

Radiative  
Dirac neutrino  
masses via  $\mathbb{Z}_4$   
from 3211

# Radiative Dirac neutrino masses with $\mathbb{Z}_4$ symmetry from $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$

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Brief review  
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Anomalies

$\mathbb{Z}_4$

$m_\nu$

Mass  
Spectrum

$0\nu 4\beta$

Dark Matter  
Candidates

Conclusions

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In collaboration with Prof. Dr. Sin Kyu Kang and Arnab Dasgupta

# Overview

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- Generate naturally small neutrino masses
- Naturally accommodate DM (Dirac, Majorana, or bosonic)
- Connect the existence of DM to  $m_\nu \neq 0$  neutrino masses (Scotogenicness)
- $0\nu 4\beta$  as leading order of  $0\nu 2n\beta$  processes

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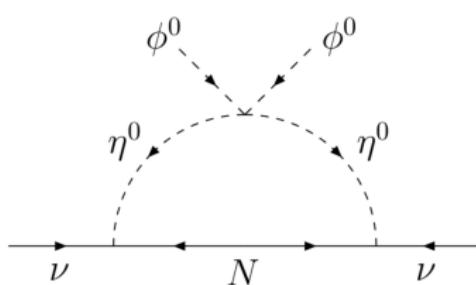
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## d-5 Weinberg

operator:  $\frac{f_{ij}}{2\Lambda} (\nu_i \phi^0 - l_i \phi^+) (\nu_j \phi^0 - l_j \phi^+) + \text{h.c.} \xrightarrow{*} \text{Majorana neutrino masses}$

- See-saw-1, Add SM singlet fermion,  $m_\nu = -\frac{m_D^2}{M_N}$
- See-saw-2, add SM triplet scalar,  $m_\nu = -\frac{f_{\mu\nu}}{M^2}$
- See-saw-3, add SM triplet fermion,  $m_\nu = -\frac{m_D^2}{M_\Sigma}$

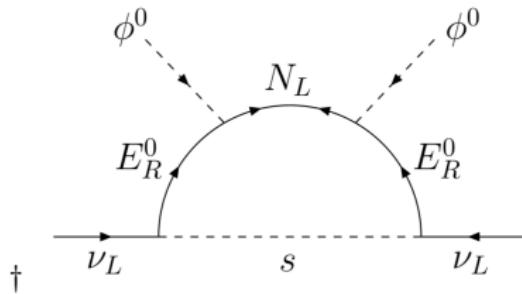
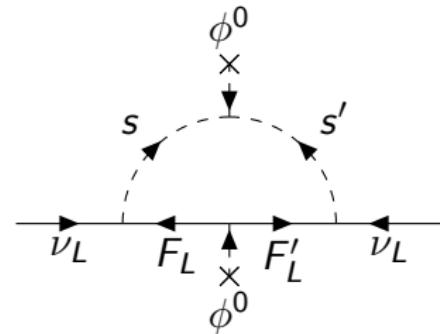


\*Phys. Rev. Lett. 43(1979), 1566; Phys. Rev. Lett. 81 (1998) 1171

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## Brief review



<sup>†</sup>Zee model (1980), Phys.Lett. 93B (1980) 389; Scot-Inverse-Seesaw Phys.Lett. B737 (2014) 280-282.

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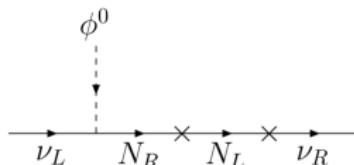
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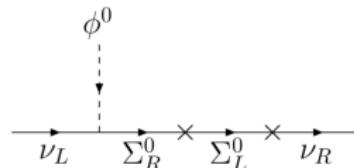
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- Insert a Dirac fermion singlet  $N$  which does not transform under  $\mathcal{S}$ , then break  $\mathcal{S}$  softly by the dimension-three  $\bar{\nu}_R N_L$  term.



- Insert a Dirac fermion triplet  $(\Sigma^+, \Sigma^0, \Sigma^-)$  which does not transform under  $\mathcal{S}$ , then break  $\mathcal{S}$  and  $SU(2)_L \times U(1)$  together spontaneously to obtain the dimension-three  $\bar{\nu}_R \Sigma_L^0$  term.



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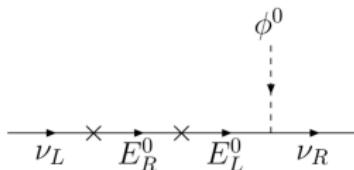
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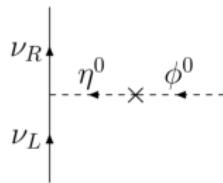
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- Insert a Dirac fermion doublet  $(E^0, E^-)$  which transforms as  $\nu_R$  under  $\mathcal{S}$ , then break  $\mathcal{S}$  softly by the dimension-three  $(\bar{E}^0 \nu_L + E^+ e^-)$  term.



- Insert a scalar doublet  $(\eta^+, \eta^0)$  which transforms as  $\nu_R$  under  $\mathcal{S}$ , then break  $\mathcal{S}$  softly by the dimension-two  $(\eta^- \phi^+ + \bar{\eta}^0 \phi^0)$  term.



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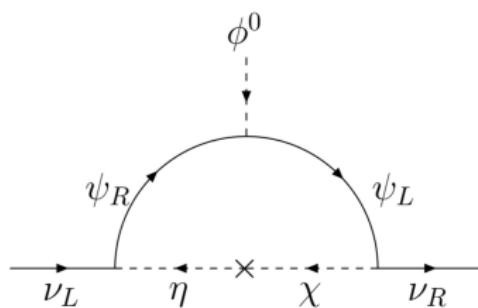
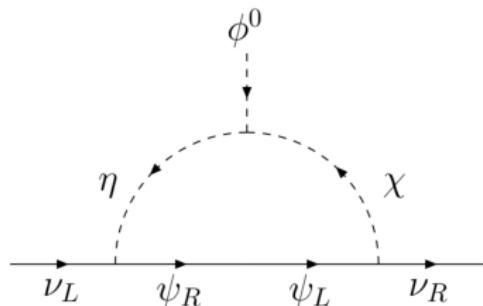
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# Model particle content

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Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$	Flavor
Q	<b>3</b>	2	$\frac{1}{6}$	$\frac{1}{3}$	3
$u^c$	<b><math>\bar{3}</math></b>	1	$-\frac{2}{3}$	$-\frac{1}{2}$	3
$d^c$	<b><math>\bar{3}</math></b>	1	$\frac{1}{3}$	$-\frac{1}{3}$	3
L	<b>1</b>	2	$-\frac{1}{2}$	-1	3
$e^c$	<b>1</b>	1	1	1	3
$\nu^c$	<b>1</b>	1	0	5	3
N	<b>1</b>	1	0	-6	3
$N^c$	<b>1</b>	1	0	2	3
$\Psi_1$	<b>1</b>	1	0	13	3
$\Psi_2$	<b>1</b>	1	0	-14	$2 \times 3$
$\Psi_3$	<b>1</b>	1	0	15	3
H	<b>1</b>	2	$\frac{1}{2}$	0	1
$\eta$	<b>1</b>	2	$\frac{1}{2}$	-1	1
$\chi$	<b>1</b>	1	0	1	1
S	<b>1</b>	1	0	2	1
$S_4$	<b>1</b>	1	0	4	1

# Lagrangian

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$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = & \mathcal{L}_{\text{Yuk}}^{\text{SM}} - \bar{N}_{aR} Y_L^{ab} L_{bi} \eta_j \epsilon^{ij} - \bar{\nu}_{aR} Y_R^{ab} N_{bL} \chi \\ & - \bar{N}_{aR} Y_{ND}^{ab} N_{bL} S_4 - N_{aR} Y_{NM}^{ab} N_{bR} S_4 \\ & - \Psi_{1aL} Y_{12}^{ab} \Psi_{2bL} \chi - \Psi_{2aL} Y_{23}^{ab} \Psi_{3bL} \chi^* \\ & - \Psi_{1aL} Y_{13}^{ab} \Psi_{3bL} \frac{(S_4^*)^7}{\Lambda^6} - \Psi_{2aL} Y_{22}^{ab} \Psi_{2bL} \frac{S_4^7}{\Lambda^6} + \text{H.c.}\end{aligned}$$

# Potential

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$$\begin{aligned} V(H, \eta, \chi, S, S_4) = & -m_H^2 H^\dagger H + \frac{1}{2} \lambda_H (H^\dagger H)^2 + m_\eta^2 \eta^\dagger \eta + \frac{1}{2} \lambda_\eta (\eta^\dagger \eta)^2 \\ & + m_\chi^2 \chi^* \chi + \frac{1}{2} \lambda_\chi (\chi^* \chi)^2 + m_S^2 S^* S + \frac{1}{2} \lambda_S (S^* S)^2 \\ & - m_{S_4}^2 S_4^* S_4 + \frac{1}{2} \lambda_{S_4} (S_4^* S_4)^2 + \lambda_{H\eta} (H^\dagger H) (\eta^\dagger \eta) \\ & + \lambda'_{H\eta} (H^\dagger \eta) (\eta^\dagger H) + \lambda_{H\chi} (H^\dagger H) (\chi^* \chi) \\ & + \lambda_{HS} (H^\dagger H) (S^* S) + \lambda_{HS_4} (H^\dagger H) (S_4^* S_4) \\ & + \lambda_{\eta\chi} (\eta^\dagger \eta) (\chi^* \chi) + \lambda_{\eta S} (\eta^\dagger \eta) (S^* S) \\ & + \lambda_{\eta S_4} (\eta^\dagger \eta) (S_4^* S_4) + \lambda_{\chi S} (\chi^* \chi) (S^* S) + \lambda_{\chi S_4} (\chi^* \chi) (S_4^* S_4) \\ & + \lambda_{SS_4} (S^* S) (S_4^* S_4) + [\mu_H (H^\dagger \eta) \chi + \mu_S \chi^2 S^* + \mu_{S_4} S^2 S_4^* \\ & + \lambda_1 (H^\dagger \eta) \chi^* S + \lambda_2 \chi^2 S S_4^* + \text{H.c.}] \end{aligned}$$

# Triangle Anomalies

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$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$	$SU(3)_C^2 U_{1_Y}$	$SU(3)_L^2$	$U_{1_Y}$	$U_{1_Y^3}$	$U_{1_Y}$	$U_{1_{B-L}^2}$	$SU(3)_C^2$	$U_{1_{B-L}}$	$U_{1_{B-L}}$	$SU(3)_L^2$	$U_{1_Y^2}$	$U_{1_{B-L}}$	$U_{1_{B-L}}^3$	$SU(3)_C^3$
$Q - \mathbf{3} \otimes \mathbf{2} \otimes \frac{1}{3} \otimes \frac{1}{3}$	$\frac{1}{6}$		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$u_R^C - \mathbf{\bar{3}} \otimes \mathbf{1} \otimes -\frac{2}{3} \otimes -\frac{1}{3}$	$-\frac{1}{3}$		$0$	$-\frac{8}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$0$	$0$	$-\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$d_R^C - \mathbf{\bar{3}} \otimes \mathbf{1} \otimes \frac{1}{3} \otimes -\frac{1}{3}$	$\frac{1}{6}$		$0$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$0$	$0$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$
$L - \mathbf{1} \otimes \mathbf{2} \otimes -\frac{1}{2} \otimes -1$	$0$		$-\frac{1}{4}$	$-\frac{1}{4}$	$-1$	$0$	$0$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$0$
$e_R^C - \mathbf{1} \otimes \mathbf{1} \otimes 1 \otimes 1$	$0$		$0$	$1$	$1$	$0$	$0$	$0$	$0$	$1$	$1$	$1$	$0$	$0$
$\nu_R^C - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes 5$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$125$	$0$
$N_L - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes -6$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$-216$	$0$
$N_R^C - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes 2$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$8$	$0$
$\Psi_{1L} - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes 13$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$2197$	$0$
$\Psi_{2L} - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes -14$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$-2744$	$0$
$\Psi_{3L} - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes -14$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$-2744$	$0$
$\Psi_{3L} - \mathbf{1} \otimes \mathbf{1} \otimes 0 \otimes 15$	$0$		$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$3375$	$0$
Total:	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$

§

$$\sum Q_{B-L} \Big|_{\text{Grav.}} = 3 \left( 2 \times \frac{1}{3} - \frac{1}{3} - \frac{1}{3} \right) - 2 \times 1 + 1 + 5 - 6 + 2 + 13 - 2 \times 14 + 15 = 0$$

§ Generated using Susyno:Renato M. Fonseca, Calculating the renormalisation group equations of a SUSY model with Susyno, Computer Physics Communications 183 (2012) 2298

# Residual symmetries

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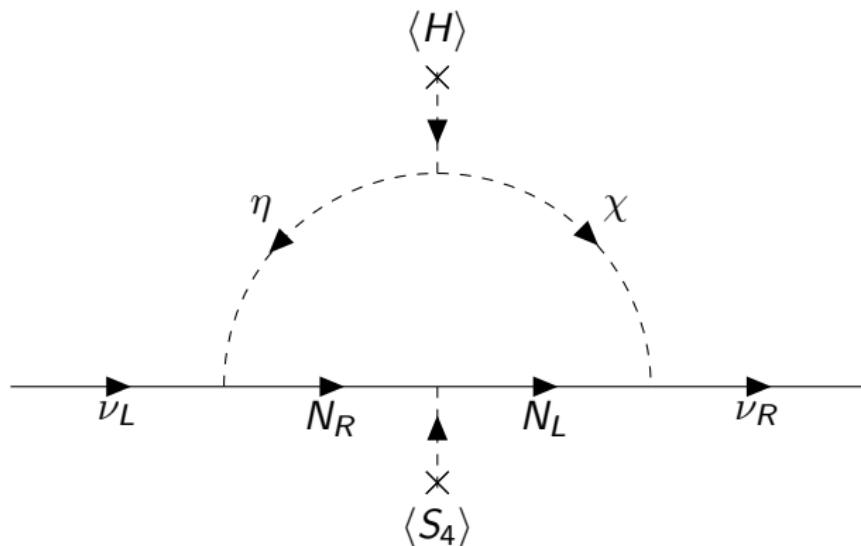
$\mathbb{Z}_4$	Fields
$1$	$H, S_4$
$w^*$	$\nu^a, e^a, \Psi_{1L}^{a*}, \Psi_{3R}^a, \eta^0, \eta^+, \chi^*$
$w^2$	$N_{L,R}^a, \Psi_{i2L}^a, S$
Global $U(1)_{(B-L)}$	Fields
$\frac{1}{3}$	$u^a, d^a$

## Radiative neutrino mass generation

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# Radiative neutrino mass generation

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$$m_\nu^{ab} = \frac{s_N c_N s_{\eta\chi} c_{\eta\chi}}{16\pi^2} Y_L^{ac} \left\{ m_{N_1} \left[ F\left(\frac{m_{\xi_1}^2}{m_{N_1}^2}\right) - F\left(\frac{m_{\xi_2}^2}{m_{N_1}^2}\right) \right] + m_{N_2} \left[ F\left(\frac{m_{\xi_2}^2}{m_{N_2}^2}\right) - F\left(\frac{m_{\xi_1}^2}{m_{N_2}^2}\right) \right] \right\}_{cd} Y_R^{db}$$

$$F(x) = \frac{x}{1-x} \ln x$$

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$$\mathcal{L}_N = (\bar{N}_L^c, \bar{N}_R) \begin{pmatrix} 0 & Y_{ND} v_4 \\ Y_{ND} v_4 & Y_{NM}^\dagger v_4 \end{pmatrix} \begin{pmatrix} N_L \\ N_R^c \end{pmatrix} + \text{H.c.}$$

$$m_{N_{1,2}} = \frac{v_4}{2} \left( |Y_{NM}| \pm \sqrt{|Y_{NM}|^2 + 4|Y_{ND}|^2} \right),$$

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} N_L \\ N_R^c \end{pmatrix},$$

$$\tan(2\theta_N) = -2 \frac{|Y_{ND}|}{|Y_{NM}|}.$$

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$\Psi_i$  get their masses in one of two ways:

- Effectively:  $m_{\Psi_{1,3}}^{ab} = Y_{13}^{ab} \frac{\langle S_8 \rangle^7}{\Lambda^6}$  and  $m_{\Psi_{22}}^{\alpha\beta} = Y_{22}^{\alpha\beta} \frac{\langle S_8 \rangle^7}{\Lambda^6}$  with  $a, b = 1 - 3$  and  $\alpha, \beta = 1 - 6$ , respectively.
- Add scalar  $U(1)_{B-L} \sim 28 \rightarrow \Psi_{1L}^a Y_{13}^{ab} \Psi_{3L}^b S_{28}^*$  and  $\Psi_{2L}^a Y_{22}^{ab} \Psi_{2L}^b S_{28}$ . Potential  $m = 0$  Goldstone mode  $\rightarrow$  more scalars.

# Scalar Masses

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$$\frac{\partial V(s_i)}{\partial v_i} \Big|_{v_i \in \{v, v_4\}} = 0 \rightarrow$$

$$\lambda_H v^2 + \lambda_{HS_4} v_4^2 - 2m_H^2 = 0,$$

$$\lambda_{S_4} v_4^2 + \lambda_{HS_4} v^2 - 2m_{S_4}^2 = 0.$$

3 sectors:  $(1, w, w^2 \sim \mathbb{Z}_4)$

# $\mathbb{1} \sim \mathbb{Z}_4$ scalar sector

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$$(Re[H^0], Re[S_4]) \rightarrow \begin{pmatrix} \lambda_H v^2 & \lambda_{HS_4} v v_4 \\ \lambda_{HS_4} v v_4 & \lambda_{S_4} v_4^2 \end{pmatrix}$$

$$m_{1,2}^2 = \frac{(\lambda_H v^2 + \lambda_{S_4} v_4^2) \pm \sqrt{(\lambda_H v^2 - \lambda_{S_4} v_4^2)^2 + 4\lambda_{HS_4}^2 v^2 v_4^2}}{2}$$

$$\tan(2\theta_{H^0 S_4}) = \frac{2\lambda_{HS_4} v v_4}{\lambda_H v^2 - \lambda_{S_4} v_4^2},$$

$$\text{with } \begin{pmatrix} s_h \\ s_H \end{pmatrix} = \begin{pmatrix} \cos\theta_{H^0 S_4} & \sin\theta_{H^0 S_4} \\ -\sin\theta_{H^0 S_4} & \cos\theta_{H^0 S_4} \end{pmatrix} \begin{pmatrix} Re[H^0] \\ Re[S_4] \end{pmatrix}.$$

$Im[H^0]$ ,  $Im[S_4]$ , and  $H^\pm$  correspond to would-be Nambu-Goldstone bosons

$$m_{\eta^\pm}^2 = m_\eta^2 + \lambda_{H\eta} v^2/2 + \lambda_{\eta S_4} v_4^2/2.$$

# $w, w^* \sim \mathbb{Z}_4$ scalar sector

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$$\begin{aligned} Re/Im[(\eta, \chi)] \rightarrow & \begin{pmatrix} m_\eta^2 + (\lambda_{H\eta} + \lambda_{H\eta 2}) \frac{v^2}{2} + \lambda_\eta s_4 \frac{v_4^2}{2} & \pm \mu_H \frac{v}{\sqrt{2}} \\ \pm \mu_H \frac{v}{\sqrt{2}} & m_\chi^2 + \lambda_{H\chi} \frac{v^2}{2} + \lambda_\chi s_4 \frac{v_4^2}{2} \end{pmatrix} \\ m_{1,2}^2 = & \frac{1}{2} \left[ m_\eta^2 + m_\chi^2 + (\lambda_{H\eta} + \lambda_{H\eta 2} + \lambda_{H\chi}) \frac{v^2}{2} + (\lambda_\eta s_4 + \lambda_\chi s_4) \frac{v_4^2}{2} \right] \\ & \pm \frac{1}{2} \sqrt{\left[ m_\eta^2 - m_\chi^2 + (\lambda_{H\eta} + \lambda_{H\eta 2} - \lambda_{H\chi}) \frac{v^2}{2} + (\lambda_\eta s_4 - \lambda_\chi s_4) \frac{v_4^2}{2} \right]^2 + 4\mu_H^2 \frac{v^2}{2}} \end{aligned}$$

# $w, w^* \sim \mathbb{Z}_4$ scalar sector

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$$\tan(2\theta_{\eta\chi R/I}) = \frac{\pm 2\mu_H \frac{v}{\sqrt{2}}}{m_\eta^2 - m_\chi^2 + (\lambda_{H\eta} + \lambda_{H\eta 2} - \lambda_{H\chi}) \frac{v^2}{2} + (\lambda_{\eta S_4} - \lambda_{\chi S_4}) \frac{v_4^2}{2}},$$

with  $\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\eta\chi} & \sin\theta_{\eta\chi} \\ -\sin\theta_{\eta\chi} & \cos\theta_{\eta\chi} \end{pmatrix} \begin{pmatrix} \eta^0 \\ \chi \end{pmatrix}.$

# $w^2 \sim \mathbb{Z}_4$ scalar sector

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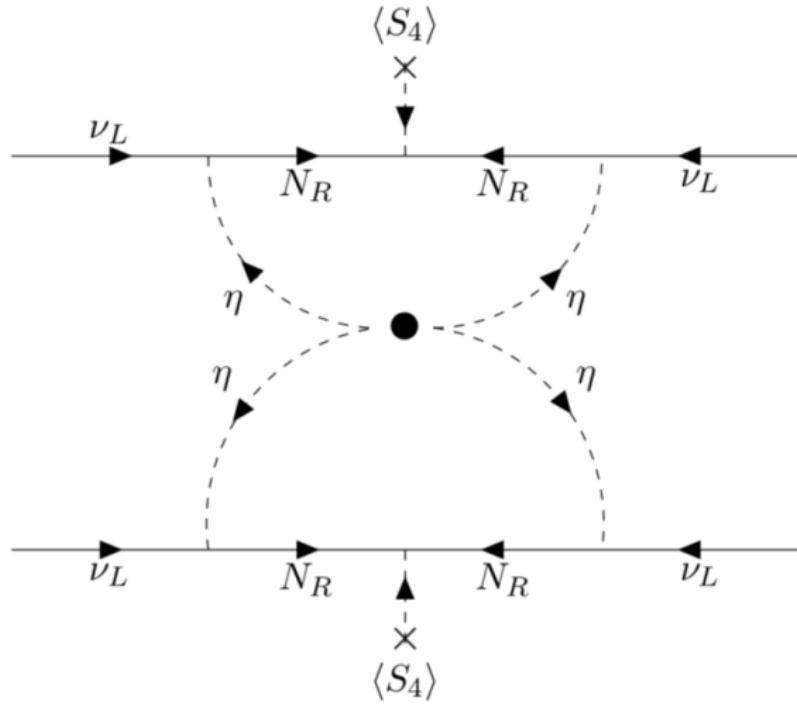
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$$m_{\text{Re}[S]}^2 = m_S^2 + \lambda_{HS} \frac{v^2}{2} + \lambda_{SS_4} \frac{v_4^2}{2} + \sqrt{2} \mu_{S_4} v_4,$$

$$m_{\text{Im}[S]}^2 = m_S^2 + \lambda_{HS} \frac{v^2}{2} + \lambda_{SS_4} \frac{v_4^2}{2} - \sqrt{2} \mu_{S_4} v_4.$$

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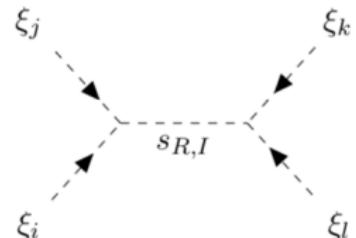
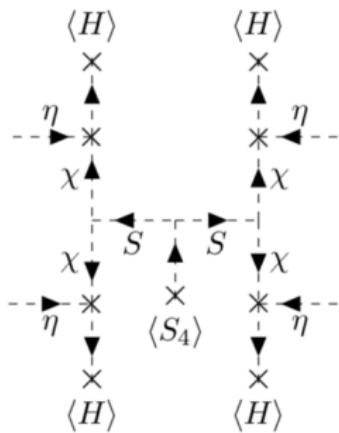
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$$\left( \frac{\mu_{ij}^R \mu_{kl}^R}{m_{s_R}^2} + \frac{\mu_{ij}^I \mu_{kl}^I}{m_{s_I}^2} \right) \xi_i \xi_j \xi_k \xi_l + \text{H.c.}$$

$$\mu_{ij}^R = \begin{pmatrix} s^2 & cs \\ cs & c^2 \end{pmatrix} \left( \sqrt{2}\mu_S + \lambda_2 \nu_4 \right)$$

$$\mu_{ij}^I = \begin{pmatrix} s^2 & cs \\ cs & c^2 \end{pmatrix} i \left( -\sqrt{2}\mu_S + \lambda_2 \nu_4 \right)$$

$$\frac{Q^{abcd}}{\Lambda^2} (\nu_a \nu_b) (\nu_c \nu_d)$$

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$$\frac{Q^{abcd}}{\Lambda^2} = \imath c_4 Y_L^{a\alpha} (v_s)_j \left[ s_N^2 \frac{F(x_{l1}, y_{lj}, x_{j1})}{m_{N_1}} + c_N^2 \frac{F(x_{l2}, y_{lj}, x_{j2})}{m_{N_2}} \right]_{\alpha\beta} (M_s)_{jl} (v_s)_l Y_L^{\beta b} \times$$

$$Y_L^{c\gamma} (v_s)_i \left[ s_N^2 \frac{F(x_{k1}, y_{ki}, x_{i1})}{m_{N_1}} + c_N^2 \frac{F(x_{k2}, y_{ki}, x_{i2})}{m_{N_2}} \right]_{\gamma\delta} (M_s)_{ik} (v_s)_k Y_L^{\delta d} \times$$

$$\frac{\left(m_{S_l}^2 - m_{S_R}^2\right) \left(2\mu_s^2 + \lambda_2^2 v_4^2\right) + \left(m_{S_l}^2 + m_{S_R}^2\right) 2\sqrt{2}\mu_S \lambda_2 v_4}{m_{S_l}^2 m_{S_R}^2},$$

$$v_S = \begin{pmatrix} \cos(\theta_{\eta\chi}) \\ -\sin(\theta_{\eta\chi}) \end{pmatrix},$$

$$M_S = \begin{pmatrix} \sin^2(\theta_{\eta\chi}) & \sin(\theta_{\eta\chi}) \cos(\theta_{\eta\chi}) \\ \cos(\theta_{\eta\chi}) \sin(\theta_{\eta\chi}) & \cos^2(\theta_{\eta\chi}) \end{pmatrix}$$

$$F(x, y, z) = \frac{\ln x}{(1-z)(1-x)} - \frac{\ln y}{(1-z)(1-y)}$$

$$x_{ij} = \frac{m_i^2}{m_{N_j}^2}, \quad y_{ij} = \frac{m_i^2}{m_j^2},$$

$$c_4 = \begin{cases} \frac{4!}{3!} & i = j = k = l | i, j, k, l \in \{1, 2\} \\ 4 & (i = j \wedge k = l) \vee (i = k \wedge j = l) \vee (i = l \wedge j = k) | i, j, k, l \in \{1, 2\} \end{cases}.$$

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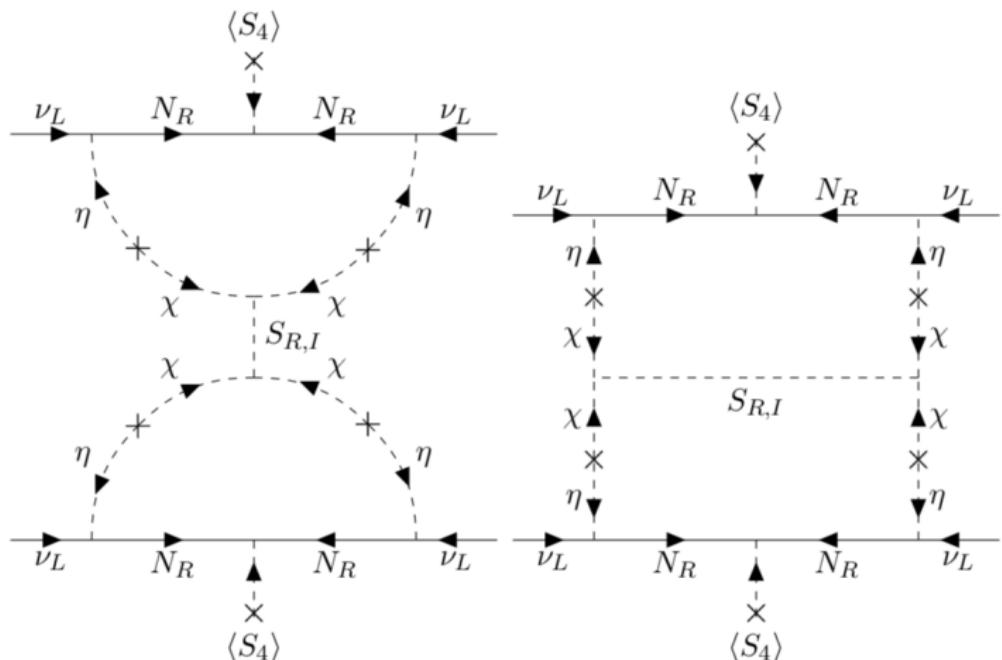
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$$\tau_{1/2}^{0\nu4\beta} > 3.2 \times 10^{21} \text{ years}$$

$$\tau_{1/2}^{-1} = \Gamma_{0\nu4\beta} = G_{0\nu4\beta} |A_{0\nu4\beta}|^2 = G_{0\nu4\beta} \left( \frac{G_F^4}{q^4} \frac{Q_{0\nu4\beta}}{\Lambda^2} \right)^2$$

$$= Q^{11} \left( \frac{G_F^4}{q^4} \frac{Q_{0\nu4\beta}}{\Lambda^2} \right)^2 q^{18}$$

$$\frac{Q_{0\nu4\beta}}{\Lambda^2} \leq (\tau_{1/2} Q^{11} q^{10} G_F^8)^{-1/2} = 7.8 \times 10^{18} \text{ TeV}^{-2}$$

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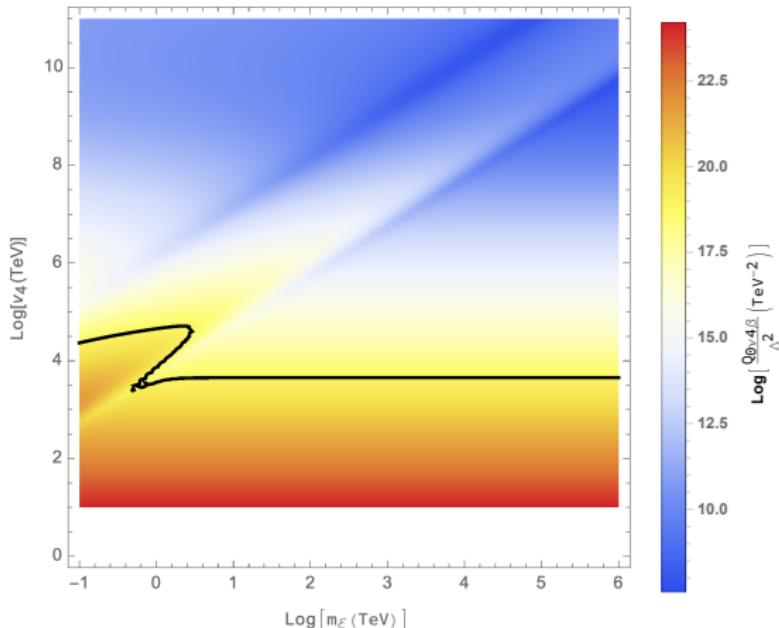
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$\frac{Q_{0\nu 4\beta}}{\Lambda^2}$  vs.  $m_\xi$  and  $v_4$  for  $\mu_S = 10^9$  TeV,  $\frac{m\xi_2}{m\xi_1} - 1 = 10^{-2}$ ,  $\frac{Y_D}{Y_M} = 10^{-3}$ ,  $Y_{M,L} \sim O(1)$ ,  $\theta_\xi = \pi/4$ , and

$m_{S_R(I)} = 0.8(2)$  TeV. Solid curve corresponds to current half-life constraint on  $\frac{Q_{0\nu 4\beta}}{\Lambda^2}$  from NEMO-3 and KURF

# Neutrinoless quadruple beta decay

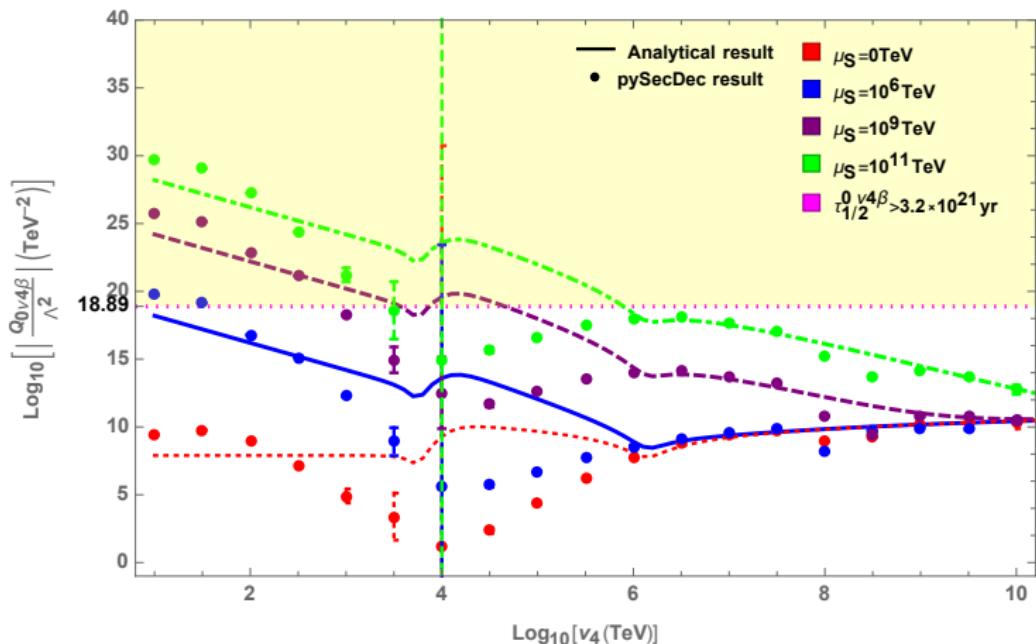
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Comparison of numerically obtained result from pySecDec (solid circles) with analytically derived formula (curves) for different values of  $v_4$ , and  $\mu_S$  for  $m_\xi = 1 \text{ TeV}$ ,  $\frac{m_\xi 2}{m_\xi 1} - 1 = 10^{-2}$ ,  $\frac{Y_D}{Y_M} = 10^{-3}$ ,  $Y_{M,L} \sim O(1)$ ,  $\lambda_2 \sim O(1)$ ,  $\theta_\xi = \pi/4$ , and  $m_{S_R(I)} = 0.8(2) \text{ TeV}$ .

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- Majorana type:  $N_i$
- Dirac type:  $\Psi_{1,3}$
- Real scalar:  $s_{R,I}$

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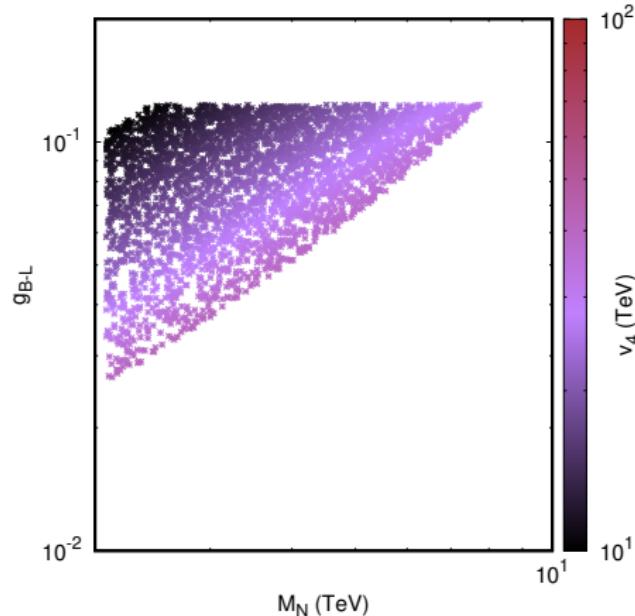
$m_\nu$

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Scatter plot of the  $U(1)_{B-L}$  gauge coupling  $g_{B-L}$  versus the DM mass  $|m_N|$  for  $\frac{Y_D}{Y_M} \ll 1$  and  
 $10 \text{ TeV} v_4 100 \text{ TeV}$   
 $m_N \approx -v_4 \frac{Y_D^2}{Y_M} = 2.2 \sim 7.8 \text{ TeV}$

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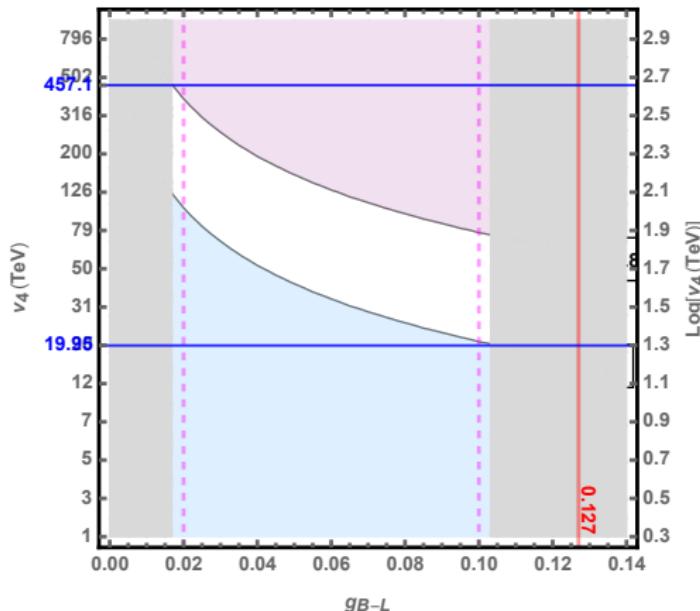
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$$\Gamma(S \rightarrow \nu\nu) = \frac{\sum |A|^2}{16\pi m_S}$$

$$= \frac{m_S}{16\pi} \left[ \left| \sum_{ijk} \mu_{ij}^x C_0(0, 0, m_S^2, m_j, m_k, m_i) A_L \right|^2 + \left| \sum_{ijk} \mu_{ij}^x C_0(0, 0, m_S^2, m_j, m_k, m_i) A_R \right|^2 \right]$$

$$A_L = (s_N, c_N)_k^2 Y_L m_{N_k} Y_L \begin{pmatrix} c_\xi^2 & -s_\xi c_\xi \\ -s_\xi c_\xi & s_\xi^2 \end{pmatrix}$$

$$A_R = (c_N, -s_N)_k^2 Y_R^* m_{N_k} Y_R^* \begin{pmatrix} s_\xi^2 & s_\xi c_\xi \\ s_\xi c_\xi & c_\xi^2 \end{pmatrix}$$

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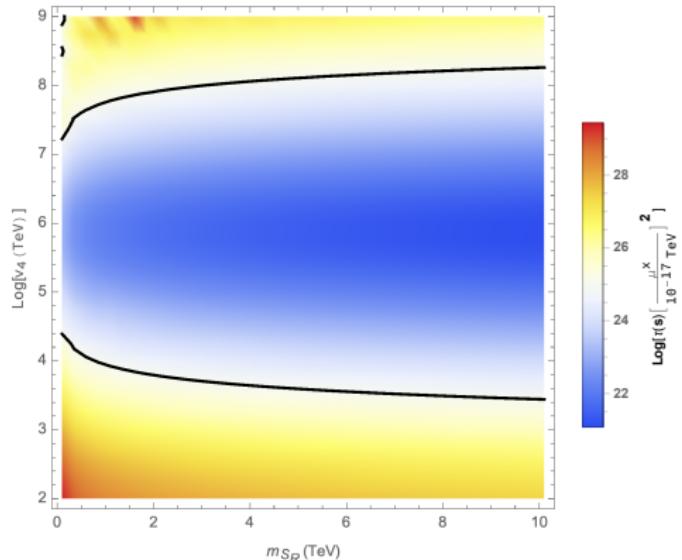
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$S_{R,I}$  lifetime vs.  $v_4$ ,  $m_S$ , and  $\mu^\chi$  for  $\frac{m_{\xi_1}}{m_{\xi_2}} - 1 = 10^{-2}$  and  $Y_{M,L} \sim O(1)$ ,  $Y_R \sim 10^{-4}$ ,  $\theta_\xi = \pi/4$ , and  $Y_D = 10^{-2}$ . The solid line indicates the lower limit on  $\tau_s > 10^{25} \text{ s}$

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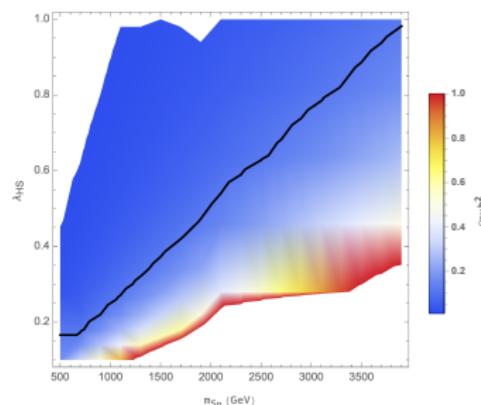
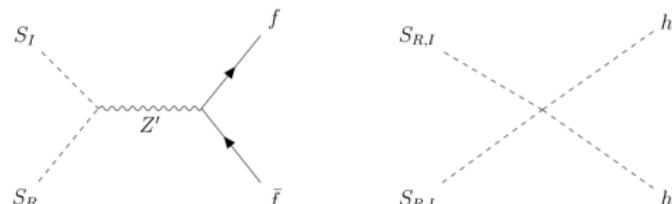
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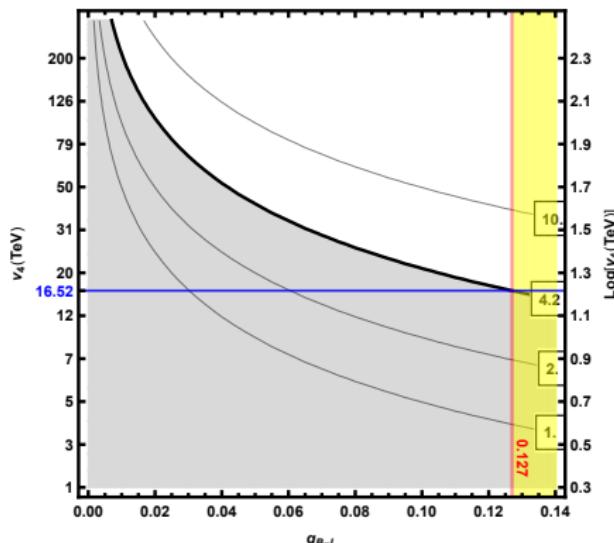
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Unitarity and collider constraints,  $g_{B-L} < 0.127$  and  $M'_Z > 4.2$  TeV are imposed. Contours represent  $M'_Z$  in TeV units

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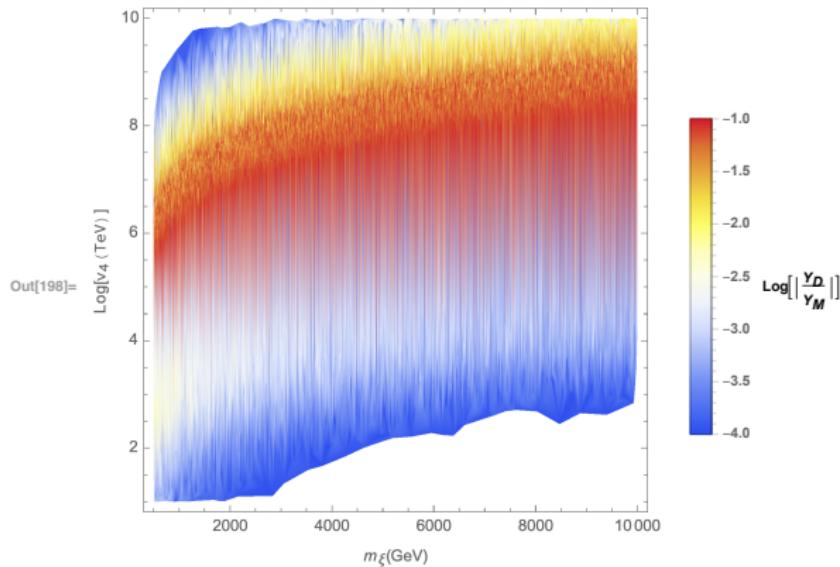
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Correlation between  $\nu_4$  and  $\frac{Y_D}{Y_M}$  for  $\frac{m_{\xi_1}}{m_{\xi_2}} - 1 = 10^{-2}$ ,  $Y_{M,L} \sim O(1)$ ,  $Y_R \sim 10^{-4}$  and  $\theta_\xi = \pi/4$

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- $\mathbb{G}_{SM} \times U(1)_{B-L}$  extension
- Radiative Dirac neutrino masses
- multiple Dirac or Majorana DM candidates
- One symmetry responsible for DM stability and Diracness of the neutrinos
- $0\nu 4\beta$  dominant  $0\nu 2n\beta$  process

Thank you!

# Inetrations¶

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#	Operator	Dim.	Self con.??	Repeated fields	Symmetry and number of parameters
1	$H(C) H(R)$	2	True		(1, 1)
2	$\eta(C) \eta(R)$	2	True		(1, 1)
3	$x(C) x(R)$	2	True		(1, 1)
4	$S(C) S(R)$	2	True		(1, 1)
5	$S_4(C) S_4(R)$	2	True		(1, 1)
6	$H(C) \eta(R) x(R)$	3	False		(1, 1)
7	$x(C) x(C) S(R)$	3	False	$x(C)$	(S, 1, 1)
8	$S(C) S(C) S_4(R)$	3	False	$S(C)$	(S, 1, 1)
9	$u(C) Q(R) H(R)$	4	False		(9, 1)
10	$d(C) Q(R) H(C)$	4	False		(9, 1)
11	$e(R) L(C) H(R)$	4	False		(9, 1)
12	$L(R) N_k(C) \eta(R)$	4	False		(9, 1)
13	$v(R) N_k(C) x(C)$	4	False		(9, 1)
14	$N_L(R) N_k(C) S_4(R)$	4	False		(9, 1)
15	$N_R(R) N_k(R) S_4(R)$	4	False	$N_k(R)$	(S, 6, 1)
16	$g_{2L}(C) g_{2L}(C) x(C)$	4	False		(9, 1)
17	$g_{2L}(R) g_{2L}(R) x(C)$	4	False		(9, 1)
18	$H(C) H(C) H(R) H(R)$	4	True	(H(C), H(R))	((S, S), 1, 1)
19	$H(C) H(R) \eta(C) \eta(R)$	4	True		(1, 2)
20	$H(C) H(R) x(C) x(R)$	4	True		(1, 1)
21	$H(C) H(R) S(C) S(R)$	4	True		(1, 1)
22	$H(C) H(R) S_4(C) S_4(R)$	4	True		(1, 1)
23	$H(C) \eta(R) x(C) S(R)$	4	False		(1, 1)
24	$\eta(C) \eta(C) \eta(R) \eta(R)$	4	True	(\{\eta(C), \eta(R)\})	((S, S), 1, 1)
25	$\eta(C) \eta(R) x(C) x(R)$	4	True		(1, 1)
26	$\eta(C) \eta(R) S(C) S(R)$	4	True		(1, 1)
27	$\eta(C) \eta(R) S_4(C) S_4(R)$	4	True		(1, 1)
28	$x(C) x(C) x(R) x(R)$	4	True	(x(C), x(R))	((S, S), 1, 1)
29	$x(C) x(C) S(C) S_4(R)$	4	False	$x(C)$	(S, 1, 1)
30	$x(C) x(R) S(C) S(R)$	4	True		(1, 1)
31	$x(C) x(R) S_4(C) S_4(R)$	4	True		(1, 1)
32	$S(C) S(C) S(R) S(R)$	4	True	(S(C), S(R))	((S, S), 1, 1)
33	$S(C) S(R) S_4(C) S_4(R)$	4	True		(1, 1)
34	$S_4(C) S_4(C) S_4(R) S_4(R)$	4	True	(S_4(C), S_4(R))	((S, S), 1, 1)

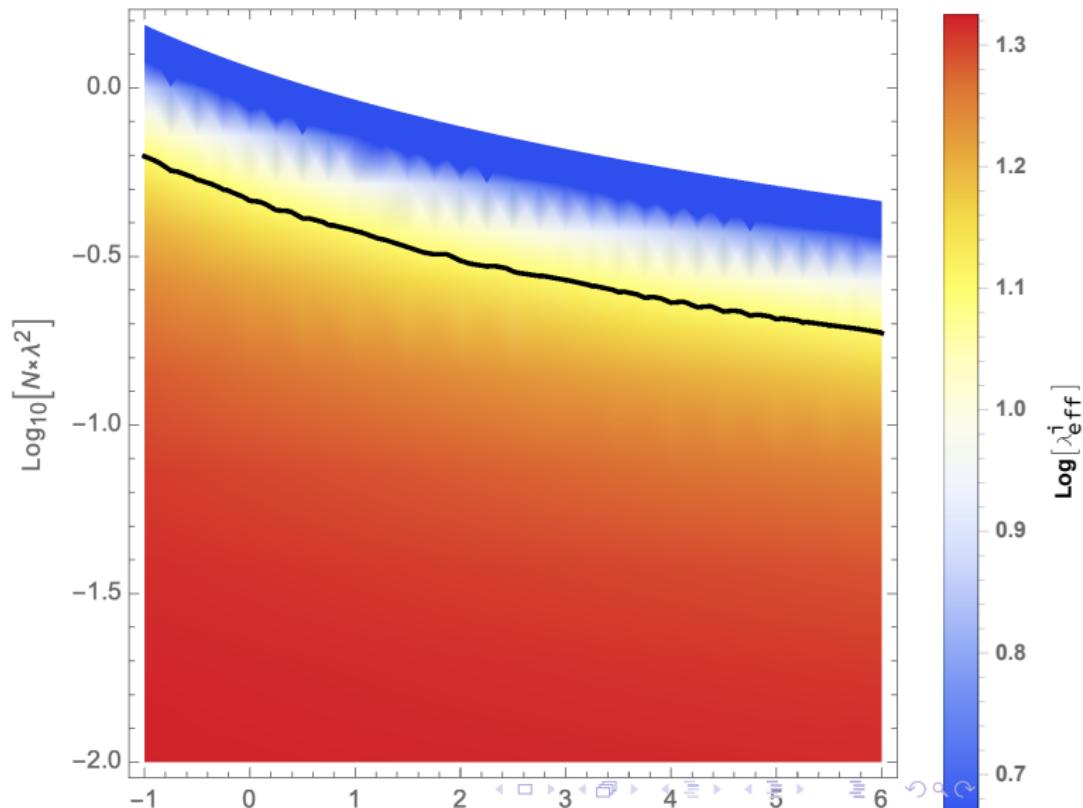
§Generated using Renato M. Fonseca, "The Sym2Int program: going from symmetries to interactions, arXiv:1703.05221 [hep-ph]



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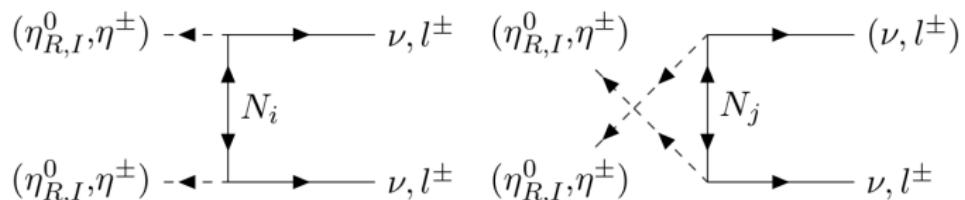
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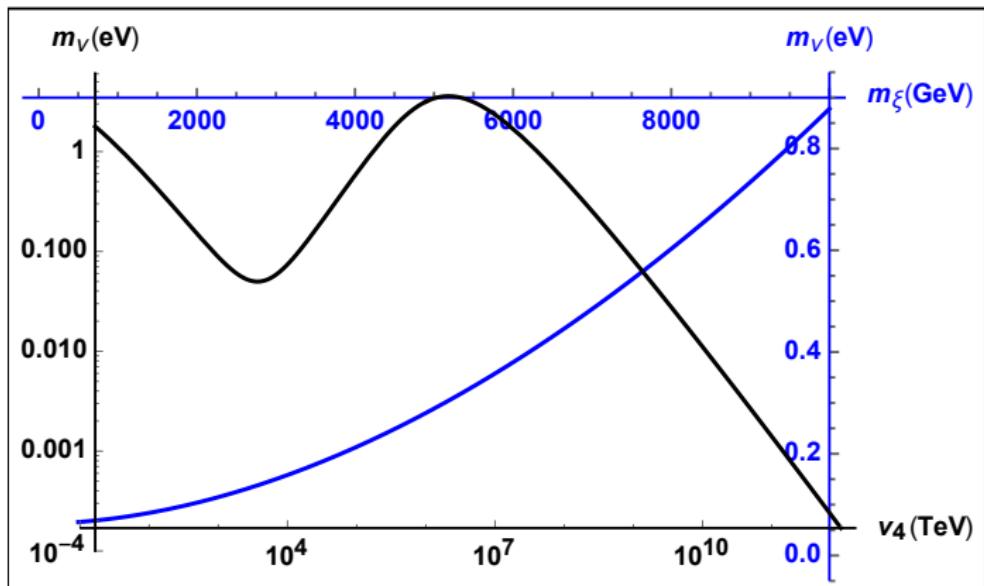
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