Problem Set #1

Thursday, 17 October 2024 12:22

1. **Elasticity and Stress. [5 pts]** Two specimens of granite are studied. The cross-sectional area of each sample is 1 cm^2. One sample is 11 cm in length and for this sample a load of 375 MPa causes a deformation of 508 microns. The stiffness *k* of the second sample is 5.08×10^7 N/m.
1.1 What is Youngs modulus E of the granite?
1.2 The stiffness k can be expressed in units of force per displacement. Write an equation for stiffness in terms of E and other variables.
1.3 How long is the second granite sample?

$A = 1\,cm^2 = 1 \cdot 10^{-4}\,m^2$ } both

$L_1 = 11\,cm = 0.11\,m$
$\sigma_1 = 375\,MPa = 3.75 \cdot 10^{8}\,Pa$
$\Delta L_1 = 508\,\mu m = 5.08 \cdot 10^{-4}\,m$
} first sample

$k_2 = 5.08 \cdot 10^{7}\,N/m$ } second sample

1.1 Young Module ← $E = \frac{\sigma}{\varepsilon}$ → stress, → strain

$\varepsilon_1 = \frac{\Delta L_1}{L_1} = 5.08 \cdot 10^{-4}\,m \,/\, 0.11\,m \cong 4.618 \cdot 10^{-3}$

$\Rightarrow E_1 = \frac{\sigma_1}{\varepsilon_1} = 3.75 \cdot 10^{8}\,Pa \,/\, 4.618 \cdot 10^{-3} \cong 8.12 \cdot 10^{10}\,Pa$
$= 81.2\,GPa$ → E for granite

1.2

stiffness ← $k = \frac{F}{\Delta L}$ → force, → deformation

using $F = \sigma A$ and $\sigma = E/\varepsilon = E \frac{\Delta L}{L}$

we can write $k = E \frac{\cancel{\Delta L}}{L} A \frac{1}{\cancel{\Delta L}} = \frac{EA}{L}$

1.3 $k = \frac{EA}{L} \rightarrow L = \frac{EA}{k}$

$L_2 = \frac{E_G A}{k_2} = \frac{8.12 \cdot 10^{10}\,Pa \cdot 10^{-4}\,m^2}{5.08 \cdot 10^{7}\,N/m} \cong 0.159\,m \cong 16\,cm$

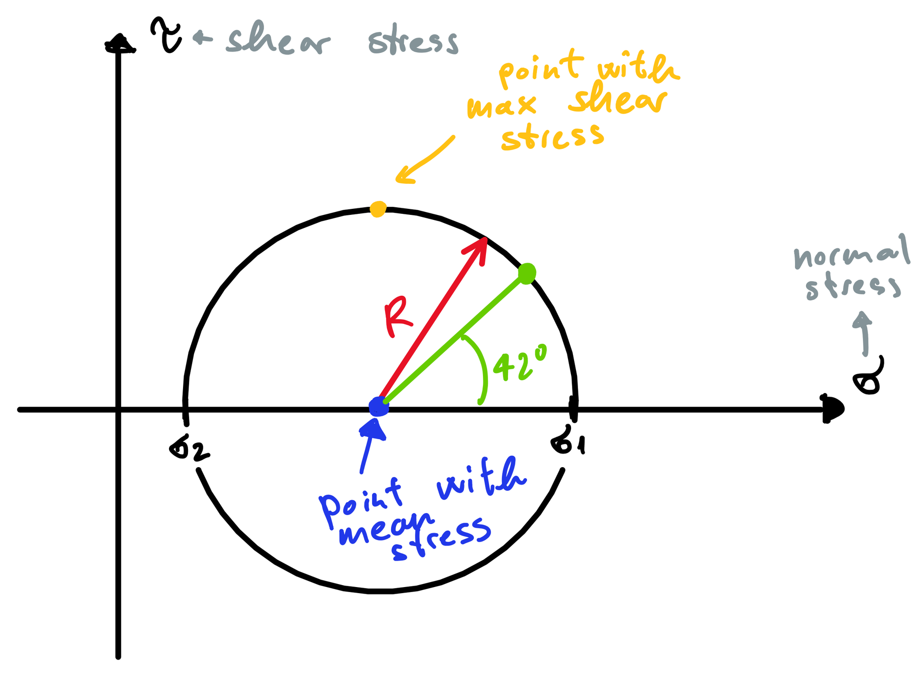
2. **Horizontal Stress. [5]** In many regions the horizontal stress in Earth's crust is produced by the vertical overburden stress and Poisson expansion. That is, we can assume uniaxial strain in the vertical direction due to the mass of the rock above with lateral strain that generates a horizontal stress. The relevant equations are written as: $\sigma_h = \sigma_v [\nu/(1-\nu)]$ where ν is Poisson's ratio, σ_h is horizontal stress, and σ_v is effective vertical stress. Assuming a Poisson ratio of 0.25 and a rock density of 2.5×10^3 kg/m^3 calculate:
2.1 The vertical effective stress at 10 km
2.2 The horizontal effective stress at 10 km

$\rho = 2.5 \cdot 10^{3}\,kg/m^3$
$\nu = 0.25$

2.1
we can calculate the vertical stress as
$\sigma_v = \rho g h = 2.5 \cdot 10^{3}\,kg/m^3 \cdot 9.81\,\cancel{m}/s^2 \cdot 10^{4}\,\cancel{m} \cong 2.45 \cdot 10^{8}\,Pa =$
$= 245\,MPa$

2.2
$\sigma_h = \sigma_v \left(\frac{\nu}{1-\nu}\right) = 2.45 \cdot 10^{8}\,Pa \left(\frac{0.25}{0.75}\right) \cong 8.17 \cdot 10^{7}\,Pa =$
$= 81.7\,MPa$

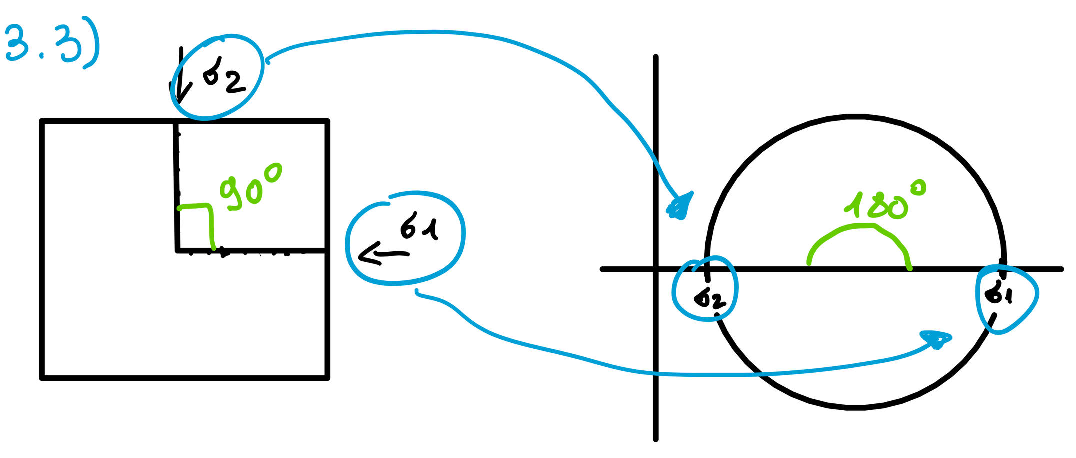
3. **Stress Analysis [5]** Construct a Mohr circle for principal stresses of σ_1=70 MPa and σ_2= 10 MPa. Determine
3.1) the maximum shear stress
3.2) the mean stress
3.3) the stresses on a plane with normal that has an angle of 21° to σ_1.
3.4) Make a detailed sketch (for example like the one in Question 5) of the principal stresses and all planes of interest (e.g., those in 3.1 and 3.3).



3.1)
τ_{max} must be equal to R
$\tau_{max} = R = |\sigma_1 - \sigma_2| / 2 = 30\,MPa$

3.2)
σ_{mean} must be the center of the Mohr circle
$\sigma_{mean} = \sigma_2 + \frac{|\sigma_1 - \sigma_2|}{2} = \sigma_2 + R = 40\,MPa$

3.3)



When we are representing angles in our Mohr circle we need to double it
$\alpha = 21° \rightarrow 2\alpha = 42°$
An then whe know that:
$\tau = R \sin(2\alpha)$, $\sigma = \sigma_{mean} + R \cos(2\alpha)$
$\Rightarrow \tau \cong 20.07\,MPa$, $\sigma \cong 62.29\,MPa$

3.4) Drawing on Top

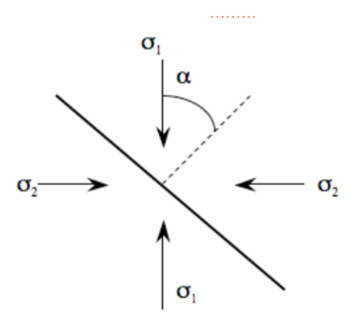
4. **Earthquake Nucleation [5]** For a fault zone in Earth's crust where shear modulus G = 30 GPa, Poisson's ratio = 0.25, and the fault normal stress is 30 MPa, if the fault obeys Slip Weakening Friction with $\Delta\mu/L$ = 1e-5/μm, what is the size of the Earthquake nucleation dimension?

4. $G = 30\,GPa$, $\nu = 0.25$?, $\sigma = 30\,MPa$, $\Delta\mu / L = 10^{-5}\,\mu m^{-1}$

$r = \frac{7\pi}{24} \frac{GL}{\sigma \Delta\mu} = \frac{7\pi}{24} \frac{\cancel{30} \cdot 10^{\cancel{9}\,3}\,\cancel{Pa}}{\cancel{30} \cdot \cancel{10^{6}\,Pa}} \cdot 10^{5}\,\mu m = \frac{7\pi}{24} \cdot 10^{8}\,\mu m \cong 91.62\,m$

↑ nucleation size

5. **Coulomb Failure [15]** Starting from the Coulomb failure criterion and a stress state $\sigma_1 > \sigma_2$, derive the optimum angle α for Coulomb failure in terms of the relationship between α and the Coulomb parameters τ, μ' and C, where α is the angle between σ_1 and the normal to the failure plane, μ' is the coefficient of internal friction and C is the cohesion. State carefully whether the optimum angle α depends on μ' and/or C.
(hint: derivatives will be useful; failure occurs when the difference between τ and ($\mu' \sigma_n$) is maximum)



5. coulomb failure criterion → $\tau = C + \mu' \sigma$ cohesion, internal friction

$R = \frac{(\sigma_1 - \sigma_2)}{2}$ >0 , $\tau(\alpha) = R \sin(2\alpha)$, $\sigma(\alpha) = \sigma_2 + R + R\cos(2\alpha)$

$\tau(\alpha) - c - \mu' \sigma(\alpha) =: f(\alpha)$ must maximum

$f(\alpha) = R \sin(2\alpha) - c - \mu' \sigma_2 - \mu' R - \mu' R \cos(2\alpha)$ → is indipendent of C

$f'(\alpha) = 2R\cos(2\alpha) + 2\mu' R \sin(2\alpha)$ ← set to zero

$\rightarrow \cos(2\alpha) = -\mu' \sin(2\alpha) \rightarrow$

$\rightarrow \frac{\cos(2\alpha)}{\sin(2\alpha)} = -\mu' \rightarrow \frac{1}{\tan(2\alpha)} = -\mu' \rightarrow$

$\rightarrow \tan(2\alpha) = -1/\mu' \rightarrow \bar{\alpha} = \frac{1}{2} \arctan\left(-\frac{1}{\mu'}\right)$

But, is it a Maximum?

with this formula, if $\mu' < 0$, $f(\bar{\alpha})$ is a minimum but that doesn't care because we could take the module of the difference.
Maybe, set $\mu' \geq 0$ we could use $f(\alpha) = \mu' \sigma - \tau$ for a better formula → $\bar{\alpha} = 1/2 \arctan(1/\mu')$