



UNSW
SYDNEY

Introduction to Neutrino Mass Models

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Neutrino mass terms



RH neutrinos allow to generate Dirac neutrino masses like for charged fermions

$$-\overline{L}_L \tilde{H} Y_\nu \nu_R + \text{h.c.} \xrightarrow{\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}} -\overline{\nu}_L M_\nu^D \nu_R + \text{h.c.} \quad \text{with} \quad M_\nu^D = Y_\nu^\dagger \frac{v}{\sqrt{2}}$$

Bounds on the absolute neutrino mass scale:

- $m_{\nu_e} < 450$ meV (KATRIN)
- $|m_{ee}| < 28 - 122$ meV (KamLAND-Zen)
- $\sum_i m_i < 120$ meV (cosmology)

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- $\sum_i m_i < 120$ meV (cosmology)
- Additional term allowed in Lagrangian: Majorana mass term for ν_R : $\nu_R^T C \nu_R$
 - ▶ implies Majorana neutrinos (see discussion below)
 - ▶ additional symmetry required to forbid term!
- Dirac neutrino masses are less “natural”

$$Y_\nu = \frac{\sqrt{2} m_\nu}{v} \ll 10^{-12} \ll y_e \ll 1$$

Yukawa couplings Y_ν are extremely tiny!

- SM Lagrangian features four accidental symmetries:
 1. baryon number (B),
 2. lepton flavour numbers ($L_{e,\mu,\tau}$)
- Only (total) lepton number $L = L_e + L_\mu + L_\tau$ may be conserved



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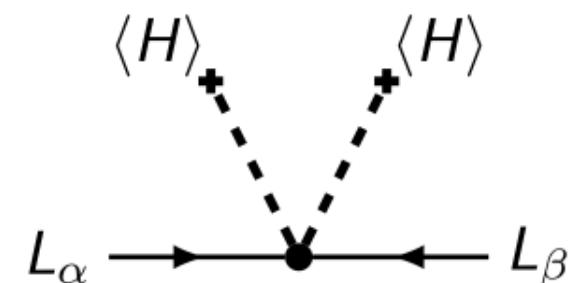


There is physics beyond the SM. If it is heavier than the electroweak scale it may be parameterised in terms of effective operators → **Standard Model as an effective field theory (SMEFT)**

Weinberg operator $\mathcal{L} \supset -\frac{1}{2}\kappa_{ij}(L_i \varepsilon H)^T C(L_j \varepsilon H) + \text{h.c.}$

- is the unique dimension-5 operator in SMEFT
- is symmetric, i.e. $\kappa_{ij} = \kappa_{ji}$
- is $\Delta L = 2$ and generates Majorana neutrino masses after EWSB

$$(M_\nu^M)_{ij} = \kappa_{ij} \frac{v^2}{2} \quad \text{with} \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$



Tiny neutrino masses explained by high scale of operator

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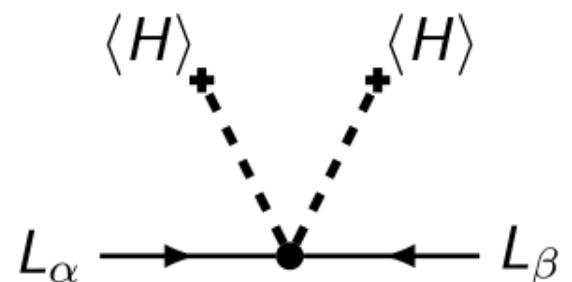
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Tiny neutrino masses explained by high scale of operator



What is the UV completion
of the Weinberg operator?

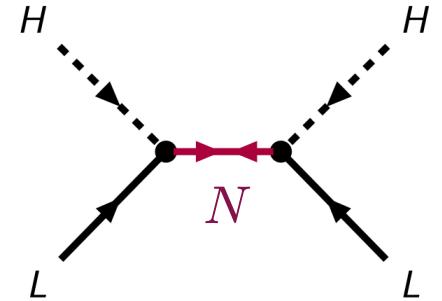
Seesaw models

Type I seesaw model

SM with RH neutrinos allows a Majorana mass term

$$-\mathcal{L} \supset \frac{1}{2} \nu_R^T C M \nu_R + \overline{\nu_R} Y_\nu^\dagger L \varepsilon H + \text{h.c.}$$

It results in effective interaction of 2 lepton doublets with 2 Higgs



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At energy scales $E \ll M$, we can expand the fermionic N propagator

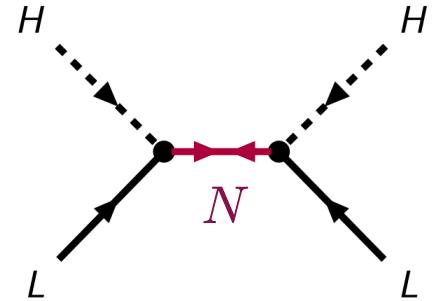
$$\frac{i(p+M)}{p^2 - M^2 + i\varepsilon} \simeq -\frac{i}{M} \left(1 + \frac{p}{M} + \dots \right)$$

The leading order term results in the Weinberg operator

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} (Y_\nu^* M^{-1} Y_\nu^\dagger)_{ij} (L_i \varepsilon H)^T C (L_j \varepsilon H) \\ \rightarrow \kappa_{ij} &= -(Y_\nu^* M^{-1} Y_\nu^\dagger)_{ij} \end{aligned}$$

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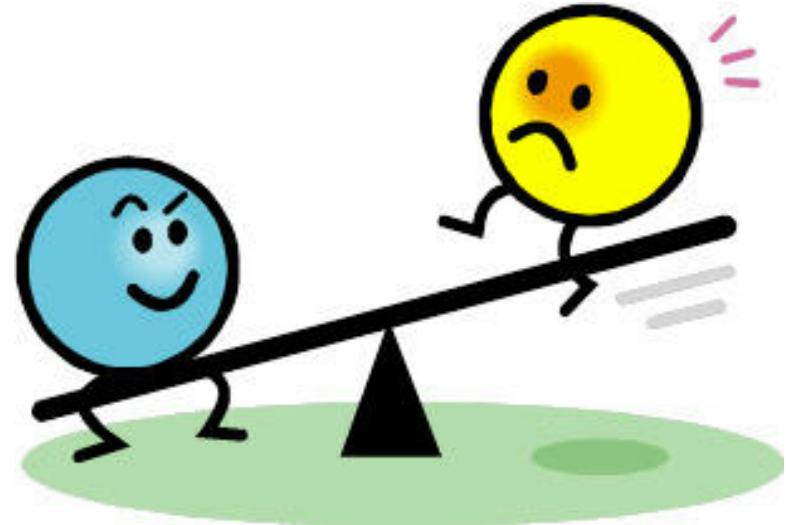
Seesaw models

Seesaw model prediction $\kappa_{ij} = -(\mathbf{Y}_\nu^* \mathbf{M}^{-1} \mathbf{Y}_\nu^\dagger)_{ij}$
for Weinberg operator $-\frac{1}{2}\kappa_{ij}(L_i \varepsilon H)^T C(L_j \varepsilon H)$

Consequently neutrino masses $\nu_L^T m_\nu C \nu_L$ are

$$m_\nu^\dagger = -\left(M_\nu^D M^{-1} (M_\nu^D)^T \right)$$

Hermitian conjugation due to definitions of Y_ν & m_ν



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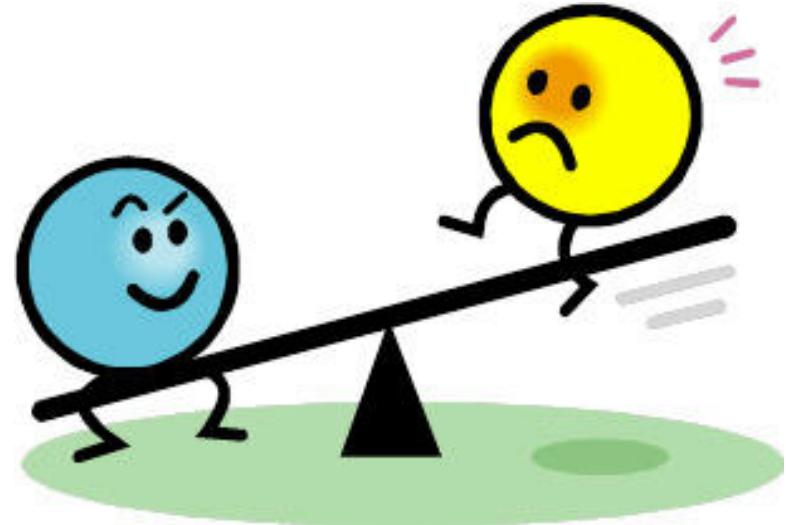
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- This is the **seesaw mechanism**
 - Light neutrinos explained by heavy RH neutrinos, $m_\nu \propto \frac{1}{M}$
 - For $Y_\nu \sim 1$, and $M \sim 10^{14}$ GeV $\rightarrow m_\nu \sim 0.1$ eV
- Heavy ν_R masses close to scale of grand unification
- Simple elegant explanation of tiny neutrino masses
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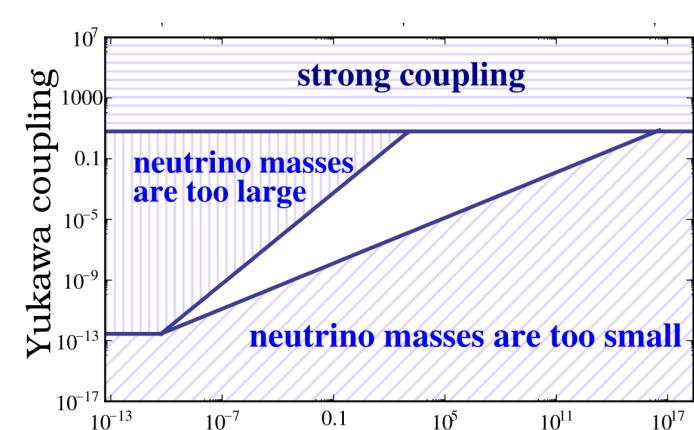
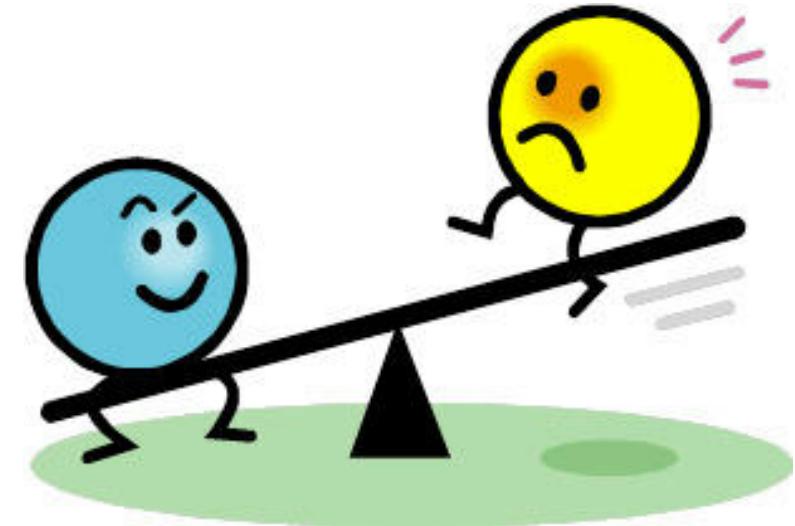
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- Heavy ν_R masses close to scale of grand unification
- Simple elegant explanation of tiny neutrino masses
- Two RH neutrinos sufficient to explain neutrino masses



[1204.5379]

Comments on seesaw phenomenology

Seesaw models

- The seesaw formula also holds for light ν_R with $|M| < m_W$.
- This can be seen by considering the neutral fermion mass matrix

$$-\mathcal{L} \supset \frac{1}{2} \nu_R^T C M \nu_R + \bar{L} Y_\nu \tilde{H} \nu_R + \text{h.c.}$$

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$$\begin{aligned}-\mathcal{L} \supset & \frac{1}{2}\nu_R^T CM\nu_R + \overline{LY}_\nu \tilde{H}\nu_R + \text{h.c.} = \frac{1}{2}\overline{\nu_R} M^\dagger \nu_R^c + \overline{\nu_R} (M_\nu^D)^\dagger \nu_L + \dots \\ & = \frac{1}{2}\overline{\nu_R^{cc}} M^\dagger \nu_R^c + \overline{\nu_R^{cc}} (M_\nu^D)^\dagger \nu_L + \dots = \frac{1}{2}(\nu_R^c)^T CM^\dagger \nu_R^c + (\nu_R^c)^T C(M_\nu^D)^\dagger \nu_L + \dots \\ & = \frac{1}{2}N^T CM^\dagger N + \dots \quad \text{with} \quad N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} 0 & (M_\nu^D) \\ (M_\nu^D)^T & M \end{pmatrix}\end{aligned}$$

- The diagonalisation is trivial in the 1+1 flavour case for $|M_\nu^D| \ll |M|$. There is **one large eigenvalue M** and **one small eigenvalue $m_\nu^\dagger = -\frac{(M_\nu^D)^2}{M}$** . Light neutrino masses in the 3+3 flavour case are given by the seesaw formula

$$m_\nu^\dagger = -M_\nu^D M^{-1} (M_\nu^D)^T$$

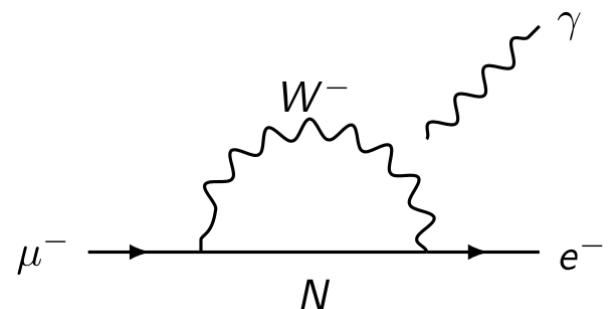
- The **active-sterile mixing angle** is approximately $\theta \sim |U| \sim \frac{M_\nu^D}{M}$.

How can we test the seesaw mechanism?

- Lepton flavour violating rare decay $\mu \rightarrow e\gamma$

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_i U_{\mu i}^* U_{ei} \frac{m_{\nu i}^2}{m_W^2} \right|^2 \lesssim 10^{-54}$$

- constant term in m_ν drops out due to GIM mechanism (unitarity of PMNS matrix)
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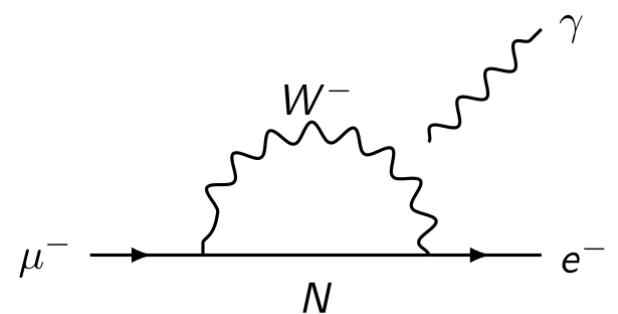
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- constant term in m_ν drops out due to GIM mechanism (unitarity of PMNS matrix)
- similar for $\mu \rightarrow 3e$ and τ decays
- no noteworthy contribution to quark flavour physics
- **Extremely difficult to test**

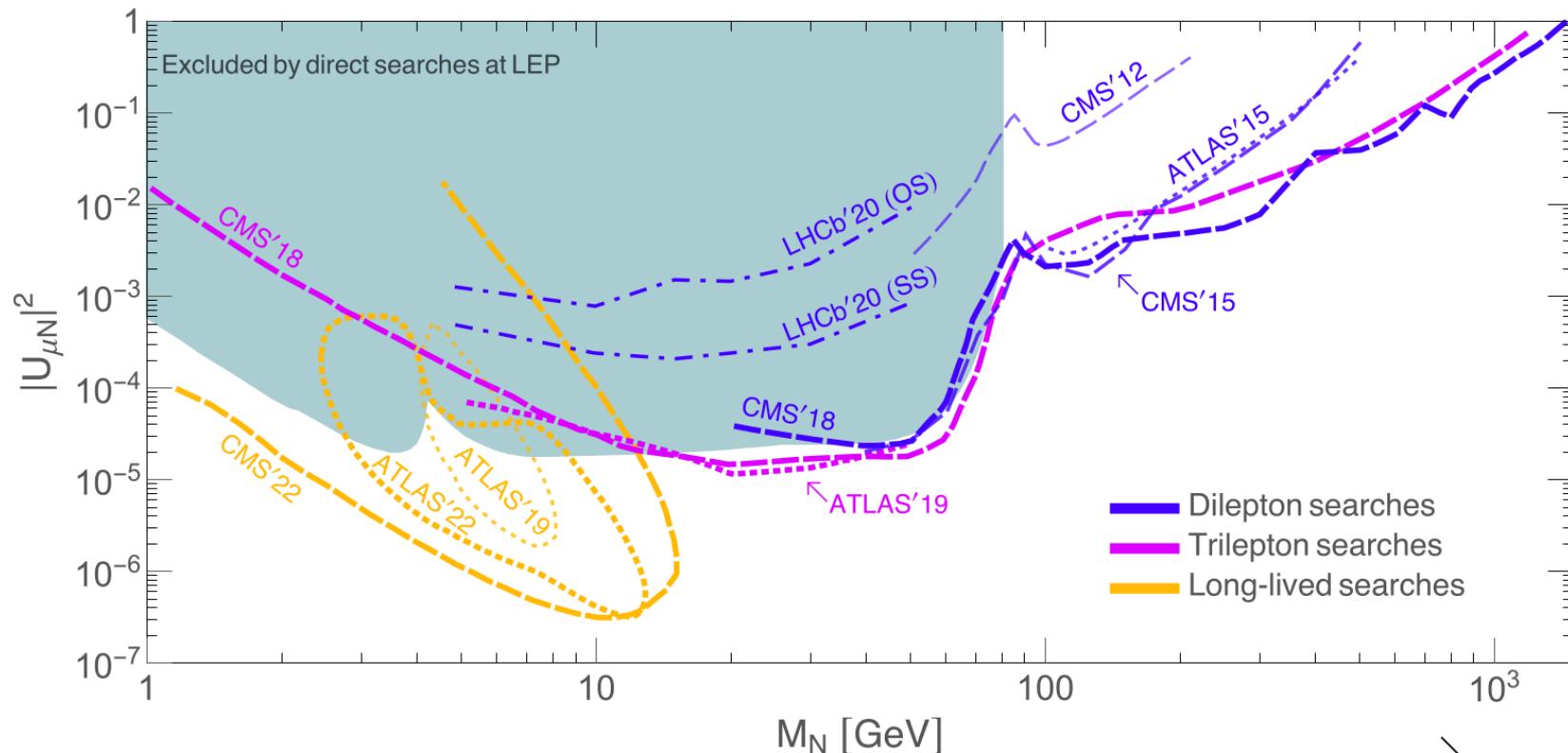
This changes in extensions of the minimal seesaw model. Examples are

- SUSY seesaw due to slepton soft mass terms
- left-right symmetric models due to RH gauge bosons
- **inverse seesaw model (see below)**



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Seesaw models

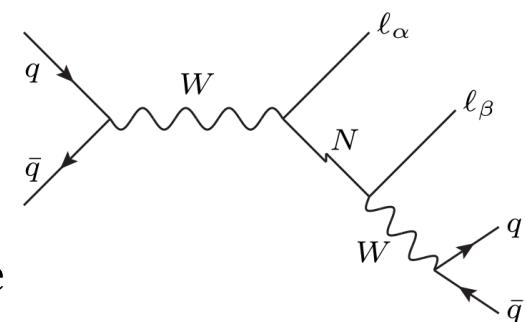


[2208.13882]

seesaw estimate

$$|U|^2 \sim 10^{-12} \frac{50\text{GeV}}{M_N}$$

- LHC and other colliders are probing seesaw model
- long-lived particle searches sensitive to small Yukawa couplings
- current limits > 3 orders of magnitude away from seesaw estimate



What if there are more than 3 sterile neutrinos?

We add **3 RH neutrinos** $\nu_R \equiv N^c$ and **3 gauge singlet fermions** S . This is an arbitrary distinction in the SM, but may be motivated in grand unified theories like SO(10), where the RH neutrinos are part of the 16-plet. The neutral fermion mass matrix is

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ . & 0 & M_{NS} \\ . & . & \mu_S \end{pmatrix} \quad \text{with} \quad \mathcal{N} = \begin{pmatrix} \nu_L \\ N \\ S \end{pmatrix} \quad \text{with} \quad N \equiv \nu_R^c$$

[left lower triangle fixed by symmetry]

For $\mu_S \gg M_{NS} \gg m_D$ we apply twice the seesaw formula to obtain light neutrino masses

$$m_\nu^{\text{DS}} = m_D (M_{NS}^{-1})^T \mu_S M_{NS}^{-1} m_D^T$$

This is commonly known as **double seesaw**.

A well-motivated setup could be $\mu_S \sim M_{\text{Planck}}$, $M_{NS} \sim M_{\text{GUT}}$, $m_D \sim m_W \Rightarrow m_\nu^{\text{DS}} \sim \text{eV}$

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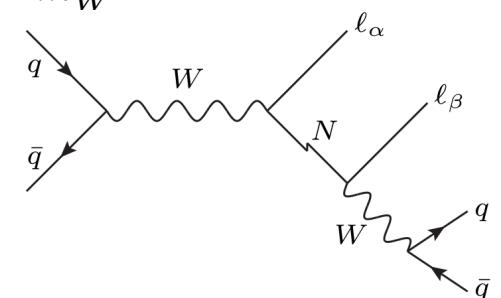
The same result applies for $\mu_S \ll m_D \ll M_{NS}$ which is referred to as **inverse seesaw**.

- Lepton number is only broken by μ_S which can be arbitrarily small
 - Smallness of neutrino masses explained by tiny μ_S
 - It is technically natural, i.e. quantum corrections do not spoil it
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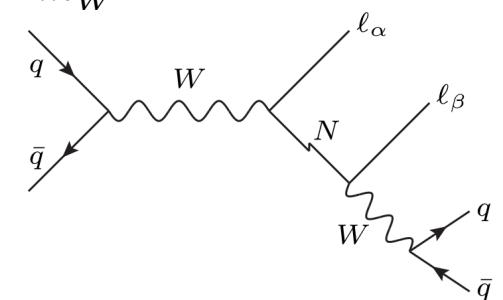
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- Inverse seesaw has large active-sterile mixing angles $U_{\alpha i}$ with $i = 4, \dots$
 - possibly large contributions to $\mu \rightarrow e\gamma$ since $\text{Br} = \frac{3\alpha}{32\pi} \left| \sum_i U_{\mu i}^* U_{ei} \frac{m_i^2}{m_W^2} \right|^2$
 - sterile neutrinos easier to produce at colliders
 - **but** we don't expect to see \cancel{L} since it is suppressed by μ_S/M_{NS}



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 - sterile neutrinos easier to produce at colliders
 - **but** we don't expect to see \cancel{L} since it is suppressed by μ_S/M_{NS}
- Inverse seesaw is easier to test experimentally



$$\mathcal{M} = \begin{pmatrix} 0 & m_D & \underline{\mu_{\nu S}} \\ \cdot & \underline{\mu_{NN}} & M_{NS} \\ \cdot & \cdot & \underline{\mu_S} \end{pmatrix} \quad \text{with} \quad \mathcal{N} = \begin{pmatrix} \nu_L \\ N \\ S \end{pmatrix}$$

lepton number violation

- $\nu_L - \nu_L$ element zero due to EW symmetry
- m_D and M_{NS} preserve lepton number L
- underlined elements $\underline{\mu_X}$ violate lepton number L with $\Delta L = 2$
- We distinguish 3 variants (and their combinations), depending on which lepton number violating entries are non-zero
 - double/inverse seesaw $\mu_S \neq 0$ (depending on hierarchy)
 - linear seesaw $\nu_{\nu S} \neq 0$
 - radiative inverse seesaw $\nu_{NN} \neq 0$

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Linear seesaw: $\mu_{\nu S} \neq 0$

$$m_\nu^{\text{LS}} = - \left[m_D (\mu_{\nu S} M_{NS}^{-1})^T + (m_{\nu S} M_{NS}^{-1}) m_D^T \right]$$

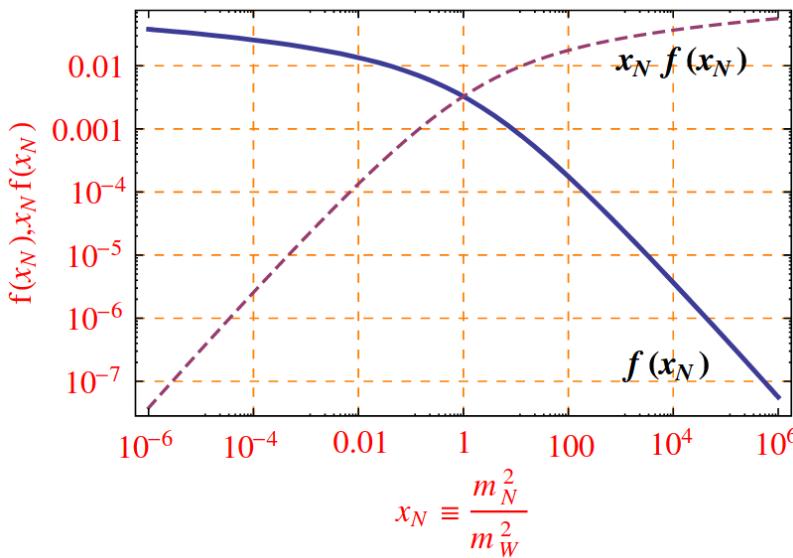
- neutrino mass is linear in the Dirac mass m_D
- small $\mu_{\nu S}$ is technically natural if it is the only source of \mathcal{L}

Radiative inverse seesaw

Seesaw models

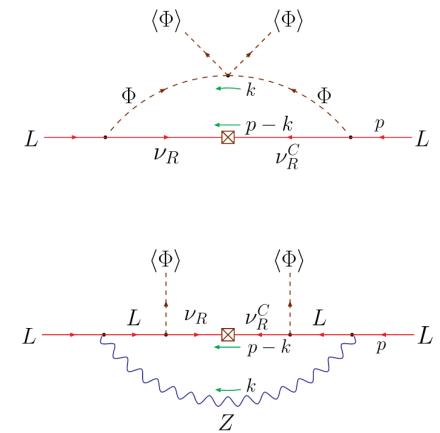
- simplest case: $\mu_{NN} \neq 0$ and $\mu_S = \mu_{\nu S} = 0$ [1209.4051]
- active neutrinos are massless at tree level
- neutrino masses generated at 1-loop $m_\nu^{1L} \simeq \frac{f(x_N)}{m_W^2} m_D \mu_{NN} m_D^T$

$$f(x_N) = \frac{\alpha_2}{16\pi} \left(\frac{x_H}{x_N - x_H} \ln\left(\frac{x_N}{x_H}\right) + \frac{3x_Z}{x_N - x_Z} \ln\left(\frac{x_N}{x_Z}\right) \right)$$



consider $\mu_{NN} \gg M_{NS} \gg m_D$

- heavy sterile neutrinos of mass μ_{NN} with $\mathcal{N}_1 \simeq N$
- light sterile ν 's, mass $M_{NS}^T \mu_{NN}^{-1} M_{NS}$ and $\mathcal{N}_2 \simeq S$
- very light active neutrinos $\mathcal{N}_3 \simeq \nu_L$



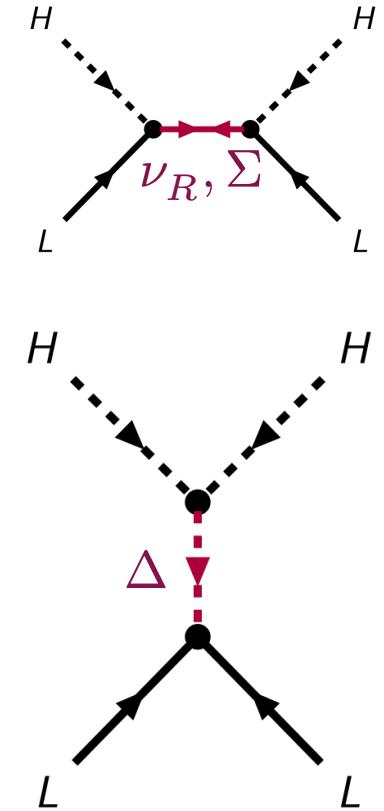
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Minimal tree-level UV completions of Weinberg operator

Seesaw models

- (Type 1) seesaw model is not only UV completion of Weinberg operator $LLHH$
- SU(2) group theory: $(LH) \sim 1 + 3$
 - ▶ $(LH)(LH)$ features two possible singlets:
 - Type 1 seesaw – $\nu_R \sim (1, 1, 0)$ Minkowski 1977
 - Type 3 seesaw – $\Sigma \sim (1, 3, 0)$ Foot, Lew, He, Joshi 1989
- More SU(2) group theory: $(LL) \sim 1 + 3$ and $(HH) \sim 3$

Why is $(HH) \sim 1 + 3$?



Minimal tree-level UV completions of Weinberg operator

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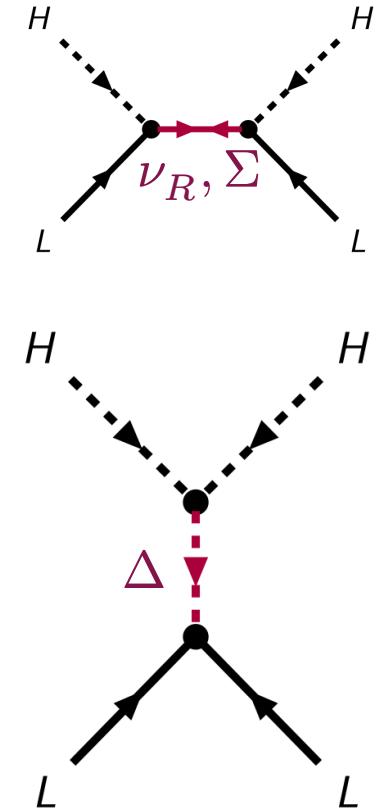
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Why is $(HH) \sim 1 + 3$?

- ▶ Type 2 seesaw – $\Delta \sim (1, 3, -1)$ Mohapatra, Senjanovic, Magg, Wetterich, Lazarides, Shafi, Schechter, Valle

Type 3 seesaw

- $\Sigma \sim (1, 3, 0)$ is Weyl fermion with 3 components, $\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)^T$
- Neutrino mass generation via Σ^0 exactly the same as in type 1 seesaw
- Σ^\pm form a vector-like lepton which mixes with SM charged leptons
 - ▶ Compared to type 1 seesaw, type 3 seesaw has richer phenomenology



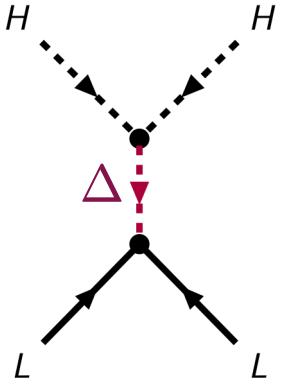
We introduce one electroweak triplet scalar $\Delta \sim (1, 3, 1)$. A convenient parametrisation is in form of a hermitian 2×2 matrix

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

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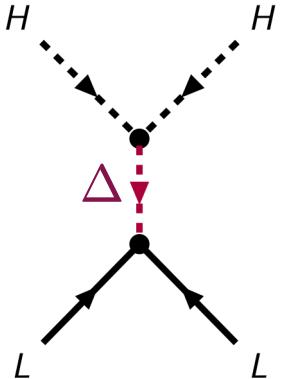
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It is in the adjoint representation of $SU(2)$, $\Delta \rightarrow U\Delta U^\dagger$. The relevant Lagrangian terms are

$$-\mathcal{L} = \frac{1}{2} \overline{L^c} Y_\Delta \varepsilon \Delta L + \mu_\Delta H^T \varepsilon \Delta^\dagger H + \text{h.c.}$$

symmetric, $Y_\Delta = Y_\Delta^T$ mass dimension 1

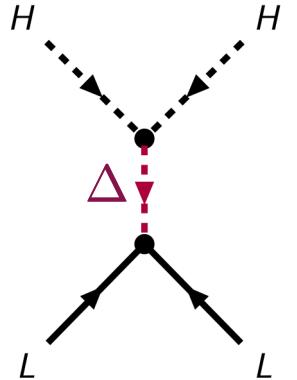
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For $E \ll M_\Delta$, Δ can be integrated to produce the Weinberg operator with $\kappa = -\frac{2\mu_\Delta Y_\Delta}{M_\Delta^2}$. The same result is obtained when considering the VEV of Δ : $\langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}$ with $v_\Delta = \frac{2\mu_\Delta}{M_\Delta^2} \frac{v^2}{2}$

The VEV v_Δ is strongly constrained by the ρ parameter (ratio of electroweak gauge boson masses): $v_\Delta \lesssim O(1 \text{ GeV})$

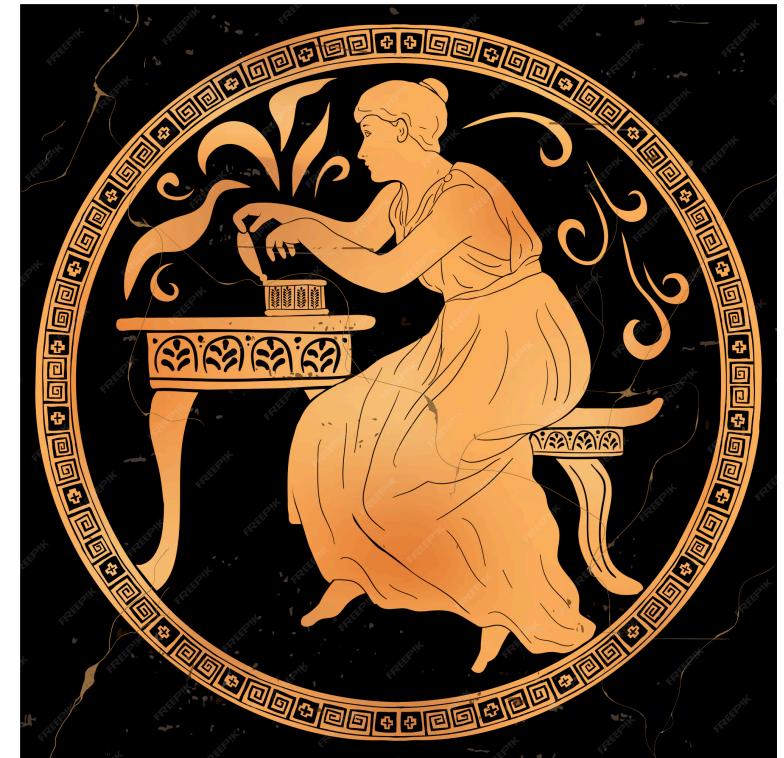
Radiative neutrino mass models

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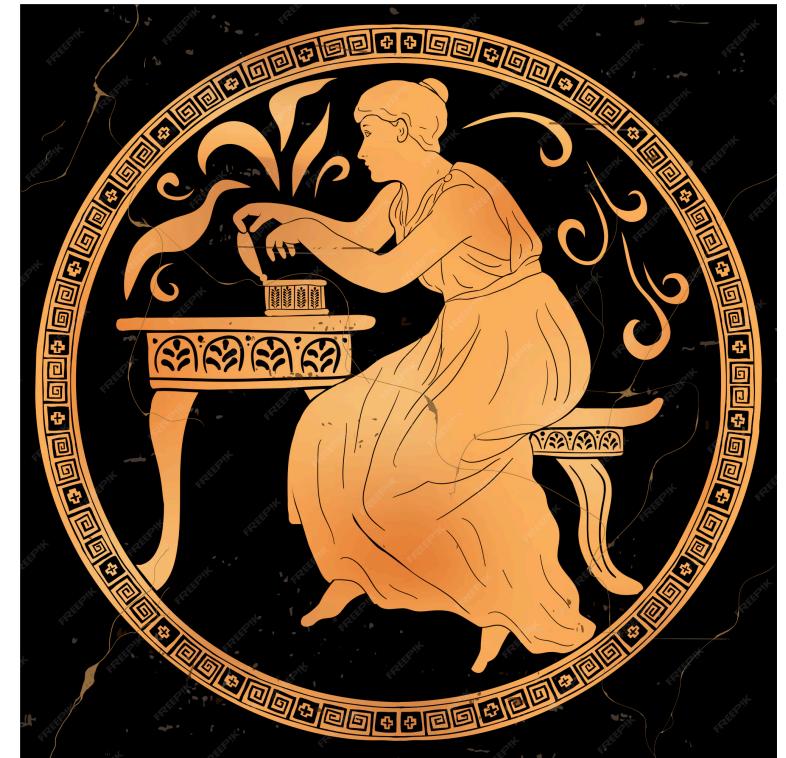
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 - more complicated than seesaw models
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How many loops should we consider? → up to 3 loops

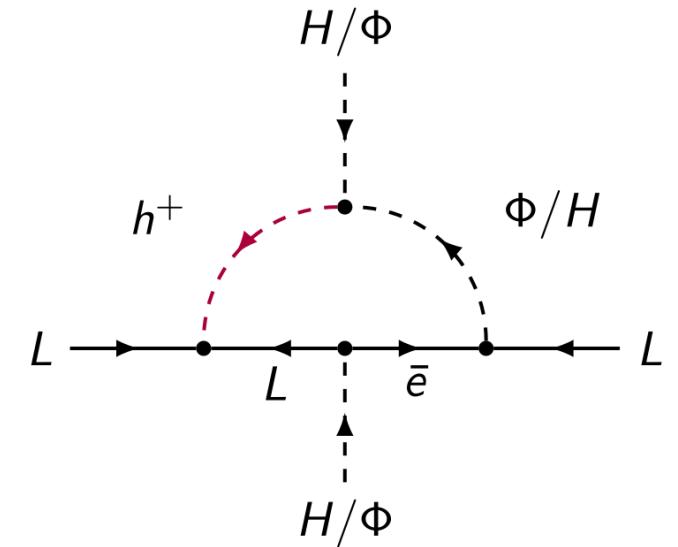


- Two new scalars:
 - singly charged scalar $h^+ \sim (1, 1, 1)$
 - second electroweak doublet scalar $\Phi \sim (1, 2, \frac{1}{2})$

$$-\mathcal{L} \supset \overline{L^c} f \varepsilon L h + \overline{e_R} Y_2 \Phi^\dagger L + \mu h^{+*} H \Phi$$

$\Delta L = 2$ in presence of f, Y_2, μ, M_e

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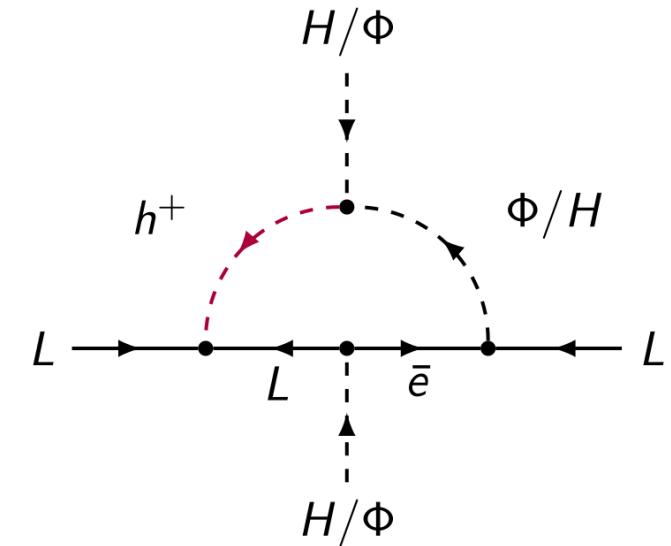


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- Without loss of generality $\langle \Phi \rangle = 0$

$$m_\nu = -\frac{\sin(2\varphi)}{16\pi^2} (\mathbf{f} M_e Y_2 + Y_2^T M_e \mathbf{f}^T) \ln\left(\frac{m_2^2}{m_1^2}\right) \quad \text{with} \quad \sin(2\varphi) \simeq \frac{\sqrt{2}v\mu}{m_2^2 - m_1^2}$$

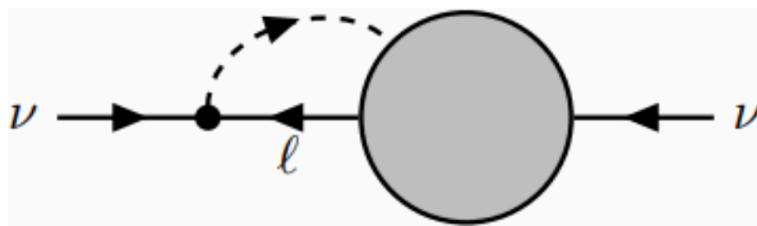
with $M_e = \text{diag, charged scalar masses } m_i$

- Simplest variant (Zee-Wolfenstein model) with Z_2 symmetry ruled out

Models with singly-charged scalar

Radiative neutrino mass models

- singly charged scalar $h \sim (1, 1, 1)$ has anti-symmetric Yukawa interaction $\overline{L^c} f \varepsilon L h$
- f has a zero eigenvalue with eigenvector $v_\alpha = \varepsilon_{\alpha\beta\gamma} f_{\beta\gamma}$.
- If one of these lepton doublets is an external leg and neutrino mass is described by



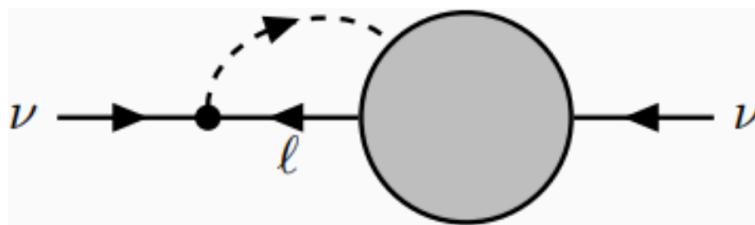
$$m_\nu = X f - f X^T$$

where X describes the gray blob.

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- The zero eigenvalue implies

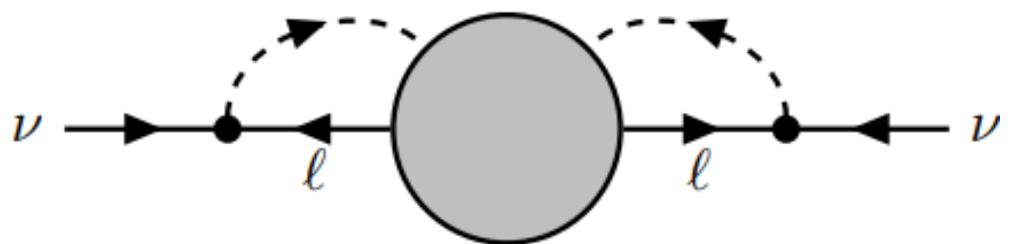
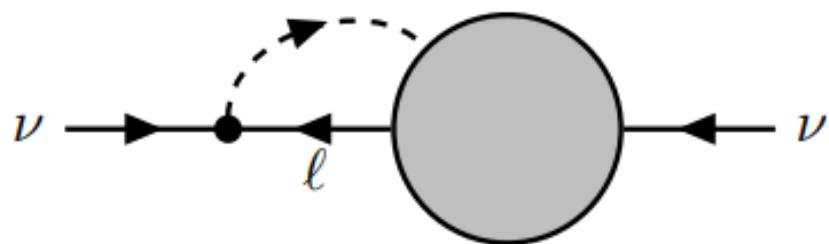
$$0 = v^T m_\nu v = v^T U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger v = \left(\sqrt{m_\nu^{\text{diag}}} U_{\text{PMNS}}^\dagger v \right)^T \left(\sqrt{m_\nu^{\text{diag}}} U_{\text{PMNS}}^\dagger v \right) \equiv w^T w$$

- one constraint on f from low energy parameters irrespective of other UV physics

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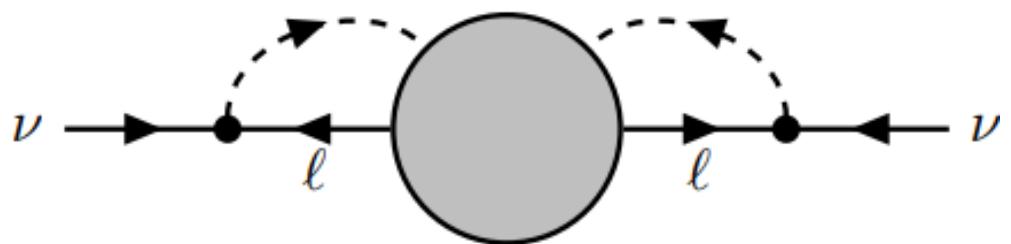
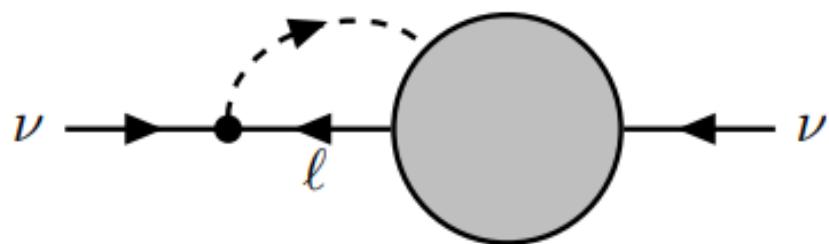
constraint

$$0 = \left(\sqrt{m_\nu^{\text{diag}}} U_{\text{PMNS}}^\dagger v \right)^T \left(\sqrt{m_\nu^{\text{diag}}} U_{\text{PMNS}}^\dagger v \right)$$

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quadratic: $m_\nu = f S f$ with symmetric S

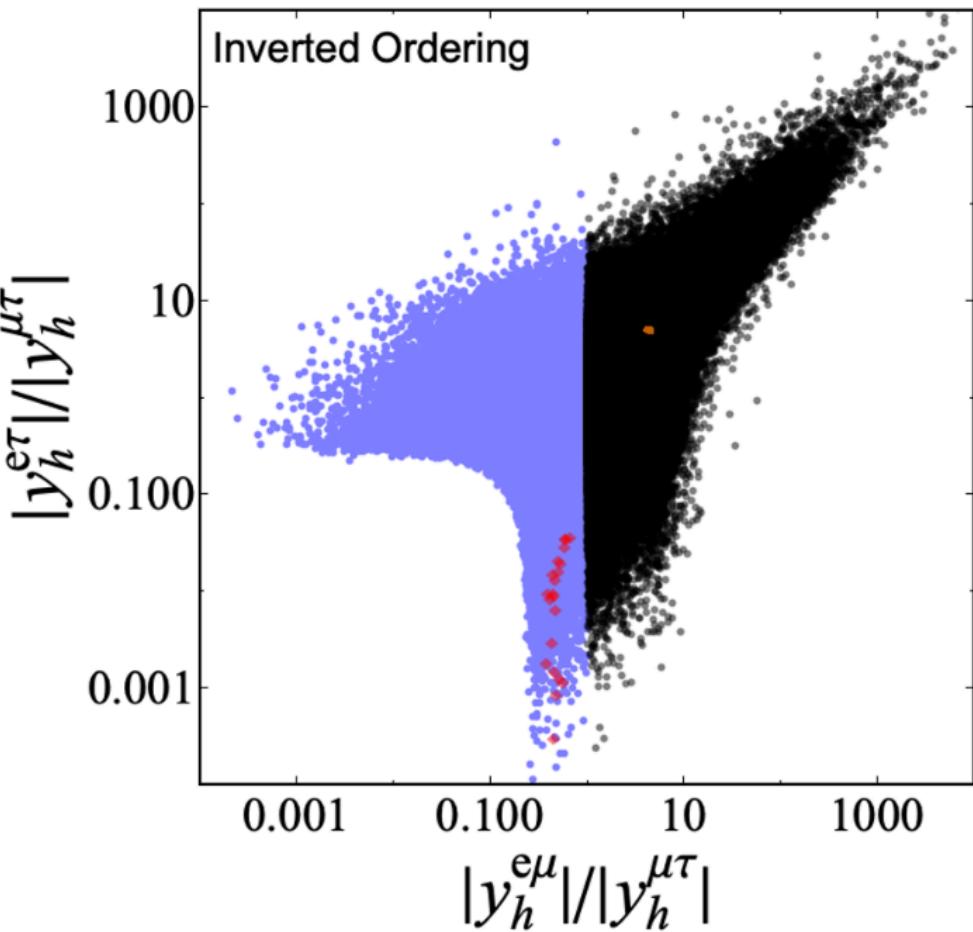
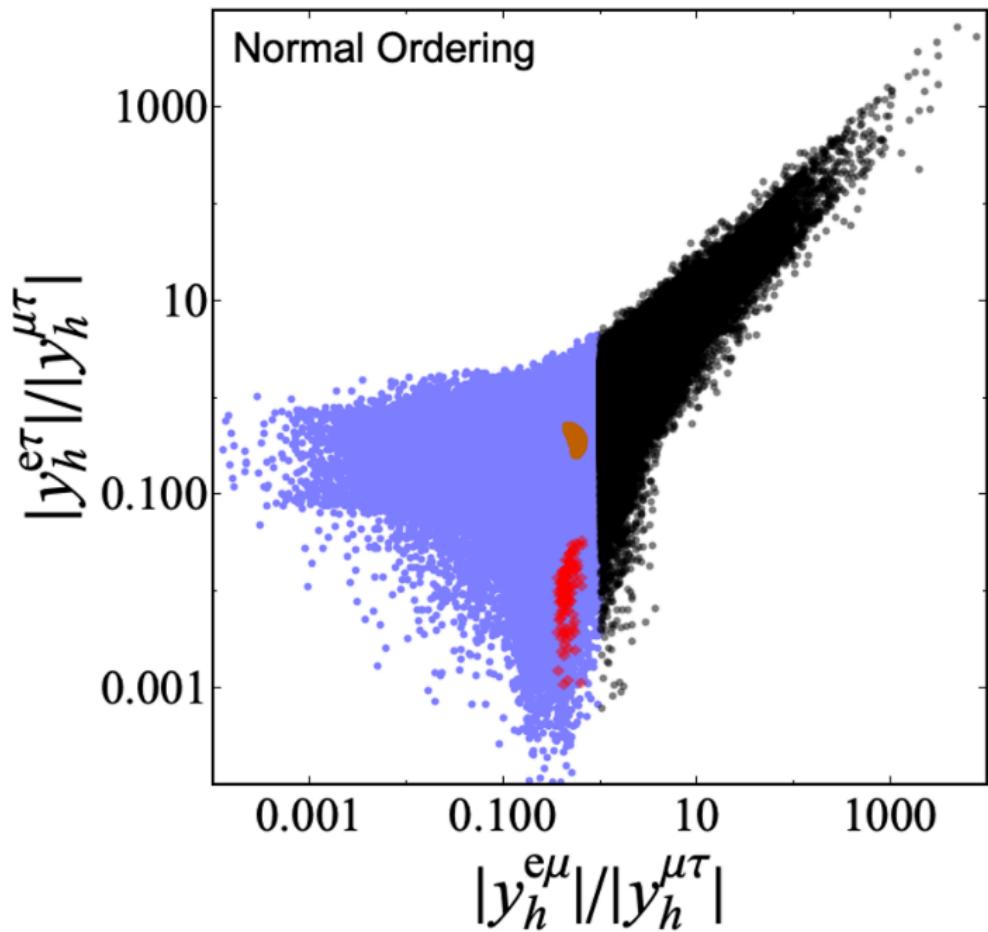
- Zero eigenvalue implies

$$0 = m_\nu v \rightarrow m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger v = 0$$

- **two conditions** on f independent of other UV physics.

Models with singly-charged scalar

Radiative neutrino mass models



- overall scale undetermined
- quadratic case (brown) very constrained

- black (blue) $\frac{g_\tau}{g_e}$ explained at $3(2)\sigma$
- red: Cabibbo angle anomaly

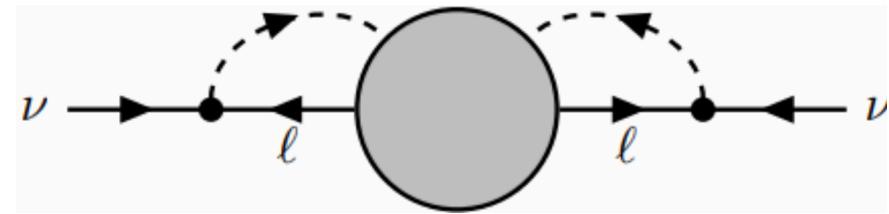
$$y_h \equiv f$$

Model with singly-charged scalar – quadratic case

Radiative neutrino mass models

- neutrino mass $m_\nu = f S f^\dagger$ with symmetric S
- two conditions on f : $m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger v = 0$

$$\frac{f^{e\tau}}{f^{\mu\tau}} = \frac{t_{12}c_{23} + s_{13}s_{23}e^{i\delta}}{c_{13}}$$

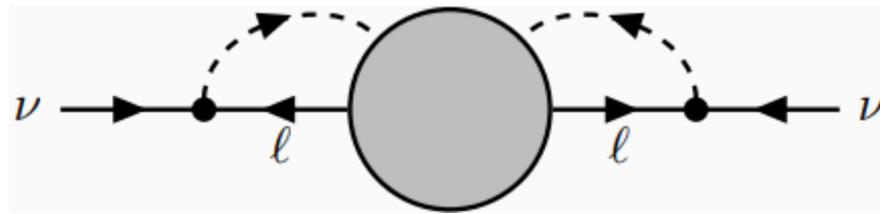


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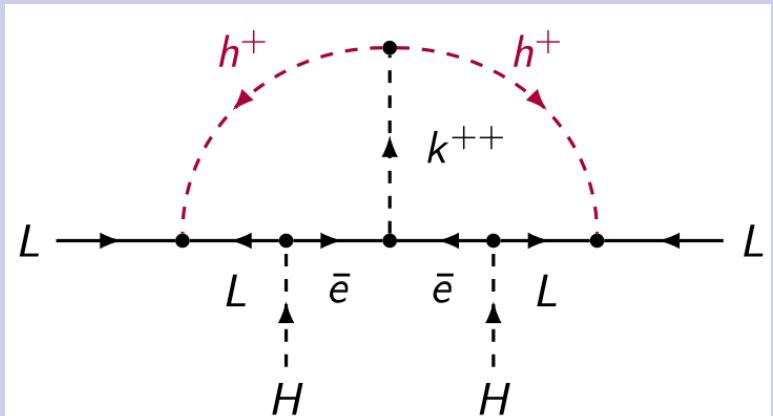
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Zee-Babu model

- example for quadratic case
- $$-\mathcal{L} = \overline{L^c} f \varepsilon L h + \overline{e_R^c} g e_R k^{++} + \mu (k^{++})^\dagger h^+ h^+$$

- neutrino mass approximately given by

$$m_\nu \simeq -\frac{\mu}{48\pi^2 M^2} f M_e g^\dagger M_e f$$

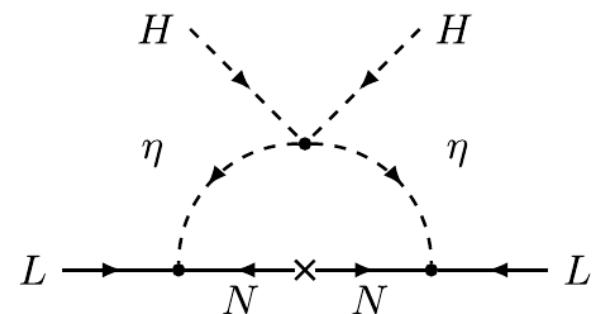


Radiative seesaw model – scotogenic model

Radiative neutrino mass models

- additional particles
 - SM singlet fermions $N \sim (1, 1, 0)_-$
 - electroweak doublet scalar $\eta \sim (1, 2, \frac{1}{2})_-$
- forbid seesaw contribution using Z_2 symmetry

$$Z_2 : \quad N \rightarrow -N \quad \text{and} \quad \eta \rightarrow -\eta$$



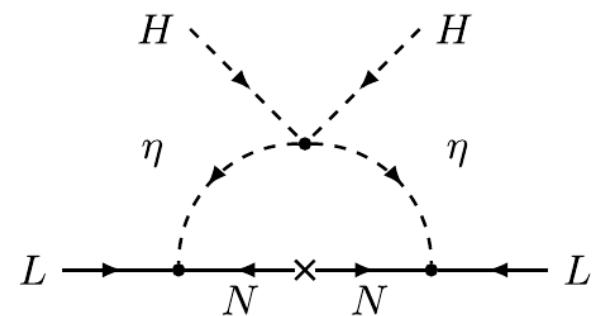
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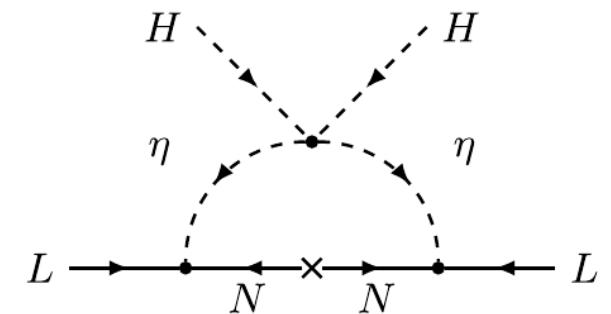
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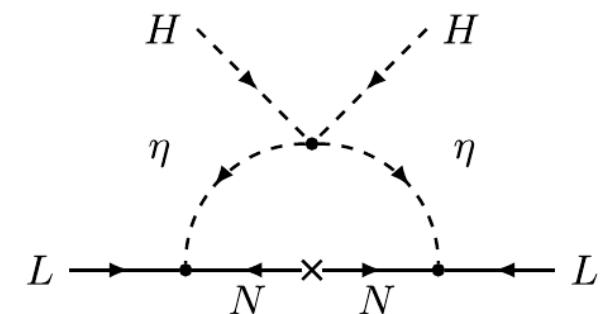
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Radiative seesaw model – scotogenic model

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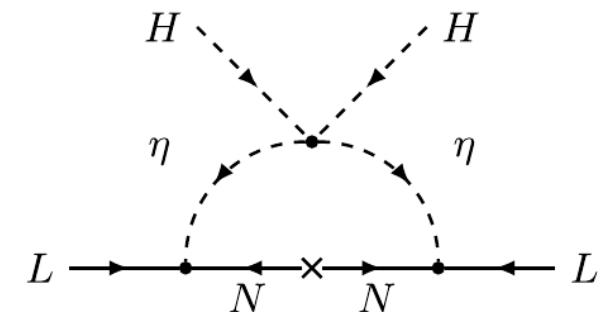
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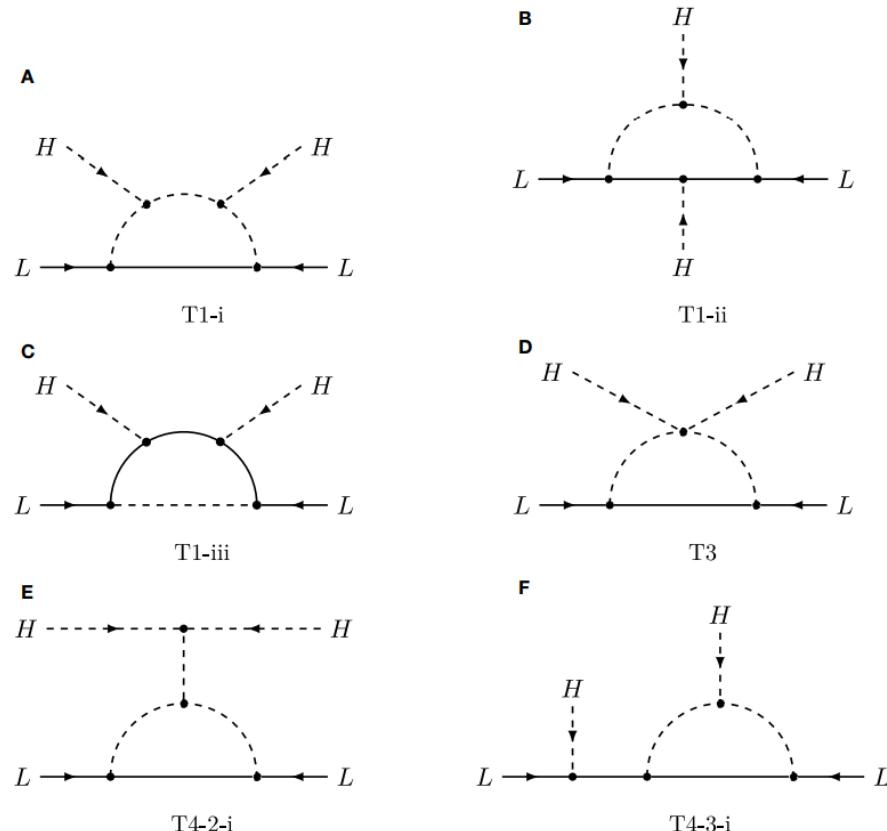
The radiative seesaw model is also often called scoto-genic (“created from darkness”) model.

Systematic approaches

Loop-level UV completions of Weinberg operator

Systematic approaches

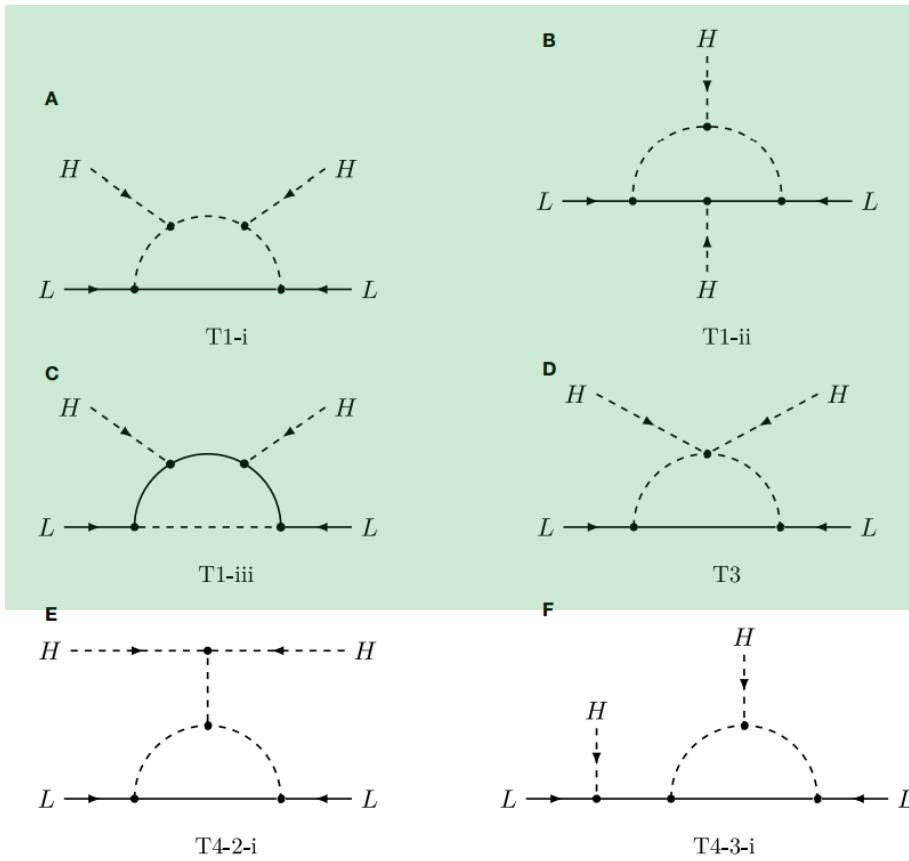
Bonnet+ [1204.5862] identified **six viable 1-loop topologies** for Weinberg operator



Loop-level UV completions of Weinberg operator

Systematic approaches

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- exclude topologies which necessarily are accompanied by tree level neutrino masses
- bottom two topologies T4-2-i, T4-3-i don't work either, require symmetry, no viable model known \Rightarrow 4 viable topologies, e.g.
 - ▶ Zee model – T1-ii
 - ▶ scotogenic model – T3
- general study enabled to systematically consider variants of scotogenic model, see Restrepo+ [1308.3655]
- topologies exist for
 - ▶ 2-loop Aristazabal+ [1411.7038] (incl loop integrals) and
 - ▶ 3-loop Cepedello+ [1807.00629]

Which operators are $\Delta L = 2$?

Kobach [1604.05726]

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- We use LH 2-component Weyl spinors $f = Q, L, u, d, e$ and daggered spinors f^\dagger
- Hypercharge conservation Y_f hypercharge of f

$$0 = \sum_f 2Y_f(N_f - N_{f^\dagger})$$

$$= \frac{1}{3}(N_Q - N_{Q^\dagger}) - \frac{4}{3}(N_u - N_{u^\dagger}) + \frac{2}{3}(N_d - N_{d^\dagger}) - (N_L - N_{L^\dagger}) + 2(N_e - N_{e^\dagger}) + (N_H - N_{H^\dagger})$$

- non-Abelian gauge groups $SU(3) \times SU(2)$ don't result in new constraints

$$\frac{1}{3}(N_Q + N_{d^\dagger} + N_{u^\dagger}) - \frac{1}{3}(N_{Q^\dagger} + N_d + N_u) \in \mathbb{Z}$$

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- Lorentz invariance N_D is number of covariant derivatives

$$N_D \text{ even} \Leftrightarrow \sum_{f \neq H} N_f \text{ even} \wedge \sum_{f \neq H} N_{f^\dagger} \text{ even}$$

- Lepton and baryon number

$$\Delta L = (N_L - N_{L^\dagger}) - (N_e - N_{e^\dagger}) \in \mathbb{Z}$$

$$\Delta B = \frac{1}{3}(N_Q + N_{u^\dagger} + N_{d^\dagger}) - \frac{1}{3}(N_{Q^\dagger} + N_u + N_d) \in \mathbb{Z}$$

- Operator dimension X field strength tensor

$$d = N_H + N_{H^\dagger} + N_D + 2N_X + \frac{3}{2} \sum_f N_f$$

- Using hypercharge condition and definitions of ΔB and ΔL , we find

$$d = 3(N_{Q^\dagger} - N_{u^\dagger} + N_{d^\dagger} + N_{L^\dagger} + N_{e^\dagger}) + N_D + 6N_u - 2N_H + 4N_{H^\dagger} + 2N_X + 3\left(\frac{1}{2}\Delta B + \frac{3}{2}\Delta L\right)$$

- Green highlighted part is always even, since N_D even $\Leftrightarrow \sum_f N_{f^\dagger}$ even

d even $\Leftrightarrow \frac{1}{2}(\Delta B - \Delta L)$ even

In particular for $\Delta B = 0$,

d even $\Leftrightarrow \Delta \frac{L}{2}$ even implies $\Delta L = 2 \rightarrow d$ odd

Weinberg-like operators

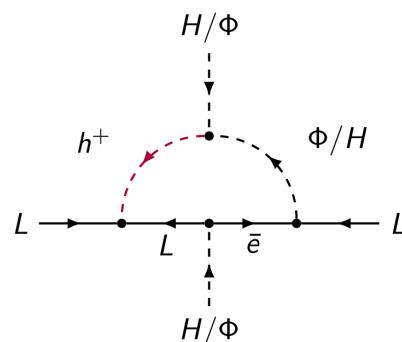
$$O_1 = L^i H^j \epsilon_{ij} L^k H^l \epsilon_{kl}$$

$$O'_1 = O_1 H^\dagger H$$

$$O''_1 = O_1 (H^\dagger H)^2$$

$$O_1^{(n)} = O_1 (H^\dagger H)^n$$

Zee model



Zee model Zee-Babu model

$$O_2 = L^i L^j L^k \bar{e} H^l \epsilon_{ij} \epsilon_{kl}$$

$$O_{4a} = L^i L^j Q_i^\dagger \bar{u}^\dagger H^k \epsilon_{jk}$$

$$O_9 = L^i L^j L^k \bar{e} L^l \bar{e} \epsilon_{ij} \epsilon_{kl}$$

$$O_{11a} = L^i L^j Q^k \bar{d} Q^l \bar{d} \epsilon_{ij} \epsilon_{kl}$$

$$O_{12a} = L^i L^j Q_i^\dagger \bar{u}^\dagger Q_j^\dagger \bar{u}^\dagger$$

...

$$O_{59} = L^i Q^j \bar{d} \bar{d} \bar{e}^\dagger \bar{u}^\dagger H^k H_i^\dagger \epsilon_{jk}$$

operators up to dimension 11 classified

$$O_{3a} = L^i L^j Q^k \bar{d} H^l \epsilon_{ij} \epsilon_{kl}$$

$$O_{4b} = L^i L^j Q_k^\dagger \bar{u}^\dagger H^k \epsilon_{ij}$$

$$O_{3a} = L^i L^j Q^k \bar{d} H^l \epsilon_{ik} \epsilon_{jl}$$

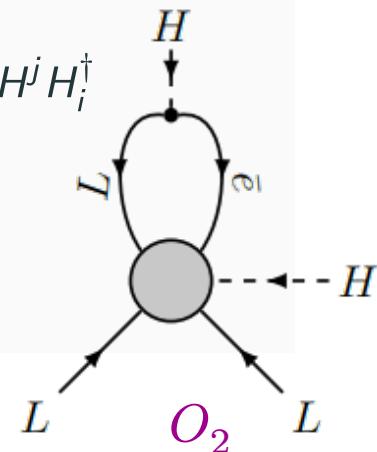
$$O_8 = L^i \bar{d} \bar{e}^\dagger \bar{u}^\dagger H^j \epsilon_{ij}$$

$$O_{10} = L^i L^j L^k \bar{e} Q^l \bar{d} \epsilon_{ij} \epsilon_{kl}$$

$$O_{11b} = L^i L^j Q^k \bar{d} Q^l \bar{d} \epsilon_{ik} \epsilon_{jl}$$

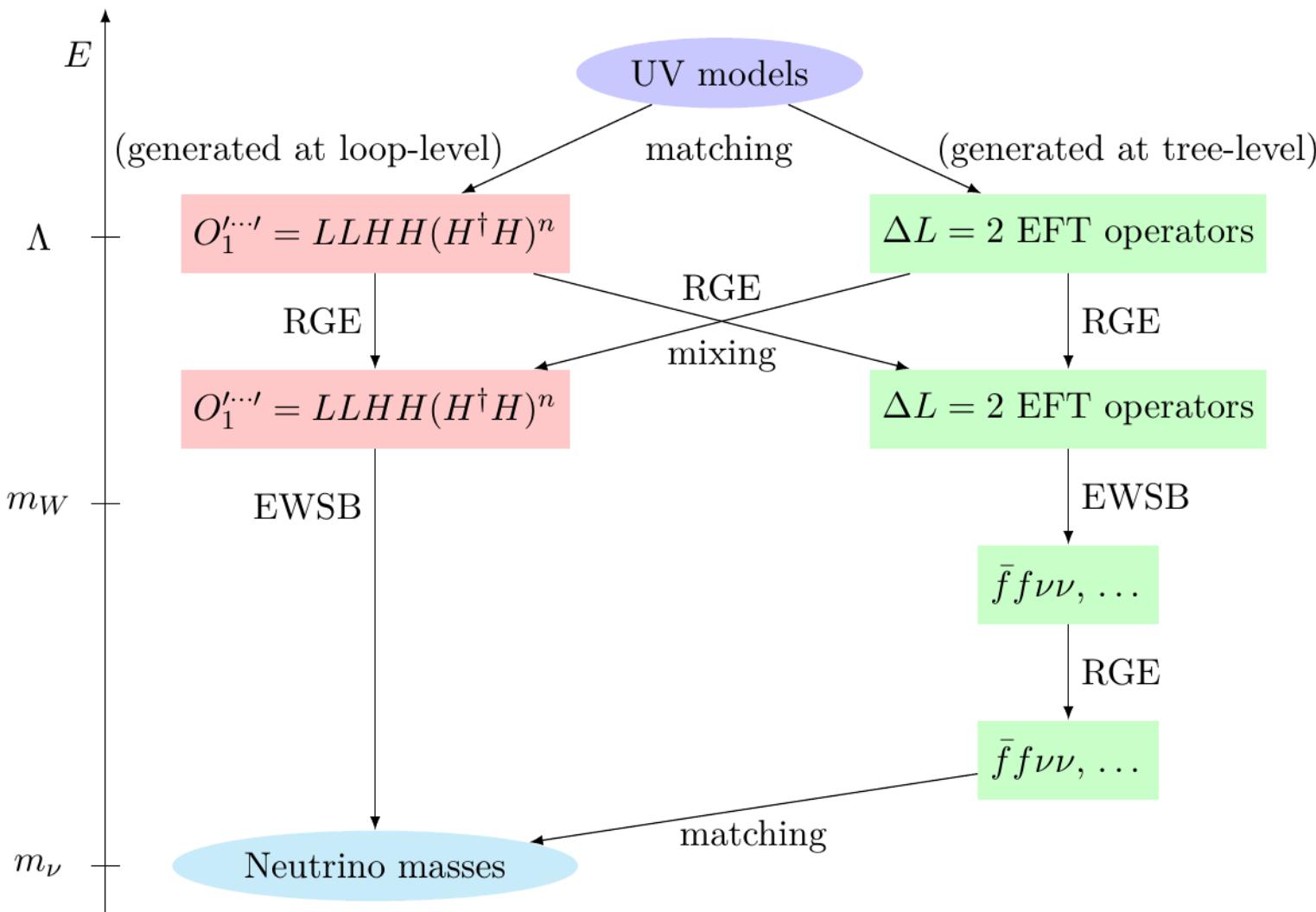
$$O_{12b} = L^i L^j Q_k^\dagger \bar{u}^\dagger Q_l^\dagger \bar{u}^\dagger \epsilon_{ij} \epsilon^{kl}$$

$$O_{60} = L^i \bar{d} Q_j^\dagger \bar{u}^\dagger \bar{e}^\dagger \bar{u}^\dagger H^j H_i^\dagger$$



Effective field theory

Systematic approaches



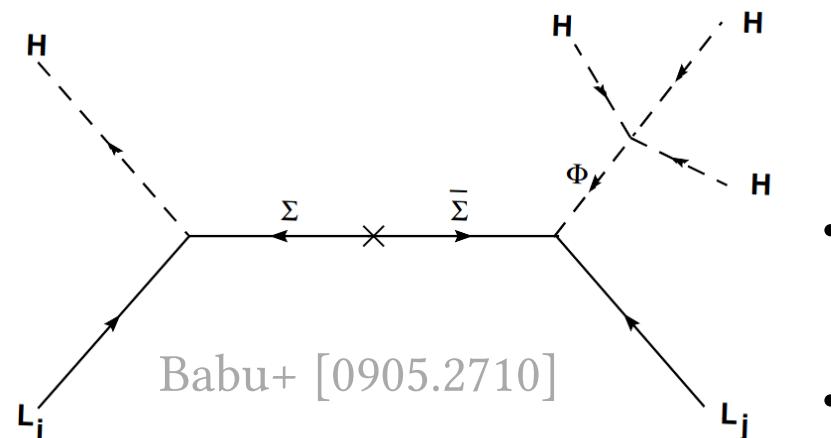
lowest dimensional Weinberg-like operator $O_1^{(n)}$ **dominates** due to power counting

1. Are there examples where O'_1 dominates over O_1 ?
2. Why is it meaningful to consider general $\Delta L = 2$ operators?

Example for Weinberg-like operator

Systematic approaches

- new particles
 - vector-like lepton $\Sigma \sim (1, 3, 1)$
 - scalar $\Phi \sim (1, 4, \frac{3}{2})$
- Lagrangian
$$\mathcal{L} = \overline{c} Y L \tilde{H} + \overline{\Sigma} \Phi \overline{Y} L + \lambda_5 H^3 \Phi + \text{h.c.}$$
- neutrinos mass
 - Weinberg operator not generated at tree level
 - Dimension-7 operator O'_1 generated at tree level

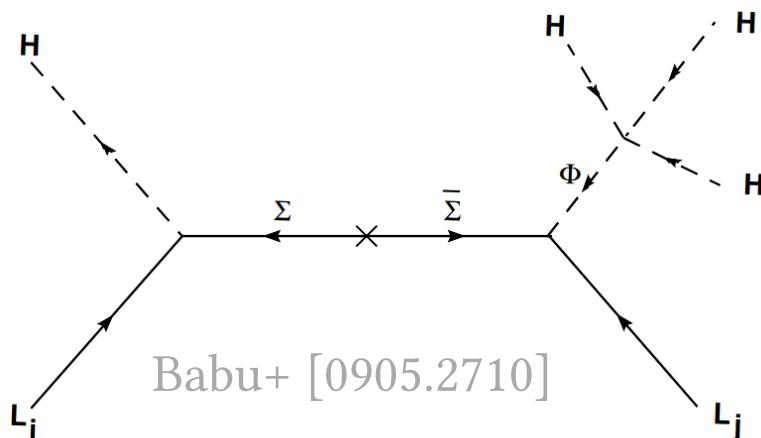


$$m_\nu = \frac{\lambda_5 v^4}{4 M_\Phi^2 M_\Sigma} (Y \overline{Y}^T + \overline{Y} Y^T)$$

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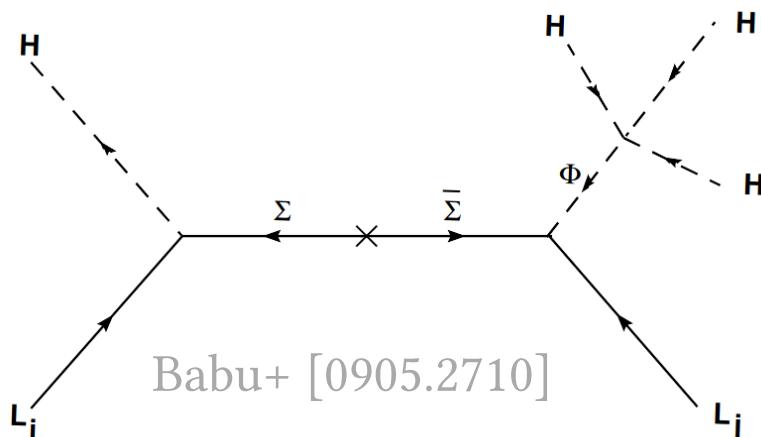
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- Weinberg operator generated at 1-loop level via topology T3 → becomes important for large M_Φ, M_Σ
- several highly charged particles at low scale

Why is it meaningful to consider general $\Delta L = 2$ operators?

-

-

Why is it meaningful to consider general $\Delta L = 2$ operators?

- useful classification which
 - provides information about other SM particles and
 - their phenomenology
- **caveat:** it generally requires to consider UV completions. The $\Delta L = 2$ operator only describes $\Delta L = 2$ processes

Dimension-7 $\Delta L = 2$ operators

Systematic approaches

- Babu+ [hep-ph/0106054] classified $\Delta L = 2$ operators (excl cov. derivatives, field strength tensors) up to dimension-11 (parentheses indicate isospin contractions)

$$O'_1 = (LH)(LH)H^\dagger H \quad O_2 = (LL)(LH)\bar{e} \quad O_{3a} = (LL)(QH)\bar{d} \quad O_{3b} = (LQ)(LH)\bar{d}$$

$$O_{4a} = (LQ^\dagger)(LH)\bar{u}^\dagger \quad O_{4b} = (LL)(Q^\dagger H)\bar{u}^\dagger \quad O_8 = L\bar{e}u\bar{d}H$$

minimal tree-level UV completions

- 2 viable topologies (external solid lines can be both scalar or fermion)



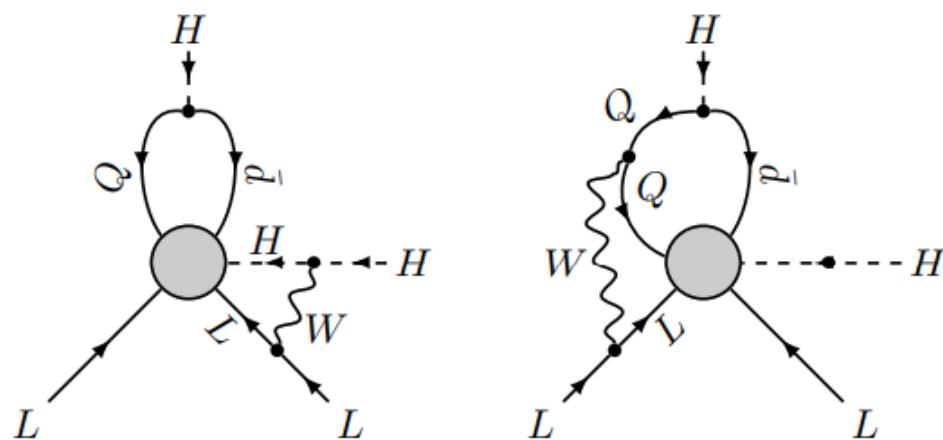
generic comments

- singly charged scalar for O_2, O_{3a}, O_{4b}
 - ▶ only small number of different particles (12)
 - ▶ allows to study their phenomenology and apply to models
- O'_1 only generated via 2nd topology (see above)
- Isospin structure prevents 1-loop neutrino masses for O_{3a} and O_{4b}
- O_8 generates neutrino masses only at 2-loop order



Dimension-7 $\Delta L = 2$ operators

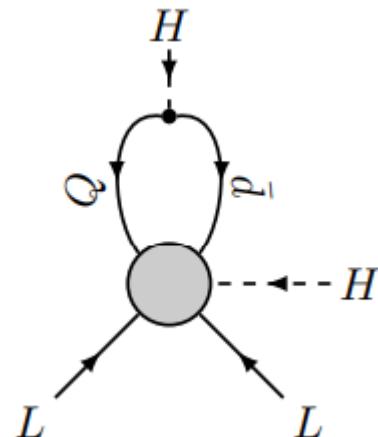
Systematic approaches

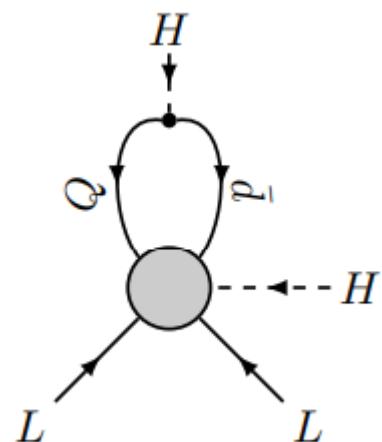
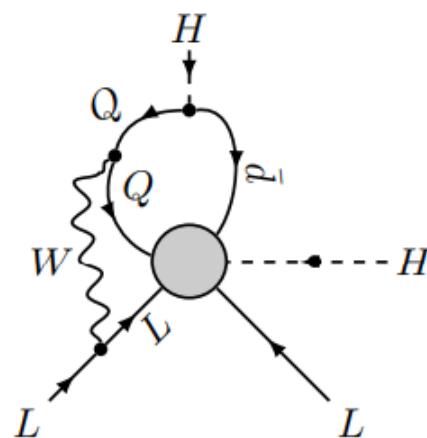
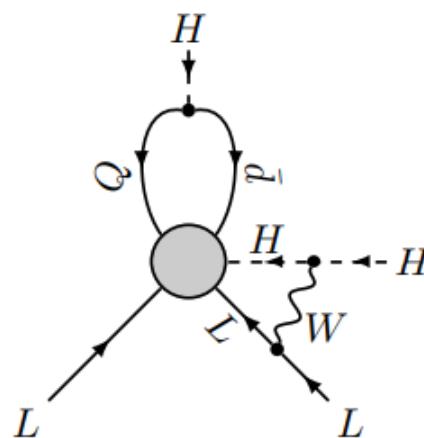


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-





$$O_{3a} = (LL)(QH)\bar{d} \quad O_{3b} = (LQ)(LH)\bar{d}$$

generic comments

- Isospin structure prevents 1-loop neutrino masses for O_{3a} and O_{4b}
- Weinberg operator can be estimated by closing off the loops of the loop diagrams without specifying UV model
- This is **NOT** an estimate within EFT, but the estimate for the size of the loop-induced Weinberg operator from matching of the UV theory.

Neutrino masses

Classification in terms of effective $\Delta L = 2$ operators

Babu, Leung hep-ph/0106054; deGouvea, Jenkins 0708.1344 Bonnet, Hernandez, Ota, Winter 0907.3143

$$m_\nu \simeq \frac{c_R v^2}{(16\pi^2)^l \Lambda} , \text{ with}$$

Loop factor

$$c_R \simeq \prod_i g_i \times \epsilon \times \left(\frac{v^2}{\Lambda^2}\right)^n$$

μ/Λ

$LLHH(H^\dagger H)^n$

$$m_\nu \gtrsim 0.05 \text{eV} \Rightarrow \begin{cases} l = 1 \rightarrow \Lambda < 10^{12} \text{GeV} \\ l = 2 \rightarrow \Lambda < 10^{10} \text{GeV} \\ l = 3 \rightarrow \Lambda < 10^8 \text{GeV} \end{cases}$$

→ no information on $\Delta L = 0$ processes

Systematic construction of models

Angel, Rodd, Volkas 1212.5862; Cai, Clarke, MS, Volkas 1308.0463; Gargalionis, Volkas 2009.13537

Bonnet, Hirsch, Ota, Winter 1204.5862; Aristizabal Sierra, Degee, Dorame, Hirsch 1411.7038; Cepedello, Fonseca, Hirsch 1807.00629

Gargalionis, Volkas 2009.13537: "exploding! $\Delta L = 2$ operators" ... "1000s of models"

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$\Delta L = 2$ operators

- general statements about $\Delta L = 2$ processes
- no information about other processes

UV models

- enables to study phenomenology
- too many models! not enough time

Is it possible to combine the two approaches?

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Is it possible to combine the two approaches? → simplified models

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UV models

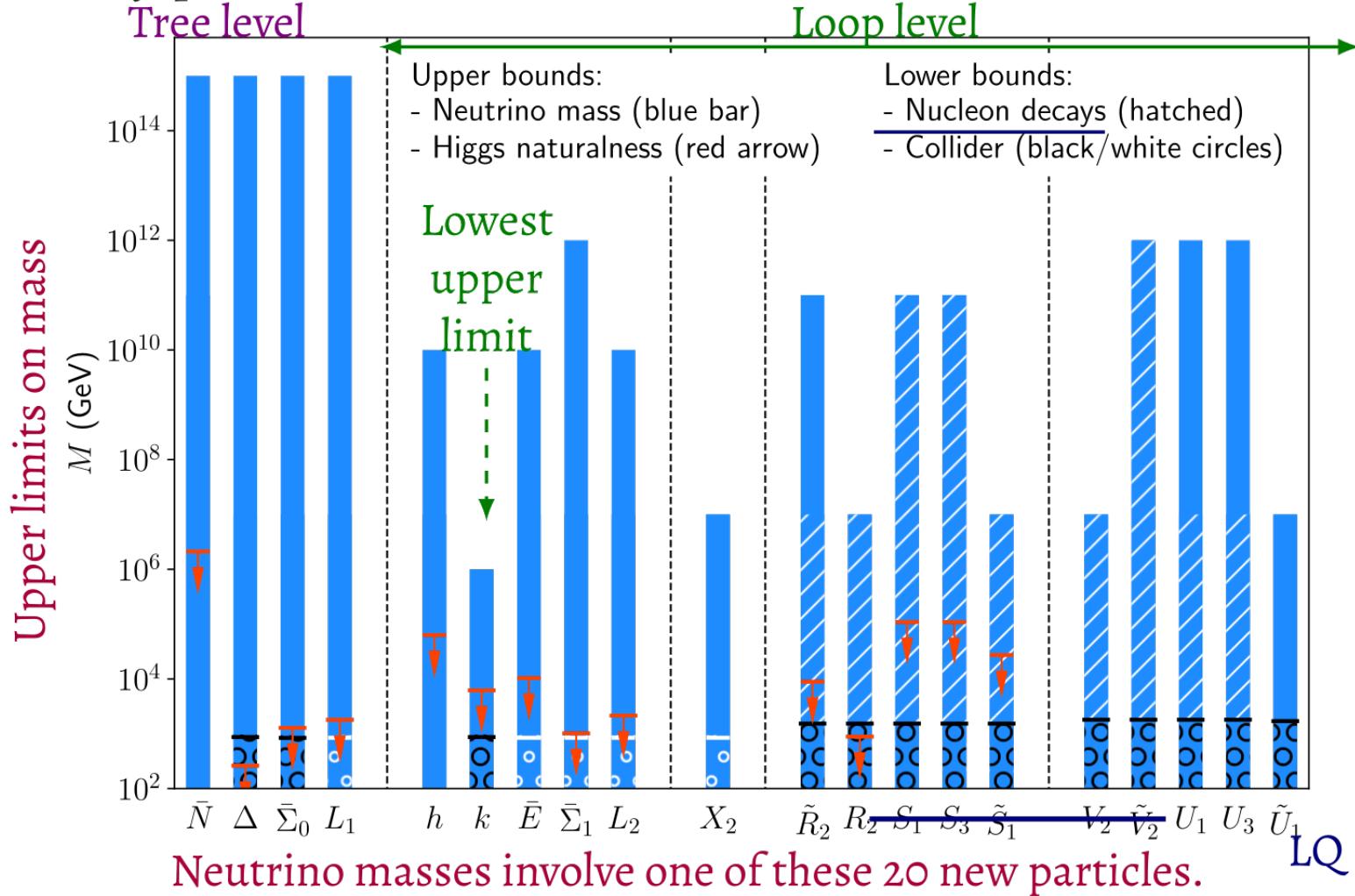
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Is it possible to combine the two approaches? → simplified models

Idea:

1. neutrino mass requires at least one new particle X (mass M) coupled to SM lepton(s), carrying L (and maybe B)
2. QFT: L is violated (by two units) via new operators at scale Λ which encode the (model-dependent) UV physics
3. Majorana neutrino masses $m_\nu \propto \Lambda^{-1}$ are generated
4. $m_\nu > 0.05$ eV & $M < \Lambda$ provide conservative upper bound on M
5. L -conserving phenomenology mostly determined by renormalisable $\Delta L = 0$ operators

Summary plot

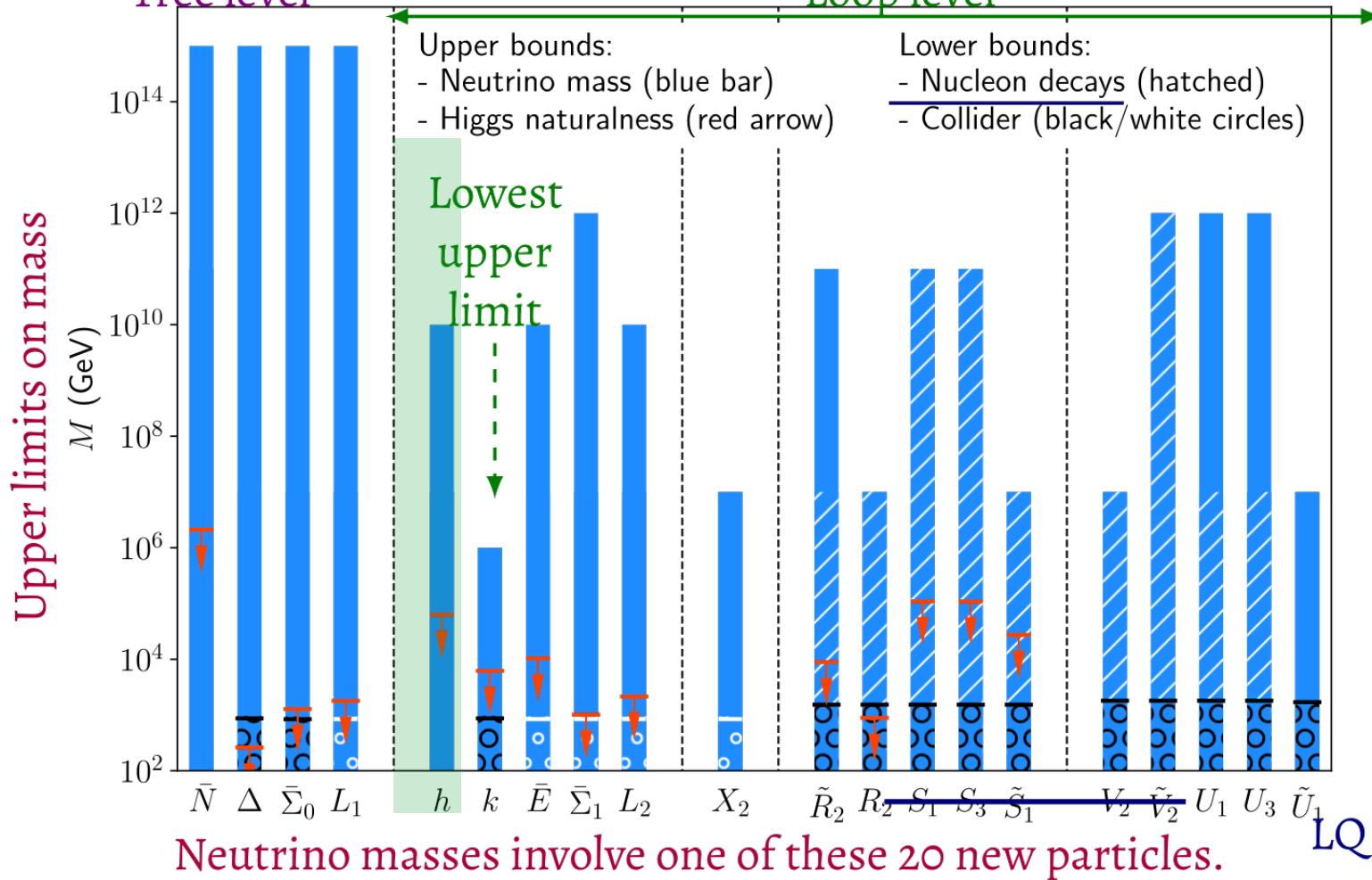


Summary plot

Tree level

Juan Herrero-García, MS 1903.10552 [Eur.Phys.J. C79 (2019) no.11, 938]

Loop level



Questions?