



# Introduction to Neutrino Physics

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# Introduction

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# LH and RH neutrinos

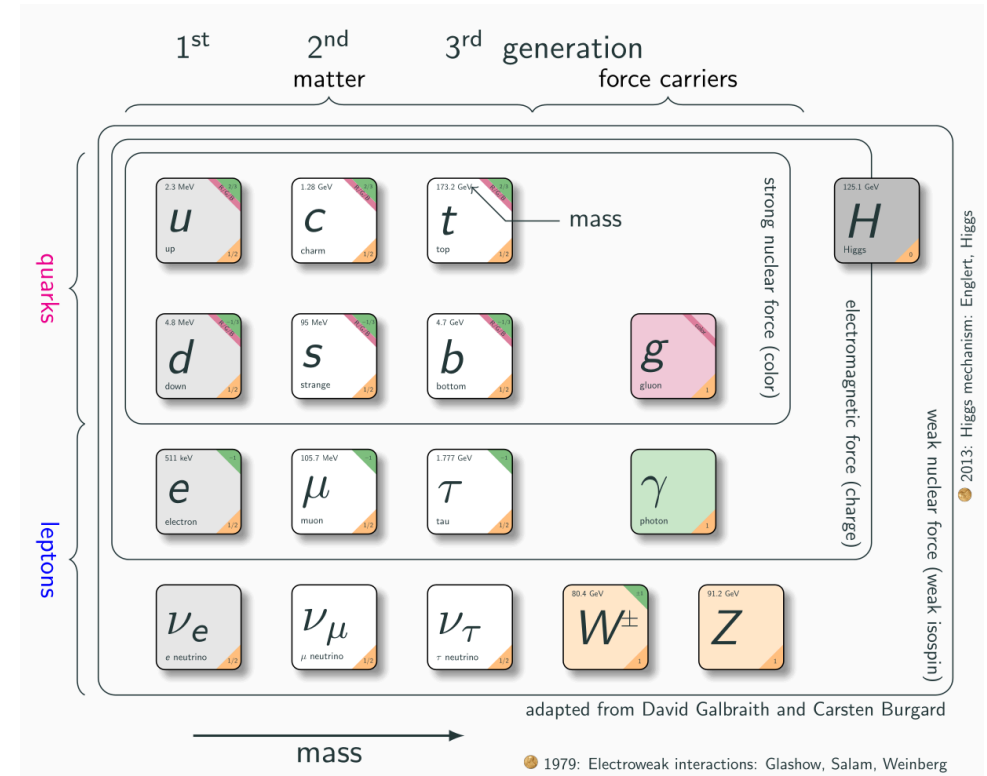
In the **Standard Model** there are only left-handed (LH) neutrinos which are part of electroweak lepton doublets  $L_\alpha$

$$L_e = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad L_\mu = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad L_\tau = \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$$

LH neutrinos are negative chirality states

$$\gamma_5 \nu_L = -\nu_L$$

For **massless neutrinos**, negative chirality states are negative helicity states

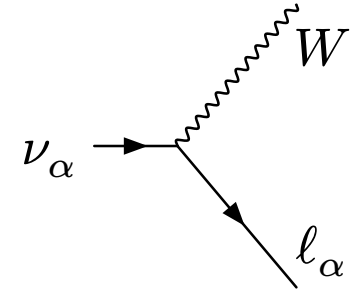


**helicity** of (anti)particles:  $h^{(v)} = (-)\frac{1}{2}\sigma \cdot \hat{p}$

# LH and RH neutrinos

**For massive neutrinos:**

Negative chirality states have small admixture of positive helicity state.



## Dirac neutrinos

		helicity	$\ell^-$ prod.	$\ell^+$ prod.
$\nu$		$-\frac{1}{2}$	1	0
$\bar{\nu}$		$-\frac{1}{2}$	0	$\left(\frac{E}{m}\right)^2 \ll 1$
$\nu$		$\frac{1}{2}$	$\left(\frac{E}{m}\right)^2 \ll 1$	0
$\bar{\nu}$		$\frac{1}{2}$	0	1

## Majorana neutrinos

Neutrinos may be their own antiparticles, i.e.  $\nu = \nu^c$

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	$-\frac{1}{2}$	1	0
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# LH and RH neutrinos

When SM was formulated, neutrinos were considered massless  $\rightarrow$  RH neutrinos not needed

$$-\mathcal{L} = \overline{L}_L \Phi Y_e e_R + \overline{Q}_L \Phi Y_d d_R + \overline{Q}_L \tilde{\Phi} Y_u u_R + \text{h.c.} \quad \text{with } \tilde{\Phi} \equiv \varepsilon \Phi^*$$

Neutrino masses may be introduced similar to charged fermions:

$$-\mathcal{L}_\nu = \overline{L}_L \tilde{\Phi} Y_\nu \nu_R + \text{h.c.}$$

**Q:** How do RH neutrinos transform with SM gauge group  $\text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y$ ?

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The number of RH neutrinos is **not** fixed. At least two RH neutrinos are required to explain the observed neutrino oscillations.

# Dirac or Majorana neutrinos?

A central question when discussing neutrinos is

## Dirac or Majorana neutrinos?

Assume neutrino masses are generated like charged fermion masses. Then, after electroweak symmetry breaking

$$-\bar{L}\tilde{\Phi}Y_{\nu}\nu_R + \text{h.c.} \xrightarrow{\langle\Phi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}} -\bar{\nu}_L M_{\nu}^D \nu_R + \text{h.c.} \quad \text{with } M_{\nu}^D = Y_{\nu} \frac{v}{\sqrt{2}}$$

This is a **Dirac mass term** and, thus, the neutrino is a **Dirac particle** with 4 spinor components, 2 from  $\nu_L$  and 2 from  $\nu_R$ , like all the charged fermions in the SM.

We can define the **(total) lepton number**

$$L = L_e + L_{\mu} + L_{\tau}$$

$L_{\alpha}$  are individual lepton numbers



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- All LH leptons and RH charged leptons have  $L = +1$
- All leptonic anti-particles have lepton number  $L = -1$
- Quarks, gauge bosons and the Higgs have lepton number  $L = 0$
- All terms of the SM conserve lepton number, in particular

$$-\mathcal{L} = \overline{L}_L \Phi Y_e e_R + \overline{L}_L \tilde{\Phi} Y_\nu \nu_R + \text{h.c.}$$

# Dirac or Majorana neutrinos?

Since neutrinos are electrically neutral and colorless, there is another possibility – they can be their own antiparticles

$$\nu = \nu^c \equiv C\bar{\nu}^T$$

$$C = i\gamma^2\gamma^0$$

Such particles are called **Majorana particles**: LH and RH neutrino fields are **not** independent

$$\nu_L^c \equiv (\nu_L)^c$$

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A Majorana particle has only 2 independent components and not 4.

Using this, we can write down a Majorana mass term

$$-\mathcal{L} = \frac{1}{2}\overline{\nu_L^c}M_\nu^M\nu_L + \text{h.c.} \quad \text{with} \quad M_\nu^M = (M_\nu^M)^T$$

- any Majorana mass term **breaks** lepton number by two units:  $\Delta L = 2$
- it **cannot** arise from a renormalisable gauge invariant operator in the SM (see lecture 2)

# Neutrino interactions

The charged lepton mass term is

$$-\mathcal{L} \supset \overline{e}_L M_e e_R + \text{h.c.}$$

$$\left. \begin{array}{l} e_L \rightarrow L_e e_L \\ e_R \rightarrow R_e e_R \end{array} \right\} \Rightarrow M_e \rightarrow L_e^\dagger M_e R_e = \text{diag}$$

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$$\mathcal{L} = -\frac{g}{2\sqrt{2}} W_\mu^- \overline{\ell}_L \gamma^\mu \nu_L - \frac{g}{\cos \theta_w} Z_\mu \overline{\nu}_L \gamma^\mu \nu_L$$

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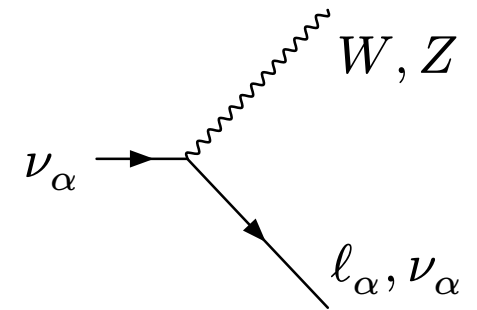
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$$\nu_L \rightarrow L_\nu \nu_L \Rightarrow L_\nu^\dagger M_\nu^M L_\nu^* = \text{diag}$$

$$\begin{aligned} \mathcal{L} &= -\frac{g}{2\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu \nu_L - \frac{g}{\cos \theta_w} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L \\ &\rightarrow -\frac{g}{2\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu \underbrace{L_e^\dagger L_\nu}_{U_{\text{PMNS}}} \nu_L - \frac{g}{\cos \theta_w} Z_\mu \bar{\nu}_L \gamma^\mu \underbrace{L_\nu^\dagger L_\nu}_1 \nu_L \end{aligned}$$





# Neutrino interactions

The **PMNS (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix** is defined as

$$U_{\text{PMNS}} = L_e^\dagger L_\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{R_{23}(\theta_{23})U_{13}(\theta_{13}, \delta)R_{12}(\theta_{12})P(\alpha_1, \alpha_2)}_{\hat{U}_{\text{PMNS}}}$$

and relates the flavour eigenstates  $\hat{\nu}_{L\alpha}$  with the mass eigenstates  $\nu_{Li}$ :  $\hat{\nu}_{L\alpha} = (U_{\text{PMNS}})_{\alpha i} \nu_{Li}$

The PMNS matrix is parameterised in terms of 3 angles  $\theta_{ij}$  and 1 Dirac CP phase  $\delta$ . For Majorana neutrinos there are 2 additional Majorana phases  $\alpha_i$ .

- $R_{ij}$  is a rotation in the i-j plane by  $\theta_{ij}$ , similarly  $U_{ij}$  with  $\pm s_{13} \rightarrow \pm s_{13} e^{\mp i\delta}$ , for example

$$U_{13} = \begin{pmatrix} c_{13} & s_{13} e^{-i\delta} \\ & 1 \\ -s_{13} e^{i\delta} & c_{13} \end{pmatrix}$$

- The Majorana phases  $\alpha_i$  are multiplied from the right as  $P = \text{diag}(e^{\frac{i}{2}\alpha_1}, e^{\frac{i}{2}\alpha_2}, 1)$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha_1} & & \\ & e^{\frac{i}{2}\alpha_2} & \\ & & 1 \end{pmatrix}$$

Commonly, the mixing angles are denoted

- solar mixing angle  $\theta_{12} \sim 34^\circ$
- atmospheric mixing angle  $\theta_{23} \sim 45^\circ$
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There is currently only a hint for a non-zero Dirac CP phase  $\delta$ . It can also be expressed in terms of the **Jarlskog invariant**

$$J_{\text{CP}} = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin(2\theta_{13}) \cos(\theta_{13}) \sin(\delta)$$

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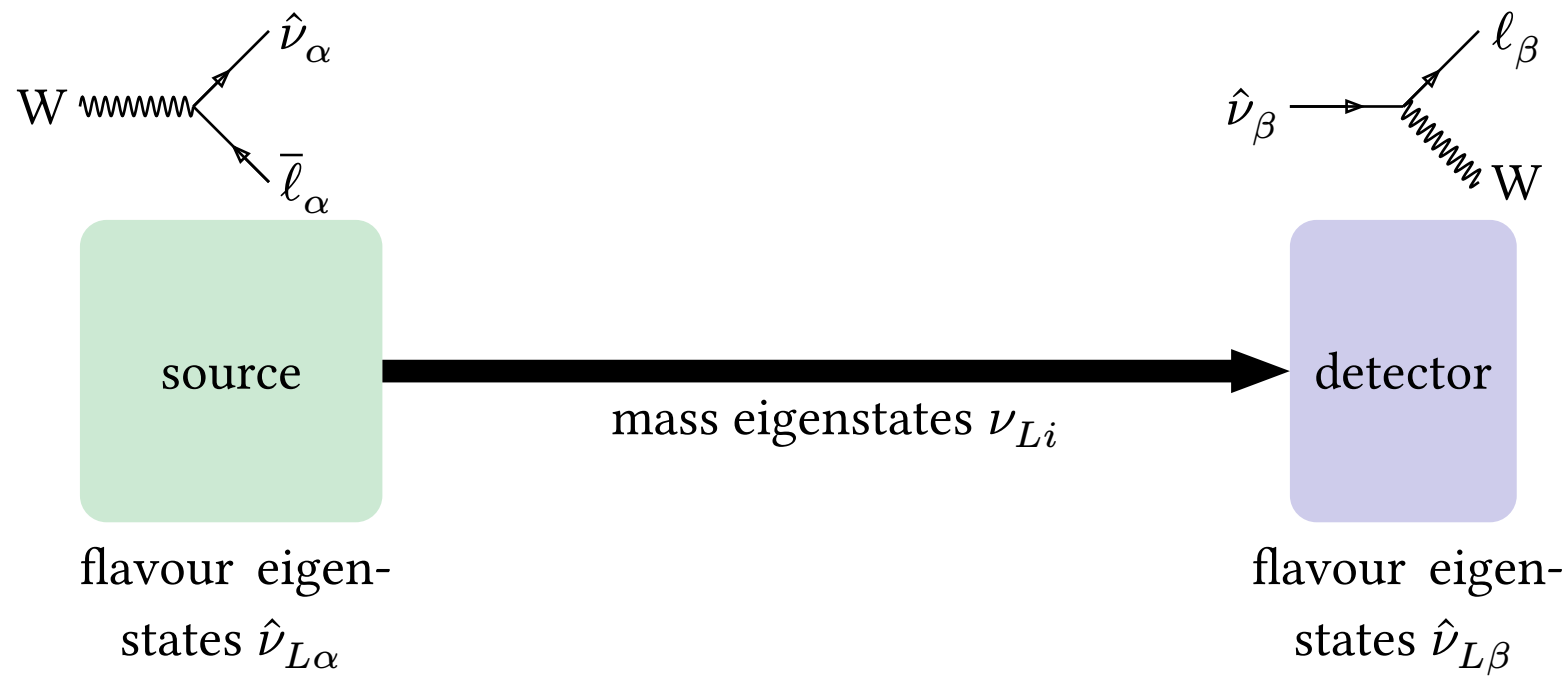
If neutrinos are *Majorana particles*, there are two additional phases, the so-called **Majorana phases**  $\alpha_i$ .

If one of the neutrinos is massless, there is only one physical Majorana phase.

# Neutrino oscillations

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# Basic picture



Neutrinos are produced and detected as flavour eigenstates and propagate as mass eigenstates.

The same picture applies for antineutrinos.

# Two-flavour oscillations

We consider **two-flavour neutrino oscillations in vacuum**. The PMNS matrix is parameterised by one angle  $\theta$  and possibly a Majorana phase  $\alpha$

$$U_{\text{PMNS}} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha} & \\ & 1 \end{pmatrix}$$

We denote the **flavour** eigenstate  $\nu_{Le}$  and  $\nu_{L\mu}$  and the **mass** eigenstates  $\nu_{L1}$  and  $\nu_{L2}$ :

$$|\nu_{Le}\rangle = c_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + s_\theta |\nu_{L2}\rangle$$

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We use **plane waves in quantum mechanics** for the calculation of neutrino oscillations.

**What is the probability to detect a muon neutrino  $\nu_{L\mu}$  in the detector if the source produced an electron neutrino  $\nu_{Le}$ ?**

The probability to detect  $\nu_{L\mu}$  at time  $T$  from a  $\nu_{Le}$  produced in the source is

$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_{L\mu} | \nu_{Le}(T) \rangle|^2$$

## flavour eigenstates

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In the mass basis the Hamiltonian is diagonal and thus time evolution is described by

$$|\nu_{Li}(t)\rangle = e^{-iE_i t} |\nu_{Li}\rangle$$

Hence we find for the time-evolved electron neutrino state

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and consequently the probability amplitude is

$$\langle \nu_{L\mu} | \nu_{Le}(T) \rangle = \left( -s_\theta e^{-\frac{i}{2}\alpha} \langle \nu_{L1} | + c_\theta \langle \nu_{L2} | \right) \left( c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 T} |\nu_{L1}\rangle + s_\theta e^{-iE_2 T} |\nu_{L2}\rangle \right)$$

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**Note that the Majorana phase  $\alpha$  dropped out!** This is a general result: Majorana phases generally drop out since they always show up together with a mass eigenstate  $e^{\frac{i}{2}\alpha_i} | \nu_{Li} \rangle$  and thus drop out of the probability amplitude due to the orthogonality of the mass eigenstates.

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$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= | -s_\theta c_\theta e^{-iE_1 T} + c_\theta s_\theta e^{-iE_2 T} |^2 \\ &= s_\theta^2 c_\theta^2 | 1 - e^{-i(E_2 - E_1)T} |^2 \end{aligned}$$



$$P(\nu_e \rightarrow \nu_\mu) = s_\theta^2 c_\theta^2 |1 - e^{-i(E_2 - E_1)T}|^2$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= s_\theta^2 c_\theta^2 \left| 1 - e^{-i(E_2 - E_1)T} \right|^2 \\ &= \frac{\sin^2(2\theta)}{4} \left( 2 - 2 \cos \left( 2 \frac{(E_2 - E_1)T}{2} \right) \right) \end{aligned}$$

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In the ultra-relativistic limit ( $E \gg m_i$ ), the neutrino energy difference

$$E_2 - E_1 = \sqrt{\mathbf{p}_2^2 + m_2^2} - \sqrt{\mathbf{p}_1^2 + m_1^2} \cong \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E} \quad \text{using } E \simeq E_i \simeq |\mathbf{p}_i|$$

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Thus the oscillation probability becomes ( $T \cong L$ )

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$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

## Non-zero oscillation probability requires

- non-zero mass squared difference  $\Delta m_{21}^2$ 
  - there is no flavour change for **vanishing or degenerate masses**
- 
-

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## Non-zero oscillation probability requires

- non-zero mass squared difference  $\Delta m_{21}^2$ 
  - there is no flavour change for **vanishing or degenerate masses**
- non-zero mixing angle  $\theta$ , i.e. neutrino flavour eigenstates cannot be and mass eigenstates
- a finite distance  $L$  between source and detector

Maximum flavour conversion probability may be reached for **maximal mixing**  $\theta = \frac{\pi}{4}$

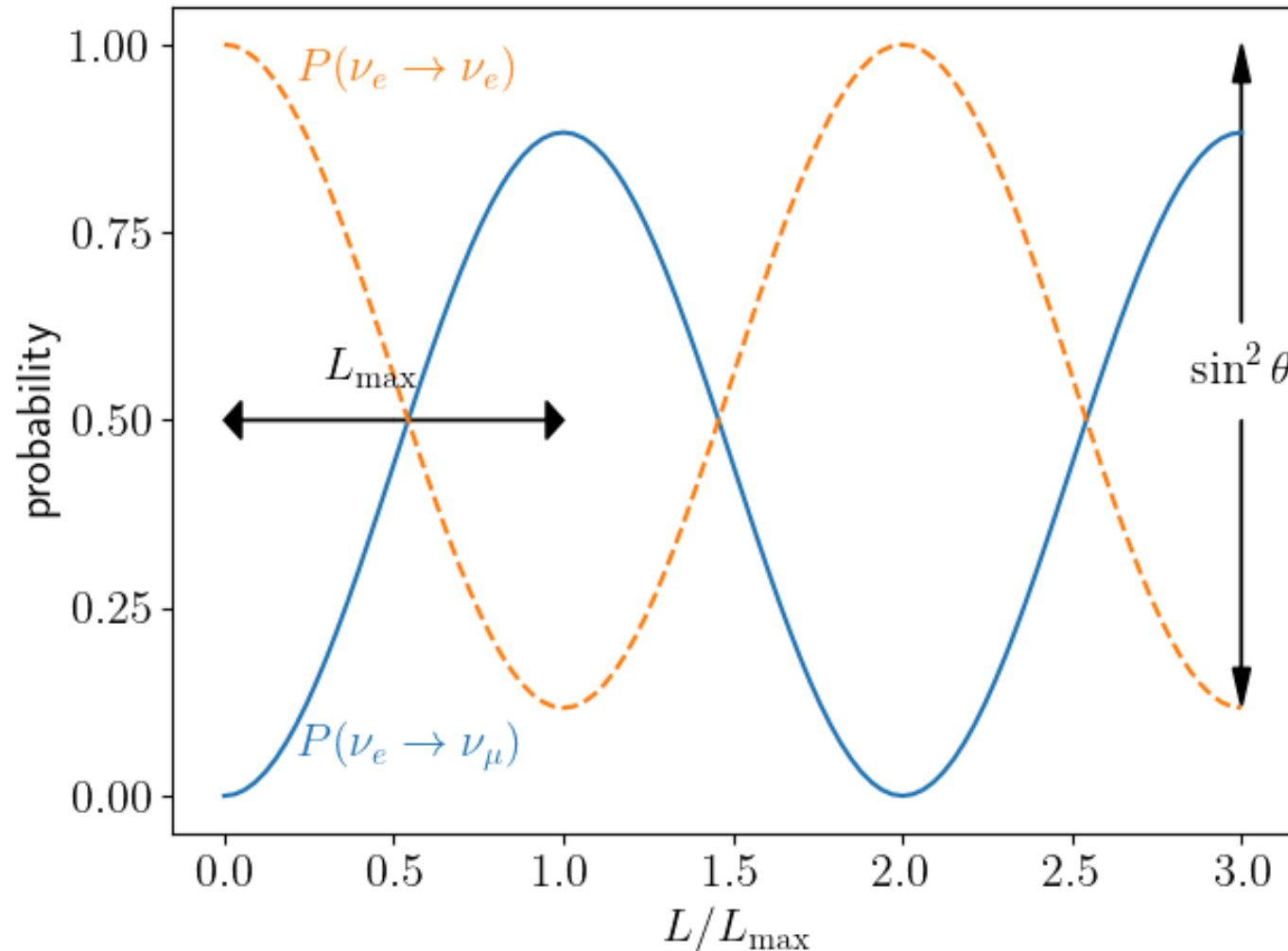
Conservation of probability implies  $P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) = 1$  and thus

$$P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_\mu) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Similar expressions are obtained for the other oscillation probabilities with  $\nu_e \leftrightarrow \nu_\mu$



# Two-flavour oscillations



- oscillation probability

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- 1st oscillation maximum

$$L_{\text{max}} = \frac{2\pi E}{\Delta m_{21}^2}$$

- maximum osc probability

$$P_{\text{max}} = \sin^2(2\theta)$$

# Two-flavour oscillations

- degeneracies

$$\Delta m_{21}^2 \rightarrow -\Delta m_{21}^2 \quad \text{and} \quad \theta \rightarrow \frac{\pi}{2} - \theta$$

## oscillation probability

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- antineutrinos

$$|\bar{\nu}_{Le}\rangle = c_\theta e^{-\frac{i}{2}\alpha} |\bar{\nu}_{L1}\rangle + s_\theta |\bar{\nu}_{L2}\rangle$$

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$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$$

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- (Total) lepton number is preserved in neutrino oscillations

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- if oscillation length < position uncertainty in source/detector

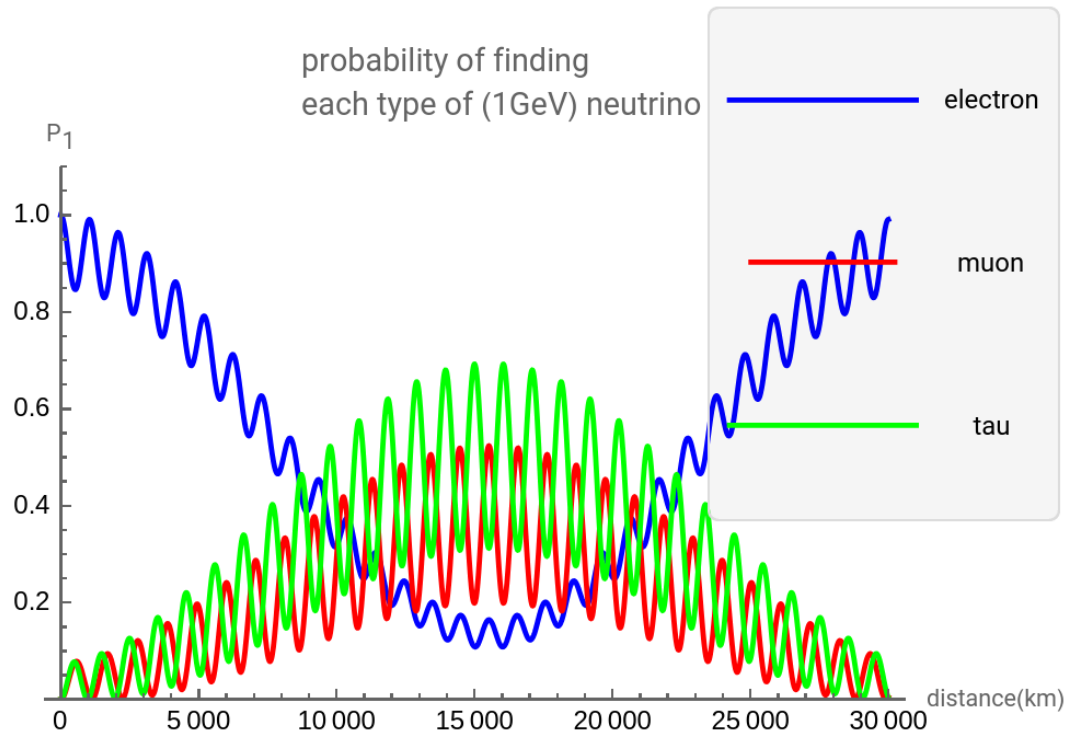
consider **average probability**

$$P(\nu_e \rightarrow \nu_\mu) = \frac{1}{2} \sin^2(2\theta)$$

# Three-flavour oscillations in vacuum

The **three-flavour neutrino oscillation probability** can be compactly written as

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) - 2 \sum_{i < j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{2E} \right)$$



[Wolfram neutrino oscillations demonstration]



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- first line same for  $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$
-

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- first line same for  $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$
- third term describes **CP violation**

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) - P(\nu_\alpha \rightarrow \nu_\beta) \\ = 4 \sum_{i < j} \operatorname{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{2E} \right)$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) - 2 \sum_{i < j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{2E} \right)$$

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- no CP violation if  $\alpha = \beta$

**CP violation in lepton sector can be much larger than in quark sector!**

$$J_{\text{CP}}^{\text{quark}} = (3.12_{-0.12}^{+0.13}) \cdot 10^{-5}$$

$$J_{\text{CP}}^{\text{lepton}} = (3.31 - 3.35) \cdot 10^{-2} \sin(\delta)$$

**Jarlskog invariant**

$$J_{\text{CP}} = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin(2\theta_{13}) \cos(\theta_{13}) \sin(\delta)$$

# Matter effect

- Effective vacuum Hamiltonian is diagonal in mass basis

$$H_{\text{vac}}^{(m)} = U_{\text{PMNS}}^\dagger H_{\text{vac}}^{(f)} U_{\text{PMNS}} = \frac{M_\nu^\dagger M_\nu}{2E} \equiv \frac{M^2}{2E}$$

- Electroweak interactions generate an effective potential for neutrinos
- Hamiltonian in flavour basis is

$$H^{(f)} = H_{\text{vac}}^{(f)} + V^{(f)}$$

- effective potential from NC interactions is flavour universal

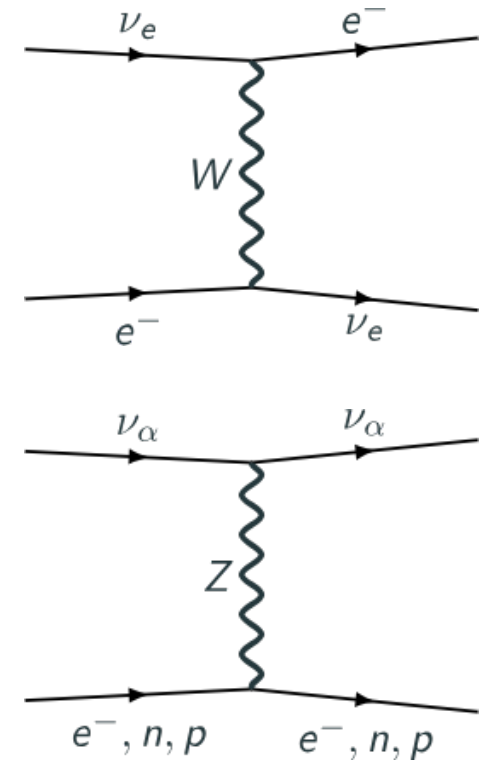
→ no effect unless there are light sterile  $\nu$ 's

- matter potential due to electron density  $n_e$

$$V_{ee}^{(f)} = \pm \sqrt{2} G_F n_{e(x)}$$

- vacuum mass eigenstates are not eigenstates of Hamiltonian in matter

→ need to find mass eigenstates for neutrinos in matter



# Matter effect

- find mass eigenstates of neutrino Hamiltonian in matter

$$H^{(f)} = H_{\text{vac}}^{(f)} + V^{(f)} \quad \text{with} \quad V_{ee}^{(f)} = \pm \sqrt{2} G_F n_{e(x)}$$

- define mixing matrix  $\tilde{U}$  with

$$\nu^{(f)}(x) = \tilde{U}(x) \tilde{\nu}^{(m)}(x) \quad \text{and} \quad H^{(f)} = \frac{1}{2E} \tilde{U}(x) \tilde{M}^2 \tilde{U}^\dagger(x)$$

# Matter effect

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- neutrino propagation is described by

$$i \frac{d\tilde{\nu}^{(m)}}{dx} = \left[ \frac{\tilde{M}^2}{2E} - i \tilde{U}^\dagger(x) \frac{d\tilde{U}(x)}{dx} \right] \tilde{\nu}^{(m)}(x)$$

- the second term describes the change of mass eigenstates. It can be neglected, if the matter potential is slowly varying  $L = \frac{4\pi E}{\Delta \tilde{M}^2} \ll \left( \frac{d \ln n_{e(x)}}{dx} \right)^{-1}$ .
- This is called *adiabatic approximation*. In this approximation, we observe unitary evolution and the magnitude of the mass eigenstates remains unchanged.

# Matter effect

- in the adiabatic approximation neutrino propagation is described by

$$i \frac{d\tilde{\nu}^{(m)}}{dx} = \frac{\tilde{M}^2}{2E} \tilde{\nu}^{(m)}(x)$$

- Effective neutrino mixing angle and mass squared difference in matter

$$\sin(2\tilde{\theta}) = \frac{\sin(2\theta)}{\sqrt{\sin^2(2\theta) + C^2}} \quad \text{and} \quad \Delta\tilde{m}^2 = \Delta m^2 \sqrt{\sin^2(2\theta) + C^2}$$

with the parameter  $C(x) = \cos(2\theta) - \frac{2\sqrt{2}G_F n_e(x)E}{\Delta m^2}$

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with the parameter  $C(x) = \cos(2\theta) - \frac{2\sqrt{2}G_F n_e(x)E}{\Delta m^2}$

- Matter effect breaks degeneracies

$$\Delta m^2 \rightarrow -\Delta m^2 \quad \text{and} \quad \theta \rightarrow \frac{\pi}{2} - \theta$$



Effective neutrino mixing angle and mass squared difference in matter

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with the parameter  $C(x) = \cos(2\theta) - \frac{2\sqrt{2}G_F n_e(x)E}{\Delta m^2}$

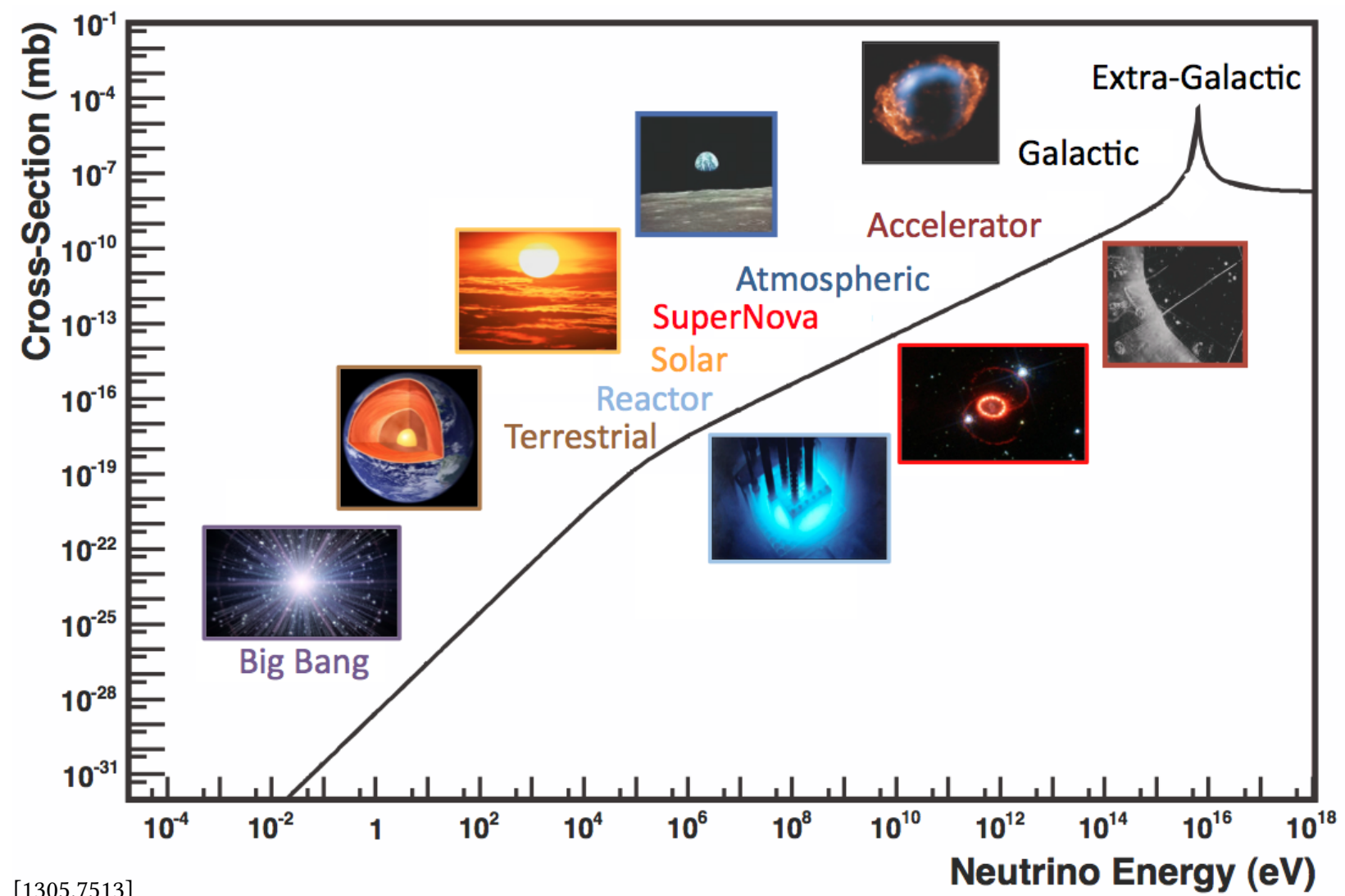
Mikheyev-Smirnov-Wolfenstein (MSW) resonance occurs for  $C(x) = 0$  or equivalently

$$\Delta m^2 \cos(2\theta) = 2\sqrt{2}G_F n_e(x)E$$

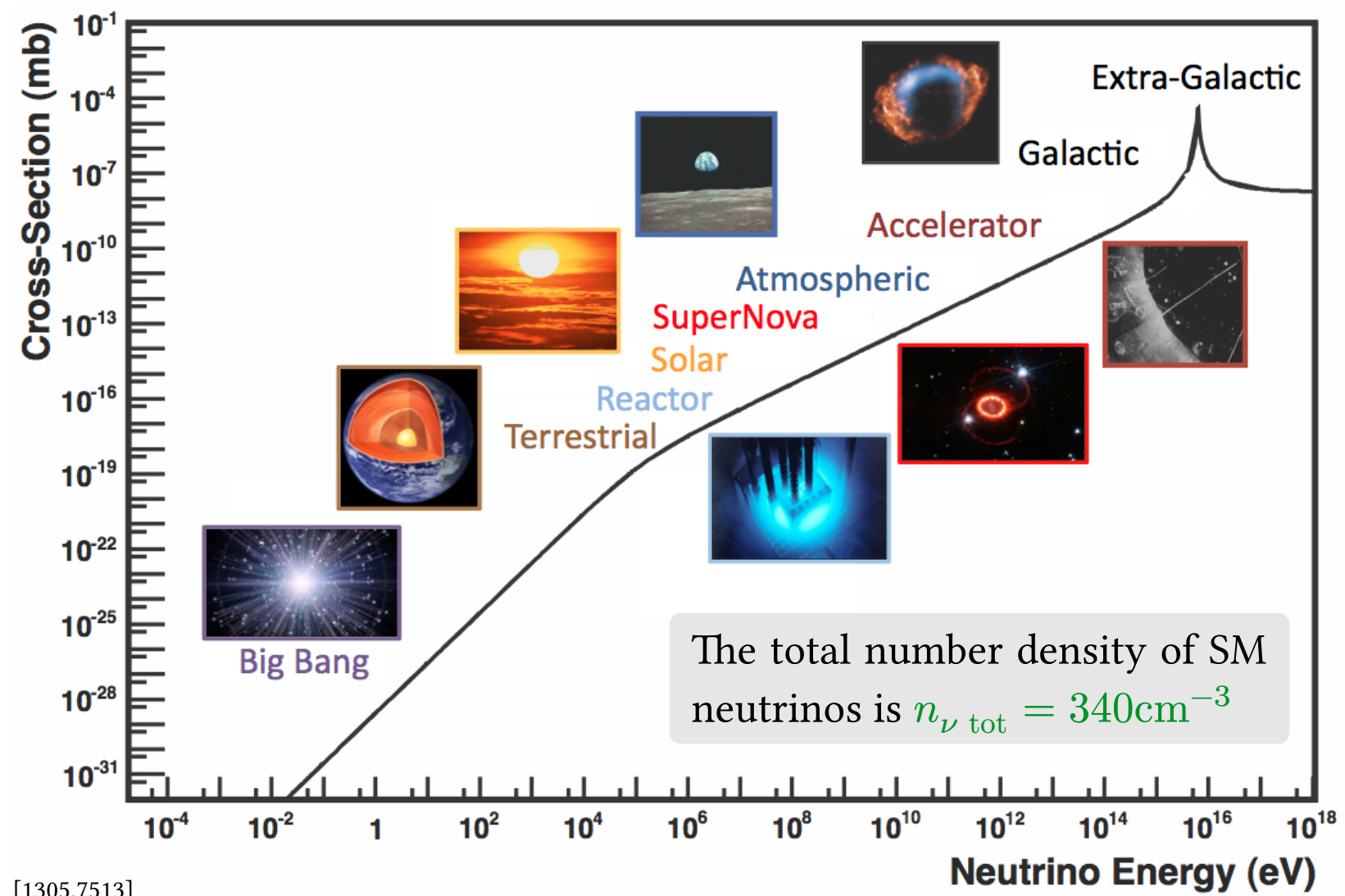
In this case the effective neutrino mixing angle becomes maximal  $\sin(2\tilde{\theta}) = 1$  and thus oscillations are maximally efficient.

# Experimental overview

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[1305.7513]



[1305.7513]

# Brief history

1920s Ellis: beta decay spectrum is continuous

1959 Cowan-Reines neutrino experiment:  $\nu$  detection 🏆 1995

1962 Lederman, Schwartz, Steinberger:  $\nu_\mu$  detection 🏆 1988

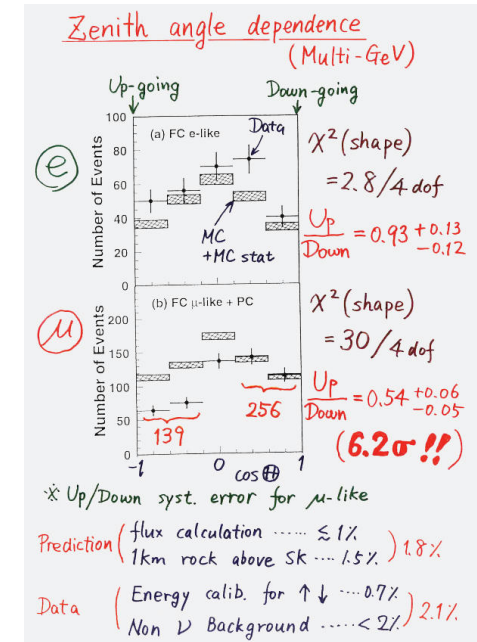
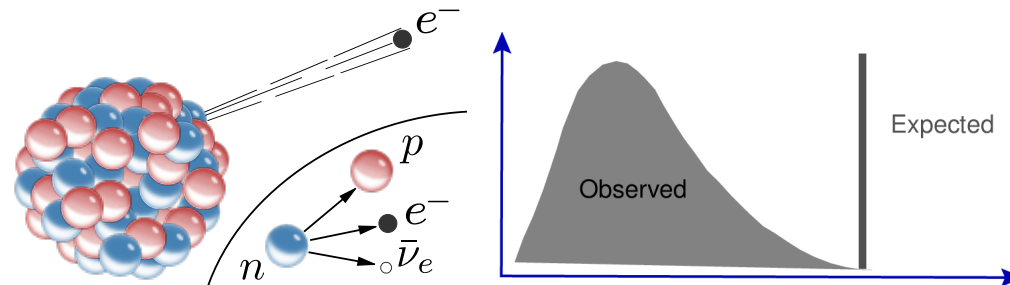
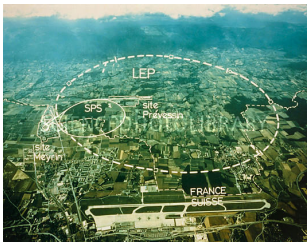
1989 Large Electron Positron collider: Z decay width  $\Rightarrow N_\nu = 2.984 \pm 0.008$

1998 Super-Kamiokande: atmospheric neutrino oscillations 🏆 2015

2001 SNO: solar neutrino oscillations 🏆 2015

2012 Daya Bay, RENO, Double CHOOZ:  $\theta_{13}$

2017 COHERENT:  $\text{CE}\nu\text{NS}$



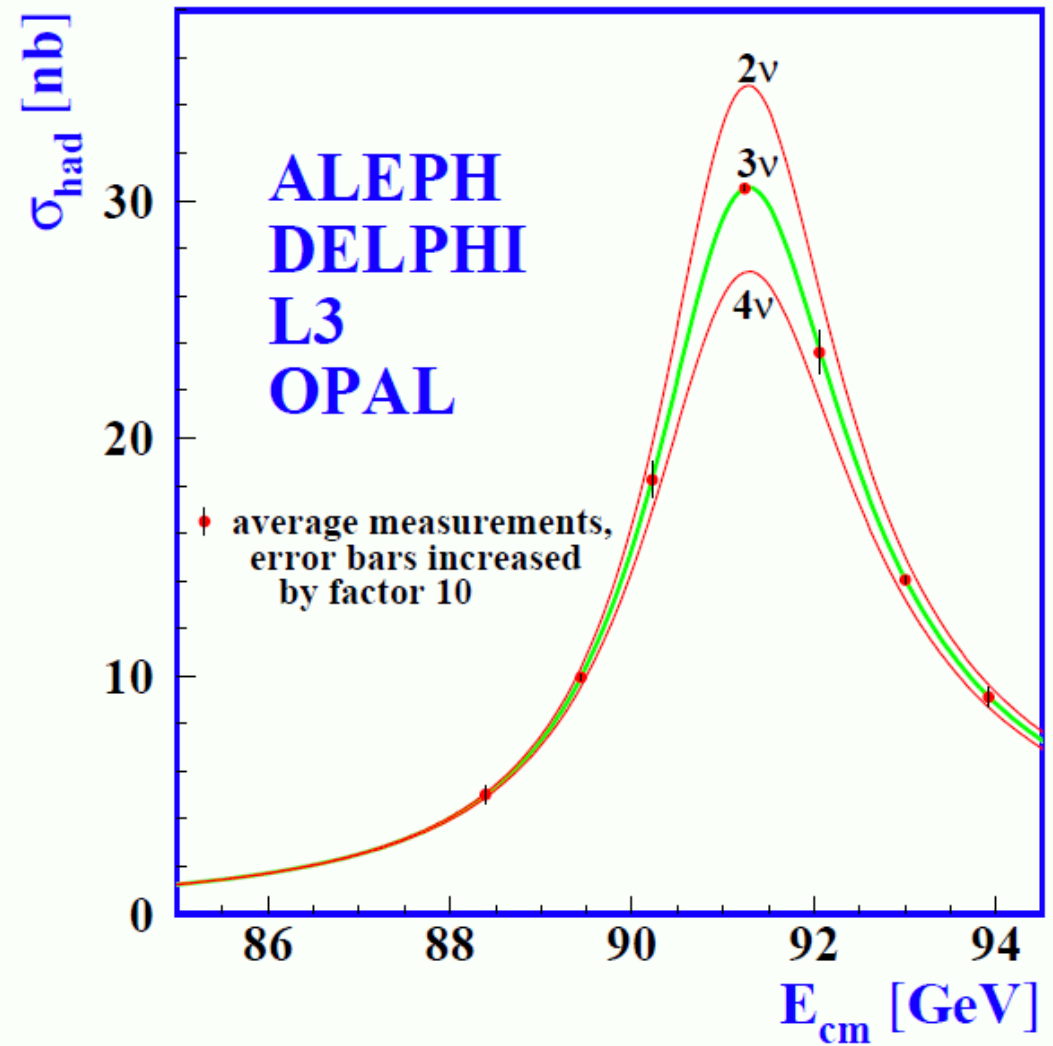
# Number of neutrinos

SM neutrinos couple to the  $Z$  boson and thus  $Z \rightarrow \nu_\alpha \bar{\nu}_\alpha$  with  $\alpha = e, \mu, \tau$ .

Experimental proof of the **existence of three light active neutrinos** with mass less than half of the  $Z$  boson mass has been obtained by precisely measuring  $Z$  boson decays.

LEP inferred the invisible partial width  $\Gamma(Z \rightarrow \text{inv})$  of the  $Z$  boson by precisely measuring the total width and in hadronic decays. The combination of four LEP experiments yields

$$N_\nu = 2.984 \pm 0.008$$



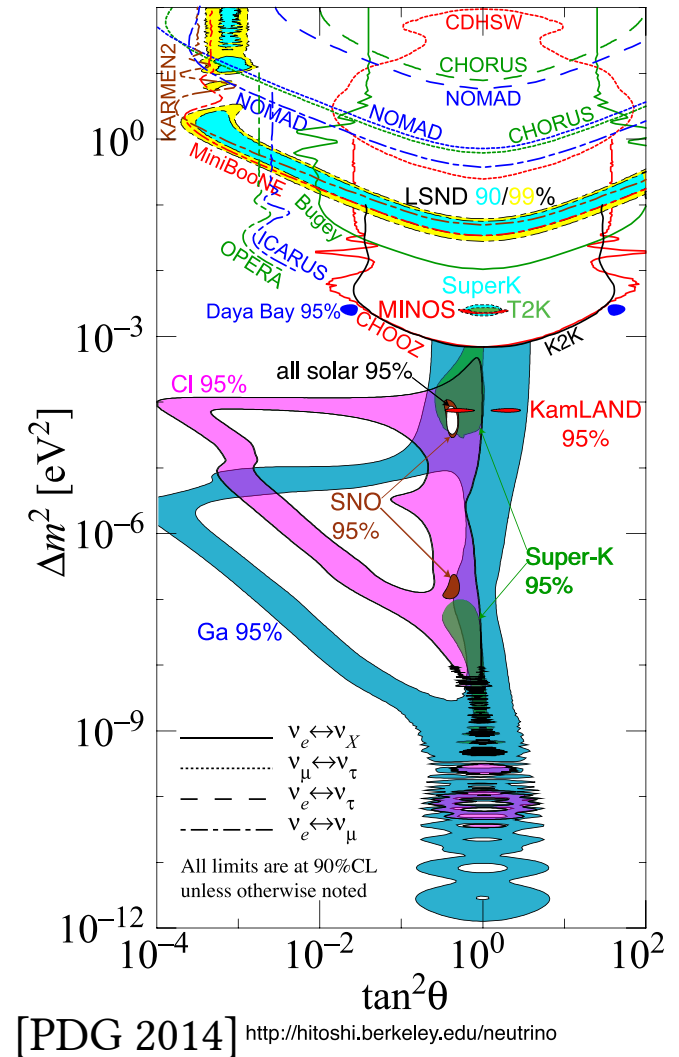
# Neutrino oscillations

There are many neutrino oscillation experiments as illustrated in the figure on the right which captures the situation in 2014, more than 10 years ago. I will not discuss individual neutrino oscillation experiments. The following is a review of the lepton mixing parameters and mass squared differences of a **global fit** to neutrino oscillation data.

There are three research groups which perform these fits

- **IFIC group** (P.F. de Salas, D.V. Forero, C.A. Ternes, M. Tortola, J.W.F. Valle, S. Gariazzo, P. Parinez-Mirave, O. Mena)
- **Bari group** (F. Capozzi, E. Lisi, A. Marrone, A. Palazzo)
- **Spain/USA-Germany group** (I. Esteban, C. Gonzalez Garcia, M. Maltoni, T. Schwetz, A. Zhou)

The latest results from the third group which the following is based on can be accessed via <http://www.nu-fit.org>.



NuFIT 5.3 (2024)

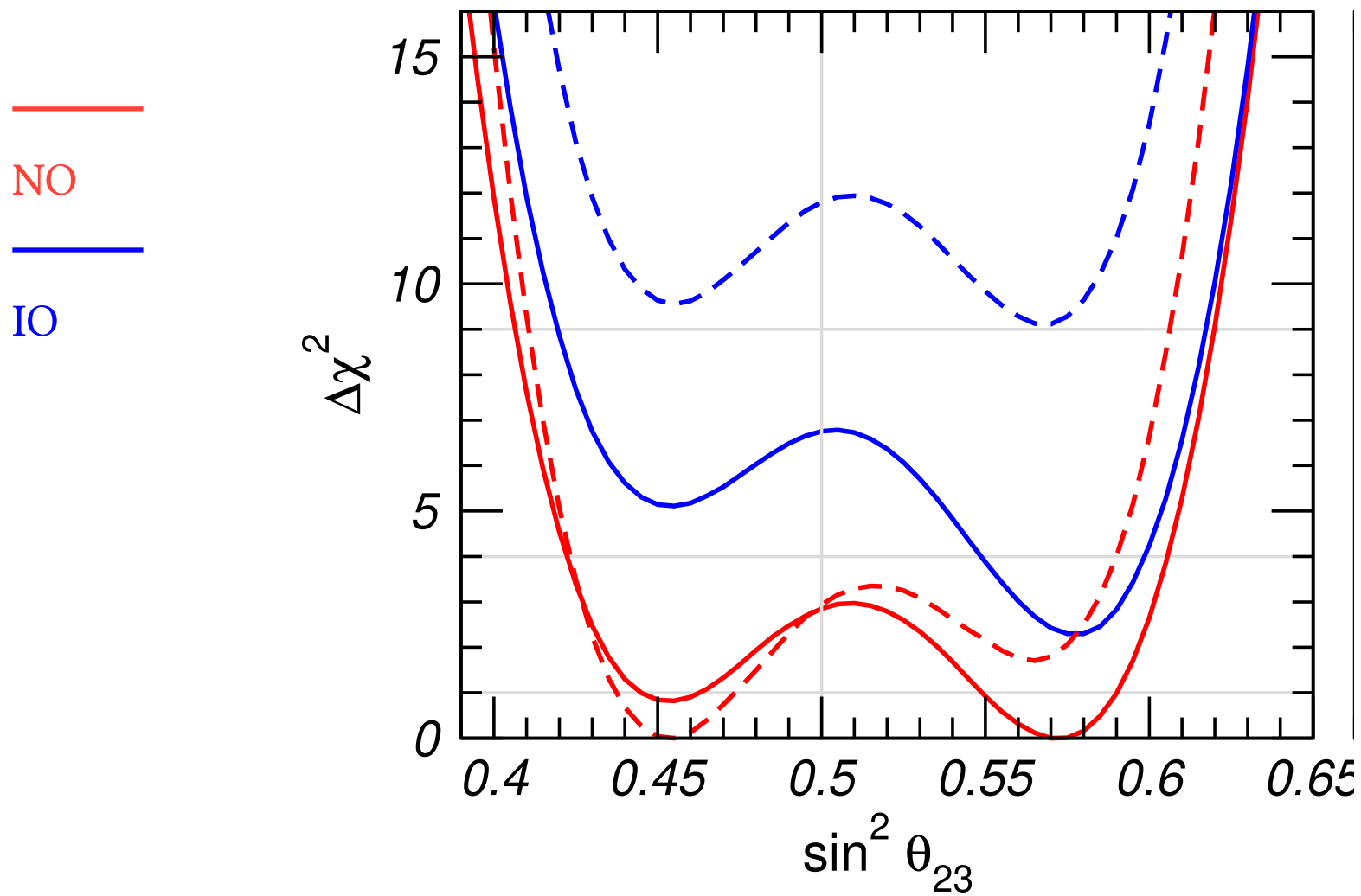
without SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 2.3$ )	
		bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range
	$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.344$
	$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \rightarrow 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \rightarrow 35.94$
	$\sin^2 \theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \rightarrow 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \rightarrow 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \rightarrow 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \rightarrow 52.1$
	$\sin^2 \theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \rightarrow 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \rightarrow 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \rightarrow 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \rightarrow 8.90$
	$\delta_{\text{CP}}/^\circ$	$197^{+41}_{-25}$	$108 \rightarrow 404$	$286^{+27}_{-32}$	$192 \rightarrow 360$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \rightarrow 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \rightarrow 8.03$
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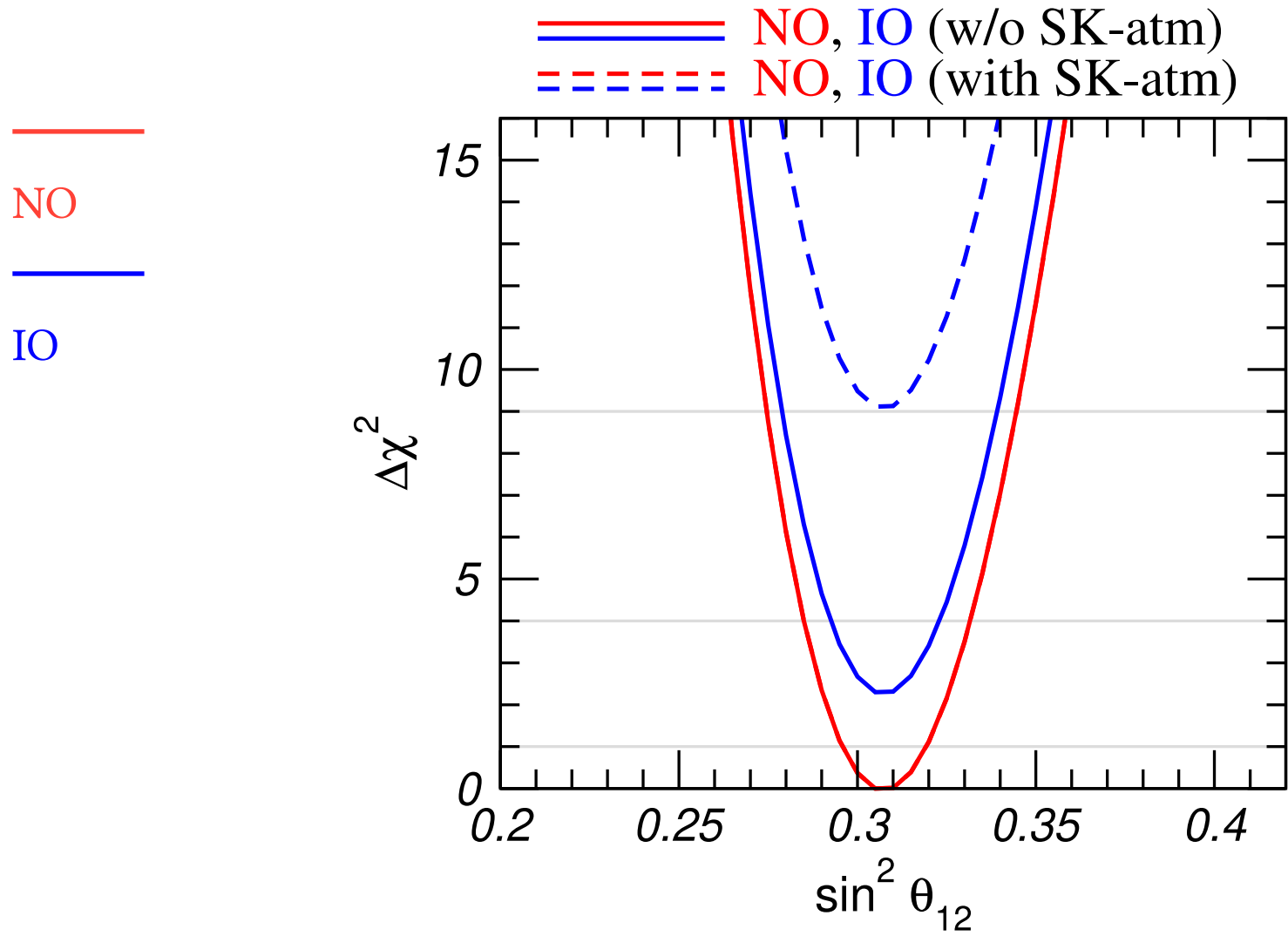
Atmospheric mixing  $\theta_{23}$  is the largest and can be maximal  $45^\circ$



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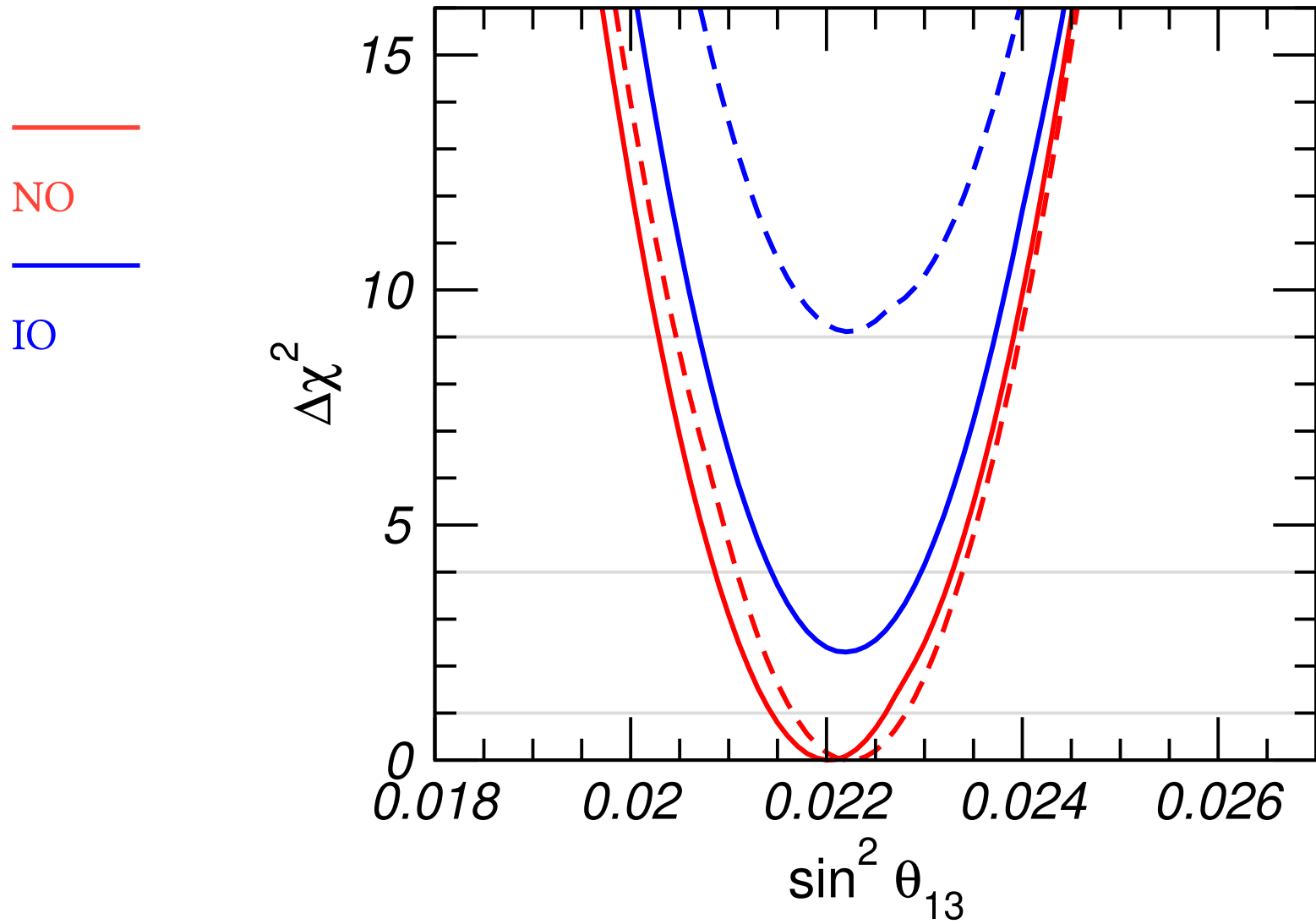
Solar mixing  $\theta_{12}$  is the second largest but cannot be maximal



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Reactor mixing  $\theta_{13}$  has been best measured and is of the size of the Cabibbo angle

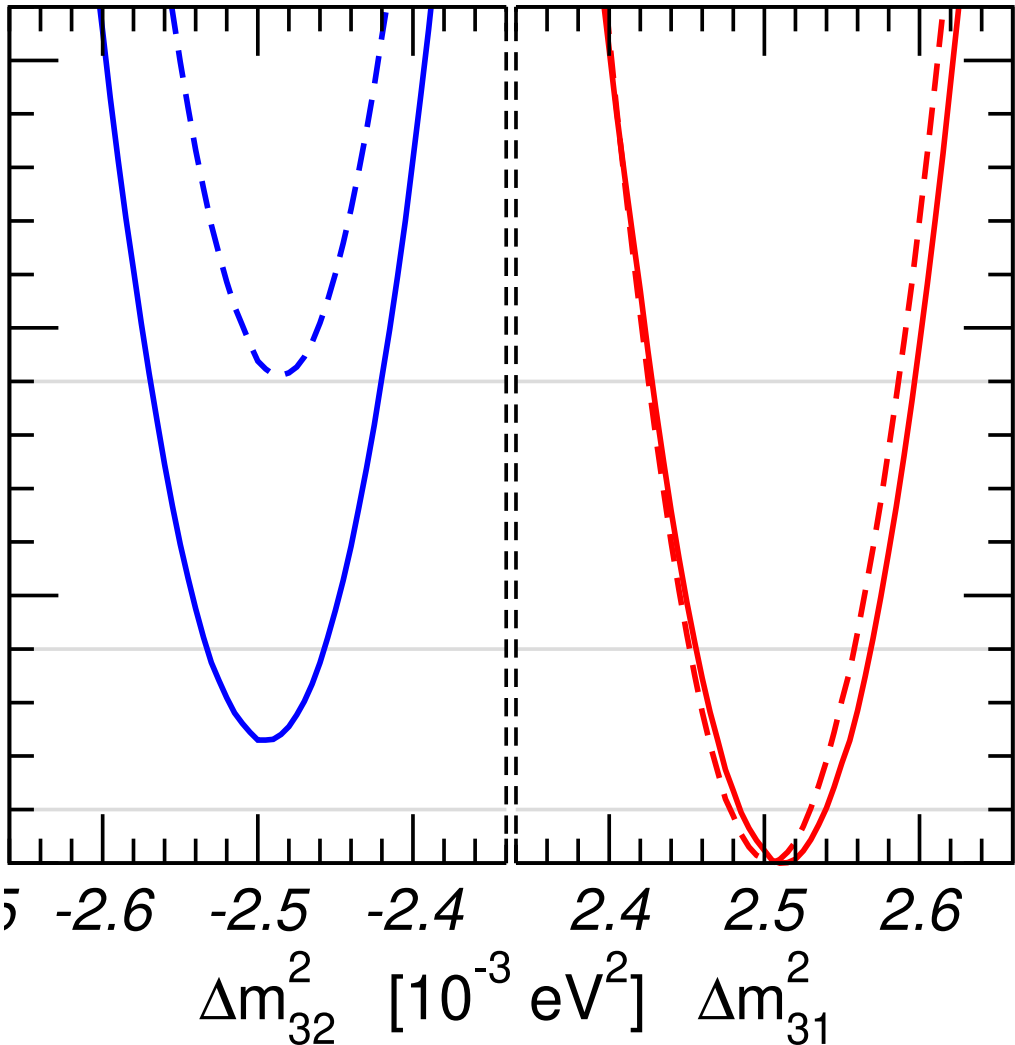
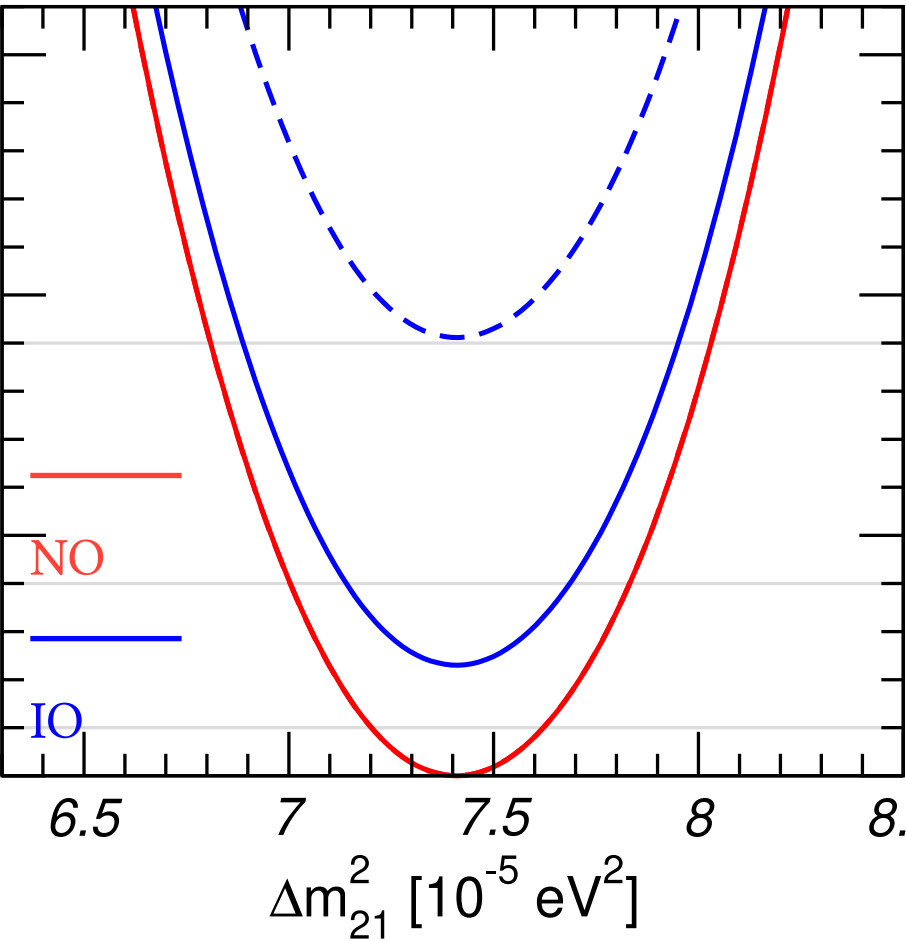


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Mass ordering, i.e. sign of  $\Delta m_{3i}^2$  is one of the aims of DUNE and Hyper-K.

NuFIT 5.3 (2024)

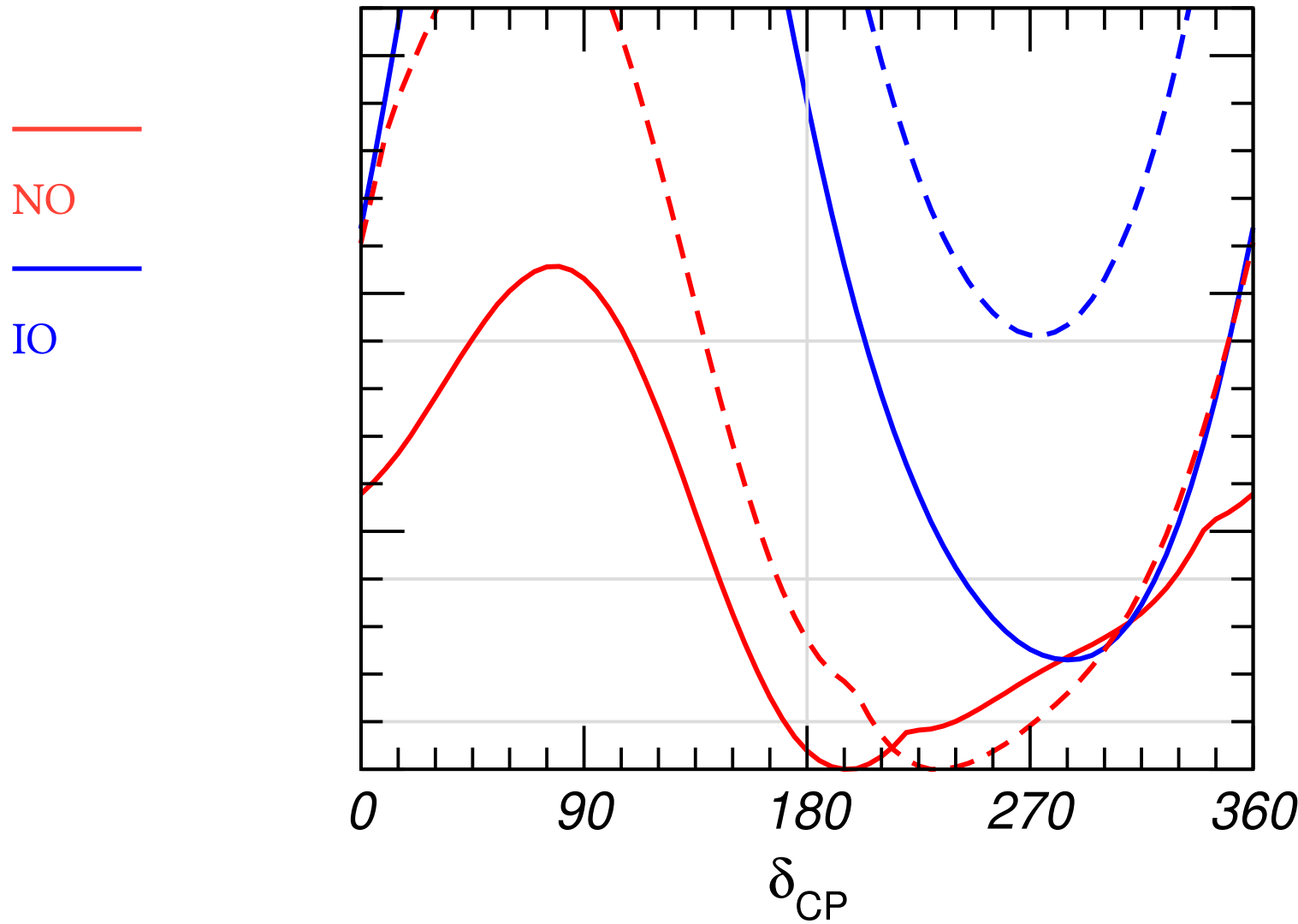


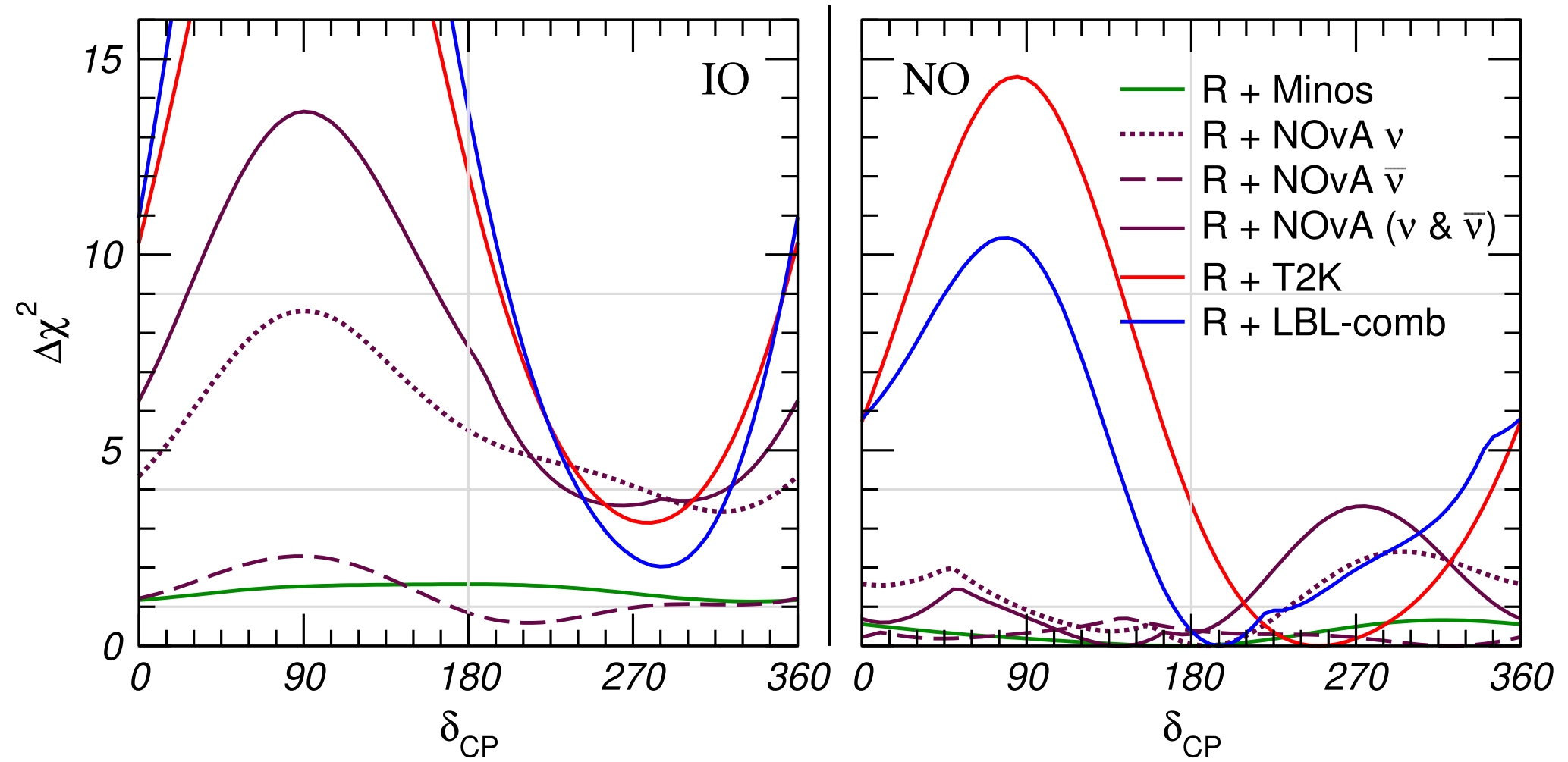


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Dirac phase  $\delta$  has not been measured yet. It may be large

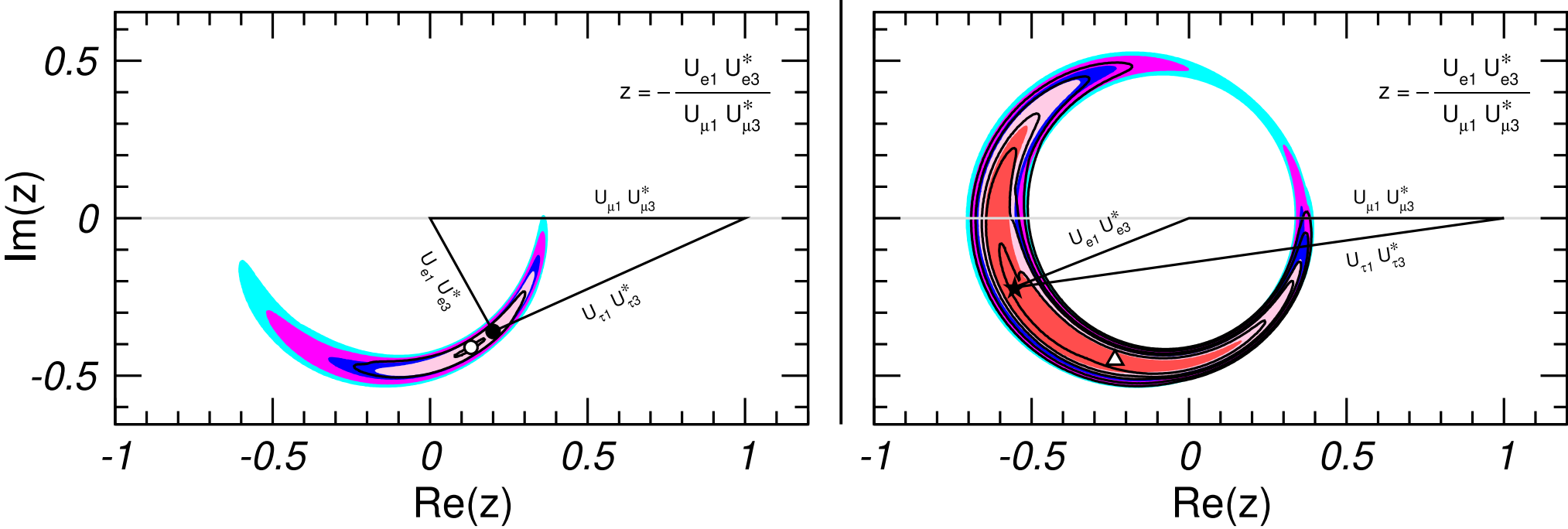




NuFIT 5.3 (2024)

IO

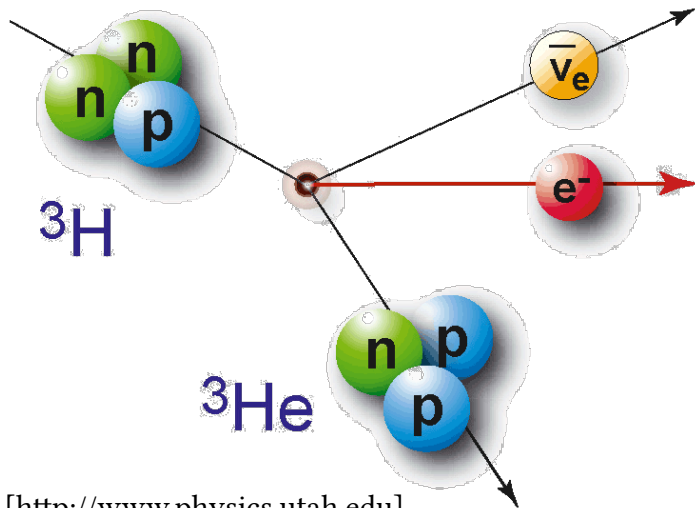
NO



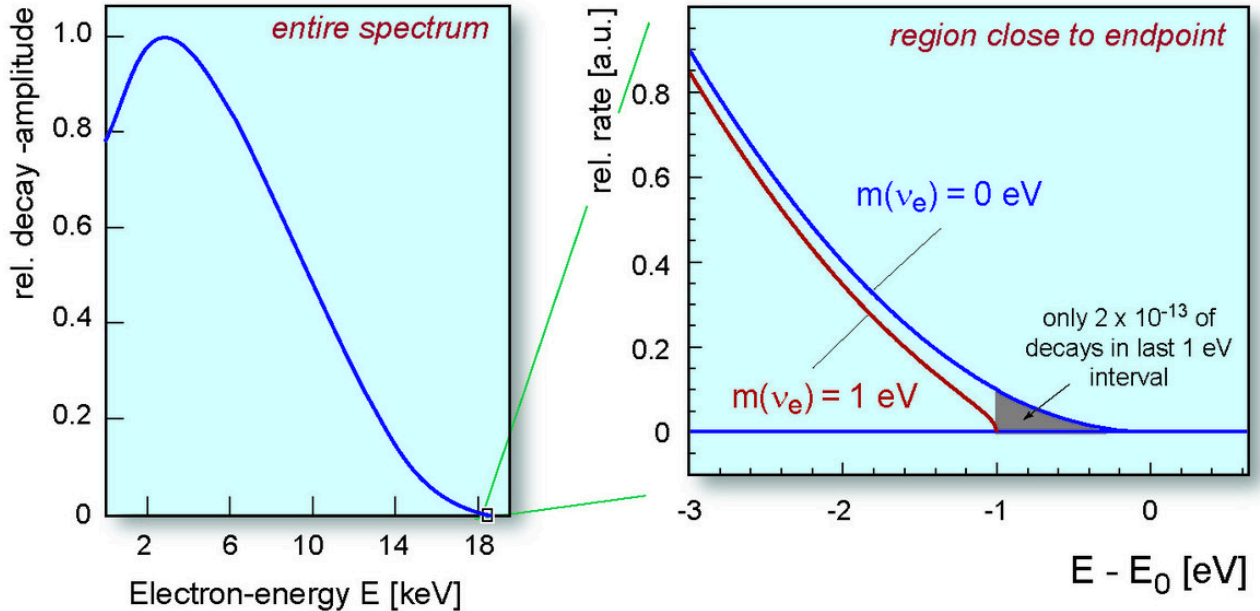
Area of the unitary triangle is a measure of CP violation.

# Tritium beta decay

A model-independent way to measure the absolute neutrino mass scale is to use the kinematics of beta decay, in particular  **$\beta$  decay of tritium** is well suited due to its **low  $Q$  value** of  $Q_\beta = 18.6$  keV.



[<http://www.physics.utah.edu>]

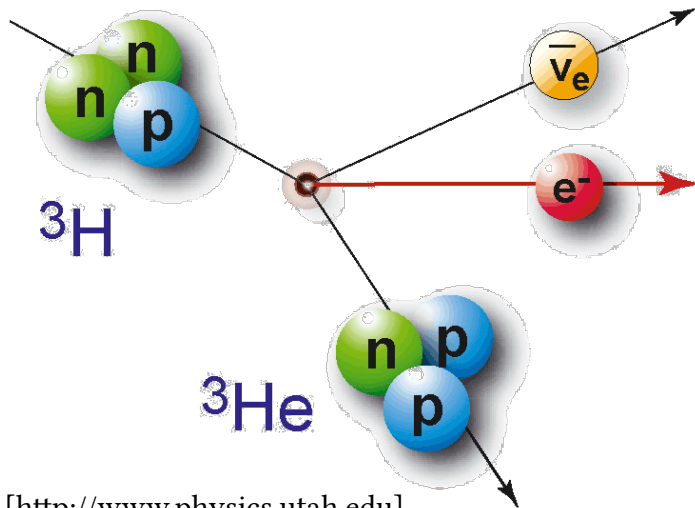


If energy resolution worse than mass splitting

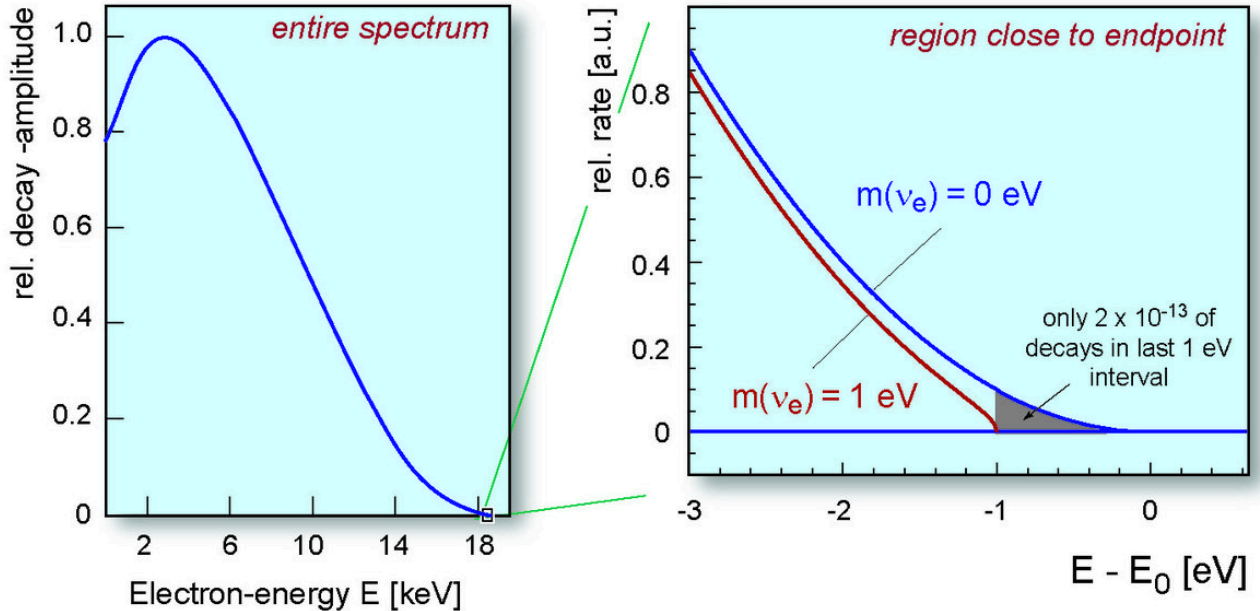
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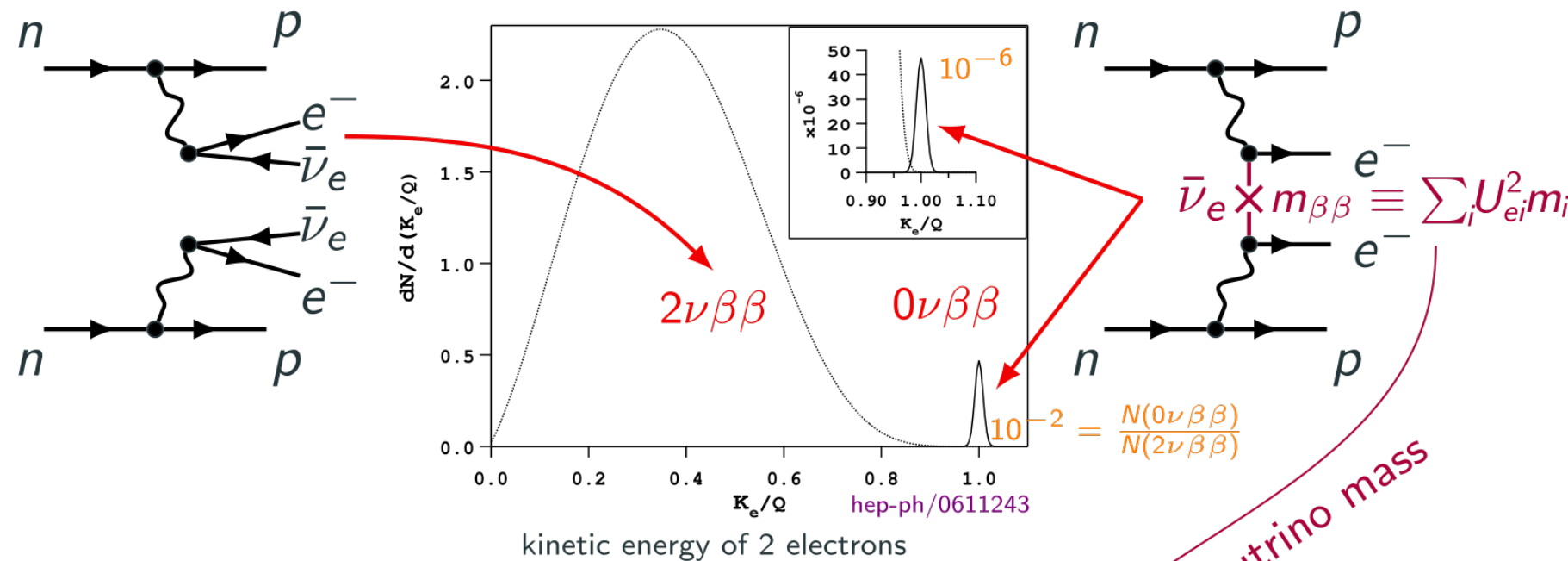
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If energy resolution worse than mass splitting

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**KATRIN experiment:**  $\sqrt{\sum_j |U_{ej}|^2 m_j^2} < 0.45$  eV  
[2406.13516]



lifetime

nuclear physics

$$T_{1/2}^{0\nu} \propto \frac{1}{|M_{\text{nucl}}|^2} \frac{1}{\langle m_{\beta\beta} \rangle^2}$$

effective neutrino mass

- Neutrinoless double beta ( $0\nu 2\beta$ ) decay violates lepton number  $\Delta L = 2$
- It only occurs for Majorana neutrinos and relates to the half life like

$$\frac{1}{T_{\frac{1}{2}}^{0\nu}} = \underbrace{G^{0\nu}}_{\text{phase space}} \underbrace{|M_{\text{nucl}}|^2}_{\text{NME}} |m_{\beta\beta}|^2$$

where  $|m_{\beta\beta}| = |c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{-2i\delta} m_3|$



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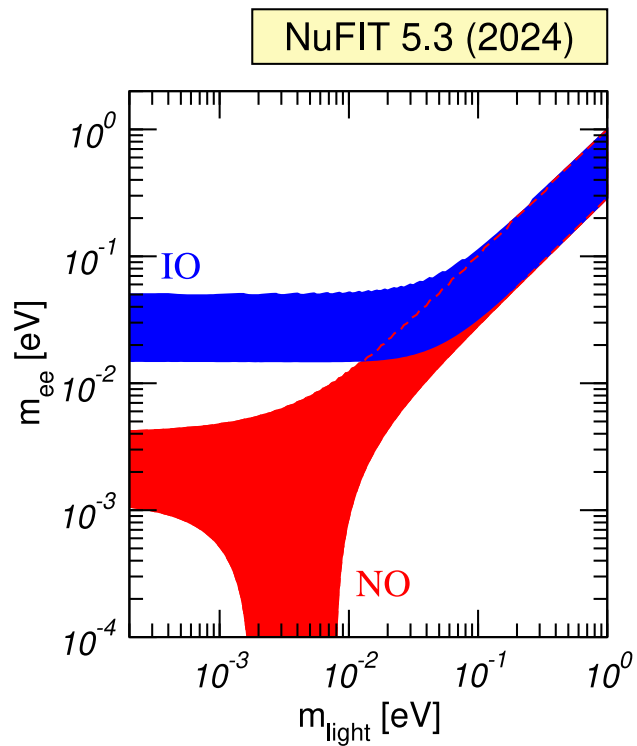
## Experimental limits at 90% CL

isotope	<sup>76</sup> Ge	<sup>100</sup> Mo	<sup>130</sup> Te	<sup>132</sup> Xe
$T_{\frac{1}{2}}^{0\nu} [10^{25} \text{ years}]$	19	0.3	3.8	38
experiment	LEGEND	AMoRE	CUORE	KamLAND-Zen

**Neutrino mass limit:**  
assuming light neutrino dominance

KamLAND-Zen provides the strongest limit

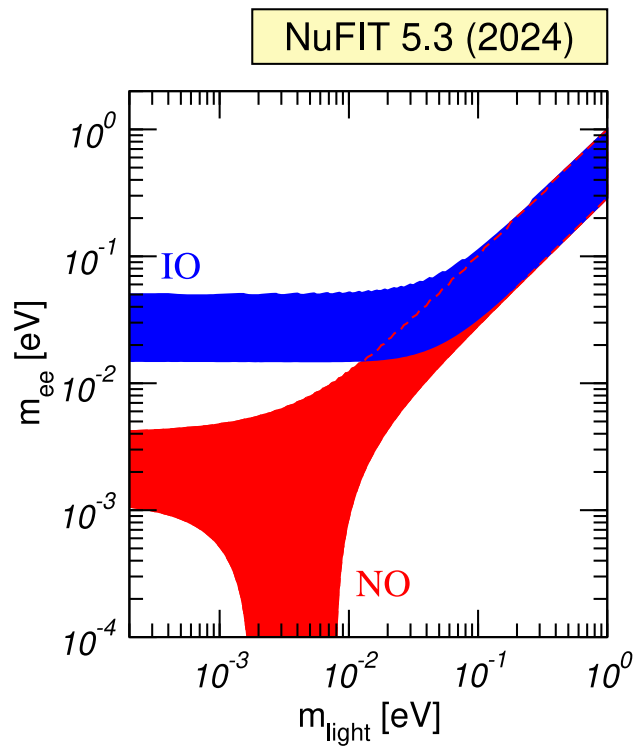
$$|m_{\beta\beta}| < 28 - 122 \text{ meV}$$



- future experiments probe  $m_{ee} \gtrsim 10^{-2}$  eV
- 
- 

- Cancellation possible for NO

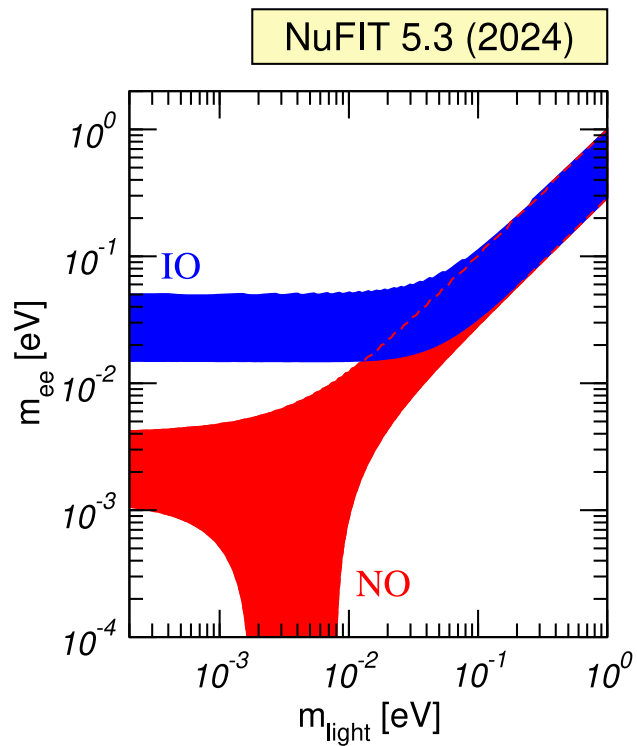
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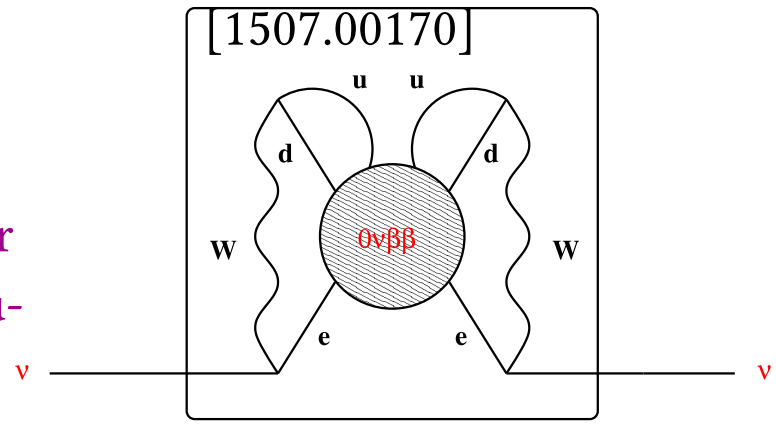
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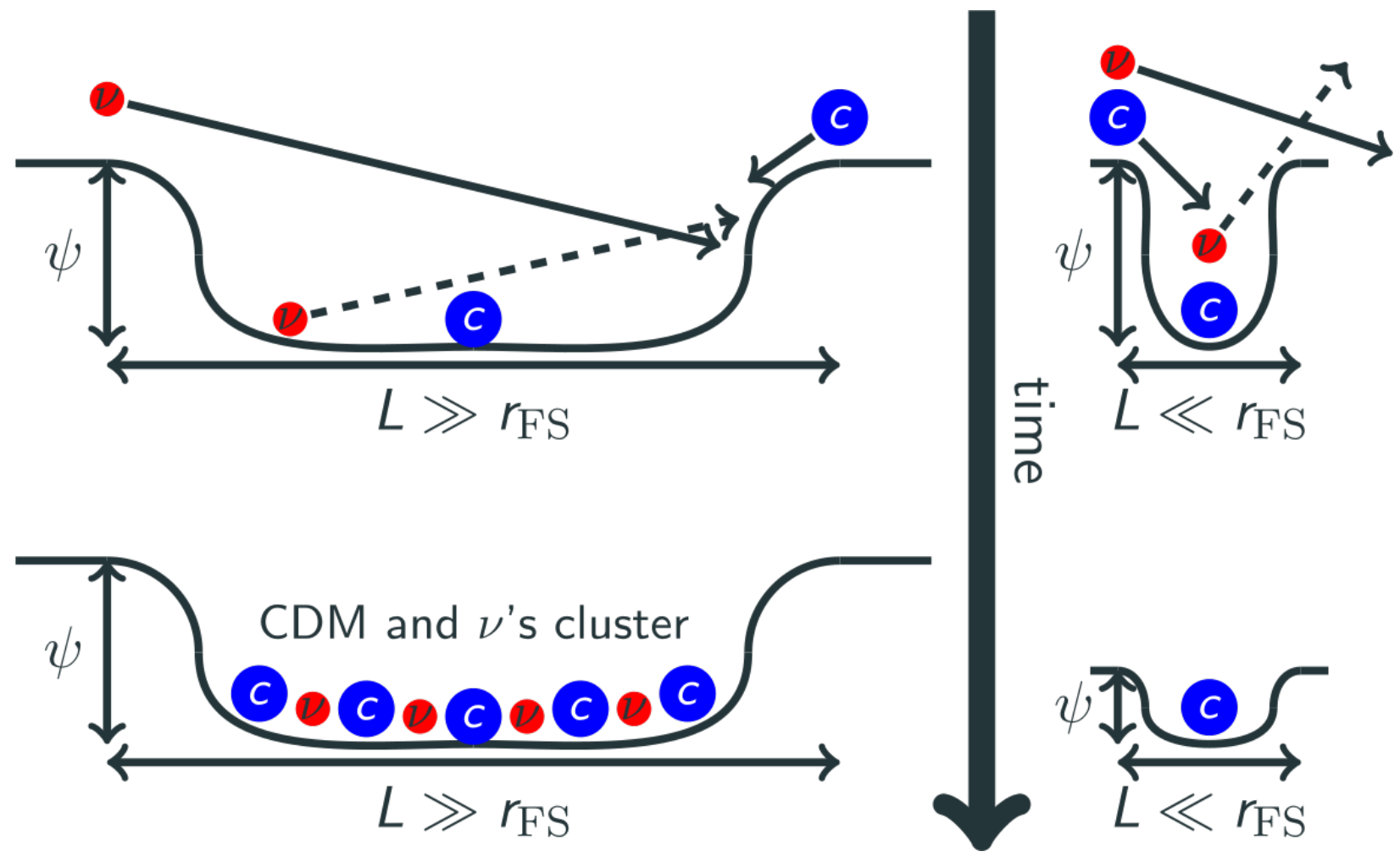
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- Note there may be other contributions to  $0\nu2\beta$  decay, e.g. SUSY neutralino, RH  $W$  boson exchange, ...
- **What does detection of  $0\nu2\beta$  decay imply for neutrino masses?**

Black-box theorem:  
(Schechter-Valle)

- neutrinos are Majorana
- Black-box theorem however only predicts a tiny neutrino mass tiny

$$m_\nu < 10^{-24} \text{ eV}$$

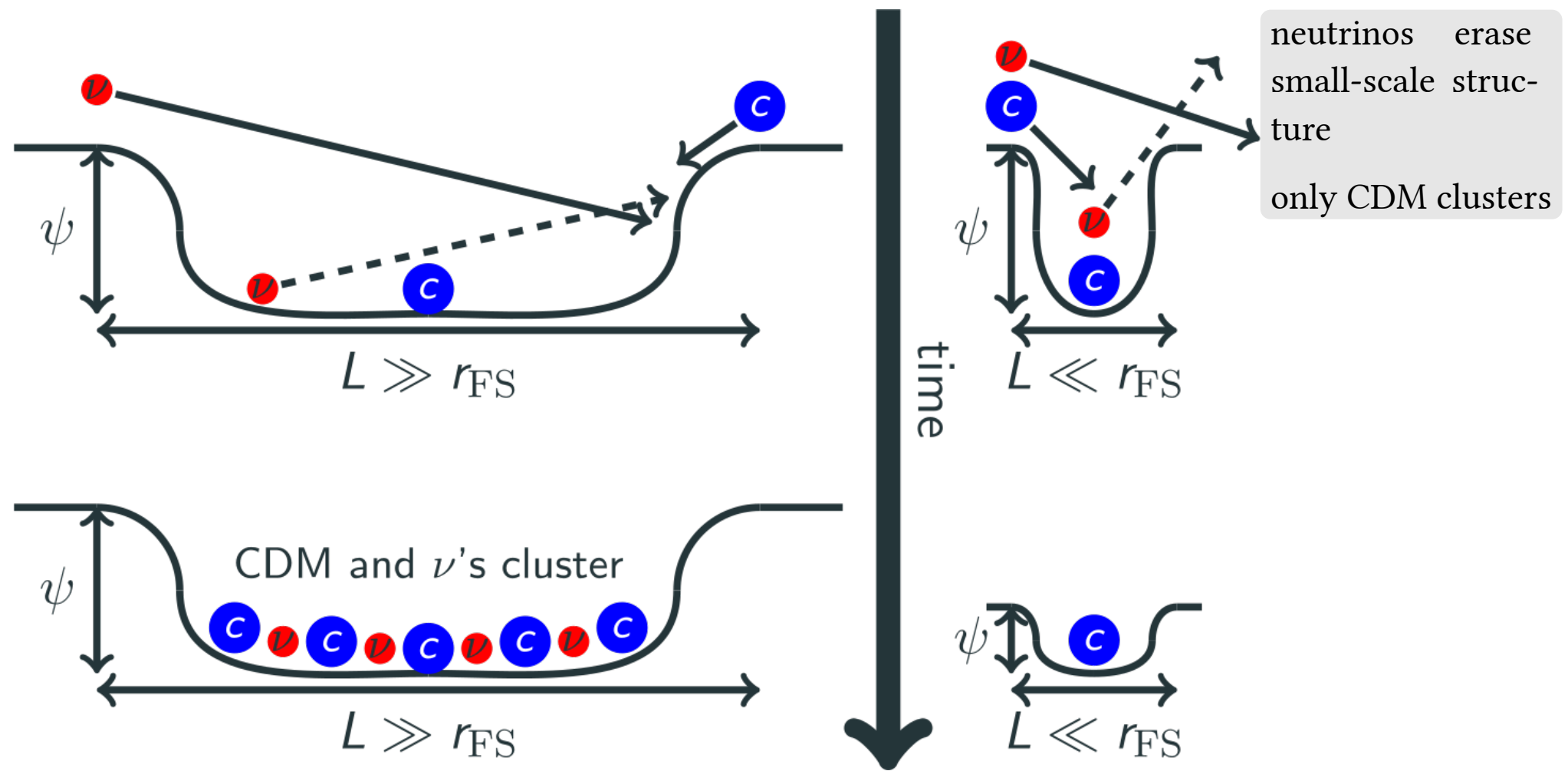




$L$  is size of astrophysical object

$r_{\text{FS}} = \int_{t_{\text{in}}}^{t_0} dt \frac{\langle v(t) \rangle}{a(t)}$  is free-streaming scale

# Neutrino masses affect cosmological structure formation Experimental overview



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Neutrinos have a large effect on cosmology: the formation of large scale structure (LSS) as explained on the previous slide, big bang nucleosynthesis and the Cosmic Microwave Background (CMB) provide constraints

The Planck experiment [1807.06209]

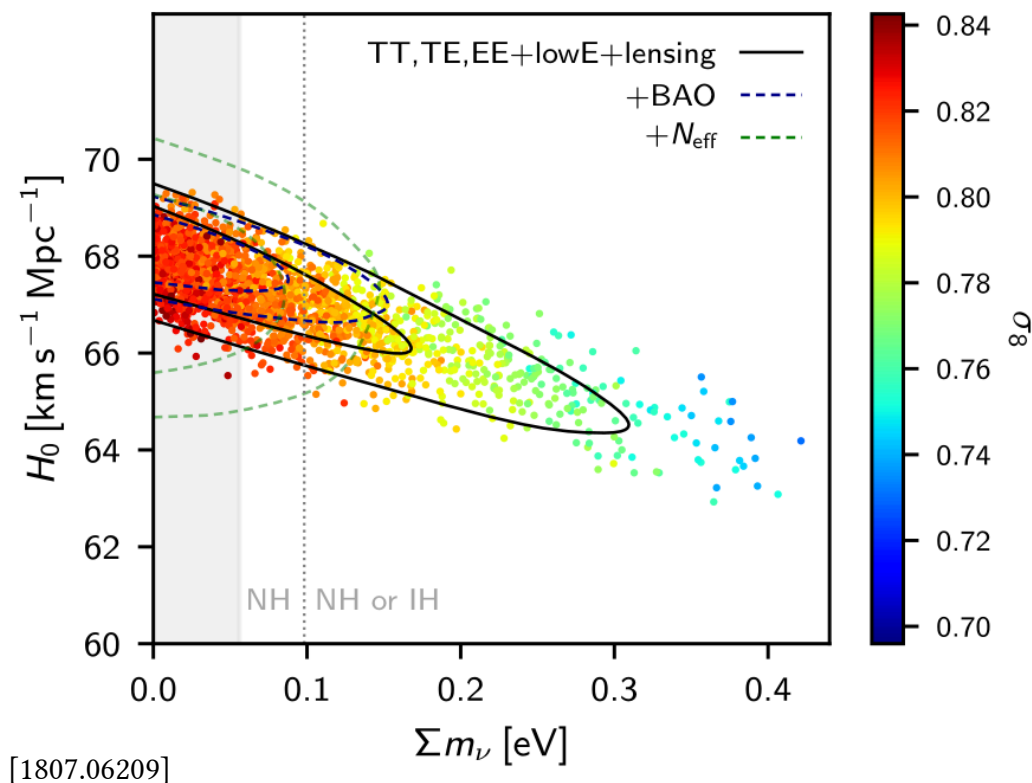
- measured the number of neutrinos

$$N_{\text{eff}} = 2.99 \pm 0.17$$

- placed an upper bound on the sum of neutrino masses

$$\sum m_\nu < 0.12 \text{ eV at 95\% CL}$$

- this puts pressure on inverted ordering



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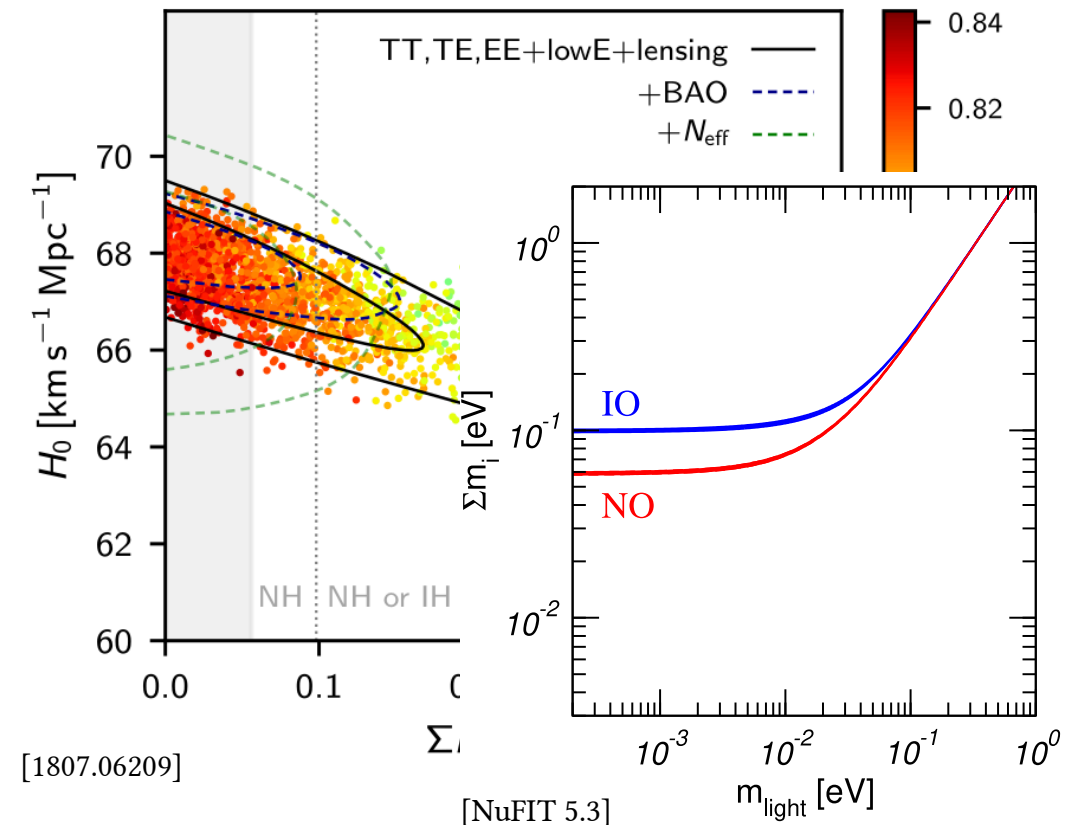
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# Open questions

Are neutrinos their own antiparticles?

What is the absolute neutrino mass scale?

Is there normal or inverted mass ordering?

Is there leptonic CP violation?

Anything beyond 3 neutrinos?



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Anything beyond 3 neutrinos?

Why are neutrinos so light?

Any explanation for the mixing angles?

