Introduction to Neutrino Mass Models

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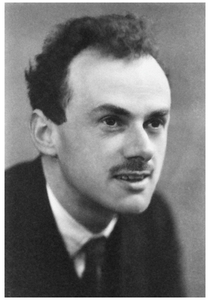
26 September 2024

Neutrino mass terms

Dirac neutrino masses



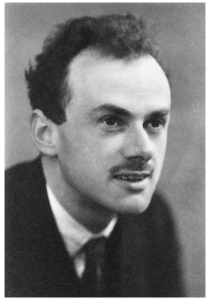
RH neutrinos allow to generate Dirac neutrino masses like for charged fermions

$$-\overline{L_L}\tilde{H}Y_\nu\nu_R + \text{h.c.} \xrightarrow{\langle H\rangle=\frac{1}{\sqrt{2}}\binom{0}{v}} -\overline{\nu_L}M_\nu^D\nu_R + \text{h.c.} \quad \text{with} \quad M_\nu^D = Y_\nu^\dagger\frac{v}{\sqrt{2}}$$

Bounds on the absolute neutrino mass scale:

- $m_{\nu_e} < 450$ meV (KATRIN)
- $|m_{ee}| < 28 - 122$ meV (KamLAND-Zen)
- $\sum_i m_i < 120$ meV (cosmology)

Dirac neutrino masses



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$$Y_\nu = \frac{\sqrt{2}m_\nu}{v} \ll 10^{-12} \ll y_e \ll 1$$

Yukawa couplings Y_ν are extremely tiny!

- Additional term allowed in Lagrangian: Majorana mass term for ν_R: $\nu_R^T C\nu_R$
  - implies Majorana neutrinos (see discussion below)
  - additional symmetry required to forbid term!
- Dirac neutrino masses are less “natural”

Majorana neutrinos

- SM Lagrangian features four accidental symmetries:
  1. baryon number (B),
  2. lepton flavour numbers ($L_{e,\mu,\tau}$)
- Only (total) lepton number $L = L_e + L_\mu + L_\tau$ may be conserved

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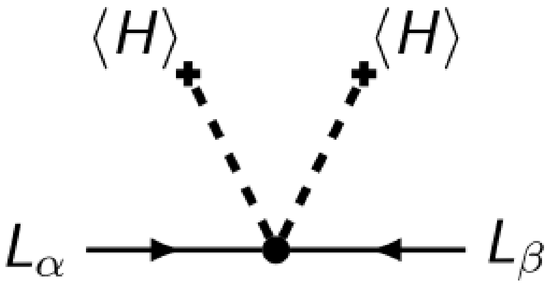
There is physics beyond the SM. If it is heavier than the electroweak scale it may be parameterised in terms of effective operators $\rightarrow$ Standard Model as an effective field theory (SMEFT)

Weinberg operator $\mathcal{L} \supset -\frac{1}{2}\kappa_{ij}(L_i \varepsilon H)^T C \left(L_j \varepsilon H\right) + \text{h.c.}$

- is the unique dimension-5 operator in SMEFT
- is symmetric, i.e. $\kappa_{ij} = \kappa_{ji}$
- is $\Delta L = 2$ and generates Majorana neutrino masses after EWSB



$$\left(M_\nu^M\right)_{ij} = \kappa_{ij}\frac{v^2}{2} \quad \text{with} \quad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

Tiny neutrino masses explained by high scale of operator

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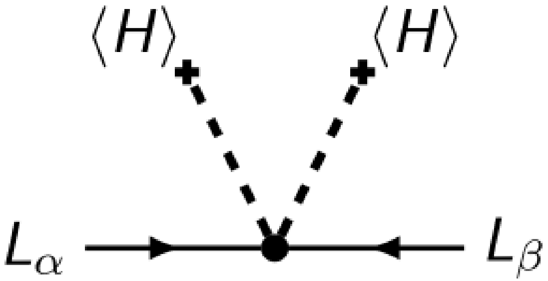


What is the UV completion of the Weinberg operator?

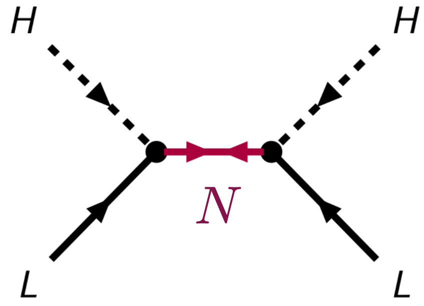
Tiny neutrino masses explained by high scale of operator

Seesaw models

Type I seesaw model

SM with RH neutrinos allows a Majorana mass term

$$-\mathcal{L} \supset \frac{1}{2}\nu_R^T C M \nu_R + \overline{\nu_R} Y_\nu^\dagger L \varepsilon H + \text{h.c.}$$



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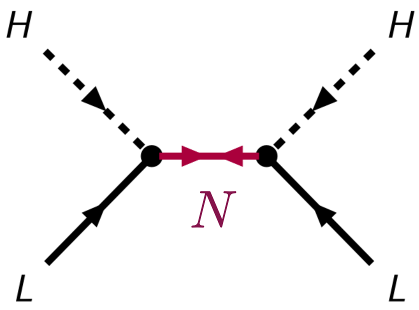

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At energy scales $E \ll M$, we can expand the fermionic N propagator

$$\frac{i(\not{p} + M)}{p^2 - M^2 + i\varepsilon} \simeq -\frac{i}{M}\left(1 + \frac{\not{p}}{M} + \dots\right)$$

The leading order term results in the Weinberg operator

$$\mathcal{L} = \frac{1}{2}\left(\boldsymbol{Y}_{\boldsymbol{\nu}}^* \boldsymbol{M}^{-1} \boldsymbol{Y}_{\boldsymbol{\nu}}^\dagger\right)_{ij} (L_i \varepsilon H)^T C (L_j \varepsilon H)$$

$$\rightarrow \quad \kappa_{ij} = -\left(\boldsymbol{Y}_{\boldsymbol{\nu}}^* \boldsymbol{M}^{-1} \boldsymbol{Y}_{\boldsymbol{\nu}}^\dagger\right)_{ij}$$

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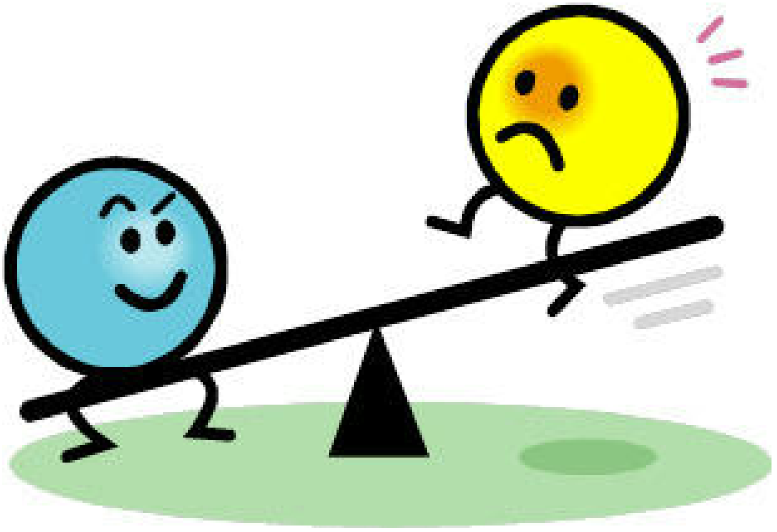
Seesaw model prediction $\kappa_{ij} = -\left(\boldsymbol{Y}_\nu^* \boldsymbol{M}^{-1} \boldsymbol{Y}_\nu^\dagger\right)_{ij}$
for Weinberg operator $-\frac{1}{2}\kappa_{ij}(L_i\varepsilon H)^T C(L_j\varepsilon H)$

Consequently neutrino masses $\nu_L^T m_\nu C\nu_L$ are

$$m_\nu^\dagger = -\left(M_\nu^D M^{-1} (M_\nu^D)^T\right)$$

Hermitian conjugation due to definitions of Y_ν & m_ν



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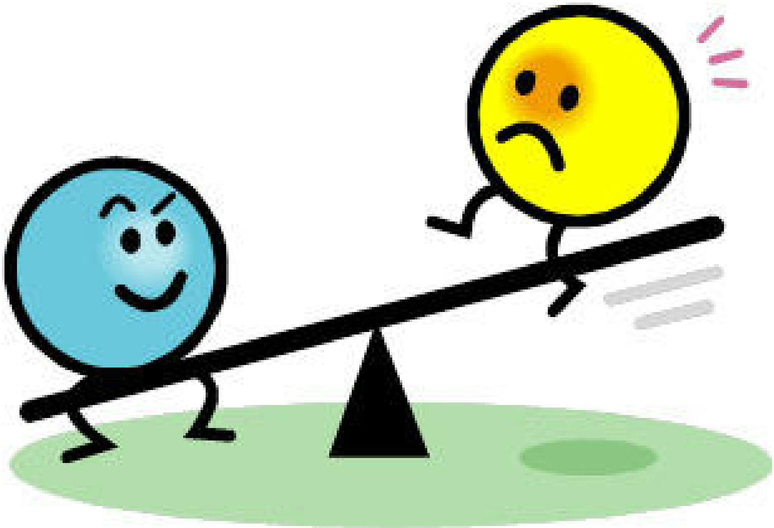

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- This is the **seesaw mechanism**
  - Light neutrinos explained by heavy RH neutrinos, $m_\nu \propto \frac{1}{M}$
  - For $Y_\nu \sim 1$, and $M \sim 10^{14}$ GeV $\rightarrow m_\nu \sim 0.1$ eV
- Heavy ν_R masses close to scale of grand unification
- Simple elegant explanation of tiny neutrino masses

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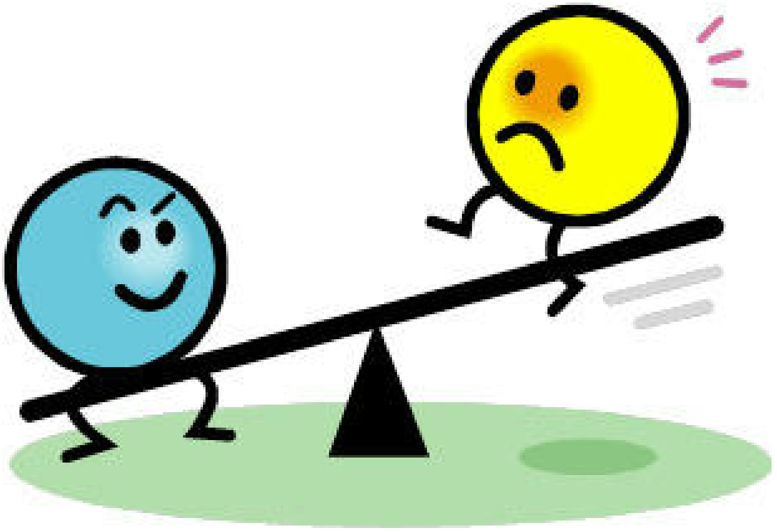

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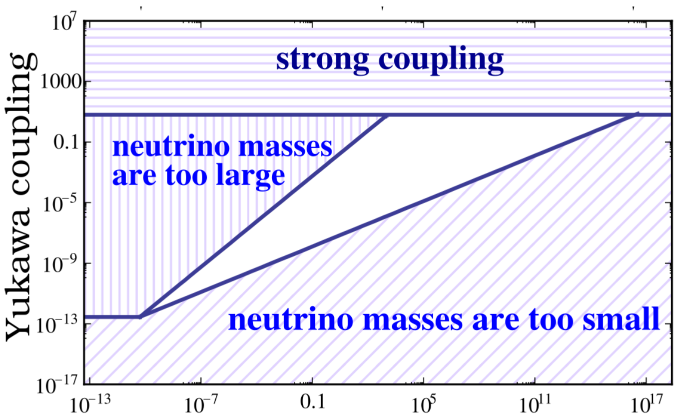
- Two RH neutrinos sufficient to explain neutrino masses

Type I seesaw model

[1204.5379]



Comments on seesaw phenomenology

- The seesaw formula also holds for light ν_R with $|M| < m_W$.
- This can be seen by considering the neutral fermion mass matrix

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&= \frac{1}{2} N^T C \mathcal{M}^\dagger N + \ldots \quad \text{with} \quad N = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} 0 & (M_\nu^D) \\ (M_\nu^D)^T & M \end{pmatrix}
\end{aligned}
$$

- The diagonalisation is trivial in the 1+1 flavour case for $|M_\nu^D| \ll |M|$. There is one large eigenvalue M and one small eigenvalue $m_\nu^\dagger = -\frac{(M_\nu^D)^2}{M}$. Light neutrino masses in the 3+3 flavour case are given by the seesaw formula

$$
m_\nu^\dagger = -M_\nu^D M^{-1} (M_\nu^D)^T
$$

Comments on seesaw phenomenology

- The active-sterile mixing angle is approximately $\theta \sim |U| \sim \frac{M_\nu^D}{M}$.

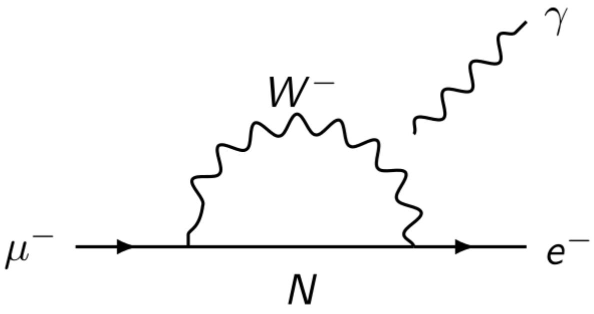
Comments on seesaw phenomenology

How can we test the seesaw mechanism?

- Lepton flavour violating rare decay $\mu \to e\gamma$

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \, |\sum_i U^*_{\mu i} U_{ei} \frac{m^2_{\nu i}}{m^2_W}|^2 \lesssim 10^{-54}$$

- constant term in m_ν drops out due to GIM mechanism (unitarity of PMNS matrix)



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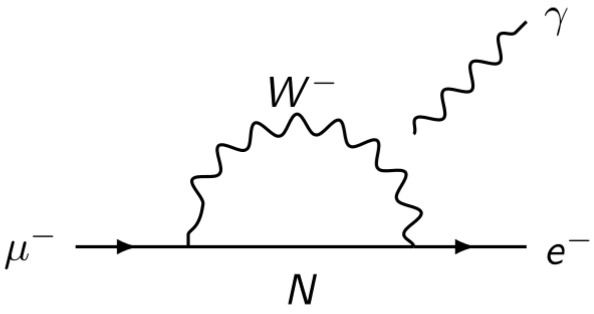
- constant term in m_ν drops out due to GIM mechanism (unitarity of PMNS matrix)
- similar for $\mu \to 3e$ and τ decays
- no noteworthy contribution to quark flavour physics
- Extremely difficult to test

This changes in extensions of the minimal seesaw model. Examples are

- SUSY seesaw due to slepton soft mass terms
- left-right symmetric models due to RH gauge bosons
- inverse seesaw model (see below)



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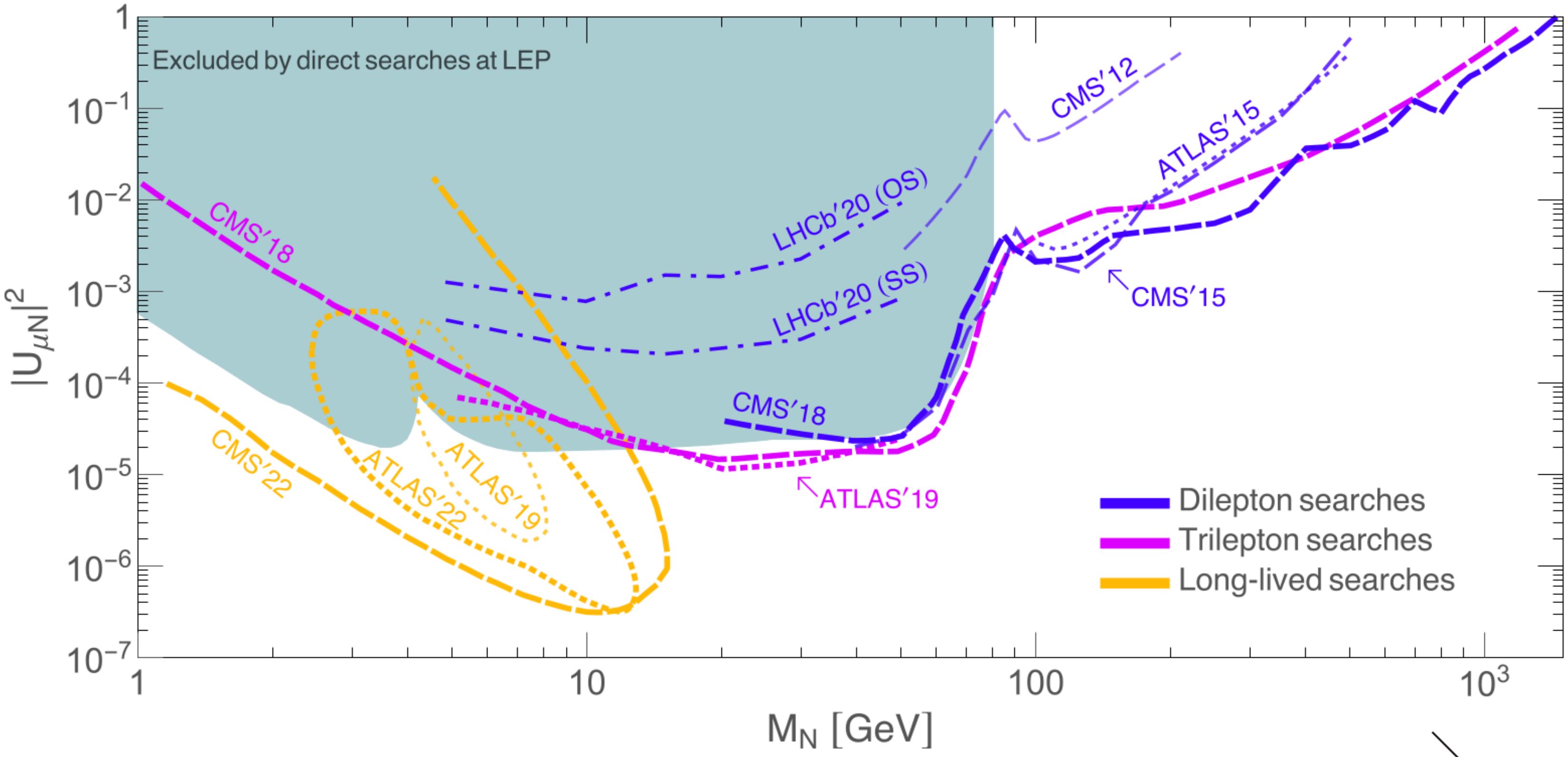
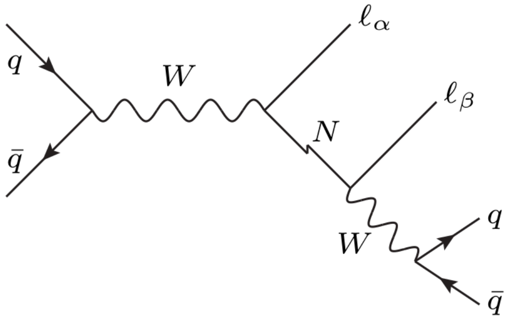


[2208.13882]

seesaw estimate

$$|U|^2 \sim 10^{-12} \frac{50\text{GeV}}{M_N}$$



- LHC and other colliders are probing seesaw model
- long-lived particle searches sensitive to small Yukawa couplings
- current limits $>$ 3 orders of magnitude away from seesaw estimate

Double seesaw

What if there are more than 3 sterile neutrinos?

We add **3 RH neutrinos** $\nu_R \equiv N^c$ and **3 gauge singlet fermions** S. This is an arbitrary distinction in the SM, but may be motivated in grand unified theories like SO(10), where the RH neutrinos are part of the 16-plet. The neutral fermion mass matrix is

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ . & 0 & M_{NS} \\ . & . & \mu_S \end{pmatrix} \quad \text{with} \quad \mathcal{N} = \begin{pmatrix} \nu_L \\ N \\ S \end{pmatrix} \quad \text{with} \quad N \equiv \nu_R^c$$

[left lower triangle fixed by symmetry]

For $\mu_S \gg M_{NS} \gg m_D$ we apply twice the seesaw formula to obtain light neutrino masses

$$m_\nu^{\mathrm{DS}} = m_D \left(M_{NS}^{-1}\right)^T \mu_S M_{NS}^{-1} m_D^T$$

This is commonly known as **double seesaw**.

A well-motivated setup could be $\mu_S \sim M_{\text{Planck}}$, $M_{NS} \sim M_{\text{GUT}}$, $m_D \sim m_W \Rightarrow m_\nu^{\mathrm{DS}} \sim \text{eV}$

Inverse seesaw

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The same result applies for $\mu_S \ll m_D \ll M_{NS}$ which is referred to as **inverse seesaw**.

- Lepton number is only broken by μ_S which can be arbitrarily small
  - Smallness of neutrino masses explained by tiny μ_S
  - It is technically natural, i.e. quantum corrections do not spoil it

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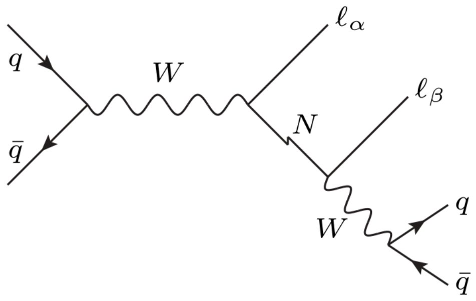
  - Smallness of neutrino masses explained by tiny μ_S
  - It is technically natural, i.e. quantum corrections do not spoil it
- Inverse seesaw has large active-sterile mixing angles $U_{\alpha i}$ with $i = 4, \ldots$
  - possibly large contributions to $\mu \to e\gamma$ since $\mathrm{Br} = \frac{3\alpha}{32\pi} \left| \sum_i U_{\mu i}^* U_{ei} \frac{m_i^2}{m_W^2} \right|^2$
  - sterile neutrinos easier to produce at colliders
  - **but** we don't expect to see $\not{L}$ since it is suppressed by μ_S / M_{NS}



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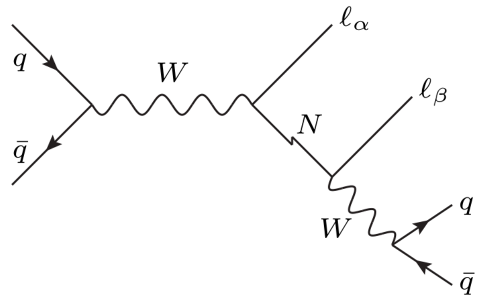
  - **but** we don't expect to see $\not{L}$ since it is suppressed by μ_S / M_{NS}
- Inverse seesaw is easier to test experimentally



Linear seesaw and variants

$$\mathcal{M} = \begin{pmatrix} \underline{0} & m_D & \underline{\mu_{\nu S}} \\ \cdot & \underline{\mu_{NN}} & M_{NS} \\ \cdot & \cdot & \underline{\mu_S} \end{pmatrix} \quad \text{with} \quad \mathcal{N} = \begin{pmatrix} \nu_L \\ N \\ S \end{pmatrix}$$

lepton number violation

- $\nu_L - \nu_L$ element zero due to EW symmetry
- m_D and M_{NS} preserve lepton number L
- underlined elements $\underline{\mu_X}$ violate lepton number L with $\Delta L = 2$
- We distinguish 3 variants (and their combinations), depending on which lepton number violating entries are non-zero
  - double/inverse seesaw $\mu_S \neq 0$ (depending on hierarchy)
  - linear seesaw $\nu_{\nu S} \neq 0$
  - radiative inverse seesaw $\nu_{NN} \neq 0$

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Linear seesaw: $\mu_{\nu S} \neq 0$

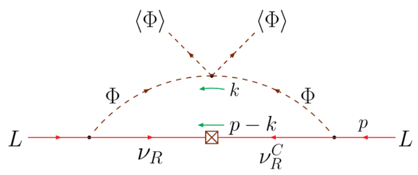
$$m_\nu^{\text{LS}} = -\left[m_D\left(\mu_{\nu S} M_{NS}^{-1}\right)^T + \left(m_{\nu S} M_{NS}^{-1}\right) m_D^T\right]$$

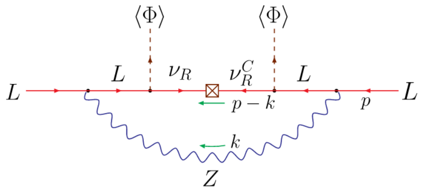
- neutrino mass is linear in the Dirac mass m_D
- small $\mu_{\nu S}$ is technically natural if it is the only source of $\cancel{L}$

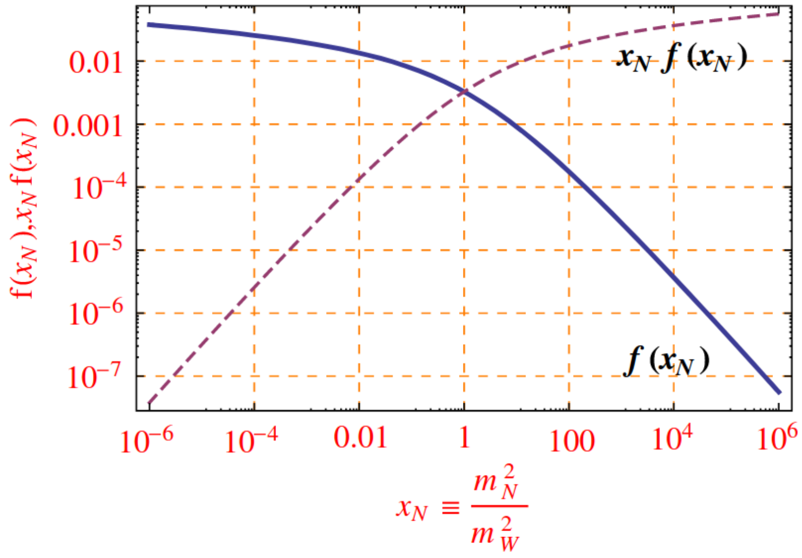
Radiative inverse seesaw

- simplest case: $\mu_{NN} \neq 0$ and $\mu_S = \mu_{\nu S} = 0$ [1209.4051]
- active neutrinos are massless at tree level
- neutrino masses generated at 1-loop $m_\nu^{1L} \simeq \frac{f(x_N)}{m_W^2} m_D \mu_{NN} m_D^T$

$$f(x_N) = \frac{\alpha_2}{16\pi}\left(\frac{x_H}{x_N - x_H}\ln\left(\frac{x_N}{x_H}\right) + \frac{3x_Z}{x_N - x_Z}\ln\left(\frac{x_N}{x_Z}\right)\right)$$







consider $\mu_{NN} \gg M_{NS} \gg m_D$

- heavy sterile neutrinos of mass μ_{NN} with $\mathcal{N}_1 \simeq N$
- light sterile ν's, mass $M_{NS}^T \mu_{NN}^{-1} M_{NS}$ and $\mathcal{N}_2 \simeq S$
- very light active neutrinos $\mathcal{N}_3 \simeq \nu_L$

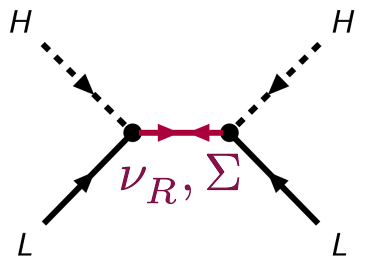
$$\mathcal{M} = \begin{pmatrix} \underline{0} & m_D & \underline{\mu_{\nu S}} \\ \cdot & \underline{\mu_{NN}} & M_{NS} \\ \cdot & \cdot & \underline{\mu_S} \end{pmatrix}$$

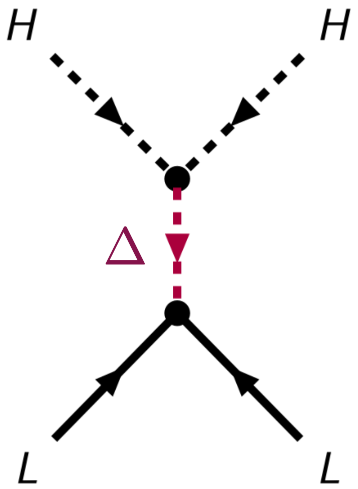
Minimal tree-level UV completions of Weinberg operator

- (Type 1) seesaw model is not only UV completion of Weinberg operator $LLHH$
- SU(2) group theory: $(LH) \sim 1 + 3$
  - $(LH)(LH)$ features two possible singlets:
    - Type 1 seesaw – $\nu_R \sim (1, 1, 0)$ Minkowski 1977
    - Type 3 seesaw – $\Sigma \sim (1, 3, 0)$ Foot, Lew, He, Joshi 1989
- More SU(2) group theory: $(LL) \sim 1 + 3$ and $(HH) \sim 3$

Why is $(HH) \nsim 1 + 3$?

  - Type 2 seesaw – $\Delta \sim (1, 3, -1)$ Mohapatra, Senjanovic, Magg, Wetterich, Lazarides, Shafi, Schechter, Valle





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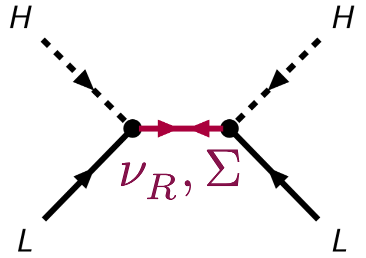

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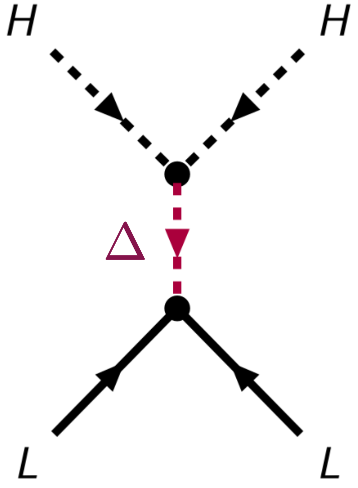

Type 3 seesaw

- $\Sigma \sim (1, 3, 0)$ is Weyl fermion with 3 components, $\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)^T$
- Neutrino mass generation via Σ^0 exactly the same as in type 1 seesaw
- $\Sigma^\pm$ form a vector-like lepton which mixes with SM charged leptons
  - Compared to type 1 seesaw, type 3 seesaw has richer phenomenology

Type II seesaw

We introduce one electroweak triplet scalar $\Delta \sim (1, 3, 1)$. A convenient parametrisation is in form of a hermitian 2×2 matrix

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

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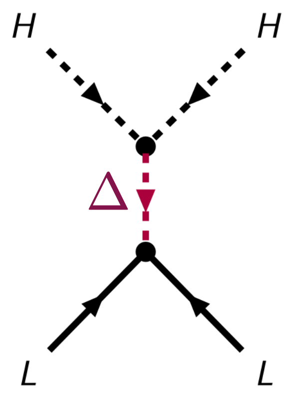

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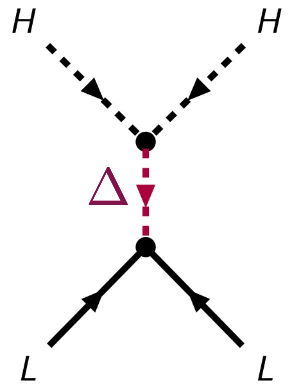

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It is in the adjoint representation of SU(2), $\Delta \to U\Delta U^\dagger$. The relevant Lagrangian terms are

$$-\mathcal{L} = \frac{1}{2}\overline{L^c}Y_\Delta \varepsilon \Delta L + \mu_\Delta H^T \varepsilon \Delta^\dagger H + \text{h.c.}$$

symmetric, $Y_\Delta = Y_\Delta^T$

mass dimension 1

$\Delta L = 2$ in presence of both couplings Y_Δ and μ_Δ

For $E \ll M_\Delta$, Δ can be integrated to produce the Weinberg operator with $\kappa = -\frac{2\mu_\Delta Y_\Delta}{M_\Delta^2}$.

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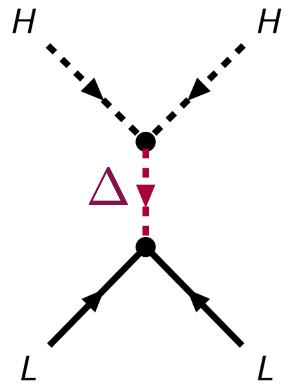

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The VEV v_Δ is strongly constrained by the ρ parameter (ratio of electroweak gauge boson masses): $v_\Delta \lesssim O(1\ \text{GeV})$

Radiative neutrino mass models

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  - Difficult to cover the theory space
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How many loops should we consider? $\rightarrow$ up to 3 loops

Zee model

- Two new scalars:
  - singly charged scalar $h^+ \sim (1,1,1)$
  - second electroweak doublet scalar $\Phi \sim \left(1,2,\frac{1}{2}\right)$

$$-\mathcal{L} \supset \overline{L^c}\boldsymbol{f}\varepsilon L h + \overline{e_R}Y_2\Phi^\dagger L + \mu h^{+*} H\Phi$$

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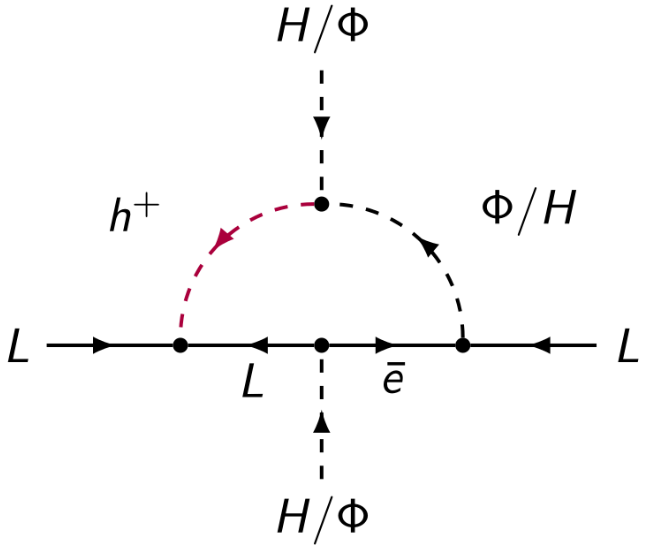

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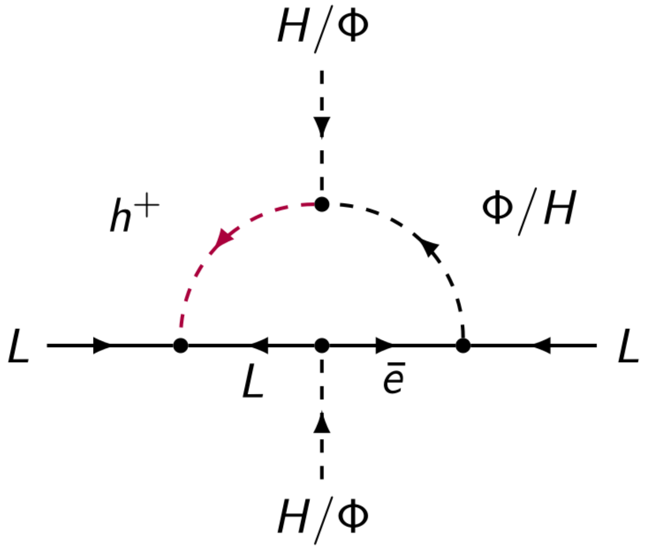


- Without loss of generality $\langle\Phi\rangle = 0$

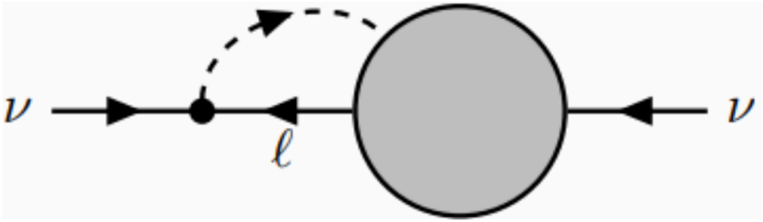
$$m_\nu = -\frac{\sin(2\varphi)}{16\pi^2}\left(\boldsymbol{f} M_e Y_2 + Y_2^T M_e \boldsymbol{f}^T\right) \ln\left(\frac{m_2^2}{m_1^2}\right) \quad \text{with} \quad \sin(2\varphi) \simeq \frac{\sqrt{2} v \mu}{m_2^2 - m_1^2}$$

with $M_e = $ diag, charged scalar masses m_i

- Simplest variant (Zee-Wolfenstein model) with Z_2 symmetry ruled out

Models with singly-charged scalar

- singly charged scalar $h \sim (1,1,1)$ has anti-symmetric Yukawa interaction $\overline{L^c}\boldsymbol{f}\varepsilon L h$
- $\boldsymbol{f}$ has a zero eigenvalue with eigenvector $v_\alpha = \varepsilon_{\alpha\beta\gamma}\boldsymbol{f}_{\beta\gamma}$.
- If one of these lepton doublets is an external leg and neutrino mass is described by



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where X describes the gray blob.

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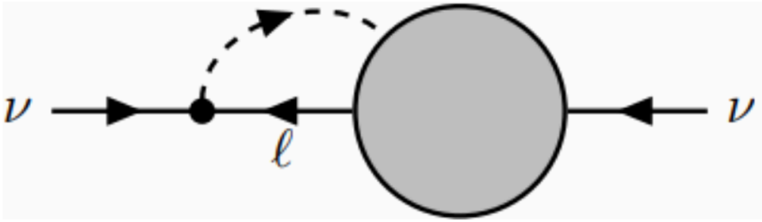

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- The zero eigenvalue implies

$$0 = v^T m_\nu v = v^T U^*_{\text{PMNS}} m_\nu^{\text{diag}} U^\dagger_{\text{PMNS}} v = \left(\sqrt{m_\nu^{\text{diag}}} U^\dagger_{\text{PMNS}} v\right)^T \left(\sqrt{m_\nu^{\text{diag}}} U^\dagger_{\text{PMNS}} v\right) \equiv w^T w$$

- one constraint on $\boldsymbol{f}$ from low energy parameters irrespective of other UV physics

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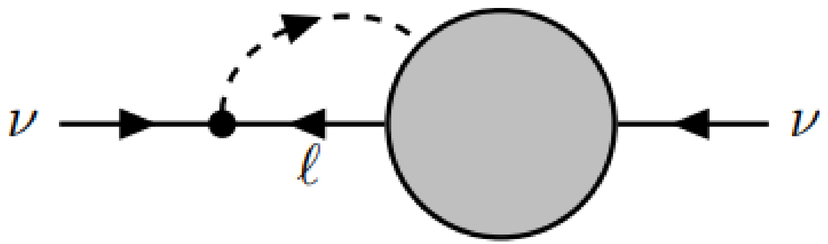

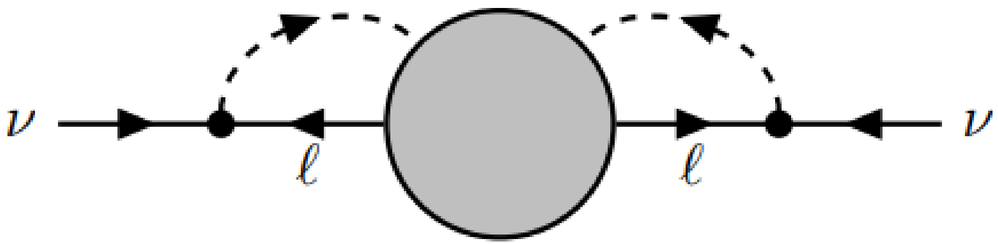

linear: $m_\nu = X\boldsymbol{f} - \boldsymbol{f}X^T$
where X describes the gray blob.

constraint

$$0 = \left(\sqrt{m_\nu^{\mathrm{diag}}}U_{\mathrm{PMNS}}^\dagger v\right)^T\left(\sqrt{m_\nu^{\mathrm{diag}}}U_{\mathrm{PMNS}}^\dagger v\right)$$

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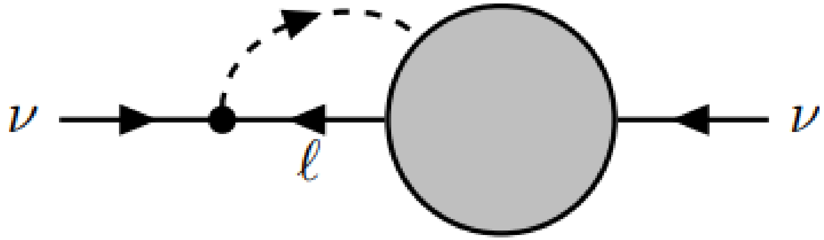

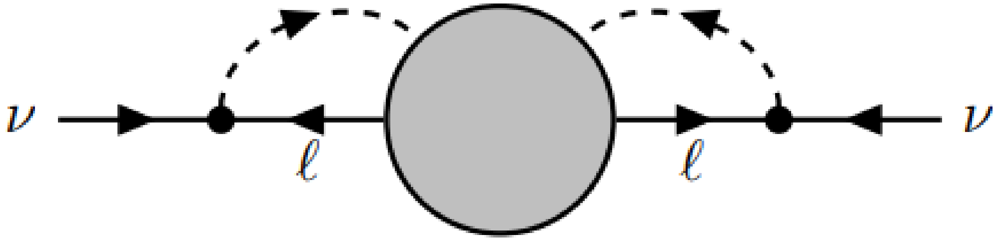

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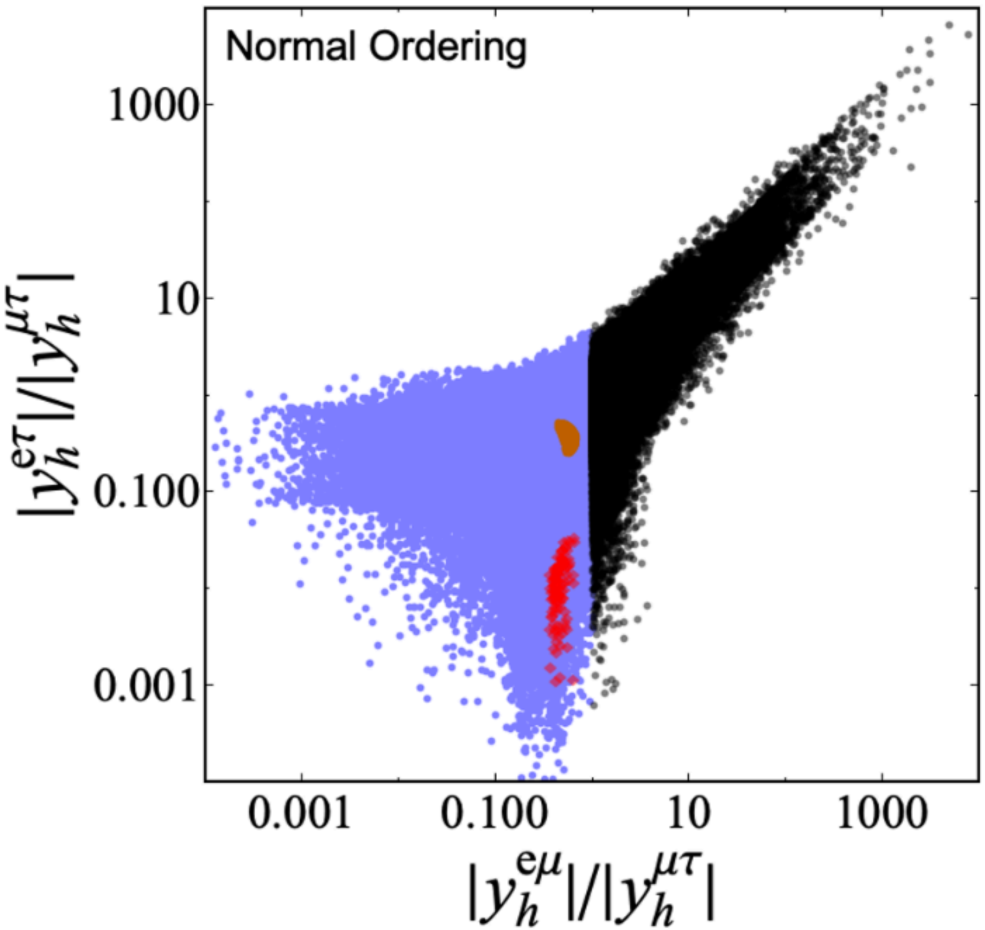
quadratic: $m_\nu = fSf$ with symmetric S

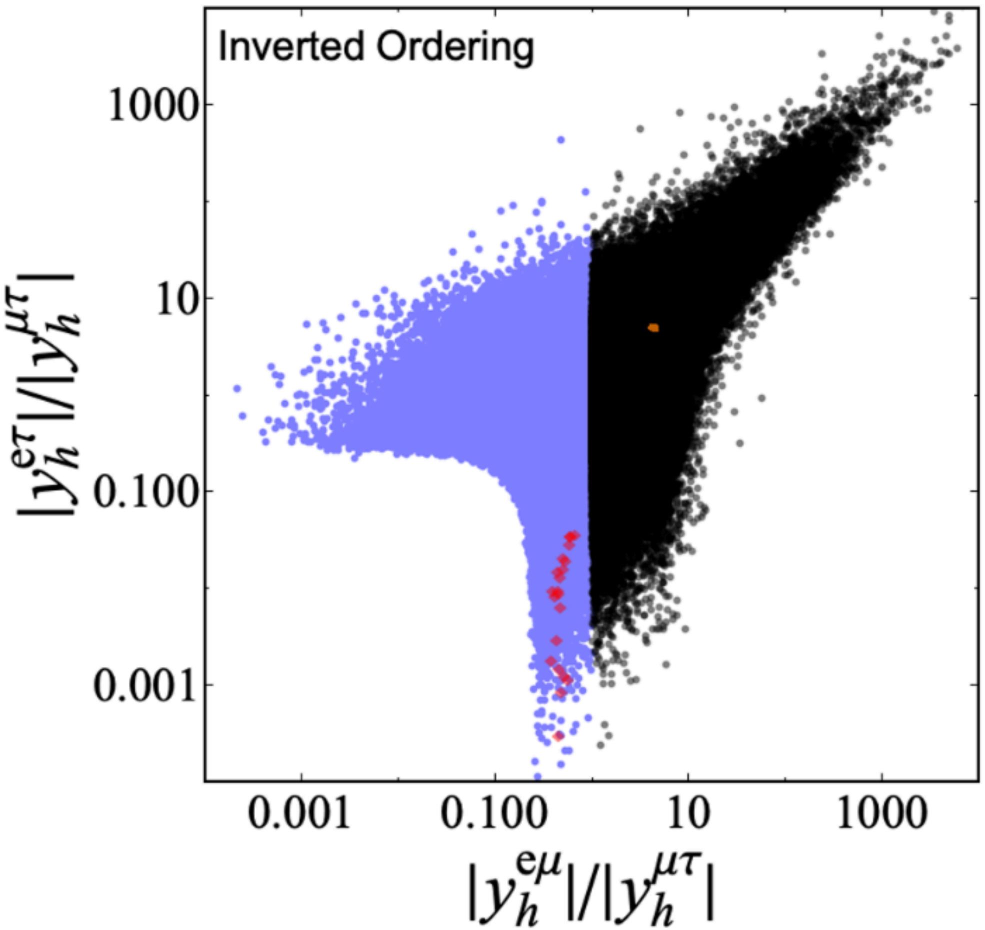
- Zero eigenvalue implies

$$0 = m_\nu v \quad \rightarrow \quad m_\nu^{\mathrm{diag}} U_{\mathrm{PMNS}}^\dagger v = 0$$

- two conditions on $\boldsymbol{f}$ independent of other UV physics.

Models with singly-charged scalar





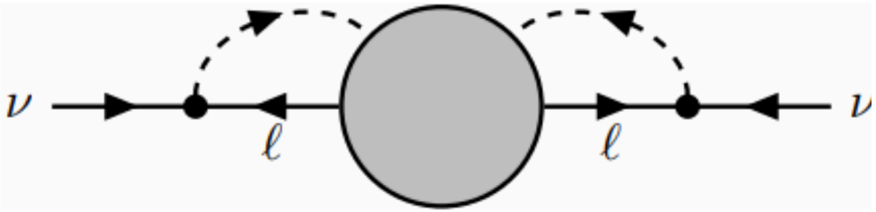
- overall scale undetermined
- quadratic case (brown) very constrained

- black (blue) $\frac{g_\tau}{g_e}$ explained at $3(2)\sigma$
- red: Cabibbo angle anomaly

$y_h \equiv f$

Model with singly-charged scalar – quadratic case

- neutrino mass $m_\nu = fSf$ with symmetric S
- two conditions on f: $m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger v = 0$



$$\frac{f^{e\tau}}{f^{\mu\tau}} = \frac{t_{12}c_{23} + s_{13}s_{23}e^{i\delta}}{c_{13}} \qquad \frac{f^{e\mu}}{f^{\mu\tau}} = \frac{t_{12}s_{23} - s_{13}c_{23}e^{i\delta}}{c_{13}}$$

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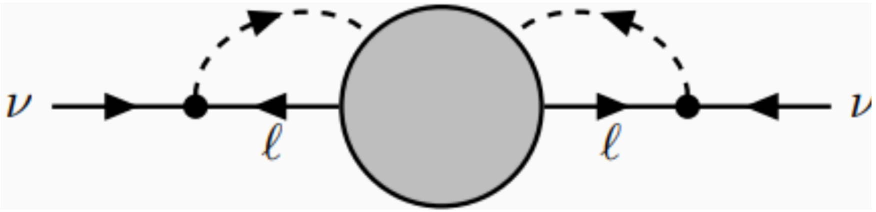

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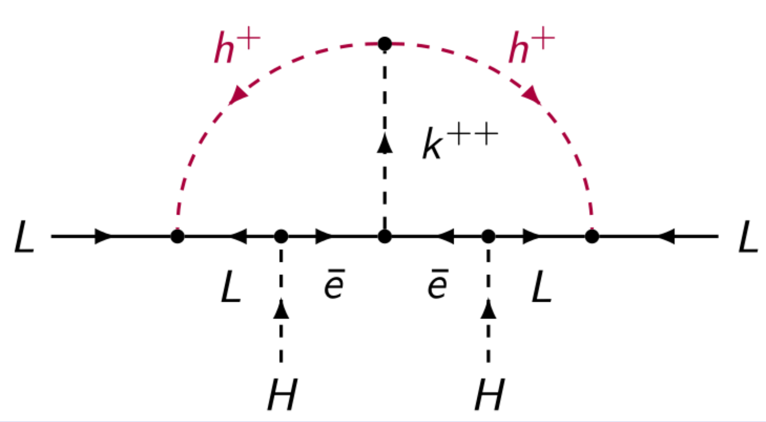
Zee-Babu model

- example for quadratic case

$$-\mathcal{L} = \overline{L^c} f \varepsilon L h + \overline{e_R^c} g e_R k^{++} + \mu (k^{++})^\dagger h^+ h^+$$

- neutrino mass approximately given by

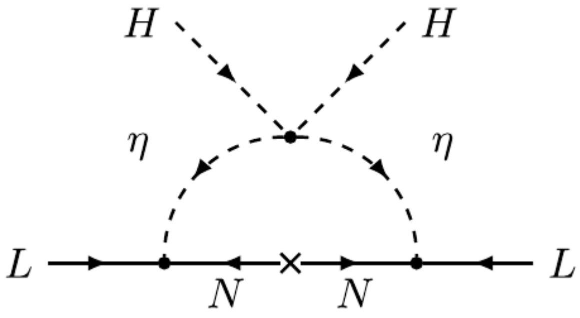
$$m_\nu \simeq -\frac{\mu}{48\pi^2 M^2} f M_e g^\dagger M_e f$$



Radiative seesaw model – scotogenic model

- additional particles
  - SM singlet fermions $N \sim (1, 1, 0)_-$
  - electroweak doublet scalar $\eta \sim \left(1, 2, \frac{1}{2}\right)_-$
- forbid seesaw contribution using Z_2 symmetry

$$Z_2 : \quad N \to -N \quad \text{and} \quad \eta \to -\eta$$



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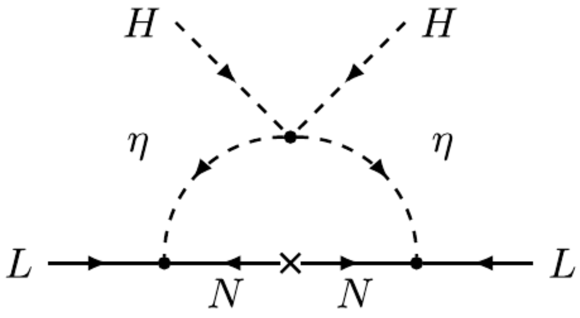

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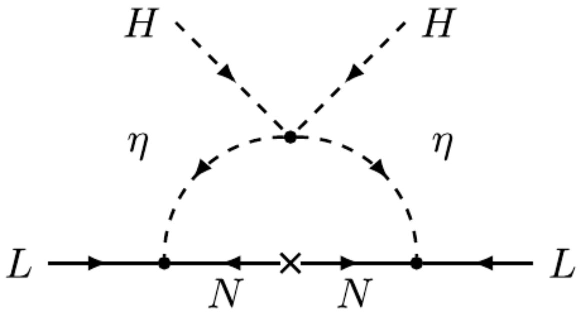

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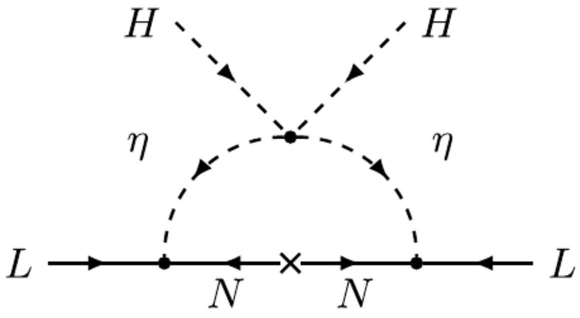

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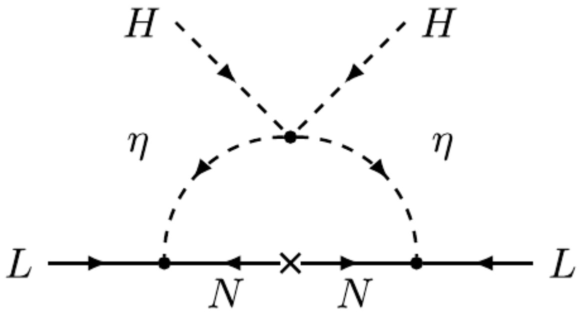

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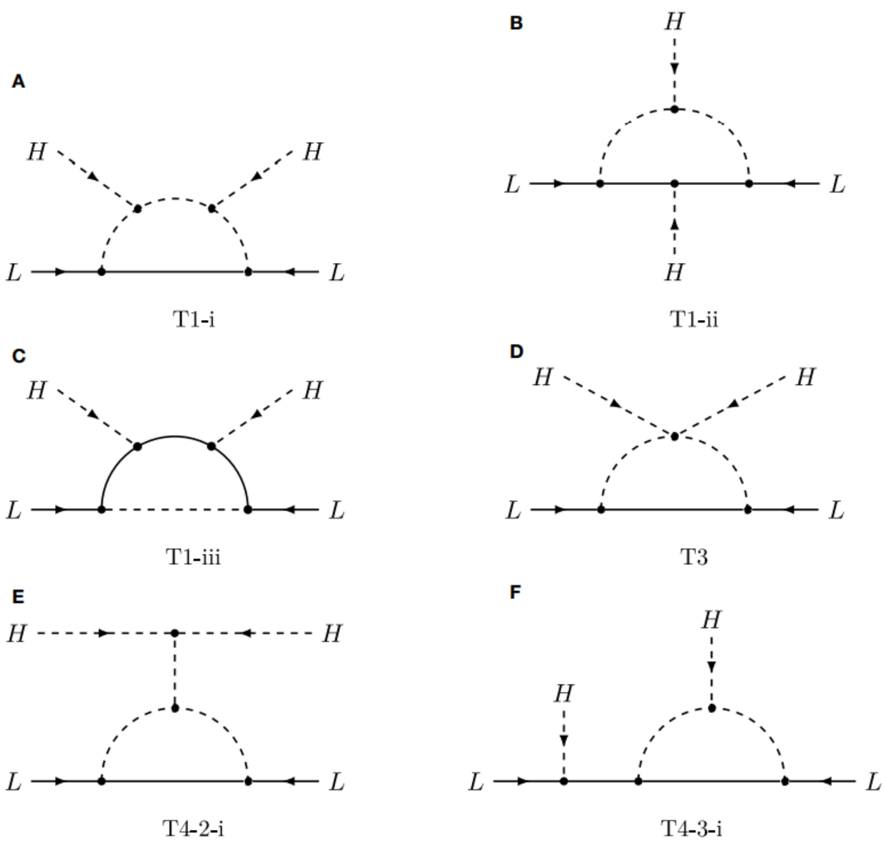
The radiative seesaw model is also often called scoto-genic ("created from darkness") model.

Systematic approaches

Loop-level UV completions of Weinberg operator

Bonnet+ [1204.5862] identified **six viable 1-loop topologies** for Weinberg operator



- exclude topologies which necessarily are accompanied by tree level neutrino masses
- bottom two topologies T4-2-i, T4-3-i don't work either, require symmetry, no viable model known ⇒ 4 viable topologies, e.g.
  - Zee model – T1-ii
  - scotogenic model – T3
- general study enabled to systematically consider variants of scotogenic model, see Restrepo+ [1308.3655]
- topologies exist for
  - 2-loop Aristazabal+ [1411.7038] (incl loop integrals) and
  - 3-loop Cepedello+ [1807.00629]

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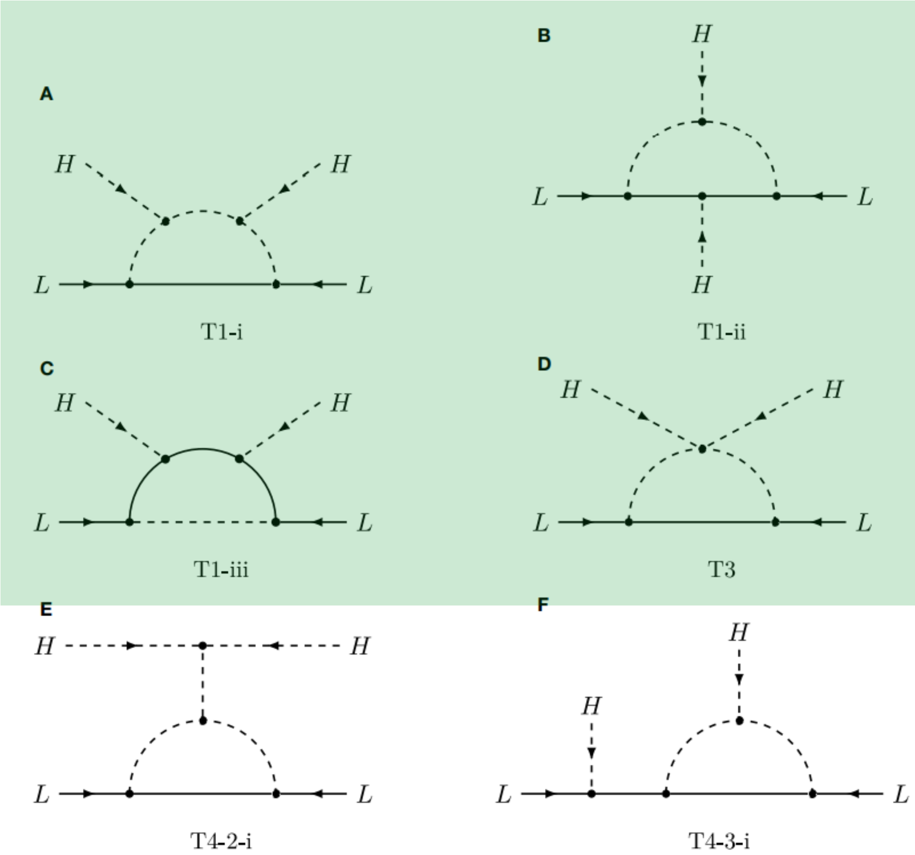

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SMEFT operators with $\Delta L = 2$

Which operators are $\Delta L = 2$?

Kobach [1604.05726]

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Which operators are $\Delta L = 2$? Kobach [1604.05726]

- We use LH 2-component Weyl spinors $f = Q, L, u, d, e$ and daggered spinors $f^\dagger$
- Hypercharge conservation Y_f hypercharge of f

$$0 = \sum_f 2Y_f \left(N_f - N_{f^\dagger}\right)$$

$$= \frac{1}{3}\left(N_Q - N_{Q^\dagger}\right) - \frac{4}{3}(N_u - N_{u^\dagger}) + \frac{2}{3}(N_d - N_{d^\dagger}) - (N_L - N_{L^\dagger}) + 2(N_e - N_{e^\dagger}) + (N_H - N_{H^\dagger})$$

- non-Abelian gauge groups SU(3) × SU(2) don't result in new constraints

$$\frac{1}{3}\left(N_Q + N_{d^\dagger} + N_{u^\dagger}\right) - \frac{1}{3}\left(N_{Q^\dagger} + N_d + N_u\right) \in \mathbb{Z}$$

$$\left(N_Q - N_{Q^\dagger}\right) - 3(N_L - N_{L^\dagger}) + (N_H - N_{H^\dagger}) \in 2\mathbb{Z}$$

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\begin{aligned}
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$$

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$$

- Lorentz invariance N_D is number of covariant derivatives

SMEFT operators with $\Delta L = 2$

$$N_D \text{ even} \Leftrightarrow \sum_{f \neq H} N_f \text{ even} \wedge \sum_{f \neq H} N_{f^\dagger} \text{ even}$$

SMEFT operators with $\Delta L = 2$

- Lepton and baryon number

$$\Delta L = (N_L - N_{L^\dagger}) - (N_e - N_{e^\dagger}) \in \mathbb{Z}$$

$$\Delta B = \frac{1}{3}\left(N_Q + N_{u^\dagger} + N_{d^\dagger}\right) - \frac{1}{3}\left(N_{Q^\dagger} + N_u + N_d\right) \in \mathbb{Z}$$

- Operator dimension X field strength tensor

$$d = N_H + N_{H^\dagger} + N_D + 2N_X + \frac{3}{2}\sum_f N_f$$

- Using hypercharge condition and definitions of ΔB and ΔL, we find

$$d = 3\left(N_{Q^\dagger} - N_{u^\dagger} + N_{d^\dagger} + N_{L^\dagger} + N_{e^\dagger}\right) + N_D + 6N_u - 2N_H + 4N_{H^\dagger} + 2N_X + 3\left(\frac{1}{2}\Delta B + \frac{3}{2}\Delta L\right)$$

- Green highlighted part is always even, since N_D even $\Leftrightarrow \sum_f N_{f^\dagger}$ even

SMEFT operators with $\Delta L = 2$

$$d \text{ even} \Leftrightarrow \frac{1}{2}(\Delta B - \Delta L) \text{ even}$$

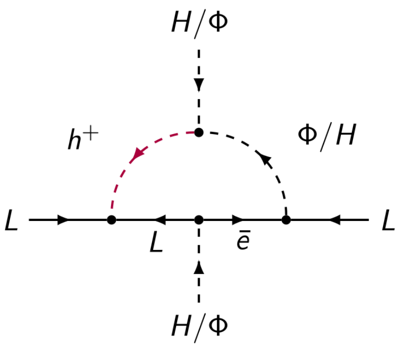
In particular for $\Delta B = 0$,
d even $\Leftrightarrow \Delta \frac{L}{2}$ even implies $\Delta L = 2 \to d$ odd

$\Delta L = 2$ operators

Weinberg-like operators

$$O_1 = L^i H^j \varepsilon_{ij} L^k H^l \varepsilon_{kl}$$
$$O_1' = O_1 H^\dagger H$$
$$O_1'' = O_1 \left(H^\dagger H\right)^2$$
$$O_1^{(n)} = O_1 \left(H^\dagger H\right)^n$$

Zee model



Zee model Zee-Babu model

$O_2 = L^i L^j L^k \bar{e} H^l \epsilon_{ij}\epsilon_{kl}$ $\quad O_{3a} = L^i L^j Q^k \bar{d} H^l \epsilon_{ij}\epsilon_{kl}$ $\quad O_{3a} = L^i L^j Q^k \bar{d} H^l \epsilon_{ik}\epsilon_{jl}$

$O_{4a} = L^i L^j Q_i^\dagger \bar{u}^\dagger H^k \epsilon_{jk}$ $\quad O_{4b} = L^i L^j Q_k^\dagger \bar{u}^\dagger H^k \epsilon_{ij}$ $\quad O_8 = L^i \bar{d} \bar{e}^\dagger \bar{u}^\dagger H^j \epsilon_{ij}$

$O_9 = L^i L^j L^k \bar{e} L^l \bar{e} \epsilon_{ij}\epsilon_{kl}$ $\quad O_{10} = L^i L^j L^k \bar{e} Q^l \bar{d} \epsilon_{ij}\epsilon_{kl}$

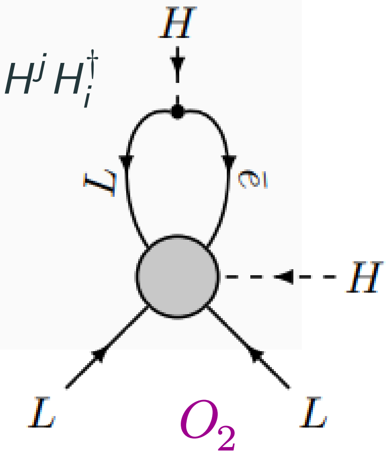
$O_{11a} = L^i L^j Q^k \bar{d} Q^l \bar{d} \epsilon_{ij}\epsilon_{kl}$ $\quad O_{11b} = L^i L^j Q^k \bar{d} Q^l \bar{d} \epsilon_{ik}\epsilon_{jl}$

$O_{12a} = L^i L^j Q_i^\dagger \bar{u}^\dagger Q_j^\dagger \bar{u}^\dagger$ $\quad O_{12b} = L^i L^j Q_k^\dagger \bar{u}^\dagger Q_l^\dagger \bar{u}^\dagger \epsilon_{ij}\epsilon^{kl}$

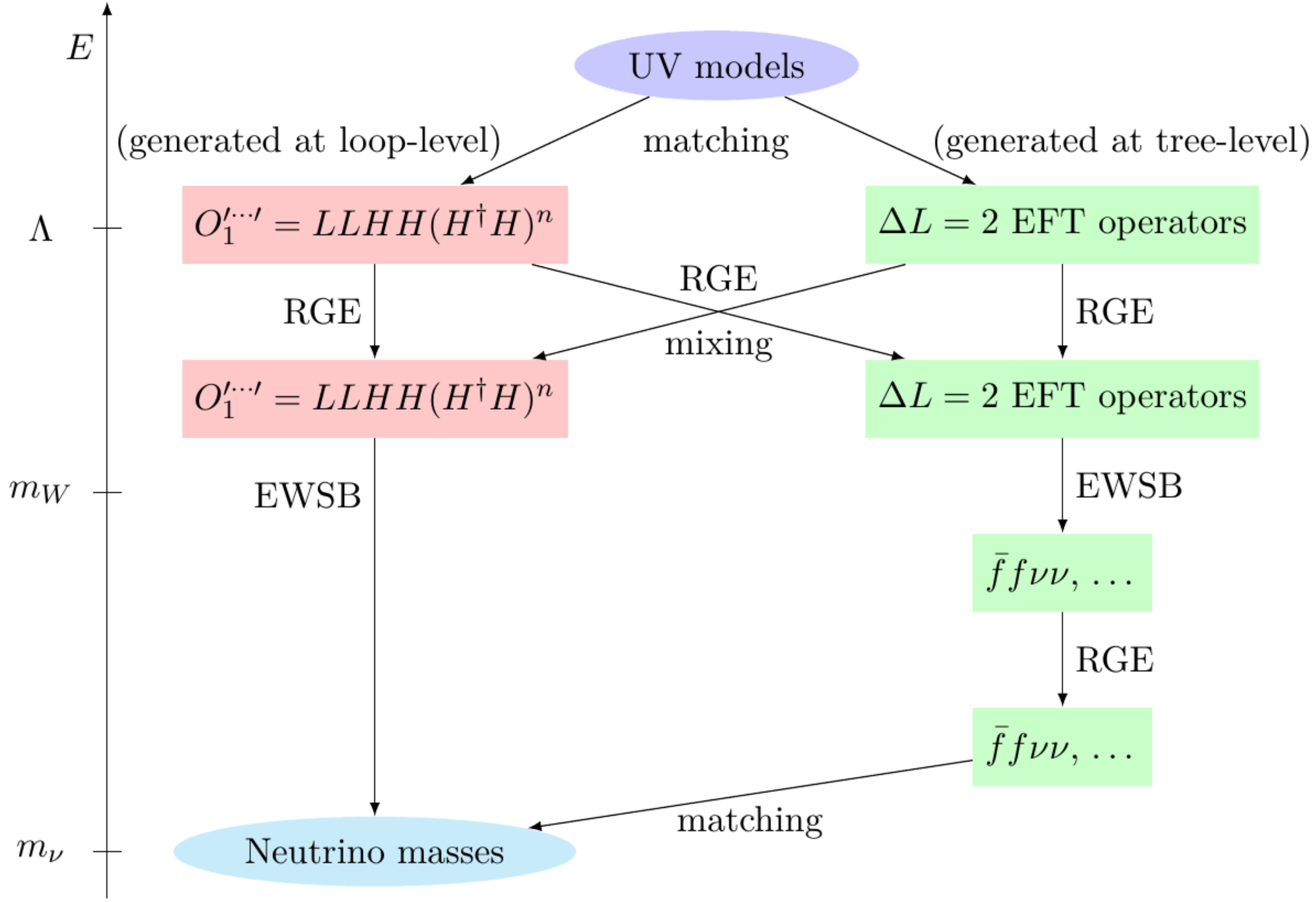
$\dots$

$O_{59} = L^i Q^j \bar{d}\bar{d}\bar{e}^\dagger \bar{u}^\dagger H^k H_i^\dagger \epsilon_{jk}$ $\quad O_{60} = L^i \bar{d} Q_j^\dagger \bar{u}^\dagger \bar{e}^\dagger \bar{u}^\dagger H^j H_i^\dagger$

operators up to dimension 11 classified



Effective field theory



lowest dimensional Weinberg-like operator $O_1^{(n)}$ **dominates** due to power counting

1. Are there examples where O_1' dominates over O_1?
2. Why is it meaningful to consider general $\Delta L = 2$ operators?

Example for Weinberg-like operator

- new particles
  - vector-like lepton $\Sigma \sim (1,3,1)$
  - scalar $\Phi \sim \left(1,4,\frac{3}{2}\right)$

$$\Sigma = \left(\Sigma^{++}, \Sigma^{+}, \Sigma^{0}\right)$$

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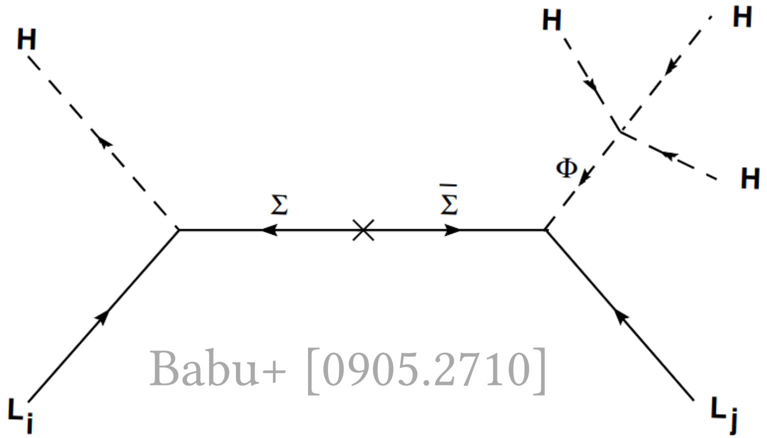

Babu+ [0905.2710]

- Lagrangian

$$\mathcal{L} = \overline{\Sigma^c} Y L \tilde{H} + \overline{\Sigma} \Phi \overline{Y} L + \lambda_5 H^3 \Phi + \text{h.c.}$$

- neutrinos mass
  - Weinberg operator not generated at tree level
  - Dimension-7 operator O_1' generated at tree level

$$m_\nu = \frac{\lambda_5 v^4}{4 M_\Phi^2 M_\Sigma} \left(Y \overline{Y}^T + \overline{Y} Y^T\right)$$

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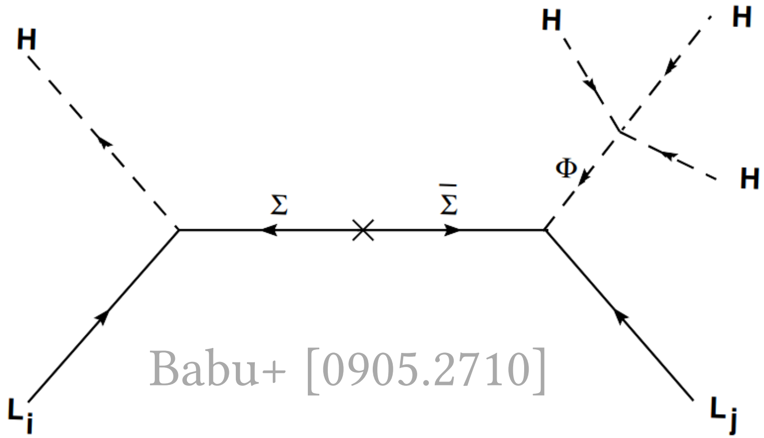

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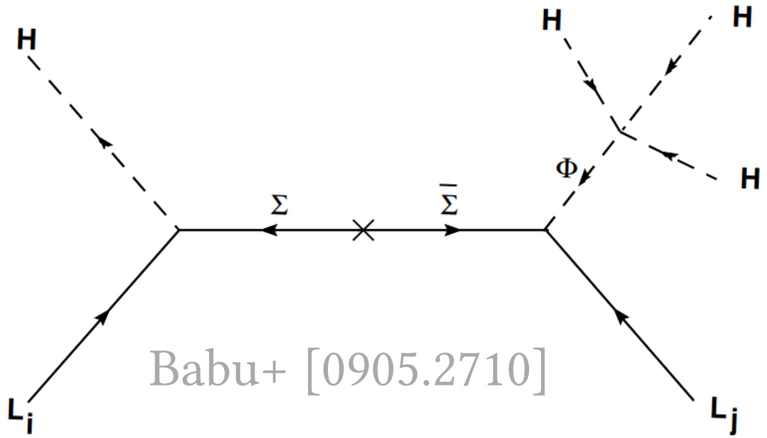

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- Weinberg operator generated at 1-loop level via topology T3 → becomes important for large M_Φ, M_Σ
- several highly charged particles at low scale

$\Delta L = 2$ operators

Why is it meaningful to consider general $\Delta L = 2$ operators?

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Why is it meaningful to consider general $\Delta L = 2$ operators?

- useful classification which
  - provides information about other SM particles and
  - their phenomenology
- **caveat:** it generally requires requires to consider UV completions. The $\Delta L = 2$ operator only describes $\Delta L = 2$ processes

Dimension-7 $\Delta L = 2$ operators

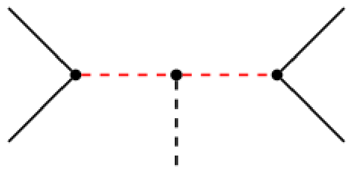
- Babu+ [hep-ph/0106054] classified $\Delta L = 2$ operators (excl cov. derivatives, field strength tensors) up to dimension-11 (parentheses indicate isospin contractions)

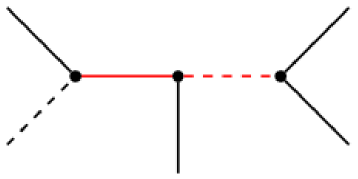
$$O_1' = (LH)(LH)H^\dagger H \qquad O_2 = (LL)(LH)\overline{e} \qquad O_{3a} = (LL)(QH)\overline{d} \qquad O_{3b} = (LQ)(LH)\overline{d}$$

$$O_{4a} = \left(LQ^\dagger\right)(LH)\overline{u}^\dagger \qquad O_{4b} = (LL)\left(Q^\dagger H\right)\overline{u}^\dagger \quad O_8 = L\overline{e}\overline{u}\overline{d}H$$

minimal tree-level UV completions

- 2 viable topologies (external solid lines can be both scalar or fermion)

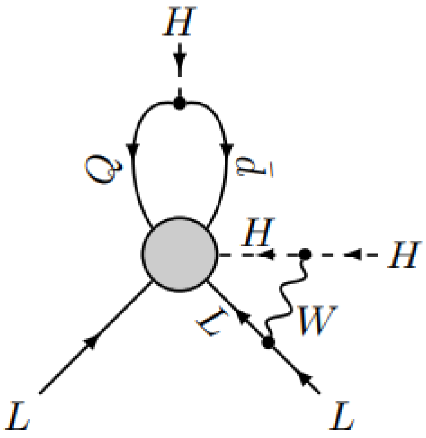


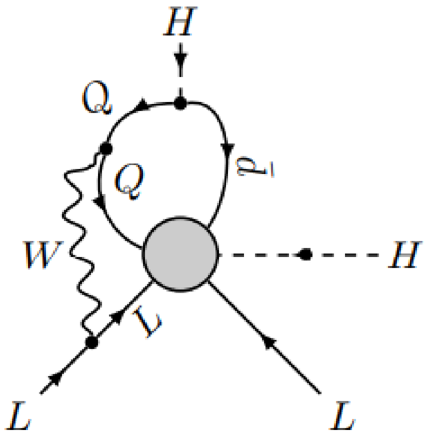


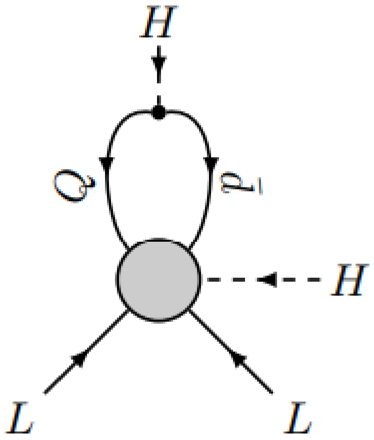
generic comments

- singly charged scalar for O_2, O_{3a}, O_{4b}
  - only small number of different particles (12)
  - allows to study their phenomenology and apply to models
- O_1' only generated via 2nd topology (see above)
- Isospin structure prevents 1-loop neutrino masses for O_{3a} and O_{4b}
- O_8 generates neutrino masses only at 2-loop order

Dimension-7 $\Delta L = 2$ operators







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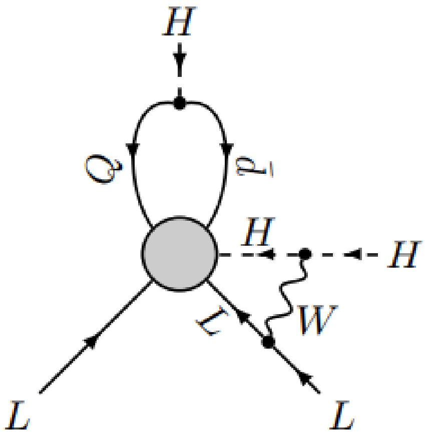

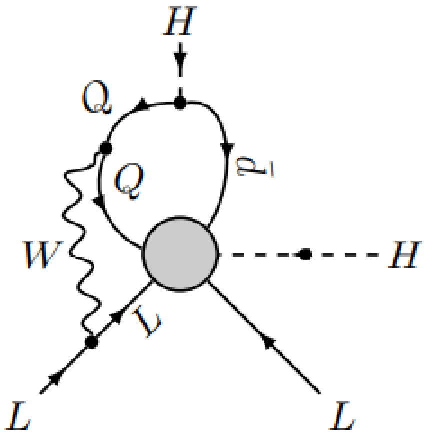

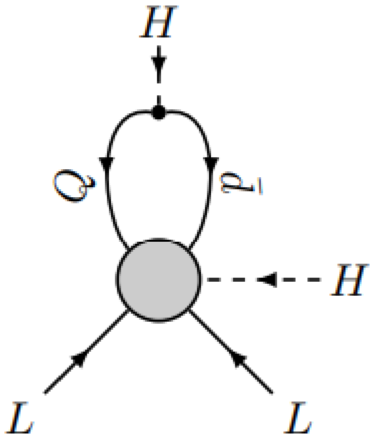

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- Isospin structure prevents 1-loop neutrino masses for O_{3a} and O_{4b}
- Weinberg operator can be estimated by closing off the loops of the loop diagrams without specifying UV model
- This is **NOT** an estimate within EFT, but the estimate for the size of the loop-induced Weinberg operator from matching of the UV theory.

Simple estimates for $\Delta L = 2$ operators

Neutrino masses

Classification in terms of effective $\Delta L = 2$ operators

Babu, Leung hep-ph/0106054; deGouvea, Jenkins 0708.1344 Bonnet, Hernandez, Ota, Winter 0907.3143

$$m_\nu \simeq \frac{c_R v^2}{(16\pi^2)^l \Lambda}\text{, with}\qquad c_R \simeq \prod_i g_i \times \epsilon \times \left(\frac{v^2}{\Lambda^2}\right)^n$$

Loop factor

μ/Λ

$LLHH(H^\dagger H)^n$

$$m_\nu \gtrsim 0.05\text{eV} \Rightarrow \begin{cases} l=1 \to \Lambda < 10^{12}\text{GeV} \\ l=2 \to \Lambda < 10^{10}\text{GeV} \\ l=3 \to \Lambda < 10^{8}\text{GeV} \end{cases}$$

$\to$ no information on $\Delta L = 0$ processes

Systematic construction of models

Angel, Rodd, Volkas 1212.5862; Cai, Clarke, MS, Volkas 1308.0463; Gargalionis, Volkas 2009.13537
Bonnet, Hirsch, Ota, Winter 1204.5862; Aristizabal Sierra, Degee, Dorame, Hirsch 1411.7038; Cepedello, Fonseca, Hirsch 1807.00629

Gargalionis, Volkas 2009.13537: "exploding! $\Delta L = 2$ operators" ..."1000s of models"

$\to$ a large number of models!

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Simplified models

$\Delta L = 2$ operators

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- no information about other processes

UV models

- enables to study phenomenology
- too many models! not enough time

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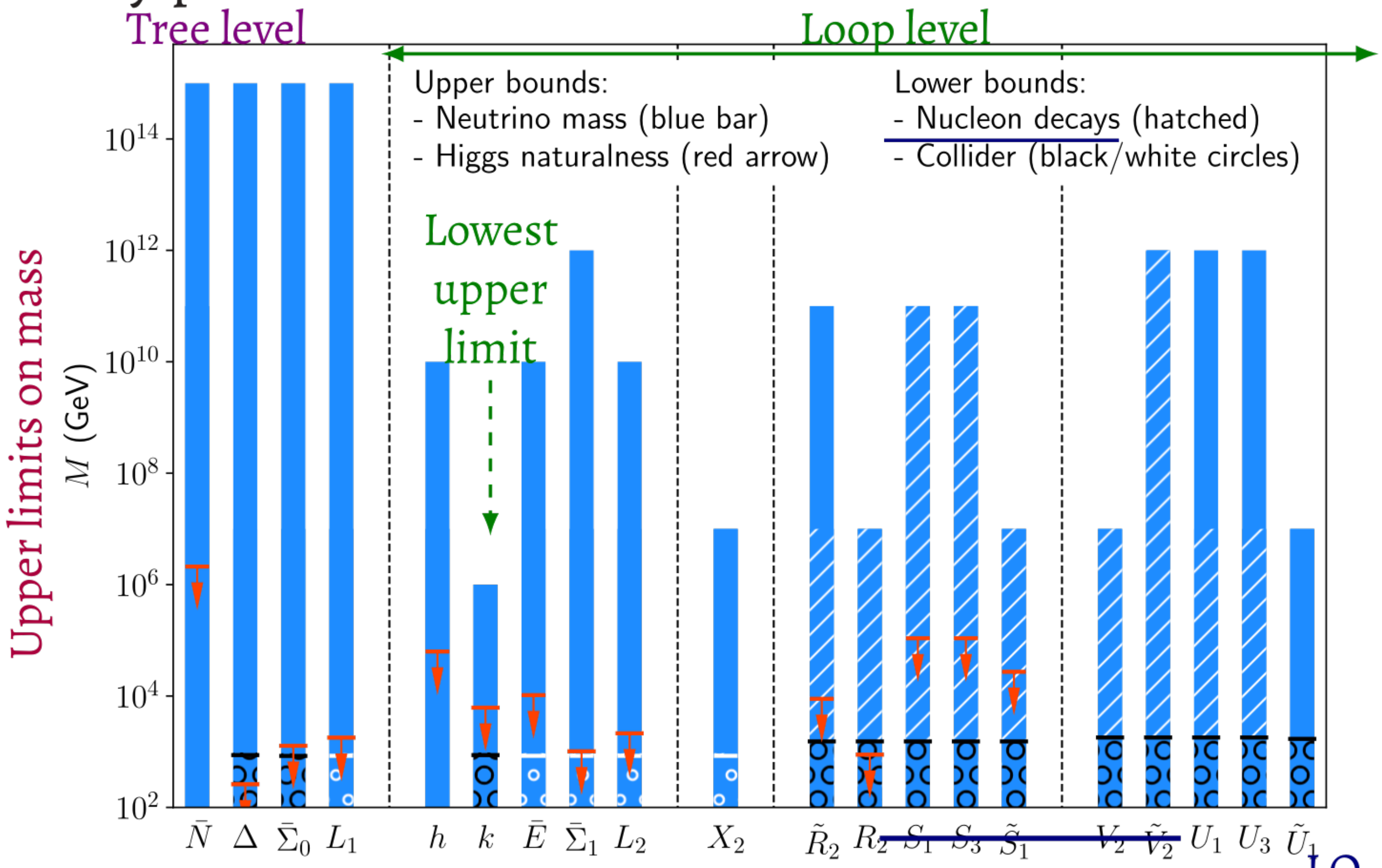
Idea:

1. neutrino mass requires at least one new particle X (mass M) coupled to SM lepton(s), carrying L (and maybe B)
2. QFT: L is violated (by two units) via new operators at scale Λ which encode the (model-dependent) UV physics
3. Majorana neutrino masses $m_\nu \propto \Lambda^{-1}$ are generated
4. $m_\nu > 0.05$ eV $\& M < \Lambda$ provide conservative upper bound on M
5. L-conserving phenomenology mostly determined by renormalisable $\Delta L = 0$ operators

Summary plot

Juan Herrero-García, MS 1903.10552 [Eur.Phys.J. C79 (2019) no.11, 938]



Neutrino masses involve one of these 20 new particles.

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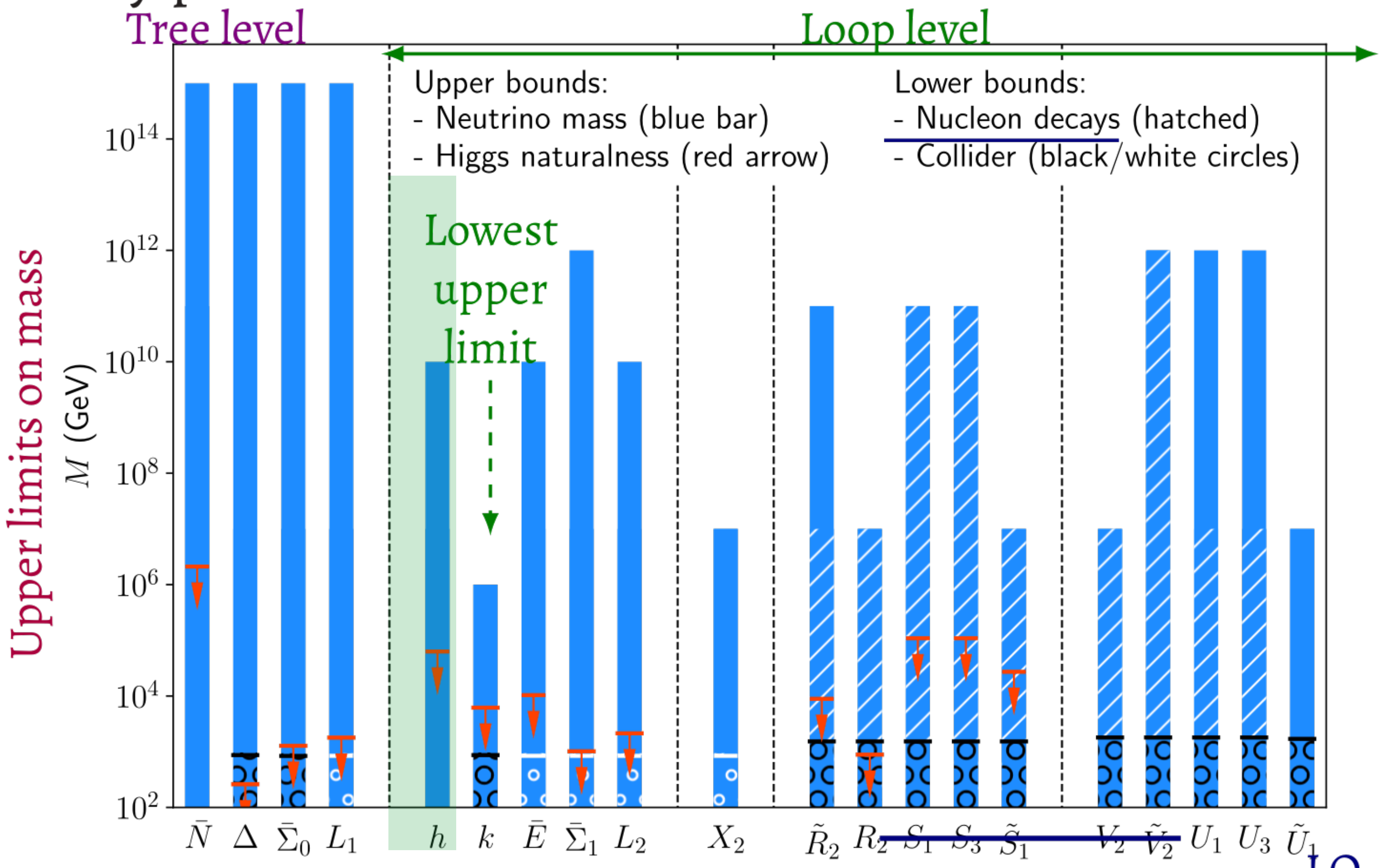

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Questions?