Introduction to Neutrino Physics

Michael Schmidt

UNSW Sydney

23 September 2024

Introduction

LH and RH neutrinos

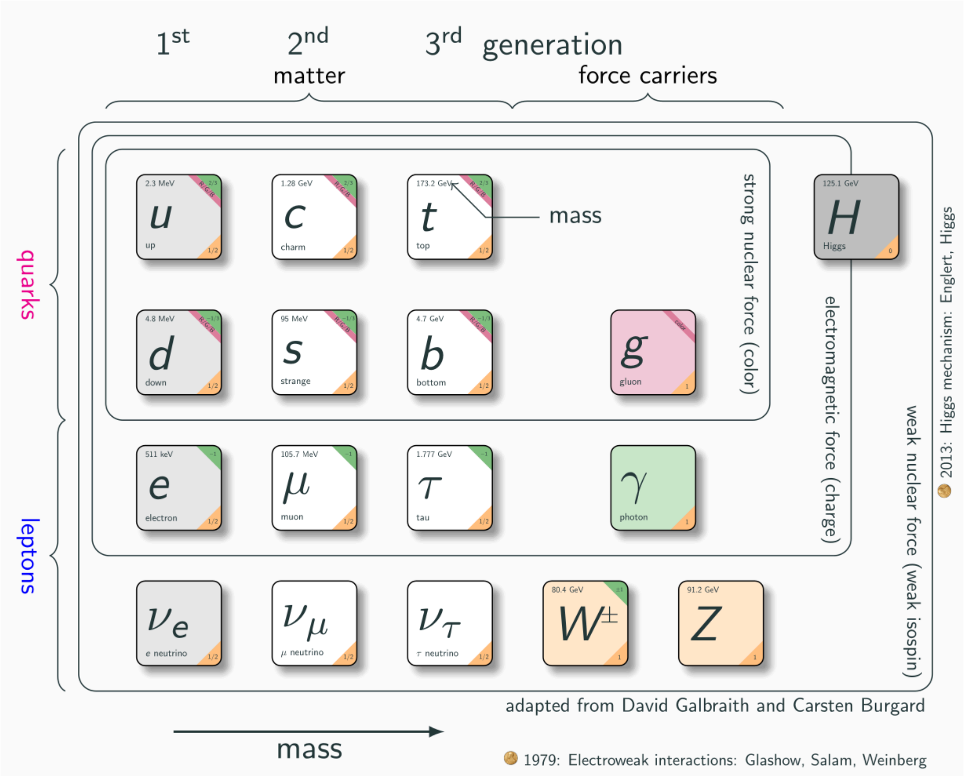
In the Standard Model there are only left-handed (LH) neutrinos which are part of electroweak lepton doublets L_α

$$L_e = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad L_\mu = \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad L_\tau = \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

LH neutrinos are negative chirality states

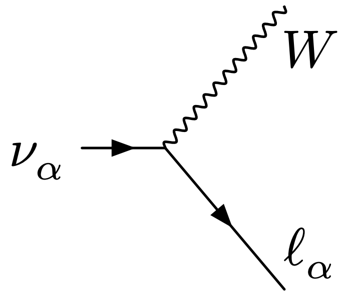
$$\gamma_5 \nu_L = -\nu_L$$

For **massless neutrinos**, negative chirality states are negative helicity states



helicity of (anti)particles: $h^{(v)} = (-)\frac{1}{2}\boldsymbol{\sigma} \cdot \hat{\boldsymbol{p}}$

LH and RH neutrinos



For massive neutrinos:

Negative chirality states have small admixture of positive helicity state.

Dirac neutrinos

	helicity	ℓ^- prod.	ℓ^+ prod.
ν	$-\frac{1}{2}$	1	0
$\overline{\nu}$	$-\frac{1}{2}$	0	$\left(\frac{m}{E}\right)^2 \ll 1$
ν	$\frac{1}{2}$	$\left(\frac{m}{E}\right)^2 \ll 1$	0
$\overline{\nu}$	$\frac{1}{2}$	0	1

Majorana neutrinos

Neutrinos may be their own antiparticles, i.e. $\nu = \nu^c$

helic-ity	ℓ^- prod.	ℓ^+ prod.
$-\frac{1}{2}$	1	$\left(\frac{m}{E}\right)^2$
$\frac{1}{2}$	$\left(\frac{m}{E}\right)^2$	1

LH and RH neutrinos

When SM was formulated, neutrinos were considered massless $\rightarrow$ RH neutrinos not needed

$$-\mathcal{L} = \overline{L_L} H Y_e e_R + \overline{Q_L} H Y_d d_R + \overline{Q_L} \tilde{H} Y_u u_R + \text{h.c.} \qquad \text{with } \tilde{H} \equiv \varepsilon H^*$$

Neutrino masses may be introduced similar to charged fermions:

$$-\mathcal{L}_\nu = \overline{L_L} \tilde{H} Y_\nu \nu_R + \text{h.c.}$$

Q: How do RH neutrinos transform with SM gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$?

LH and RH neutrinos

When SM was formulated, neutrinos were considered massless $\to$ RH neutrinos not needed

$$-\mathcal{L} = \overline{L_L} H Y_e e_R + \overline{Q_L} H Y_d d_R + \overline{Q_L} \tilde{H} Y_u u_R + \text{h.c.} \qquad \text{with } \tilde{H} \equiv \varepsilon H^*$$

Neutrino masses may be introduced similar to charged fermions:

$$-\mathcal{L}_\nu = \overline{L_L} \tilde{H} Y_\nu \nu_R + \text{h.c.}$$

Q: How do RH neutrinos transform with SM gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$?

They are **gauge singlets** $\nu_R \sim (1, 1, 0)$.

LH and RH neutrinos

When SM was formulated, neutrinos were considered massless → RH neutrinos not needed

$$-\mathcal{L} = \overline{L_L} H Y_e e_R + \overline{Q_L} H Y_d d_R + \overline{Q_L} \tilde{H} Y_u u_R + \text{h.c.} \qquad \text{with } \tilde{H} \equiv \varepsilon H^*$$

Neutrino masses may be introduced similar to charged fermions:

$$-\mathcal{L}_\nu = \overline{L_L} \tilde{H} Y_\nu \nu_R + \text{h.c.}$$

Q: How do RH neutrinos transform with SM gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times U(1)_Y$?

They are **gauge singlets** $\nu_R \sim (1, 1, 0)$.

The number of RH neutrinos is **not** fixed. At least two RH neutrinos are required to explain the observed neutrino oscillations.

nomenclature

- ν_R usually called *sterile* neutrinos, since they are gauge singlets
- ν_L usually called *active* neutrinos, since they participate in electroweak interactions

Dirac or Majorana neutrinos?

A central question when discussing neutrinos is

Dirac or Majorana neutrinos?

Assume neutrino masses are generated like charged fermion masses. Then, after electroweak symmetry breaking

$$-\overline{L}\tilde{H}Y_\nu \nu_R + \text{h.c.} \xrightarrow{\langle H\rangle = \frac{1}{\sqrt{2}}\binom{0}{v}} -\overline{\nu_L} M_\nu^D \nu_R + \text{h.c.} \qquad \text{with } M_\nu^D = Y_\nu \frac{v}{\sqrt{2}}$$

This is a Dirac mass term and, thus, the neutrino is a Dirac particle with 4 spinor components, 2 from ν_L and 2 from ν_R, like all the charged fermions in the SM.

We can define the (total) lepton number

$$\boxed{L = L_e + L_\mu + L_\tau}$$

L_α are individual lepton numbers

Dirac or Majorana neutrinos?

We can define the (total) lepton number

$$L = L_e + L_\mu + L_\tau$$

L_α are individual lepton numbers

- All LH leptons and RH charged leptons have $L = +1$
- All leptonic anti-particles have lepton number $L = -1$
- Quarks, gauge bosons and the Higgs have lepton number $L = 0$
- All terms of the SM conserve lepton number, in particular

$$-\mathcal{L} = \overline{L_L} H Y_e e_R + \overline{L_L} \tilde{H} Y_\nu \nu_R + \text{h.c.}$$

Dirac or Majorana neutrinos?

Since neutrinos are electrically neutral and colorless, there is another possibility – they can be their own antiparticles

$$\boxed{\nu = \nu^c \equiv C\overline{\nu}^T} \qquad C = i\gamma^2\gamma^0$$

Such particles are called Majorana particles: LH and RH neutrino fields are **not** independent

$$\nu_L^c \equiv (\nu_L)^c$$

Dirac or Majorana neutrinos?

Since neutrinos are electrically neutral and colorless, there is another possibility – they can be their own antiparticles

$$\boxed{\nu = \nu^c \equiv C\overline{\nu}^T} \qquad C = i\gamma^2\gamma^0$$

Such particles are called Majorana particles: LH and RH neutrino fields are **not** independent

$$\nu_L^c \equiv (\nu_L)^c = (P_L\nu)^c$$

Dirac or Majorana neutrinos?

Since neutrinos are electrically neutral and colorless, there is another possibility – they can be their own antiparticles

$$\boxed{\nu = \nu^c \equiv C\overline{\nu}^T} \qquad C = i\gamma^2\gamma^0$$

Such particles are called Majorana particles: LH and RH neutrino fields are **not** independent

$$\nu_L^c \equiv (\nu_L)^c = (P_L\nu)^c = C\overline{P_L\nu}^T$$

Dirac or Majorana neutrinos?

Since neutrinos are electrically neutral and colorless, there is another possibility – they can be their own antiparticles

$$\boxed{\nu = \nu^c \equiv C\overline{\nu}^T} \qquad C = i\gamma^2\gamma^0$$

Such particles are called Majorana particles: LH and RH neutrino fields are **not** independent

$$\nu_L^c \equiv (\nu_L)^c = (P_L\nu)^c = C\overline{P_L\nu}^T = C\gamma^0 P_L \nu^*$$

Dirac or Majorana neutrinos?

Since neutrinos are electrically neutral and colorless, there is another possibility – they can be their own antiparticles

$$\boxed{\nu = \nu^c \equiv C\overline{\nu}^T} \qquad\qquad C = i\gamma^2\gamma^0$$

Such particles are called Majorana particles: LH and RH neutrino fields are **not** independent

$$\nu_L^c \equiv (\nu_L)^c = (P_L\nu)^c = C\overline{P_L\nu}^T = C\gamma^0 P_L \nu^* = P_R C\gamma^0\nu^* = P_R\nu^c$$

A Majorana particle has only 2 independent components and not 4.

Using this, we can write down a Majorana mass term

$$-\mathcal{L} = \frac{1}{2}\overline{\nu_L^c} M_\nu^M \nu_L + \text{h.c.} \qquad \text{with} \qquad M_\nu^M = \left(M_\nu^M\right)^T$$

- any Majorana mass term **breaks** lepton number by two units: $\Delta L = 2$
- it **cannot** arise from a renormalisable gauge invariant operator in the SM (see lecture 2)

Neutrino interactions

The charged lepton mass term is

$$-\mathcal{L} \supset \overline{e_L} M_e e_R + \text{h.c.}$$

$$\left.\begin{matrix} e_L \to L_e e_L \\ e_R \to R_e e_R \end{matrix}\right\} \Rightarrow \quad M_e \to L_e^\dagger M_e R_e = \text{diag}$$

Dirac neutrinos

$$-\mathcal{L} \supset \overline{\nu_L} M_\nu^D \nu_R + \text{h.c.}$$

$$\left.\begin{matrix} \nu_L \to L_\nu \nu_L \\ \nu_R \to R_\nu \nu_R \end{matrix}\right\} \Rightarrow L_\nu^\dagger M_\nu^D R_\nu = \text{diag}$$

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} W_\mu^- \overline{\ell_L} \gamma^\mu \nu_L - \frac{g}{\cos\theta_w} Z_\mu \overline{\nu_L} \gamma^\mu \nu_L$$

Neutrino interactions

The charged lepton mass term is

$$-\mathcal{L} \supset \overline{e_L} M_e e_R + \text{h.c.}$$

$$\left.\begin{aligned} e_L &\to L_e e_L \\ e_R &\to R_e e_R \end{aligned}\right\} \Rightarrow \quad M_e \to L_e^\dagger M_e R_e = \text{diag}$$

Dirac neutrinos

$$-\mathcal{L} \supset \overline{\nu_L} M_\nu^D \nu_R + \text{h.c.}$$

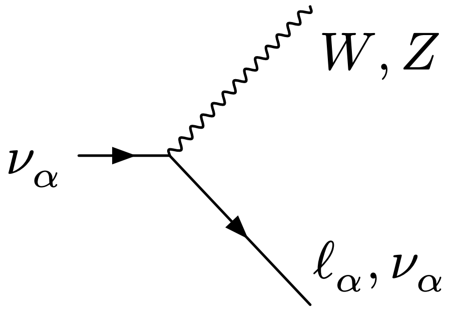
$$\left.\begin{aligned} \nu_L &\to L_\nu \nu_L \\ \nu_R &\to R_\nu \nu_R \end{aligned}\right\} \Rightarrow L_\nu^\dagger M_\nu^D R_\nu = \text{diag}$$

Majorana neutrinos

$$-\mathcal{L} \supset \overline{\nu_L} M_M \nu_L^c + \text{h.c.}$$

$$\nu_L \to L_\nu \nu_L \Rightarrow L_\nu^\dagger M_\nu^M L_\nu^* = \text{diag}$$

$$\begin{aligned} \mathcal{L} &= -\frac{g}{2\sqrt{2}} W_\mu^- \overline{\ell_L} \gamma^\mu \nu_L - \frac{g}{\cos\theta_w} Z_\mu \overline{\nu_L} \gamma^\mu \nu_L \\ &\to -\frac{g}{2\sqrt{2}} W_\mu^- \overline{\ell_L} \gamma^\mu \underbrace{L_e^\dagger L_\nu}_{U_{\text{PMNS}}} \nu_L - \frac{g}{\cos\theta_w} Z_\mu \overline{\nu_L} \gamma^\mu \underbrace{L_\nu^\dagger L_\nu}_{\mathbf{1}} \nu_L \end{aligned}$$



Neutrino interactions

The **PMNS (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix** is defined as

$$U_{\mathrm{PMNS}} = L_e^\dagger L_\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12})}_{\hat{U}_{\mathrm{PMNS}}} P(\alpha_1, \alpha_2)$$

and relates the flavour eigenstates $\hat{\nu}_{L\alpha}$ with the mass eigenstates ν_{Li}: $\hat{\nu}_{L\alpha} = (U_{\mathrm{PMNS}})_{\alpha i} \nu_{Li}$

The PMNS matrix is parameterised in terms of 3 angles θ_{ij} and 1 Dirac CP phase δ. For Majorana neutrinos there are 2 additional Majorana phases α_i.

- R_{ij} is a rotation in the i-j plane by θ_{ij}, similarly U_{ij} with $\pm s_{13} \to \pm s_{13} e^{\mp i\delta}$, for example

$$U_{13} = \begin{pmatrix} c_{13} & & s_{13} e^{-i\delta} \\ & 1 & \\ -s_{13} e^{i\delta} & & c_{13} \end{pmatrix}$$

- The Majorana phases α_i are multiplied from the right as $\quad P = \mathrm{diag}\left(e^{\frac{i}{2}\alpha_1}, e^{\frac{i}{2}\alpha_2}, 1\right)$

Neutrino interactions

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha_1} & & \\ & e^{\frac{i}{2}\alpha_2} & \\ & & 1 \end{pmatrix}$$

Commonly, the mixing angles are denoted

- solar mixing angle $\theta_{12} \sim 34°$
- atmospheric mixing angle $\theta_{23} \sim 45°$
- reactor mixing angle $\theta_{13} \sim 8.5°$

Neutrino interactions

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha_1} & & \\ & e^{\frac{i}{2}\alpha_2} & \\ & & 1 \end{pmatrix}$$

Commonly, the mixing angles are denoted

- solar mixing angle $\theta_{12} \sim 34°$
- atmospheric mixing angle $\theta_{23} \sim 45°$
- reactor mixing angle $\theta_{13} \sim 8.5°$

There is currently only a hint for a non-zero Dirac CP phase δ. It can also be expressed in terms of the **Jarlskog invariant**

$$J_{\mathrm{CP}} = \frac{1}{8}\sin(2\theta_{12})\sin(2\theta_{23})\sin(2\theta_{13})\cos(\theta_{13})\sin(\delta)$$

Neutrino interactions

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha_1} & & \\ & e^{\frac{i}{2}\alpha_2} & \\ & & 1 \end{pmatrix}$$

Commonly, the mixing angles are denoted

- solar mixing angle $\theta_{12} \sim 34°$
- atmospheric mixing angle $\theta_{23} \sim 45°$
- reactor mixing angle $\theta_{13} \sim 8.5°$

There is currently only a hint for a non-zero Dirac CP phase δ. It can also be expressed in terms of the **Jarlskog invariant**

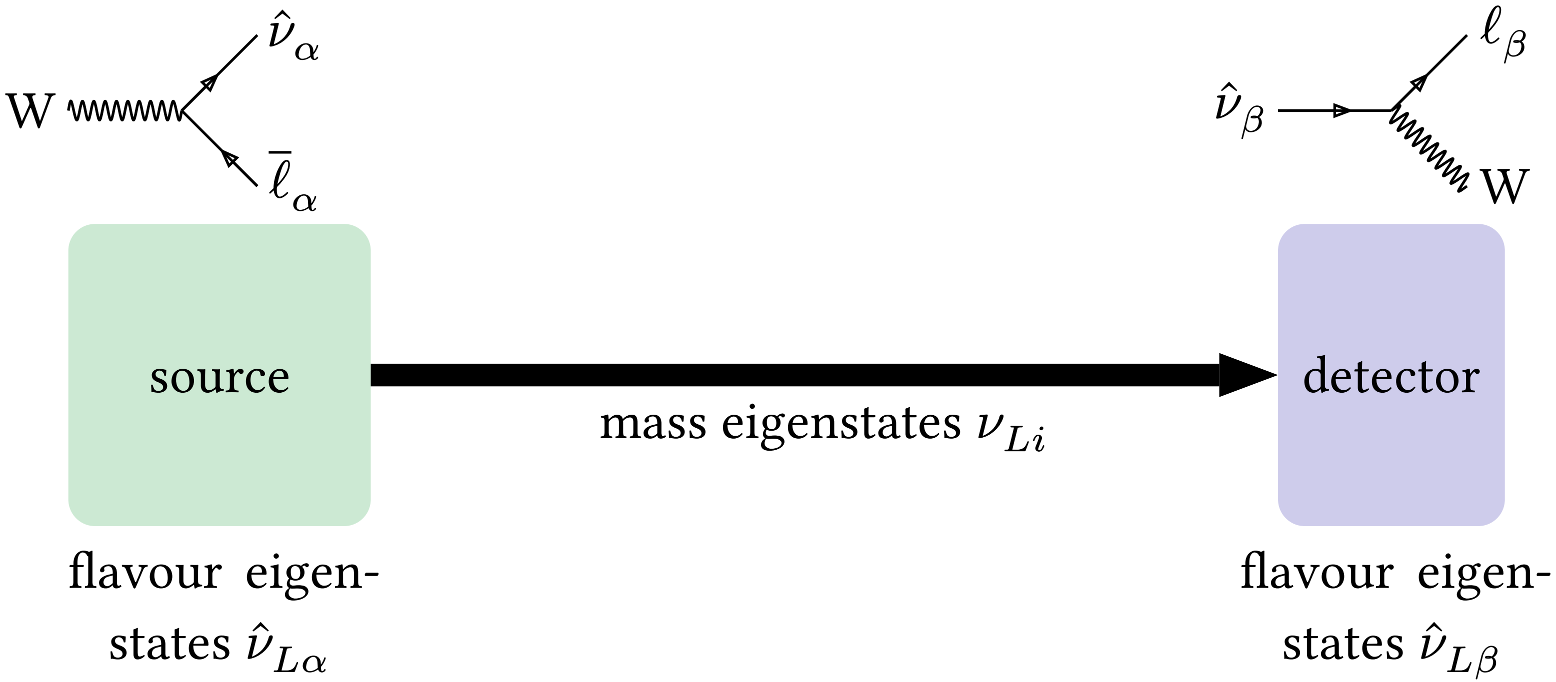
$$J_{\mathrm{CP}} = \frac{1}{8}\sin(2\theta_{12})\sin(2\theta_{23})\sin(2\theta_{13})\cos(\theta_{13})\sin(\delta)$$

If neutrinos are *Majorana particles*, there are two additional phases, the so-called **Majorana phases α_i**.

If one of the neutrinos is massless, there is only one physical Majorana phase.

Neutrino oscillations

Basic picture



Neutrinos are produced and detected as flavour eigenstates and propagate as mass eigenstates.

The same picture applies for antineutrinos.

Two-flavour oscillations

We consider **two-flavour neutrino oscillations in vacuum**. The PMNS matrix is parameterised by one angle θ and possibly a Majorana phase α

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha} & \\ & 1 \end{pmatrix}$$

We denote the **flavour** eigenstate ν_{Le} and $\nu_{L\mu}$ and the **mass** eigenstates ν_{L1} and ν_{L2}:

$$|\nu_{Le}\rangle = c_\theta e^{\frac{i}{2}\alpha}|\nu_{L1}\rangle + s_\theta|\nu_{L2}\rangle$$

$$|\nu_{L\mu}\rangle = -s_\theta e^{\frac{i}{2}\alpha}|\nu_{L1}\rangle + c_\theta|\nu_{L2}\rangle$$

Two-flavour oscillations

We consider **two-flavour neutrino oscillations in vacuum**. The PMNS matrix is parameterised by one angle θ and possibly a Majorana phase α

$$U_{\text{PMNS}} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha} & \\ & 1 \end{pmatrix}$$

We denote the **flavour** eigenstate ν_{Le} and $\nu_{L\mu}$ and the **mass** eigenstates ν_{L1} and ν_{L2}:

$$\begin{aligned} |\nu_{Le}\rangle &= c_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + s_\theta |\nu_{L2}\rangle \\ |\nu_{L\mu}\rangle &= -s_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + c_\theta |\nu_{L2}\rangle \end{aligned}$$

We use **plane waves in quantum mechanics** for the calculation of neutrino oscillations.

Two-flavour oscillations

We consider **two-flavour neutrino oscillations in vacuum**. The PMNS matrix is parameterised by one angle θ and possibly a Majorana phase α

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} e^{\frac{i}{2}\alpha} & \\ & 1 \end{pmatrix}$$

We denote the **flavour** eigenstate ν_{Le} and $\nu_{L\mu}$ and the **mass** eigenstates ν_{L1} and ν_{L2}:

$$|\nu_{Le}\rangle = c_\theta e^{\frac{i}{2}\alpha}|\nu_{L1}\rangle + s_\theta|\nu_{L2}\rangle$$
$$|\nu_{L\mu}\rangle = -s_\theta e^{\frac{i}{2}\alpha}|\nu_{L1}\rangle + c_\theta|\nu_{L2}\rangle$$

We use **plane waves in quantum mechanics** for the calculation of neutrino oscillations.

What is the probability to detect a muon neutrino $\nu_{L\mu}$ in the detector if the source produced an electron neutrino ν_{Le}?

Two-flavour oscillations

The probability to detect $\nu_{L\mu}$ at time T from a ν_{Le} produced in the source is

$$P(\nu_e \to \nu_\mu) = |\langle \nu_{L\mu} | \nu_{Le}(T) \rangle|^2$$

flavour eigenstates

$$|\nu_{Le}\rangle = c_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + s_\theta |\nu_{L2}\rangle$$
$$|\nu_{L\mu}\rangle = -s_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + c_\theta |\nu_{L2}\rangle$$

Two-flavour oscillations

The probability to detect $\nu_{L\mu}$ at time T from a ν_{Le} produced in the source is

$$P(\nu_e \to \nu_\mu) = |\langle \nu_{L\mu} | \nu_{Le}(T) \rangle|^2$$

flavour eigenstates

$$|\nu_{Le}\rangle = c_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + s_\theta |\nu_{L2}\rangle$$
$$|\nu_{L\mu}\rangle = -s_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + c_\theta |\nu_{L2}\rangle$$

In the mass basis the Hamiltonian is diagonal and thus time evolution is described by

$$|\nu_{Li}(t)\rangle = e^{-iE_i t} |\nu_{Li}\rangle$$

Hence we find for the time-evolved electron neutrino state

$$|\nu_{Le}(t)\rangle = c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 t} |\nu_{L1}\rangle + s_\theta e^{-iE_2 t} |\nu_{L2}\rangle$$

Two-flavour oscillations

The probability to detect $\nu_{L\mu}$ at time T from a ν_{Le} produced in the source is

$$P(\nu_e \to \nu_\mu) = |\langle \nu_{L\mu} | \nu_{Le}(T) \rangle|^2$$

flavour eigenstates

$$|\nu_{Le}\rangle = c_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + s_\theta |\nu_{L2}\rangle$$
$$|\nu_{L\mu}\rangle = -s_\theta e^{\frac{i}{2}\alpha} |\nu_{L1}\rangle + c_\theta |\nu_{L2}\rangle$$

In the mass basis the Hamiltonian is diagonal and thus time evolution is described by

$$|\nu_{Li}(t)\rangle = e^{-iE_i t} |\nu_{Li}\rangle$$

Hence we find for the time-evolved electron neutrino state

$$|\nu_{Le}(t)\rangle = c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 t} |\nu_{L1}\rangle + s_\theta e^{-iE_2 t} |\nu_{L2}\rangle$$

and consequently the probability amplitude is

$$\langle \nu_{L\mu} | \nu_{Le}(T) \rangle = \left(-s_\theta e^{-\frac{i}{2}\alpha} \langle \nu_{L1}| + c_\theta \langle \nu_{L2}| \right) \left(c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 T} |\nu_{L1}\rangle + s_\theta e^{-iE_2 T} |\nu_{L2}\rangle \right)$$

Two-flavour oscillations

The probability amplitude is

$$\left\langle \nu_{L\mu} \middle| \nu_{Le}(T) \right\rangle = \left(-s_\theta e^{-\frac{i}{2}\alpha} \langle \nu_{L1}| + c_\theta \langle \nu_{L2}|\right)\left(c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 T} |\nu_{L1}\rangle + s_\theta e^{-iE_2 T} |\nu_{L2}\rangle\right)$$

Using orthogonality of the mass eigenstates $\left\langle \nu_{Li} \middle| \nu_{Lj} \right\rangle = \delta_{ij}$ we find

Two-flavour oscillations

The probability amplitude is

$$\left\langle \nu_{L\mu} \middle| \nu_{Le}(T) \right\rangle = \left(-s_\theta e^{-\frac{i}{2}\alpha} \langle \nu_{L1}| + c_\theta \langle \nu_{L2}|\right)\left(c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 T} |\nu_{L1}\rangle + s_\theta e^{-iE_2 T} |\nu_{L2}\rangle\right)$$

Using orthogonality of the mass eigenstates $\left\langle \nu_{Li} \middle| \nu_{Lj} \right\rangle = \delta_{ij}$ we find

$$\left\langle \nu_{L\mu} \middle| \nu_{Le}(T) \right\rangle = -s_\theta c_\theta e^{-iE_1 T} + c_\theta s_\theta e^{-iE_2 T}$$

Note that the Majorana phase α dropped out! This is a general result: Majorana phases generally drop out since they always show up together with a mass eigenstate $e^{\frac{i}{2}\alpha_i} |\nu_{Li}\rangle$ and thus drop out of the probability amplitude due to the orthogonality of the mass eigenstates.

Two-flavour oscillations

The probability amplitude is

$$\left\langle \nu_{L\mu} \middle| \nu_{Le}(T) \right\rangle = \left(-s_\theta e^{-\frac{i}{2}\alpha} \langle \nu_{L1}| + c_\theta \langle \nu_{L2}|\right)\left(c_\theta e^{\frac{i}{2}\alpha} e^{-iE_1 T} |\nu_{L1}\rangle + s_\theta e^{-iE_2 T} |\nu_{L2}\rangle\right)$$

Using orthogonality of the mass eigenstates $\left\langle \nu_{Li} \middle| \nu_{Lj} \right\rangle = \delta_{ij}$ we find

$$\left\langle \nu_{L\mu} \middle| \nu_{Le}(T) \right\rangle = -s_\theta c_\theta e^{-iE_1 T} + c_\theta s_\theta e^{-iE_2 T}$$

Note that the Majorana phase α dropped out! This is a general result: Majorana phases generally drop out since they always show up together with a mass eigenstate $e^{\frac{i}{2}\alpha_i} |\nu_{Li}\rangle$ and thus drop out of the probability amplitude due to the orthogonality of the mass eigenstates.

The oscillation probability is thus given by

$$P\left(\nu_e \to \nu_\mu\right) = \left| - s_\theta c_\theta e^{-iE_1 T} + c_\theta s_\theta e^{-iE_2 T} \right|^2$$

Two-flavour oscillations

The probability amplitude is

$$\left\langle\nu_{L\mu}\middle|\nu_{Le}(T)\right\rangle = \left(-s_\theta e^{-\frac{i}{2}\alpha}\langle\nu_{L1}| + c_\theta\langle\nu_{L2}|\right)\left(c_\theta e^{\frac{i}{2}\alpha}e^{-iE_1T}|\nu_{L1}\rangle + s_\theta e^{-iE_2T}|\nu_{L2}\rangle\right)$$

Using orthogonality of the mass eigenstates $\left\langle\nu_{Li}\middle|\nu_{Lj}\right\rangle = \delta_{ij}$ we find

$$\left\langle\nu_{L\mu}\middle|\nu_{Le}(T)\right\rangle = -s_\theta c_\theta e^{-iE_1T} + c_\theta s_\theta e^{-iE_2T}$$

Note that the Majorana phase α dropped out! This is a general result: Majorana phases generally drop out since they always show up together with a mass eigenstate $e^{\frac{i}{2}\alpha_i}|\nu_{Li}\rangle$ and thus drop out of the probability amplitude due to the orthogonality of the mass eigenstates.

The oscillation probability is thus given by

$$\begin{aligned} P\left(\nu_e \to \nu_\mu\right) &= |-s_\theta c_\theta e^{-iE_1T} + c_\theta s_\theta e^{-iE_2T}|^2 \\ &= s_\theta^2 c_\theta^2 \mid 1 - e^{-i(E_2-E_1)T} \mid^2 \end{aligned}$$

Two-flavour oscillations

$$P\left(\nu_e \to \nu_\mu\right) = s_\theta^2 c_\theta^2 \;|\; 1 - e^{-i(E_2-E_1)T} \;\;|^2$$

Two-flavour oscillations

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) &= s_\theta^2 c_\theta^2 \mid 1 - e^{-i(E_2-E_1)T} \mid^2 \\
&= \frac{\sin^2(2\theta)}{4}\left(2 - 2\cos\left(2\frac{(E_2-E_1)T}{2}\right)\right)
\end{aligned}
$$

Two-flavour oscillations

$$
\begin{aligned}
P\left(\nu_e \to \nu_\mu\right) &= s_\theta^2 c_\theta^2 \mid 1 - e^{-i(E_2 - E_1)T} \mid^2 \\
&= \frac{\sin^2(2\theta)}{4} \left(2 - 2\cos\left(2\frac{(E_2 - E_1)T}{2}\right)\right) \\
&= \sin^2(2\theta) \sin^2\left(\frac{(E_2 - E_1)T}{2}\right)
\end{aligned}
$$

In the ultra-relativistic limit ($E \gg m_i$), the neutrino energy difference

$$
E_2 - E_1 = \sqrt{\boldsymbol{p}_2^2 + m_2^2} - \sqrt{\boldsymbol{p}_1^2 + m_1^2} \cong \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E} \quad \text{using} \quad E \simeq E_i \simeq |\boldsymbol{p}_i|
$$

Two-flavour oscillations

$$P(\nu_e \to \nu_\mu) = s_\theta^2 c_\theta^2 \mid 1 - e^{-i(E_2 - E_1)T} \mid^2$$
$$= \frac{\sin^2(2\theta)}{4}\left(2 - 2\cos\left(2\frac{(E_2 - E_1)T}{2}\right)\right)$$
$$= \sin^2(2\theta)\sin^2\left(\frac{(E_2 - E_1)T}{2}\right)$$

In the ultra-relativistic limit ($E \gg m_i$), the neutrino energy difference

$$E_2 - E_1 = \sqrt{\boldsymbol{p}_2^2 + m_2^2} - \sqrt{\boldsymbol{p}_1^2 + m_1^2} \cong \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E} \quad \text{using} \quad E \simeq E_i \simeq |\boldsymbol{p}_i|$$

Thus the oscillation probability becomes ($T \cong L$)

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Two-flavour oscillations

$$P(\nu_e \to \nu_\mu) = s_\theta^2 c_\theta^2 \mid 1 - e^{-i(E_2 - E_1)T} \mid^2$$
$$= \frac{\sin^2(2\theta)}{4}\left(2 - 2\cos\left(2\frac{(E_2 - E_1)T}{2}\right)\right)$$
$$= \sin^2(2\theta)\sin^2\left(\frac{(E_2 - E_1)T}{2}\right)$$

In the ultra-relativistic limit ($E \gg m_i$), the neutrino energy difference

$$E_2 - E_1 = \sqrt{\boldsymbol{p}_2^2 + m_2^2} - \sqrt{\boldsymbol{p}_1^2 + m_1^2} \cong \frac{m_2^2 - m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E} \quad \text{using} \quad E \simeq E_i \simeq |\boldsymbol{p}_i|$$

Thus the oscillation probability becomes ($T \cong L$)

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) = \sin^2(2\theta)\sin^2\left(1.27\frac{\Delta m_{21}^2[\mathrm{eV}^2]L[\mathrm{km}]}{E[\mathrm{GeV}]}\right)$$

Two-flavour oscillations

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Non-zero oscillation probability requires

- non-zero mass squared difference Δm_{21}^2
  - there is no flavour change for **vanishing or degenerate masses**

Two-flavour oscillations

Two-flavour oscillations

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Non-zero oscillation probability requires

- non-zero mass squared difference Δm_{21}^2
  - there is no flavour change for **vanishing or degenerate masses**
- non-zero mixing angle θ, i.e. neutrino flavour eigenstates cannot be and mass eigenstates

Two-flavour oscillations

Two-flavour oscillations

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Non-zero oscillation probability requires

- non-zero mass squared difference Δm_{21}^2
  - there is no flavour change for **vanishing or degenerate masses**
- non-zero mixing angle θ, i.e. neutrino flavour eigenstates cannot be and mass eigenstates
- a finite distance L between source and detector

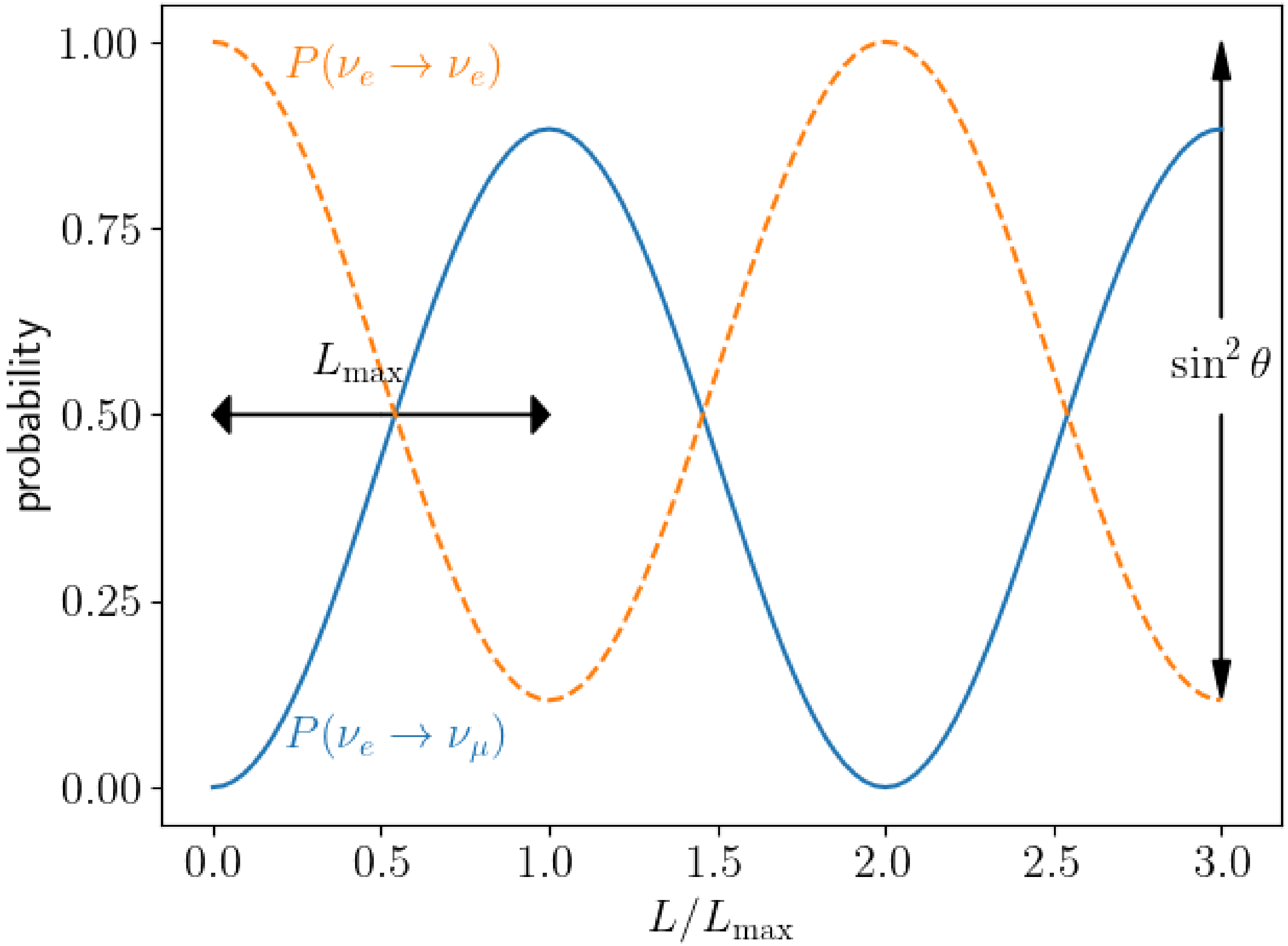
Maximum flavour conversion probability may be reached for **maximal mixing** $\boldsymbol{\theta = \frac{\pi}{4}}$

Conservation of probability implies $P(\nu_e \to \nu_e) + P(\nu_e \to \nu_\mu) = 1$ and thus

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

Similar expressions are obtained for the other oscillation probabilities with $\nu_e \leftrightarrow \nu_\mu$

Two-flavour oscillations



- oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

- 1st oscillation maximum

$$L_{\text{max}} = \frac{2\pi E}{\Delta m^2_{21}}$$

- maximum osc probability

$$P_{\text{max}} = \sin^2(2\theta)$$

Two-flavour oscillations

- degeneracies

$$\Delta m_{21}^2 \to -\Delta m_{21}^2 \quad \text{and} \quad \theta \to \frac{\pi}{2} - \theta$$

oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- antineutrinos

$$\left|\overline{\nu}_{Le}\right\rangle = c_\theta e^{-\frac{i}{2}\alpha}\left|\overline{\nu}_{L1}\right\rangle + s_\theta\left|\overline{\nu}_{L2}\right\rangle$$

$$\left|\overline{\nu}_{L\mu}\right\rangle = -s_\theta e^{-\frac{i}{2}\alpha}\left|\overline{\nu}_{L1}\right\rangle + c_\theta\left|\overline{\nu}_{L2}\right\rangle$$

Two-flavour oscillations

- degeneracies

$$\Delta m_{21}^2 \to -\Delta m_{21}^2 \quad \text{and} \quad \theta \to \frac{\pi}{2} - \theta$$

- CPT: Always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$$

oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- antineutrinos

$$|\overline{\nu}_{Le}\rangle = c_\theta e^{-\frac{i}{2}\alpha} |\overline{\nu}_{L1}\rangle + s_\theta |\overline{\nu}_{L2}\rangle$$

$$|\overline{\nu}_{L\mu}\rangle = -s_\theta e^{-\frac{i}{2}\alpha} |\overline{\nu}_{L1}\rangle + c_\theta |\overline{\nu}_{L2}\rangle$$

Two-flavour oscillations

- degeneracies

$$\Delta m_{21}^2 \to -\Delta m_{21}^2 \quad \text{and} \quad \theta \to \frac{\pi}{2} - \theta$$

- CPT: Always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$$

- CP: For 2-flavours always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$$

oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- antineutrinos

$$|\overline{\nu}_{Le}\rangle = c_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + s_\theta|\overline{\nu}_{L2}\rangle$$

$$|\overline{\nu}_{L\mu}\rangle = -s_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + c_\theta|\overline{\nu}_{L2}\rangle$$

Two-flavour oscillations

- degeneracies

$$\Delta m_{21}^2 \to -\Delta m_{21}^2 \quad \text{and} \quad \theta \to \frac{\pi}{2} - \theta$$

- CPT: Always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$$

- CP: For 2-flavours always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$$

- T: CP conservation $\leftrightarrow$ T conservation

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- antineutrinos

$$|\overline{\nu}_{Le}\rangle = c_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + s_\theta|\overline{\nu}_{L2}\rangle$$

$$|\overline{\nu}_{L\mu}\rangle = -s_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + c_\theta|\overline{\nu}_{L2}\rangle$$

Two-flavour oscillations

- degeneracies

$$\Delta m^2_{21} \to -\Delta m^2_{21} \quad \text{and} \quad \theta \to \frac{\pi}{2} - \theta$$

- CPT: Always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$$

- CP: For 2-flavours always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$$

- T: CP conservation $\leftrightarrow$ T conservation

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

- (Total) lepton number is preserved in neutrino oscillations

oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

- antineutrinos

$$|\overline{\nu}_{Le}\rangle = c_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + s_\theta|\overline{\nu}_{L2}\rangle$$
$$|\overline{\nu}_{L\mu}\rangle = -s_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + c_\theta|\overline{\nu}_{L2}\rangle$$

Two-flavour oscillations

- degeneracies

$$\Delta m^2_{21} \to -\Delta m^2_{21} \quad \text{and} \quad \theta \to \frac{\pi}{2} - \theta$$

- CPT: Always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha)$$

- CP: For 2-flavours always preserved

$$P(\nu_\alpha \to \nu_\beta) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$$

- T: CP conservation $\leftrightarrow$ T conservation

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

- (Total) lepton number is preserved in neutrino oscillations

oscillation probability

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

- antineutrinos

$$|\overline{\nu}_{Le}\rangle = c_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + s_\theta|\overline{\nu}_{L2}\rangle$$
$$|\overline{\nu}_{L\mu}\rangle = -s_\theta e^{-\frac{i}{2}\alpha}|\overline{\nu}_{L1}\rangle + c_\theta|\overline{\nu}_{L2}\rangle$$

- if oscillation length $<$ position uncertainty in source/detector

consider **average probability**

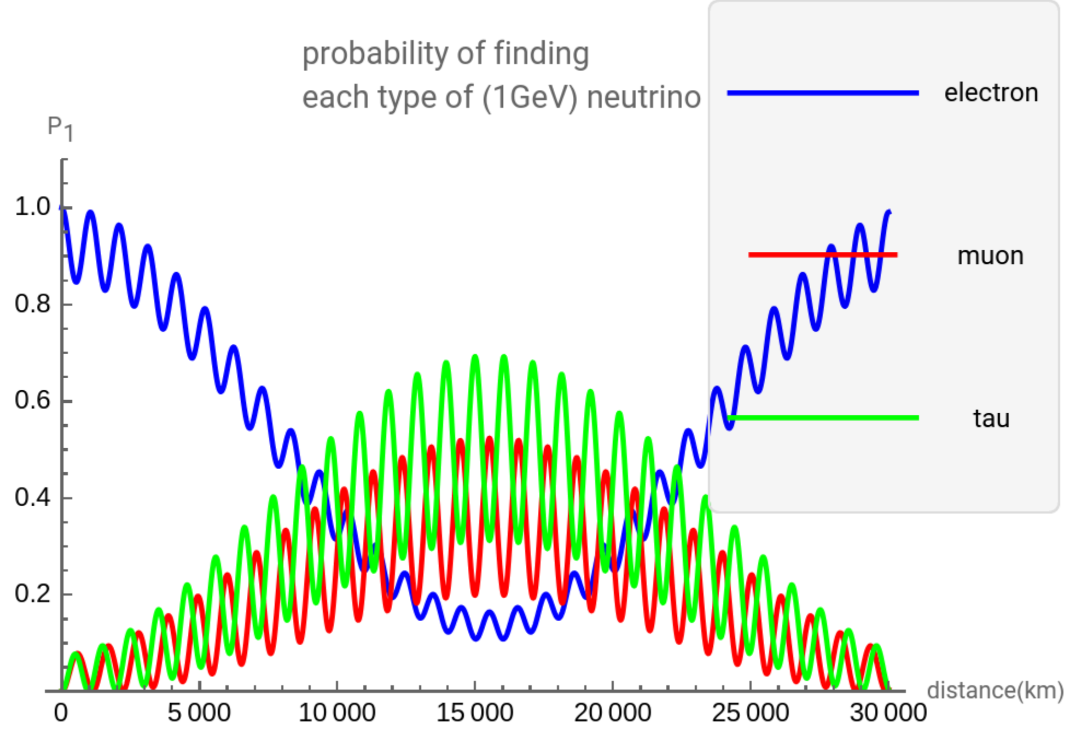
$$P(\nu_e \to \nu_\mu) = \frac{1}{2}\sin^2(2\theta)$$

Three-flavour oscillations in vacuum

The **three-flavour neutrino oscillation probability** can be compactly written as

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

$$-2\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$



[Wolfram neutrino oscillations demonstration]

Three-flavour oscillations in vacuum

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right)$$
$$-2\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{2E}\right)$$

- first line same for $P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right)$

Three-flavour oscillations in vacuum

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$
$$-2\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

- first line same for $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$
- third term describes **CP violation**

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) - P(\nu_\alpha \to \nu_\beta)$$
$$= 4\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

Three-flavour oscillations in vacuum

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

$$-2\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

- first line same for $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$
- third term describes **CP violation**

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) - P(\nu_\alpha \to \nu_\beta)$$

$$= 4\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

- no CP violation if $\alpha = \beta$

CP violation in lepton sector can be much larger than in quark sector!

$$J_{\mathrm{CP}}^{\mathrm{quark}} = \left(3.12^{+0.13}_{-0.12}\right) \cdot 10^{-5}$$

$$J_{\mathrm{CP}}^{\mathrm{lepton}} = (3.31 - 3.35) \cdot 10^{-2} \sin(\delta)$$

Jarlskog invariant

$$J_{\mathrm{CP}} = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin(2\theta_{13}) \cos(\theta_{13}) \sin(\delta)$$

Matter effect

- Effective vacuum Hamiltonian is diagonal in mass basis

$$H_{\rm vac}^{(m)} = U_{\rm PMNS}^{\dagger} H_{\rm vac}^{(f)} U_{\rm PMNS} = \frac{M_\nu^\dagger M_\nu}{2E} \equiv \frac{M^2}{2E}$$

- Electroweak interactions generate an effective potential for neutrinos
- Hamiltonian in flavour basis is

$$H^{(f)} = H_{\rm vac}^{(f)} + V^{(f)}$$

- effective potential from NC interactions is flavour universal

$\rightarrow$ no effect unless there are light sterile ν's

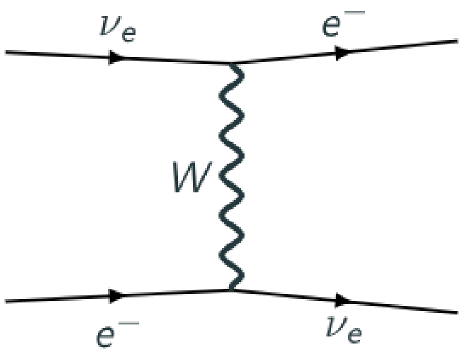
- matter potential due to electron density n_e

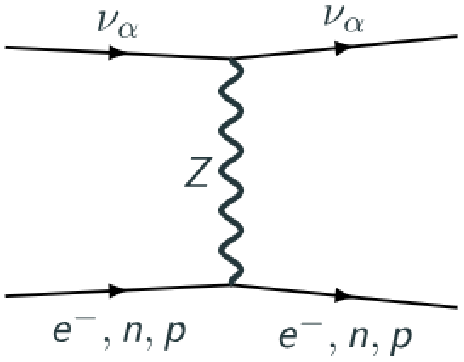
$$V_{ee}^{(f)} = \pm\sqrt{2} G_F n_{e(x)}$$

- vacuum mass eigenstates are not eigenstates of Hamiltonian in matter

Matter effect

→ need to find mass eigenstates for neutrinos in matter





Matter effect

- find mass eigenstates of neutrino Hamiltonian in matter

$$H^{(f)} = H^{(f)}_{\text{vac}} + V^{(f)} \qquad \text{with} \qquad V^{(f)}_{ee} = \pm\sqrt{2} G_F n_{e(x)}$$

- define mixing matrix $\tilde{U}$ with

$$\nu^{(f)}(x) = \tilde{U}(x)\tilde{\nu}^{(m)}(x) \qquad \text{and} \qquad H^{(f)} = \frac{1}{2E}\tilde{U}(x)\widetilde{M}^2\tilde{U}^\dagger(x)$$

Matter effect

- find mass eigenstates of neutrino Hamiltonian in matter

$$H^{(f)} = H^{(f)}_{\text{vac}} + V^{(f)} \qquad \text{with} \qquad V^{(f)}_{ee} = \pm\sqrt{2}G_F n_{e(x)}$$

- define mixing matrix $\tilde{U}$ with

$$\nu^{(f)}(x) = \tilde{U}(x)\tilde{\nu}^{(m)}(x) \qquad \text{and} \qquad H^{(f)} = \frac{1}{2E}\tilde{U}(x)\widetilde{M}^2\tilde{U}^\dagger(x)$$

- neutrino propagation is described by

$$i\frac{\mathrm{d}\tilde{\nu}^{(m)}}{\mathrm{d}x} = \left[\frac{\widetilde{M}^2}{2E} - i\tilde{U}^\dagger(x)\frac{\mathrm{d}\tilde{U}(x)}{\mathrm{d}x}\right]\tilde{\nu}^{(m)}(x)$$

- the second term describes the change of mass eigenstates. It can be neglected, if the matter potential is slowly varying $L = \frac{4\pi E}{\Delta\widetilde{M}^2} \ll \left(\frac{\mathrm{d}\ln n_{e(x)}}{\mathrm{d}x}\right)^{-1}$.
- This is called *adiabatic approximation*. In this approximation, we observe unitary evolution and the magnitude of the mass eigenstates remains unchanged.

Matter effect

- in the adiabatic approximation neutrino propagation is described by

$$i\frac{\mathrm{d}\tilde{\nu}^{(m)}}{\mathrm{d}x} = \frac{\widetilde{M}^2}{2E}\tilde{\nu}^{(m)}(x)$$

- Effective neutrino mixing angle and mass squared difference in matter

$$\sin\left(2\tilde{\theta}\right) = \frac{\sin(2\theta)}{\sqrt{\sin^2(2\theta) + C^2}} \qquad \text{and} \qquad \Delta\tilde{m}^2 = \Delta m^2\sqrt{\sin^2(2\theta) + C^2}$$

with the parameter $C(x) = \cos(2\theta) - \frac{2\sqrt{2}G_F n_e(x)E}{\Delta m^2}$

Matter effect

- in the adiabatic approximation neutrino propagation is described by

$$i\frac{\mathrm{d}\tilde{\nu}^{(m)}}{\mathrm{d}x} = \frac{\widetilde{M}^2}{2E}\tilde{\nu}^{(m)}(x)$$

- Effective neutrino mixing angle and mass squared difference in matter

$$\sin\left(2\tilde{\theta}\right) = \frac{\sin(2\theta)}{\sqrt{\sin^2(2\theta) + C^2}} \qquad \text{and} \qquad \Delta\tilde{m}^2 = \Delta m^2\sqrt{\sin^2(2\theta) + C^2}$$

with the parameter $C(x) = \cos(2\theta) - \frac{2\sqrt{2}G_F n_e(x)E}{\Delta m^2}$

- Matter effect breaks degeneracies

$$\Delta m^2 \nleftrightarrow -\Delta m^2 \qquad \text{and} \qquad \theta \nleftrightarrow \frac{\pi}{2} - \theta$$

Matter effect

Effective neutrino mixing angle and mass squared difference in matter

$$\sin\left(2\tilde{\theta}\right) = \frac{\sin(2\theta)}{\sqrt{\sin^2(2\theta) + C^2}} \qquad \text{and} \qquad \Delta\tilde{m}^2 = \Delta m^2 \sqrt{\sin^2(2\theta) + C^2}$$

with the parameter $C(x) = \cos(2\theta) - \frac{2\sqrt{2} G_F n_e(x) E}{\Delta m^2}$

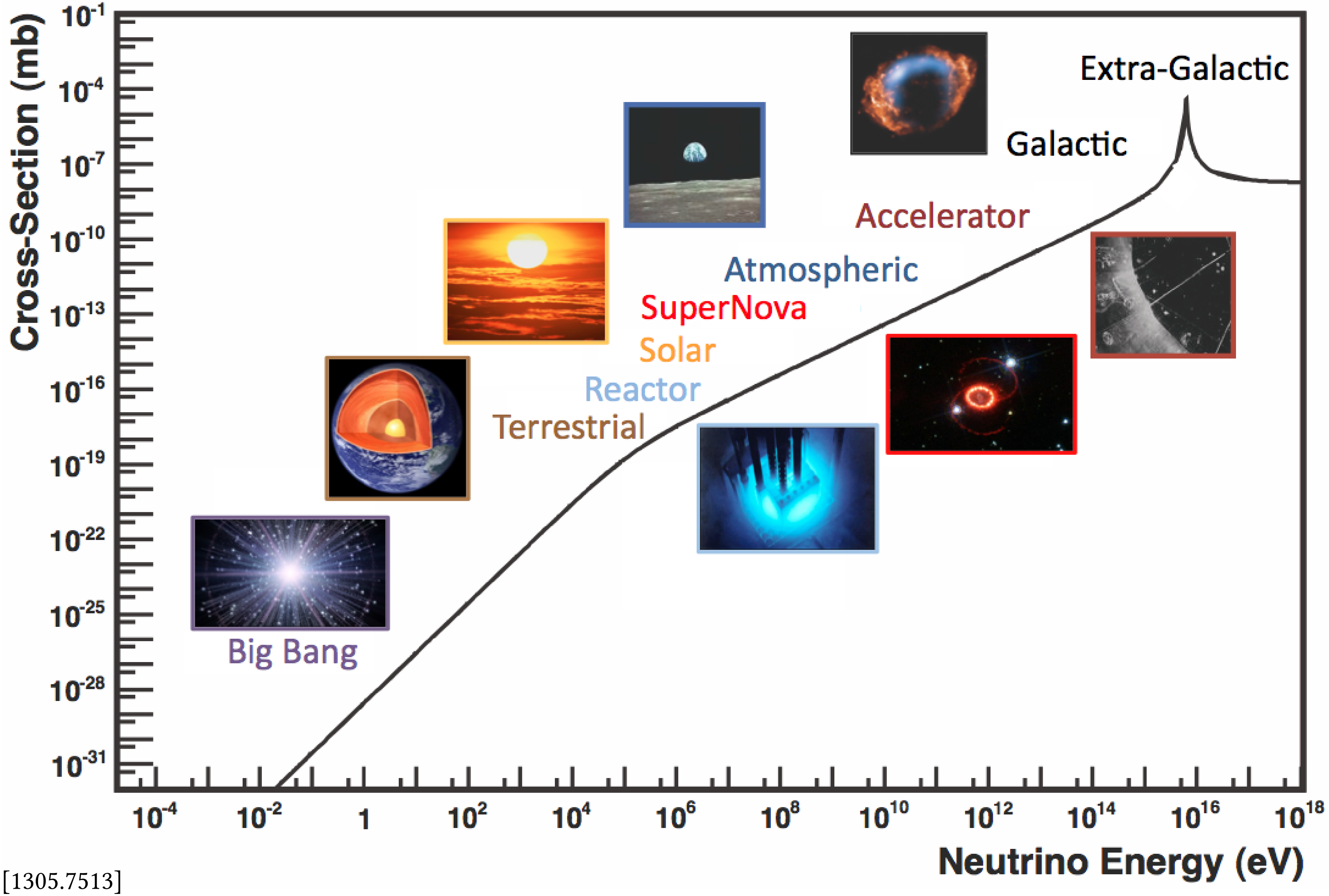
Mikheyev-Smirnov-Wolfenstein (MSW) resonance occurs for $C(x) = 0$ or equivalently

$$\Delta m^2 \cos(2\theta) = 2\sqrt{2} G_F n_e(x) E$$

In this case the effective neutrino mixing angle becomes maximal $\sin\left(2\tilde{\theta}\right) = 1$ and thus oscillations are maximally efficient.

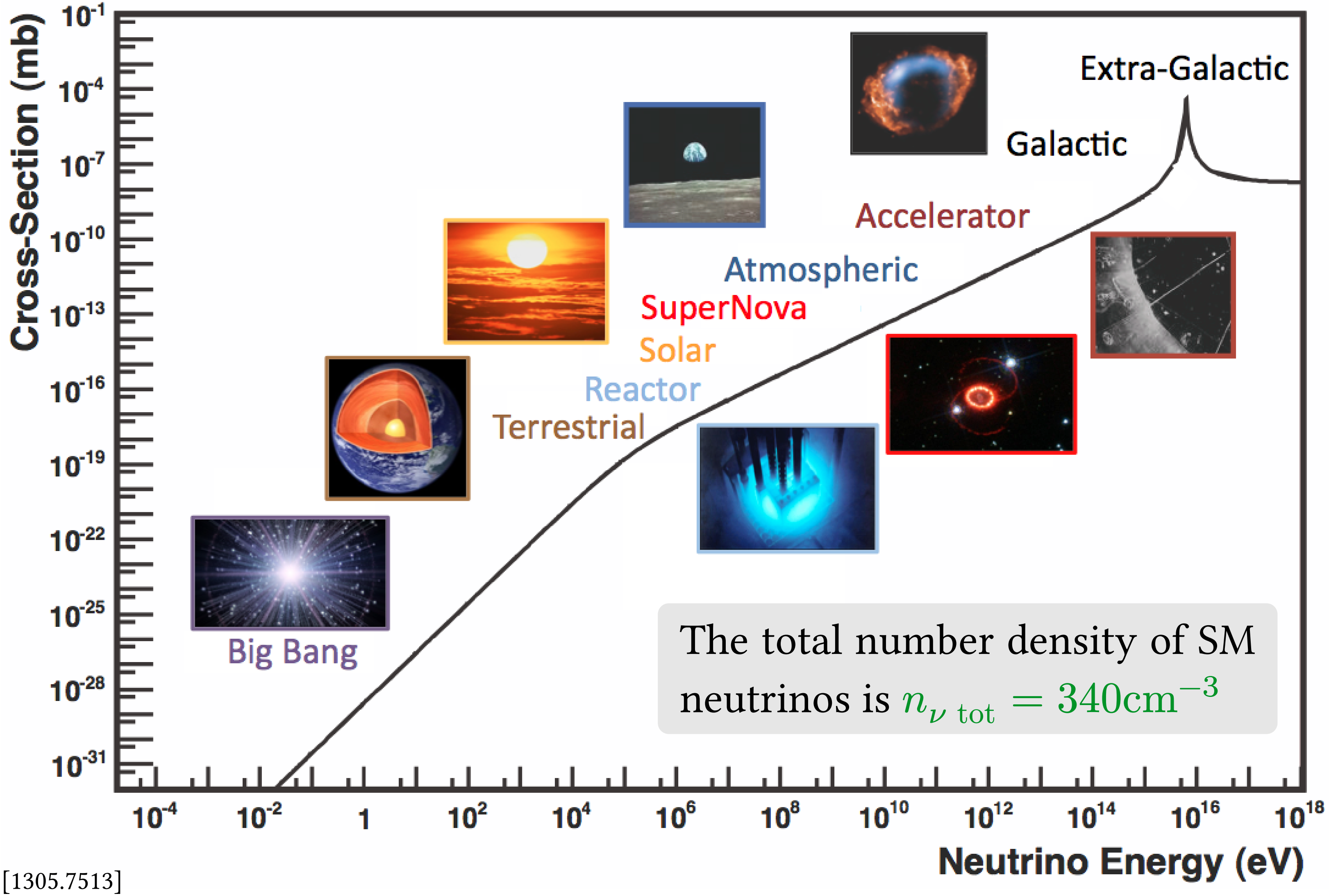
Experimental overview

Experimental evidence for neutrinos



[1305.7513]

Experimental evidence for neutrinos



The total number density of SM neutrinos is $n_{\nu\ \mathrm{tot}} = 340\mathrm{cm}^{-3}$

[1305.7513]

Brief history

1920s Ellis: beta decay spectrum is continuous

1959 Cowan-Reines neutrino experiment: ν detection 1995

1962 Lederman, Schwartz, Steinberger: ν_μ detection 1988

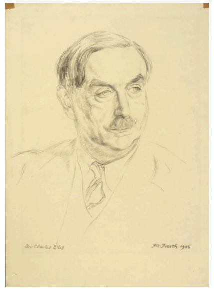
1989 Large Electron Positron collider: Z decay width $\Rightarrow N_\nu = 2.984 \pm 0.008$

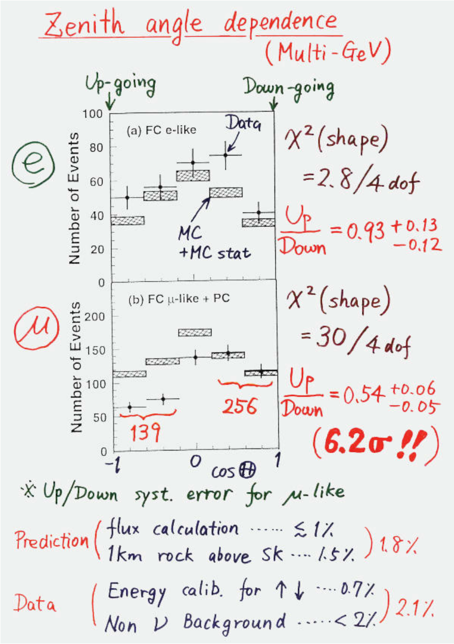
1998 Super-Kamiokande: atmospheric neutrino oscillations 2015

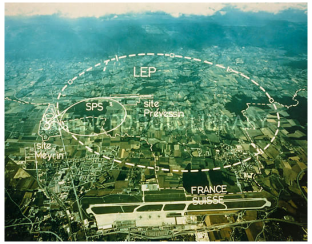
2001 SNO: solar neutrino oscillations 2015

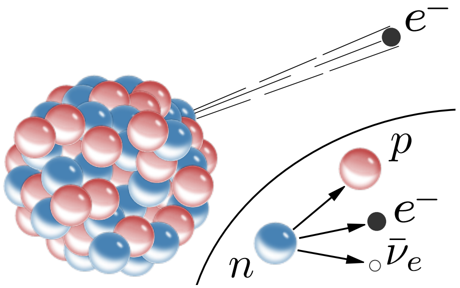
2012 Daya Bay, RENO, Double CHOOZ: θ_{13}

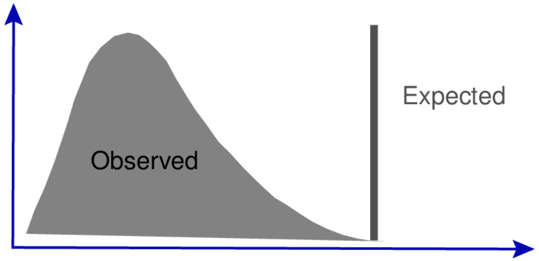
2017 COHERENT: CEνNS











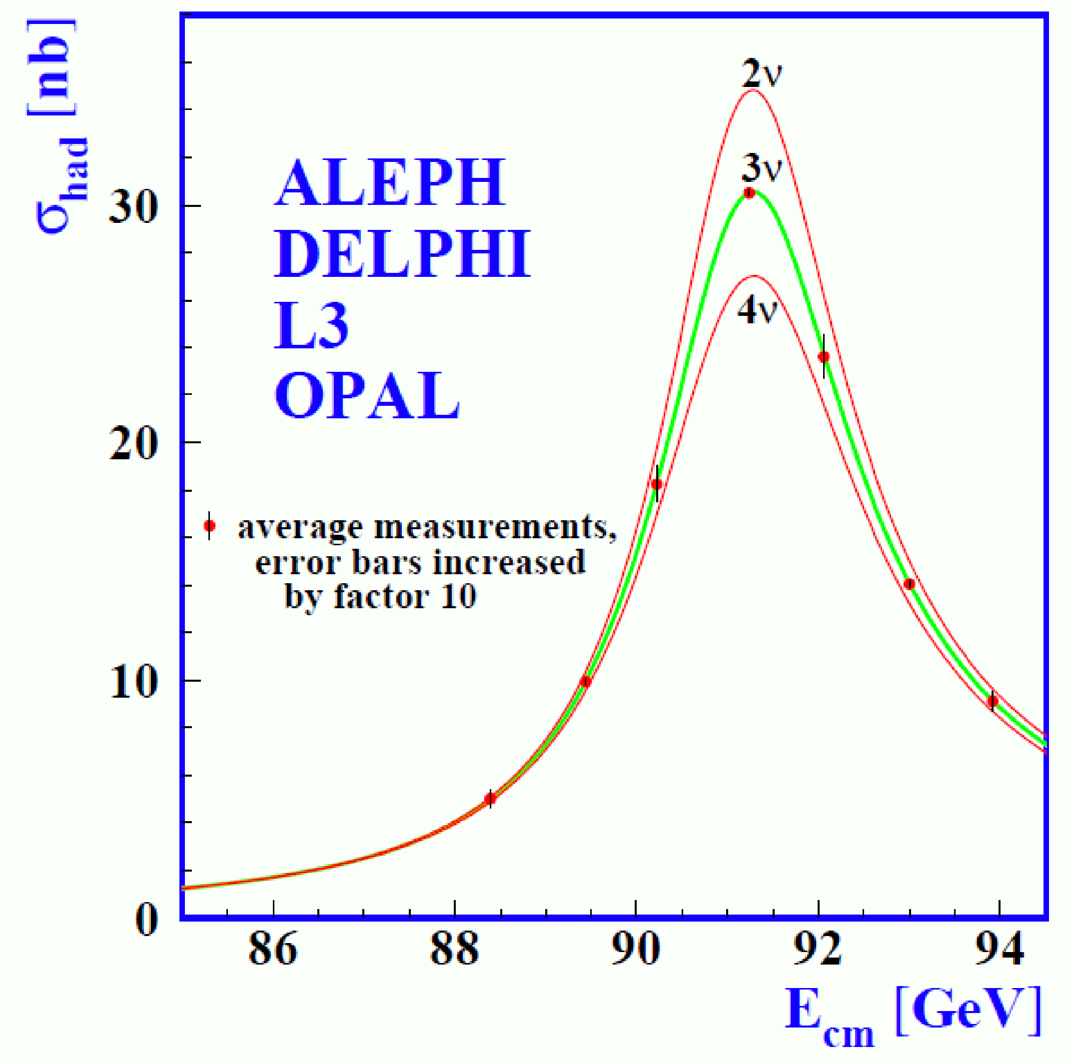
Number of neutrinos

SM neutrinos couple to the Z boson and thus $Z \to \nu_\alpha \bar{\nu}_\alpha$ with $\alpha = e, \mu, \tau$.

Experimental proof of the **existence of three light active neutrinos** with mass less than half of the Z boson mass has been obtained by precisely measuring Z boson decays.

LEP inferred the invisible partial width $\Gamma(Z \to \text{inv})$ of the Z boson by precisely measuring the total width and in hadronic decays. The combination of four LEP experiments yields

$$N_\nu = 2.984 \pm 0.008$$



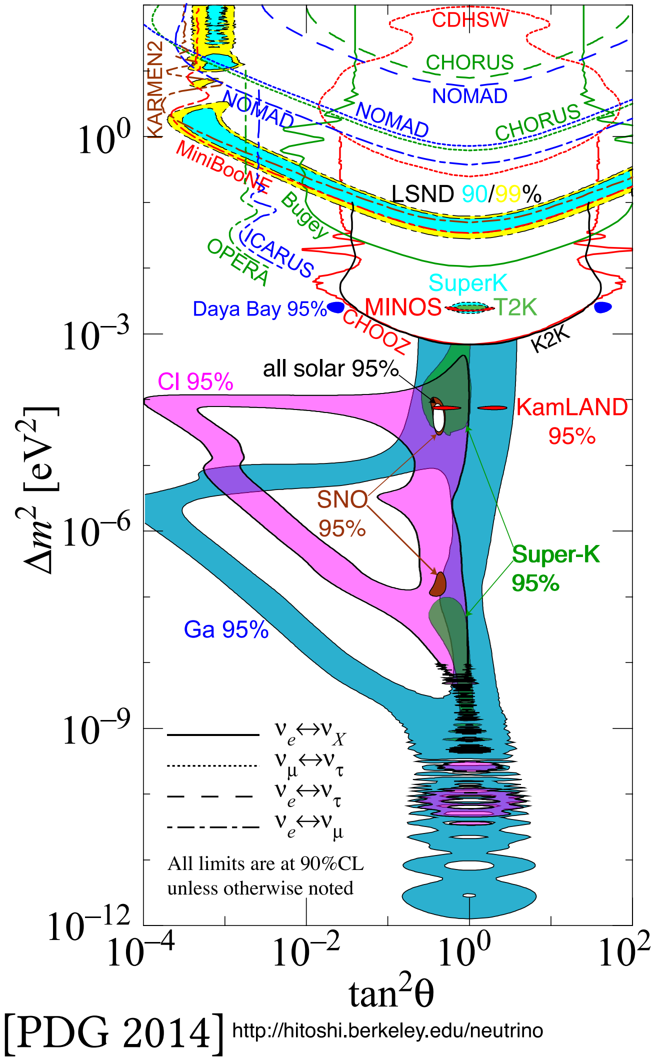
Neutrino oscillations

There are many neutrino oscillation experiments as illustrated in the figure on the right which captures the situation in 2014, more than 10 years ago. I will not discuss individual neutrino oscillation experiments. The following is a review of the lepton mixing parameters and mass squared differences of a **global fit** to neutrino oscillation data.

There are three research groups which perform these fits

- **IFIC group** (P.F. de Salas, D.V. Forero, C.A. Ternes, M. Tortola, J.W.F. Valle, S. Gariazzo, P. Parinez-Mirave, O. Mena)
- **Bari group** (F. Capozzi, E. Lisi, A. Marrone, A. Palazzo)
- **Spain/USA-Germany group** (I. Esteban, C. Gonzalez Garcia, M. Maltoni, T. Schwetz, A. Zhou)

The latest results from the third group which the following is based on can be accessed via http://www.nu-fit.org .



[PDG 2014]

Neutrino oscillations

NuFIT 5.3 (2024)

without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
$\delta_{\mathrm{CP}}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

Neutrino oscillations

NuFIT 5.3 (2024)

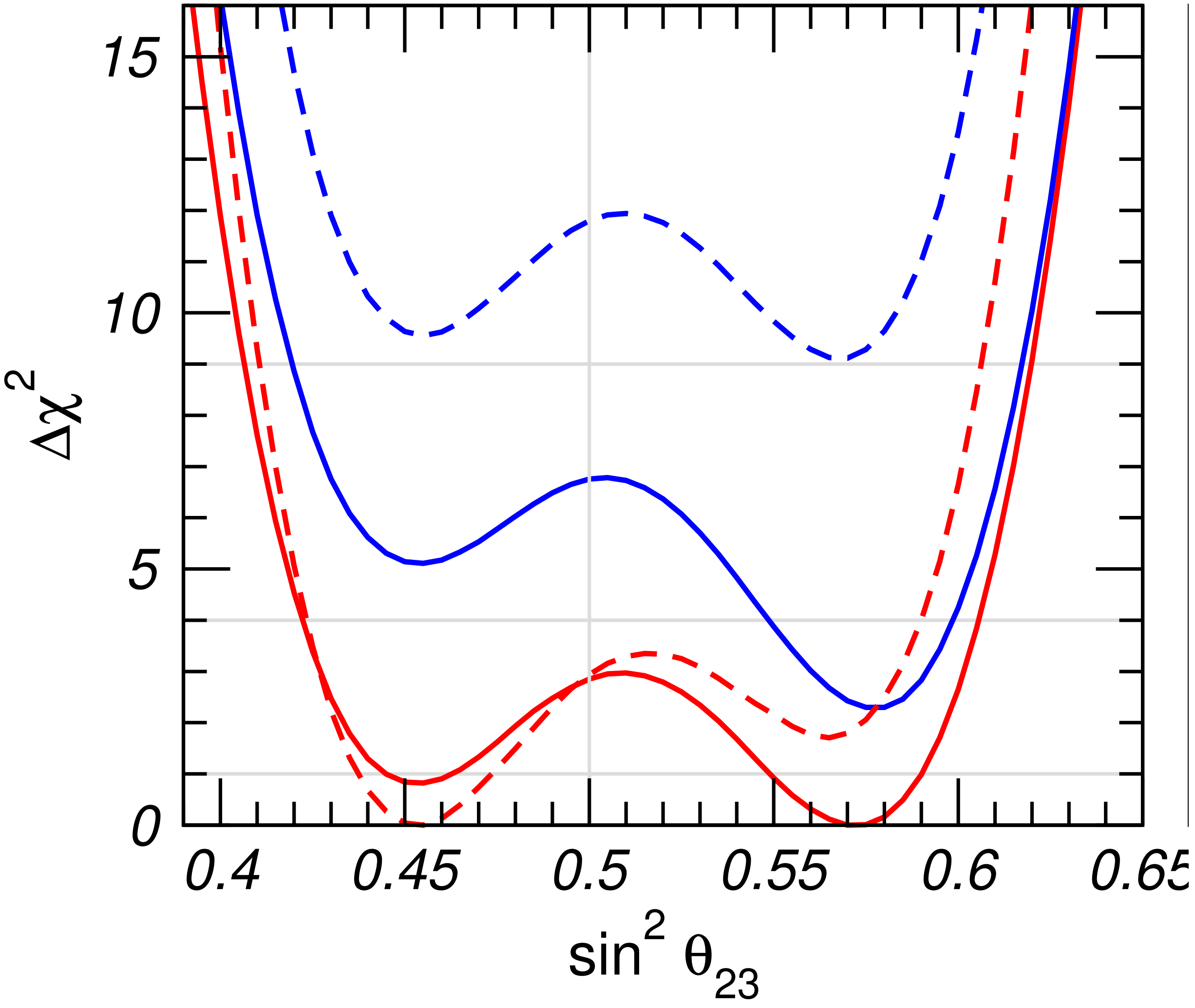
without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

Atmospheric mixing θ_{23} is the largest and can be maximal 45°

Neutrino oscillations

NO

IO



Neutrino oscillations

NuFIT 5.3 (2024)

without SK atmospheric data

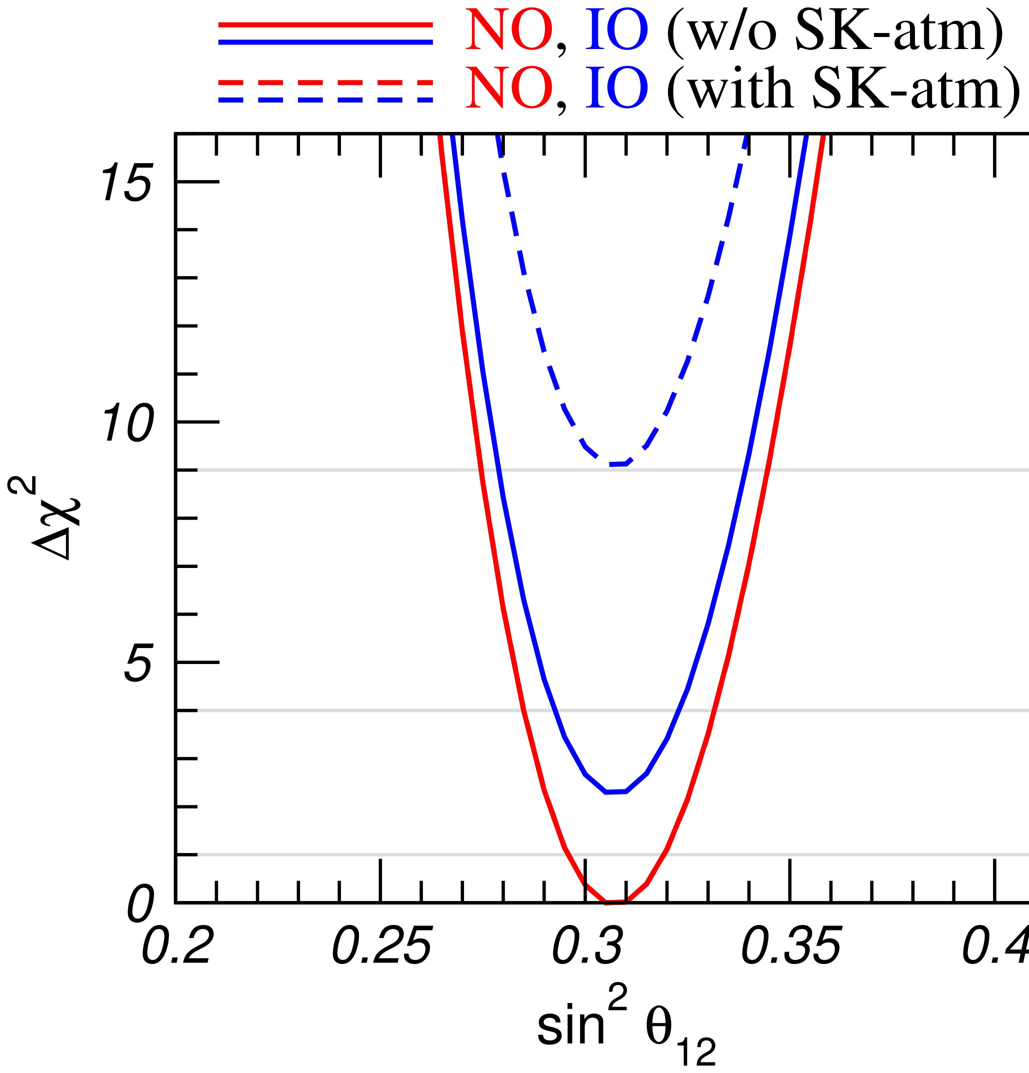
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
$\delta_{\rm CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

Solar mixing θ_{12} is the second largest but cannot be maximal

Neutrino oscillations

NO

IO



Neutrino oscillations

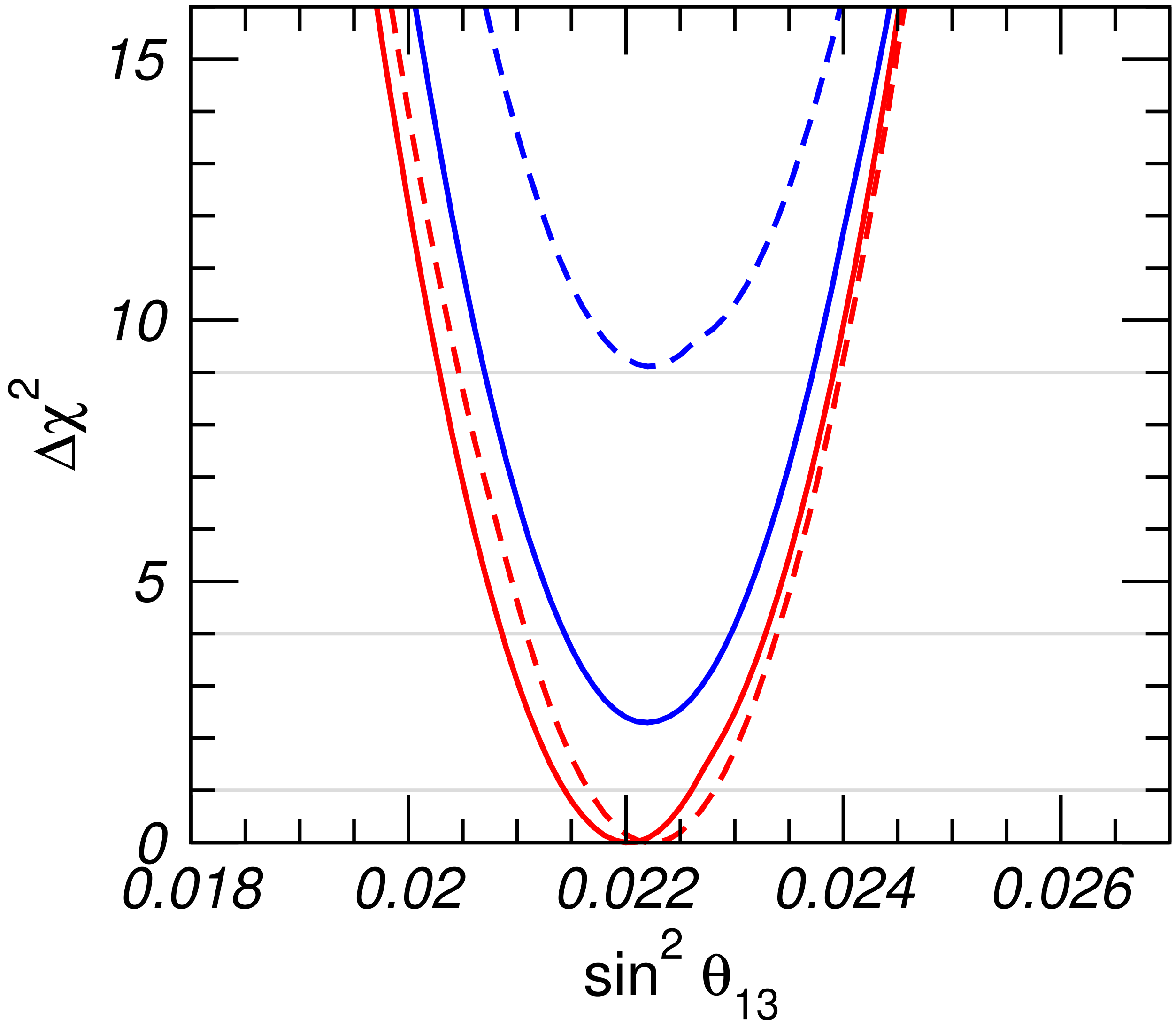
NuFIT 5.3 (2024)

without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

Reactor mixing θ_{13} has been best measured and is of the size of the Cabibbo angle

NO

IO



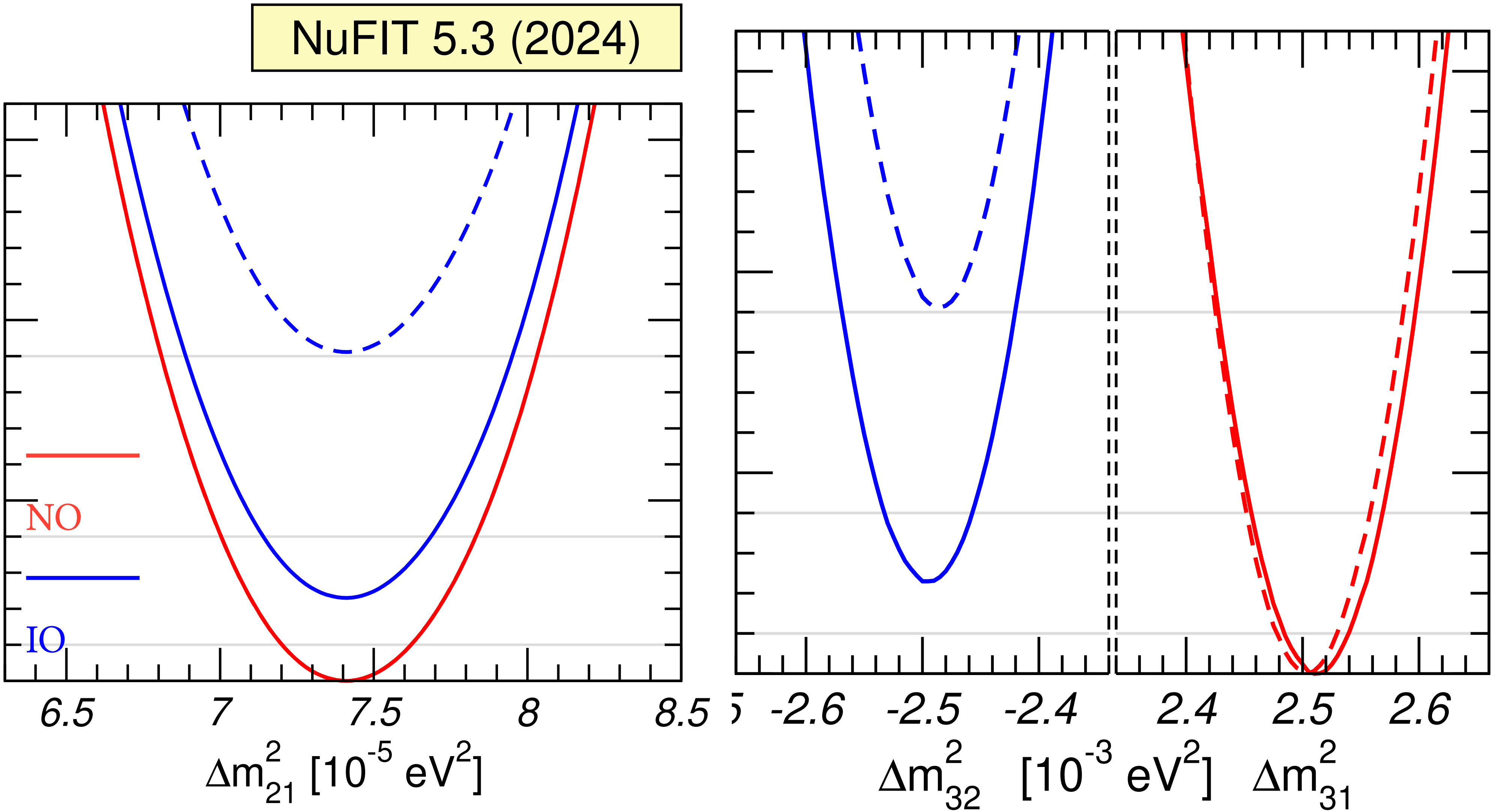
Neutrino oscillations

NuFIT 5.3 (2024)

without SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

Mass ordering, i.e. sign of Δm^2_{3i} is one of the aims of DUNE and Hyper-K.

Neutrino oscillations



Neutrino oscillations

NuFIT 5.3 (2024)

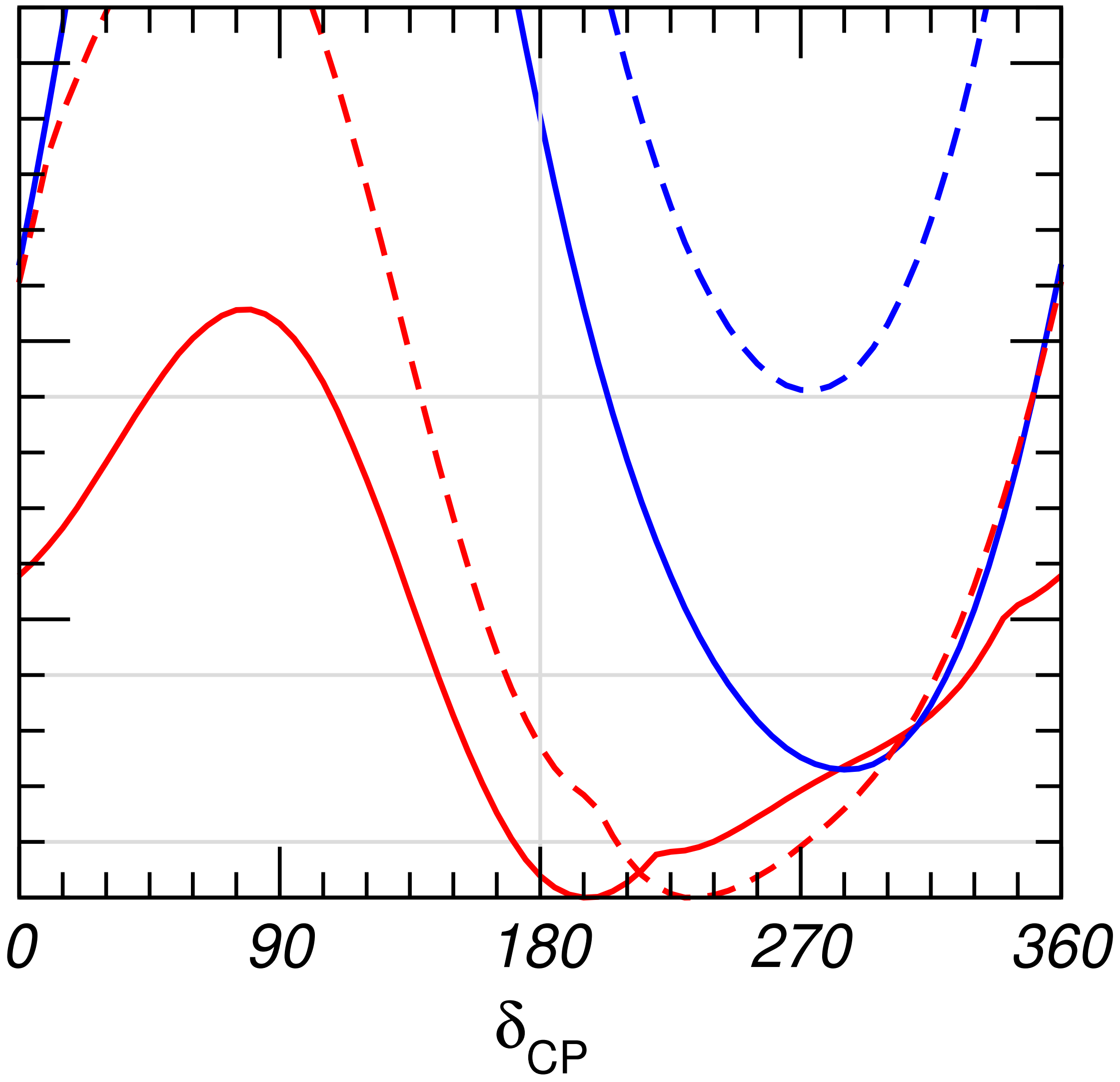
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{\rm CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$

Dirac phase δ has not been measured yet. It may be large

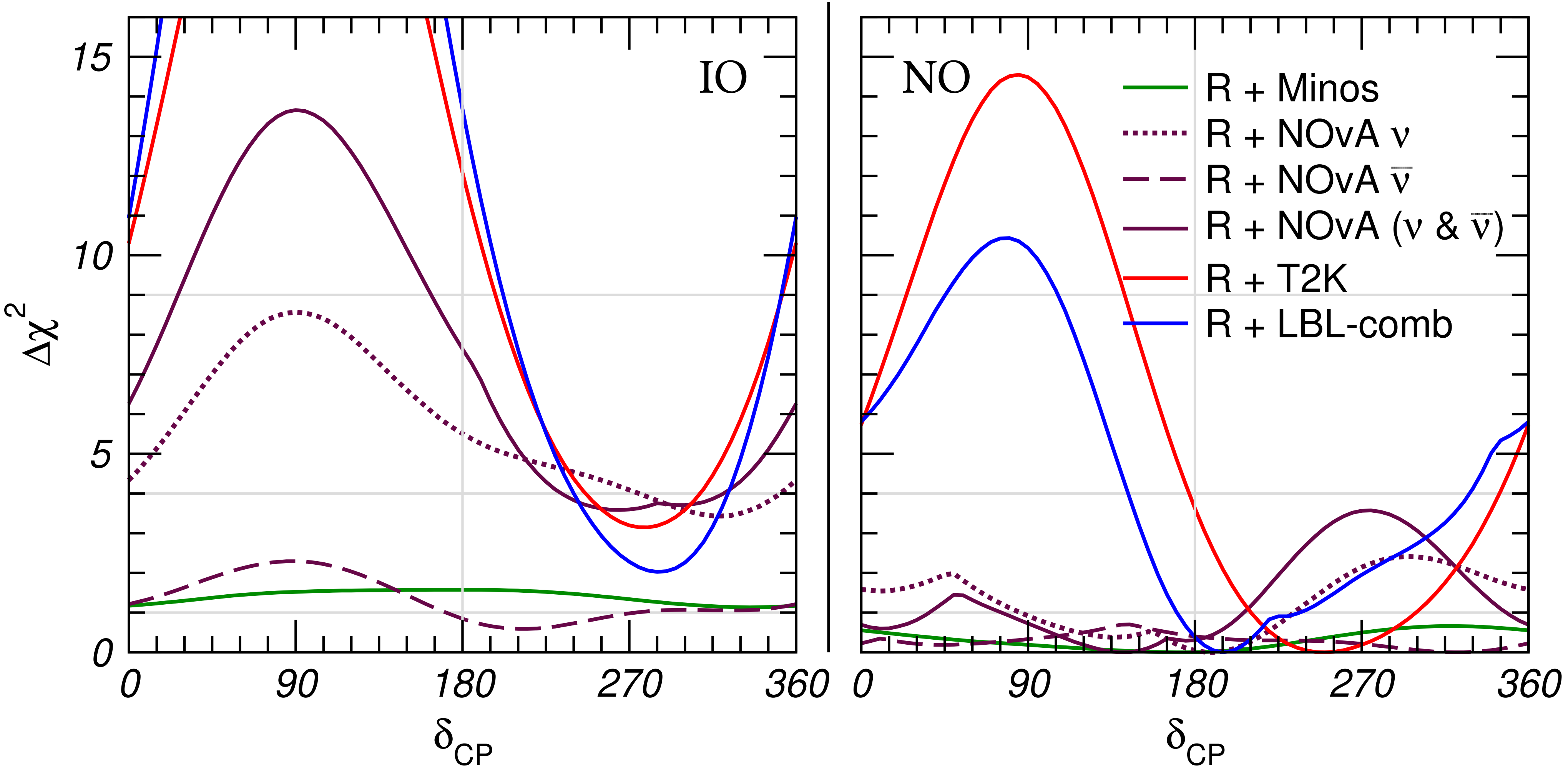
Neutrino oscillations

NO

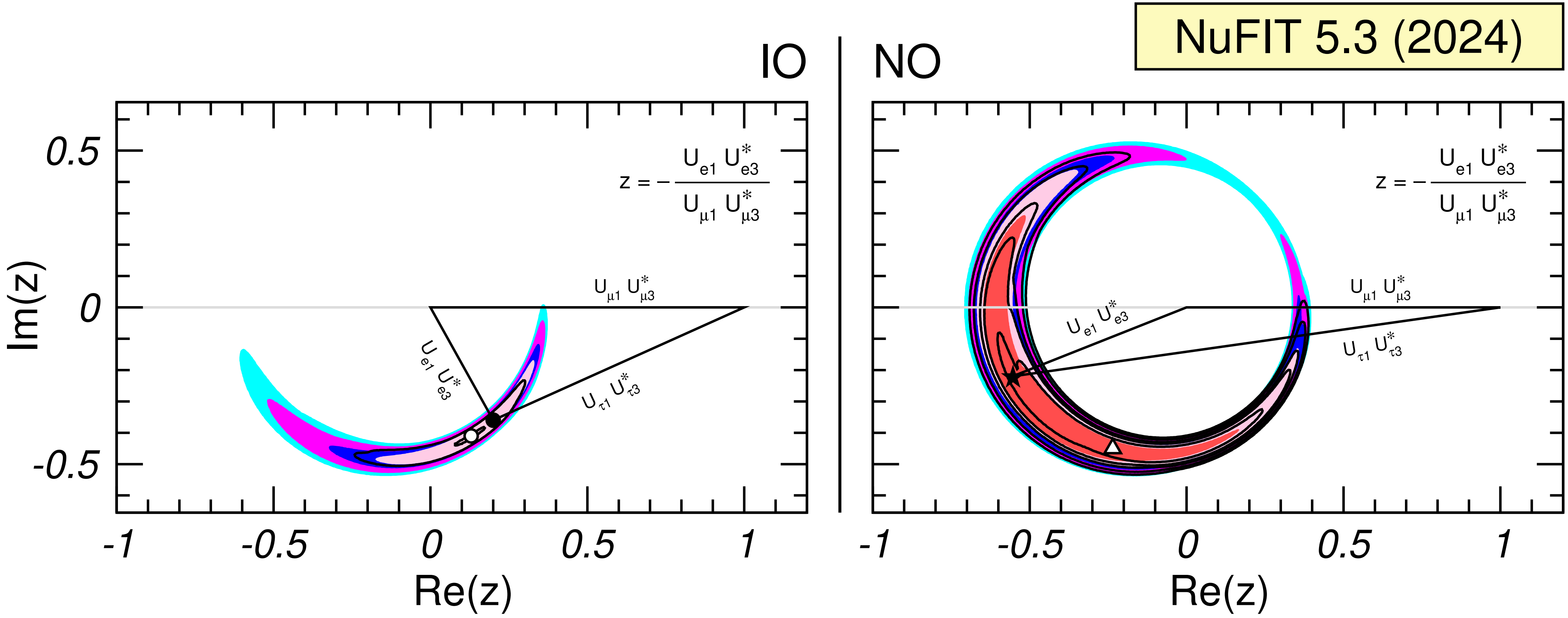
IO



Neutrino oscillations



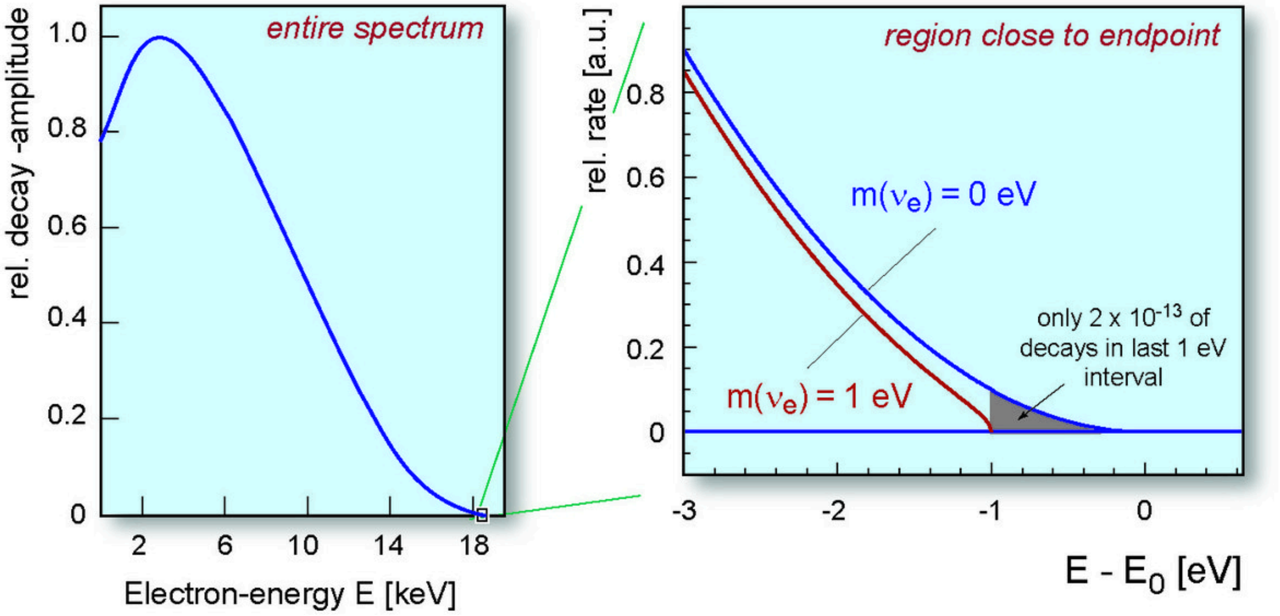
Neutrino oscillations

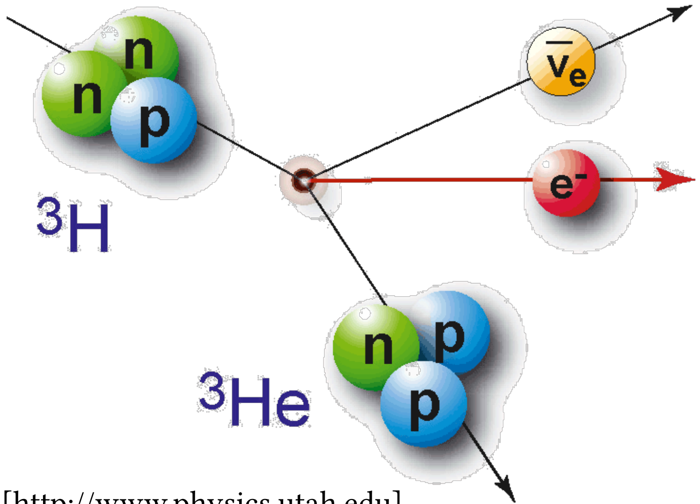


Area of the unitary triangle is a measure of CP violation.

Tritium beta decay

A model-independent way to measure the absolute neutrino mass scale is to use the kinematics of beta decay, in particular **β decay of tritium** is well suited due to its **low Q value** of $Q_\beta = 18.6$ keV.





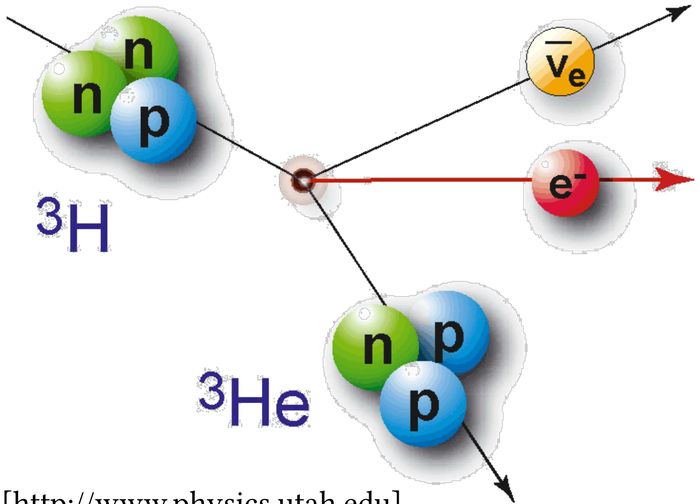
[http://www.physics.utah.edu]

If energy resolution worse than mass splitting

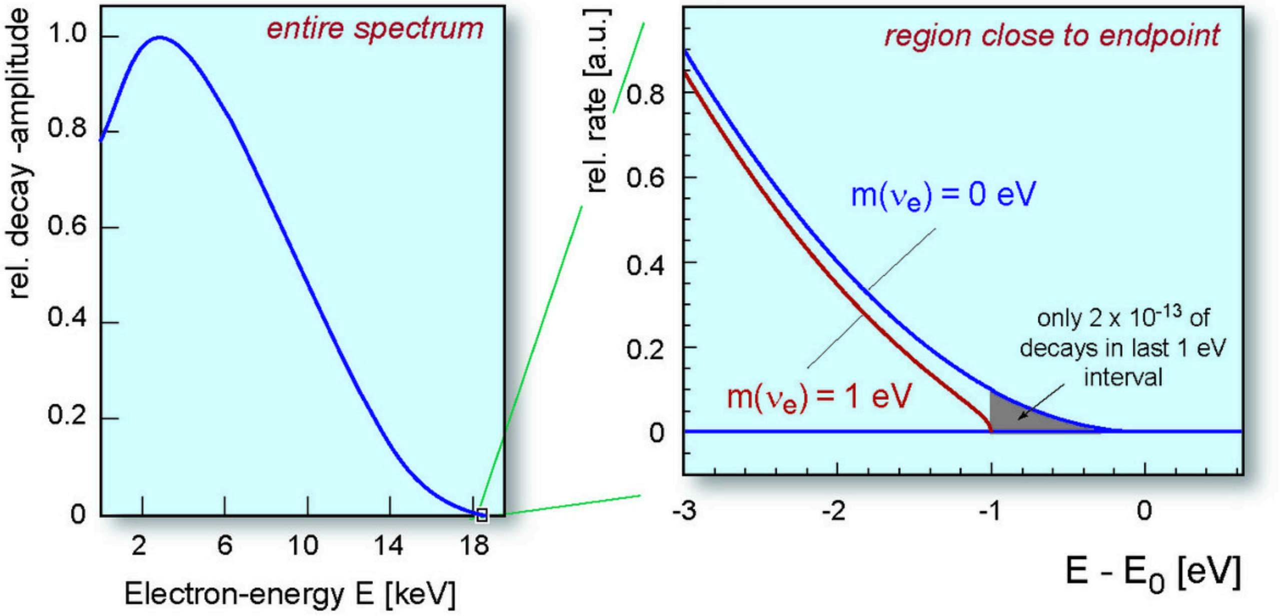
$$S(E_e) \propto (Q - E_e)\sqrt{(Q - E_e)^2 - \sum_j |U_{ej}|^2\ m_j^2}$$

Tritium beta decay

A model-independent way to measure the absolute neutrino mass scale is to use the kinematics of beta decay, in particular **β decay of tritium** is well suited due to its **low Q value** of $Q_\beta = 18.6$ keV.



[http://www.physics.utah.edu]



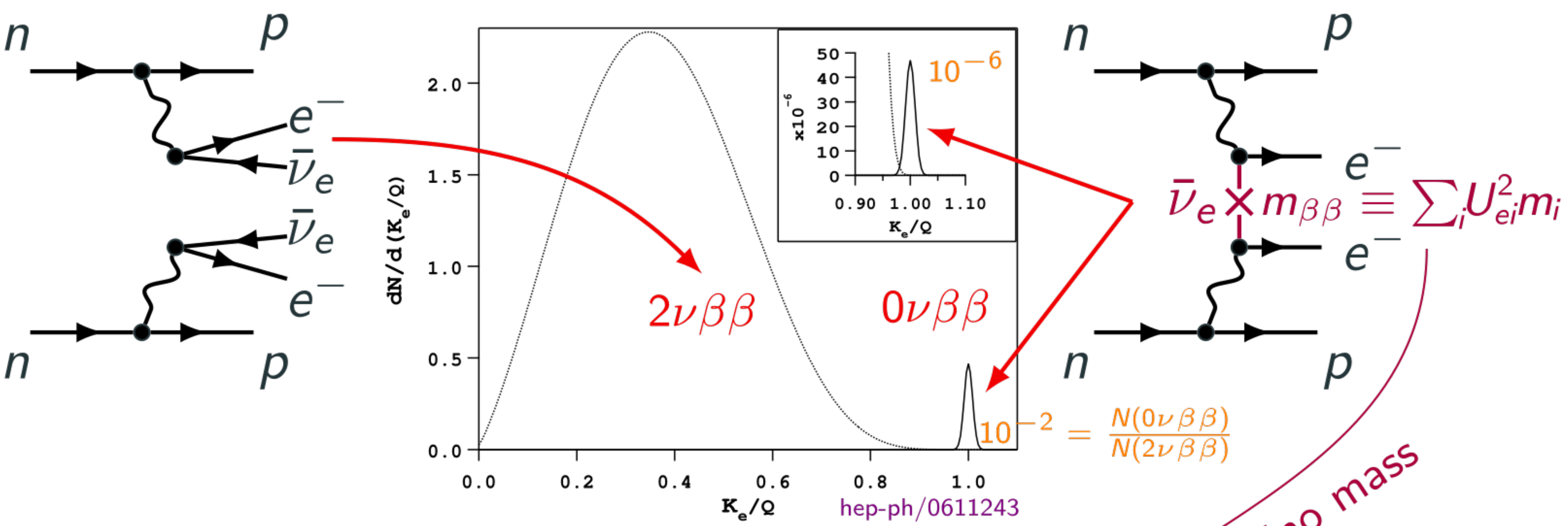
If energy resolution worse than mass splitting

$$S(E_e) \propto (Q - E_e)\sqrt{(Q-E_e)^2 - \sum_j |U_{ej}|^2\, m_j^2}$$

KATRIN experiment: $\sqrt{\sum_j |U_{ej}|^2\, m_j^2} < 0.45$ **eV**

[2406.13516]

Neutrinoless double beta decay



kinetic energy of 2 electrons

effective neutrino mass

nuclear physics

lifetime

$$T_{1/2}^{0\nu} \propto \frac{1}{|M_{\text{nucl}}|^2} \frac{1}{\langle m_{\beta\beta}\rangle^2}$$

Neutrinoless double beta decay

- Neutrinoless double beta ($0\nu 2\beta$) decay violates lepton number $\Delta L = 2$
- It only occurs for Majorana neutrinos and relates to the half life like

$$\frac{1}{T^{0\nu}_{\frac{1}{2}}} = \underbrace{G^{0\nu}}_{\text{phase space}} \underbrace{|M_{\text{nucl}}|^2}_{\text{NME}} |m_{\beta\beta}|^2$$

where $|m_{\beta\beta}| = |c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{-2i\delta} m_3|$

Neutrinoless double beta decay

- Neutrinoless double beta ($0\nu2\beta$) decay violates lepton number $\Delta L = 2$
- It only occurs for Majorana neutrinos and relates to the half life like

$$\frac{1}{T^{0\nu}_{\frac{1}{2}}} = \underbrace{G^{0\nu}}_{\text{phase space}} \underbrace{|M_{\text{nucl}}|^2}_{\text{NME}} |m_{\beta\beta}|^2$$

where $|m_{\beta\beta}| = |c^2_{12}c^2_{13}e^{i\alpha_1}m_1 + s^2_{12}c^2_{13}e^{i\alpha_2}m_2 + s^2_{13}e^{-2i\delta}m_3|$

Experimental limits at 90% CL

isotope	^{76}Ge	^{100}Mo	^{130}Te	^{132}Xe
$T^{0\nu}_{\frac{1}{2}}\,[10^{25}$ years$]$	19	0.3	3.8	38
experiment	LEGEND	AMoRE	CUORE	KamLAND-Zen

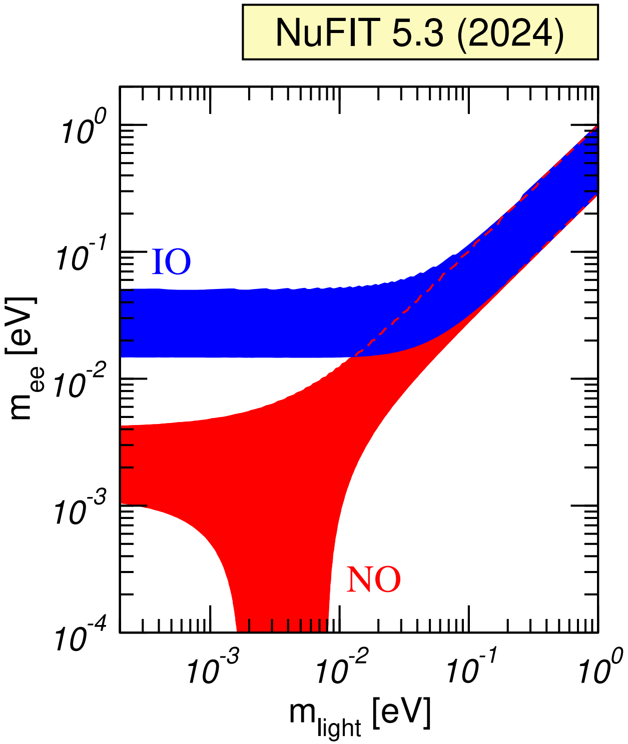
Neutrino mass limit:

assuming light neutrino dominance

KamLAND-Zen provides the strongest limit

$|m_{\beta\beta}| < 28 - 122$ meV

Neutrinoless double beta decay

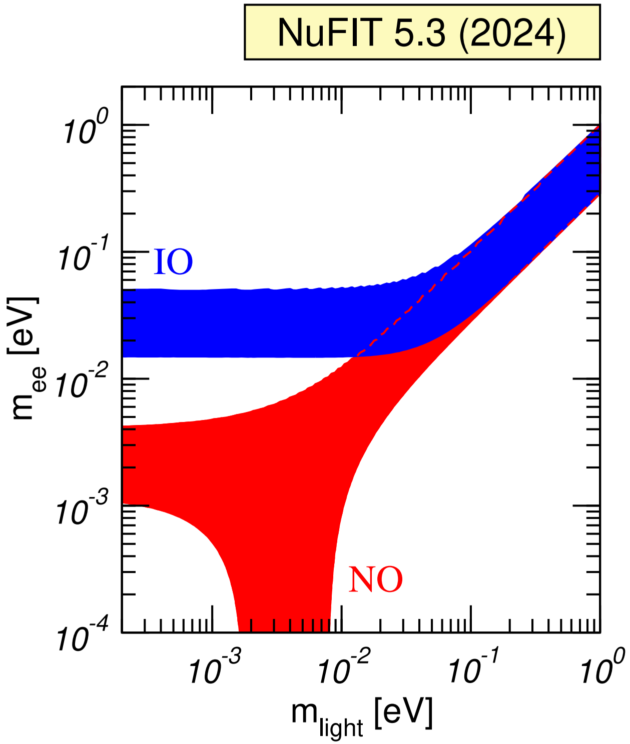


- future experiments probe $m_{ee} \gtrsim 10^{-2}$ eV

- Cancellation possible for NO

$$m_{ee} = m_{\beta\beta} = 0$$

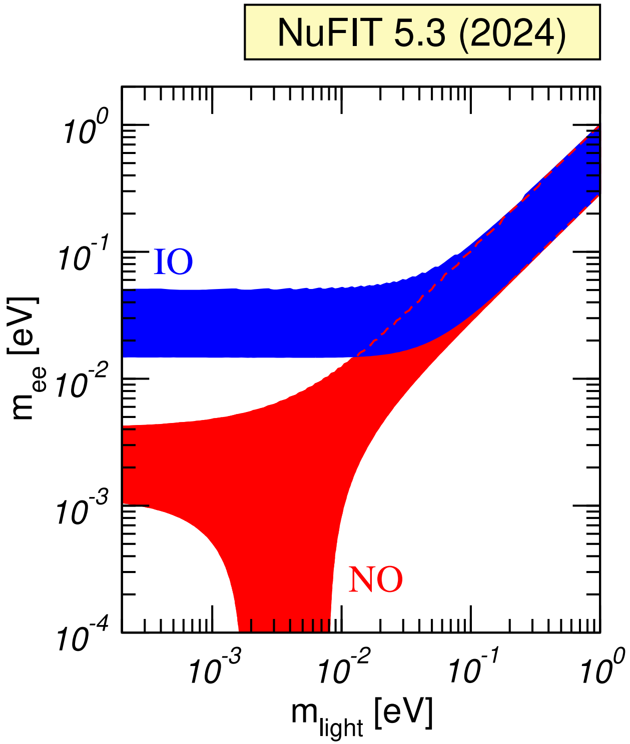
Neutrinoless double beta decay



- Cancellation possible for NO

$$m_{ee} = m_{\beta\beta} = 0$$

- future experiments probe $m_{ee} \gtrsim 10^{-2}$ eV
- Note there may be other contributions to $0\nu 2\beta$ decay, e.g. SUSY neutralino, RH W boson exchange, ...

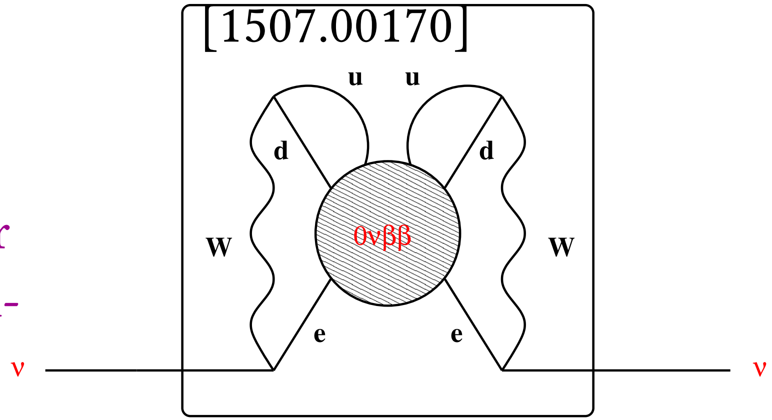


- future experiments probe $m_{ee} \gtrsim 10^{-2}$ eV
- Note there may be other contributions to $0\nu2\beta$ decay, e.g. SUSY neutralino, RH W boson exchange, ...
- **What does detection of $0\nu2\beta$ decay imply for neutrino masses?**

Black-box theorem:
(Schechter-Valle)

- neutrinos are Majorana
- Black-box theorem however only predicts a tiny neutrino mass tiny

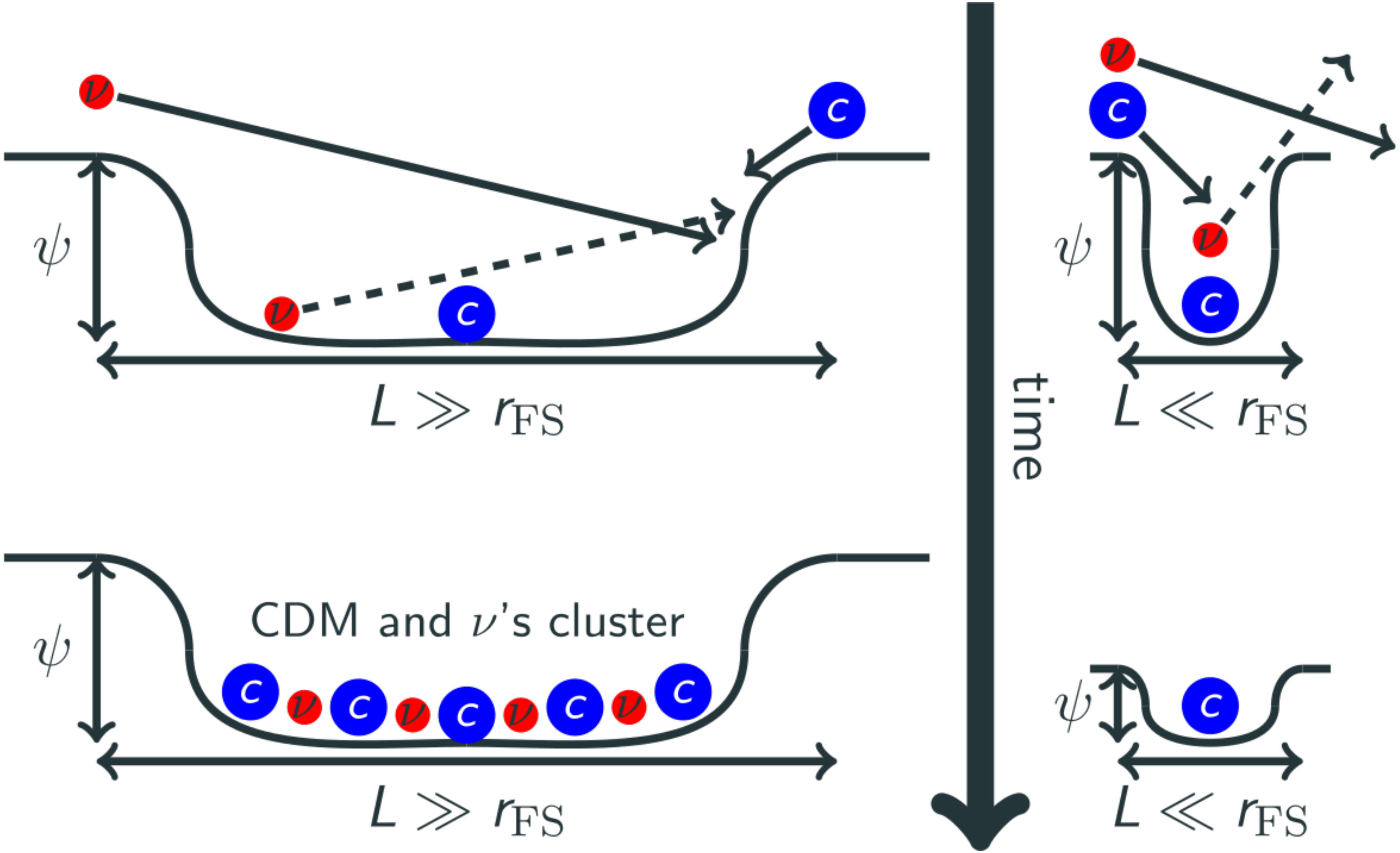
$$m_\nu < 10^{-24} \text{ eV}$$



- Cancellation possible for NO

$$m_{ee} = m_{\beta\beta} = 0$$

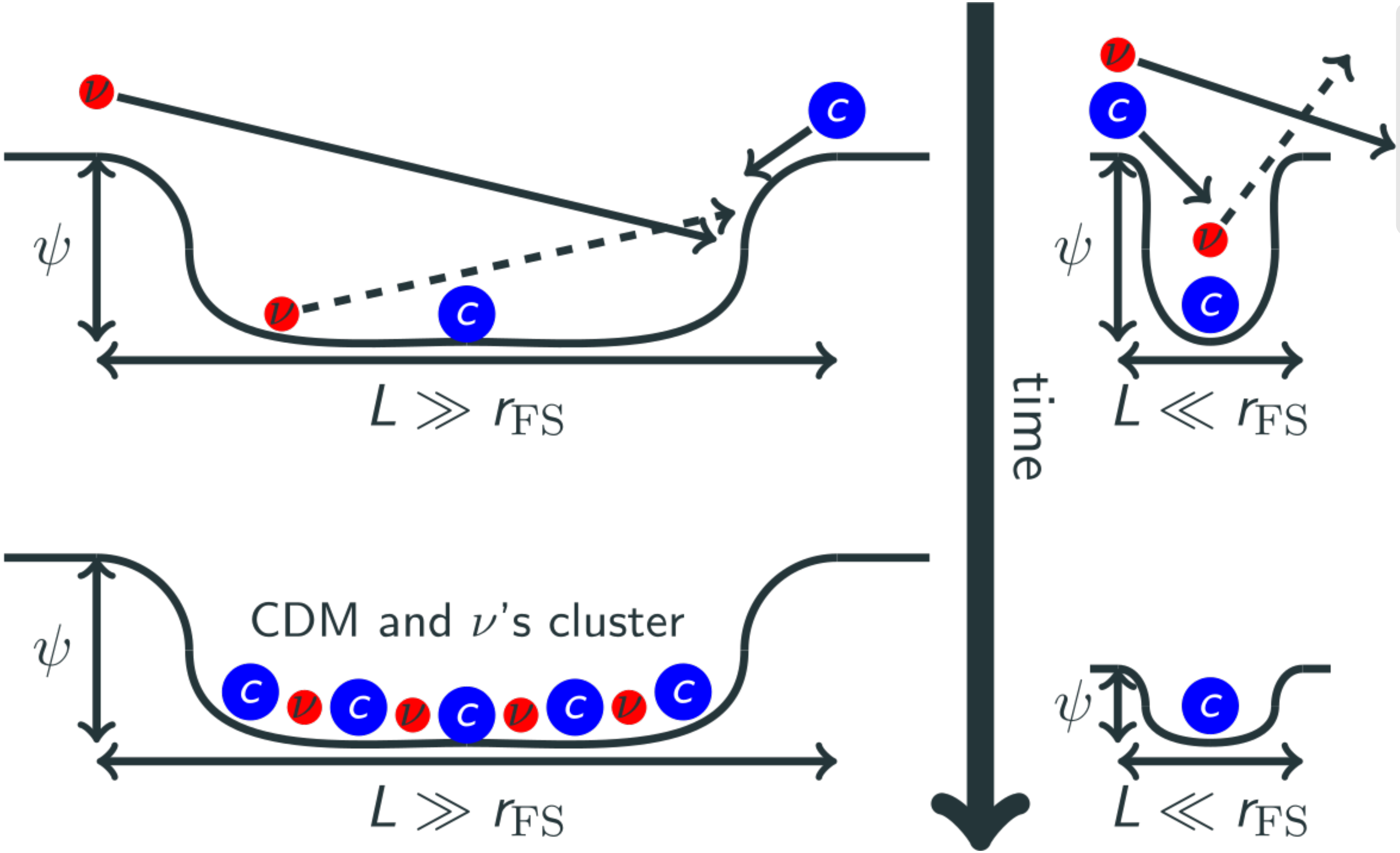
Neutrino masses affect cosmological structure formation



L is size of astrophysical object

$r_{\mathrm{FS}} = \int_{t_{\mathrm{in}}}^{t_0} \mathrm{d}t \frac{\langle v(t) \rangle}{a(t)}$ is free-streaming scale

Neutrino masses affect cosmological structure formation



neutrinos erase small-scale structure

only CDM clusters

L is size of astrophysical object

$r_{\mathrm{FS}} = \int_{t_{\mathrm{in}}}^{t_0} \mathrm{d}t \frac{\langle v(t) \rangle}{a(t)}$ is free-streaming scale

Cosmology

Neutrinos have a large effect on cosmology: the formation of large scale structure (LSS) as explained on the previous slide, big bang nucleosynthesis and the Cosmic Microwave Background (CMB) provide constraints

The Planck experiment [1807.06209]

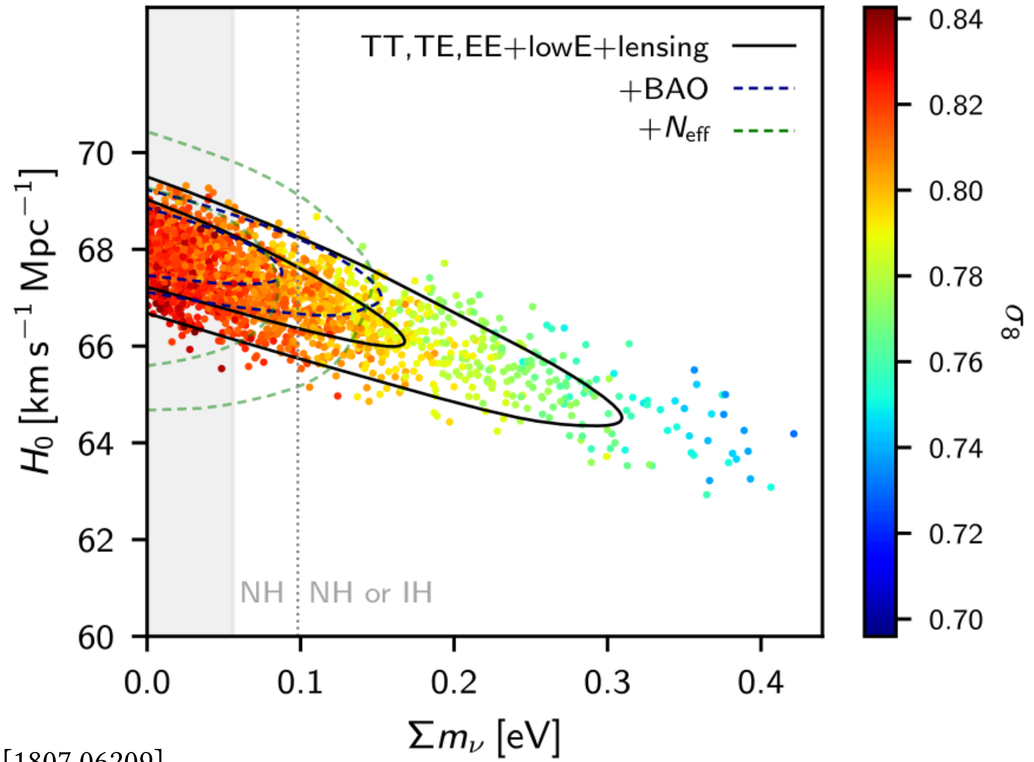
- measured the number of neutrinos

$$N_{\text{eff}} = 2.99 \pm 0.17$$

- placed an upper bound on the sum of neutrino masses

$$\sum m_\nu < 0.12 \text{ eV at } 95\% \text{ CL}$$

- this puts pressure on inverted ordering



[1807.06209]

Cosmology

Neutrinos have a large effect on cosmology: the formation of large scale structure (LSS) as explained on the previous slide, big bang nucleosynthesis and the Cosmic Microwave Background (CMB) provide constraints

The Planck experiment [1807.06209]

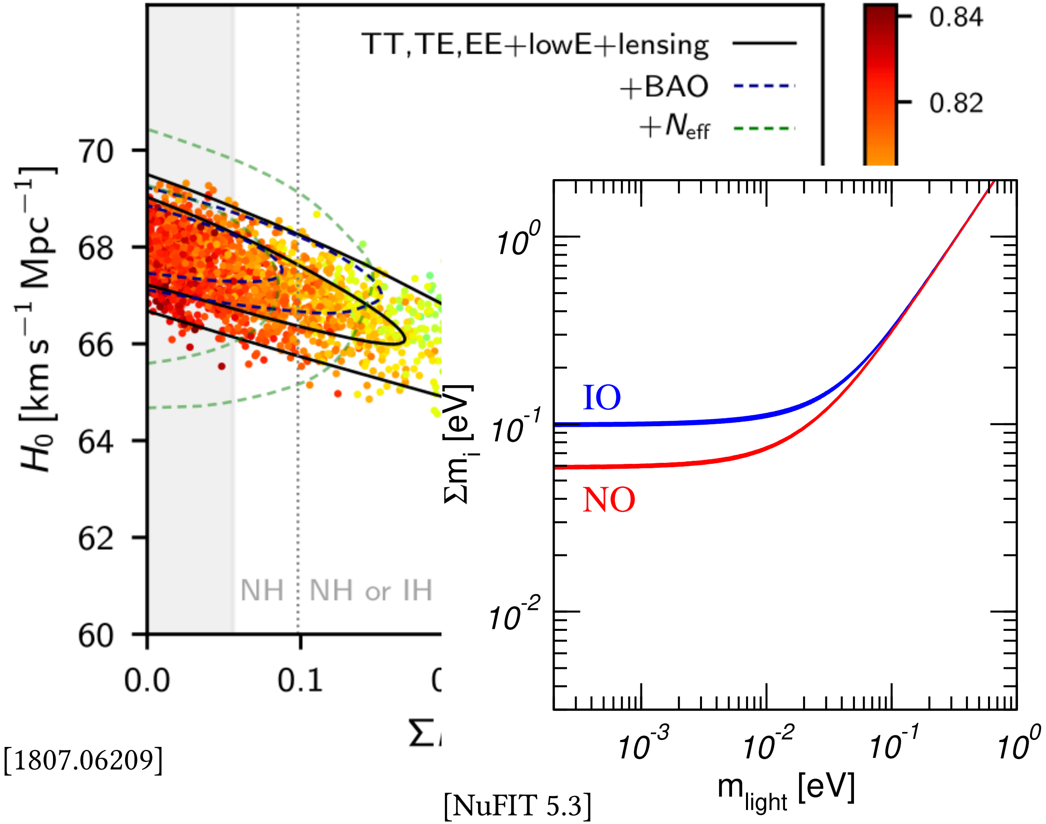
- measured the number of neutrinos

$$N_{\text{eff}} = 2.99 \pm 0.17$$

- placed an upper bound on the sum of neutrino masses

$$\sum m_\nu < 0.12 \text{ eV at } 95\% \text{ CL}$$

- this puts pressure on inverted ordering



[1807.06209]

[NuFIT 5.3]

Open questions

Are neutrinos their own antiparticles?

What is the absolute neutrino mass scale?

Is there normal or inverted mass ordering?

Is there leptonic CP violation?

Anything beyond 3 neutrinos?

Open questions

Are neutrinos their own antiparticles?

What is the absolute neutrino mass scale?

Is there normal or inverted mass ordering?

Is there leptonic CP violation?

Anything beyond 3 neutrinos?

Why are neutrinos so light?

Any explanation for the mixing angles?