z_1 - Z)}{\rho_1} & 1 & \lambda & 0 & 0 & 0 & 0 & \\ -\frac{(X_2 - X)}{\rho_2} & -\frac{(Y_2 - Y)}{\rho_2} & -\frac{(Z_2 - Z)}{\rho_2} & 0 & 0 & 1 & \lambda & 0 & 0 & \\ -\frac{(X_3 - X)}{\rho_3} & -\frac{(Y_3 - Y)}{\rho_3} & -\frac{(Z_3 - Z)}{\rho_3} & 0 & 0 & 0 & 0 & 1 & \lambda & \cdots \\ -\frac{(X_i - X)}{\rho_i} & -\frac{(Y_i - Y)}{\rho_i} & -\frac{(Z_i - Z)}{\rho_i} & 0 & 0 & 0 & 0 & 0 & 0 & \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ cdt_1 \\ N_1 \\ cdt_2 \\ N_2 \\ cdt_3 \\ N_3 \\ cdt_i \\ N_i \end{bmatrix}$$

Difficult to separate ambiguity from clocks – Hence not possible to resolve amb in single point mode

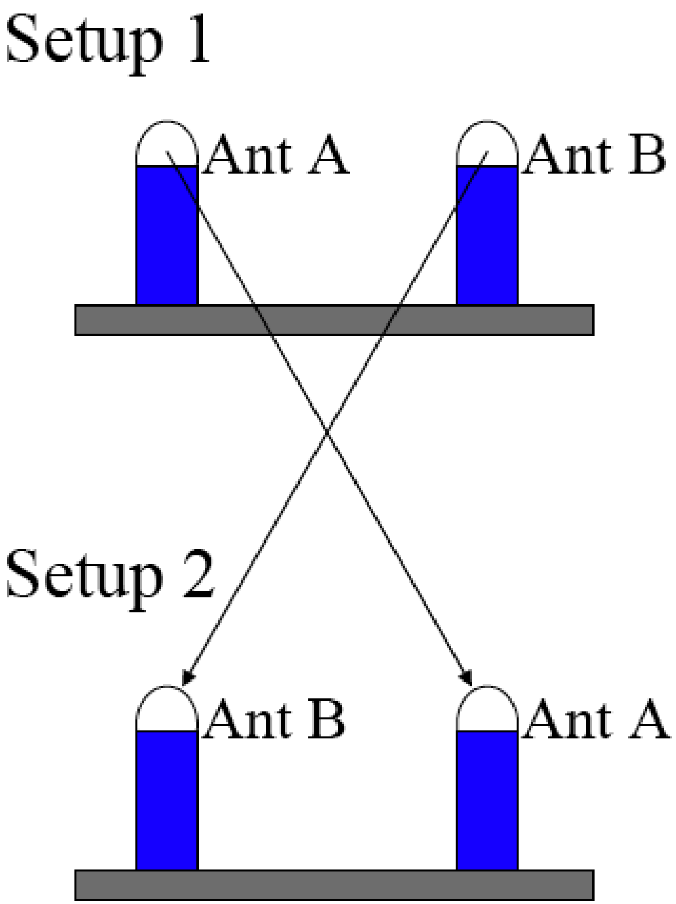
Double Difference Math Model

$$\Delta\nabla\Phi = \Delta\nabla\rho + \Delta\nabla d\rho + \lambda\Delta\nabla N - \Delta\nabla d_{ion} + \Delta\nabla d_{trop} + \varepsilon_{\Delta\nabla\Phi\,noise} + \varepsilon_{\Delta\nabla\Phi\,multipath}$$

$$\begin{bmatrix} \frac{d(\rho_1-\rho_2)}{dX} & \frac{d(\rho_1-\rho_2)}{dY} & \frac{d(\rho_1-\rho_2)}{dZ} & \lambda & 0 \\ \frac{d(\rho_1-\rho_3)}{dX} & \frac{d(\rho_1-\rho_3)}{dY} & \frac{d(\rho_1-\rho_3)}{dZ} & 0 & \lambda\cdots \\ \frac{d(\rho_1-\rho_i)}{dX} & \frac{d(\rho_1-\rho_i)}{dY} & \frac{d(\rho_1-\rho_i)}{dZ} & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ N_1 \\ N_2 \\ N_i \end{bmatrix}$$

Clocks are removed in the difference

Antenna Swapping



Setup 1

$$\nabla\Delta\,\Phi_{ab} = \nabla\Delta\rho_{ab} + \nabla\Delta\,N_{ab}$$

Setup 2

$$\nabla\Delta\,\Phi_{ba} = \nabla\Delta\rho_{ba} + \nabla\Delta\,N_{ba}$$

Important property

$$\nabla\Delta\,N_{ab} = -\nabla\Delta\,N_{ba}$$

Sum of Setup 1 and 2 (estimate baseline)

$$\nabla\Delta\,\Phi_{ab} + \nabla\Delta\,\Phi_{ba} = \nabla\Delta\rho_{ab} + \nabla\Delta\rho_{ba}$$

Difference of Setup 1 and 2 (estimate ambiguities)

$$\nabla\Delta\,\Phi_{ab} - \nabla\Delta\,\Phi_{ba} = \nabla\Delta\rho_{ab} - \nabla\Delta\rho_{ba} + 2\nabla\Delta\,N_{ab}$$

Main Objective: Use static positioning to assist AR

- Semi-Kinematic Positioning

  Remain static on one point until ambiguities are fixed. Then maintain phase lock once in motion {Returning to the initial point yields same position if correct ambiguities were fixed correctly and are maintained, although accuracy could deteriorate along trajectory if the latter extends far from the reference station}

- Kinematic Rapid Positioning

  Remain static on one point for some time

  Commence with survey

  Return to point when survey is complete

- Multiple occupations of the same point

Basic AR Strategies

- There are two basic types of geometry-based ambiguity resolution strategies (in this context, “geometry” refers to the position information)
  - Position-domain approaches
    - Use a position to determine the corresponding set of ambiguities
    - Once the ambiguities are determined, a small correction is applied to the “initial” position
  - Ambiguity-domain approaches
    - Estimate the ambiguities as real values in a Kalman filter and then try to determine the corresponding integer set
    - Once the integer set is determined, the fixed position is computed
- Both methods have the same general procedure but are implemented differently with their own strengths and weaknesses
- Geometry-free strategies try to resolve the ambiguities on a satellite-by-satellite basis without considering the position
  - Conceptually very simple but challenging to implement with real data due to code noise, code multipath and ionosphere

Geometry-Based AR Steps

1. Define a search space and reduce it to as small as possible
2. Calculate the sum of squared residuals
3. Reject sets below a threshold
4. Test it against the second best solution
5. Compute the final, “fixed ambiguity”, position

What is the Search Space?

- The ***search space*** is the space that needs to be searched for the correct integer ambiguity combination
  - For position-domain approaches, the search space is defined as the 3D volume within which the true position is believed to reside
  - For ambiguity-domain approaches, the search space is the n-dimensional volume within which the ambiguity is believed to reside

- For ambiguity resolution to be successful, the search space should include the correct position/ambiguities
  - Practically, this would require an infinite-sized space
  - Instead, the search space is defined such that the probability that it contains the true position/ambiguity is “sufficiently high” (as defined by the user and/or application under consideration)

Least-Squares Residuals

- Measurement residuals are used to assess the 'fit' of the integer ambiguities to the carrier phase data
- Computed as:

$$r = H\delta + w$$

where r is the vector of residuals

H is the design matrix

δ is the parameter correction vector

w is the misclosure vector (due to linearization of non-linear system

Model assumes ambiguities are fixed to a set of integers in the search space

- Sum-of-Squared (SOS) Residuals is a global indicator of residual magnitude

$$\Omega = r^T C_{zl}^{-1} r$$

where Ω is the SOS and $\mathrm{C}_{\mathrm{r}}^{-1}$ is the measurement weight matrix

Testing Integer Ambiguity Sets

- Goal is to find integer set that minimizes Ω
  - Assumes that errors are Gaussian
- Ω is computed for each measurement epoch – assumes ≥ 5 satellites to generate residuals
- Ω values are summed over time to consider total measurement redundancy, i.e.

$$\Omega_{Total_{t_k}} = \sum_{i=t_0}^{i=t_k} \Omega_i$$

- When a floating ambiguity filter is used, a modified Ω can be computed as

$$\Omega'_{total} = (\nabla\Delta N_{float} - \nabla\Delta N_{\text{int}})^T C_{\hat{N}}^{-1} (\nabla\Delta N_{float} - \nabla\Delta N_{\text{int}})$$

where $\Omega_{total} = \Omega_{float} + \Omega'_{total}$ *and* Ω_{float} is the SOS of the float solution

- *Either* Ω'_{total} or Ω_{total} can be used to assess the fit of the integer set

Selecting the Integer Ambiguity Set

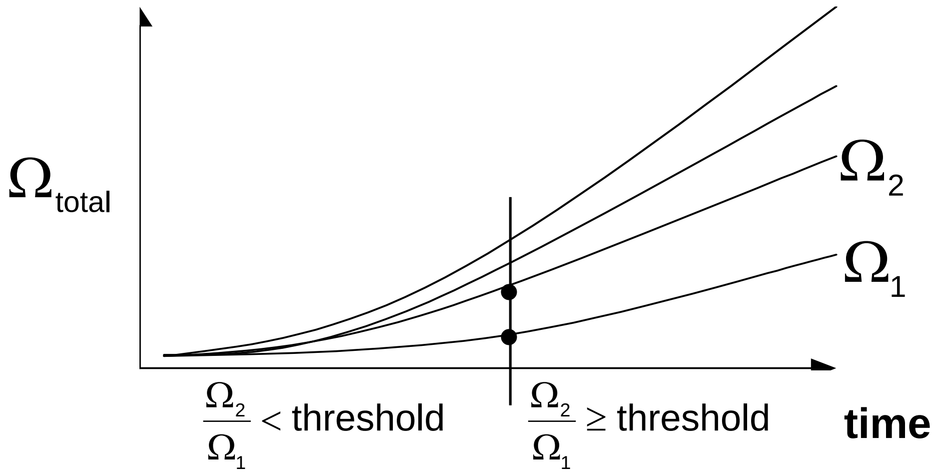
- A discriminator test is used to select an integer set
- Usually a ratio is used, e.g.

$$\frac{\Omega_{total_2}}{\Omega_{total_1}} > \text{threshold}_1$$

- where Ω_1 is the smallest SOS and Ω_2 is the second smallest SOS (pass if ratio is > threshold)

- An alternate is $$\frac{\Omega'_{total_2}}{\Omega'_{total_1}} > \text{threshold}_2$$

- Generally a threshold of 2 or 3 is used - the larger the threshold, the higher the reliability



Why Validate?

- Incorrect ambiguity resolution may not manifest itself in the residuals immediately (hence the challenge!)
- Generally, a change in satellite geometry is needed such that the carrier phase ranging errors due to an incorrect fix manifest themselves in the position domain
- Incorrect ambiguities generally have a larger effect on residuals as time passes
- Only the correct ambiguity set will maintain a reasonably small SOS residuals

- The ambiguity set the minimizes the sum of squared residuals may not be the correct set
- The set must be tested before it is accepted
- There are many tests however these are the most common

Global Test

$$\hat{\sigma}_0^2 = \frac{\hat{v}^T P \hat{v}}{u} \qquad \chi^2_{u;\frac{\alpha}{2}} \le \frac{u\hat{\sigma}_0^2}{\sigma_0^2} \ge \chi^2_{u;1-\frac{\alpha}{2}}$$

redundancy

a-priori variance

Ratio Test

$$\frac{\hat{\sigma}_0^2}{\hat{\sigma}_0'^2} \ge F_{r_1;r_2;\alpha}$$

$\hat{\sigma}_0^2$ - Second Smallest

$\hat{\sigma}_0'^2$ - Smallest

Computing the Fixed Solution

- For position-domain approaches, once the ambiguities are known a carrier phase-based range measurement is computed directly

$$\nabla\Delta\Phi - \lambda\nabla\Delta N = \nabla\Delta\rho + \nabla\Delta d\rho - \nabla\Delta d_{ion} + \nabla\Delta d_{trop}$$

The position can be computed using standard estimation techniques (e.g., least-squares or Kalman filtering)

- For ambiguity-domain approaches, the fixed ambiguities can be removed from the state vector, thus leaving a “fixed ambiguity” solution for the remaining states (*b*)

$$b_{fixed} = b - C_{b,N} C_N^{-1} (N - N_{fixed})$$

$$C_{b_{fixed}} = C_b - C_{b,N} C_N^{-1} C_{N,b} + C_{b,N} C_N^{-1} C_{N_{fixed}} C_N^{-1} C_{N,b}$$

$$\approx C_b - C_{b,N} C_N^{-1} C_{N,b}$$

Covariance matrix of the fixed ambiguities is difficult to determine and is usually neglected

Position-Domain Approaches

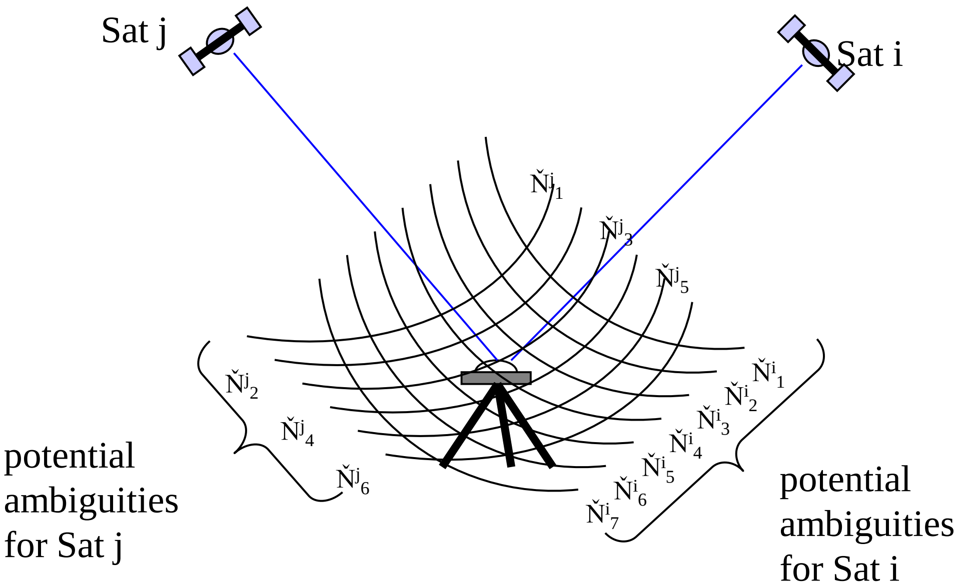
Overview

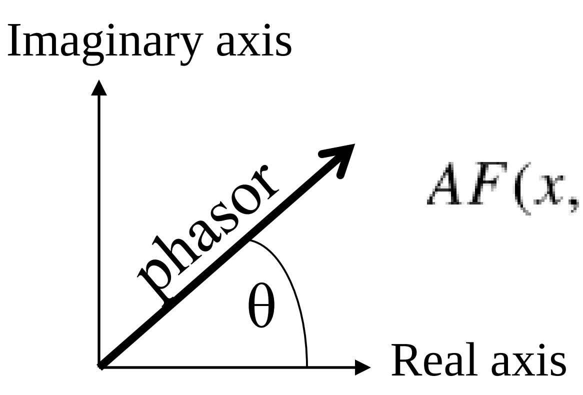
- Position-domain AR approaches are generally easier to understand than ambiguity-domain approaches
- Basic concept is to use a position estimate to compute the ambiguities which can then be tested and validated
- Two common position-domain approaches
- Ambiguity Function Method (AFM)
- Least Squares Ambiguity Search Technique (LSAST)

Erickson, C. (1992) Investigations of C/A code and carrier measurements and techniques for rapid static GPS. MSc Thesis, UCGE Report No. 20044.

Ambiguity Function Method (AFM) (1/2)

At the correct user position the sum of the distances from the user to the closest integer ambiguity will be minimized.





$$AF(x,y,z)=\frac{\sum_{k=1}^{nep}\sum_{j=1}^{nsat-1}\cos\left(2\pi\left(\nabla\Delta\Phi_{obs}^{kj}-\nabla\Delta\rho_{calc}^{kj}(x,y,z)\right)\right)}{nep(nsat-1)}$$

Ambiguity Function Method (AFM) (2/2)

- Define a search grid in the position domain
- For each location in the grid compute

$$AF(x,y,z) = \frac{\sum_{k=1}^{nep} \sum_{j=1}^{nsat-1} \cos\left(2\pi\left(\nabla\Delta\Phi_{obs}^{kj} - \nabla\Delta\rho_{calc}^{kj}(x,y,z)\right)\right)}{nep(nsat-1)}$$

- Choose the largest AF using a validation criteria

Advantages:

•Invariant to cycle slips

Disadvantages:

•Requires a lot of processing power

•Susceptible to blunders and noise.

Mader, G.L. (1992) Rapid Static and Kinematic GPS using the Ambiguity Function Technique. J. of Geophysical Research, 97, B3, 3271-3283.

Least Squares Ambiguity Search Technique (LSAST)

- Search only for the observations required for a unique position solution
- The remaining ambiguities are rounded to the nearest integer conforming to the position solution
- Only the initial set of observations need to be searched making it computationally efficient

Steps for LSAST

- Calculate code solution
- Determine ambiguity sets for primary observations using code solution
- Determine the position correction with the fixed ambiguities of one set of the primary observations
- Determine the fixed ambiguities of the secondary observations by rounding them to the closet integers
- Compute the complete fixed solution using a sequential combination of the primary and secondary observations
- Select the ambiguity set with the lowest variance factor above some threshold

- Ref: R Hatch(1990) Instantaneous Ambiguity Resolution. Proceedings of KIS 1990 Symposium (Banff, Canada, Sepember 1990), Springer.

Least Squares Ambiguity Search Technique (LSAST) Overview

Code Solution

Determine search range for primary obs.

Set i

Compute position using primary obs

$$\delta_p = (A_p^T C_{l_p}^{-1} A_p)^{-1} A_p^T C_{l_p}^{-1} w_p$$

Compute secondary Amb. Using position

$$N_s = n\mathrm{int}(\nabla\Delta\phi - \nabla\Delta\rho_{calc} / \lambda)$$

Compute position Using all obs

$$\delta_c = \delta_p - N_c^{-1} A_s^T C_{l_s} (w_s + A_s \delta_p)$$

$$N_c = (N_p + A_s^T C_{l_s}^{-1} A_s)$$

is set i the Last set?

no

yes

Select the set with the smallest variance factor above the threshold

Advantages:

- Much faster than searching all ambiguity sets

Disadvantages:

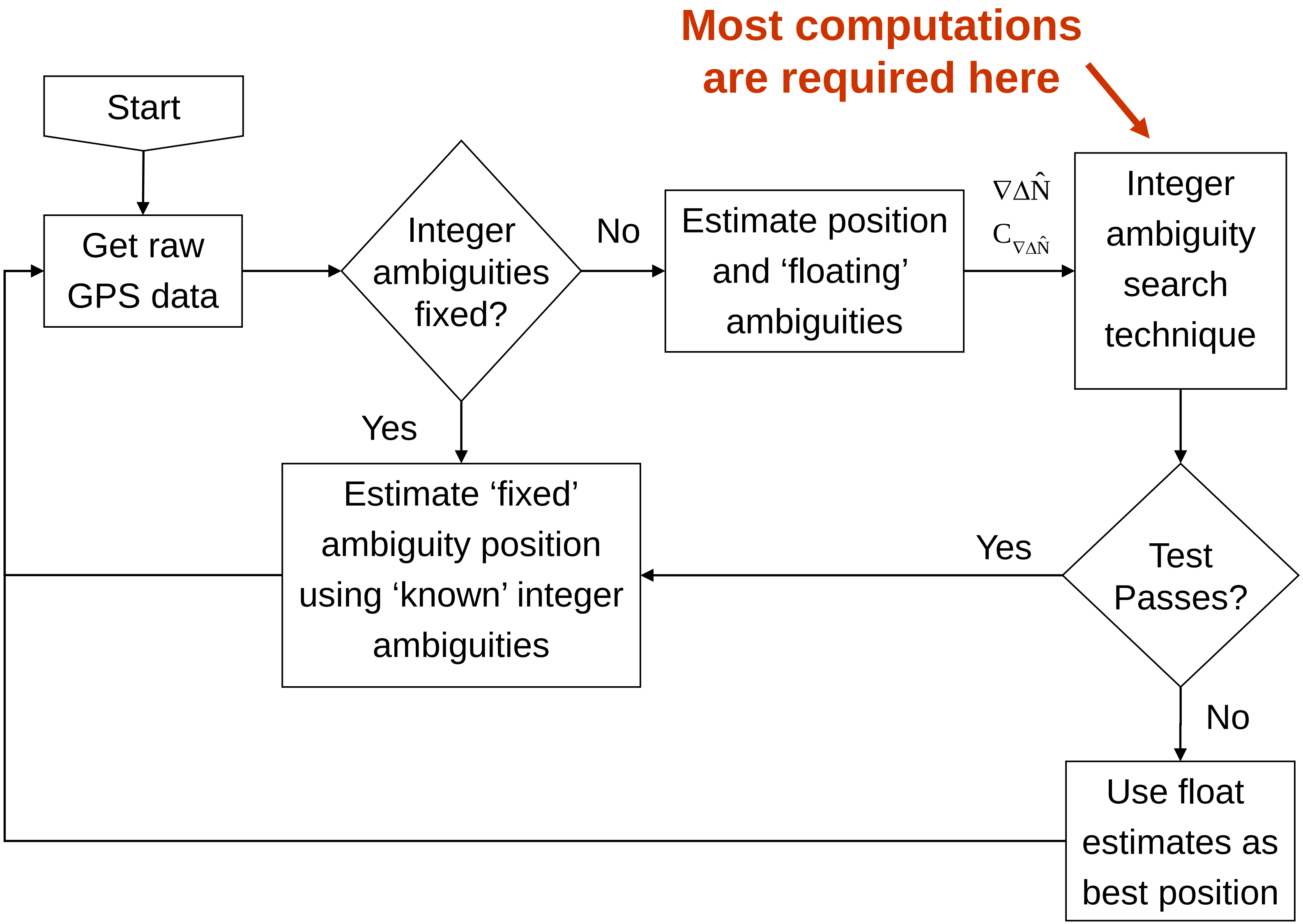
- Poor geometry of the primary satellites can have a large impact on the solution
- If used epoch-by-epoch based in RT, it is susceptible to blunders and noise

Overview

- Ambiguity-domain approaches first try to estimate the ambiguities as real (“float”) values
- The integer ambiguities are then searched based on the float estimates and the corresponding variance-covariance matrix
- A benefit of ambiguity-domain approaches is that a float ambiguity solution is available even if the ambiguities cannot be reliably resolved as integers
  - Allows the multipath and noise benefits of the carrier phase to still be included in the position solution
  - In some situations, a float ambiguity solution may be sufficient and/or more reliable

Ambiguity-Domain Approaches

Flowchart



Initial Double Difference Math Model

$$\Delta\nabla\Phi \approx \Delta\nabla\rho + \lambda\Delta\nabla N$$

$$\begin{bmatrix} \frac{d(p_1 - p_2)}{dx} & \frac{d(p_1 - p_2)}{dy} & \frac{d(p_1 - p_2)}{dz} & \lambda & 0 \\ \frac{d(p_1 - p_3)}{dx} & \frac{d(p_1 - p_3)}{dy} & \frac{d(p_1 - p_3)}{dz} & 0 & \lambda \cdots \\ \frac{d(p_1 - p_i)}{dx} & \frac{d(p_1 - p_i)}{dy} & \frac{d(p_1 - p_i)}{dz} & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ N_1 \\ N_2 \\ N_i \end{bmatrix}$$

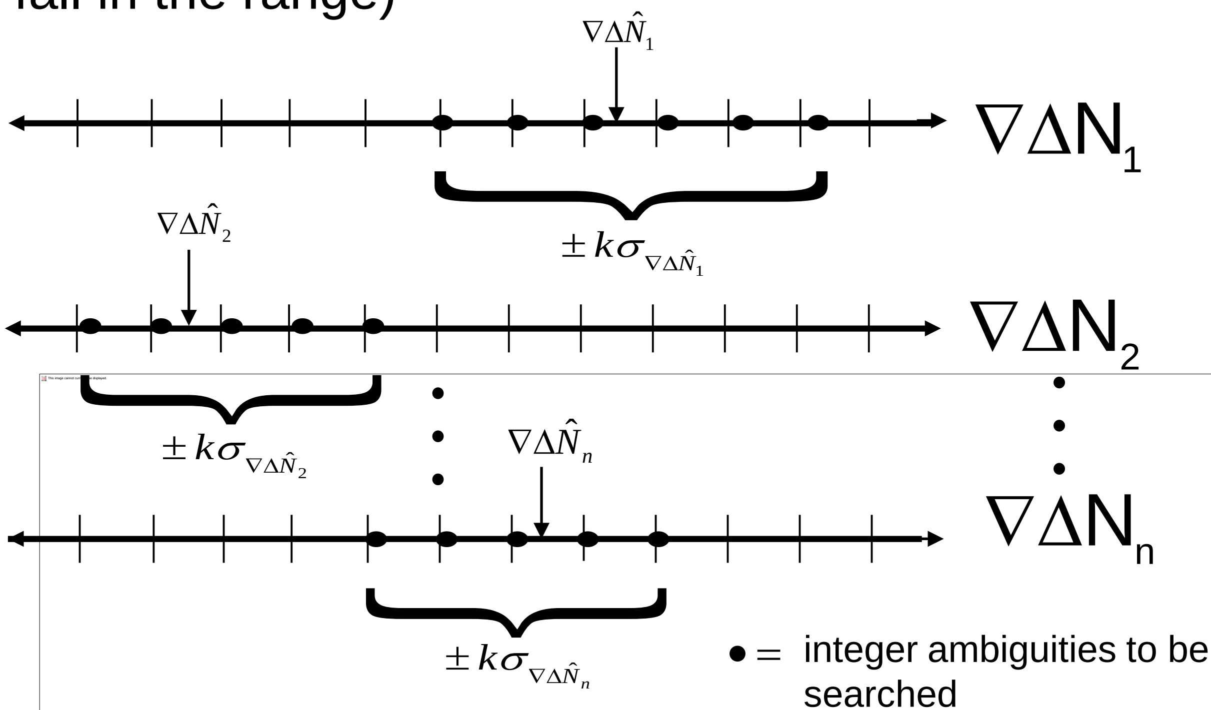
Ambiguities are estimated in the state vector along with the position (and/or other parameters of interest, if any)

Defining the Search Space

True integers must fall within search space within the range

$$\nabla\Delta\hat{N} - k\sigma_{\nabla\Delta\hat{N}} \le \nabla\Delta N_{\text{int}} \le \nabla\Delta\hat{N} + k\sigma_{\nabla\Delta\hat{N}}$$

where k is a scalar (e.g. 3 for 99.7% probability that the true integer will fall in the range)



- E.g. if there are 8 SVs (7 DDs) and 5 integers to test per DD, there would be 5^7=78,125 integers to test!

Overview of Search Strategies

- Some common methods for ambiguity-domain AR include
  - Rounding
  - Bank of Kalman Filters
  - Fast Ambiguity Resolution Approach (FARA)
  - Cholesky Search Technique
  - Fast Ambiguity Search Filter (FASF)
  - Least Squares Ambiguity Decorrelation Adjustment (LAMBDA)

Rounding

- As the name suggests, this approach simply rounds the ambiguities to the integer value
- First ambiguity method
- Fastest of all methods (no searching required)
- Generally works very poorly unless the float filter is given a long time to converge
  - Several hours
  - Assumes no cycle slips
- No reliability measure

Bank of Kalman Filters

Procedure:

1. Process one Kalman Filter solution for each integer ambiguity set within the search space
2. Reject diverging solutions

Disadvantages

- Very time consuming
- Computationally very expensive
- Requires many epochs to reject diverging solutions

Fast Ambiguity Resolution Approach (FARA)

- Considers the correlation between pairs of floating ambiguities
- Integer ambiguities to be tested must fall within the following range

$$\nabla\Delta\hat{N} - \xi_{t,df,1-\alpha/2}\sigma_{\nabla\Delta\hat{N}} \leq \nabla\Delta N_{\text{int}} \leq \nabla\Delta\hat{N} + \xi_{t,df,1-\alpha/2}\sigma_{\nabla\Delta\hat{N}}$$

where $\xi_{t,df,1-\alpha/2}$ is the student t distribution for df degrees of freedom and a significance level of α

- Secondly, a pair of integer ambiguities must also satisfy the following relationship

$$\nabla\Delta\hat{N}_{ij} - \xi_{t,df,1-\alpha/2}\sigma_{\nabla\Delta\hat{N}_{ij}} \leq \nabla\Delta N_{i_{\text{int}}} - \nabla\Delta N_{j_{\text{int}}} \leq \nabla\Delta\hat{N}_{ij} + \xi_{t,df,1-\alpha/2}\sigma_{\nabla\Delta\hat{N}_{ij}}$$

where

$$\sigma_{\nabla\Delta\hat{N}_{ij}} = \sqrt{\sigma^2_{\nabla\Delta\hat{N}_i} - 2\sigma_{\nabla\Delta\hat{N}_i\nabla\Delta\hat{N}_j} + \sigma^2_{\nabla\Delta\hat{N}_j}}$$

and $\sigma_{\nabla\Delta\hat{N}_i\nabla\Delta\hat{N}_j}$ is the covariance between the two float solutions from the covariance matrix

Reference: Frei, E., and G. Beutler (1990), Rapid Static Positioning Based on the Fast Ambiguity Resolution Approach ‘FARA’: Theory and First Results, Manuscripta Geodaetica, Vol. 15, pp. 325-356.

FARA Example

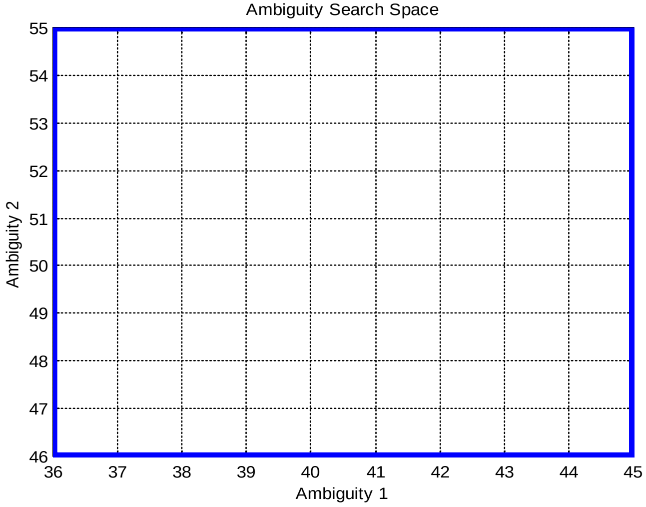
Ambiguity 1:

40.093 cycles → $3\sigma_1$ = 5 cycles

Ambiguity 2:

50.378 cycles → $3\sigma_2$ = 5 cycles

$3\sigma_{12}$ = 2 cycles



Number of combinations: 100

Ambiguity 1 - 2:

-10.285 cycles → $3\sigma_{12}$ = 7 cycles

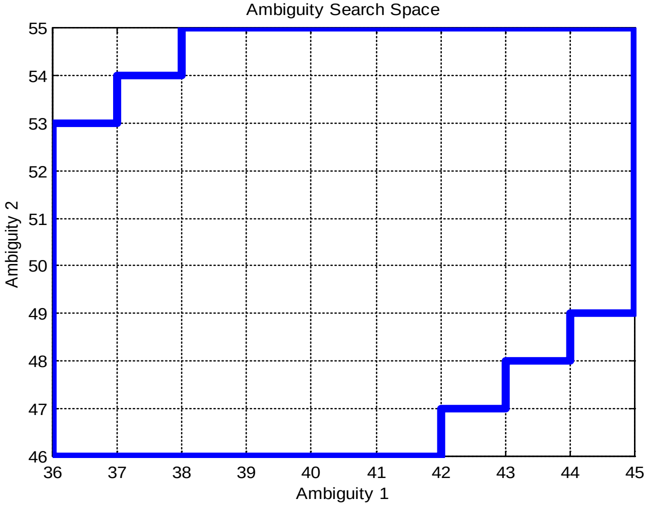
Ambiguity Range:

-17.285 ⇔ -3.285

$N_1 = 35$ $N_2 = 55$

$N_{12} = -20$

-20 < -17.285 outside of range



Number of combinations: 91

Only marginally effective

Cholesky Search (1/2)

Optimize the computation of the sum of squared residuals

Cholesky decomposition property: C = SS^T

$$\underset{1xn}{r}^T \underset{nxn}{C_z^{-1}} \underset{nx1}{r}$$

$$\underset{1xn}{r}^T \underset{nxn}{S} \underset{nxn}{S^T} \underset{nx1}{r}$$

$$\underset{1xn}{A} = \underset{1xn}{r}^T \underset{nxn}{S}$$

$$\underset{1xn}{r}^T \underset{nxn}{C_l^{-1}} \underset{nx1}{r} = \underset{1xn}{A} \underset{nx1}{A^T}$$

$$AA^T = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$= a_1^2 + a_2^2 + \cdots + a_n^2$$

$$= \sum_{i=1}^{n} a_i^2$$

At each addition, test that the accumulated sum is smaller than the second smallest. If it fails then reject it without further computation.

Cholesky Search (2/2)

$$\Omega_1 = \Omega + (k - K\hat{x})^T (K(A^T PA)^{-1} K^T)^{-1} (k - K\hat{x})$$

Where;

- Ω_1 - Sum of squared residuals for fixed ambiguities
- Ω - Sum of squared residuals for the float solution
- k - vector of integer ambiguity values
- K - transformation matrix from x to k
- $\hat{x}$ - float ambiguity values

Advantages:

Shortens the time required to search all of the ambiguity sets.

Fast Ambiguity Search Filter (FASF)

If correlation exists between ambiguities A and B then fixing A will reduce the variance of B

Reduction of Covariance: $$C_{\hat{x}'} = C_{\hat{x}} - \frac{c_n c_n^T}{(C_{\hat{x}})_{n,n}}$$

Adjustment of the State: $$\hat{x}' = \hat{x} - \frac{c_n(\hat{N}_n - N_{n_{\mathrm{int}}})}{(C_{\hat{x}})_{n,n}}$$

Example:

$$\hat{x}' = \hat{x} - \frac{c_n(\hat{N}_n - N_{n_{int}})}{(C_{\hat{x}})_{n,n}} \qquad C'_{\hat{x}} = C_{\hat{x}} - \frac{c_n c_n^T}{(C_{\hat{x}})_{n,n}}$$

Float Solution

$$\hat{x} = \begin{bmatrix} 2.3 \\ 3.2 \end{bmatrix}$$

Fix to 3.0

$$C_{\hat{x}} = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2.3 \\ 3.2 \end{bmatrix} - \frac{0.2}{5}\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2.3 - \frac{2}{25} \\ 3.2 - 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 2.22 \\ 3.0 \end{bmatrix}$$

Better estimate

$$= \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} 2 \\ 5 \end{bmatrix}\begin{bmatrix} 2 & 5 \end{bmatrix}$$

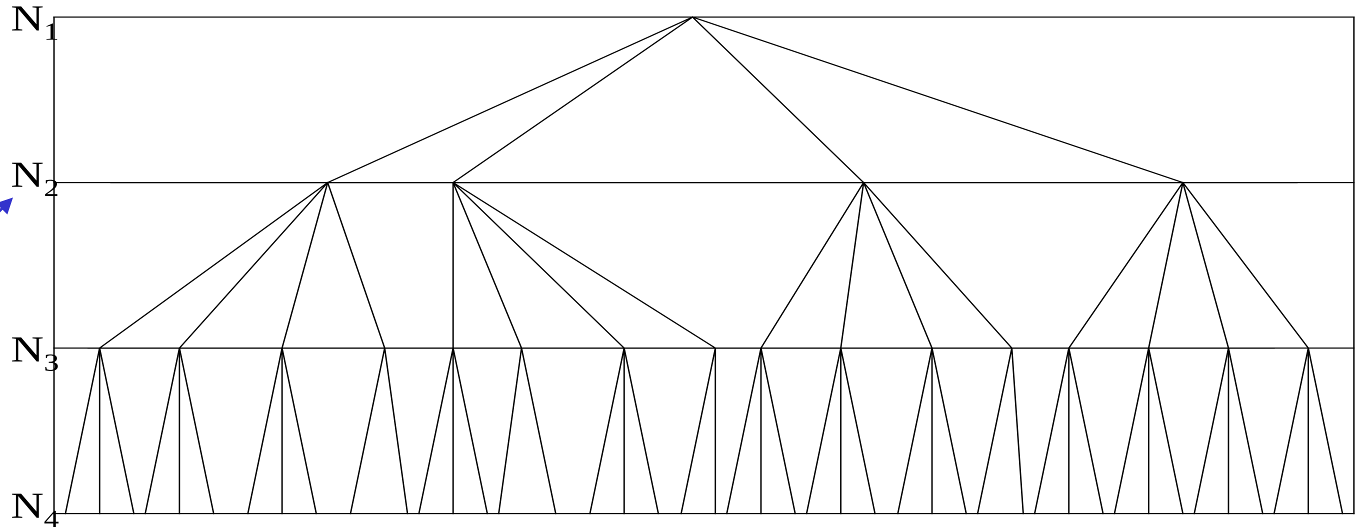
$$= \begin{bmatrix} 4 - \frac{4}{5} & 0 \\ 0 & 0 \end{bmatrix}$$

Reduced variance

FASF - Recursive Computation of the Search Range (RCSR)

Fixing N_1 to a value gives a range of values for N_2

Each fix of N2 gives a different test set for N3 (and so on)



The sum of squared residuals for each of the sets is compared to find the best set.

Advantage:

Can greatly reduce the number of ambiguity sets to test

Ref: Chen, D., and G. Lachapelle (1995) A Comparison of the FASF and Least-Squares Search Algorithms for Ambiguity Resolution On The Fly. **Navigation**, Journal of The Institute of Navigation, 42, 2, 371-390

Least Squares Ambiguity Decorrelation Adjustment (LAMBDA)

- Re-parameterization of the ambiguity states and covariance matrix
- Multi-dimensional confidence ellipsoid is severely elongated due to high correlation between ambiguities
- Transforms the ambiguity states to minimize correlation and reduce the search space
  - Minimizes the correlation in the transformed ambiguities
  - Integer transformation
  - Volume preserving (i.e., every transformed ambiguity set has a unique original set)
- Considers the full correlation between ambiguity states

Decorrelate the second ambiguity from the first

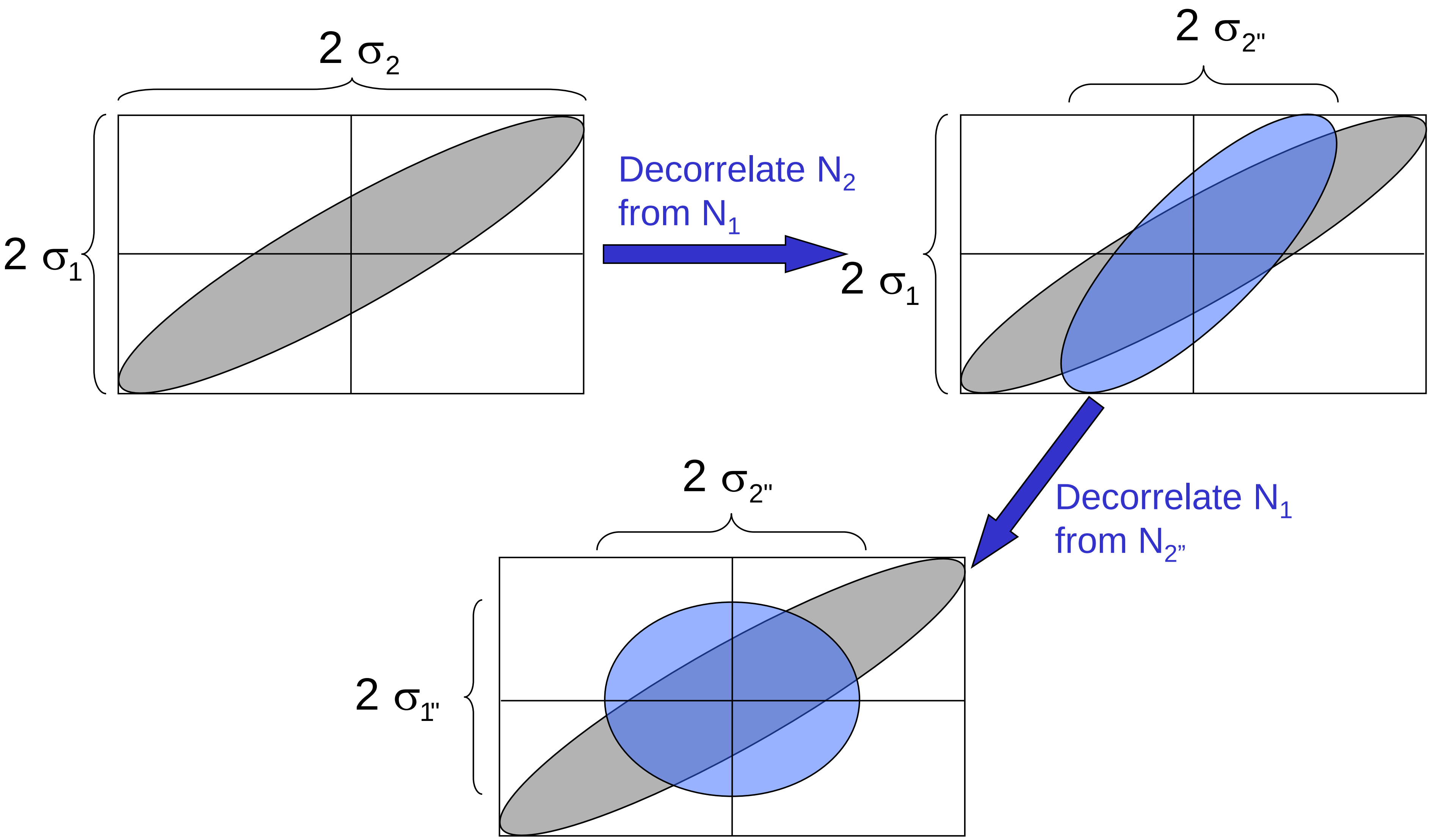
$$\begin{bmatrix} N_1 \\ N_{2''} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \mathrm{int}(-\sigma_{2,1}\sigma_1^{-2}) & 1 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

Decorrelate the first ambiguity from the transformed second

$$\begin{bmatrix} N_{1''} \\ N_{2''} \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{int}(-\sigma_{1,2''}\sigma_{2''}^{-2}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} N_1 \\ N_{2''} \end{bmatrix}$$

Reference: Teunissen, P.J.G. (1994), *A New Method for Fast Carrier Phase Ambiguity Estimation*, Proc. of IEEE PLANS, Las Vegas, pp. 562-573.

LAMBDA Decorrelation Example



LAMBDA Search

From a simplified Cholesky search: The sum of squared residuals is

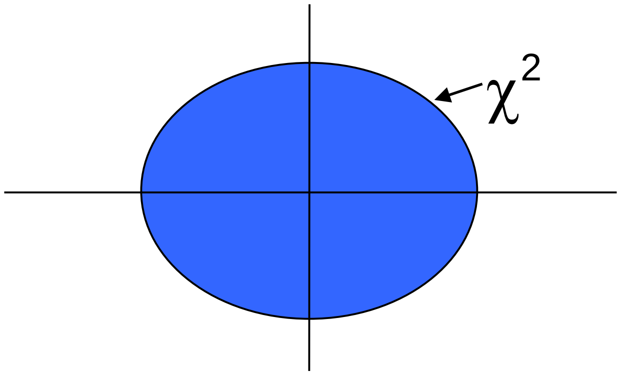
$$\Omega_{fixed} = \Omega_{float} + (N_{fixed} - N)^T C_N^{-1} (N_{fixed} - N)$$

Minimizing Ω_{fixed} as a function of N_{fixed} is independent of Ω_{float} therefore

To minimize Ω_{fixed} we must minimize $(N_{fixed} - N)^T C_N^{-1} (N_{fixed} - N)$

Search the ambiguities based on the function

$$(N_{fixed} - N)^T C_N^{-1} (N_{fixed} - N) \le \chi^2$$



Defining the search space by the minimum sum of squared residuals ensures that if only one ambiguity set is found then that set is the best because, by definition, all other ambiguity sets will have a higher sum of squared residuals.

Geometry-Free Ambiguity Resolution

General Overview

- All of the previous methods were geometry-based methods because they considered the effect of the position, either directly in the position-domain approaches or by estimating the position in the ambiguity-domain approaches
- Geometry-free methods ignore the position information altogether
  - Ambiguities are resolved on a satellite-by-satellite basis using each satellites' pseudorange and carrier phase measurements
  - Basic concept is as follows

$$P = \rho + \varepsilon_P$$
$$\varphi\lambda = \rho + N + \varepsilon_\varphi$$

$$\Rightarrow \quad P - \varphi\lambda = -N + \varepsilon_P - \varepsilon_\varphi$$

If the errors are small relative to the carrier wavelength, the float ambiguities can be resolved directly and can be rounded to integers

General Approach

- Due to the presence of errors (primarily the ionosphere and code noise and multipath) the model is generally modified as follows, for the L1 case

$$x^T = \begin{bmatrix} \rho & I_{L1} & N_{L1} \end{bmatrix}$$

$$\begin{bmatrix} P_{L1} \\ \varphi_{L1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1/\lambda_{L1} & -\lambda_{L1}/\lambda_{L1}^2 & 1 \end{bmatrix} \begin{bmatrix} \rho \\ I_{L1} \\ N_{L1} \end{bmatrix} + \begin{bmatrix} \nu_P \\ \nu_\varphi \end{bmatrix}$$

- If multiple frequencies are to be considered, the state vector is augmented by the new ambiguities

$$x^T = \begin{bmatrix} \rho & I_{L1} & N_{L1} & N_{L2} \end{bmatrix}$$

$$\begin{bmatrix} P_{L1} \\ \varphi_{L1} \\ P_{L2} \\ \varphi_{L2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1/\lambda_{L1} & -\lambda_{L1}/\lambda_{L1}^2 & 1 & 0 \\ 1 & \lambda_{L2}^2/\lambda_{L1}^2 & 0 & 0 \\ 1/\lambda_{L2} & -\lambda_{L2}/\lambda_{L1}^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \rho \\ I_{L1} \\ N_{L1} \\ N_{L2} \end{bmatrix} + \begin{bmatrix} \nu_{P_{L1}} \\ \nu_{\varphi_{L2}} \\ \nu_{P_{L2}} \\ \nu_{\varphi_{L2}} \end{bmatrix}$$

- The “range” contains all of the geometric terms (geometric range, troposphere, residual satellite clock, residual receiver clock, etc.)
- An ionosphere pseudo-observation is sometimes added to constrain the estimates to reasonable values (based on ionosphere activity and baseline length)

Motivation

- Selection of an integer ambiguity set is ultimately based on statistical values
  - Sum of squared residuals
  - Ratio of SOS residuals

- In general therefore, the probably of fixing the ambiguities correctly is therefore not 100%

- An incorrect fix of one cycle on one satellite can cause the position error to far exceed the estimated accuracy of the system (from the position covariance matrix)
  - Hazardously Misleading Information (HMI)

- Safety-critical applications need to ensure the ambiguity resolution process is correct with a high level of confidence

PCF References

- O'Keefe, K., M. Petovello, G. Lachapelle and M.E. Cannon (2006) Assessing probability of correct ambiguity resolution in the presence of time-correlated errors. **Navigation**, U.S. Institute of Navigation, 53, 4, 269-282.
- Kondo, K. (2003) *Optimal Success/Error Rate and Its Calculation in Resolution of Integer Ambiguities in Carrier Phase Positioning of Global Positioning System (GPS) and Global Navigation Satellite System (GNSS)*, Proceedings of ION Annual Meeting 2003, Albuquerque, NM, pp. 176-187.
- Teunissen, P.J.G. (1998) *Success probability of integer GPS ambiguity rounding and bootstrapping*, Journal of Geodesy, Vol 72, pp. 606-612. http://www.springerlink.com/link.asp?id=100435
- Teunissen, P.J.G. (2000) *The success rate and precision of GPS ambiguities*, Journal of Geodesy, Vol 74, pp. 321-326. http://www.springerlink.com/link.asp?id=100435
- Teunissen, P.J.G. (2001) *GNSS Ambiguity Bootstrapping: Theory and Application*, Proceedings of KIS 2001, Department of Geomatics Engineering, The University of Calgary, pp. 246-254.
- Teunissen, P.J.G., D. Odijk and P. Joosten (1998) *A Probabilistic Evaluation of Correct GPS Ambiguity Resolution*, Proceedings of ION GPS 1998, Nashville, TN, pp. 1315-1323.

Stochastic Ionospheric Modeling (SIM)

- Use of an ionospheric weighted model:
  - Estimates ionosphere states using code and carrier phase measurements and
  - External ionospheric observations ('pseudo-observations')
  - System:

$$
\begin{cases}
\Delta\nabla\phi_1 = \Delta\nabla\rho + \lambda_1\Delta\nabla N_1 - I_1 + \varepsilon_{\phi_1} \\
\Delta\nabla\phi_2 = \Delta\nabla\rho + \lambda_2\Delta\nabla N_2 - \dfrac{f_1^2}{f_2^2} I_1 + \varepsilon_{\phi_2} \\
\Delta\nabla\phi_3 = \Delta\nabla\rho + \lambda_3\Delta\nabla N_3 - \dfrac{f_1^2}{f_3^2} I_1 + \varepsilon_{\phi_3} \\
\Delta\nabla P_1 = \Delta\nabla\rho + I_1 + \varepsilon_{P_1} \\
\Delta\nabla P_2 = \Delta\nabla\rho + \dfrac{f_1^2}{f_2^2} I_1 + \varepsilon_{P_2} \\
I = I_1
\end{cases}
$$

Observation

States: position, ambiguities and ionospheric errors

Unmodeled Errors (Noise, Multipath, Tropospheric and Orbital Errors)

Odijk, D. (2002) Fast Precise GPS positioning in the presence of ionospheric delays. Doctoral Thesis, Publ. on Geodesy 52, Netherlands Geodetic Commission

Partial Ambiguity Fixing

- The method is conceptually simple:
  - Assume N satellites tracked, in which case there are N-1 DD ambiguities to resolve
  - For the simple case, the state (unknowns) vector consists of 3 position differences and N-1 ambiguities for a total of N+2 states
  - If all ambiguities can be resolved, the state vector reduces to 3 states, yielding maximum observability and best accuracy
  - During the ambiguity resolution process, some ambiguities are resolved earlier than others, depending on several factors, resulting in the reduction of the state vector
  - As the dimension of the state vector shrinks, observability and accuracy increase
  - In many situations (e.g. some satellites heavily impacted by the ionosphere), only ambiguity subsets can be resolved as integer. If the subset is deemed adequate, a solution based only on it can be selected or, alternatively, a mixed fixed/float integer solution is used.

Overview

- Several AR strategies are reviewed/tested in the sequel
  - Eight fixed and float strategies
  - Backward processing and comparison with forward processing
  - Averaging with more than one base station
  - Comparison of different software packages
  - Batch methods

Eight AR Strategies

Strategy	Ambiguity	Observables	Ionosphere Bias
1: Single Frequency	N_1	CP_1, P	Not Parameterized
2: Wide Lane (WL)	N_{WL}	CP_1, CP_2, P	
3: L1 and L2	$N_1 \ N_2$	CP_1, CP_2, P	
4: L1 and WL	$N_1 \ N_{WL}$	CP_1, CP_2, P	
5: IF fixed	$N_1 \ N_{WL}$ (IF Fixed)	CP_1, CP_2, P	Ionosphere-Free Combination
6: IF Float	N_{IF} (IF Float)	CP_1, CP_2, P	
7: L1 and L2 with SIM	$N_1 \ N_2 \ I_1$	CP_1, CP_2, P	Stochastic Ionosphere Modeling
8: L1 and WL with SIM	$N_1 \ N_{WL} \ I_1$	CP_1, CP_2, P	

CP_1 – L1 Carrier Phase
CP_2 – L2 Carrier Phase
P – Pseudo-range Code
SIM - Stochastic Ionosphere Modeling

LAMBDA decorrelation and search is used to fix ambiguities when applicable.

Liu, J., M.E. Cannon, Pl. Alves, M.G. Petovello, G. Lachapelle, G. MacGougan and L. DeGroot (2003) A Performance Comparison of Single and Dual Frequency GPS Ambiguity Resolution Strategies. **GPS Solutions**, 7, 2, 87-100

Strategy 1: Single Frequency

Observations: CP_1, P   States: Position, Velocity, N_1

L1 code and phase are used to observe position, velocity and L1 ambiguities

$$CP_1 = \frac{\rho}{\lambda_1} + N_1 + e(cp_1)$$

$$P = \rho + e(P)$$

PROS	simple model, low noise and ionospheric error in metres, applicable to single frequency receivers
CONS	difficult to resolve the L1 integer ambiguties for long baselines or in periods of high ionospheric activity

Strategy 2: Wide Lane

Observations: CP_1, CP_2, P States: Position, Velocity, N_{WL}

L1 code and wide lane (WL) phase are used to observe position, velocity and WL ambiguities

$$CP_{WL} = CP_1 - CP_2$$

$$N_{WL} = N_1 - N_2$$

$$CP_{WL} = \frac{\rho}{\lambda_{WL}} + N_{WL} + e(cp_{WL})$$

$$P = \rho + e(P)$$

PROS	easy to resolve ambiguities due to large wavelength (86cm) to ionospheric error ratio (in cycles)
CONS	ionospheric error in meters larger than the L1 observable, nearly six times noisier than the L1 in metres, noisy position estimates expected

Ambiguity Resolution Strategies

Strategy 3: L1 and L2

Observations: CP_1, CP_2, P States: Position, Velocity, N_1, N_2

L1 code and L1 and L2 phase are used to observe position, velocity and L1 and L2 ambiguities

$$CP_1 = \frac{\rho}{\lambda_1} + N_1 + e(cp_1)$$

$$CP_2 = \frac{\rho}{\lambda_2} + N_2 + e(cp_2)$$

$$P = \rho + e(P)$$

PROS	more system redundancy, the carrier phase noise minimized because no frequency combinations formed between L1 and L2.
CONS	L2 has a higher ionospheric error in metres than L1 and WL, expected to suffer significantly from ionospheric error in periods of high ionospheric activity

Strategy 4: L1 and WL

Observations: CP_1, CP_2, P   States: Position, Velocity, N_1, N_{WL}

L1 code and L1 and L2 phase are used to observe position, velocity and L1 and WL ambiguities

$$CP_1 = \frac{\rho}{\lambda_1} + N_1 + e(cp_1)$$

$$CP_2 = \frac{\rho}{\lambda_2} + N_1 - N_{WL} + e(cp_2)$$

$$P = \rho + e(P)$$

PROS	fast convergence of WL ambiguties, more system redundancy
CONS	expected to suffer significantly from ionospheric error in periods of high ionospheric activity

Strategy 5: IF Fixed

Observations: CP_1, CP_2, P   States: Position, Velocity, N_{WL}, N_1

1. L1 code and WL phase are used to observe position, velocity and WL ambiguities.
2. When the WL ambiguities are determined then IF phase are used with WL to observe ionosphere free L1.
3. When the L1 ambiguities are determined then IF phase with fixed ambiguities is used to observed position and velocity.

$$CP_{WL} = \frac{\rho}{\lambda_{WL}} + N_{WL} + e(cp_{WL})$$

$$P = \rho + e(P)$$

$$\Longrightarrow \quad CP_1 - \frac{\lambda_1}{\lambda_2} CP_2 - \frac{\lambda_1}{\lambda_2} N_{WL} = \frac{\rho}{\lambda_{IF}} + \frac{\lambda_2 - \lambda_1}{\lambda_2} N_1 + e(cp_{IF})$$

PROS	free from first-order ionospheric error for the determination of N_1 and the positions
CONS	three times noise level as L1 (in meters) , and very short effective N_1 wavelength (10.7 cm)

Strategy 6: IF Float

Observations: CP_1, CP_2, P States: Position, Velocity, N_{IF}

L1 code and IF phase are used to observe position, velocity and IF ambiguities

$$CP_1 - \frac{\lambda_1}{\lambda_2} CP_2 = \frac{\rho}{\lambda_{IF}} + N_{IF} + e(cp_{IF})$$

$$P = \rho + e(P)$$

PROS	float-valued IF ambiguity should be unbiased, no risk of resolving wrong ambiguities
CONS	high noise level of the IF combinations, long convergence time

7

Strategy 7: L1 and L2 with SIM

Observations: CP_1, CP_2, P, I_0 States: Position, Velocity, N_1, N_2, I_1

L1 code and L1 and L2 phase are used to observe position, velocity, L1 and L2 ambiguities, and ionosphere bias

$$CP_1 = \frac{\rho}{\lambda_1} + N_1 - I_1 + e(cp_1)$$

$$CP_2 = \frac{\rho}{\lambda_2} + N_2 - \frac{\lambda_2}{\lambda_1^2} I_1 + e(cp_2)$$

$$P = \rho + I_1 + e(P)$$

$$I_0 = I_1$$

$$I_0 \sim (0, \sigma^2)$$

PROS	estimator largely unbiased, position estimates not influenced by ionospheric error, observation noise is kept minimum
CONS	weaker solutions as additional states are estimated, slow filter convergence

Strategy 8: L1 and WL with SIM

Observations: CP_1, CP_2, P — States: Position, Velocity, N_1, N_{WL}

L1 code and L1 and L2 phase are used to observe position, velocity, L1 and WL ambiguities, and ionosphere bias

$$CP_1 = \frac{\rho}{\lambda_1} + N_1 - I_1 + e(cp_1)$$

$$CP_2 = \frac{\rho}{\lambda_2} + N_1 - N_{WL} - \frac{\lambda_2}{\lambda_1^2} I_1 + e(cp_2)$$

$$P = \rho + I_1 + e(P)$$

$$I_0 = I_1$$

$$I_0 \sim (0, \sigma^2)$$

PROS	fast convergence of WL ambiguties, estimator largely unbiased, position estimates not influenced by ionospheric error, observation noise is kept minimum
CONS	weaker solutions as additional states are estimated, slow filter convergence

Batch Methods

- Most software are developed to process data sequentially (usually forward, sometime forward and backward)
- Above (Forward) is better suited for real-time environment
- Use of least-squares or Kalman filtering
- In batch mode, data segments, large or small, can be processed simultaneously
    - Effects of the ionosphere and troposphere can be better modeled
    - Ambiguities can be better determined
- Often used for very precise static applications with long data sessions (surveying, precise slow movement monitoring, etc)
- Can be used for kinematic applications
- Example: Bernese software

CASE STUDY
Carrier-Phase Positioning

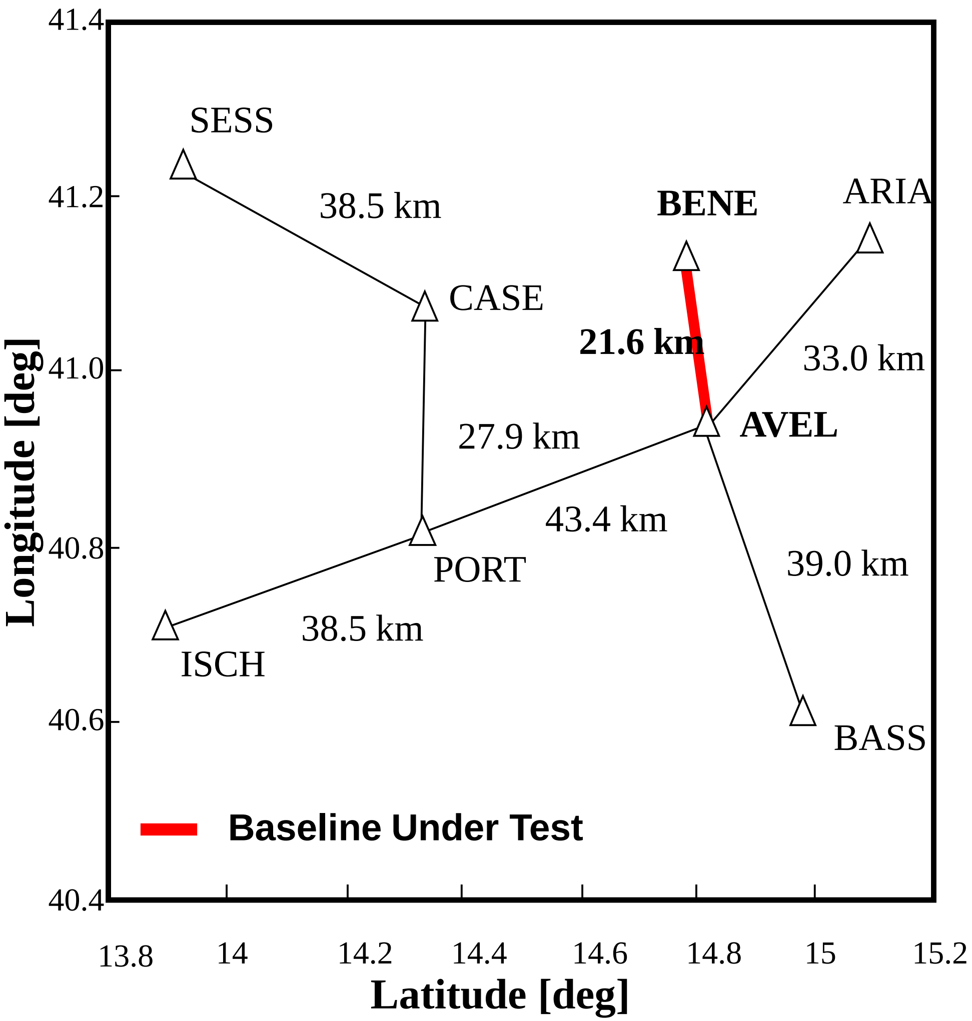
- Static data collection of several days with geodetic receivers, with a data rate of 1s
- Batch processing with Bernese software to obtain station positions and ambiguities
- Kinematic positioning simulated using epoch-by-epoch forward processing with Univ. of Calgary's FLYKIN+™ software
- Comparison of kinematic versus batch mode to assess kinematic performance

LIU, J., P. ALVES, M. PETOVELLO, G. MacGOUGAN, L. de GROOT, M.E. CANNON, and G. LACHAPELLE (2002) Development and Testing of an Optimal Cascading Scheme to Resolve Multi-Frequency Carrier Phase Ambiguities. Proceedings of GPS2002 (Session F2, Portland, OR, 24-27 September), The Institute of Navigation, 933-944.

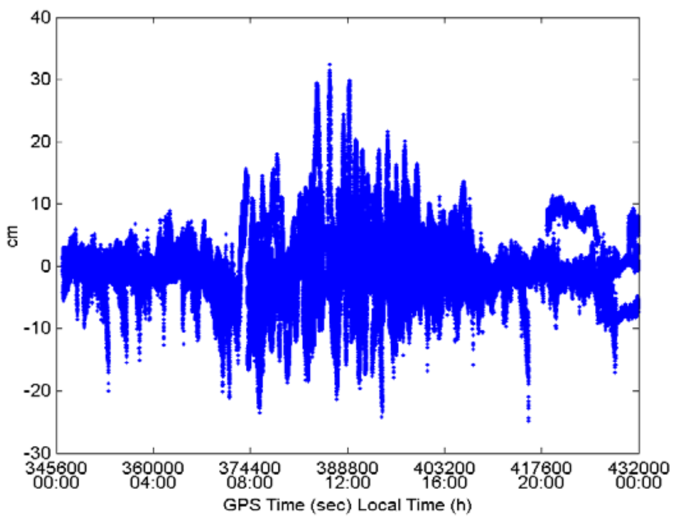
PUGLIANO, G. (2002) Tecnica GPS Multi-Reference Station - Principi E Applicazione Del Systema MultiRef. PhD Thesis, Universita' Degli Studi di Napoli Parthenope (www.geomatics.ucalgary.ca/GradTheses.html)

Italian (Campagna) Carrier-Phase Positioning Dataset

Avel-to-Bene Baseline (22 km)



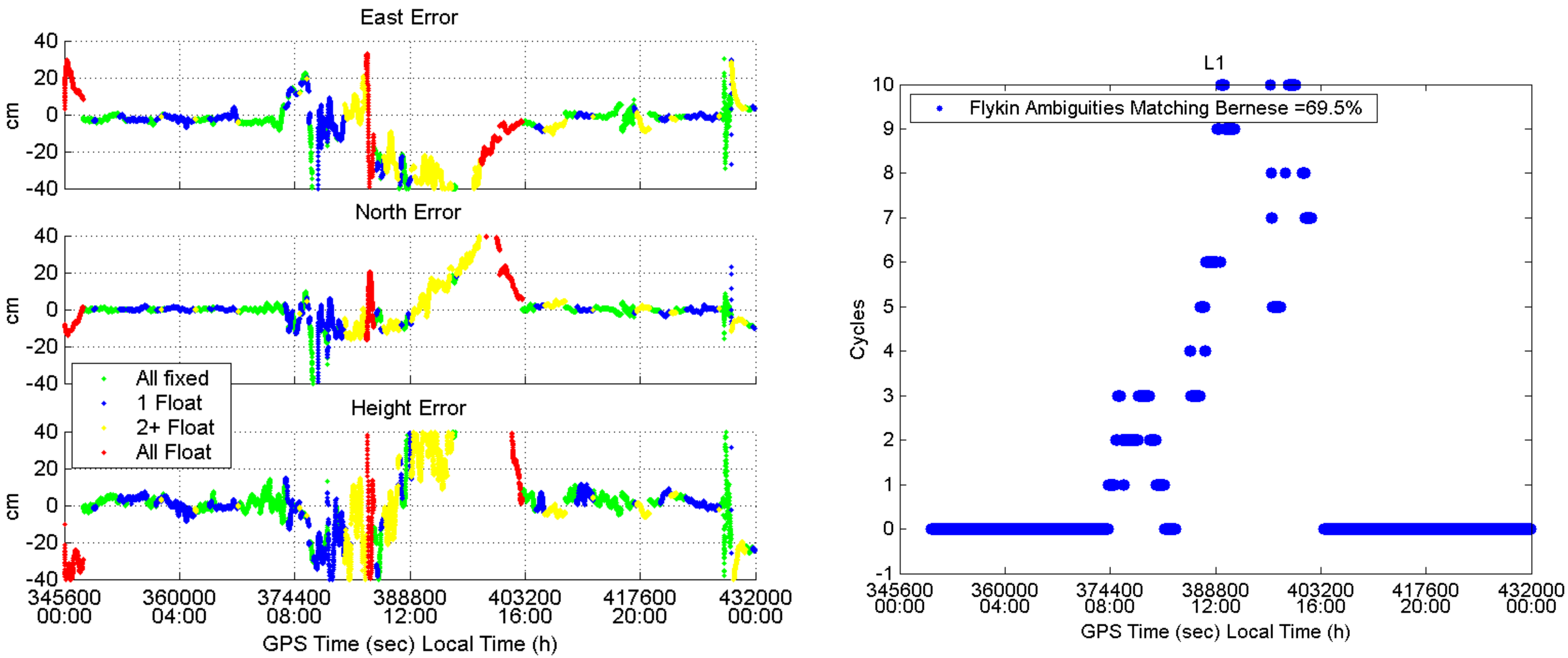
Italian Dataset Information	
Network	**Campania**
Location	**Italy**
Date	**02/07/02**
Duration	**24 hrs**
Data Rate	**1 Hz**
Elev Mask	**15°**



DD Ionospheric Bias

Italian Carrier-Phase Positioning Dataset

Single Frequency, 22 Km Baseline



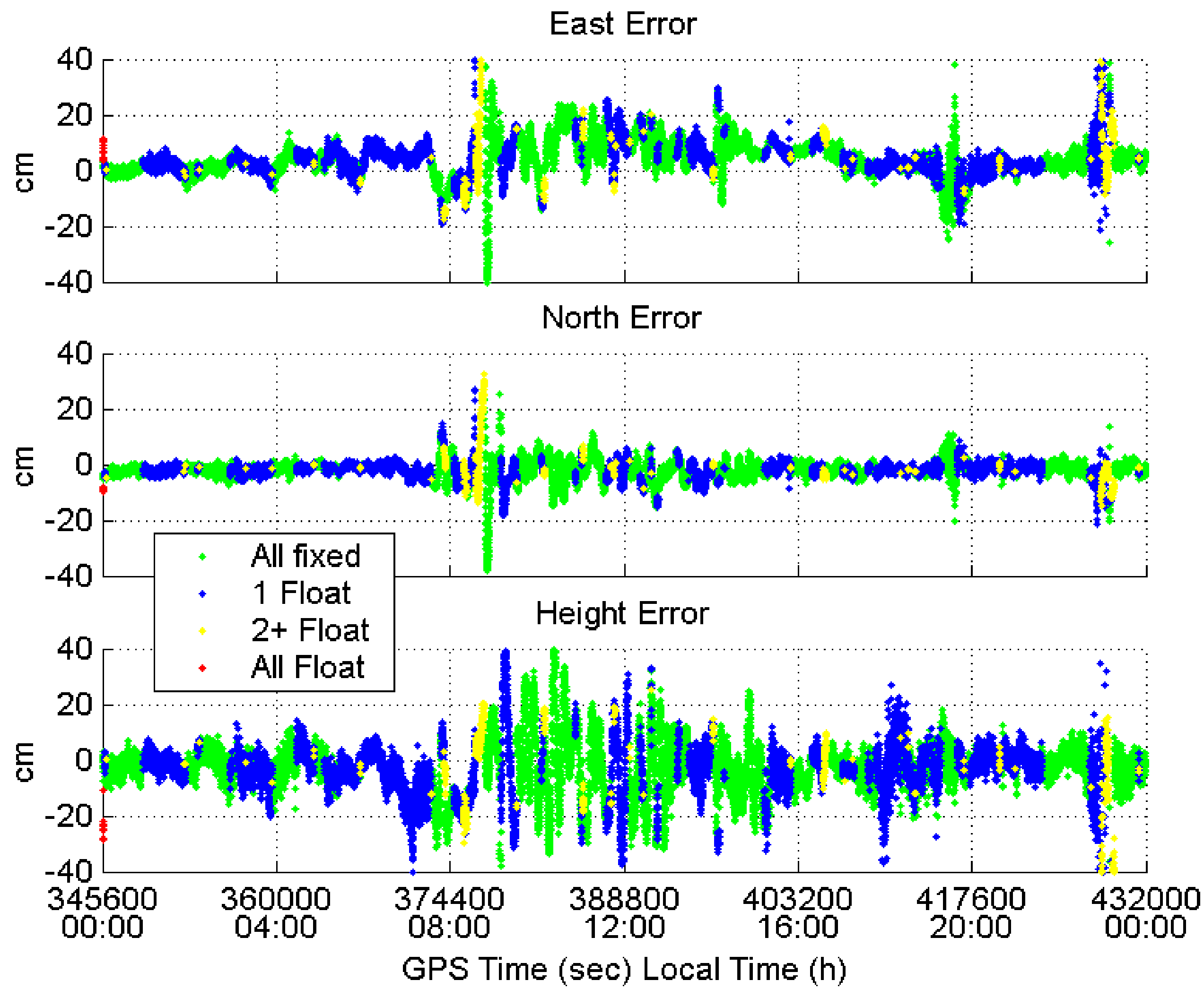
Poor performance during high mid-day ionospheric activity.

Italian Carrier-Phase Positioning Dataset

Wide lane, 22 Km Baseline

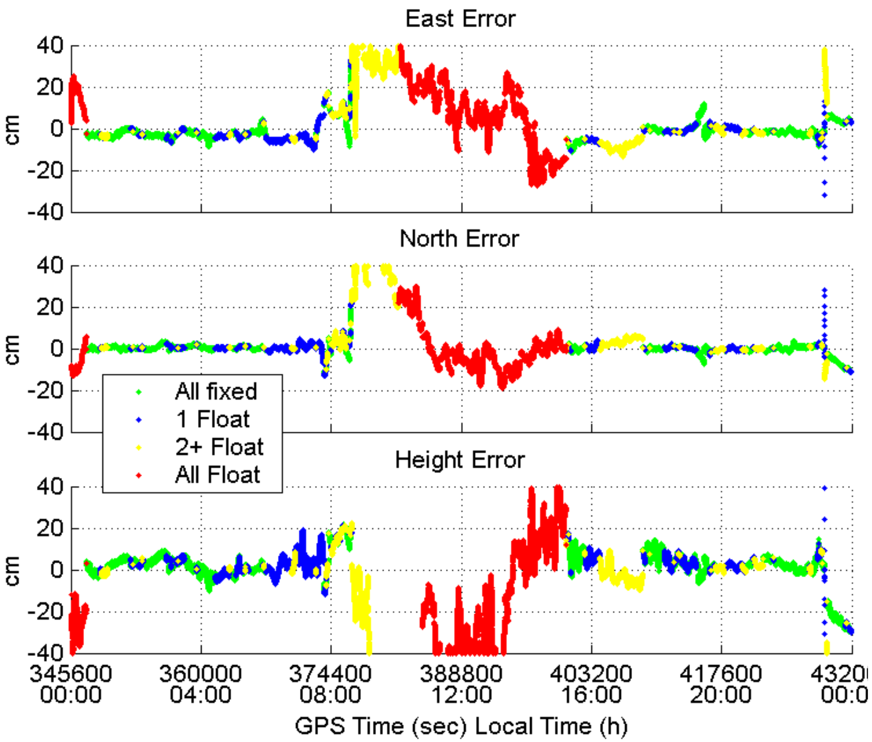
Wide lane results are much less biased than the single frequency results because all the fixed WL ambiguities are correct.

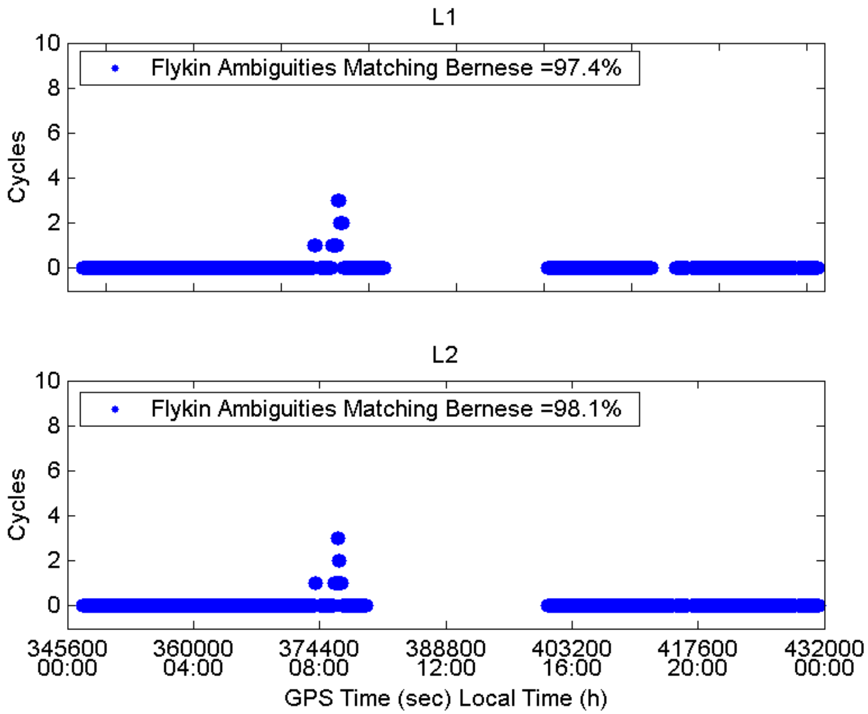
Wide lane is much noiser than L1 only.



Italian Carrier-Phase Positioning Dataset

L1 and L2, 22 Km Baseline

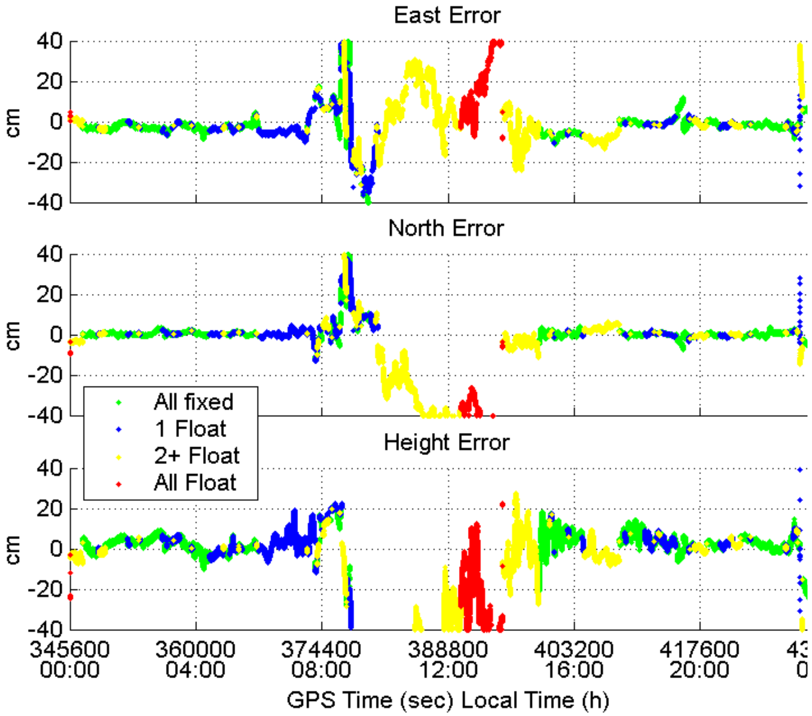


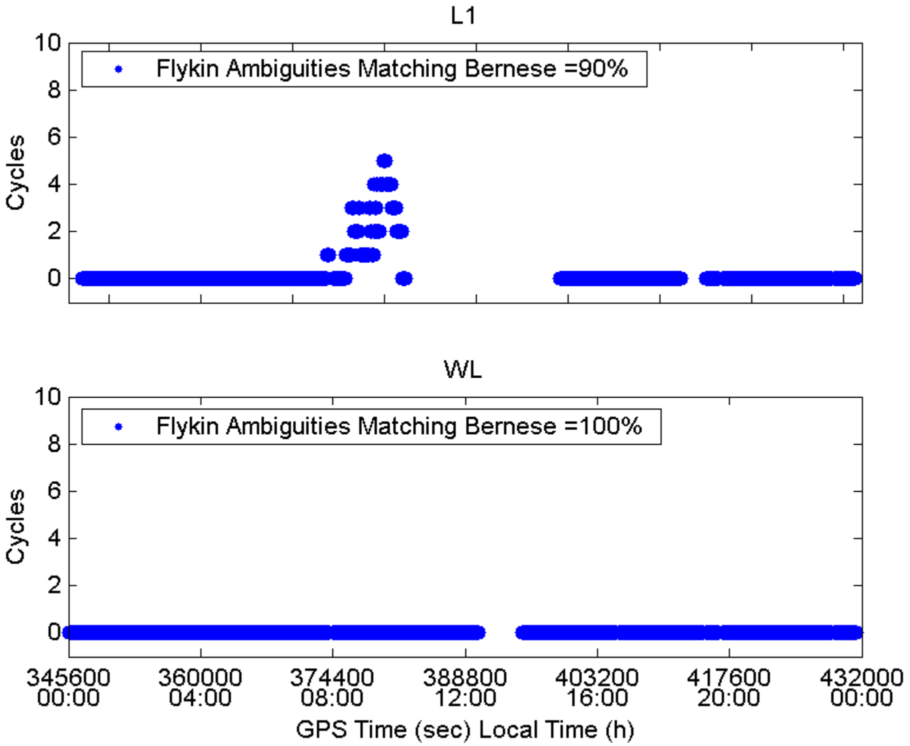


Poor performance during high midday ionospheric activity.

Italian Carrier-Phase Positioning Dataset

L1 and WL, 22 Km Baseline



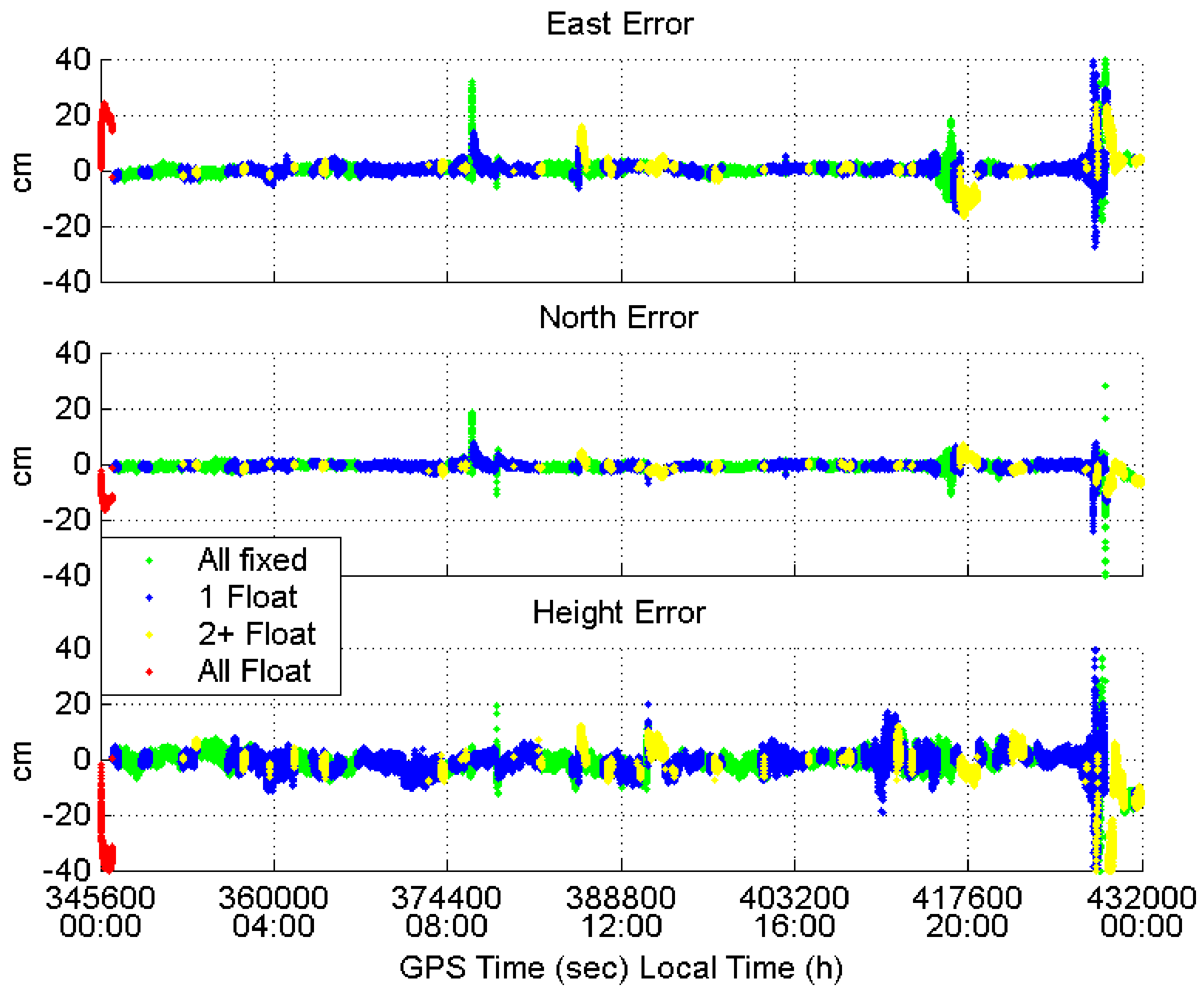


Poor performance during high midday ionospheric activity.

Italian Carrier-Phase Positioning Dataset

IF Fixed, 22 Km Baseline

IF Fixed results are very good. All midday ionosphere characteristics are removed.

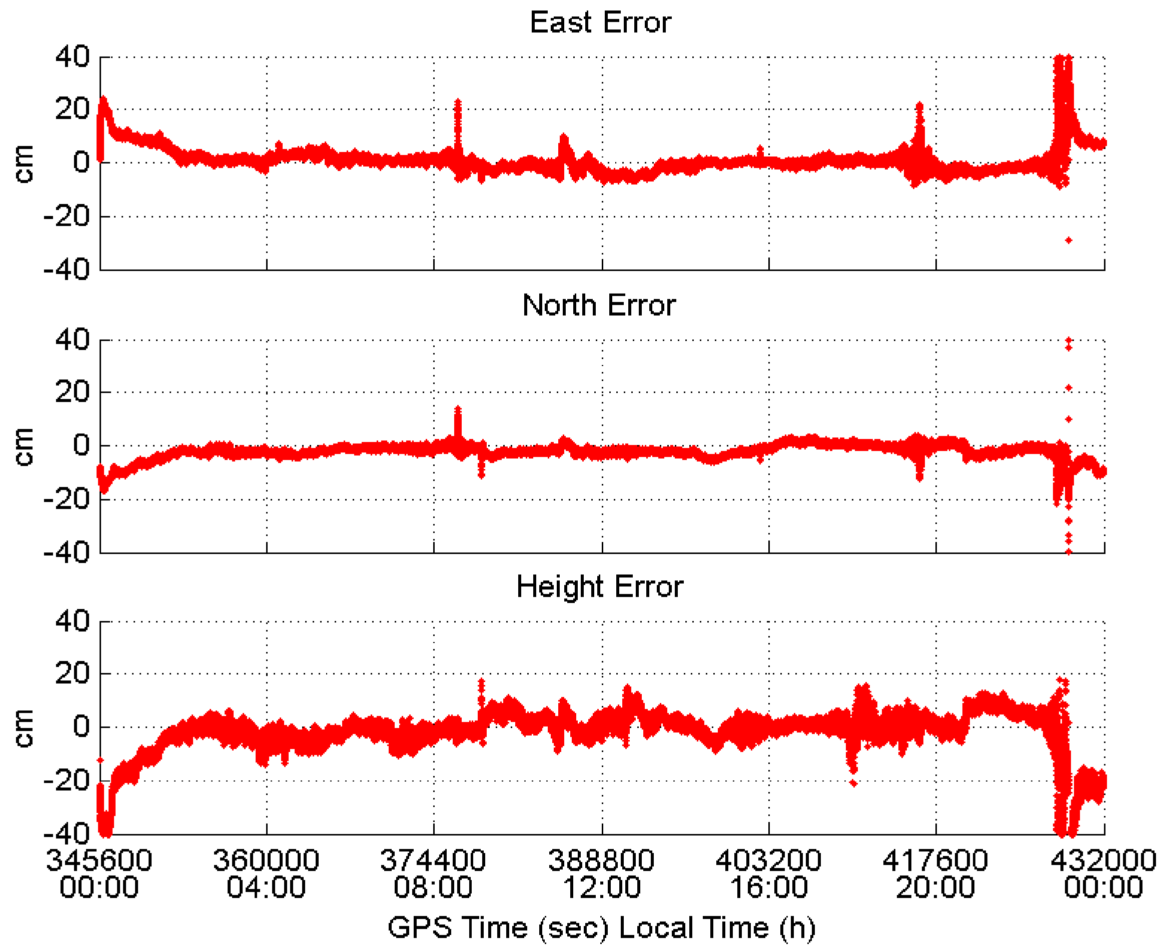


Italian Carrier-Phase Positioning Dataset

IF Float, 22 Km Baseline

IF Float results are very good. All midday ionosphere characteristics are removed.

Convergence is much slower than with IF Fixed.

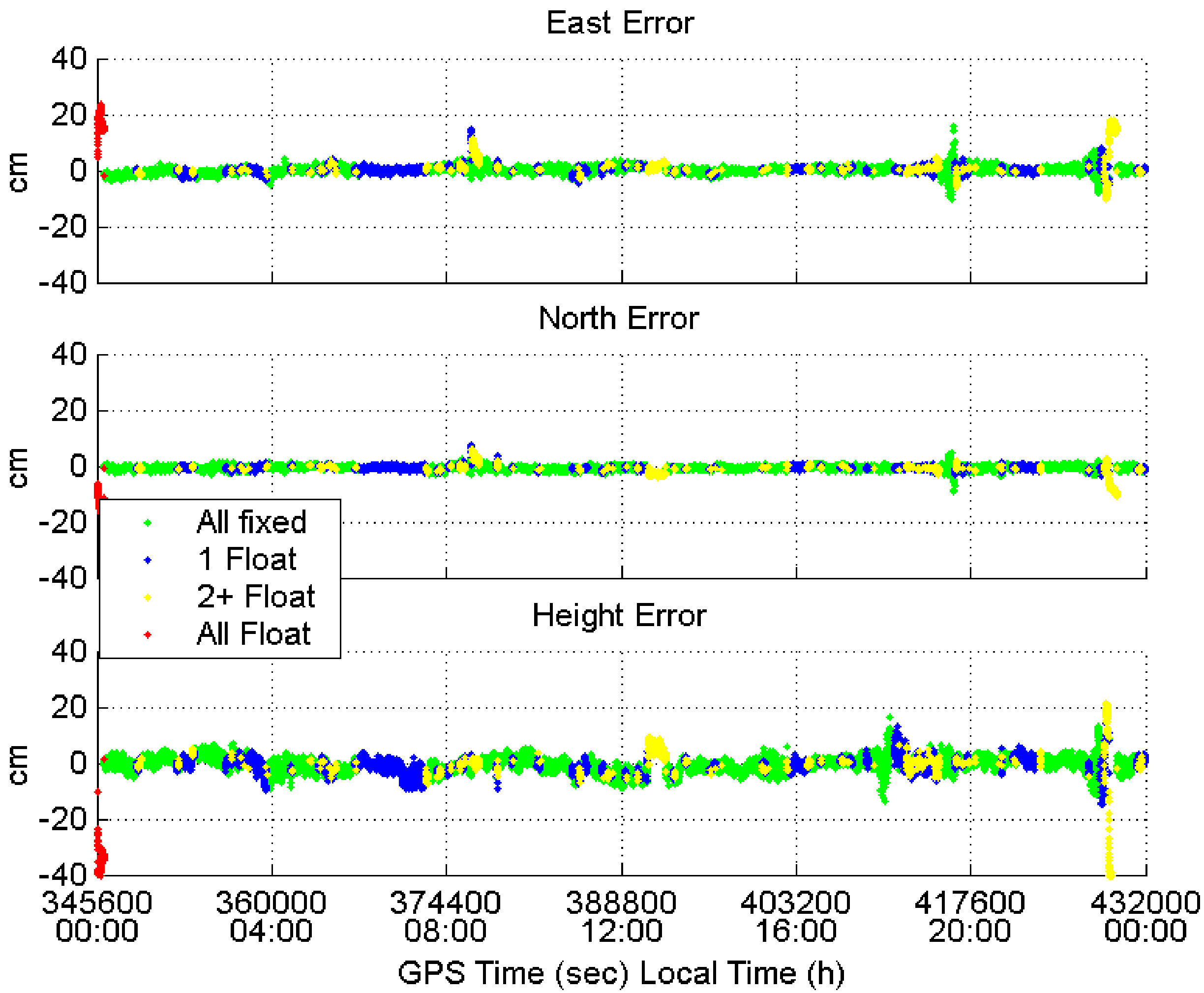


Italian Carrier-Phase Positioning Dataset

L1 and L2 with SIM, 22 Km Baseline

L1 and L2 with SIM is comparable to the IF Fixed solution but with less noise.

All midday ionosphere characteristics are removed.

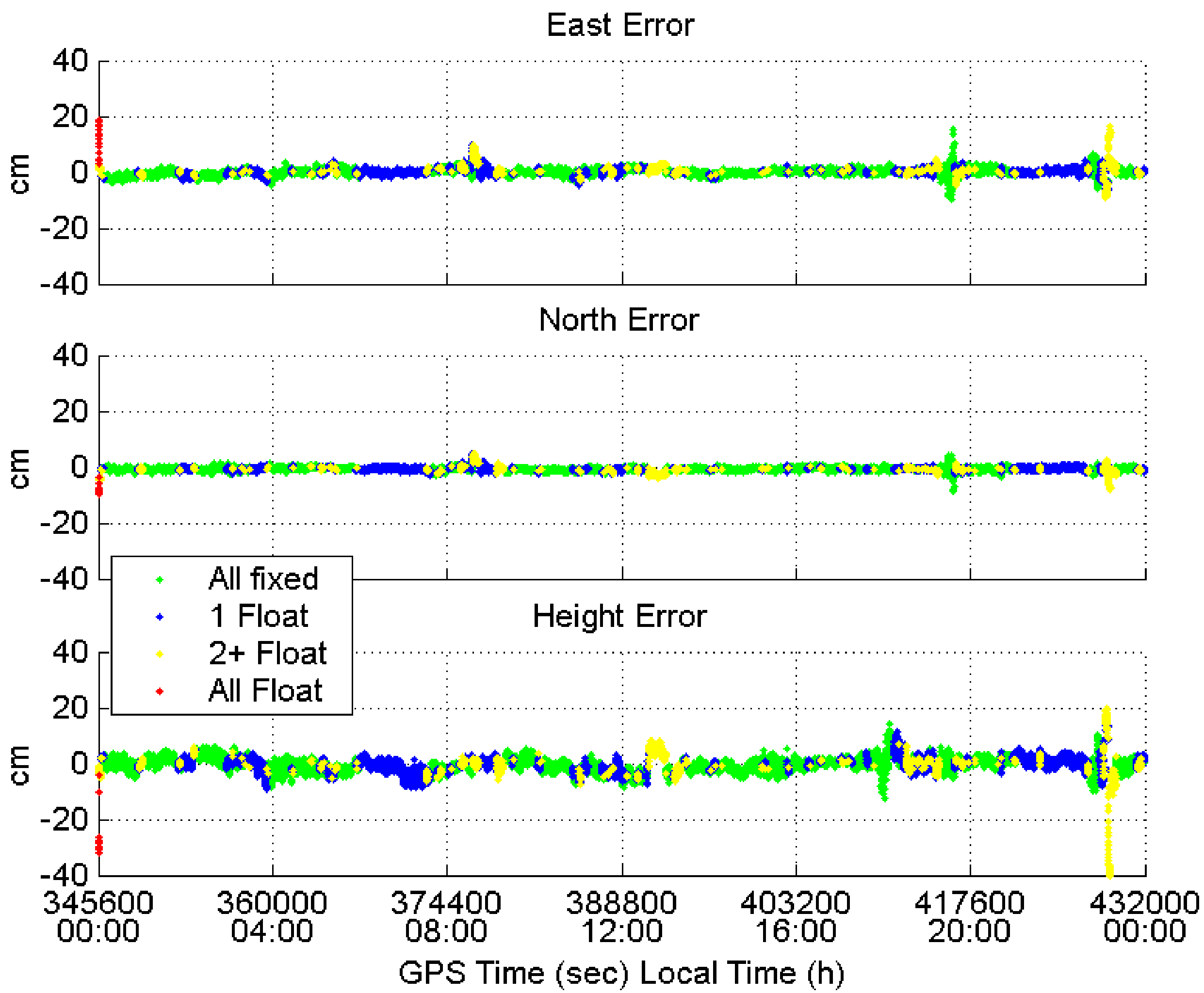


Italian Carrier-Phase Positioning Dataset

L1 and WL with SIM, 22 Km Baseline

L1 and WL with SIM is slightly better than L1 and L2 with SIM

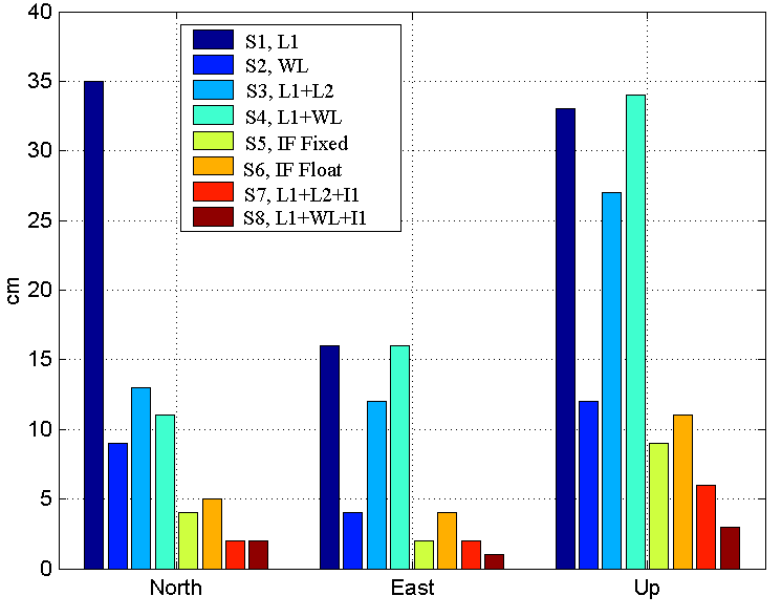
All midday ionosphere characteristics are removed.



Italian Carrier-Phase Positioning Dataset

RMS Statistics Summary, 22 Km Baseline

Position RMS(cm), All points



Percentage of correctly Fixed Ambiguity(%)

	S1	S2	S3	S4	S5	S6	S7	S8
N_1	69.5		97.4	90	100		100	100
N_2			98.1				100	
N_{WL}		100		100	100			100

Testing of Forward vs Backward vs two-base methods with Different Software

- Two independent GPS software used
  - NovAtel's GrafNav and University of Calgary's FLYKIN+.
  - Forward and reverse processing
  - One and two base stations

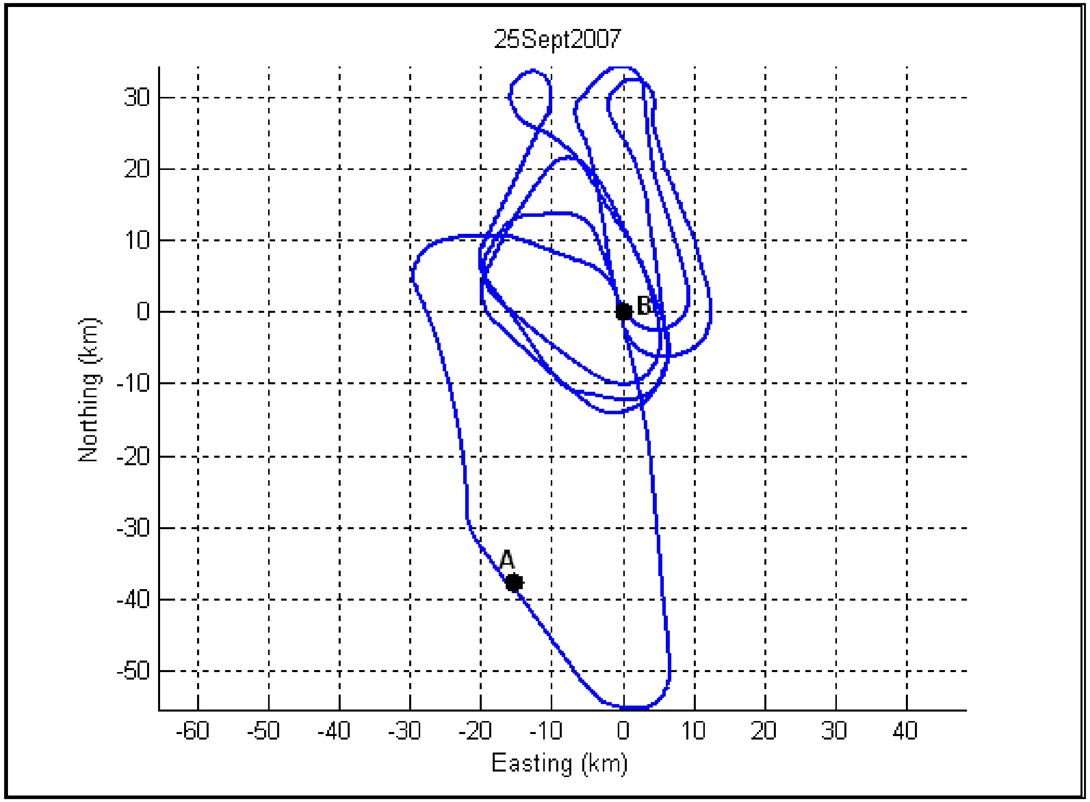
GPS Processing Summary (FWD = Forward Processing, REV = Reverse Processing, CMB = Forward/Reverse Combined Processing)

Software	Base Station		
	B	A	A & B
GrafNav	FWD, REV, CMB	FWD, REV, CMB	FWD, REV, CMB
FLYKIN+™ (L1&L2)	FWD	FWD	FWD
FLYKIN+™ (WL)	FWD	FWD	FWD

Liu, J., M.E. Cannon, Pl. Alves, M.G. Petovello, G. Lachapelle, G. MacGougan and L. DeGroot (2003) A Performance Comparison of Single and Dual Frequency GPS Ambiguity Resolution Strategies. **GPS Solutions**, 7, 2, 87-100

Aircraft Test Description

- Light jet aircraft, up to 300 km/hr, 100 minutes, Sep07,period of low ionospheric activity, latitude of about 55 degrees.
- Dual-frequency high performance GPS receivers at two base stations (A and B) separated by 40 km. Data logged at 5 Hz.



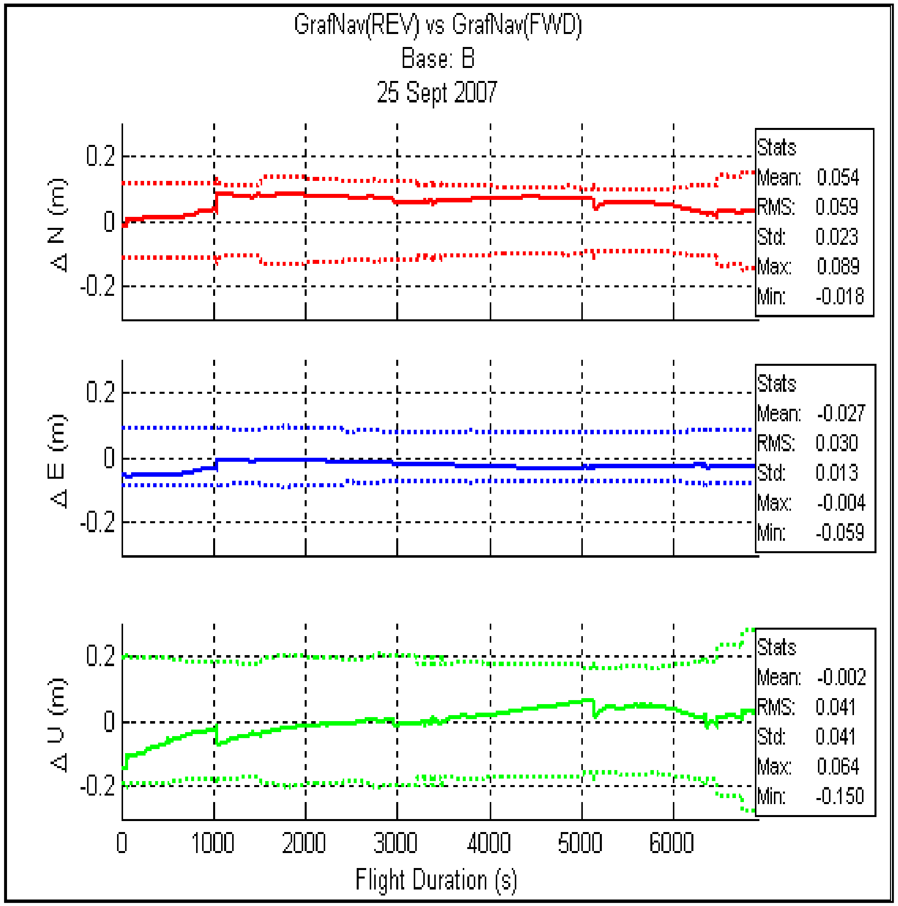
Data Courtesy of the Canadian Forces, Aerospace Engineering Test Establishment, Cold Lake, Alberta

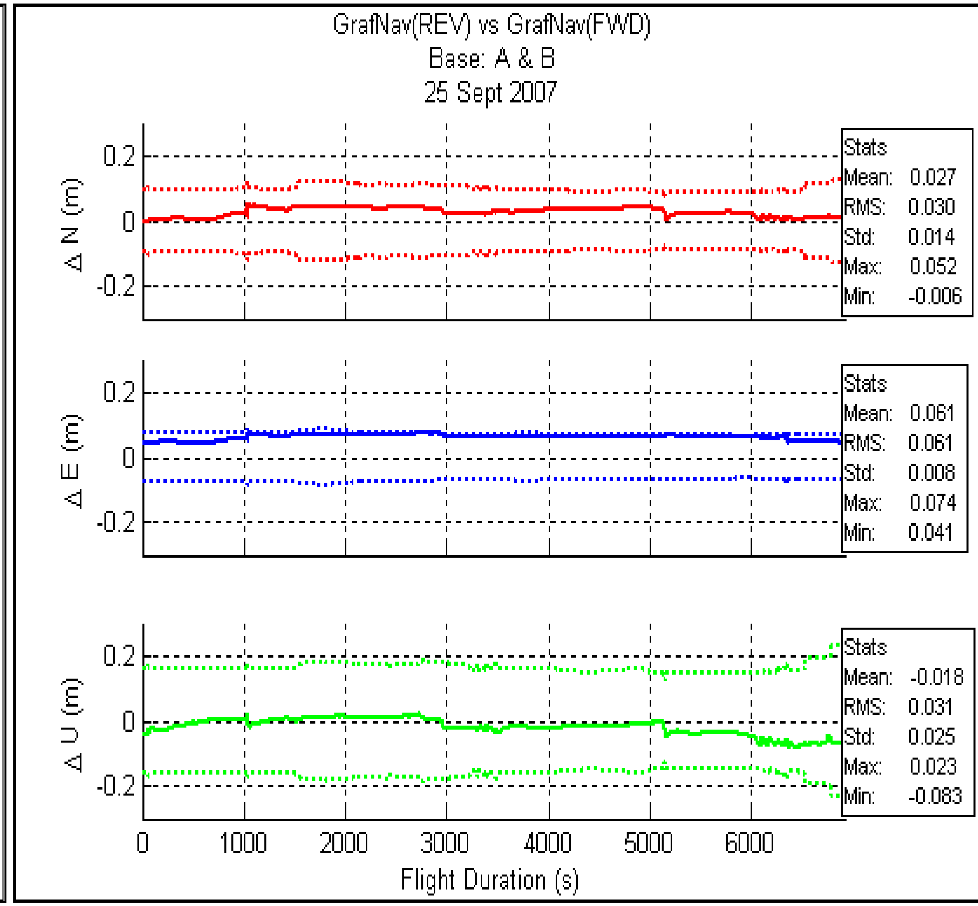
Data Analysis Strategy

- Assess the GrafNav solutions by comparing their forward and reverse solutions using different combinations of base stations
- Assess the FLYKIN+™ solutions by comparing the results obtained with L1&L2 and widelane ambiguities using different base stations
- Compare the GrafNav and FLYKIN+™ solutions
- GrafNav solution had a “fixed” status throughout the entire kinematic portion of the data. Although no information is provided regarding the number or type of fixed ambiguities, the fact that the solutions were “fixed” is indeed promising

GrafNav (Rev) Vs GrafNav (For)

- The solid line is the position agreement and the dotted lines represent the estimated 3σ bounds (99.7%) in metres
- Similar results for Base A





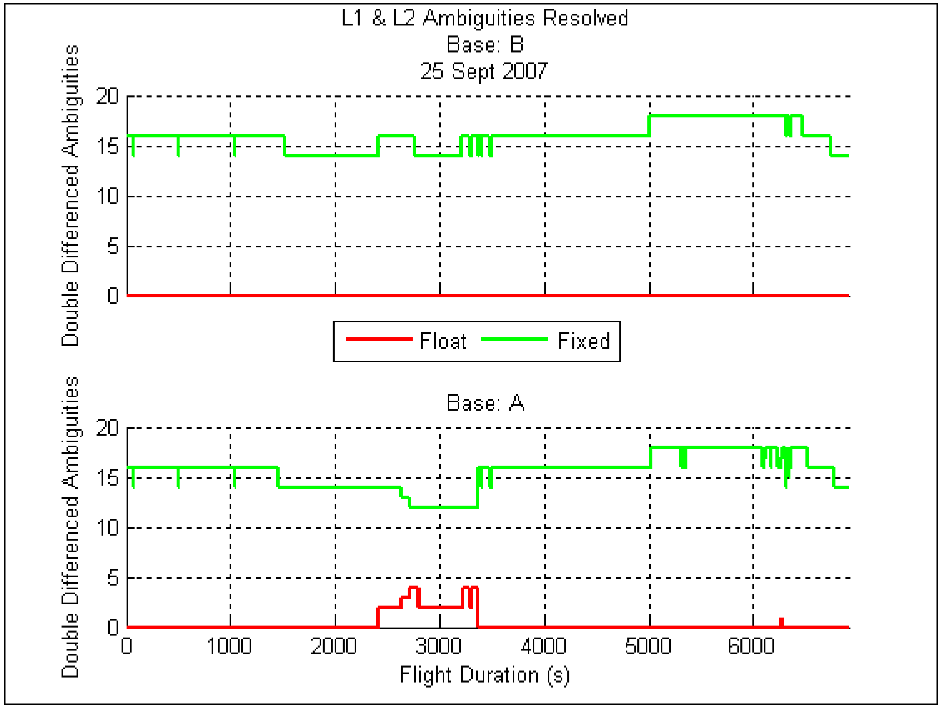
GrafNav Consistency Statistics

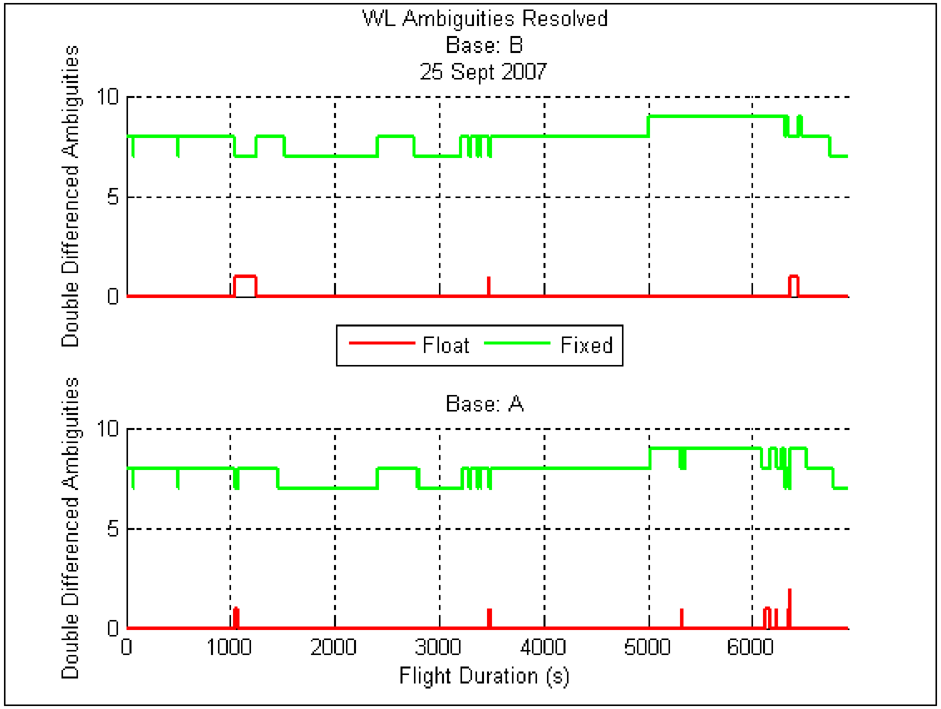
RMS Position Agreement of Different Solutions Obtained using GrafNav and Different Combinations of Base Stations

Solution 1	Solution 2	RMS Agreement (m)		
		North	East	Vertical
GrafNav FWD B	GrafNav REV A	0.059	0.030	0.041
GrafNav FWD (A)	GrafNav REV (A)	0.019	0.052	0.033
GrafNav FWD (B + A)	GrafNav REV (B + A)	0.030	0.061	0.031
GrafNav FWD (B)	GrafNav FWD (A)	0.074	0.048	0.051

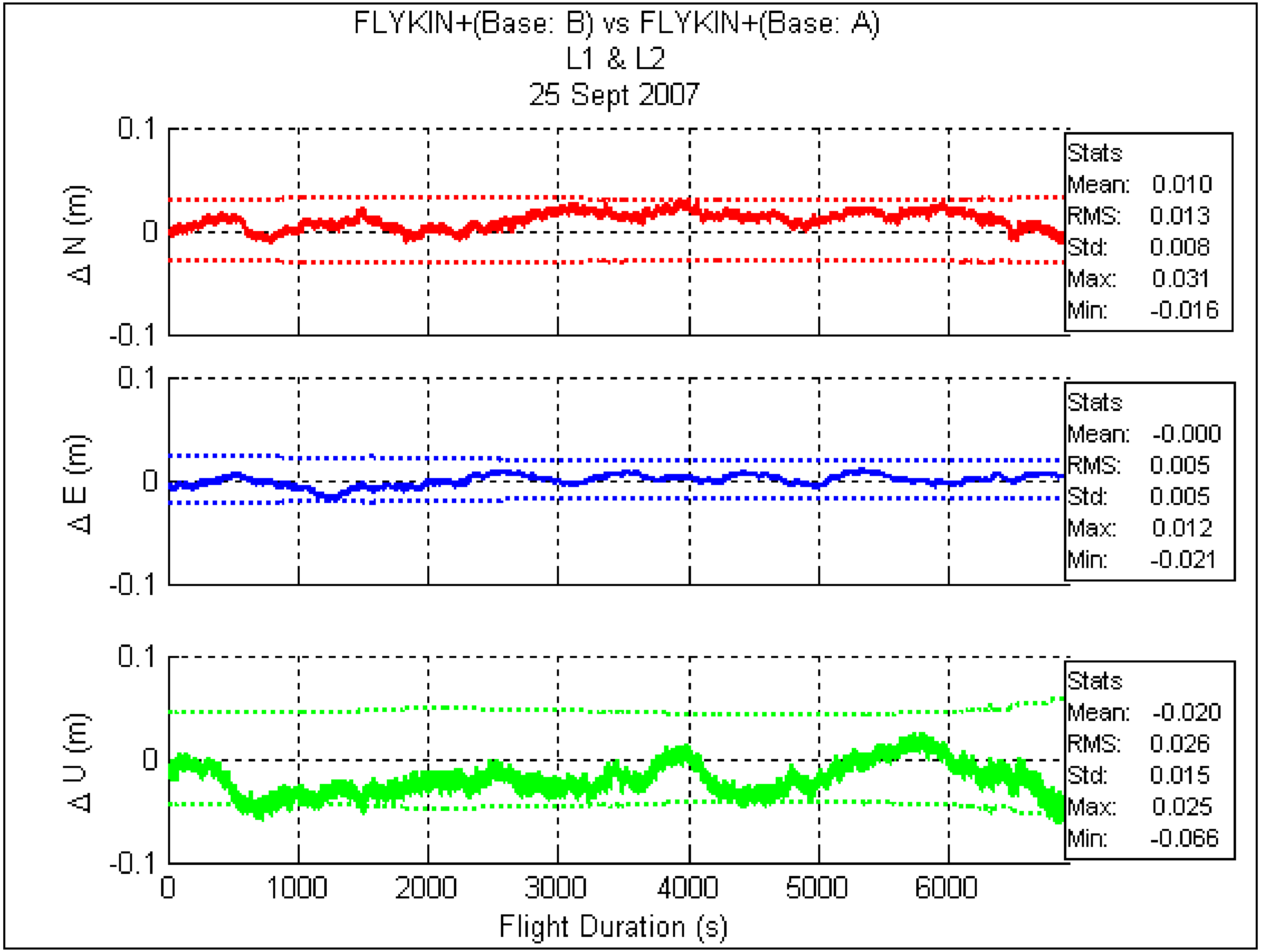
FLYKIN+™ - Resolved Ambiguities

Fixed/float approach during short intervals





FLYKIN+™ Position Agreement (L1 & L2) - Base A vs B



FLYKIN+™ Consistency Statistics

RMS Position Agreement of Different Solutions Obtained using FLYKIN+™

Solution 1	Solution 2	RMS Agreement (m)		
		North	East	Vertical
FLYKIN+™ L1&L2 (B)	FLYKIN+™ L1&L2 (A)	0.013	0.005	0.026
FLYKIN+™ WL (B)	FLYKIN+™ WL (A)	0.040	0.014	0.039

FLYKIN+™ vs GrafNav Solution Statistics

- ***The solutions are consistent to within 3 to 9 cm***

RMS Position Agreement Between FLYKIN+™ and GrafNav Solutions

Solution 1	Solution 2	RMS Agreement (m)		
		North	East	Vertical
FLYKIN+™ L1&L2 (B)	GrafNav CMB (B)	0.033	0.025	0.075
FLYKIN+™ WL (B)	GrafNav CMB (B)	0.027	0.031	0.083
FLYKIN+™ L1&L2 (A)	GrafNav CMB (A)	0.046	0.020	0.048
FLYKIN+™ WL (A)	GrafNav CMB (A)	0.026	0.026	0.078